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What Comes Beyond the Standard Models

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The Members of the Organizing Committee of the International Workshop "What Comes Beyond the Standard Models", Bled, Slovenia, state that the articles published in the Proceedings to the 16^{th} Workshop "What Comes Beyond the Standard Models", Bled, Slovenia are refereed at the Workshop in intense in-depth discussions.

Contents

Preface

The series of workshops on ”What Comes Beyond the Standard Models?” started in 1998 with the idea of Norma and Holger for organizing a real workshop, in which participants would spend most of the time in discussions, confronting different approaches and ideas. It is the sixteenth workshop which took place this year in the picturesque town of Bled by the lake of the same name, surrounded by beautiful mountains and offering pleasant walks and mountaineering.

In our very open minded, friendly, cooperative, long, tough and demanding discussions several physicists and even some mathematicians have contributed. Most of topics presented and discussed in our Bled workshops concern the proposals how to explain physics beyond the so far accepted and experimentally confirmed both standard models - in elementary particle physics and cosmology. Although most of participants are theoretical physicists, many of them with their own suggestions how to make the next step beyond the accepted models and theories, experts from experimental laboratories were very appreciated, helping a lot to understand what do measurements really tell and which kinds of predictions can best be tested.

The (long) presentations (with breaks and continuations over several days), followed by very detailed discussions, have been extremely useful at least for the organizers. We hope and believe, however, that this is the case also for most of participants, including students. Many a time, namely, talks turned into very pedagogical presentations in order to clarify the assumptions and the detailed steps, analysing the ideas, statements, proofs of statements and possible predictions, confronting participants’ proposals with the proposals in the literature or with proposals of the other participants, so that all possible weak points of the proposals showed up very clearly. The ideas therefore seem to develop in these years considerably faster than they would without our workshops.

In the sixteen years experiments made a large step. Among the most important ones is also the LHC confirmation that the scalar field, the Higgs, is like other fermionic and bosonic fields - just a field.

Can it happen, however, as asked by Anatoli Romaniouk, that at the LHC no important (from the point of view to show the next step beyond the *standard model*) new degree of freedom will be measured since all the so far offered proposals which try to understand the assumptions of the *standard model* are non realisable at the reachable energy regime or even wrong?

But, is the Higgs really one field or just the effective presentation of several scalar fields, what would explain the Yukawa couplings which are unavoidably

connected with the question about the origin of families? The answer to these and several other questions is offered by the *spin-charge-family* theory, predicting the fourth family, coupled to the observed three families, which will be measured at the LHC together with several additional scalar fields. The theory predicts also the stable fifth family, which is the candidate for forming the dark matter. It is reported in this (and also previous) proceedings.
The experiments might observe also that the space time is more than 3+1 dimensional. The supersymmetric extension of the standard model of elementary fermionic and bosonic fields and string theories also "see" more than one scalar field, predicting in addition new particles and noticeable extra dimensions, as it is explained in this proceedings.
The experimentalists might trust theoretical predictions much more, if the overlap among different proposed theories would be clearly analysed and recognized. And any proposed theory would be much more trustable if it would not be designed to answer just one of many open questions.
There are inventive new predictions originating already in the *standard model*, such as the prediction that there exist a cluster of twelve top quarks and anti-quarks (or may be members of the fourth family as well) forming a bound state due to the Yukawa couplings, which might explain the Dark matter. And there are searching for the answers to the questions why Nature has made a choice of the observed charges and dimension which might help to make a choice which of proposed theories, offering new steps beyond the *standar model* are the most trustable.
The prediction that there are atoms composed of -2 charged clusters of the fifth family quarks bound by Coulomb force with the $+2$ charged primordial helium nuclei in nuclear-interacting OHe atoms, which form the dark matter, needs a stable fifth family.
At high energy scales cosmology and elementary particle physics start to be inextricably connected. The dark matter had essential influence on formation of galaxies and so might have the primordial black holes.
Direct measurements of the dark matter, made in Gran Sasso by DAMA/LIBRA, and presented and discussed at Bled workshops, and of other direct measurements, among them CDMS and XENON experiments also presented and discussed at Bled, although not yet in agreement with each others, but to the organizers understanding also not in disagreement, might resolve the problem about the origin of the dark matter.
We only can understand Nature through theories which are confirmed by experiments within experimental inaccuracy. Future experiments will confirm or disprove today proposals and their predictions. But even if not confirmed, new and different ways of looking at the open problems in physics will certainly help to find next better steps beyond the standard models. In particular it looks like that it is essential to understand the symmetries and possible reasons for their conservation, which was also one of the main topics of this year and previous Bled workshops.
Bled Workshops owe their success to participants who have at Bled in the heart of Slovene Julian Alps enabled friendly and active sharing of information, yet their success was boosted by vidoeconferences. Questions and answers as well as

lectures enabled by M.Yu. Khlopov via Virtual Institute of Astroparticle Physics (www.cosmovia.org) of APC have in ample discussions helped to resolve many dilemmas.
The reader can find the records of all the talks delivered by cosmovia since Bled 2009 on `www.cosmovia.org` in `Previous - Conferences`. The four talks, by Ignatios Antoniadis (Mass hierarchy and physics beyond the standard model), Riccardo Cerulli, on behalf of DAMA collaboration (Results from DAMA/LIBRA and perspectives), Anatoli Romaniouk (Status of the ATLAS experiment) and Norma Mankoč Borštnik (Predictions from the spin-charge-family theory), can be accessed directly at
`http://viavca.in2p3.fr/what_comes_beyond_the_standard_models_16th.html`

Let us conclude this preface by thanking cordially and warmly to all the participants, present personally or through the teleconferences at the Bled workshop, for their excellent presentations and in particular for really fruitful discussions and the good and friendly working atmosphere.
The workshops take place in the house gifted to the Society of Mathematicians, Physicists and Astronomers of Slovenia by the Slovenian mathematician Josip Plemelj, well known to the participants by his work in complex algebra.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(the Organizing comittee)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(the Editors)

Ljubljana, December 2013

1 Predgovor (Preface in Slovenian Language)

Serija delavnic ”Kako preseči oba standardna modela, kozmološkega in elektrošibkega” (”What Comes Beyond the Standard Models?”) se je začela leta 1998 z idejo Norme in Holgerja, da bi organizirali delavnice, v katerih bi udeleženci v izčrpnih diskusijah kritično soočili različne ideje in teorije. Letos smo imeli šestnajsto delavnico v mestu Bled ob slikovitem jezeru, kjer prijetni sprehodi in pohodi na čudovite gore, ki kipijo nad mestom, ponujajo priložnosti in vzpodbudo za diskusije.
K našim zelo odprtim, prijateljskim, dolgim in zahtevnim diskusijam, polnim iskrivega sodelovanja, je prispevalo veliko fizikov in celo nekaj matematikov. Večina predlogov teorij in modelov, predstavljenih in diskutiranih na naših blejskih delavnicah, išče odgovore na vprašanja, ki jih v fizikalni skupnosti sprejeta in s številnimi poskusi potrjena standardni model osnovnih fermionskih in bozonskih polj ter kozmološki standardni model puščata odprta. Čeprav je večina udeležencev teoretičnih fizikov, mnogi z lastnimi idejami kako narediti naslednji korak onkraj sprejetih modelov in teorij, so še posebej dobrodošli predstavniki eksperimentalnih laboratorijev, ki nam pomagajo v odprtih diskusijah razjasniti resnično sporočilo meritev in katere napovedi lahko poskusi najzanesljiveje preverijo.
Organizatorji moramo priznati, da smo se na blejskih delavnicah v (dolgih) predstavitvah (z odmori in nadaljevanji čez več dni), ki so jim sledile zelo podrobne diskusije naučili veliko, morda več kot večina udeležencev. Upamo in verjamemo, da to velja tudi za večino udeležencev, tudi za študente. Velikokrat so se predavanja spremenila v zelo pedagoške predstavitve, ki so pojasnile predpostavke in podrobne korake, soočile predstavljene predloge s predlogi v literaturi ali s predlogi ostalih udeležencev ter jasno pokazale, kje utegnejo tičati šibke točke predlogov. Zdi se, da so se ideje v teh letih razvijale bistveno hitreje, zahvaljujoč prav tem delavnicam.
V teh šestnajstih letih so eksperimenti napravili velike korake. Med najpomembnejšimi dosežki je potrditev LHC, da je skalarno polje, Higgsov delec, prav tako polje kot ostala fermionska in bozonska polja.
Anatoli Romaniouk se je vprašal, ali se lahko zgodi, da ne bodo na LHC izmerili ničesar, kar bi pomagalo razložiti privzetke *standardnega modela*, ker nobeden dosedaj predstavljen predlog za razlago predpostavk tega modela pri dosegljivih energijah ni smiselen.
Toda, ali je Higgsov skalar res samo eno polje ali pa je le efektivna predstavitev več skalarnih polj, kar bi lahko pojasnilo Yukawine sklopitve, ko najdemo odgovor na vprašanje o izvoru družin? Odgovor na to in mnoga druga vprašanja ponuja teorija *spinov-nabojev-družin*, ki napoveduje četrto družino, sklopljeno z opaženimi

tremi, ki bo izmerjena na LHC skupaj z več skalarnimi polji. Ta teorija napoveduje tudi stabilno peto družino, ki je kandidat za tvorbo temne snovi. O tem poroča ta in prejšnji zborniki. Poskusi na LHC bodo morda potrdili, da ima prostor-čas več kot (3+1) razsežnost.

Tudi v supersimetrični razširitvi *standardnega modela* osnovnih fermionskih in bozonskih polj in teoriji strun "vidijo" več kot eno samo skalarno polje, napovedujejo nove delce in opazne dodatne dimenzije, kot je razloženo v tem zborniku.

Eksperimentalni fiziki bi morda bolj zaupali napovedim teorije, če bi se prekrivanje različnih teorij jasno analiziralo in razumelo. Predlagana teorija bi bila deležna večjega zaupanja, če ne bi bila zasnovana kot odgovor na eno samo od mnogih odprtih vpraăsanj.

Že v okviru standardnega modela se pojavljajo inovativne nove napovedi, kot je napoved, da obstojajo stabilne gruče dvanajstih t kvarkov in antikvarkov (ali morda celo članov četrte družine), ki zaradi Yukawinih sklopitev tvorijo vezano stanje in so morda razlaga za temno snov. V zborniku najdemo predlog, kako iskati odgovor na vprašanje, zakaj je Narava izbrala doslej opažene naboje, simetrije in dimenzijo prostora-časa, ki bo morda pomagal izbrati med predlogi teorij tistega, ki ponuja najboljšo pot za razumevanje odprtih vprašanj *standardnega modela*.

Napoved, da temno snov sestavljajo gruče z nabojem -2 iz pete družine kvarkov vezanih s coulombsko silo s prvinskimi helijevimi jedri z nabojem $+2$ v atome OHe, ki interagirajo z jedrsko silo, potrebuje obstoj pete družine.

Pri visokih energijah sta kozmologija in fizika osnovnih delcev neločljivo povezani. Temna snov je bistveno vplivala na nastanek galaksij, in morda prav tako tudi prvinske črne luknje. Direktne meritve temne snovi obetajo razložiti, iz česa je temna snov. Meritve z DAMA/LIBRA v Gran Sassu, ki so bile predstavljene in diskutirane na blejskih delavnicah, in druge direktne meritve, med njimi eksperimenta CDMS in XENON, ki sta tudi bila predstavljena in diskutirana na Bledu, sicer niso v medsebojnem ujemanju, vendar po razumevanju organizatorjev delavnice tudi ne v direktnem nasprotju.

Naravo lahko razumemo le tako, da postavljamo teorije, ki jih preverjamo s poskusi. Prihodnji poskusi bodo potrdili ali ovrgli predloge teorij in njihove napovedi. Četudi bi ne potrdili nobene od doslej predlaganih, pa novi in različni pogledi na odprte probleme v fiziki pomagajo poiskati nove in primernejše korake, ki bodo pojasnili privzetke standardnih modelov. Posebej pomembno se zdi razumevanje simetrij in možnih razlogov za njihovo ohranitev in nastanek, kar je ena glavnih tem te in prejšnjih blejskih delavnic.

Četudi so k uspehu „Blejskih delavnic" največ prispevali udeleženci, ki so na Bledu omogočili prijateljsko in aktivno izmenjavo mnenj v osrčju slovenskih Julijcev, so k uspehu prispevale tudi videokonference, ki so povezale delavnice z laboratoriji po svetu. Vprašanja in odgovori ter tudi predavanja, ki jih je v zadnjih letih omogočil M.Yu. Khlopov preko Virtual Institute of Astroparticle Physics (www.cosmovia.org, APC, Pariz), so v izčrpnih diskusijah pomagali razčistiti marsikatero dilemo.

Bralec najde zapise vseh predavanj, objavljenih preko "cosmovia" od leta 2009, na `www.cosmovia.org` v povezavi `Previous - Conferences`. Štiri letošnja predavanja, predavanje Ignatiosa Antoniadisa (Mass hierarchy and physics beyond the standard model), Riccarda Cerullija, člana skupine DAMA (Results from DAMA/LIBRA and perspectives), Anatolija Romaniouka (Status of the ATLAS experiment) in Norme Mankoč Borštnik (Predictions from the spin-charge-family theory), so dostopna na

`http://viavca.in2p3.fr/what_comes_beyond_the_standard_models_16th.html`

Naj zaključimo ta predgovor s prisrčno in toplo zahvalo vsem udeležencem, prisotnim na Bledu osebno ali preko videokonferenc, za njihova predavanja in še posebno za zelo plodne diskusije in odlično vzdušje.
Delavnica poteka v hiši, ki jo je Društvu matematikov, fizikov in astronomov Slovenije zapustil v last slovenski matematik Josip Plemelj, udeležencem delavnic, ki prihajajo iz različnih koncev sveta, dobro poznan po svojem delu v kompleksni algebri.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(Organizacijski odbor)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(uredniki)

Ljubljana, grudna (decembra) 2013

Talk Section

All talk contributions are arranged alphabetically with respect to the authors' names.

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1 Mass Hierarchy and Physics Beyond the Standard Model

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Abstract. I discuss the status of the mass hierarchy problem and prospects for beyond the Standard Model physics in the light of the Higgs scalar discovery at the LHC and the experimental searches for new physics. In particular, I will discuss in this context low energy supersymmetry and large extra dimensions with low string scale.

Povzetek. Izmerjeno Higgsovo skalarno polje, ki ga je napovedal *standardni model*, kliče po oceni stanja teorij v fiziki osnovnih delcev. Predstavim problem hierarhije mas doslej poznanih osnovnih delcev in napovedi za meritve na LHC, ki jih ponujata supersimetrična teorija - napoveduje supersimetrične partnerje doslej poznanim delcem in poljem - in teorija strun - ki napoveduje, da bodo meritve potrdile obstoj več kot stirih dimenzij.

1.1 Introduction

During the last few decades, physics beyond the Standard Model (SM) was guided from the problem of mass hierarchy. This can be formulated as the question of why gravity appears to us so weak compared to the other three known fundamental interactions corresponding to the electromagnetic, weak and strong nuclear forces. Indeed, gravitational interactions are suppressed by a very high energy scale, the Planck mass $M_P \sim 10^{19}$ GeV, associated to a length $l_P \sim 10^{-35}$ m, where they are expected to become important. In a quantum theory, the hierarchy implies a severe fine tuning of the fundamental parameters in more than 30 decimal places in order to keep the masses of elementary particles at their observed values. The reason is that quantum radiative corrections to all masses generated by the Higgs vacuum expectation value (VEV) are proportional to the ultraviolet cutoff which in the presence of gravity is fixed by the Planck mass. As a result, all masses are "attracted" to about 10^{16} times heavier than their observed values.

Besides compositeness, there are two main theories that have been proposed and studied extensively during the last years, corresponding to different approaches of dealing with the mass hierarchy problem. (1) Low energy supersymmetry with all superparticle masses in the TeV region. Indeed, in the limit of exact supersymmetry, quadratically divergent corrections to the Higgs self-energy

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are exactly cancelled, while in the softly broken case, they are cutoff by the supersymmetry breaking mass splittings. (2) TeV scale strings, in which quadratic divergences are cutoff by the string scale and low energy supersymmetry is not needed. Both ideas are experimentally testable at high-energy particle colliders and in particular at LHC.

On the other hand, the recent major discovery of the Higgs boson at the LHC with a mass around 126 GeV is so far compatible with the Standard Model within 2σ and its precision tests. It is also compatible with low energy supersymmetry, although with some degree of fine-tuning in its minimal version. Indeed, in the minimal supersymmetric Standard Model (MSSM), the lightest Higgs scalar mass m_h satisfies the following inequality:

$$m_h^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{3}{(4\pi)^2}\frac{m_t^4}{v^2}\left[\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{A_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{A_t^2}{12 m_{\tilde{t}}^2}\right)\right] \lesssim (130\mathrm{GeV})^2\,, \quad (1.1)$$

where the first term in the r.h.s. corresponds to the tree-level prediction and the second term includes the one loop corrections due to the top and stop loops. Here, $m_Z, m_t, m_{\tilde{t}}$ are the Z-boson, the top and stop quark masses, respectively, $v = \sqrt{v_1^2 + v_2^2}$ with v_i the VEVs of the two higgses, $\tan\beta = v_2/v_1$, and A_t the trilinear stop scalar coupling. Thus, a Higgs mass around 126 GeV requires a heavy stop $m_{\tilde{t}} \simeq 3$ TeV for vanishing A_t, or $A_t \simeq 3m_{\tilde{t}} \simeq 1.5$ TeV in the 'best' case. These values are obviously consistent with the present LHC bounds on supersymmetry searches, but they will certainly be probed in the next run at double energy. Theoretically, they imply a fine-tuning of the electroweak (EW) scale at the percent to per mille level. This fine-tuning can be alleviated in supersymmetric models beyond the MSSM, as discussed in the next session.

1.2 MSSM Higgs sector with dimension-five and dimension-six operators

Although extremely successful, the Standard Model or its supersymmetric version (MSSM) is not a fundamental theory, and this motivated the theoretical efforts to understand the nature of new physics beyond it. This search can be done using an effective field theory approach, in which the "new physics" is parametrised by effective operators. The power of this approach resides in arranging these operators in powers of $1/M_*$ where M_* is the scale of new physics that generated them. To improve the predictive power, one considers additional organising principles, such as: **(i)** symmetry constraints that these operators should respect, often inspired by phenomenology (for example: R-parity, lepton or baryon number conservation, etc). **(ii)** a truncation of the series of operators to a given order in the power of the inverse scale $1/M_*$. The effective low-energy Lagrangian then takes the form

$$\mathcal{L} = \mathcal{L}_0 + \sum_{i,n} \frac{c_{n,i}}{M_*^n}\,\mathcal{O}_{n,i} \quad (1.2)$$

where $\mathcal{L}_0$ is the SM or the MSSM Lagrangian; $\mathcal{O}_{n,i}$ is an operator of dimension $d = n + 4$ with the index i running over the set of operators of a given dimension;

$c_{n,i}$ are some coefficients of order $\mathcal{O}(1)$. This description is appropriate for scales E which satisfy $E \ll M_*$. Constraints from phenomenology can then be used to set bounds on the scale of new physics M_*.

Regarding the origin of operators $\mathcal{O}_{n,i}$, they can be generated classically or at the quantum level. At the classical level, this can happen by integration of some new massive states, via the equations of motion and one then generates an infinite series. This can happen even in 4D renormalisable theories; indeed, even though the low energy interaction looks nonrenormalisable, it may actually point to a renormalisable theory valid up to a much higher scale (a familiar example is the Fermi interaction). Such effective operators are also generated at the quantum level, for example following compactification of a higher dimensional theory, by the radiative corrections associated with momentum and winding modes of the compactification [1–5].

The effects of these operators on the low energy observables can be comparable to the radiative effects of light states in the SM/MSSM [6] and this shows the importance of their study. In the following we shall study these effects to the case of the MSSM Higgs sector with additional operators of dimensions $d = 5$ and $d = 6$ [6,7]. In particular we show that the mass of lightest SM-like Higgs can easily be increased close to the observed value by new physics in the region of few TeV. We then discuss the nature of the "new physics" behind the effective operators.

1.2.1 MSSM Higgs sector with d=5 and d=6 operators

In the leading order, new physics beyond the MSSM Higgs sector can manifest itself as operators of either $d = 5$ [6,12–14] or $d = 6$ [7,15] or both. If generated by the same new physics, by comparing $\mathcal{O}(1/M^*)$ and $\mathcal{O}(1/M_*^2)$ terms one can estimate when the series expansion in $1/M_*$ breaks down. There is only one operator in the Higgs sector of dimension $d = 5$:

$$\begin{aligned}\mathcal{L}_1 &= \frac{1}{M_*}\int d^2\theta\ \lambda'_H(S)\,(H_2.H_1)^2 + \text{h.c.}\\ &= 2\,\zeta_{10}\,(h_2.h_1)(h_2.F_1 + F_2.h_1) + \zeta_{11}\,m_0\,(h_2.h_1)^2 + \text{h.c},\end{aligned} \tag{1.3}$$

where $\lambda'_H(S)/M_* = \zeta_{10} + \zeta_{11}\,m_0\,\theta\theta$ and $\zeta_{10},\ \zeta_{11} \sim 1/M_*$. It can be generated by integrating out a massive gauge singlet or $SU(2)$ triplet. Indeed, in the MSSM with a massive gauge singlet, with an F-term of type $M_*\Sigma^2 + \Sigma\,H_1.H_2$, when integrating out Σ generates $\mathcal{L}_1$. With the standard notation, here $S = m_0\theta^2$ is the spurion superfield and m_0 the supersymmetry breaking scale.

We assume that $m_0 \ll M_*$, so that the effective approach is reliable. If this is not respected and the "new physics" is represented by "light" states (like the MSSM states), the $1/M_*$ expansion is not reliable and one should work in a setup where these are not integrated out.

The list of $d = 6$ operators is longer [16]:

$$\begin{aligned}
\mathcal{O}_1 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_1\ (H_1^\dagger\, e^{V_1}\, H_1)^2,\\
\mathcal{O}_2 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_2\ (H_2^\dagger\, e^{V_2}\, H_2)^2,\\
\mathcal{O}_3 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_3\ (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2^\dagger\, e^{V_2}\, H_2),\\
\mathcal{O}_4 &= \frac{1}{M_*^2}\int d^4\theta\, \mathcal{Z}_4\ (H_2.\,H_1)\,(H_2.\,H_1)^\dagger,\\
\mathcal{O}_5 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_5\ (H_1^\dagger\, e^{V_1}\, H_1)\ H_2.\,H_1 + h.c.,\\
\mathcal{O}_6 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_6\ (H_2^\dagger\, e^{V_2}\, H_2)\ H_2.\,H_1 + h.c.,\\
\mathcal{O}_7 &= \frac{1}{M_*^2}\int d^2\theta\, \mathcal{Z}_7\, \mathrm{Tr}\, W^\alpha\, W_\alpha\, (H_2\, H_1) + h.c.,\\
\mathcal{O}_8 &= \frac{1}{M_*^2}\int d^4\theta\, \mathcal{Z}_8\ (H_2\, H_1)^2 + h.c.,
\end{aligned} \qquad (1.4)$$

where $W^\alpha = (-1/4)\,\overline{D}^2 e^{-V} D^\alpha\, e^V$ is the chiral field strength of $SU(2)_L$ or $U(1)_Y$ vector superfields V_w and V_Y respectively. Also $V_{1,2} = V_w^a(\sigma^a/2) + (\mp 1/2)\, V_Y$ with the upper (minus) sign for V_1. Finally, the wavefunction coefficients are spurion dependent and have the structure

$$(1/M_*^2)\,\mathcal{Z}_i(S, S^\dagger) = \alpha_{i0} + \alpha_{i1}\, m_0\, \theta\theta + \alpha_{i1}^*\, m_0\, \overline{\theta\theta} + \alpha_{i2}\, m_0^2\, \theta\theta\overline{\theta\theta}, \qquad \alpha_{ij} \sim 1/M_*^2. \qquad (1.5)$$

Regarding the origin of these operators: $\mathcal{O}_{1,2,3}$ can be generated in the MSSM by an additional, massive $U(1)'$ gauge boson or $SU(2)$ triplets, when integrated out [12]. $\mathcal{O}_4$ can be generated by a massive gauge singlet or $SU(2)$ triplet, while $\mathcal{O}_{5,6}$ can be generated by a combination of $SU(2)$ doublets and massive gauge singlet. $\mathcal{O}_7$ is essentially a threshold correction to the gauge coupling, with a moduli field replaced by the Higgs. $\mathcal{O}_8$ exists only in non-susy case, but is generated when removing a $d = 5$ derivative operator by field redefinitions [6], so we keep it.

1.2.2 Higgs mass corrections from d = 5 and d = 6 operators.

With the above set of independent, effective operators, one finds the scalar potential V and its EW minimum which is perturbed by $\mathcal{O}(1/M_*^2)$ corrections from that of the MSSM. From V one computes the mass of CP-odd/even Higgs fields:

$$m_A^2 = (m_A^2)_{\rm MSSM} - \frac{2\,\zeta_{10}\,\mu_0\, v^2}{\sin 2\beta} + 2\, m_0\, \zeta_{11}\, v^2 + \delta m_A^2, \quad \delta m_A^2 = \mathcal{O}(1/M_*^2) \qquad (1.6)$$

for the pseudoscalar Higgs, with $(m_A^2)_{\rm MSSM}$ the MSSM value, with δm_A^2 due to $\mathcal{O}(1/M_*^2)$ corrections from $d = 5$ and $d = 6$ operators. For the CP-even Higgs one has [6,12,14]

$$
\begin{aligned}
m^2_{h,H} &= (m^2_{h,H})_{\text{MSSM}} \\
&+ (2\,\zeta_{10}\,\mu_0)\,v^2\,\sin 2\beta\left[1 \pm \frac{m_A^2 + m_Z^2}{\sqrt{\tilde{w}}}\right] + \frac{(-2\,\zeta_{11}\,m_0)\,v^2}{2}\left[1 \mp \frac{(m_A^2 - m_Z^2)\,\cos^2 2\beta}{\sqrt{\tilde{w}}}\right] \\
&+ \delta m^2_{h,H}, \qquad \text{where} \qquad \delta m^2_{h,H} = \mathcal{O}(1/M_*^2)
\end{aligned}
\tag{1.7}
$$

The upper (lower) signs correspond to h (H), and

$$\tilde{w} \equiv (m_A^2 + m_Z^2)^2 - 4\,m_A^2\,m_Z^2\,\cos^2 2\beta.$$

With this result one can show that the mass m_h can be increased near the observed value, also with the help of quantum corrections [6,12–14].

Regarding the $\mathcal{O}(1/M_*^2)$ corrections of $\delta m^2_{h,H}$, δm_A^2 and $\delta m^2_{h,H}$ of (1.6), (1.7), in the general case of including all operators and their associated supersymmetry breaking, they have a complicated form. Exact expressions can be found in [7,15]. For most purposes, an expansion of these in $1/\tan\beta$ is accurate enough. At large $\tan\beta$, $d = 6$ operators bring corrections comparable to those of $d = 5$ operators. The relative $\tan\beta$ enhancement of $\mathcal{O}(1/M_*^2)$ corrections compensates for the extra suppression that these have relative to $\mathcal{O}(1/M_*)$ operators (which involve both h_1, h_2 and are not enhanced in this limit). Note however that in some models only $d = 6$ operators may be present, depending on the details of the "new physics" generating the effective operators.

Let us present the correction $\mathcal{O}(1/M^2)$ to m_h^2 for the case m_A is kept fixed to an appropriate value. The result is, assuming $m_A > m_Z$, (otherwise δm_h^2 and δm_H^2 are exchanged):

$$
\begin{aligned}
\delta m_h^2 &= -2\,v^2\left[\alpha_{22}\,m_0^2 + (\alpha_{30} + \alpha_{40})\mu_0^2 + 2\alpha_{61}\,m_0\,\mu_0 - \alpha_{20}\,m_Z^2\right] \\
&- (2\,\zeta_{10}\,\mu_0)^2\,v^4\,(m_A^2 - m_Z^2)^{-1} \\
&+ v^2 \cot\beta\Big[(m_A^2 - m_Z^2)^{-1}\Big(4\,m_A^2\,\big((2\alpha_{21} + \alpha_{31} + \alpha_{41} + 2\alpha_{81})\,m_0\,\mu_0 \\
&+ (2\alpha_{50} + \alpha_{60})\,\mu_0^2 + \alpha_{62}\,m_0^2\big) \\
&- (2\alpha_{60} - 3\alpha_{70})\,m_A^2\,m_Z^2 - (2\alpha_{60} + \alpha_{70})\,m_Z^4\Big) \\
&+ 8\,(m_A^2 + m_Z^2)\,(\mu_0\,m_0\,\zeta_{10}\,\zeta_{11})\,v^2/(m_A^2 - m_Z^2)^2\Big] \\
&+ \mathcal{O}(1/\tan^2\beta)
\end{aligned}
\tag{1.8}
$$

The mass corrections in (1.8) must be added to the rhs of eq.(1.7) to obtain the full value of m_h^2. Together with (1.4), (1.5), these corrections identify the operators of $d = 6$ with the largest contributions, which is important for model building beyond the MSSM Higgs sector. These operators are $\mathcal{O}_{2,3,4}$ in the absence of supersymmetry breaking and $\mathcal{O}_{2,6}$ when this is broken. It is preferable, however, to increase m_h^2 by supersymmetric rather than supersymmetry-breaking effects of the effective operators, because the latter are less under control in the effective approach; also, one would favour a supersymmetric solution to the fine-tuning problem associated with increasing the MSSM Higgs mass. Therefore $\mathcal{O}_{2,3,4}$ are the leading operators, with the remark that $\mathcal{O}_2$ has a smaller effect, of order $(m_Z/\mu_0)^2$ relative to $\mathcal{O}_{3,4}$ (for similar α_{j0}, $j = 2, 3, 4$). At smaller $\tan\beta$, $\mathcal{O}_{5,6}$ can also give significant contributions, while $\mathcal{O}_7$ has a relative suppression factor $(m_Z/\mu_0)^2$. Note that we kept all operators $\mathcal{O}_i$ independent. By doing so, one can easily single out the individual contribution of each operator, which helps in model building, since not all operators are present in a specific model.

One limit to consider is that where the operators of $d = 6$ have coefficients such that their contributions add up to maximise δm_h^2. Since α_{ij} are not known, one can choose:

$$-\alpha_{22} = -\alpha_{61} = -\alpha_{30} = -\alpha_{40} = \alpha_{20} > 0 \tag{1.9}$$

In this case, at large $\tan\beta$:

$$\delta m_h^2 \approx 2\, v^2 \alpha_{20} \big[m_0^2 + 2\, m_0 \mu_0 + 2\, \mu_0^2 + m_Z^2 \big] \tag{1.10}$$

A simple numerical example is illustrative. For $m_0 = 1$ TeV, $\mu_0 = 350$ GeV, and with $v \approx 246$ GeV, one has $\delta m_h^2 \approx 2.36\, \alpha_{20} \times 10^{11}$ (GeV)^2. Assuming $M_* = 10$ TeV and ignoring $d = 5$ operators, with $\alpha_{20} \sim 1/M_*^2$ and the MSSM value of m_h taken to be its upper classical limit m_Z (reached for large $\tan\beta$), we obtain an increase of m_h from $d = 6$ operators alone of about $\Delta m_h = 12.15$ GeV to $m_h \approx 103$ GeV. An increase of α_{20} by a factor of 2.5 to $\alpha_{20} \sim 2.5/M_*^2$ would give $\Delta m_h \approx 28$ GeV to $m_h \approx 119.2$ GeV, which is already above the LEP bound. Note that this increase is realised even for a scale M_* of new physics beyond the LHC reach.

The above choice of $M_* = 10$ TeV was partly motivated by the fine-tuning results [13] and on convergence grounds: the expansion parameter of our effective analysis is m_q/M_* where m_q is any scale of the theory, in particular it can be m_0. For a susy breaking scale $m_0 \sim \mathcal{O}(1)$ TeV (say $m_0 = 3$ TeV) and $c_{1,2}$ or α_{ij} of $\mathcal{Z}_i(S, S^\dagger)$ of order unity (say $c_{1,2} = 2.5$) one has for $M_* = 10$ TeV that $c_{1,2}\, m_0/M_* = 0.75$ which is already close to unity, and at the limit of validity of the effective expansion in powers of $1/M_*$. To conclude, even for a scale of "new physics" above the LHC reach, one can still classically increase m_h to near the LHC measured value.

1.3 Strings and extra dimensions

The appropriate and most convenient framework for low energy supersymmetry and grand unification is the perturbative heterotic string. Indeed, in this theory, gravity and gauge interactions have the same origin, as massless modes of the closed heterotic string, and they are unified at the string scale M_s. As a result, the Planck mass is predicted to be proportional to M_s:

$$M_P = M_s/g\,, \tag{1.11}$$

where g is the gauge coupling. In the simplest constructions all gauge couplings are the same at the string scale, given by the four-dimensional (4d) string coupling, and thus no grand unified group is needed for unification. In our conventions $\alpha_{GUT} =$

$g^2 \simeq 0.04$, leading to a discrepancy between the string and grand unification scale M_{GUT} by almost two orders of magnitude. Explaining this gap introduces in general new parameters or a new scale, and the predictive power is essentially lost. This is the main defect of this framework, which remains though an open and interesting possibility.

The other other perturbative framework that has been studied extensively in the more recent years is type I string theory with D-branes. Unlike in the heterotic string, gauge and gravitational interactions have now different origin. The latter are described again by closed strings, while the former emerge as excitations of open strings with endpoints confined on D-branes [17]. This leads to a braneworld description of our universe, which should be localized on a hypersurface, i.e. a membrane extended in p spatial dimensions, called p-brane (see Fig. 1.1). Closed strings propagate in all nine dimensions of string theory: in those extended along the p-brane, called parallel, as well as in the transverse ones. On the contrary, open strings are attached on the p-brane. Obviously, our p-brane world must

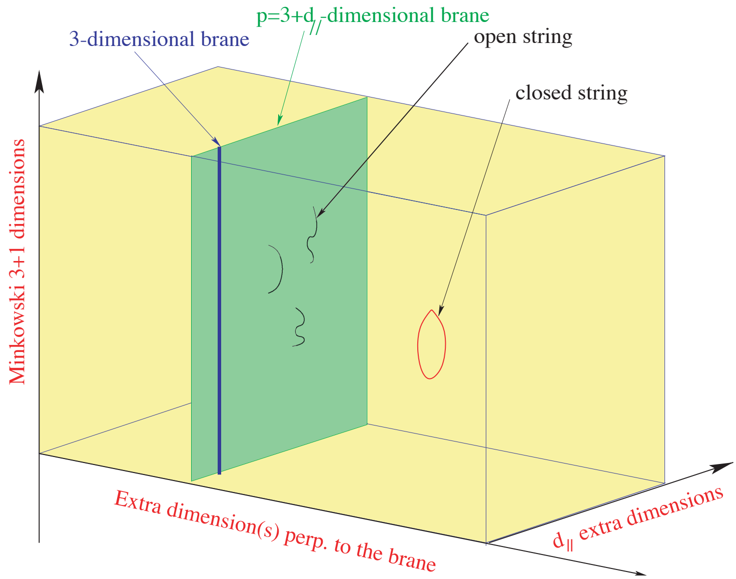

Fig. 1.1. D-brane world universe in type I string framework.

have at least the three known dimensions of space. But it may contain more: the extra $d_{\|} = p - 3$ parallel dimensions must have a finite size, in order to be unobservable at present energies, and can be as large as $\mathrm{TeV}^{-1} \sim 10^{-18}$ m [18]. On the other hand, transverse dimensions interact with us only gravitationally and experimental bounds are much weaker: their size should be less than about 0.1 mm [19]. In the following, I review the main properties and experimental signatures of low string scale models [20].

1.3.1 Framework of low scale strings

In type I theory, the different origin of gauge and gravitational interactions implies that the relation between the Planck and string scales is not linear as (1.11) of the heterotic string. The requirement that string theory should be weakly coupled, constrain the size of all parallel dimensions to be of order of the string length, while transverse dimensions remain unrestricted. Assuming an isotropic transverse space of $n = 9 - p$ compact dimensions of common radius $R_\perp$, one finds:

$$M_P^2 = \frac{1}{g_s^2} M_s^{2+n} R_\perp^n \,, \qquad g_s \simeq g^2 \,. \tag{1.12}$$

where g_s is the string coupling. It follows that the type I string scale can be chosen hierarchically smaller than the Planck mass [21,20] at the expense of introducing extra large transverse dimensions felt only by gravity, while keeping the string coupling small [20]. The weakness of 4d gravity compared to gauge interactions (ratio M_W/M_P) is then attributed to the largeness of the transverse space $R_\perp$ compared to the string length $l_s = M_s^{-1}$.

An important property of these models is that gravity becomes effectively $(4+n)$-dimensional with a strength comparable to those of gauge interactions at the string scale. The first relation of Eq. (1.12) can be understood as a consequence of the $(4+n)$-dimensional Gauss law for gravity, with $M_*^{(4+n)} = M_s^{2+n}/g^4$ the effective scale of gravity in $4+n$ dimensions. Taking $M_s \simeq 1$ TeV, one finds a size for the extra dimensions $R_\perp$ varying from 10^8 km, .1 mm, down to a Fermi for $n = 1, 2$, or 6 large dimensions, respectively. This shows that while $n = 1$ is excluded, $n \geq 2$ is allowed by present experimental bounds on gravitational forces [19,22]. Thus, in these models, gravity appears to us very weak at macroscopic scales because its intensity is spread in the “hidden” extra dimensions. At distances shorter than $R_\perp$, it should deviate from Newton’s law, which may be possible to explore in laboratory experiments.

1.3.2 Experimental implications in accelerators

We now turn to the experimental predictions of TeV scale strings. Their main implications in particle accelerators are of four types, in correspondence with the four different sectors that are generally present:

1. New compactified parallel dimensions; In this case $RM_s \gtrsim 1$, and the associated compactification scale $R_\parallel^{-1}$ would be the first scale of new physics that should be found increasing the beam energy [18,23]. The main consequence is the existence of KK excitations for all SM particles that propagate along the extra parallel dimensions. These can be produced on-shell at LHC as new resonances [24].
2. New extra large transverse dimensions and low scale quantum gravity,. The main experimental signal is gravitational radiation in the bulk from any physical process on the world-brane [25].
3. Genuine string and quantum gravity effects. Direct production of string resonances in hadron colliders leads generically to a universal deviation from

Standard Model in jet distribution [26]. In particular, the first Regge excitation of the gluon has spin 2 and a width an order of magnitude lower than the string scale, leading to a characteristic peak in dijet production; similarly, the first excitations of quarks have spin 3/2.

4. Extra U(1)'s arising generically in D-brane models as part of unitary gauge group factors. They obtain in general masses due to four- or higher-dimensional anomalies, via the so-called Green-Schwarz anomaly cancellation mechanism involving axionic fields from the closed string sector. The resulting masses are therefore suppressed by a loop factor compared to the string scale. From the low energy point of view, they gauge global symmetries of the Standard Model, such as the baryon and lepton number. An important property of the anomaly cancellation mechanism is that the anomalous U(1) gauge bosons acquire masses leaving behind the corresponding global symmetries unbroken in perturbation theory. Thus, this is a way to guarantee proton stability (from unbroken baryon number) and avoid large Majorana neutrino masses (from unbroken lepton number) due to dimension-5 operators involving two higgses and two leptons that are suppressed only by the TeV string scale. Such extra U(1)s have interesting properties and distinct experimental signatures [27–29].
5. Concerning possible micro-black hole production, note that a string size black hole has a horizon radius $r_H \sim 1$ in string units, while the Newton's constant behaves as $G_N \sim g_s^2$. It follows that the mass of a d-dimensional black hole is [30]: $M_{BH} \sim r_H^{d/2-1}/G_N \simeq 1/g_s^2$. Using the value of the SM gauge couplings $g_s \simeq g^2 \sim 0.1$, one finds that the energy threshold M_{BH} of micro-black hole production is about four orders of magnitude higher than the string scale, implying that one would produce 10^4 string states before reaching M_{BH}.

1.3.3 Electroweak symmetry breaking

Non-supersymmetric TeV strings offer also a framework to realize gauge symmetry breaking radiatively. Indeed, from the effective field theory point of view, one expects quadratically divergent one-loop contributions to the masses of scalar fields. The divergences are cut off by M_s and if the corrections are negative, they can induce electroweak symmetry breaking and explain the mild hierarchy between the weak and a string scale at a few TeV, in terms of a loop factor [31]. More precisely, in the minimal case of one Higgs doublet H, the scalar potential is:

$$V = \lambda(H^\dagger H)^2 + \mu^2(H^\dagger H)\,, \tag{1.13}$$

where λ arises at tree-level. Moreover, in any model where the Higgs field comes from an open string with both ends fixed on the same brane stack, it is given by an appropriate truncation of a supersymmetric theory. On the other hand, μ^2 is generated at one loop:

$$\mu^2 = -\varepsilon^2\, g^2\, M_s^2\,, \tag{1.14}$$

where ε is a loop factor that can be estimated from a toy model computation and varies in the region $\epsilon \sim 10^{-1} - 10^{-3}$.

Indeed, consider for illustration a simple case where the whole one-loop effective potential of a scalar field can be computed. We assume for instance one extra dimension compactified on a circle of radius $R > 1$ (in string units). An interesting situation is provided by a class of models where a non-vanishing VEV for a scalar (Higgs) field ϕ results in shifting the mass of each KK excitation by a constant $a(\phi)$:

$$M_m^2 = \left(\frac{m + a(\phi)}{R}\right)^2 , \tag{1.15}$$

with m the KK integer momentum number. Such mass shifts arise for instance in the presence of a Wilson line, $a = q\oint \frac{dy}{2\pi}A$, where A is the internal component of a gauge field and q the charge of a given state under the corresponding generator. A straightforward computation shows that the ϕ-dependent part of the one-loop effective potential is given by [32]:

$$V_{eff} = -\text{Tr}(-)^F \frac{R}{32\pi^{3/2}} \sum_n e^{2\pi i n a} \int_0^\infty dl\; l^{3/2} f_s(l)\; e^{-\pi^2 n^2 R^2 l} \tag{1.16}$$

where $F = 0, 1$ for bosons and fermions, respectively. We have included a regulating function $f_s(l)$ which contains for example the effects of string oscillators. To understand its role we will consider the two limits $R >> 1$ and $R << 1$. In the first case only the $l \to 0$ region contributes to the integral. This means that the effective potential receives sizable contributions only from the infrared (field theory) degrees of freedom. In this limit we would have $f_s(l) \to 1$. For example, in the string model considered in [31]:

$$f_s(l) = \left[\frac{1}{4l}\frac{\theta_2}{\eta^3}(il + \frac{1}{2})\right]^4 \to 1 \qquad \text{for} \qquad l \to 0, \tag{1.17}$$

and the field theory result is finite and can be explicitly computed. As a result of the Taylor expansion around $a = 0$, we are able to extract the one-loop contribution to the coefficient of the term of the potential quadratic in the Higgs field. It is given by a loop factor times the compactification scale [32]. One thus obtains $\mu^2 \sim g^2/R^2$ up to a proportionality constant which is calculable in the effective field theory. On the other hand, if we consider $R \to 0$, which by T-duality corresponds to taking the extra dimension as transverse and very large, the one-loop effective potential receives contributions from the whole tower of string oscillators as appearing in $f_s(l)$, leading to squared masses given by a loop factor times M_s^2, according to eq. (1.14).

More precisely, from the expression (1.16), one finds:

$$\varepsilon^2(R) = \frac{1}{2\pi^2}\int_0^\infty \frac{dl}{(2\,l)^{5/2}} \frac{\theta_2^4}{4\eta^{12}} \left(il + \frac{1}{2}\right) R^3 \sum_n n^2 e^{-2\pi n^2 R^2 l} . \tag{1.18}$$

For the asymptotic value $R \to 0$ (corresponding upon T-duality to a large transverse dimension of radius $1/R$), $\varepsilon(0) \simeq 0.14$, and the effective cut-off for the mass term is M_s, as can be seen from Eq. (1.14). At large R, $\mu^2(R)$ falls off as $1/R^2$, which is the effective cut-off in the limit $R \to \infty$, as we argued above, in agreement with field theory results in the presence of a compactified extra dimension.In fact, in the limit $R \to \infty$, an analytic approximation to $\varepsilon(R)$ gives:

$$\varepsilon(R) \simeq \frac{\varepsilon_\infty}{M_s\,R} , \qquad\qquad \varepsilon_\infty^2 = \frac{3\,\zeta(5)}{4\,\pi^4} \simeq 0.008 . \tag{1.19}$$

The potential (1.13) has the usual minimum, given by the VEV of the neutral component of the Higgs doublet $v = \sqrt{-\mu^2/\lambda}$. Furthermore, from (1.14), one can compute M_s in terms of the Higgs mass $m_H^2 = -2\mu^2$: $M_s = \frac{m_H}{\sqrt{2}\, g\varepsilon}$, yielding naturally values in the TeV range.

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2 DAMA/LIBRA Results and Perspectives

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Abstract. The DAMA/LIBRA experiment is composed by about 250 kg of highly radiopure NaI(Tl). It is in operation at the underground Gran Sasso National Laboratory of the INFN. The main aim of the experiment is to investigate the Dark Matter (DM) particles in the Galactic halo by exploiting the model independent DM annual modulation signature. The DAMA/LIBRA experiment and the former DAMA/NaI (the first generation experiment having an exposed mass of about 100 kg) have released results corresponding to a total exposure of 1.17 ton $\times$ yr over 13 annual cycles; they have provided a model independent evidence of the presence of DM particles in the galactic halo at 8.9 σ C.L.. The results of a further annual cycle, concluding the DAMA/LIBRA–phase1, have been released after this Workshop and are not included here. In the fall 2010 an important upgrade of the experiment have been performed. All the PMTs of the NaI(Tl) detectors have been replaced with new ones having higher quantum efficiency with the aim to decrease the software energy threshold considered in the data analysis. The perspectives of the running DAMA/LIBRA–phase2 will be shortly summarized.

Povzetek. Experiment, poznan pod imenom DAMA/LIBRA, meri lastnosti galaktičnih delcev, jakost toka katerih niha s periodo kroženja Zemlje okoli Sonca. Postavljen je v podzemeljskem laboratoriju, ki nosi ime Gran Sasso National Laboratory in spada pod INFN. Gradi ga približno 250 kg zelo čistega (brez radioktivnih primesi) NaI(Tl). Prednik tega eksperimenta, DAMA/Na, je imel 100 kg eksponirane mase. Oba skupaj ponujata rezultate, ki ustrezajo celotni ekspoziciji 1.17 ton $\times$ let v trinajstletnih ciklusih. Rezultati so neodvisni od modelov, ki pojasnjujejo iz česa je temna snov. Z zanesljivostjo 8.9 σ potrjujejo obstoj delcev temne snovi v galaksiji. Rezultati naslednjega letnega cikla, ti zaključujejo fazo 1 poskusa DAMA/LIBRA, v to poročilo niso vključeni. V jeseni 2010 je bil eksperiment posodobljen. Fotopomnoževalke v detektorjih z NaI(Tl) so, da bi po večali kvantni izkoristek, zamenjali z novimi in s tem znižali energijski prag, uporabljen v programih za analizo meritev. Predstavimo tudi potekajočo drugo fazo eksperimenta DAMA/LIBRA.

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2.1 The DAMA/LIBRA set-up

The DAMA project develops and uses low background scintillators. It consists of the following experimental set-ups: i) DAMA/NaI ($\simeq$ 100 kg of highly radiopure NaI(Tl)) that took data for 7 annual cycles and completed its data taking on July 2002 [1–6]; ii) DAMA/LXe, $\simeq$ 6.5 kg liquid Kr-free Xenon enriched either in ^{129}Xe or in ^{136}Xe [7]; iii) DAMA/R&D, a facility dedicated to tests on prototypes and to perform experiments developing and using various kinds of low background crystal scintillators in order to investigate various rare processes [8]; iv) DAMA/Ge, where sample measurements and measurements dedicated to the investigation of several rare processes are carried out as well as in the LNGS STELLA facility [9]; v) DAMA/CRYS, a new small set-up to test prototype detectors; vi) the second generation DAMA/LIBRA set-up, $\simeq$ 250 kg highly radiopure NaI(Tl)) [10–18]. Many rare processes have been studied with these set-ups obtaining competitive results.

The main apparatus, DAMA/LIBRA, is investigating the presence of DM particles in the galactic halo by exploiting the model independent DM annual modulation signature. In fact, as a consequence of its annual revolution around the Sun, which is moving in the Galaxy traveling with respect to the Local Standard of Rest towards the star Vega near the constellation of Hercules, the Earth should be crossed by a larger flux of Dark Matter particles around ~2 June (when the Earth orbital velocity is summed to the one of the solar system with respect to the Galaxy) and by a smaller one around ~2 December (when the two velocities are subtracted). This DM annual modulation signature is very distinctive since the effect induced by DM particles must simultaneously satisfy all the following requirements: (1) the rate must contain a component modulated according to a cosine function; (2) with one year period; (3) with a phase that peaks roughly around ~ 2nd June; (4) this modulation must be present only in a well-defined low energy range, where DM particles can induce signals; (5) it must be present only in those events where just a single detector, among all the available ones in the used set-up, actually "fires" (*single-hit* events), since the probability that DM particles experience multiple interactions is negligible; (6) the modulation amplitude in the region of maximal sensitivity has to be $\lesssim$7% in case of usually adopted halo distributions, but it may be significantly larger in case of some particular scenarios such as e.g. those in refs. [19,20]. At present status of technology it is the only model independent signature available in direct Dark Matter investigation that can be effectively exploited.

The DAMA/LIBRA data released at time of this Workshop correspond to six annual cycles for an exposure of 0.87 ton$\times$yr [11,12]. Considering these data together with those previously collected by DAMA/NaI over 7 annual cycles (0.29 ton$\times$yr), the total exposure collected over 13 annual cycles is 1.17 ton$\times$yr; this is orders of magnitude larger than the exposures typically collected in the field.

The DAMA/NaI set up and its performances are described in ref. [1,3,5,21], while the DAMA/LIBRA set-up and its performances are described in ref. [10,12]. The sensitive part of the DAMA/LIBRA set-up is made of 25 highly radiopure NaI(Tl) crystal scintillators placed in a 5-rows by 5-columns matrix; each crystal is coupled to two low background photomultipliers working in coincidence at single photoelectron level. The detectors are placed inside a sealed copper box flushed with HP nitrogen and surrounded by a low background and massive shield made of Cu/Pb/Cd-foils/polyethylene/paraffin; moreover, about 1 m concrete (made from the Gran Sasso rock material) almost fully surrounds (mostly outside the barrack) this passive shield, acting as a further neutron moderator. The installation has a 3-fold levels sealing system which excludes the detectors from environmental air. The whole installation is air-conditioned and the temperature is continuously monitored and recorded. The detectors' responses range from 5.5 to 7.5 photoelectrons/keV. Energy calibrations with X-rays/γ sources are regularly carried out down to few keV in the same conditions as the production runs. A software energy threshold of 2 keV is considered. The experiment takes data up to the MeV scale and thus it is also sensitive to high energy signals. For all the details see ref. [10].

2.2 Short summary of the results

Several analyses on the model-independent DM annual modulation signature have been performed (see Refs. [11–13] and references therein). Here Fig. 2.1 shows the time behaviour of the experimental residual rates of the *single-hit* events collected by DAMA/NaI and by DAMA/LIBRA in the (2–6) keV energy interval [11,12]. The superimposed curve is the cosinusoidal function: $A \cos \omega(t - t_0)$ with

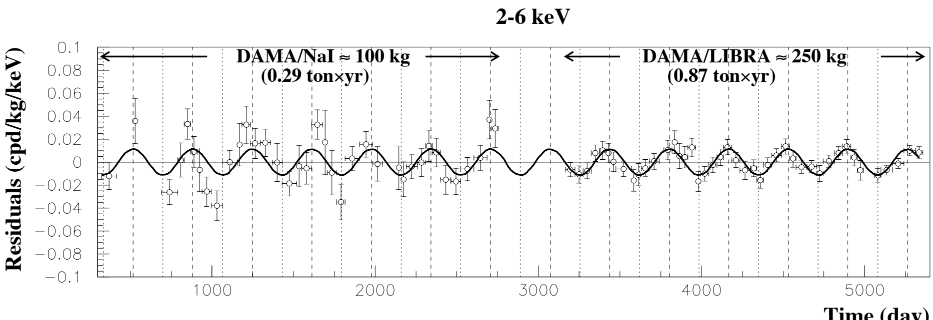

Fig. 2.1. Experimental model-independent residual rate of the *single-hit* scintillation events, measured by DAMA/NaI over seven and by DAMA/LIBRA over six annual cycles in the (2 – 6) keV energy interval as a function of the time [5,21,11,12]. The zero of the time scale is January 1^{st} of the first year of data taking. The experimental points present the errors as vertical bars and the associated time bin width as horizontal bars. See refs. [11,12] and text.

a period $T = \frac{2\pi}{\omega} = 1$ yr, with a phase $t_0 = 152.5$ day (June 2^{nd}), and modulation amplitude, A, obtained by best fit over the 13 annual cycles. The hypothesis of absence of modulation in the data can be discarded [11,12] and, when the period and the phase are released in the fit, values well compatible with those expected for a DM particle induced effect are obtained; for example, in the cumulative (2–6) keV energy interval: $A = (0.0116 \pm 0.0013)$ cpd/kg/keV, $T = (0.999 \pm 0.002)$ yr and $t_0 = (146 \pm 7)$ day. Summarizing, the analysis of the *single-hit* residual rate favours the presence of a modulated cosine-like behaviour with proper features at 8.9σ C.L.[12].

The same data of Fig. 2.1 have also been investigated by a Fourier analysis obtaining a clear peak corresponding to a period of 1 year [12]; this analysis in other energy regions shows instead only aliasing peaks. It is worth noting that for this analysis the original formulas in Ref. [23] have been slightly modified in order to take into account for the different time binning and the residuals errors (see e.g. Ref. [13]).

Moreover, in order to verify absence of annual modulation in other energy regions and, thus, to also verify the absence of any significant background modulation, the time distribution in energy regions not of interest for DM detection has also been investigated: this allowed the exclusion of background modulation in the whole energy spectrum at a level much lower than the effect found in the lowest energy region for the *single-hit* events [12]. A further relevant investigation has been done by applying the same hardware and software procedures, used to acquire and to analyse the *single-hit* residual rate, to the *multiple-hits* events in which more than one detector "fires". In fact, since the probability that a DM particle interacts in more than one detector is negligible, a DM signal can be present just in the *single-hit* residual rate. Thus, this allows the study of the background behaviour in the same energy interval of the observed positive effect. The result of the analysis is reported in Fig. 2.2 where it is shown the residual rate of the *single-hit* events measured over the six DAMA/LIBRA annual cycles, as collected in a single annual cycle, together with the residual rates of the *multiple-hits* events, in the same considered energy interval. A clear modulation is present in the *single-hit*

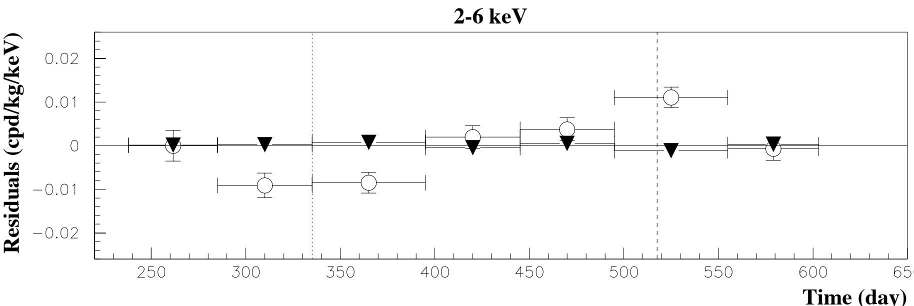

Fig. 2.2. Experimental residual rates over the six DAMA/LIBRA annual cycles for *single-hit* events (open circles) (class of events to which DM events belong) and for *multiple-hit* events (filled triangles) (class of events to which DM events do not belong). The initial time of the figure is taken on August 7^{th}. The experimental points present the errors as vertical bars and the associated time bin width as horizontal bars. See text and refs. [11,12].

events, while the fitted modulation amplitudes for the *multiple-hits* residual rate are well compatible with zero [12]. Similar results were previously obtained also for the DAMA/NaI case [21]. Thus, again evidence of annual modulation with proper features, as required by the DM annual modulation signature, is present in the *single-hit* residuals (events class to which the DM particle induced events belong), while it is absent in the *multiple-hits* residual rate (event class to which only background events belong). Since the same identical hardware and the same identical software procedures have been used to analyze the two classes of events, the obtained result offers an additional strong support for the presence of a DM particle component in the galactic halo further excluding any side effect either from hardware or from software procedures or from background.

The annual modulation present at low energy has also been analyzed by depicting the differential modulation amplitudes, S_m, as a function of the energy; the S_m is the modulation amplitude of the modulated part of the signal obtained by maximum likelihood method over the data, considering $T = 1$ yr and $t_0 = 152.5$ day. The S_m values are reported as function of the energy in Fig. 2.3. It can be

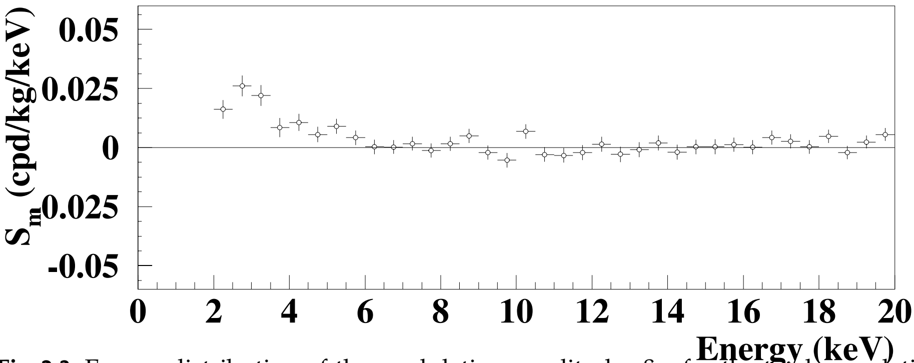

Fig. 2.3. Energy distribution of the modulation amplitudes S_m for the total cumulative exposure 1.17 ton×yr obtained by maximum likelihood analysis. The energy bin is 0.5 keV. A clear modulation is present in the lowest energy region, while S_m values compatible with zero are present just above. See refs. [11,12] and text.

inferred that a positive signal is present in the (2–6) keV energy interval, while S_m values compatible with zero are present just above; in particular, the S_m values in the (6–20) keV energy interval have random fluctuations around zero with χ^2 equal to 27.5 for 28 degrees of freedom. It has been also verified that the measured modulation amplitudes are statistically well distributed in all the crystals, in all the annual cycles and energy bins; these and other discussions can be found in ref. [12].

Many other analyses and discussions can be found in Refs. [11–13] and references therein. Both the data of DAMA/LIBRA and of DAMA/NaI fulfil all the requirements of the DM annual modulation signature.

Careful investigations on absence of any significant systematics or side reaction have been quantitatively carried out (see e.g. Ref. [5,3,10–12,17,13,24–30], and references therein). No systematics or side reactions able to mimic the signature (that is, able to account for the measured modulation amplitude and simultaneously satisfy all the requirements of the signature) has been found or suggested by anyone over more than a decade.

The obtained DAMA model independent evidence is compatible with a wide set of scenarios regarding the nature of the DM candidate and related astrophysical, nuclear and particle Physics. For examples some given scenarios and parameters are discussed e.g. in Ref. [2–5,11,13]. Further large literature is available on the topics (see for example in Ref [13]). Moreover, both the negative results and all the possible positive hints, achieved so-far in the field, are largely compatible with the DAMA model-independent DM annual modulation results in many scenarios considering also the existing experimental and theoretical uncertainties; the same holds for indirect approaches; see e.g. arguments in Ref. [13] and references therein. As an example in Fig. 2.4 there are shown allowed regions for DM candidates interacting by elastic scattering on target-nuclei with spin-independent coupling, including also some of the existing uncertainties [31].

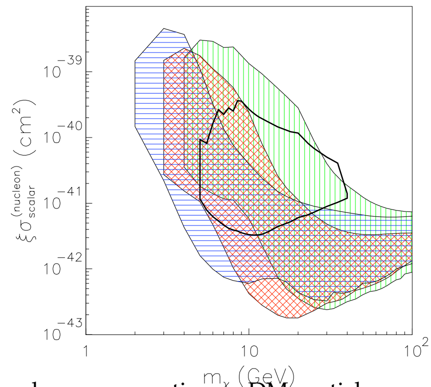

Fig. 2.4. Regions in the nucleon cross section vs DM particle mass plane allowed by DAMA for a DM candidate interacting via spin-independent elastic scattering on target-nucleus; three different instances for the Na and I quenching factors have been considered: i) without including the channeling effect [(green) vertically-hatched region], ii) by including the channeling effect [(blue) horizontally-hatched region)], and iii) without the channeling effect considering energy-dependence of Na and I quenching factors [31] [(red) crosshatched region]. The velocity distributions and the same uncertainties as in refs. [5,21] are considered here. These regions represent the domain where the likelihood-function values differ more than 7.5 σ from the null hypothesis (absence of modulation). The allowed region obtained for the CoGeNT experiment, including the same astrophysical models as in refs. [5,21] and assuming for simplicity a fixed value for the Ge quenching factor and a Helm form factor with fixed parameters, is also reported by a (black) thick solid line. This region includes configurations whose likelihood-function values differ more than 1.64 σ from the null hypothesis (absence of modulation). For details see ref. [31].

2.3 DAMA/LIBRA–phase2 and perspectives

A first upgrade of the DAMA/LIBRA set-up was performed in September 2008. One detector was recovered by replacing a broken PMT and a new optimization of some PMTs and HVs was done; the transient digitizers were replaced with new ones (the U1063A Acqiris 8-bit 1GS/s DC270 High-Speed cPCI Digitizers) having better performances and a new DAQ with optical read-out was installed. The DAMA/LIBRA–phase1 concluded its data taking in this configuration on 2010; the data of the last (seventh) annual cycle of this phase1 have been released after this Workshop [35].

A further and more important upgrade has been performed at the end of 2010 when all the PMTs have been replaced with new ones having higher Quantum Efficiency (Q.E.), realized with a special dedicated development by HAMAMATSU co.. Details on the developments and on the reached performances in the operative conditions are reported in Ref. [18]. We remind that up to October 2010 low background PMTs, developed by EMI-Electron Tubes with dedicated R&D, were used; the light yield and other response features already allowed a software energy threshold of 2 keV in the data analysis. The feasibility to decrease the software energy threshold below 2 keV in the new configuration has been demonstrated[18].

Since the fulfillment of this upgrade, the DAMA/LIBRA–phase2 is continuously running in order: (1) to increase the experimental sensitivity lowering the software energy threshold of the experiment; (2) to improve the corollary investigation on the nature of the DM particle and related astrophysical, nuclear and particle physics arguments; (3) to investigate other signal features. This requires long and heavy full time dedicated work for reliable collection and analysis of very large exposures, as DAMA collaboration has always done.

Another upgrade at the end of 2012 was successfully concluded: new-concept preamplifiers were installed, with suitable operative and electronic features; in particular, they allow the direct connection of the signal to the relative channel of the Transient Digitizer (TD).

Moreover, further improvements are planned. In particular, new trigger modules have been prepared and ready to be installed. A further simplification of the electronic chain has been proposed and funded; for such purpose a new electronic module, New Linear FiFo (NLF), has been designed. It will allow – among the others – a significant reduction of the number of used NIM slots with definitive advantage.

In the future DAMA/LIBRA will also continue its study on several other rare processes [14–16] as also the former DAMA/NaI apparatus did [6].

Finally, further improvements to increase the sensitivity of the set-up are under evaluation; in particular, the use of high Q.E. and ultra-low background PMTs directly coupled to the NaI(Tl) crystals is considered[1]. This possible configuration will allow a further large improvement in the light collection and a further lowering of the software energy threshold. Moreover, efforts towards a possible highly radiopure NaI(Tl) "general purpose" experiment (DAMA/1ton) having full sensitive mass of 1 ton (we already proposed in 1996 as general purpose set-up) are continuing in various aspects.

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3 Revisiting Trace Anomalies in Chiral Theories

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Abstract. This is a report on work in progress about gravitational trace anomalies. We review the problem of trace anomalies in chiral theories in view of the possibility that such anomalies may contain not yet considered CP violating terms. The research consists of various stages. In the first stage we examine chiral theories at one-loop with external gravity and show that a (CP violating) Pontryagin term appears in the trace anomaly in the presence of an unbalance of left and right chirality. However the imaginary coupling of such term implies a breakdown of unitarity, putting a severe constraint on such type of models. In a second stage we consider the compatibility of the presence of the Pontryagin density in the trace anomaly with (local) supersymmetry, coming to an essentially negative conclusion.

Povzetek. To je poročilo o raziskavah gravitacijskih slednih anomalij. Pri tem nas posebej zanima, kaj lahko sledne anomalije prispevajo h kršitvi simetrije CP. Najprej obravnavamo kiralne teorije v prisotnosti (zunanjega) gravitacijskega polja v enozančnem približku. Pokažemo, da se v sledni anomaliji pojavi Pontrjaginov člen, ta krši CP simetrijo, kadar število levoročnih in desnoročnih brezmasnih delcev ni v ravnovesju. Vendar modeli z imaginarno sklopitvijo takega člena s poljem niso unitarni. V drugem koraku obravnavamo skladnost Pontrjaginove gostote v sledni anomaliji v modelih z lokalno supersimetrijo in to možnost v bistvu zavrnemo.

3.1 Introduction

We revisit trace anomalies in theories coupled to gravity, an old subject brought back to people's attention thanks to the importance acquired recently by conformal field theories both in themselves and in relation to the AdS/CFT correspondence. What has stimulated specifically this research is the suggestion by [1] that trace anomalies may contain a CP violating term (the Pontryagin density). It is well known that a basic condition for baryogenesis is the existence of CP nonconserving reactions in an early stage of the universe. Many possible mechanisms for this have been put forward, but to date none is completely satisfactory. The appearance of a CP violating term in the trace anomaly of a theory weakly coupled to gravity may provide a so far unexplored new mechanism for baryogenesis.

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Let us recall that the energy-momentum tensor in field theory is defined by $T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$. Under an infinitesimal local rescaling of the matrix: $\delta g_{\mu\nu} = 2\sigma g_{\mu\nu}$ we have

$$\delta S = \frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu} = -\int d^4x\sqrt{-g}\sigma T_\mu{}^\mu.$$

If the action is invariant, classically $T_\mu{}^\mu = 0$, but at one loop (in which case S is replaced by the one-loop effective action W) the trace of the e.m. tensor is generically nonvanishing. In D=4 it may contain, in principle, beside the Weyl density (square of the Weyl tensor)

$$\mathcal{W}^2 = \mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 2\mathcal{R}_{nm}\mathcal{R}^{nm} + \frac{1}{3}\mathcal{R}^2 \tag{3.1}$$

and the Gauss-Bonnet (or Euler) one,

$$E = \mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 4\mathcal{R}_{nm}\mathcal{R}^{nm} + \mathcal{R}^2, \tag{3.2}$$

another nontrivial piece, the Pontryagin density,

$$P = \frac{1}{2}\left(\epsilon^{nmlk}\mathcal{R}_{nmpq}\mathcal{R}_{lk}{}^{pq}\right) \tag{3.3}$$

Each of these terms appears in the trace with its own coefficient:

$$T_\mu{}^\mu = aE + c\mathcal{W}^2 + eP \tag{3.4}$$

The coefficient a and c are known at one-loop for any type of matter. The coefficient of (3.3) has not been sufficiently studied yet. The purpose of this paper is to fill up this gap. The plan of our research consists of three stages. To start with we analyse the one loop calculation of the trace anomaly in chiral models. Both the problem and the relevant results are not new: the trace anomaly contains beside the square Weyl density and the Euler density also the Pontryagin density. What is important is that the e coefficient is purely imaginary. This entails a violation of unitarity at one-loop and, consequently, introduces an additional criterion for a theory to be acceptable. The latter is similar to the analogous criterion for chiral gauge and gravitational anomalies, which is since long a selection criterion for acceptable theories. A second stage of our research concerns the compatibility between the appearance of the Pontryagin term in the trace anomaly and supersymmetry. Since it is hard to supersymmetrize the above three terms and relate them to one another in a supersymmetric context, the best course is to consider a conformal theory in 4D coupled to (external) $N = 1$ supergravity formulated in terms of superfields and find all the potential superconformal anomalies. This will allow us to see whether (3.3) can be accommodated in an anomaly supermultiplet as a trace anomaly member. The result of our analysis seems to exclude this possibility. Finally, a third stage of our research is to analyse the possibility that the Pontryagin density appears in the trace anomaly in a nonperturbative way, for instance via an AdS/CFT correspondence as suggested in [1].

In this contribution we will consider the first two issues above. In the next section we will examine the problem of the one-loop trace anomaly in a prototype chiral theory. Section 3.3 is devoted to the compatibility of the Pontryagin term in the trace anomaly with supersymmetry.

3.2 One-loop trace anomaly in chiral theories

The model we will consider is the simplest possible one: a left-handed spinor coupled to external gravity in 4D. The action is

$$S = \int d^4x \sqrt{|g|}\, i\bar{\psi}_L \gamma^m (\nabla_m + \frac{1}{2}\omega_m)\psi_L \tag{3.5}$$

where $\gamma^m = e_a^m \gamma^a$, ∇ ($m, n, \ldots$ are world indices, $a, b, \ldots$ are flat indices) is the covariant derivative with respect to the world indices and ω_m is the spin connection:

$$\omega_m = \omega_m^{ab}\Sigma_{ab}$$

where $\Sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ are the Lorentz generators. Finally $\psi_L = \frac{1+\gamma_5}{2}\psi$. Classically the energy-momentum tensor

$$T_{\mu\nu} = \frac{i}{2}\bar{\psi}_L \gamma_\mu \overset{\leftrightarrow}{\nabla}_\nu \psi_L \tag{3.6}$$

is both conserved on shell and traceless. At one loop to make sense of the calculations one must introduce regulators. The latter generally break both diffeomorphism and conformal invariance. A careful choice of the regularization procedure may preserve diff invariance, but anyhow breaks conformal invariance, so that the trace of the e.m. tensor takes the form (3.4), with specific nonvanishing coefficients a, c, e. There are various techniques to calculate the latter: cutoff, point splitting, Pauli-Villars, dimensional regularizations. Here we would like to briefly recall the heat kernel method utilized in [2] and in references cited therein (a more complete account will appear elsewhere). Denoting by D the relevant Dirac operator in (3.5) one can prove that

$$\delta W = -\int d^4x\sqrt{-g}\sigma T_\mu{}^\mu = -\frac{1}{16\pi^2}\int d^4x\sqrt{-g}\sigma b_4\left(x, x; D^\dagger D\right).$$

Thus

$$T_\mu{}^\mu = b_4\left(x, x; D^\dagger D\right) \tag{3.7}$$

The coefficient $b_4\left(x, x; D^\dagger D\right)$ appear in the heat kernel. The latter has the general form

$$K(t, x, y; \mathcal{D}) \sim \frac{1}{(4\pi t)^2} e^{-\frac{\sigma(x,y)}{2t}} \left(1 + t b_2(x, y; \mathcal{D}) + t^2 b_4(x, y; \mathcal{D}) + \cdots\right),$$

where $\mathcal{D} = D^\dagger D$ and $\sigma(x, y)$ is the half square length of the geodesic connecting x and y, so that $\sigma(x, x) = 0$. For coincident points we therefore have

$$K(t, x, x; \mathcal{D}) \sim \frac{1}{16\pi^2}\left(\frac{1}{t^2} + \frac{1}{t} b_2(x, x; \mathcal{D}) + b_4(x, x; \mathcal{D}) + \cdots\right). \tag{3.8}$$

This expression is divergent for $t \to 0$ and needs to be regularized. This can be done in various ways. The finite part, which we are interested in, has been calculated first by DeWitt, [3], and then by others with different methods. The results are reported in [2]. For a spin $\frac{1}{2}$ left-handed spinor as in our example one gets

$$b_4\left(x, x; D^{\dagger}D\right) = \frac{1}{180 \times 16\pi^2} \int d^4x\sqrt{-g}\left(a\, E_4 + c\, W^2 + e\ P\right) \tag{3.9}$$

with

$$a = \frac{11}{4}, \qquad c = -\frac{9}{2}, \qquad e = \frac{15}{4} \tag{3.10}$$

This result was obtained with an entirely Euclidean calculation. Turning to the Minkowski the actual e.m trace at one loop is

$$T_{\mu}{}^{\mu} = \frac{1}{180 \times 16\pi^2}\left(\frac{11}{4}\, E + c\, \mathcal{W}^2 + i\frac{15}{4}\, P\right) \tag{3.11}$$

As pointed out above the important aspect of (3.11) is the i appearing in front of the Pontryagin density. The origin of this imaginary coupling is easy to trace. It comes from the trace of gamma matrices including a γ_5 factor. In 4d, while the trace of an even number of gamma matrices, which give rise to first two terms in the RHS of (3.11), is a real number, the trace of an even number of gamma's multiplied by γ_5 is always imaginary. The Pontryagin term comes precisely from the latter type of traces. It follows that, as a one loop effect, the energy momentum tensor becomes complex, and, in particular, since T_0^0 is the Hamiltonian density, we must conclude that unitarity is not preserved in this type of theories. Exactly as chiral gauge theories with nonvanishing chiral gauge anomalies are rejected as sick theories, also chiral models with complex trace anomalies are not acceptable theories. For instance the old-fashioned standard model with massless left-handed neutrinos is in this situation. This model, provided it has an UV fixed point, has a complex trace anomaly and breaks unitarity. This is avoided in the modern formulation of the electroweak interactions by the addition of a right-handed neutrino (for each flavor), or, alternatively, by using Majorana neutrinos. So, in hindsight, one could have predicted massive neutrinos.

In general we can say that in models with a chirality unbalance a problem with unitarity may arise due to the trace anomaly and has to be carefully taken into account.

3.3 Pontryagin density and supersymmetry

In this section we discuss the problem posed by the possible appearance of the Pontryagin term in the trace anomaly: is it compatible with supersymmetry? It is a well known fact that trace anomalies in supersymmetric theories are members of supermultiplets, to which also the Abelian chiral anomaly belongs. Thus one way to analyse this issue would be to try and supersymmetrize the three terms (3.1,3.2) and (3.3) and see whether they can be accommodated in supermultiplets. This direct approach, however, is far from practical. What we will do, instead, is to consider a conformal theory in 4D coupled to (external) supergravity formulated in terms of superfields, and find all the potential superconformal anomalies. This will allow us to see whether (3.3) can be accommodated in an anomaly supermultiplet as a trace anomaly member.

3.3.1 Minimal supergravity

The most well known model of $N = 1$ supergravity in $D = 4$ is the so-called *minimal supergravity*, see for instance [4]. The superspace of $N = 1$ supergravity is spanned by the supercoordinates $Z^M = (x^m, \theta^\mu, \bar\theta_{\dot\mu})$. In this superspace one introduces a superconnection, a supertorsion and the relevant supercurvature. To determine the dynamics one imposes constraints on the supertorsion. Such constraints are not unique. A particular choice of the latter, the *minimal* constraints, define the minimal supergravity model, which can be formulated in terms of the superfields $R(z)$, $G_a(z)$ and $W_{\alpha\beta\gamma}(z)$. R and $W_{\alpha\beta\gamma}$ are chiral while G_a is real. One also needs the antichiral superfields $R^+(z)$ and $\bar W_{\dot\alpha\dot\beta\dot\gamma}(z)$. $W_{\alpha\beta\gamma}$ is completely symmetric in the spinor indices $\alpha, \beta, \ldots$. These superfields satisfy themselves certain constraints. Altogether the independent degrees of freedom are 12 bosons + 12 fermions. One can define superconformal transformations in terms of a parameter superfield σ. For instance

$$\begin{aligned}\delta R &= (2\bar\sigma - 4\sigma)R - \frac{1}{4}\nabla_{\dot\alpha}\nabla^{\dot\alpha}\bar\sigma\\ \delta G_a &= -(\sigma+\bar\sigma)G_a + i\nabla_a(\bar\sigma - \sigma)\\ \delta W_{\alpha\beta\gamma} &= -3\sigma W_{\alpha\beta\gamma}\end{aligned}$$

To find the possible superconformal anomalies we use a cohomological approach. Having in mind a superconformal matter theory coupled to a $N = 1$ supergravity, we define the functional operator that implements these transformations, i.e.

$$\Sigma = \int_{x\theta} \delta\chi_i \frac{\delta}{\delta\chi_i}$$

where χ_i represent the various superfields in the game and $_{x\theta}$ denotes integration $d^4xd^4\theta$. This operator is nilpotent: $\Sigma^2 = 0$. As a consequence it defines a cohomology problem. The cochains are integrated local expressions of the superfields and their superderivatives, invariant under superdiffeomorphism and local super-Lorentz transformations. Candidates for superconformal anomalies are nontrivial cocycles of Σ which are not coboundaries, i.e. integrated local functionals Δ_σ, linear in σ, such that

$$\Sigma\,\Delta_\sigma = 0, \qquad \text{and} \qquad \Delta_\sigma \neq \Sigma\,\mathcal{C}$$

for any integrated local functional $\mathcal{C}$ (not containing σ).

The complete analysis of all the possible nontrivial cocycles of the operator Σ was carried out in [5]. It was shown there that the latter can be cast into the form

$$\Delta_\sigma = \int_{x\theta}\left[\frac{E(z)}{-8R(z)}\,\sigma(z)\,\mathcal{S}(z) + \text{h.c.}\right] \tag{3.12}$$

where $\mathcal{S}(z)$ is a suitable chiral superfield, and all the possibilities for $\mathcal{S}$ were classified. For supergravity alone (without matter) the only nontrivial possibilities turn out to be:

$$\mathcal{S}_1(z) = W^{\alpha\beta\gamma}W_{\alpha\beta\gamma} \qquad \text{and} \qquad \mathcal{S}_2(z) = (\bar{\nabla}_{\dot{\alpha}}\bar{\nabla}^{\dot{\alpha}} - 8R)(G_a G^a + 2RR^+) \tag{3.13}$$

(the operator $(\bar{\nabla}_{\dot{\alpha}}\bar{\nabla}^{\dot{\alpha}} - 8R)$ maps a real superfield into a chiral one).

It is well-known that the (3.12) cocycles contain not only the trace anomaly, but a full supermultiplet of anomalies. The local expressions of the latter are obtained by stripping off the corresponding parameters from the integrals in (3.12).

In order to recognize the ordinary field content of the cocycles (3.13) one has to pass to the component form. This is done by choosing the lowest components of the supervielbein as follows:

$$E_M{}^A(z)\Big|_{\theta=\bar{\theta}=0} = \begin{pmatrix} e_m{}^a(x) & \frac{1}{2}\psi_m{}^\alpha(x) & \frac{1}{2}\bar{\psi}_{m\dot{\alpha}}(x) \\ 0 & \delta_\mu{}^\alpha & 0 \\ 0 & 0 & \delta^{\dot{\mu}}{}_{\dot{\alpha}} \end{pmatrix}$$

where $e_m{}^a$ are the usual 4D vierbein and $\psi_m{}^\alpha(x), \bar{\psi}_{m\dot{\alpha}}(x)$ the gravitino field components. Similarly one identifies the independent components of the other superfields (the lowest component of R and G_a). For σ we have

$$\sigma(z) = \omega(x) + i\alpha(x) + \sqrt{2}\Theta^\alpha\chi_\alpha(x) + \Theta^\alpha\Theta_\alpha(F(x) + iG(x)) \tag{3.14}$$

where Θ^α are Lorentz covariant anticommuting coordinates, [4]. The component fields of (3.14) identify the various anomalies in the cocycles (3.13). In particular ω is the parameter of the ordinary conformal transformations and α the parameter of the chiral transformations. They single out the corresponding anomalies. At this point it is a matter of algebra to write down the anomalies in component. Retaining for simplicity only the metric we obtain the *ordinary* form of the cocycles. This is

$$\Delta_\sigma^{(1)} \approx \tag{3.15}$$
$$\int_x e\Big\{\omega\Big(\mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 2\mathcal{R}_{nm}\mathcal{R}^{nm} + \frac{1}{3}\mathcal{R}^2\Big) - \frac{1}{2}\,\alpha\,\epsilon^{nmlk}\mathcal{R}_{nmpq}\mathcal{R}_{lk}{}^{pq}\Big\}$$

for the first cocycle ($\approx$ denotes precisely the ordinary form), and

$$\Delta_\sigma^{(2)} = 4\int_x e\,\omega\big(\frac{2}{3}\mathcal{R}^2 - 2\mathcal{R}_{nm}\mathcal{R}^{nm}\big) \tag{3.16}$$

for the second. Taking a suitable linear combination of the two we get

$$\Delta_\sigma^{(1)} + \frac{1}{2}\Delta_\sigma^{(2)} \approx \tag{3.17}$$
$$\int_x e\Big\{\omega\big(\mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 4\mathcal{R}_{nm}\mathcal{R}^{nm} + \mathcal{R}^2\big) - \frac{1}{2}\,\alpha\,\epsilon^{nmlk}\mathcal{R}_{nmpq}\mathcal{R}_{lk}{}^{pq}\Big\}$$

We see that (3.15) contain $\mathcal{W}^2$ while (3.17) contains the Euler density in the terms proportional to ω (trace anomaly). They both contain the Pontryagin density in the term proportional to α (chiral anomaly).

In conclusion $\Delta_\sigma^{(1)}$ corresponds to a multiplet of anomalies, whose first component is the Weyl density multiplied by ω, accompanied by the Pontryagin density (the Delbourgo-Salam anomaly) multiplied by α. On the other hand $\Delta_\sigma^{(2)}$ does not contain the Pontryagin density and the part linear in ω is a combination of the Weyl and Gauss-Bonnet density. None of them contains the Pontryagin density in the trace anomaly part. Therefore we must conclude that, as far as $N = 1$ minimal supergravity is concerned, our conclusion about the compatibility between the Pontryagin density as a trace anomaly terms and local supersymmetry, is negative.

3.3.2 Other nonminimal supergravities

As previously mentioned the minimal model of supergravity is far from unique. There are many other choices of the supertorsion constraints, beside the minimal one. Most of them are connected by field redefinitions and represent the same theory. But there are choices that give rise to different dynamics. This is the case for the nonminimal 20+20 and 16+16 models. In the former case one introduces two new spinor superfields T_α and $\bar{T}_{\dot\alpha}$, while setting $R = R^+ = 0$. This model has 20+20 degrees of freedom. The bosonic dofs are those of the minimal model, excluding R and R^+, plus 10 additional ones which can be identified with the lowest components of the superfields $S = \mathcal{D}^\alpha T_\alpha - (n+1)T^\alpha T_\alpha$ and $\bar{S}$, $\bar{\mathcal{D}}_{\dot\alpha} T_\alpha$ and $\mathcal{D}_\alpha T_{\dot\alpha}$. The superconformal parameter is a generic complex superfield Σ constrained by the condition

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha + (n+1)T^\alpha \mathcal{D}_\alpha)\left[3n(\bar{\Sigma} - \Sigma) - (\bar{\Sigma} + \Sigma)\right] = 0$$

where n is a numerical parameter. It is easy to find a nontrivial cocycle of this symmetry

$$\Delta^{(1)}_{n.m.} = \int_{x,\theta} E\,\Sigma\, W^{\alpha\beta\gamma} W_{\alpha\beta\gamma} \frac{\bar{T}_{\dot\alpha}\bar{T}^{\dot\alpha}}{\bar{S}^2} + \text{h.c.}$$

and to prove that its ordinary component form is, up to a multiplicative factor,

$$\Delta^{(1)}_\Sigma \approx$$
$$\frac{1}{4}\int_x e\Big\{\omega\left(\mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 2\mathcal{R}_{nm}\mathcal{R}^{nm} + \frac{1}{3}\mathcal{R}^2\right) - \frac{1}{2}\,\alpha\,\epsilon^{nmlk}\mathcal{R}_{nmpq}\mathcal{R}_{lk}{}^{pq}\Big\}$$

where $\omega + i\alpha$ is the lowest component of the superfield Σ. That is, the same ordinary form as $\Delta_\sigma^{(1)}$. As for other possible cocycles they can be obtained from the minimal supergravity ones by way of superfield redefinitions. To understand this point one should remember what was said above: different models of supergravity are defined by making a definite choice of the torsion constraints and, after such a choice, by identifying the dynamical degrees of freedom. This is the way minimal and nonminimal models are introduced. However it is possible to transform the choices of constraints into one another by means of linear transformations of the supervierbein and the superconnection, [7,8]:

$$E'_M{}^A = E_M{}^B X_B{}^A, \qquad E'_A{}^M = X^{-1}{}_A{}^B E_B{}^M, \qquad \Phi'_{MA}{}^B = \Phi_{MA}{}^B + \chi_{MA}{}^B$$

for suitable $X_A{}^B$ and $\chi_{MA}{}^B$. This was done in [6] and will not be repeated here. The result is a very complicated form for the cocycle $\Delta^{(2)}_{n.m.}$, derived from $\Delta^{(2)}_\sigma$. However the ordinary component form is the same for both.

As for the 16+16 nonminimal supergravity, it is obtained from the 20+20 model by imposing

$$T_\alpha = \mathcal{D}_\alpha \psi, \qquad T_{\dot\alpha} = \mathcal{D}_{\dot\alpha} \psi$$

where ψ is a (dimensionless) real superfield. The independent bosonic dofs are the lowest component of $S, \bar{S}$, $c_{\alpha\dot\alpha}$ and $G_{\alpha\dot\alpha}$, beside the metric. The superconformal transformation are expressed in terms of a real vector superfield L and an arbitrary chiral superfield Λ satisfying the constraint

$$\left(\mathcal{D}^\alpha \mathcal{D}_\alpha + (n+1) T^\alpha \mathcal{D}_\alpha\right)(2L + (3n+1)\Lambda) = 0.$$

The derivation of the nontrivial superconformal cocycles is much the same as for the previous model. The end result is two cocycles whose form, in terms of superfields, is considerably complicated, but whose ordinary form is the same as $\Delta^{(1)}_\sigma$ and $\Delta^{(2)}_\sigma$.

At this point we must clarify whether the cocycles we have found in 20+20 and 16+16 nonminimal supergravities are the only ones. In [6] a systematic cohomological search of such nontrivial cocycles has not been done, the reason being that when dimensionless fields, like ψ and $\bar{\psi}$, are present in a theory a polynomial analysis is not sufficient (and a non-polynomial one is of course very complicated). But we can argue as follows: consider a nontrivial cocycle in nonminimal or 16+16 nonminimal supergravity; it can be mapped to a minimal cocycle which either vanishes or coincides with the ones classified in [5]. There is no other possibility because in minimal supergravity there are no dimensionless superfields (apart from the vielbein) and the polynomial analysis carried out in [5] is sufficient to identify all cocycles. We conclude that the 20+20 and 16+16 nonminimal nontrivial cocycles, which reduce in the ordinary form to a nonvanishing expression, correspond to $\Delta^{(1)}_\sigma$ and $\Delta^{(2)}_\sigma$ in minimal supergravity and only to them.

None of these cocycles contains the Pontryagin density in the trace anomaly part. Therefore we must conclude that, as far as $N = 1$ minimal and nonminimal supergravity is concerned, our conclusion about the compatibility between the Pontryagin density as a trace anomaly terms and local supersymmetry, is negative.

3.4 Conclusion

A component of the trace anomaly which appear in chiral theories (the Pontryagin density) may have interesting implications. It is a CP violating term and, as such, it could be an interesting mechanism for baryogenesis. At one loop, as we have seen, this term violates unitarity and the only use we can make of it is as a selection criterion for phenomenological models with an UV fixed point. If, on the other hand, by some other kind of mechanism still to be discovered, this term appears in the trace of the em tensor with a real coefficient, it may become very interesting as a CP violating term. In the last section we have seen that, however, this is incompatible with supersymmetry. In other words, if such mechanism exists, it can become effective only after supersymmetry breaking. The search for the P term continues.

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4 Can We Predict the Fourth Family Masses for Quarks and Leptons?

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Abstract. In the ref. [1–4] four massless families of quarks and leptons before the electroweak break are predicted. Mass matrices of all the family members demonstrate in this proposal the same symmetry, determined by the family groups. There are scalar fields - two $SU(2)$ triplets, the gauge fields of the family quantum numbers, and three singlets, the gauge fields of the three charges (Q, Q' and Y')- all doublets with respect to the weak charge, which determine mass matrices on the tree level and, together with other contributions, also beyond the tree level. The symmetry of mass matrices remains unchanged for all loop corrections. The three singlets are, in loop corrections also together with other contributors, responsible for the differences in properties of the family members. Taking into account by the *spin-charge-family* theory proposed symmetry of mass matrices for all the family members and simplifying study by assuming that mass matrices are Hermitian and real and mixing matrices real, we fit free parameters of mass matrices to experimental data within the experimental accuracy. Calculations are in progress.

Povzetek. Teorija spina-nabojev-družin napoveduje [1–4], preden se zlomi elektrošibka simetrija, štiri brezmasne družine kvarkov in leptonov. Masne matrike vseh članov družin imajo po zlomitvi enako simetrijo, ki jo določajo družinska kvantna števila: Vsak spinor nosi družinski kvantni števili dveh grup $SU(2)$, nosi pa tudi kvantna števila člana družine. Pri zlomitvi simetrije sodelujejo skalarna polja, ki so tripletna umeritvena polja bodisi ene od dveh grup $SU(2)$ (družinska simetrija), ali pa singletna umeritvena polja treh nabojev (Q, Q' in Y'), ki razlikujejo med člani posamezne družine. Vsa skalarna polja so dubleti glede na šibki naboj, nosijo pa tudi hiper naboj. Simetrije masnih matrik se ohranjajo v vseh zančnih popravkih. Trije singleti določajo, v zančnih popravkih skupaj z ostalimi prispevki, razlike v lastnostih članov družin. Problem poenostavimo s predpostavko, da so masne matrike hermitske in realne in mešalne matrike realne. Zahtevana simetrija masnih matrik ima enako število prostih parametrov kot je doslej izmerjenih podatkov (dvakrat po tri mase in mešalna matrika - za kvarke in leptone). Napake podatkov omogočijo določitev le intervalov za vrednosti parametrov. Iz masnih matrik določimo lastnosti članov četrte družine. Računi so v teku.

4.1 Introduction

There are several attempts in the literature to reconstruct mass matrices of quarks and leptons out of the observed masses and mixing matrices and correspondingly to learn more about properties of fermion families [8]. The most popular is the

$n \times n$ matrix close to the democratic one, predicting that $(n-1)$ families must be very light in comparison with the n^{th} one. Most of attempts treat neutrinos differently than the other family members, relying on the Majorana part, the Dirac part and the "sea-saw" mechanism. Most often are the number of families taken to be equal to the number of the so far observed families, while symmetries of mass matrices are chosen in several different ways [9]. Also possibilities with four families are discussed [12].

In this paper we follow the prediction of the *spin-charge-family* theory [1–4,7] that there are four massless families above the electroweak break and that the scalar fields - the two triplets carrying the family charges in the adjoint representations and the three singlets carrying the charges of the family members (Q, Q' and Y') - all doublets with respect to the weak charge, cause (after getting nonzero vacuum expectation values) the electroweak break. Assuming that the contributions of all the scalar (and in loop corrections also of other) fields to mass matrices of fermions are real and symmetric, we are left with the following symmetry of mass matrices

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha}, \tag{4.1}$$

the same for all the family members $\alpha \in \{u, d, \nu, e\}$. In appendix 4.5.1 the evaluation of this mass matrix is presented and the symmetry commented. A change of phases of the left handed and the right handed basis - there are $(2n-1)$ free choices - manifests in a change of phases of mass matrices.

The differences in the properties of the family members originate in the different charges of the family members and correspondingly in the different couplings to the corresponding scalar and gauge fields.

We fit (sect. 4.3.2) the mass matrix Eq. (4.1) with 6 free parameters of any family member 6 to the so far observed properties of quarks and leptons within the experimental accuracy. That is: *For a pair of either quarks or leptons, we fit twice* 6 *free parameters of the two mass matrices to twice three so far measured masses and to the corresponding mixing matrix*. Since we have the same number of free parameters (two times 6 for each pair, since the mass matrices are assumed to be real) as there are measured quantities (two times 3 masses and 6 angles of the orthogonal mixing matrix under a simplification that the mixing matrix is real and Hermitian), we would predict the fourth family masses uniquely, provided that the measured quantities are accurate. The $n-1$ submatrix of any unitary matrix determine the unitary matrix uniquely for $n \geq 4$. The experimental inaccuracy enable to determine only the interval for the fourth family masses.

If the prediction of the *spin-charge-family* theory, that there are four families which manifest in the massless basis the symmetry of Eq. (4.1), is correct, we expect that enough accurate experimental data for the properties of the so far observed three families will offer narrow enough intervals for the fourth family masses.

We treat all the family members, the quarks and the leptons, equivalently. We also estimate the contributions of the fourth family members to the mesons decays in dependence of the fourth family masses, taking into account also the estimations of the refs. [15] [1]. However, we must admit that our estimations are so far pretty rough.

In sect. 4.3.1 we check on a toy model how accurate must be the experimental data that enable the prediction of the fourth family masses: For two ”known” mass matrices, obeying the symmetry of Eq. (4.1), which lead approximately to the experimental data, we calculate masses and the mixing matrix. Then, taking the mixing matrix and twice three lower masses as an input, we look back for the starting two mass matrices with the required symmetry, allowing for the three lower families ”experimental” inaccuracy. In the same section we then estimate the fourth family masses. So far the results are preliminary. Although we spent quite a lot of efforts to make the results transparent and trustable, the numerical procedure to take into account the experimental inaccuracy of data is not yet good enough to allow us to determine the interval of the fourth family masses, even not for quarks, so that all the results are very preliminary.

Still we can say that the so far obtained support the prediction of the *spin-charge-family* theory that there are four families of quarks and leptons, the mass matrices of which manifest the symmetry determined by the family groups – the same for all the family members, quarks and leptons. The mass matrices are quite close to the ”democratic” ones, in particular for leptons.

Since the mass matrices offer an insight into the properties of the scalar fields, which determine mass matrices (together with other fields), manifesting effectively as the observed Higgs and the Yukawa couplings, we hope to learn about the properties of these scalar fields also from the mass matrices of quarks and leptons.

In appendix 4.5 we offer a very brief introduction into the *spin-charge-family* theory, which the reader, accepting the proposed symmetry of mass matrices without knowing the origin of this symmetry, can skip.

In sect. 4.2 the procedure to fit free parameters of mass matrices (Eq. (4.1) to the experimental data is discussed. We comment our studies in sect. 4.4.

4.2 Procedure used to fit free parameters of mass matrices to experimental data

Matrices, following from the *spin-charge-family* theory might not be Hermitian (appendix 4.6). We, however, simplify our study, presented in this paper, by assuming that the mass matrix for any family member, that is for the quarks and the leptons, is real and symmetric. We take the simplest phases up to signs, which depend on the choice of phases of the basic states, as discussed in appendices 4.5.1 [2].

[1] M.I.Vysotsky and A.Lenz comment in their very recent papers that the fourth family is excluded provided that one assumes the *standard model* with one scalar field (the Higgs) and extends the number of families from three to four while using loop corrections when evaluating the decay properties of the Higgs. We have, however, several scalar fields and first estimates show that the fourth family quarks might have masses close to 1 TeV.

[2] In the ref. [17] we made a similar assumption, except that we allow that the symmetry on the tree level of mass matrices might be changed in loop corrections. We got in that study dependence of mass matrices and correspondingly mixing matrices for quarks on masses of the fourth family.

The matrix elements of mass matrices, with the loop corrections in all orders taken into account, manifesting the symmetry of Eq. (4.1), are in this paper taken as free parameters.

Let us first briefly overview properties of mixing matrices, a more detailed explanation of which can be found in subsection 4.2.1 of this section.

Let M^α, α denotes the family member ($\alpha = u, d, \nu, e$), be the mass matrix in the massless basis (with all loop corrections taken into account). Let $V_{\alpha\beta} = S^\alpha S^{\beta\dagger}$, where α represents either the u-quark and β the d-quark, or α represents the ν-lepton and β the e-lepton, denotes a (in general unitary) mixing matrix of a particular pair.

For $n \times n$ matrix ($n = 4$ in our case) it follows:
i. If a known submatrix $(n-1) \times (n-1)$ of an unitary matrix $n \times n$ with $n \geq 4$ is extended to the whole unitary matrix $n \times n$, the n^2 unitarity conditions determine $(2(2(n-1)+1))$ real unknowns completely. If the submatrix $(n-1) \times (n-1)$ of an unitary matrix is made unitary by itself, then we loose the information.
ii. If the mixing matrix is assumed to be orthogonal, then the $(n-1) \times (n-1)$ submatrix contains all the information about the $n \times n$ orthogonal matrix to which it belongs and the $n(n+1)/2$ conditions determine the $2(n-1)+1$ real unknowns completely for any n.
If the submatrix of the orthogonal matrix is made orthogonal by itself, then we loose the information.

We make in this paper, to simplify the present study, several assumptions [7], presented already in the introduction. In what follows we present the procedure used in our study and repeat the assumptions.

1. If the mass matrix M^α is Hermitian, then the unitary matrices S^α and T^α, introduced in appendix 4.6 to diagonalize a non Hermitian mass matrix, differ only in phase factors depending on phases of basic vectors and manifesting in two diagonal matrices, $F^{\alpha\, S}$ and $F^{\alpha\, T}$, corresponding to the left handed and the right handed basis, respectively. For Hermitian mass matrices we therefore have: $T^\alpha = S^\alpha\, F^{\alpha\, S} F^{\alpha\, T\, \dagger}$. By changing phases of basic vectors we can change phases of $(2n-1)$ matrix elements.
2. We take the diagonal matrices $\mathcal{M}^\alpha_d$ and the mixing matrices $V_{\alpha\beta}$ from the available experimental data. The mass matrices M^α in Eq. (4.1) have, if they are Hermitian and real, 6 free real parameters $(a^\alpha, a^\alpha_1, a^\alpha_2, b^\alpha, e^\alpha, d^\alpha)$.
3. We limit the number of free parameters of the mass matrix of each family member α by taking into account n relations among free parameters, in our case $n = 4$, determined by the invariants

$$I^\alpha_1 = -\sum_{i=1,4} m^\alpha_i, \quad I^\alpha_2 = \sum_{i>j=1,4} m^\alpha_i\, m^\alpha_j,$$
$$I^\alpha_3 = -\sum_{i>j>k=1,4} m^\alpha_i\, m^\alpha_j\, m^\alpha_k, \quad I^\alpha_4 = m^\alpha_1\, m^\alpha_2\, m^\alpha_3\, m^\alpha_4\,, \tag{4.2}$$

which are expressions appearing at powers of λ_α, $\lambda^4_\alpha + \lambda^3_\alpha I_1 + \lambda^2_\alpha I_2 + \lambda^1_\alpha I_3 + \lambda^0_\alpha I_4 = 0$, in the eigenvalue equation. The invariants are fixed, within the experimental accuracy of the data, by the observed masses of quarks and leptons and by the fourth family mass, if we make a choice of it. In appendix 4.2.2 we present the relations among the reduced number of free parameters for a chosen m_4^α. There are $(6-4)$ free parameters left for each mass matrix.

4. The diagonalizing matrices S^α and S^β, each depending on the reduced number of free parameters, are for real and symmetric mass matrices orthogonal. They follow from the procedure

$$M^\alpha = S^\alpha \mathbf{M}_d^\alpha T^{\alpha\,\dagger}\,, \quad T^\alpha = S^\alpha F^{\alpha\,S} F^{\alpha\,T\,\dagger}\,,$$
$$\mathbf{M}_d^\alpha = (m_1^\alpha, m_2^\alpha, m_3^\alpha, m_4^\alpha)\,, \tag{4.3}$$

provided that S^α and S^β fit the experimentally observed mixing matrices $V^\dagger_{\alpha\beta}$ within the experimental accuracy and that M^α and M^β manifest the symmetry presented in Eq. (4.1). We keep the symmetry of the mass matrices accurate. One can proceed in two ways.

$$A.: \quad S^\beta = V^\dagger_{\alpha\beta} S^\alpha\,, \qquad B.: \quad S^\alpha = V_{\alpha\beta} S^\beta\,,$$
$$A.: \quad V^\dagger_{\alpha\beta}\, S^\alpha\, \mathbf{M}_d^\beta\, S^{\alpha\dagger} V_{\alpha\beta} = M^\beta\,, \qquad B.: \quad V_{\alpha\beta}\, S^\beta\, \mathbf{M}_d^\alpha\, S^{\beta\dagger} V^\dagger_{\alpha\beta} = M^\alpha\,. \tag{4.4}$$

In the case A. one obtains from Eq. (4.3), after requiring that the mass matrix M^α has the desired symmetry, the matrix S^α and the mass matrix M^α $(= S^\alpha \mathbf{M}_d^\alpha S^{\alpha\dagger})$, from where we get the mass matrix $M^\beta = V^\dagger_{\alpha\beta}\, S^\alpha\, \mathbf{M}_d^\beta\, S^{\alpha\dagger} V_{\alpha\beta}$. In case B. one obtains equivalently the matrix S^β, from where we get M^α $(= V_{\alpha\beta}\, S^\beta\, \mathbf{M}_d^\alpha\, S^{\beta\dagger} V^\dagger_{\alpha\beta})$. We use both ways iteratively taking into account the experimental accuracy of masses and mixing matrices.

5. Under the assumption of the present study that the mass matrices are real and symmetric, the orthogonal diagonalizing matrices S^α and S^β form the orthogonal mixing matrix $V_{\alpha\beta}$, which depends on at most 6 $(= \frac{n(n-1)}{2})$ free real parameters (appendix 4.6). Since, due to what we have explained at the beginning of this section, the experimentally measured matrix elements of the 3×3 submatrix of the 4×4 mixing matrix (if not made orthogonal by itself) determine the 4×4 mixing matrix - within the experimental accuracy - completely, also the fourth family masses are determined, again within the experimental accuracy. We must not forget, however, that the assumption of the real and symmetric mass matrices, leading to orthogonal mixing matrices, might not be an acceptable simplification, since we do know that the 3×3 submatrix of the mixing matrix has one complex phase, while the unitary 4×4 has three complex phases. (In the next step of study, with hopefully more accurate experimental data, we shall relax conditions on hermiticity of mass matrices and correspondingly on orthogonality of mixing matrices). We expect that too large experimental inaccuracy leave the fourth family masses in the present study quite undetermined, in particular for leptons.

6. We study quarks and leptons equivalently. The difference among family members originate on the tree level in the eigenvalues of the operators $(Q^\alpha, Q'^\alpha, Y'^\alpha)$, which in loop corrections together with other contributors in all orders con-

tribute to all mass matrix elements and cause the difference among family members [3].

Let us conclude. If the mass matrix of a family member obeys the symmetry required by the *spin-charge-family* theory, which in a simplified version (as it is taken in this study) is real and symmetric, the matrix elements of the mixing matrices of quarks and leptons are correspondingly real, each of them with $\frac{n(n-1)}{2}$ free parameters. These six parameters of each mixing matrix are, within the experimental inaccuracy, determined by the three times three experimentally determined submatrix. After taking into account three so far measured masses of each family member, the six parameters of each mass matrix reduce to three. Twice three free parameters are within the experimental accuracy correspondingly determined by the 3×3 submatrix of the mixing matrix. The fourth family masses are correspondingly determined - within the experimental accuracy.

The assumption that the two 3×3 mixing matrices are unitary would lead to the loss of the information about the 4×4 mixing matrix. This is the case also if we take the orthogonalized version of the 3×3 mixing matrices.

Since neither the measured masses nor the measured mixing matrices are determined accurately enough to reproduce the 4×4 mixing matrices, we can expect that the masses and mixing matrix elements of the fourth family will be determined only within some quite large intervals.

4.2.1 Submatrices and their extensions to unitary and orthogonal matrices

In this appendix well known properties of $n \times n$ matrices, extended from $(n-1) \times (n-1)$ submatrices are discussed. We make a short overview of the properties, needed in this paper, although all which will be presented here, is the knowledge on the level of text books.

Any $n \times n$ complex matrix has $2n^2$ free parameters. The $n + 2n(n-1)/2$ unitarity requirements reduce the number of free parameters to n^2 $(= 2n^2 - (n + 2n(n-1)/2))$.

Let us assume a $(n-1) \times (n-1)$ known submatrix of the unitary matrix. The submatrix can be extended to the unitary matrix by $(2 \times [2(n-1)+1])$ real parameters of the last column and last line. The n^2 unitarity conditions on the whole matrix reduce the number of unknowns to $(2(2n-1) - n^2)$. For $n = 4$ and higher the $(n-1) \times (n-1)$ submatrix contains all the information about the unitary $n \times n$ matrix.

The ref. [6] proposes a possible extension of an $(n-1) \times (n-1)$ unitary matrix $V_{(n-1)(n-1)}$ into $n \times n$ unitary matrices V_{nn}.

The choice of phases of the left and the right basic states which determine the unitary matrix (like this is the case with the mixing matrices of quarks and leptons) reduces the number of free parameters for $(2n-1)$. Correspondingly is the number of free parameters of such an unitary matrix equal to $n^2-(2n-1)$, which manifests

[3] There are also Majorana like terms contributing in higher order loop corrections [3] which might strongly influence in particular the neutrino mass matrix.

in $\frac{1}{2}n(n-1)$ real parameters and $\frac{1}{2}(n-1)(n-2)$ $(= n^2 - \frac{1}{2}n(n-1) - (2n-1))$ phases (which determine the number of complex parameters).

Any real $n \times n$ matrix has n^2 free parameters which the $\frac{1}{2}n(n+1)$ orthogonality conditions reduce to $\frac{1}{2}n(n-1)$. The $(n-1)\times(n-1)$ submatrix of this orthogonal matrix can be extended to this $n \times n$ orthogonal matrix with $[2(n-1)+1]$ real parameters. The $\frac{1}{2}n(n+1)$ orthogonality conditions reduce these $[2(n-1)+1]$ free parameters to $(2n-1-\frac{1}{2}n(n+1))$, which means that the $(n-1)\times(n-1)$ submatrix of an $n \times n$ orthogonal matrix determine properties of its $n \times n$ orthogonal matrix completely. Any $(n-1)\times(n-1)$ submatrix of an orthogonal matrix contains all the information about the whole matrix for any n. Making the submatrix of the orthogonal matrix orthogonal by itself one looses the information about the $n \times n$ orthogonal matrix.

4.2.2 Free parameters of mass matrices after taken into account invariants

It is useful for numerical evaluation purposes to take into account for each family member its mass matrix invariants (sect. 4.2), expressible with three within the experimental accuracy known masses, while we keep the fourth one as a free parameter. We shall make a choice of a^α instead of the fourth family mass.

We shall skip in this section the family member index α and introduce new parameters as follows

$$a, b\,, \quad f = d + e\,, \quad g = d - e\,, \quad q = \frac{a_1 + a_2}{\sqrt{2}}\,, \quad r = \frac{a_1 - a_2}{\sqrt{2}}\,. \tag{4.5}$$

After making a choice of $a\frac{I_1}{4}$, that is of the fourth family mass, four invariants of Eq. (4.2) reduce the number of free parameters to 2. The four invariants therefore relate six parameters leaving three of them, the a included as a free parameter, undetermined. There are for each pair of family members the measured mixing matrix elements, assumed in this paper to be orthogonal and correspondingly determined by six parameters, which then fixes these two times 3 parameters. The (accurately enough) measured 3×3 submatrix of the (assumed to be orthogonal) 4×4 mixing matrix namely determines these 6 parameters within the experimental accuracy.

Using the starting relation among the invariants and introducing into them new parameters (a, b, f, g, q, r) from Eq. (4.5) we obtain

$$\begin{aligned}
a &= \frac{I_1}{4}\,,\\
I_2' &= -I_2 + 6a^2 - q^2 - r^2 - 2b^2 = f^2 + g^2\,,\\
I_3' &= -\frac{1}{2b}(I_3 - 2aI_2 + 4a^2) = f^2 - g^2\,,\\
I_4' &= I_4 - aI_3 + a^2 I_2 - 3a^4\\
&= \frac{1}{4}(q^2 - r^2)^2 + (q^2 + r^2)b^2 + \frac{1}{2}(q^2 - r^2)\cdot(\pm)\cdot[\pm]\,2gf\\
&\quad + b^2(f^2 + g^2) + \frac{1}{4}(2gf)^2\,.
\end{aligned} \tag{4.6}$$

We eliminate, using the first two equations, the parameters f and g, expressing them as functions of I'_2 and I'_3, which depend, for a particular family member, on the three known masses, the parameter a and the three parameters r, q and b. We are left with the four free parameters (a, b, q, r) and the below relation among these parameters

$$\begin{aligned}&\{-\frac{1}{2}(q^4+r^4)+(-2b^2+\frac{1}{2}(-I_2+6a^2-2b^2))(q^2+r^2)\\&+(I'_4-\frac{1}{4}((-I_2+6a^2-2b^2)^2+I'^2_3)+b^2(-I_2+6a^2-2b^2))\}^2\\&=-\frac{1}{4}(q^2-r^2)^2((-I_2+6a^2-2b^2-(q^2+r^2))^2-I'^2_3)\,,\end{aligned} \tag{4.7}$$

which reduces the number of free parameters to 3. These 3 free parameters must be determined, together with the corresponding three parameters of the partner, from the measured mixing matrix.

We eliminate one of the 4 free parameters in Eq. (4.7) by solving the cubic equation for, let us make a choice, q^2

$$\alpha q^6+\beta q^4+\gamma q^2+\delta=0\,. \tag{4.8}$$

Parameter $(\alpha, \beta, \gamma, \delta)$ depend on the 3 remaining free parameters (a, b, r) and the three, within experimental accuracy, known masses.

To reduce the number of free parameters from the starting 6 in Eq. (4.1) to the 3 left after taking into account invariants of each mass matrix, we look for the solution of Eq (4.8) for all allowed values for (a, b, r). We make a choice for a in the interval of (a_{min}, a_{max}), determined by the requirement that a, which solves the equations, is a real number. Allowing only real values for parameters f and g we end up with the equation

$$-I_2+6a^2-2b^2-(q^2+r^2)>|\frac{I_3+8a^3-2aI_2}{2b}|\,, \tag{4.9}$$

which determines the maximal positive b for $q=0=r$ and also the minimal positive value for b. For each value of the parameter a the interval (b_{min}, b_{max}), as well as the interval $(r_{min}=0, r_{max})$, follow when taking into account experimental values for the three lower masses.

4.3 Numerical results

Taking into account the assumptions and the procedure explained in sect. 4.2 and in the ref. [7] we are looking for the 4×4 in this paper taken to be real and symmetric mass matrices for quarks and leptons, obeying the symmetry of Eq. (4.1) and manifesting properties – masses and mixing matrices – of the so far observed three families of quarks and leptons in agreement with the experimental limits for the appearance of the fourth family masses and mixing matrix elements to the lower three families, as presented in the refs. [16,15]. We also take into account our so far made rough estimations of possible contributions of the fourth family members to the decay of mesons. More detailed estimations are in progress.

We hope that we shall be able to learn from the mass matrices of quarks and leptons also about the properties of the scalar fields, which cause masses of quarks and leptons, manifesting effectively so far as the measured Higgs and Yukawa couplings.

First we test the predicting power of our model in dependence of the experimental inaccuracy of masses and mixing matrices on a toy model: Starting with two known mass matrices with the symmetry of Eq. (4.1) we calculate masses and from the two diagonalizing matrices also the mixing matrix. ¿From the known masses and mixing matrix, for which we allow ”experimental inaccuracy”, we check how does the reproducibility of the two starting mass matrices depend on the ”experimental inaccuracy” and how does the ” experimental inaccuracy” influence the fourth family masses.

Then we take the 3×3 measured mixing matrices for quarks and leptons and the measured masses, all with the experimental inaccuracy. Taking into account that the 3×3 submatrix of the unitary 4×4 matrix determines, if measured accurately enough, the 4×4 matrix, we look for the twice 4×4 mass matrices with the symmetry of Eq. (4.1), and correspondingly for the fourth family masses, for quarks and leptons.

When extending the two so far measured 3×3 submatrices of the 4×4 mixing matrices we try to take into account as many experimental data as possible.

4.3.1 Checking on a toy model how much does the symmetry of mass matrices (Eq. (4.1)) limit the fourth family properties

We check in this subsection on a toy model the reproducibility of the starting two mass matrices from the known two times three lower masses (say m_{u_i}, m_{d_i} , $i = (1,2,3)$) and the 3×3 submatrix (say $(V_{ud})_{i,j}$, $i,j = (1,2,3)$) of the 4×4 unitary mixing matrix in dependence of the inaccuracy allowed for m_{u_i}, m_{d_i} , $i = (1,2,3)$ and $(V_{ud})_{i,j}$, $i,j = (1,2,3)$.

We take the following two mass matrices, chosen so that they reproduce to high extent the measured properties of quarks (masses and mixing matrix) for some experimentally acceptable values for the fourth family masses and also the corresponding mixing matrix elements.

$$\mathcal{M}^{toy_u} = \begin{pmatrix} 220985. & 119365. & 120065. & 204610. \\ 119365. & 218355. & 204610. & 120065. \\ 120065. & 204610. & 192956. & 119365. \\ 204610. & 120065. & 119365. & 190325. \end{pmatrix},$$

$$\mathcal{M}^{toy_d} = \begin{pmatrix} 175825. & 174262. & 174290. & 175709. \\ 174262. & 175839. & 175709. & 174290. \\ 174290. & 175709. & 175640. & 174262. \\ 175709. & 174290. & 174262. & 175654. \end{pmatrix}. \tag{4.10}$$

Diagonalizing these two mass matrices we find the following twice four masses

$$\mathbf{M}_d^{toy_u}/MeV/c^2 = (1.3, 620., 172000., 650000.)\,,$$
$$\mathbf{M}_d^{toy_d}/MeV/c^2 = (2.9, 55., 2900., 700000.)\,, \tag{4.11}$$

and the mixing matrix

$$V_{toy_{ud}} = \begin{pmatrix} -0.97286 & -0.22946 & -0.02092 & 0.02134 \\ 0.23019 & -0.97205 & -0.04607 & -0.00287 \\ 0.00976 & 0.04965 & -0.99872 & -0.00045 \\ 0.02143 & 0.00213 & -0.00013 & 0.99977 \end{pmatrix}. \tag{4.12}$$

In order to simulate experimental inaccuracies (intervals of values for twice three lower masses and for the matrix elements of the 3×3 submatrix of the above unitary 4×4 matrix) and test the influence of these inaccuracies on the fourth family masses, we change the fourth family mass m_{u_4} in the interval $((300-1200))$ GeV and check the accuracy with which the matrix elements of the 3×3 submatrix of the 4×4 unitary matrix are reproduced. We measure the averaged inaccuracy in σ's [4]. We keep in Table 4.1 the d_4 mass equal to 700 GeV.

m_{u_4}/GeV	300	500	600	650	700	800	1200
"exp. inacc"/σ	4.0	1.0	0.29	0.0	0.25	0.66	1.6

Table 4.1. The average inaccuracy in σ of the mixing matrix elements of the 3×3 submatrix of the unitary quark mixing matrix (Eq.(4.12)) in dependence of the fourth family mass of the $m_{toy_{u_4}}$-quark. $m_{toy_{d_4}}$ mass is kept equal to 700 GeV.

Let us add that the accuracy, with which the 3×3 submatrix of the 4×4 mixing matrix is reproduced, depends much less on $m_{toy_{d_4}}$ than it does on $m_{toy_{u_4}}$ in this toy model case.

We use this experience when evaluating intervals, within which the fourth family masses appear when taking into account the inaccuracies of the experimental data.

4.3.2 Numerical results for the observed quarks and leptons with mass matrices obeying Eq. (4.1)

We take for the quark and lepton masses the experimental values [16], recalculated to the Z boson mass scale. We take from [16] also the experimentally declared inaccuracies for the so far measured 3×3 mixing matrices, taken in our calculations as submatrices of the 4×4 unitary mixing matrices and pay attention on the experimentally allowed values for the fourth family masses and other limitations presented in refs. [15]. We also have started to make our own rough estimations for limitations which follow from the meson decays to which the fourth family members participate. Our estimations are in progress.

The numerical procedure, tested in the toy model and working well in this case, must still be adapted to take experimental inaccuracies into account in a way to be able to see which values within the experimentally allowed ones are the

[4] We define σ as the difference of the reproduced mixing matrix elements and the exact matrix elements, following from the starting two mass matrices.

most trustable from the point of view of the symmetries of the 4×4 mass matrices predicted by the *spin-charge-family* theory.

Although the accurate enough mixing matrices and masses of quarks and leptons are essential for the prediction of the fourth family members masses, we still hope that even with the present accuracy of the experimental data the intervals for the fourth family masses shall not be too large, in particular not for quarks, for which the data are much more accurate than for leptons. Let us point out that from so far obtained results we are not yet able to predict the fourth family mass intervals, which would be reliable enough.

We therefore present some preliminary results. Let us point out that all the mass matrices manifest within a factor less then 2 the "democratic" view. This is, as expected, more and more the case, the higher might be the fourth family masses, and in particular is true for the leptons.

- For quarks we take [16]:
  1. The quark mixing matrix [16] $V_{ud} = S^u\, S^{d\,\dagger}$

$$|V_{ud}| = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 & |V_{u_1 d_4}| \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 & |V_{u_2 d_4}| \\ 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 & |V_{u_3 d_4}| \\ |V_{u_4 d_1}| & |V_{u_4 d_2}| & |V_{u_4 d_3}| & |V_{u_4 d_4}| \end{pmatrix}, \tag{4.13}$$

  determining for each assumed and experimentally allowed set of values for the mixing matrix elements of the 3×3 submatrix the corresponding fourth family mixing matrix elements ($|V_{u_i d_4}|$ and $|V_{u_4 d_j}|$) from the unitarity condition for the 4×4 mixing matrix.
  2. The masses of quarks are taken at the energy scale of M_Z, while we take the fourth family masses as free parameters. We allow the values from 300 GeV up to more than TeV to see the influence of the experimental inaccuracy on the fourth family masses.

$$\begin{aligned} \mathbf{M}^u_d/\mathrm{MeV}/c^2 &= (1.27 + 0.50 - 0.42, 619 \pm 84, 171\,700. \pm 3\,000., \\ &\quad m^{u_4} > 335\,000.)\,, \\ \mathbf{M}^d_d/\mathrm{MeV}/c^2 &= (2.90 + 1.24 - 1.19, 55 + 16 - 15, 2\,890. \pm 90., \\ &\quad m^{d_4} > 300\,000.)\,. \end{aligned} \tag{4.14}$$

- For leptons we take [16]:
  1. We evaluate 3×3 matrix elements from the data [16]

$$\begin{aligned} &7.05 \cdot 10^{-17} \le \Delta(\mathbf{m}_{21}/\mathrm{MeV}/c^2)^2 \le 8.34 \cdot 10^{-17}\,, \\ &2.07 \cdot 10^{-15} \le \Delta(\mathbf{m}_{(31),(32)}/\mathrm{MeV}/c^2)^2 \le 2.75 \cdot 10^{-15}\,, \\ &0.25 \le \sin^2\theta_{12} \le 0.37\,, \quad 0.36 \le \sin^2\theta_{23} \le 0.67\,, \\ &\sin^2\theta_{13} < 0.035(0.056)\,, \quad \sin^2 2\theta_{13} = 0.098 \pm 0.013\,, \end{aligned} \tag{4.15}$$

  which means that $\frac{\pi}{4} - \frac{\pi}{10} \le \theta_{23} \le \frac{\pi}{4} + \frac{\pi}{10}$, $\frac{\pi}{5.4} - \frac{\pi}{10} \le \theta_{12} \le \frac{\pi}{4} + \frac{\pi}{10}$, $\theta_{13} < \frac{\pi}{13}$.

This reflects in the lepton mixing matrix $V_{\nu e} = S^{\nu}\, S^{e\,\dagger}$

$$|V_{\nu e}| = \begin{pmatrix} 0.8224 & 0.5200 & 0.1552 & |V_{\nu_1 e_4}| \\ 0.3249 & 0.7239 & 0.6014 & |V_{\nu_2 e_4}| \\ 0.4455 & 0.4498 & 0.7704 & |V_{\nu_3 e_4}| \\ |V_{\nu_4 e_1}| & |V_{\nu_4 e_2}| & |V_{\nu_4 e_3}| & |V_{\nu_4 e_4}| \end{pmatrix}\,, \tag{4.16}$$

determining for each assumed value for any mixing matrix element within the experimentally allowed inaccuracy the corresponding fourth family mixing matrix elements ($|V_{\nu_i e_4}|$ and $|V_{\nu_4 e_j}|$) from the unitarity condition for the 4×4 mixing matrix.

2. The masses of leptons are taken from [16] while we take the fourth family masses as free parameters, checking how much does the experimental inaccuracy influence a possible prediction for the fourth family leptons masses and how does this prediction agree with experimentally allowed values [16,15] for the fourth family lepton masses.

$$\begin{aligned} &\mathbf{M}^{\nu}_{d}/\mathrm{MeV}/c^2 = (1 \cdot 10^{-9},\, 9 \cdot 10^{-9},\, 5 \cdot 10^{-8},\, m^{\nu_4} > 90\,000.)\,, \\ &\mathbf{M}^{e}_{d}/\mathrm{MeV}/c^2 = (0.486\,570\,161 \pm 0.000\,000\,042, \\ &\;102.718\,135\,9 \pm 0.000\,009\,2,\; 1746.24 \pm 0.20, m^{e_4} > 102\,000\,)\,. \end{aligned} \tag{4.17}$$

Following the procedure explained in sect. 4.2 we look for the mass matrices for the u-quarks and the d-quarks and the ν-leptons and the e-leptons by requiring that the mass matrices reproduce experimental data while manifesting symmetry of Eq. (4.1), predicted by the *spin-charge-family* theory.

We look for several properties of the obtained mass matrices: **i.** We test the influence of the experimentally declared inaccuracy of the 3×3 submatrices of the 4×4 mixing matrices and of the twice 3 measured masses on the prediction of the fourth family masses. **ii.** We look for how could different choices for the masses of the fourth family members limit the inaccuracy of particular matrix elements of the mixing matrices or the inaccuracy of the three lower masses of family members. **iii.** We test how close to a democratic mass matrix are the obtained mass matrices in dependence of the fourth family masses.

The numerical procedure, used in this contribution, is designed for quarks and leptons.

In the two next subsections 4.3.2, 4.3.2 we present some preliminary results for 4×4 mass matrices as they follow from the *spin-charge-family* theory for quarks and leptons, respectively.

Mass matrices for quarks Searching for mass matrices with the symmetries of Eq. (4.1) to determine the interval for the fourth family quark masses in dependence of the values of the mixing matrix elements within the experimental inaccuracy, we have not yet found a trustable way to extract which experimental inaccuracies of the mixing matrix elements should be taken more and which less "seriously". We also need to evaluate more accurately the experimental limitations for the fourth family masses, originating in decay properties of mesons and other experiments. Although in the toy model case the "inaccuracy" of the matrix elements leads very clearly to the right fourth family masses, this is not the case when the experimental data for the 3×3 mixing matrix elements are known within the accuracy from 0.02% to 12%. The so far obtained results can not yet make the choice among less or more trustable experimental values: We can not yet make more accurate choice for those data which have large experimental inaccuracies.

We are still trying to improve our the procedure of searching for the masses of the fourth family quarks.

Let us still present two cases to demonstrate how do quark mass matrices change with respect to the fourth family masses: The first two mass matrices lead to the fourth family masses $m_{u_4} = 300$ GeV and $m_{d_4} = 700$ GeV, while the second two lead to the fourth family masses $m_{u_4} = 1\,200$ GeV and $m_{d_4} = 700$ GeV.

-

$$M^{u} = \begin{pmatrix} 402673. & 256848. & 267632. & 329419. \\ 256848. & 402393. & 329419. & 267632. \\ 267632. & 329419. & 283918. & 256848. \\ 329419. & 267632. & 256848. & 283638. \end{pmatrix},$$

$$M^{d} = \begin{pmatrix} 176784. & 174262. & 174524. & 175473. \\ 174262. & 176816. & 175473. & 174524. \\ 174524. & 175473. & 174663. & 174262. \\ 175473. & 174524. & 174262. & 174695. \end{pmatrix}, \tag{4.18}$$

$$V_{ud} = \begin{pmatrix} 0.97365 & 0.22296 & 0.00225 & -0.04782 \\ 0.22276 & -0.97412 & 0.03818 & -0.00444 \\ 0.01071 & -0.03671 & -0.99927 & -0.0001 \\ 0.04761 & 0.00634 & 0.00018 & 0.99885 \end{pmatrix}. \tag{4.19}$$

The corresponding masses are

$$\mathbf{M}^{u}_{d}/\mathrm{MeV}/c^2 = (1.29957, 620.002, 172\,000., 300\,000.)\,,$$
$$\mathbf{M}^{d}_{d}/\mathrm{MeV}/c^2 = (2.88508, 55.024, 2\,899.99, 700\,000.)\,. \tag{4.20}$$

-

$$M^{u} = \begin{pmatrix} 351427. & 256907. & 257179. & 342730. \\ 256907. & 342353. & 342730. & 257179. \\ 257179. & 342730. & 343958. & 256907. \\ 342730. & 257179. & 256907. & 334884. \end{pmatrix},$$

$$M^{d} = \begin{pmatrix} 175762. & 174263. & 174289. & 175708. \\ 174263. & 175581. & 175708. & 174289. \\ 174289. & 175708. & 175898. & 174263. \\ 175708. & 174289. & 174263. & 175717. \end{pmatrix}, \tag{4.21}$$

$$V_{ud} = \begin{pmatrix} -0.9743 & 0.22521 & -0.00366 & 0.00383 \\ 0.22515 & 0.97325 & -0.04567 & 0.00299 \\ -0.00672 & -0.04532 & -0.99895 & -0.00019 \\ 0.00305 & -0.00378 & -0.00004 & 0.99999 \end{pmatrix}. \tag{4.22}$$

The corresponding masses are

$$\begin{aligned}\mathbf{M}^u_d/\text{MeV}/c^2 &= (1.29957, 620.002, 172\,000., 1\,200\,000.)\,,\\ \mathbf{M}^d_d/\text{MeV}/c^2 &= (2.88508, 55.024, 2\,899.99, 700\,000.)\,.\end{aligned} \tag{4.23}$$

We notice:
i. In both cases the required symmetry, Eq. (4.1), is (on purpose) kept very accurate.
ii. In both cases the mass matrices of quarks look quite close to the "democratic" matrix, in the second case slightly more than in the first case.
iii. The mixing matrix elements are in the second case much closer (within the experimental values are V_{11}, V_{12}, V_{13} and V_{32}, almost within the experimental values are V_{21}, V_{22} and V_{33}) to the experimentally allowed values, than in the first case (almost within the experimentally allowed values are only V_{21}, V_{22} and V_{23}).

These results suggest that the fourth family masses $m_{u_4} = 1\,200$ GeV and $m_{d_4} = 700$ GeV are much more trustable than $m_{u_4} = 300$ GeV and $m_{d_4} = 700$ GeV.

Mass matrices for leptons We present here results for leptons, manifesting properties of the lepton mass matrices. These results are less informative than those for quarks, since the experimental results are for leptons mixing matrix much less accurate than in the case of quarks and also masses are known less accurately.

We have

$$\begin{aligned}M^\nu &= \begin{pmatrix} 14\,021. & 14\,968. & 14\,968. & -14\,021.\\ 14\,968. & 15\,979. & 15\,979. & -14\,968.\\ 14\,968. & 15\,979. & 15\,979 & -14\,968.\\ -14\,021. & -14\,968. & -14\,968. & 14\,021.\end{pmatrix},\\ M^e &= \begin{pmatrix} 28\,933. & 30\,057. & 29\,762. & -27\,207.\\ 30\,057. & 32\,009. & 31\,958. & -29\,762.\\ 29\,762. & 31\,958. & 32\,009. & -30\,057.\\ -27\,207. & -29\,762. & -30\,057. & 28\,933.\end{pmatrix},\end{aligned} \tag{4.24}$$

which leads to the mixing matrix $V_{\nu e}$

$$V_{\nu e 1} = \begin{pmatrix} 0.82363 & 0.54671 & -0.15082 & 0.\\ -0.50263 & 0.58049 & -0.64062 & 0.\\ -0.26268 & 0.60344 & 0.75290 & 0.\\ 0. & 0. & 0. & 0.\end{pmatrix}, \tag{4.25}$$

and the masses

$$\begin{aligned}\mathbf{M}^\nu_d/\text{MeV}/c^2 &= (5\cdot 10^{-9}\,, 1\cdot 10^{-8}\,, 4.9\cdot 10^{-8}\,, 60\,000.)\,,\\ \mathbf{M}^e_d/\text{MeV}/c^2 &= (0.510999\,, 105.658\,, 1\,776.82\,120\,000)\,.\end{aligned} \tag{4.26}$$

We did not adapt lepton masses to Z_m mass scale. Zeros (0.) for the matrix elements concerning the fourth family members means that the values are less than 10^{-5}.

We notice:
i. The required symmetry, Eq. (4.1), is kept very accurate.

ii. The mass matrices of leptons are very close to the "democratic" matrix.
iii. The mixing matrix elements among the first three and the fourth family members are very small, what is due to our choice, since the matrix elements of the 3×3 submatrix of the 4×4 unitary matrix, predicted by the *spin-charge-family* theory are very inaccurately known.

4.4 Discussions and conclusions

One of the most interesting open questions in the elementary particle physics is: Where do the family originate? Explaining the origin of families would answer the question about the number of families possibly observable at the low energy regime, about the origin of the scalar field(s) and Yukawa couplings and would also explain differences in the fermions properties - the differences in masses and mixing matrices among family members – quarks and leptons.

Assuming that the prediction of the *spin-charge-family* theory that there are four rather than so far observed three coupled families, the mass matrices of which demonstrate in the massless basis the $SU(2) \times SU(2)$ symmetry of Eq. (4.1), the same for all the family members - the quarks and the leptons - we look in this paper for:
i. The origin of differences in the properties of the family members - quarks and leptons.
ii. The allowed intervals for the fourth family masses.
iii. The matrix elements in the mixing matrices among the fourth family members and the three already measured ones.

Our calculations presented here are preliminary and in progress.

Let us tell that there are two kinds of the scalar fields in the *spin-charge-family* theory, responsible for the masses and mixing matrices of quarks and leptons (and consequently also for the masses of the weak gauge fields): The ones which distinguish among the family members and the other ones which distinguish among the families. The differences between quarks and leptons and between u and d quarks and between ν and e leptons originate in the first kind of the scalar fields, which carry Q, Q' (the two charges which, like in the *standard model*, originate in the weak and hyper charge) and Y' (which originates in the hypercharge and in the fermion quantum number, similarly as in the $SO(10)$ models).

The existence of four coupled families seems almost unavoidable for the explanation of the properties of the neutrino families if all the family members should start from the massless basis in an equivalent way: The 4×4 mass matrix, very close to a democratic one, offers three almost massless (in comparison with the observed quarks and charged leptons masses) families and a very massive one.

Taking the symmetry of, to simplify the calculations assumed to be real and symmetric, 4×4 mass matrices, we determine 6 free parameters of any of the mass matrices by requiring that the mass matrices lead to the observed properties of quarks and leptons. In both cases the 2 times three masses and the (in this simplified study) orthogonal mixing matrix with 6 parameters, determine the 2×6 parameters (as required by the *spin-charge-family* theory) of the two mass matrices within the experimental accuracy.

The same procedure is used to study either quarks or leptons. Expected results are not only the mass matrices, but also the intervals within which masses of the fourth families should be observed and the corresponding mixing matrices.

We developed a special procedure to extract the dependence of the fourth family masses on the experimental inaccuracy of masses and mixing matrices. Our test of this procedure on a toy model, in which we first postulate two mass matrices (leading to masses and mixing matrices very close to those of quarks), calculate the masses and the mixing matrix, and then from three lowest masses and the 3×3 sub matrix of the unitary 4×4 mixing matrix calculate back the starting mass matrices and the fourth family masses, showed that the procedure leads very accurately to the starting mass matrices.

When we use the same procedure to extract the properties of the fourth family members from the experimental data within the experimental inaccuracies, the procedure was not selective enough to make useful predictions. We are improving the procedure to be able to extract the intervals of the fourth family masses in dependence of the accuracy of particular data. *Yet the here presented preliminary results show, that the masses of the fourth families quarks with* $m_{u_4} > 1$TeV *lead to the mixing matrix much closer to the experimental data than does* $m_{u_4} \approx 300$GeV.

Let us conclude this report by pointing out that even if we shall not be able to limit the mass intervals for the fourth family members strongly enough to be predictive, yet the accurate enough data for the 3×3 submatrix of the unitary mass matrix will sooner or later determine the 4×4 unitary matrix so that the predictions will be accurate enough.

4.5 APPENDIX: A brief presentation of the *spin-charge-family* theory

We present in this section a very brief introduction into the *spin-charge family* theory [1–4]. The reader can skip this appendix taking by the *spin-charge family* theory required symmetry of mass matrices of Eq. (4.1) as an input to the study of properties of the 4×4 mass matrices – with the parameters which depend on charges of the family members – and can come to this part of the paper, if and when would like to learn where do families and scalar fields possibly originate from.

Let us start by directing attention of the reader to one of the most open questions in the elementary particle physics and cosmology: Why do we have families, where do they originate and correspondingly where do scalar fields, manifesting as Higgs and Yukawa couplings, originate? The *spin-charge-family* theory is offering a possible explanation for the origin of families and scalar fields, and in addition for the so far observed charges and the corresponding gauge fields.

There are, namely, two (only two) kinds of the Clifford algebra objects: One kind, the Dirac γ^a, takes care of the spin in $d = (3+1)$, while the spin in $d \geq 4$ (rather than the total angular momentum) manifests in $d = (3+1)$ in the low energy regime as the charges. In this part the *spin-charge family* theory is like the Kaluza-Klein theory, unifying spin (in the low energy regime, otherwise the total angular momentum) and charges, and offering a possible answer to the question about the origin of the so far observed charges and correspondingly also about the so far observed gauge fields. The second kind of the Clifford algebra objects, forming the equivalent representations with respect to the Dirac kind, recognized by one of the authors (SNMB), is responsible for the appearance of families of fermions.

There are correspondingly also two kinds of gauge fields, which appear to manifest in $d = (3+1)$ as the so far observed vector gauge fields (the number of - obviously non yet observed - gauge fields grows with the dimension) and as the scalar gauge fields. The scalar fields are responsible, after gaining nonzero vacuum expectation values, for the appearance of masses of fermions and gauge bosons. They manifest as the so far observed Higgs [5] and the Yukawa couplings.

All the properties of fermions and bosons in the low energy regime originate in the *spin-charge-family* theory in a simple starting action for massless fields in $d = [1 + (d-1)]$. Fermions interact with the vielbeins $f^{\alpha}{}_{a}$ and correspondingly with the two kinds of the spin connection fields: with $\omega_{abc} = f^{\alpha}{}_{c}\, \omega_{ab\alpha}$ which are the gauge fields of $S^{ab} = \frac{i}{4}\,(\gamma^a\gamma^b - \gamma^b\gamma^a)$ and with $\tilde{\omega}_{abc} = f^{\alpha}{}_{c}\, \tilde{\omega}_{ab\alpha}$ which are the gauge fields of $\tilde{S}^{ab} = \frac{i}{4}\,(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$. $\alpha, \beta, \ldots$ is the Einstein index and $a, b, \ldots$ is the flat index. The starting action is the simplest one

$$
\begin{aligned}
S &= \int d^dx\, E\, \mathcal{L}_f + \int d^dx\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \\
\mathcal{L}_f &= \frac{1}{2}(\bar{\psi}\,\gamma^a p_{0a}\psi) + \text{h.c.} \\
p_{0a} &= f^{\alpha}{}_{a}\, p_{0\alpha} + \frac{1}{2E}\,\{p_{\alpha}, E f^{\alpha}{}_{a}\}_{-}\,, \\
p_{0\alpha} &= p_{\alpha} - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}\,, \\
R &= \frac{1}{2}\{f^{\alpha[a}f^{\beta b]}\;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^{c}{}_{b\beta})\} + \text{h.c.}\,, \\
\tilde{R} &= \frac{1}{2}\,f^{\alpha[a}f^{\beta b]}\;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\tilde{\omega}^{c}{}_{b\beta}) + \text{h.c.}\,.
\end{aligned}
\tag{4.27}
$$

Fermions, coupled to the vielbeins and the two kinds of the spin connection fields, *manifest* (after several breaks of the starting symmetries) *before the electroweak break four massless families of quarks and leptons*, the left handed fermions are weak charged and the right handed ones are weak chargeless. The vielbeins and the two kinds of the spin connection fields manifest effectively as the observed gauge fields and (those with the scalar indices in $d = (1+3)$) as several scalar fields. The mass matrices of the four family members (quarks and leptons) are after the electroweak break expressible on a tree level by the vacuum expectation values of the two kinds of the spin connection fields and the corresponding vielbeins with the scalar indices ([4,13]):

i. One kind originates in the scalar fields $\tilde{\omega}_{abc}$, manifesting as the two $SU(2)$ triplets – $\tilde{A}^{\tilde{N}_L\, i}_{s}, i = (1,2,3)\,, s = (7,8)$; $\tilde{A}^{\tilde{1}\, i}_{s}\,, i = (1,2,3)\,, s = (7,8)$; – and one singlet – $\tilde{A}^{\tilde{4}}_{s}\,, s = (7,8)$ – contributing equally to all the family members.

ii. The second kind originates in the scalar fields ω_{abc}, manifesting as three singlets – $A^{Q}_{s}, A^{Q'}_{s}, A^{Y'}$, $s = (7,8)$ – contributing the same values to all the families and distinguishing among family members. Q and Q' are the quantum numbers from the *standard model*, Y' originates in the second $SU(2)$ (a kind of a right handed "weak") charge.

All the scalar fields manifest, transforming the right handed quarks and leptons into the corresponding left handed ones [5] and contributing also to the masses of the weak bosons, as doublets with respect to the weak charge. Loop corrections, to which all the scalar and also gauge vector fields contribute coherently, change contributions of the off-diagonal and diagonal elements on the tree level, keeping the tree level symmetry of mass matrices unchanged [6].

4.5.1 Mass matrices on the tree level and beyond which manifest $SU(2) \times SU(2)$ symmetry

Let us make a choice of a massless basis ψ_i, $i = (1,2,3,4)$, for a particular family memeber α. And let us take into account the two kinds of the operators, which transform the basis vectors into one another

$$\tilde{N}^{i}_{L}\,,\; i = (1,2,3)\,, \qquad \tilde{\tau}^{i}_{L}\,,\; i = (1,2,3)\,, \tag{4.28}$$

with the properties

$$\begin{aligned}
\tilde{N}^{3}_{L}\,(\psi_1,\psi_2,\psi_3,\psi_4) &= \frac{1}{2}(-\psi_1,\psi_2,-\psi_3,\psi_4)\,,\\
\tilde{N}^{+}_{L}\,(\psi_1,\psi_2,\psi_3,\psi_4) &= (\psi_2,\ 0,\psi_4,\ 0)\,,\\
\tilde{N}^{-}_{L}\,(\psi_1,\psi_2,\psi_3,\psi_4) &= (0\ ,\psi_1,\ 0,\psi_3)\,,\\
\tilde{\tau}^{3}\,(\psi_1,\psi_2,\psi_3,\psi_4) &= \frac{1}{2}(-\psi_1,-\psi_2,\psi_3,\psi_4)\,,\\
\tilde{\tau}^{+}\,(\psi_1,\psi_2,\psi_3,\psi_4) &= (\psi_3,\psi_4,\ 0,\ 0)\,,\\
\tilde{\tau}^{-}\,(\psi_1,\psi_2,\psi_3,\psi_4) &= (\ 0,\ 0,\psi_1,\psi_2)\,.
\end{aligned} \tag{4.29}$$

This is indeed what the two $SU(2)$ operators in the *spin-charge-family* theory do. The gauge scalar fields of these operators determine, together with the corresponding coupling constants, the off diagonal and diagonal matrix elements on the tree level. In addition to these two kinds of $SU(2)$ scalars there are three $U(1)$ scalars, which distinguish among the family members, contributing on the tree level the same diagonal matrix elements for all the families. In loop corrections in all orders the symmetry of mass matrices remains unchanged, while the three $U(1)$ scalars,

[5] It is the term $\gamma^0\gamma^s\,\phi^{Ai}_{s}$, where ϕ^{Ai}_{s}, with $s = (7,8)$ denotes any of the scalar fields, which transforms the right handed fermions into the corresponding left handed partner [3,4,13]. This mass term originates in $\bar{\psi}\,\gamma^a p_{0a}\psi$ of the action Eq.(4.27), with $a = s = (7,8)$ and $p_{0s} = f^{\sigma}_{s}\,(p_\sigma - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\sigma} - \frac{1}{2}S^{st}\omega_{st\sigma})$.

[6] It can be seen that all the loop corrections keep the starting symmetry of the mass matrices unchanged. We have also started [3,14] with the evaluation of the loop corrections to the tree level values. This estimation has been done so far [14] only up to the first order and partly to the second order.

contributing coherently with the two kinds of $SU(2)$ scalars and all the massive fields to all the matrix elements, manifest in off diagonal elements as well. All the scalars are doublets with respect to the weak charge, contributing to the weak and the hypercharge of the fermions so that they transform the right handed members into the left handed onces.

With the above (Eq. (4.29) presented choices of phases of the left and the right handed basic states in the massless basis the mass matrices of all the family members manifest the symmetry, presented in Eq. (4.1). One easily checks that a change of the phases of the left and the right handed members, there are $(2n-1)$ possibilities, causes changes in phases of matrix elements in Eq. (4.1).

4.6 APPENDIX: Properties of non Hermitian mass matrices

This pedagogic presentation of well known properties of non Hermitian matrices can be found in many textbooks, for example [18]. We repeat this topic here only to make our discussions transparent.

Let us take a non Hermitian mass matrix M^{α} as it follows from the *spin-charge-family* theory, α denotes a family member (index $\pm$ used in the main text is dropped).

We always can diagonalize a non Hermitian M^{α} with two unitary matrices, S^{α} ($S^{\alpha\,\dagger}\, S^{\alpha} = I$) and T^{α} ($T^{\alpha\,\dagger}\, T^{\alpha} = I$)

$$S^{\alpha\,\dagger}\, M^{\alpha}\, T^{\alpha} = \mathbf{M}^{\alpha}_{d} = (m^{\alpha}_{1} \dots m^{\alpha}_{i} \dots m^{\alpha}_{n}). \tag{4.30}$$

The proof is added below.

Changing phases of the basic states, those of the left handed one and those of the right handed one, the new unitary matrices $S'^{\alpha} = S^{\alpha}\, F_{\alpha S}$ and $T'^{\alpha} = T^{\alpha}\, F_{\alpha T}$ change the phase of the elements of diagonalized mass matrices $\mathbf{M}^{\alpha}_{d}$

$$\begin{aligned}
S'^{\alpha\,\dagger}\, M^{\alpha}\, T'^{\alpha} &= F^{\dagger}_{\alpha S}\, \mathbf{M}^{\alpha}_{d}\, F_{\alpha T} = \\
&\quad diag(m^{\alpha}_{1} e^{i(\phi^{\alpha S}_{1} - \phi^{\alpha T}_{1})} \dots m^{\alpha}_{i}\, e^{i(\phi^{\alpha S}_{i} - \phi^{\alpha T}_{i})}, \dots m^{\alpha}_{n}\, e^{i(\phi^{\alpha S}_{n} - \phi^{\alpha T}_{n})})\,, \\
F_{\alpha S} &= diag(e^{-i\phi^{\alpha S}_{1}}, \dots, e^{-i\phi^{\alpha S}_{i}}, \dots, e^{-i\phi^{\alpha S}_{n}})\,, \\
F_{\alpha T} &= diag(e^{-i\phi^{\alpha T}_{1}}, \dots, e^{-i\phi^{\alpha T}_{i}}, \dots, e^{-i\phi^{\alpha T}_{n}})\,.
\end{aligned} \tag{4.31}$$

In the case that the mass matrix is Hermitian T^{α} can be replaced by S^{α}, but only up to phases originating in the phases of the two basis, the left handed one and the right handed one, since they remain independent.

One can diagonalize the non Hermitian mass matrices in two ways, that is either one diagonalizes $M^{\alpha} M^{\alpha\,\dagger}$ or $M^{\alpha\dagger} M^{\alpha}$

$$\begin{aligned}
(S^{\alpha\dagger} M^{\alpha} T^{\alpha})(S^{\alpha\dagger} M^{\alpha} T^{\alpha})^{\dagger} &= S^{\alpha\dagger} M^{\alpha} M^{\alpha\,\dagger} S^{\alpha} = \mathbf{M}^{\alpha 2}_{dS}\,, \\
(S^{\alpha\dagger} M^{\alpha} T^{\alpha})^{\dagger}(S^{\alpha\dagger} M^{\alpha} T^{\alpha}) &= T^{\alpha\dagger} M^{\alpha\,\dagger} M^{\alpha} T^{\alpha} = \mathbf{M}^{\alpha 2}_{dT}\,, \\
\mathbf{M}^{\alpha\,\dagger}_{dS} = \mathbf{M}^{\alpha}_{dS}\,, &\quad \mathbf{M}^{\alpha\,\dagger}_{dT} = \mathbf{M}^{\alpha}_{dT}\,.
\end{aligned} \tag{4.32}$$

One can prove that $\mathbf{M}^{\alpha}_{dS} = \mathbf{M}^{\alpha}_{dT}$. The proof proceeds as follows. Let us define two Hermitian (H^{α}_{S} , H^{α}_{T}) and two unitary matrices (U^{α}_{S} , H^{α}_{T})

$$\begin{aligned} H^{\alpha}_{S} &= S^{\alpha}\mathbf{M}^{\alpha}_{dS}S^{\alpha\,\dagger}\,, \qquad H^{\alpha}_{T} = T^{\alpha}\mathbf{M}^{\alpha\dagger}_{dT}T^{\alpha\,\dagger}\,,\\ U^{\alpha}_{S} &= H^{\alpha-1}_{S}M^{\alpha}\,, \qquad U^{\alpha}_{T} = H^{\alpha-1}_{T}M^{\alpha\,\dagger}\,, \end{aligned} \tag{4.33}$$

It is easy to show that $H^{\alpha\,\dagger}_{S} = H^{\alpha}_{S}$, $H^{\alpha\,\dagger}_{T} = H^{\alpha}_{T}$, $U^{\alpha}_{S}\,U^{\alpha\,\dagger}_{S} = I$ and $U^{\alpha}_{T}\,U^{\alpha\,\dagger}_{T} = I$. Then it follows

$$\begin{aligned} S^{\alpha\dagger}\,H^{\alpha}_{S}\,S^{\alpha} &= \mathbf{M}^{\alpha}_{dS} = \mathbf{M}^{\alpha\,\dagger}_{dS} = S^{\alpha\dagger}\,M^{\alpha}\,U^{\alpha\,-1}_{S}\,S^{\alpha} = S^{\alpha\dagger}\,M^{\alpha}\,T^{\alpha}\,,\\ T^{\alpha\dagger}\,H^{\alpha}_{T}\,T^{\alpha} &= \mathbf{M}^{\alpha}_{dT} = \mathbf{M}^{\alpha\,\dagger}_{dT} = T^{\alpha\dagger}\,M^{\alpha\,\dagger}\,U^{\alpha\,-1}_{T}\,T^{\alpha} = T^{\alpha\dagger}\,M^{\alpha\dagger}\,S^{\alpha}\,, \end{aligned} \tag{4.34}$$

where we recognized $U^{\alpha\,-1}_{S}\,S^{\alpha} = T^{\alpha}$ and $U^{\alpha\,-1}_{T}\,T^{\alpha} = S^{\alpha}$. Taking into account Eq. (4.31) the starting basis can be chosen so, that all diagonal masses are real and positive.

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5 On Emergent SUSY Gauge Theories

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Abstract. We present the basic features of emergent SUSY gauge theories where an emergence of gauge bosons as massless vector Nambu-Goldstone modes is triggered by the spontaneously broken supersymmetry rather than the physically manifested Lorentz violation. We start considering the supersymmetric QED model extended by an arbitrary polynomial potential of massive vector superfield that induces the spontaneous SUSY violation in the visible sector. As a consequence, a massless photon appears as a companion of a massless photino emerging as a goldstino in the tree approximation, and remains massless due to the simultaneously generated special gauge invariance. This invariance is only restricted by the supplemented vector field constraint invariant under supergauge transformations. Meanwhile, photino being mixed with another goldstino appearing from a spontaneous SUSY violation in the hidden sector largely turns into the light pseudo-goldstino. Such pseudo-goldstonic photinos considered in an extended supersymmetric Standard Model framework are of a special observational interest that, apart from some indication of the QED emergence nature, may appreciably extend the scope of SUSY breaking physics being actively studied in recent years.

Povzetek. Predstavim osnovne lastnosti umeritvenih teorij, "emergent supersymmetry", pri katerih postanejo brezmasni vektorski Nambu-Goldstonovi bozoni umeritvena polja, sproži pa njihov nastanek spontano zlomljena supersimetrija in ne kršitev Lorentzove invariance. Najprej predstavim supersimetrični model kvantne elektrodinamike, ki ga posplošim s tem, da dopustim za masivni vektorski superpotencial polinom poljubne stopnje. To polje sproži spontani zlom supersimetrije v opazljivem sektorju. Pojavita se brezmasni foton in njegov spremljevalec, prav tako brezmasni fotino. Fotino, na drevesnem nivoju je to brezmasni goldstino, ostane brezmasen tudi po kvantnih popravkih zaradi posebne spontano nastale umeritvene invariance. To invarianco omejuje samo dopolnjen pogoj na vektorsko polje, ki pa je invarianten na superumeritvene transformacije. Fotino postane lahki psevdo-goldstino, ko tvori superpozicijo s še enim goldstinom, ki se pojavi ob spontani zlomitvi supersimetrije v skritem sektorju. Ti psevdo-goldstonski fotini iz razširjenih supersimetričnih modelov Standardnega modela, so posebej zanimivi za meritve. Te lahko, poleg potrditve, da se kvantna elektrodinamika v teh teorijah pojavi spontano, podprejo supersimetrične teorije, ki so v zadnjih letih zelo popularne.

5.1 Introduction

It is long believed that spontaneous Lorentz invariance violation (SLIV) may lead to an emergence of massless Nambu-Goldstone modes [1] which are identified

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with photons and other gauge fields appearing in the Standard Model. This idea [2] supported by a close analogy with the dynamical origin of massless particle excitations for spontaneously broken internal symmetries has gained new impetus [3–7] in recent years.

In this connection, one important thing to notice is that, in contrast to the spontaneous violation of internal symmetries, SLIV seems not to necessarily imply a physical breakdown of Lorentz invariance. Rather, when appearing in a gauge theory framework, this may ultimately result in a noncovariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory. In substance, the SLIV ansatz, due to which the vector field develops a vacuum expectation value (vev)

$$< A_\mu(x) > \; = n_\mu M \tag{5.1}$$

(where n_μ is a properly-oriented unit Lorentz vector, $n^2 = n_\mu n^\mu = \pm 1$, while M is the proposed SLIV scale), may itself be treated as a pure gauge transformation with a gauge function linear in coordinates, $\omega(x) = n_\mu x^\mu M$. From this viewpoint gauge invariance in QED leads to the conversion of SLIV into gauge degrees of freedom of the massless Goldstonic photon emerged.

A good example for such a kind of the "inactive" SLIV is provided by the nonlinearly realized Lorentz symmetry for underlying vector field $A_\mu(x)$ through the length-fixing constraint

$$A_\mu A^\mu = n^2 M^2 \; . \tag{5.2}$$

This constraint in the gauge invariant QED framework was first studied by Nambu a long ago [8], and in more detail in recent years [9–13]. The constraint (5.2) is in fact very similar to the constraint appearing in the nonlinear σ-model for pions [14], $\sigma^2 + \pi^2 = f_\pi^2$, where f_π is the pion decay constant. Rather than impose by postulate, the constraint (5.2) may be implemented into the standard QED Lagrangian L_{QED} through the invariant Lagrange multiplier term

$$L_{tot} = L_{QED} - \frac{\lambda}{2}\left(A_\mu A^\mu - n^2 M^2\right) \tag{5.3}$$

provided that initial values for all fields (and their momenta) involved are chosen so as to restrict the phase space to values with a vanishing multiplier function $\lambda(x)$, $\lambda = 0$ [1].

One way or another, the constraint (5.2) means in essence that the vector field A_μ develops the vev (5.1) and Lorentz symmetry $SO(1,3)$ breaks down to $SO(3)$ or $SO(1,2)$ depending on whether the unit vector n_μ is time-like ($n^2 > 0$) or space-like ($n^2 < 0$). The point, however, is that, in sharp contrast to the nonlinear σ model for pions, the nonlinear QED theory, due to gauge invariance in the starting Lagrangian L_{QED}, ensures that all the physical Lorentz violating effects turn out to be non-observable. Actually, as was shown in the tree [8] and one-loop approximations [9], the nonlinear constraint (5.2) implemented as a supplementary condition appears in essence as a possible gauge choice for the vector field A_μ, while the S-matrix remains unaltered under such a gauge convention. So, as

[1] Otherwise, as was shown in [15] (see also [12]), it might be problematic to have the ghost-free QED model with a positive Hamiltonian.

generally expected, the inactive SLIV inspired by the length-fixing constraint (5.2), while producing an ordinary photon as a true Goldstonic vector boson (a_μ)

$$A_\mu = a_\mu + n_\mu (M^2 - n^2 a^2)^{\frac{1}{2}} \ , \ n_\mu a_\mu = 0 \ \ (a^2 \equiv a_\mu a^\mu) \ , \tag{5.4}$$

leaves physical Lorentz invariance intact[2]. Later similar result was also confirmed for spontaneously broken massive QED [10], non-Abelian theories [11] and tensor field gravity [13].

From this point of view, emergent gauge theories induced by the inactive SLIV mechanism are in fact indistinguishable from conventional gauge theories. Their Goldstonic nature could only be seen when taking the gauge condition of the length-fixing constraint type (5.2). Any other gauge, e.g. Coulomb gauge, is not in line with Goldstonic picture, since it breaks Lorentz invariance in an explicit rather than spontaneous way. As to an observational evidence in favor of emergent theories the only way for inactive SLIV to cause physical Lorentz violation would be if gauge invariance in these theories appeared slightly broken in an explicit, rather than spontaneous, way. Actually, such a gauge symmetry breaking, induced by some high-order operators, leads in the presence of SLIV to deformed dispersion relations for matter and gauge fields involved. This effect typically appears proportional to powers of the ratio M/M_P, so that for some high value of the SLIV scale M it may become physically observable even at low energies. Though one could speculate about some generically broken or partial gauge symmetry [16], this seems to be too high price for an actual Lorentz violation which may stem from SLIV[3]. And, what is more, is there really any strong theoretical reason left for the Lorentz invariance to be physically broken, if the Goldstonic gauge fields are anyway generated through the "safe" inactive SLIV models which recover conventional Lorentz invariance?

Nevertheless, it may turn out that SLIV is not the only reason why massless photons could dynamically appear, if spacetime symmetry is further enlarged. In this connection, special interest may be related to supersymmetry. Actually, as we try to show below, the situation is changed remarkably in the SUSY inspired emergent models which, in contrast to non-SUSY analogues, could naturally have some clear observational evidence. We argue that a generic source for massless photons may be spontaneously broken supersymmetry rather than physically

[2] Indeed, the nonlinear QED contains a plethora of Lorentz and CPT violating couplings when it is expressed in terms of the pure Goldstonic photon modes a_μ. However, the contributions of all these couplings to physical processes completely cancel out among themselves.

[3] In this connection, the simplest possibility could be a conventional QED Lagrangian extended by the vector field potential energy terms, $L = L_{QED} - \frac{\lambda}{4}\left(A_\mu A^\mu - n^2 M^2\right)^2$, where λ is a coupling constant. This Lagrangian being sometimes referred to as the "bumblebee" model (see [7] and references therein) is in a sense a linear version of the nonlinear QED appearing in the limit $\lambda \to \infty$. Actually, both of models are physically equivalent in the infrared energy domain, where the Higgs mode is considered infinitely massive. However, as we see shortly, whereas the nonlinear QED model successfully matches supersymmetry, the "bumblebee" model cannot be conceptually realized in the SUSY context.

manifested spontaneous Lorentz violation [17]. Towards this end, we consider supersymmetric QED model extended by an arbitrary polynomial potential of massive vector superfield that induces the spontaneous SUSY violation[4]. As a consequence, a massless photon emerges as a companion of a massless photino being Goldstone fermion in the broken SUSY phase in the visible sector (section 2). Remarkably, this masslessness appearing at the tree level is further protected against radiative corrections by the simultaneously generated special gauge invariance. This invariance is only restricted by the supplemented vector field constraint invariant under supergauge transformations (section 3). Meanwhile, photino being mixed with another goldstino appearing from a spontaneous SUSY violation in the hidden sector largely turns into the light pseudo-goldstino whose physics seems to be of special interest (section 4). And finally, we conclude (section 5).

5.2 Extended supersymmetric QED

We now consider the supersymmetric QED extended by an arbitrary polynomial potential of a general vector superfield $V(x,\theta,\overline{\theta})$ which in the standard parametrization [18] has a form

$$V(x,\theta,\overline{\theta}) = C(x) + i\theta\chi - i\overline{\theta}\overline{\chi} + \frac{i}{2}\theta\theta S - \frac{i}{2}\overline{\theta\theta}S^{*} \\ -\theta\sigma^{\mu}\overline{\theta}A_{\mu} + i\theta\theta\overline{\theta}\overline{\lambda'} - i\overline{\theta\theta}\theta\lambda' + \frac{1}{2}\theta\theta\overline{\theta\theta}D', \tag{5.5}$$

where its vector field component A_{μ} is usually associated with a photon. Note that, apart from the conventional photino field λ and the auxiliary D field , the superfield (5.5) contains in general the additional degrees of freedom in terms of the dynamical C and χ fields and nondynamical complex scalar field S (we have used the brief notations, $\lambda' = \lambda + \frac{i}{2}\sigma^{\mu}\partial_{\mu}\overline{\chi}$ and $D' = D + \frac{1}{2}\partial^{2}C$ with $\sigma^{\mu} = (1, \overrightarrow{\sigma})$ and $\overline{\sigma}^{\mu} = (1, -\overrightarrow{\sigma})$). The corresponding SUSY invariant Lagrangian may be written as

$$\mathcal{L} = L_{SQED} + \sum_{n=1} b_{n}V^{n}|_{D} \tag{5.6}$$

where terms in this sum (b_n are some constants) for the vector superfield (5.5) are given through the $V^{n}|_{D}$ expansions into the component fields . It can readily be checked that the first term in this expansion appears to be the known Fayet-Iliopoulos D-term, while other terms only contain bilinear, trilinear and quadrilinear combination of the superfield components A_{μ}, S, λ and χ, respectively[5]. Actually, there appear higher-degree terms for the scalar field component

[4] It is worth noting that all the basic arguments related to the present QED example can be then straightforwardly extended to the Standard Model.

[5] Note that all terms in the sum in (5.6) except Fayet-Iliopoulos D-term explicitly break gauge invariance which is then recovered for Goldstonic gauge modes. Without loss of generality, we may restrict ourselves to the third degree superfield polynomial in the Lagrangian $\mathcal{L}$ (5.6) to eventually have a theory with dimesionless coupling constants for component fields. However, for completeness sake, it seems better to proceed with a general case.

$C(x)$ only. Expressing them all in terms of the C field polynomial

$$P(C) = \sum_{n=1} \frac{n}{2} b_n C^{n-1}(x) \tag{5.7}$$

and its first three derivatives

$$P'_C \equiv \frac{\partial P}{\partial C}\,, \quad P''_C \equiv \frac{\partial^2 P}{\partial C^2}\,, \quad P'''_C \equiv \frac{\partial^3 P}{\partial C^3} \tag{5.8}$$

one has for the whole Lagrangian $\mathcal{L}$

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^{\mu}\partial_{\mu}\overline{\lambda} + \frac{1}{2}D^2 \\ &+ P\left(D + \frac{1}{2}\partial^2 C\right) + P'_C\left(\frac{1}{2}SS^* - \chi\lambda' - \overline{\chi}\overline{\lambda'} - \frac{1}{2}A_{\mu}A^{\mu}\right) \\ &+ \frac{1}{2}P''_C\left(\frac{i}{2}\overline{\chi\chi}S - \frac{i}{2}\chi\chi S^* - \chi\sigma^{\mu}\overline{\chi}A_{\mu}\right) + \frac{1}{8}P'''_C(\chi\chi\overline{\chi\chi})\,.\end{aligned} \tag{5.9}$$

where, for more clarity, we still omitted matter superfields in the model reserving them for section 4. As one can see, extra degrees of freedom related to the C and χ component fields in a general vector superfield $V(x, \theta, \overline{\theta})$ appear through the potential terms in (5.9) rather than from the properly constructed supersymmetric field strengths, as is appeared for the vector field A_μ and its gaugino companion λ.

Varying the Lagrangian $\mathcal{L}$ with respect to the D field we come to

$$D = -P(C) \tag{5.10}$$

that finally gives the following potential energy for the field system considered

$$U(C) = \frac{1}{2}[P(C)]^2\,. \tag{5.11}$$

The potential (5.11) may lead to the spontaneous SUSY breaking in the visible sector provided that the polynomial P (5.7) has no real roots, while its first derivative has,

$$P \neq 0\,, \ P'_C = 0. \tag{5.12}$$

This requires $P(C)$ to be an even degree polynomial with properly chosen coefficients b_n in (5.7) that will force its derivative P'_C to have at least one root, $C = C_0$, in which the potential (5.11) is minimized and supersymmetry is spontaneously broken. As an immediate consequence, that one can readily see from the Lagrangian $\mathcal{L}$ (5.9), a massless photino λ being Goldstone fermion in the broken SUSY phase make all the other component fields in the superfield $V(x, \theta, \overline{\theta})$, including the photon, to also become massless. However, the question then arises whether this masslessness of photon will be stable against radiative corrections since gauge invariance is explicitly broken in the Lagrangian (5.9). We show below that it may the case if the vector superfield $V(x, \theta, \overline{\theta})$ would appear to be properly constrained.

5.3 Constrained vector superfield

We have seen above that the vector field A_μ may only appear with bilinear mass terms in the polynomially extended Lagrangian (5.9). Hence it follows that the "bumblebee" model mentioned above[4] with nontrivial vector field potential containing both a bilinear mass term and a quadrilinear stabilizing term can in no way be realized in the SUSY context. Meanwhile, the nonlinear QED model, as will become clear below, successfully matches supersymmetry.

Let us constrain our vector superfield $V(x,\theta,\overline{\theta})$ by analogy with constrained vector field in the nonlinear QED model (see (5.3)). This can be done again through the invariant Lagrange multiplier term simply adding it to the above Lagrangian (5.6)

$$\mathcal{L}_{tot} = \mathcal{L} + \frac{1}{2}\Lambda(V - C_0)^2|_D \tag{5.13}$$

where $\Lambda(x,\theta,\overline{\theta})$ is some auxiliary vector superfield, while C_0 is the constant background value of the C field for which potential U (5.11) has the SUSY breaking minimum (5.12) in the visible sector.

We further find for the Lagrange multiplier term in (5.13) that (denoting $\widetilde{C} \equiv C - C_0$)

$$\begin{aligned}\Lambda(V - C_0)^2|_D = {} & C_\Lambda\left[\widetilde{C}D' + \left(\frac{1}{2}SS^* - \chi\lambda' - \overline{\chi}\overline{\lambda'} - \frac{1}{2}A_\mu A^\mu\right)\right] \\ & + \chi_\Lambda\left[2\widetilde{C}\lambda' + i(\chi S^* + i\sigma^\mu\overline{\chi}A_\mu)\right] + \overline{\chi}_\Lambda[2\widetilde{C}\overline{\lambda'} - i(\overline{\chi}S - i\chi\sigma^\mu A_\mu)] \\ & + \frac{1}{2}S_\Lambda\left(\widetilde{C}S^* + \frac{i}{2}\overline{\chi\chi}\right) + \frac{1}{2}S^*_\Lambda\left(\widetilde{C}S - \frac{i}{2}\chi\chi\right) \\ & + 2A^\mu_\Lambda(\widetilde{C}A_\mu - \chi\sigma_\mu\overline{\chi}) + 2\lambda'_\Lambda(\widetilde{C}\chi) + 2\overline{\lambda}'_\Lambda(\widetilde{C}\overline{\chi}) + \frac{1}{2}D'_\Lambda\widetilde{C}^2\end{aligned} \tag{5.14}$$

where

$$C_\Lambda,\ \chi_\Lambda,\ S_\Lambda,\ A^\mu_\Lambda,\ \lambda'_\Lambda = \lambda_\Lambda + \frac{i}{2}\sigma^\mu\partial_\mu\overline{\chi}_\Lambda,\ D'_\Lambda = D_\Lambda + \frac{1}{2}\partial^2 C_\Lambda \tag{5.15}$$

are the component fields of the Lagrange multiplier superfield $\Lambda(x,\theta,\overline{\theta})$ in the standard parametrization (5.5). Varying the Lagrangian (5.13) with respect to these fields and properly combining their equations of motion

$$\frac{\partial\mathcal{L}_{tot}}{\partial\left(C_\Lambda,\chi_\Lambda,S_\Lambda,A^\mu_\Lambda,\lambda_\Lambda,D_\Lambda\right)} = 0 \tag{5.16}$$

we find the constraints which put on the V superfield components

$$C = C_0,\ \ \chi = 0,\ \ A_\mu A^\mu = SS^*, \tag{5.17}$$

being solely determined by the spontaneous SUSY breaking in the visible sector (5.12)

$$P'_C|_{C=C_0} = 0\,. \tag{5.18}$$

Again, as before in non-SUSY case (5.3), we only take a solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (5.15) that will provide a ghost-free theory with a positive Hamiltonian.

Now substituting the constraints (5.17, 5.18) into the total Lagrangian $\mathcal{L}_{tot}$ (5.13, 5.9) we eventually come to the basic Lagrangian in the broken SUSY phase

$$\mathcal{L}_{tot}^{br} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^{\mu}\partial_{\mu}\overline{\lambda} + \frac{1}{2}D^2 + P(C_0)D\ , \ A_{\mu}A^{\mu} = SS^* \tag{5.19}$$

being supplemented by by the vector field constraint, as indicated. So, for the constrained vector superfield,

$$\widehat{V}(x,\theta,\overline{\theta}) = C_0 + \frac{i}{2}\theta\theta S - \frac{i}{2}\overline{\theta\theta}S^* - \theta\sigma^{\mu}\overline{\theta}A_{\mu} + i\theta\theta\overline{\theta\lambda} - i\overline{\theta\theta}\theta\lambda + \frac{1}{2}\theta\theta\overline{\theta\theta}D, \tag{5.20}$$

we have the almost standard SUSY QED Lagrangian with the same states - photon, photino and an auxiliary scalar D field - in its gauge supermultiplet, while another auxiliary complex scalar field S gets only involved in the vector field constraint. The linear (Fayet-Iliopoulos) D-term with the effective coupling constant $P(C_0)$ in (5.19) shows that the supersymmetry in the theory is spontaneosly broken due to which the D field acquires the vev, $D = -P(C_0)$. Taking the nondynamical S field in the constraint (5.17) to be some constant background field (for a more formal discussion, see below) we come to the SLIV constraint (5.2) which we discussed above regarding an ordinary non-supersymmetric QED theory (sec.1). As is seen from this constraint in (5.19), one may only have a time-like SLIV in the SUSY framework but never a space-like one. There also may be a light-like SLIV, if the S field vanishes[6]. So, any possible choice for the S field corresponds to the particular gauge choice for the vector field A_{μ} in an otherwise gauge invariant theory. Thus, a massless photon emerging first as a companion of a massless photino (being Goldstone fermion in the broken SUSY phase) remains massless due to this gauge invariance.

We conclude by showing that our extended Lagrangian $\mathcal{L}_{tot}$ (5.13, 5.9), underlying the emergent QED model, is SUSY invariant, and also the constraints (5.17) on the field space appearing due to the Lagrange multiplier term in (5.13) are consistent with the supersymmetry. The first part of this assertion is somewhat immediate since the Lagrangian $\mathcal{L}_{tot}$, aside from the standard supersymmetric QED part L_{SQED} (5.6), only contains D-terms of various vector superfield products. They are, by definition, invariant under conventional SUSY transformations [18] which for the component fields (5.5) of a general superfield $V(x,\theta,\overline{\theta})$ (5.5) are

[6] Indeed, this case, first mentioned in [8], may also mean spontaneous Lorentz violation with a nonzero vev $< A_{\mu} > = (\widetilde{M}, 0, 0, \widetilde{M})$ and Goldstone modes $A_{1,2}$ and $(A_0 + A_3)/2$ $-\widetilde{M}$. The "effective" Higgs mode $(A_0 - A_3)/2$ can be then expressed through Goldstone modes so that the light-like condition $A_{\mu}^2 = 0$ is satisfied.

witten as

$$\delta_\xi C = i\xi\chi - i\overline{\xi}\overline{\chi}\ ,\ \delta_\xi\chi = \xi S + \sigma^\mu\overline{\xi}(\partial_\mu C + iA_\mu)\ ,\ \frac{1}{2}\delta_\xi S = \overline{\xi}\overline{\lambda} + \overline{\sigma}_\mu\partial^\mu\chi\ ,$$
$$\delta_\xi A_\mu = \xi\partial_\mu\chi + \overline{\xi}\partial_\mu\overline{\chi} + i\xi\sigma_\mu\overline{\lambda} - i\lambda\sigma_\mu\overline{\xi}\ ,\ \delta_\xi\lambda = \frac{1}{2}\xi\sigma^\mu\overline{\sigma}^\nu F_{\mu\nu} + \xi D\ ,$$
$$\delta_\xi D = -\xi\sigma^\mu\partial_\mu\overline{\lambda} + \overline{\xi}\sigma^\mu\partial_\mu\lambda\ . \tag{5.21}$$

However, there may still be left a question whether the supersymmetry remains in force when the constraints (5.17) on the field space are "switched on" thus leading to the final Lagrangian $\mathcal{L}^{br}_{tot}$ (5.19) in the broken SUSY phase with the both dynamical fields C and χ eliminated. This Lagrangian appears similar to the standard supersymmetric QED taken in the Wess-Zumino gauge, except that the supersymmetry is spontaneously broken in our case. In the both cases the photon stress tensor $F_{\mu\nu}$, photino λ and nondynamical scalar D field form an irreducible representation of the supersymmetry algebra (the last two line in (5.21)). Nevertheless, any reduction of component fields in the vector superfield is not consistent in general with the linear superspace version of supersymmetry transformations, whether it be the Wess-Zumino gauge case or our constrained superfield (5.20). Indeed, a general SUSY transformation does not preserve the Wess-Zumino gauge: a vector superfield in this gauge acquires some extra terms when being SUSY transformed. The same occurs with our constrained superfield as well. The point, however, is that in the both cases a total supergauge transformation

$$V \to V + i(\Omega - \Omega^*)\ , \tag{5.22}$$

where Ω is a chiral superfield gauge transformation parameter, can always restore the superfield initial form. Actually, the only difference between these two cases is that whereas the Wess-Zumino supergauge leaves an ordinary gauge freedom untouched, in our case this gauge is unambiguously fixed in terms of the above vector field constraint (5.17). However, this constraint is valid under SUSY transformations provided that the scalar field components φ and F in the Ω are properly chosen. Actually, the non-trivial part of the $\widehat{V}$ superfield transformation which can not be gauged away from the emergent theory (5.19) has the form

$$\widehat{V} \to \widehat{V} + i\theta\theta F - i\overline{\theta}\overline{\theta}F^* - 2\theta\sigma^\mu\overline{\theta}\partial_\mu\varphi\ . \tag{5.23}$$

according to which its vector and scalar field components transform as

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu(2\varphi)\ ,\ \ S \to S' = S + 2F\ . \tag{5.24}$$

It can be immediately seen that our basic Lagrangian $\mathcal{L}^{br}_{tot}$ (5.19) being gauge invariant and containing no the scalar S field is automatically invariant under either of these two transformations individually. In contrast, the supplementary vector field constraint (5.17), though it is also turned out to be invariant under supergauge transformations (5.24), but only if they are made jointly. Indeed, for any choice of the scalar φ in (5.24) there can always be found such a scalar F (and vice versa) that the constraint remains invariant

$$A_\mu A^\mu = SS^* \to A'_\mu A'^\mu = S'S'^* \tag{5.25}$$

In other words, the vector field constraint is invariant under supergauge transformations (5.24) but not invariant under an ordinary gauge transformation. As a result, in contrast to the Wess-Zumino case, the supergauge fixing in our case will also lead to the ordinary gauge fixing. We will use this supergauge freedom to reduce the S field to some constant background value and find the final equation for the gauge function $\varphi(x)$. So, for the parameter field F chosen in such a way to have

$$S' = S + 2F = Me^{i\alpha(x)} \,, \tag{5.26}$$

where M is some constant mass parameter (and $\alpha(x)$ is an arbitrary phase), we come in (5.25) to

$$(A_\mu - 2\partial_\mu \varphi)(A^\mu - 2\partial^\mu \varphi) = M^2 \,. \tag{5.27}$$

that is precisely our old SLIV constraint (5.2) being varied by the gauge transformation (5.24). Recall that this constraint, as was thoroughly discussed in Introduction (sec.1), only fixes gauge (to which such a gauge function $\varphi(x)$ has to satisfy), rather than physically breaks gauge invariance.

To summarize, it was shown that the spontaneous SUSY breaking constraints on the allowed configurations of the physical fields (5.17) in a general polynomially extended Lagrangian (5.13) are entirely consistent with the supersymmetry. In the broken SUSY phase one eventually comes to the standard SUSY QED type Lagrangian (5.19) being supplemented by the vector field constraint invariant under supergauge transformations. One might think that, unlike the gauge invariant linear (Fayet-Iliopoulos) superfield term, the quadratic and higher order superfield terms in the starting Lagrangian (5.13) would seem to break gauge invariance. However, this fear proved groundless. Actually, as was shown above in the section, this breaking amounts to the gauge fixing determined by the nonlinear vector field constraint mentioned above. It is worth noting that this constraint formally follows from the SUSY invariant Lagrange multiplier term in (5.13) for which is required the phase space to be restricted to vanishing values of all the multiplier component fields (5.15). The total vanishing of the multiplier superfield provides the SUSY invariance of such restrictions. Any non-zero multiplier component field left in the Lagrangian would immediately break supersymmetry and, even worse, would eventually lead to ghost modes in the theory and a Hamiltonian unbounded from below.

5.4 Spontaneous SUSY breaking in visible and hidden sectors: photino as pseudo-goldstino

Let us now turn to matter superfields which have not yet been included in the model. In their presence the spontaneous SUSY breaking in the visible sector, which fundamentally underlies our approach, might be phenomenologically ruled out by the well-known supertrace sum rule [18] for actual masses of quarks and leptons and their superpartners[7]. However, this sum rule is acceptably relaxed

[7] Note that an inclusion of direct soft mass terms for scalar superpartners in the model would mean in general that the visible SUSY sector is explicitly, rather than spontaneosly,

when taking into account large radiative corrections to masses of supersymmetric particles that proposedly stem from the hidden sector. This is just what one may expect in conventional supersymmetric theories with the standard two-sector paradigm, according to which a hidden sector is largely responsible for SUSY breaking, and the visible sector feels this SUSY breaking indirectly via messenger fields [18]. In this way SUSY can indeed be spontaneously broken at the tree level as well that ultimately leads to a double spontaneous SUSY breaking pattern in the model considered.

We may suppose, just for uniformity, only D-term SUSY breaking both in visible and hidden sectors[8]. Properly, our supersymmetric QED model may be further extended by some extra local $U'(1)$ symmetry which is proposed to be broken at very high energy scale M' (for some appropriate anomaly mediated scenario, see [19] and references therein). It is natural to think that due to the decoupling theorem all effects of the $U'(1)$ are suppressed at energies $E << M'$ by powers of $1/M'$ and only the D'-term of the corresponding vector superfield $V'(x, \theta, \overline{\theta})$ remains in essence when going down to low energies. Actually, this term with a proper choice of messenger fields and their couplings naturally provides the M_{SUSY} order contributions to masses of scalar superpartners.

As a result, the simplified picture discussed above (in sections 2 and 3) is properly changed: a strictly massless fermion eigenstate, the true goldstino ζ_g, should now be some mix of the visible sector photino λ and the hidden sector goldstino λ'

$$\zeta_g = \frac{\langle D \rangle \lambda + \langle D' \rangle \lambda'}{\sqrt{\langle D \rangle^2 + \langle D' \rangle^2}} \,. \tag{5.28}$$

where $\langle D \rangle$ and $\langle D' \rangle$ are the corresponding D-component vevs in the visible and hidden sectors, respectively. Another orthogonal combination of them may be referred to as the pseudo-goldstino ζ_{pg},

$$\zeta_{pg} = \frac{\langle D' \rangle \lambda - \langle D \rangle \lambda'}{\sqrt{\langle D \rangle^2 + \langle D' \rangle^2}} \,. \tag{5.29}$$

In the supergravity context, the true goldstino ζ_g is eaten through the super-Higgs mechanism to form the longitudinal component of the gravitino, while the pseudo-goldstino ζ_{pg} gets some mass proportional to the gravitino mass from supergravity effects. Due to large soft masses required to be mediated, one may generally expect that SUSY is much stronger broken in the hidden sector than in the visible one, $\langle D' \rangle >> \langle D \rangle$, that means in turn the pseudo-goldstino ζ_{pg} is largely the photino λ,

$$\zeta_{pg} \simeq \lambda \,. \tag{5.30}$$

These pseudo-goldstonic photinos seem to be of special observational interest in the model that, apart from some indication of the QED emergence nature, may

broken that would immediately invalidate the whole idea of the massless photons as the zero Lorentzian modes triggered by the spontaneously broken supersymmetry.

[8] In general, both D- and F-type terms can be simultaneously used in the visible and hidden sectors (usually just F-term SUSY breaking is used in both sectors [18]).

shed light on SUSY breaking physics. The possibility that the supersymmetric Standard Model visible sector might also spontaneously break SUSY thus giving rise to some pseudo-goldstino state was also considered, though in a different context, in [20,21]. Normally, if the visible sector possesses the R-symmetry which is preserved in the course of the mediation, then the pseudo-goldstino mass is protected up to the supergravity effects which violate R-symmetry. As a result, the pseudo-goldstino mass appears proportional to the gravitino mass, and, eventually, the same region of parameter space simultaneously solves both gravitino and pseudo-goldstino overproduction problems in the early universe [21].

Apart from cosmological problems, many other sides of new physics related to pseudo-goldstinos appearing through the multiple SUSY breaking were also studied recently (see [20–22] and references therein). The point, however, is that there have been exclusively used non-vanishing F-terms as the only mechanism of the visible SUSY breaking in models considered. In this connection, our pseudo-goldstonic photinos solely caused by non-vanishing D-terms in the visible SUSY sector may lead to somewhat different observational consequences. One of the most serious differences belongs to Higgs boson decays provided that our QED model is further extended to supersymmetric Standard Model. For the cosmologically safe masses of pseudo-goldstino and gravitino ($\lesssim 1keV$, as typically follows from R-symmetric gauge mediation) these decays are appreciably modified. Actually, the dominant channel becomes the conversion of the Higgs boson (say, the lighter CP-even Higgs boson h^0) into a conjugated pair of corresponding pseudo-sgoldstinos ϕ_{pg} and $\overline{\phi}_{pg}$ (being superpartners of pseudo-goldstinos ζ_{pg} and $\overline{\zeta}_{pg}$, respectively), $h^0 \to \phi_{pg} + \overline{\phi}_{pg}$, once it is kinematically allowed. This means that the Higgs boson will dominantly decay invisibly for F-term SUSY breaking in a visible sector [21]. By contrast, for the D-term SUSY breaking case considered here the roles of pseudo-goldstino and pseudo-sgoldstino are just played by photino and photon, respectively, that could make the standard two-photon decay channel of the Higgs boson to be even somewhat enhanced. In the light of recent discovery of the Higgs-like state [23] just through its visible decay modes, the F-term SUSY breaking in the visible sector seems to be disfavored by data, while D-term SUSY breaking is not in trouble with them.

5.5 Concluding remarks

It is well known that spontaneous Lorentz violation in general vector field theories may lead to an appearance of massless Nambu-Goldstone modes which are identified with photons and other gauge fields in the Standard Model. Nonetheless, it may turn out that SLIV is not the only reason for emergent massless photons to appear, if spacetime symmetry is further enlarged. In this connection, a special link may be related to supersymmetry that we tried to argue here by the example of supersymmetric QED that can be then straightforwardly extended to the Standard Model.

The main conclusion which has appeared in the SUSY context is that spontaneous Lorentz violation caused by an arbitrary potential of vector superfield $V(x, \theta, \overline{\theta})$ never goes any further than some noncovariant gauge constraint put on its vector field component $A_\mu(x)$ associated with a photon. This allows to think that physical Lorentz invariance is somewhat protected by SUSY, thus only admitting the "condensation" of the gauge degree of freedom in the vector field A_μ. The point, however, is that even in this case when SLIV is "inactive" it inevitably leads to the generation of massless photons as vector Nambu-Goldstone modes provided that SUSY itself is spontaneously broken. In this sense, a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz noninvariance.

To see how this idea may work we considered supersymmetric QED model extended by an arbitrary polynomial potential of a general vector superfield that induces the spontaneous SUSY violation in the visible sector. In the broken SUSY phase one eventually comes to the standard SUSY QED type Lagrangian (5.19) being supplemented by the vector field constraint invariant under supergauge transformations. As result, a massless photon appears as a companion of a massless photino which emerges in fact as the Goldstone fermion state in the tree approximation. However, being mixed with another goldstino appearing from a spontaneous SUSY violation in the hidden sector this state largely turns into the light pseudo-goldstino. Remarkably, the photon masslessness appearing at the tree level is further protected against radiative corrections by the simultaneously generated special gauge invariance. This invariance is only restricted by the nonlinear gauge condition put on vector field values, $A_\mu A^\mu = |S|^2$, so that any possible choice for the nondynamical S field corresponds to the particular gauge choice for the vector field A_μ in an otherwise gauge invariant theory. The point, however, is that this nonlinear gauge condition happens at the same time to be the SLIV type constraint which treats in turn the physical photon as the Lorentzian NG mode. So, figuratively speaking, the photon passes through three evolution stages being initially the massive vector field component of a general vector superfield (5.9), then the three-level massless companion of the Goldstonic photino in the broken SUSY stage (5.12) and finally the generically massless state as the emergent Lorentzian mode in the inactive SLIV stage (5.17).

As to pseudo-goldstonic photinos appeared in the model, they seem to be of special observational interest that, apart from some indication of the QED emergence nature, may appreciably extend the scope of SUSY breaking physics being actively discussed in recent years. In contrast to all previous considerations with non-vanishing F-terms as a mechanism of visible SUSY breaking, our pseudo-goldstonic photinos caused by non-vanishing D-terms in the visible SUSY sector will lead to somewhat different observational consequences. These and related points certainly deserve to be explored in greater detail.

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6 Coupling Electromagnetism to Global Charge

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Abstract. It is shown that an alternative to the standard scalar QED is possible. In this new version there is only global gauge invariance as far as the charged scalar fields are concerned although local gauge invariance is kept for the electromagnetic field. The electromagnetic coupling has the form $j_\mu(A^\mu + \partial^\mu B)$ where B is an auxiliary field and the current j_μ is A_μ independent so that no "sea gull terms" are introduced. In a model of this kind spontaneous breaking of symmetry does not lead to photon mass generation, instead the Goldstone boson becomes a massless source for the electromagnetic field. Infrared questions concerning the theory when spontaneous symmetry breaking takes place and generalizations to global vector QED are discussed. In this framework Q-Balls and other non topological solitons that owe their existence to a global $U(1)$ symmetry can be coupled to electromagnetism and could represent multiply charged particles now in search in the LHC. Finally, we give an example where an "Emergent" Global Scalar QED can appear from an axion photon system in an external magnetic field.

Povzetek. Pokažem, da obstaja alternativa standardni skalarni teroriji kvantne elektrodinamike. V tej novi različici velja za nabita skalarna polja samo globalna umeritvena invarianca, lokalno umeritveno invarianco pa zahtevamo za eletromagnetno polje. Elektromagnetna sklopitev ima obliko $j_\mu(A^\mu + \partial^\mu B)$, kjer je B pomožno polje, tok j_μ je neodvisen od A_μ, zaradi česar so členi tipa "sea gull" v teoriji motenj enaki nič. Pri spontani zlomitvi simetrije ostane foton brez mase, Goldstoneov bozon pa postane brezmasni izvor elektromagnetnega polja. V prispevku obravnavam probleme, ki jih ima ob spontani zlomitvi simetrije ta teorija v infrardečem območju ter njeno posplošitev do globalne vektorske teorije kvantne elektrodinamike. V tem okviru se lahko krogle "Q" in ostali netopološi solitoni, ki dolgujejo svoj obstoj globalni simetriji $U(1)$, sklopijo z elektromagnetnim poljem in bi lahko predstavljali večkratno nabite delce, ki jih trenutno iščejo na LHC. Na koncu podamo primer, kako lahko "porajajočo" globalno skalarno teorija kvantne elektrodinamike izpeljemo iz sistema aksion-foton v zunanjem magnetnem polju.

6.1 Introduction

In this paper it will be shown that an alternative to the standard scalar QED is possible. In this new version there is only global gauge invariance as far as the charged scalar fields are concerned although local gauge invariance is kept for the electromagnetic field, we call this new model Global scalar QED. The electromagnetic coupling has the form $j_\mu(A^\mu + \partial^\mu B)$ where B is an auxiliary field and the current j_μ is A_μ independent so that no "sea gull terms" are introduced. In

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a model of this kind spontaneous breaking of symmetry does not lead to photon mass generation, instead the Goldstone boson becomes a massless source for the electromagnetic field, Infrared questions concerning the theory when spontaneous symmetry breaking takes place and generalizations to global vector QED are discussed.

In this framework Q-Balls [1] and other non topological solitons [2] that owe their existence to a global $U(1)$ symmetry can be coupled to electromagnetism and could represent multiply charged particles now in search in the LHC [3].

Finally, we give an example where an "Emergent" Global Scalar QED can appear from an axion photon system in an external magnetic field.

6.2 Conventional scalar QED and its sea gulls

In conventional scalar QED, we "minimally couple" a globally invariant action (under global phase transformations). To be concrete, for a complex scalar field ψ with mass, m whose Lagrangian density can be represented in relativistic invariant form in the absence of interactions to electromagnetism as

$$\mathcal{L} = \hbar^2 g^{\mu\nu} \frac{\partial\psi^*}{\partial x^\mu} \frac{\partial\psi}{\partial x^\nu} - m^2 c^2 \psi^* \psi \tag{6.1}$$

Then, in the standard scalar QED model we introduce the electromagnetic interaction with scalar charged particles by introducing the minimal coupling in the Lagrangian for charged particles (see Eq. 6.1). As we recall, minimal coupling requires that we let the momentum p_μ be replaced by $p_\mu \to p_\mu - eA_\mu$ where $p_\mu = -i\ \hbar \frac{\partial}{\partial x^\mu}$ and where A_μ is the electromagnetic 4-vector whose Lagrangian is given by

$$\mathcal{L}_{\mathcal{EM}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{6.2}$$

with $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. We can now write the total Lagrangian after using the minimal coupling substitution into Eq. 6.1

$$\mathcal{L}_{\mathcal{T}} = g^{\mu\nu} \left[(\hbar \frac{\partial}{\partial x^\mu} - ieA_\mu)\psi^*][(\hbar \frac{\partial}{\partial x^\nu} + ieA_\nu)\psi \right] - m^2 c^2 \psi^* \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{6.3}$$

This leads to the equation of motion for the scalar field ψ

$$(i\ \hbar \frac{\partial}{\partial t} - e\phi)^2 \psi = (\frac{c\ \hbar}{i} \nabla - e\mathbf{A})^2 \psi + m^2 c^4 \psi \tag{6.4}$$

This equation and the lagrangian density from which it is derived are invariant under local gauge transformations:

$$\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\chi; \quad \phi \to \phi' = \phi - \frac{1}{c}\frac{\partial\chi}{\partial t} \quad \text{with} \quad \psi \to \exp[\frac{ie\chi}{\hbar c}]\psi \tag{6.5}$$

Furthermore the electromagnetic field satisfies the Maxwell's equations where the electric charge density ρ and the current density $\mathbf{j}(x)$ are given by (now set

$c = \hbar = 1$).

$$\rho(x) = i(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}) - 2e\phi\psi^*\psi \text{ and } \mathbf{j}(x) = -i(\psi^* \nabla \psi - \psi \nabla \psi^*) - 2e\mathbf{A}\psi^*\psi \tag{6.6}$$

There is an example, the BCS theory of superconductivity [4], where the effective theory in terms of the composite Cooper pairs retains the local gauge invariance which involves the local phase transformations of the composite scalar, however we may ask if this is a general rule, may be not.

When thinking of the electromagnetic interactions of pions, the quadratic dependence of the interactions on the potentials characterises the sea gull behaviour of standard scalar QED. As pointed out by Feynman [5], it is somewhat puzzling that spinor electrodynamics does not lead to any of such sea gulls . Considering that the microscopic description of charged pions is really the spinor electrodynamics of quarks, shouldn't we search for an effective scalar electrodynamics devoid of sea gulls?, is this possible?. In the next section we will see that this can be achieved in global scalar QED. The Global Scalar QED could address other questions as well. like the electromagnetic coupling of Q-balls and can "emerge" as an effective description of a system of axions and photons in an external field.

6.3 Global Scalar QED

There are many possible motivations for departing from the scheme implied by the minimal coupling, which leads to scalar QED. For example, if the complex scalar field is to describe a pion, since the macroscopic hadron is a very non local construction in terms of the fundamental quark fields and gluon fields as has been revealed from both the theoretical point of view [6] and from the experimental point of view [7] and in fact we may have several alternative candidates for the pion wave function (and any such proposal could give rise to a different effective theory), we do not necessarily have to keep a local gauge invariance in terms of the composite scalar fields (that would describe the hadrons), although global phase invariance must be respected. Also local gauge transformations for the photon should be mantained. Other possible use of deviating from the the minimal coupling scheme, as we will see, could be to couple Q-Ball type solitons to electromagnetism. Finally, we will give an example where an "Emergent" Global Scalar QED can appear from an axion photon system in an external magnetic field.

We work therefore with the following lagrangian density

$$\mathcal{L} = g^{\mu\nu} \frac{\partial \psi^*}{\partial x^\mu} \frac{\partial \psi}{\partial x^\nu} - U(\psi^*\psi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j_\mu (A^\mu + \partial^\mu B) \tag{6.7}$$

where

$$j_\mu = ie(\psi^* \frac{\partial \psi}{\partial x^\mu} - \psi \frac{\partial \psi^*}{\partial x^\mu}) \tag{6.8}$$

and where we have also allowed an arbitrary potential $U(\psi^*\psi)$ to allow for the possibility of spontaneous breaking of symmetry. The model is separately invariant under local gauge transformations

$$A^{\mu} \to A^{\mu} + \partial^{\mu}\Lambda; \quad B \to B - \Lambda \tag{6.9}$$

and the independent global phase transformations

$$\psi \to exp(i\chi)\psi \tag{6.10}$$

The use of a gauge invariant combination $(A^{\mu} + \partial^{\mu}B)$ can be utilized for the construction of mass terms[8] or both mass terms and couplings to a current defined from the gradient of a scalar in the form $(A^{\mu} + \partial^{\mu}B)\partial_{\mu}A$ [9]. Since the subject of this paper is electromagnetic couplings of photons and there is absolutely no evidence for a photon mass, we will disregard such type of mass terms and concentrate on the implications of the $(A^{\mu} + \partial^{\mu}B)j_{\mu}$ couplings.

6.4 A Double Charge Theory

As we will see the scalar QED model has two charge conservation laws associated with it. We see that Maxwell's equations are satisfied with j_{μ} being the source, that is

$$\partial^{\nu}F_{\nu\mu} = j_{\mu} \tag{6.11}$$

of course this implies $\partial^{\nu}\partial^{\mu}F_{\nu\mu} = \partial^{\mu}j_{\mu} = 0$. The same conclusion can be obtained from the equation of motion obtained from the variation with respect to B.

The Noether current obtained from the independent global phase transformations $\psi \to exp(i\chi)\psi$, χ being a constant, is

$$J_{\mu} = ie(\psi^{*}\frac{\partial\psi}{\partial x^{\mu}} - \psi\frac{\partial\psi^{*}}{\partial x^{\mu}}) + 2e(A_{\mu} + \partial_{\mu}B)\psi^{*}\psi \tag{6.12}$$

Therefore

$$j^{B}_{\mu} = J_{\mu} - j_{\mu} = 2e(A_{\mu} + \partial_{\mu}B)\psi^{*}\psi \tag{6.13}$$

is also conserved, that is $\partial^{\mu}((A_{\mu} + \partial_{\mu}B)\psi^{*}\psi) = 0$

6.5 No Klein Paradox

An interesting difference between standard scalar QED and global scalar QED appears in the case of strong fields Consider the global scalar QED equations with an external electromagnetic field potential step-function: $e(A_0 + \partial_0 B) \equiv V(x)$; $eA_i + e\partial_i B = 0$.

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

The Global QED equation in the presence of this potential is ($\hbar = 1, c = 1$)

$$-\frac{\partial^2\psi}{\partial t^2} + \nabla^2\psi - m^2\psi = 0 \tag{6.14}$$

for $x < 0$.

$$-\frac{\partial^2\psi}{\partial t^2} + 2iV_0\frac{\partial\psi}{\partial t} + \nabla^2\psi - m^2\psi = 0 \tag{6.15}$$

for $x > 0$. To solve the equation with this potential, we try solutions of the form:

$$\begin{aligned} \psi_< &\equiv \psi = e^{-iEt}[e^{ipx} + Re^{-ipx}] \ \text{ for } \ x < 0 \\ \psi_> &\equiv \psi = Te^{-iEt}e^{ip'x} \ \text{ for } \ x > 0 \end{aligned} \tag{6.16}$$

where $\psi_<$ represents a wave like solution for the Klein-Gordon field for $x < 0$ and $\psi_>$ represent the field for wave like solution for $x > 0$. R is the amplitude of that part of wave that is reflected wave while T is that part that is transmitted. We substitute $\psi_<$ and $\psi_>$, Eq. 6.16 into Eqs. 6.14 and 6.15 respectively. We thus find

$$E^2 - p^2 - m^2 = 0 \rightarrow E = +\sqrt{p^2 + m^2} \text{ for } \ x < 0 \tag{6.17}$$

since for incident wave for $x < 0$ we chose the positive sign in the square root as our boundary condition. and

$$E^2 - 2EV_0 - p'^2 - m^2 = 0 \rightarrow p' = \pm\sqrt{E(E - 2V_0) - m^2} \ \text{ for } \ x > 0 \tag{6.18}$$

We see here that from a certain positive value of V_0, $V_{0crit} = (E^2 - m^2)/2E$ and higher,p' becomes imaginary and therefore there is no transmited wave for large values of V_0, totally opposite to the behaviour of standard scalar QED, where for large enough barrier a transmitted wave is restored once again, leading to the "Klein paradox", the transmitted wave is interpreted there as pair creation process, no such process appears in global scalar QED.

6.6 Behaviour under Spontaneous breaking of symmetry, new couplings of Goldstone Bosons to Electromagnetism and associated infrared problems

The absence of quadratic terms in the vector potential implies that no mass generation for the photon takes place. Furthermore the Goldstone boson that results from this s.s.b. ,writing $\psi = \rho exp(i\theta)$, where ρ is real and positive, we obtain that the phase of the ψ field, is not eaten, it remains in the theory, in fact it couples derivatively to $(A_\mu + \partial_\mu B)$, like the A field studied in [9] and it produces a gradient type charge. In fact under s.s.b. regarding ρ as a constant, $j^\mu = 2e\rho^2\partial^\mu\theta$ the coupling $(A_\mu + \partial_\mu B)j^\mu$ implies the coupling of $(A_\mu + \partial_\mu B)$ to a gradient current, as discussed in [9].

It should be pointed out that this type of gradient current $j^\mu = 2e\rho^2\partial^\mu\theta$ for $\rho = constant$ generates an infrared problem, since the θ field now represents a massless field, which instead of being eaten becomes a source of electromagnetism. The normal way of solving for the electromagnetic field, using the Green's function method does not work straightforwardly, since the source now in Fourier space has support only in the light-cone and the Green's function has a pole like behaviour at the light-cone as well, so we encounter an undefined product of distributions. This is very similar to the solution of a forced harmonic oscillator when the external force has exactly the same frequency to that of the oscillator, that is the resonant case.

To resolve this problem, we note first that considering $F_{\nu\mu}$ as an antisymmetric tensor field (without at first considering whether this field derives from a four vector potential), then a solution of the equation $\partial^\nu F_{\nu\mu} = j_\mu$ is [1]

$$F_{\nu\mu} = \int_0^1 d\lambda \lambda^2 (x_\nu j_\mu(\lambda x) - x_\mu j_\nu(\lambda x)) \tag{6.19}$$

For a generic current the above $F_{\nu\mu}$ does not derive from a potential, however if the current is the gradient of a scalar field, the above $F_{\nu\mu}$ derives from a potential and provides a solution of the problem, where the Green's function method fails. Notice that the similarity with the the resonant case of the forced harmonic oscillator is very close, there the solution is of the form of an oscillating function times time and in the above solution we see the similar x_ν dependence appearing.

The resulting gauge potentials displays also a linear dependence on x_ν, which is interesting, since the central issue in the confinement problem for example is how to obtain potentials with linear dependence on the coordinates, although it is not clear how the very specific solution studied here is relevant to the confinement problem.

Axions are an example of Goldstone bosons with non trivial electromagnetic interactions

6.7 Global Vector QED

In this case we consider a complex vector field W_μ and consider the action

$$\mathcal{L} = -\frac{1}{4} g^{\mu\nu} g^{\alpha\beta} G_{\mu\alpha} G^*_{\nu\beta} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j_\mu (A^\mu + \partial^\mu B) + M^2 W_\mu W^{*\mu} \tag{6.20}$$

with $G^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu$ and where

$$j_\mu = ie(W^{*\alpha} G_{\alpha\mu} - W^\alpha G^*_{\alpha\mu}) \tag{6.21}$$

This model displays global phase invariance for the complex vector field W_μ and local gauge invariance for the photon and B fields (6.7), as was the case of global scalar QED. Once again, no sea gull terms are present here.

6.8 Q Balls and other global $U(1)$ solitons as electromagnetically charged Particles

An interesting situation could present itself when considering solitons as in the case of Q-Balls [1] or other non topological solitons [2], that depend on the existence of a $U(1)$ symmetry.

[1] I want to thank R. Tabensky for pointing this to me

These solitons have been found using actions like that used in Global scalar QED for the case e=0. The idea is minimizing the energy under the constraint that the charge of the system is given. This leads us to time dependent configurations with time dependence of the form

$$\psi(r, t) = \rho(r) exp(i\omega t) \tag{6.22}$$

We see that if there was a local gauge transformation that involve a local phase transformation of the complex scalar field ψ, then the phase of ψ is a totally unphysical quantity and the above eq. 6.22 becomes totally meaningless. That is not the case in global QED, for which 6.22 is meaningful.

Furthermore, the standard Q-Balls hold in the limit $e \to 0$ and also the small e case can be treated in perturbation theory, The introduction of a non zero e tends to destabilize the soliton as a consequence of the Coulomb repulsion that appears from the Q-ball having an electric charge. This effect is small for the case of small e, so we know there must be a range of parameters for which electrically coupled Q-Ball solitons exist.

6.9 "Emergent" scalar QED from a system of photons and axions in an external magnetic field

In this section we will consider how an "Emergent" scalar QED from a system of photons and axions in an external magnetic field. Such analysis was considered in [10] and in [11], where a "scalar QED analogy" was recognized. As we will discuss here, although the system of photons and axions in an external magnetic field does indeed have features that resemble scalar QED, the more close correspondance is with Global Scalar QED.

The action principle describing the relevant light pseudoscalar coupling to the photon is

$$S = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{8} \phi \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \right]. \tag{6.23}$$

We now specialize to the case where we consider an electromagnetic field with propagation along the y and z directions and where a strong magnetic field pointing in the x-direction is present. This field may have an arbitrary space dependence in y and z, but it is assumed to be time independent.

For the small perturbations, we consider only small quadratic terms in the action for the axion and the electromagnetic fields, considering a static magnetic field pointing in the x direction having an arbitrary y and z dependence and specializing to y and z dependent electromagnetic field perturbations and axion fields. This means that the interaction between the background field , the axion and photon fields reduces to

$$S_I = - \int d^4x \left[\beta \phi E_x \right], \tag{6.24}$$

where $\beta = gB(y,z)$. Choosing the temporal gauge for the photon excitations and considering only the x-polarization for the electromagnetic waves (since only this polarization couples to the axion) we get the following 2+1 effective dimensional action (A being the x-polarization of the photon, so that $E_x = -\partial_t A$)

$$S_2 = \int dydzdt \left[\frac{1}{2}\partial_\mu A \partial^\mu A + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 + \beta\phi\partial_t A\right]. \tag{6.25}$$

Since we consider only $A = A(t,y,z)$, $\phi = \phi(t,y,z)$, we have avoided the integration over x. For the same reason μ runs over t, y and z only . This leads to the equations

$$\partial_\mu \partial^\mu \phi + m^2\phi = \beta\partial_t A \tag{6.26}$$

and

$$\partial_\mu \partial^\mu A = -\beta\partial_t \phi. \tag{6.27}$$

As is well known, when choosing the temporal gauge the action principle cannot reproduce the Gauss constraint (here with a charge density obtained from the axion photon coupling) and has to be imposed as a complementary condition. However this constraint is automatically satisfied here just because of the type of dynamical reduction employed and does not need to be considered anymore.

Without assuming any particular y and z-dependence for β, but still insisting that it will be static, we see that in the case $m = 0$, we discover a continuous axion photon duality symmetry (these results were discussed previously in the 1+1 dimensional case, where only z dependence was considered in [10] and generalized for the case of two spatial dimensions in [11]), since

1. The kinetic terms of the photon and axion allow for a rotational $O(2)$ symmetry in the axion-photon field space.
2. The interaction term, after dropping a total time derivative, can also be expressed in an $O(2)$ symmetric way as follows:

$$S_I = \frac{1}{2}\int dydzdt\beta\left[\phi\partial_t A - A\partial_t \phi\right]. \tag{6.28}$$

It is easy to see that after introducing an appropriate complex field ϕ, this coupling is exactly of the global scalar QED form. The $U(1)$ axion photon symmetry is (in the infinitesimal limit)

$$\delta A = \epsilon\phi, \delta\phi = -\epsilon A, \tag{6.29}$$

where ϵ is a small number. Using Noether‘s theorem, this leads to the conserved current j_μ, with components given by

$$j_0^N = A\partial_t \phi - \phi\partial_t A - \frac{\beta}{2}(A^2 + \phi^2) \tag{6.30}$$

and

$$j_i^N = A\partial_i\phi - \phi\partial_i A. \tag{6.31}$$

Here $i = y, z$ coordinates. In order to have the exact correspondence with Global scalar QED, we must define the complex field ψ as

$$\psi = \frac{1}{\sqrt{2}}(\phi + iA), \tag{6.32}$$

we see that in terms of this complex field, the Noether charge density takes the form

$$j_0^N = i(\psi^*\partial_t\psi - \psi\partial_t\psi^*) - \beta\psi^*\psi. \tag{6.33}$$

which, as in Global scalar QED does not coincide with the current that enters in the interaction lagrangian, which is

$$j_0 = i(\psi^*\partial_t\psi - \psi\partial_t\psi^*) \tag{6.34}$$

We observe that the correspondance with standard scalar QED is approximate, only to first order in β, since (6.28) which represents the interaction of the magnetic field couples with the "axion photon density" 6.34, that does not contain β dependence.

This interaction has exactly the same form as that of the global scalar QED with an external "electric " field. In fact the magnetic field (or more precisely $\beta/2$) appears to play the role of external electric potential of Global scalar QED $e(A_0 + \partial_0 B) \equiv V(x)$ that couples to the axion photon density,6.34 which plays the role of an electric charge density, exactly as in Global Scalar QED.

From the point of view of the axion-photon conversion experiments, the symmetry (6.29) and its finite form, which is just a rotation in the axion-photon space, implies a corresponding symmetry of the axion-photon conversion amplitudes, for the limit $\omega >> m$.

In terms of the complex field, the Noether current takes the form

$$j_k^N = i(\psi^*\partial_k\psi - \psi\partial_k\psi^*). \tag{6.35}$$

6.10 Discussion and Conclusions

Discussing the new global QED makes sense from both the purely theoretical point of view, since it provides a new type of viewing interactions of charged scalar particles with electromagnetism, as well as from a phenomenological point of view, since standard scalar QED contains the sea gull contributions for which apparently do not represent any known physical process in the electrodynamics of charged pions for example, so it makes sense to build a theory without such sea gulls.

In this framework Q-Balls [1] and other non topological solitons [2] that owe their existence to a global $U(1)$ symmetry can be coupled to electromagnetism and could represent multiply charged particles now in search in the LHC.

Finally we have shown an example of an "Emergent" global scalar QED from a system of photons and axions in an external magnetic field.

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7 Initial Condition From the Action Principle, Its Application To Cosmology and To False Vacuum Bubbles

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Abstract. We study models where the gauge coupling constants, masses, etc are functions of some conserved charge in the universe. We first consider the standard Dirac action, but where the mass and the electromagnetic coupling constant are a function of the charge in the universe and afterwards extend this scalar fields. For Dirac field in the flat space formulation, the formalism is not manifestly Lorentz invariant, however Lorentz invariance can be restored by performing a phase transformation of the Dirac field. In the case where scalar field are considered, there is the new feature that an initial condition for the scalar field is derived from the action. In the case of the Higgs field, the initial condition require, that the universe be at the false vacuum state at a certain time slice, which is quite important for inflation scenarios. Also false vacuum branes will be studied in a similar approach. We discuss also the use of ”spoiling terms”, that violate gauge invariance to introduce these initial condition.

Povzetek. Obravnavava modele, v katerih so umeritvene sklopitvene konstante, mase, itd. funkcije nekega naboja, ki se v vesolju ohranja. Najprej študirava običajno Diracovo akcijo, v kateri so mase in elektromagnetna sklopitev funkcije tega naboja. Za Diracovo polje v ravnem prostoru akcija ni manifestno Lorentzovo invariantna, vendar za invarianco lahko poskrbimo s fazno transformacijo Diracovega polja. Če pa zamenjava fermione s skalarnimi polji, lahko iz akcije izpeljemo tudi začetni pogoj. Za Higgsovo polje začetni pogoj zahteva, da je vesolje ob določenem časovnem intervalu (časovni rezini) v lažnem vakuumu, kar omogoči različne možnosti za inflacijo vesolja. S podobnim pristopom se bova lotila tudi bran lažnega vakuuma. Na koncu obravnavava še uporabo „kvarnih členov”, ki kršijo umeritveno invarianco, ker ponudijo primerne začetne pogoje.

7.1 Introduction

Landau said ” The future physical theory should contain not only the basic equations but also the initial conditions for them ” [1]. In physics we deal with equation of motion that are obtained by varying the action, here the question of the initial condition or boundary condition are normally separated from the equation of motion, and by giving them both we can solve the physical problem (like in many differential equation problems where the solution is determined by the initial condition). Knowing just the equation of motion or just the initial conditions does not give the solution of the problem. From this point we are motivated to construct a model where initial conditions can be found from the fundamental rules of physics, without the need to assume them, they will be derived. Also we want to check whether the new model is consistent with causality and other requirements. One of the examples of a system where the initial conditions are indirectly known, and the question is why should the initial condition be like that is the inflaton model. Today there are many models for inflation, the models are defined by the kind of inflation potential. The question is why the initial field should have specific initial conditions.

The problem in the inflation initial condition is that there is not known proven way to start the universe from a false vacuum state with vacuum energy density higher than the present universe needed for inflation. In fact it appears counter intuitive not to start in the lower energy state. One idea, the "eternal inflation" that one may think solve the problem, in fact does not solve the problem. Guth et.al wrote in their paper [2] "*Thus inflationary models require physics other than inflation to describe the past boundary of the inflating region of space time*". In their article it was proven that in the past of the eternal inflationary model there must be a singularity. Also there are some initial singularity problems related to creation of baby universe from false vacuum, such a singularity cannot conceivably be produced in the laboratory, since it has no prior history [8], so we need some reason for such initial condition for the singularity of the creation of the universe. Also we are motivated to consider another direction in the research and study a model where the boundary conditions can follow from the action, this kind of approach can be used in a model where space like boundary condition of a system are fixed without any additional assumption, therefore fixing false vacuum boundary conditions on a brane. There are some equations in mathematical physics that constrain the possible initial condition that one can give. For example in electrodynamics , the equation $\nabla \cdot E = 4\pi\rho$ is a time independent equation for E and ρ, but tell us that we cannot give an initial value problem where $\nabla \cdot E = 4\pi\rho$ is not satisfied. We want to deal in fact with a sort of constraint equations, but which do not impose a constraint every where, but only for a surface (time like or space like) therefore providing in fact initial or boundary condition, in the next section we review some ideas on actions whose couplings depend on charges [3] which will be the basis to achieve this, when charged scalar fields are introduced (following section). We will see that generalizing the models where the gauge coupling constants, masses, etc are functions of some conserved charge in the universe may give such effect.

In a previous publication we considered the standard Dirac action, but where the mass and the electromagnetic coupling constant are a function of the charge in the universe and in this work we extend this scalar fields. This was motivated by the idea of obtaining a Mach like principle. For the Dirac field in the flat space formulation, the formalism is not manifestly Lorentz invariant, however Lorentz invariance can be restored by performing a phase transformation of the Dirac field. In the case where scalar field are considered, there is the new feature that an initial condition for the scalar field is derived from the action. In the case of the

Higgs field inflation [5], the initial condition require, that the universe be at the false vacuum state at a certain time slice, which is quite important for inflation scenarios. False vacuum branes will be studied in a similar approach.

7.2 The electromagnetic coupling constant as a function of the charge in the Dirac field

We begin by considering the action for the Dirac equation

$$S = \int d^4x\, \bar{\psi}(\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - eA_\mu\gamma^\mu - m)\psi \tag{7.1}$$

where $\bar{\psi} = \psi^\dagger\gamma^0$.However here we take the coupling constant e to be proportional to the total charge (It can be generalized and we can also consider an arbitrary function of the total charge[3]).

$$e = \lambda_e \int \psi^\dagger(\vec{y}, y^0 = t_0)\psi(\vec{y}, y^0 = t_0)\, d^3y = \lambda_e \int d^4y\, \bar{\psi}(y)\gamma^0\psi(y)\delta(y^0 - t_0) \tag{7.2}$$

and we will show that physics does not depend on the time slice $y^0 = t_0$. If we consider the fact that $\frac{\delta\bar{\psi}_a(x)}{\delta\bar{\psi}_b(z)} = \delta^4(x-z)\,\delta_{ab}$ and $\frac{\delta\psi(x)}{\delta\bar{\psi}(z)} = 0$ we get the equation of motion, where $b_e = \lambda_e(\int \bar{\psi}(x)A_\mu\gamma^\mu\psi(x)\, d^4x)$.

$$\frac{\delta S}{\delta\bar{\psi}(z)} = [i\gamma^\mu\partial_\mu - m - eA_\mu\gamma^\mu - b_e\gamma^0\delta(z^0 - t_0)]\psi(z) = 0 \tag{7.3}$$

so we can see that the last term in the equation of motion (7.3) contains $A^{GF}_\mu\gamma^\mu$ where $A^{GF}_\mu = \partial_\mu\Lambda$ and $\Lambda = b_e\theta(z^0 - t_0)$ is a pure gauge field. so the solution of this equation is

$$\psi = e^{-ib_e\theta(z^0 - t_0)}\psi_D \tag{7.4}$$

where ψ_D is the solution of the equation

$$[i\gamma^\mu\partial_\mu - m - eA_\mu\gamma^\mu]\psi_D = 0 \tag{7.5}$$

from which it follows that $j^\mu = \bar{\psi}_D\gamma^\mu\psi_D = \bar{\psi}\gamma^\mu\psi$ satisfies the local conservation law $\partial_\mu j^\mu = 0$ and therefore we obtain that $Q = \int d^3x\, j^0$ is conserved, so it does not depend on the time slice, furthermore it also follows that it is a scalar. For more examples see in referance [3]

7.3 Action which incorporates initials conditions

As we will see now that type of actions considered in the previous section, when generalizing them to include charged scalar fields can provide some initials condition for the scalar field [4]. Those actions can be produced by taking the coupling constants as a function of a conserved charge. If we use this development we can have the initial vacuum state for the universe in the inflationary model, so this initial condition will give us the initial condition for the universe corresponding to being initially at the false vacuum. Following there are some examples of actions that can produce initial conditions.

We will use the definition of the book of Anderson [10], which take the points in sub-manifold:

$$x^\mu = \Phi^\mu(\lambda_1, ..., \lambda_N) \tag{7.6}$$

the definition of the charge is:

$$\Theta = \lambda \int j_\mu d\sigma^\mu = \lambda \int j_\mu \delta^\mu(x - \Phi) d\sigma \tag{7.7}$$

where $d\sigma = \sqrt{-g} d^4x$ and $d\sigma^\mu$ is hyper-surface volume orthogonal to the normal n^μ. Φ^μ is parametric $d-1$ hyper-surface, and the delta function can be defined by $\delta^\mu = \int \delta^4(x - \Phi) d\sigma^\mu$ and the correspond step function are defined by $\theta = \int_x \delta(x - \Phi) d\sigma$. It is not hard to prove that Θ is constant [10]. The action for Higgs inflation with the general potential $V(\phi, \phi^*, \Theta)$ is[5]

$$\begin{aligned} S = \int d\sigma \sqrt{-g}\, [(\partial_\mu \phi^* + i\tfrac{g'}{2} A_\mu \phi^*)(\partial^\mu \phi - i\tfrac{g'}{2} A^\mu \phi) - V(\phi, \phi^*, \Theta) + \tfrac{\alpha}{16\pi} \phi^* \phi R] \\ -\tfrac{1}{4} \int F^{\mu\nu} F_{\mu\nu} \sqrt{-g} d\sigma + \tfrac{M_p}{2} \int \sqrt{-g} R\, d\sigma = \\ \int d\sigma \sqrt{-g}\, [(D\phi)^*(D\phi) - V(\phi, \phi^*, \Theta) + \tfrac{\alpha}{16\pi} \phi^* \phi R] - \tfrac{1}{4} \int F^{\mu\nu} F_{\mu\nu} \sqrt{-g} d\sigma \\ + \tfrac{M_p}{2} \int \sqrt{-g} R\, d\sigma \end{aligned} \tag{7.8}$$

from this by variation on ϕ^* , we get the equation of motion:

$$\begin{aligned} -\partial_\mu(\sqrt{-g} g^{\mu\nu} \partial^\nu \phi) - i\tfrac{g'}{2} \partial_\mu(\sqrt{-g} A^\mu \phi) - i\sqrt{-g} \tfrac{g'}{2} A_\mu \partial^\mu \phi \\ +\sqrt{-g} (\tfrac{g'}{2})^2 A_\mu A^\mu \phi - \sqrt{-g} \tfrac{\partial V}{\partial \phi^*} + \tfrac{\alpha}{16\pi} \phi R \\ -\lambda \sqrt{-g} (\int d\sigma \sqrt{-g}\, \tfrac{\partial V}{\partial \theta}) \delta^\mu(x - \Phi) [2i\partial_\mu \phi - g' A_\mu \phi] \\ -\lambda(\int d\sigma \sqrt{-g}\, \tfrac{\partial V}{\partial \theta}) i\phi\, \partial_\mu(\sqrt{-g} \delta^\mu(x - \Phi)) = 0 \end{aligned} \tag{7.9}$$

We can see that we have delta term, so to eliminate it we do the transformation:

$$A^\mu \longrightarrow A^\mu + \frac{2i\lambda_1 b}{g'} \delta^\mu(x - \Phi) \tag{7.10}$$

and

$$\phi = e^{\lambda_2 b \theta(x - \Phi)} \phi_0 \tag{7.11}$$

where $b = i\lambda(\int d\sigma \sqrt{-g}\, \frac{\partial V}{\partial \theta})$
So we have that:

$$\begin{aligned} -\partial_\mu(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi_0) - i\tfrac{g'}{2} \partial_\mu(\sqrt{-g} A^\mu \phi_0) - i\sqrt{-g} \tfrac{g'}{2} A_\mu \partial^\mu \phi_0 \\ +\sqrt{-g} (\tfrac{g'}{2})^2 A_\mu A^\mu \phi_0 - \sqrt{-g} \tfrac{\partial V}{\partial \phi^*} + \tfrac{\alpha}{16\pi} \phi R \\ -2b\sqrt{-g} \delta^\mu(x - \Phi) [(\lambda_2 - \lambda_1 + 1) \partial_\mu \phi_0 - i(\lambda_2 - \lambda_1 + 1) \tfrac{g'}{2} A_\mu \phi_0 \\ +0.5 b \delta_\mu(x - \Phi) \phi_0 (\lambda_2^2 - 2\lambda_1 + \lambda_1^2 + 2(-\lambda_1 + 1)\lambda_2)] \\ -b(\lambda_1 - \lambda_2 + 1) \phi_0\, \partial_\mu(\sqrt{-g} \delta^\mu(x - \Phi)) = 0 \end{aligned} \tag{7.12}$$

if we need that equation (7.12) will be like ordinary Klein Gordon equation we need that:

$$\lambda_2 - \lambda_1 + 1 = 0 \tag{7.13}$$

$$\lambda_2^2 - 2\lambda_1 + \lambda_1^2 + 2(-\lambda_1 + 1)\lambda_2 = 0 \tag{7.14}$$

for which there is no solution, so we must conclude that $\phi(\Phi) = 0$. We can take private case were the action give initial condition, were $\Theta = Q = \lambda \int j_0 \delta^0(t) d\sigma$, which give $\phi(t = 0) = 0$.

7.4 Boundary condition from spoiling terms

Some "spoiling terms" that is terms that break gauge invariance have been shown in the end do not to contribute the functional integral [11]. Here we will see that "spoiling terms" where non gauge invariant charge are introduced, have as a consequence that they induce boundary condition and these boundary condition imply the vanishing of the spoiling terms. To see this we take the action

$$S = \int d^4x\sqrt{-g}\,[(\partial^\mu \phi^* + i\tfrac{g'}{2}A^\mu \phi^*)(\partial_\mu \phi - i\tfrac{g'}{2}A_\mu \phi) - V(Q^{NGI}, \phi, \phi^*)] \\ -\tfrac{1}{4}\int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}d^4x - \tfrac{1}{16\pi G}\int \sqrt{-g}R\, d^4x \tag{7.15}$$

where we introduce the non gauge invariant charge

$$Q^{NGI} = \int d^4x\sqrt{-g}\delta(t - t_0)[\phi^* i \overset{\leftrightarrow}{\partial^0} \phi + g_1' A^0 \phi^* \phi] \tag{7.16}$$

where NGI= Non Gauge Invariant, and $g_1' \neq g'$. If we vary the action by a gauge transformation

$$A_\mu \to \partial_\mu \Lambda + A_\mu \tag{7.17}$$

$$\phi \to \phi_0 e^{ig'\Lambda} \tag{7.18}$$

all the other term of the action can not be change but

$$Q^{NGI} \to Q^{NGI} + (g' - g_1') \int d^4x\sqrt{-g}\delta(t - t_0)\partial_0 \Lambda(x)\phi^*\phi \tag{7.19}$$

So for all $\Lambda(x)$ if $V(Q^{NGI}, \phi, \phi^*)$ has a non trivial dependence on Q^{NGI} then equating the variation of the action to zero implies:

$$\phi^*(t_0)\phi(t_0) = 0 \tag{7.20}$$

Of course, this means that the theory effectively cancels the non gauge invariant terms when the variational principle is used, so gauge invariance is restored effectively. Also boundary condition which are gauge invariant are obtained.

It is interesting to compare the mechanism obtained from introducing spoiling terms to the mechanism obtained in order to climb up a potential using ghost field and may be also in this way end up at the top of potential [12].

7.5 Discussions and Conclusions

We have studied models where the gauge coupling constants, masses, etc are functions of some conserved charge in the universe. We first considered the standard Dirac action, but where the mass and the electromagnetic coupling constant are a function of the charge in the universe and afterwards extended this to scalar fields. For Dirac field in the flat space formulation, the formalism is not manifestly Lorentz invariant, however Lorentz invariance can be restored by performing a phase transformation of the Dirac field.

In the case where scalar fields are considered, there is the new feature that an initial condition for the scalar field is derived from the action. In the case of the Higgs field, the initial condition require, that the universe be at the false vacuum state at a certain time slice, which is quite important for inflation scenarios. Also false vacuum branes can be studied in a similar approach.

One should point out that it appears that not all possible boundary condition allow a formulation in this way, which is probably good, because we would like a theory of the boundary condition to restrict such possibilities.
Some ″spoiling terms″ that is terms that break gauge invariance have been shown that in the end they do not contribute to the functional integral [11]. We have seen that ″spoiling terms″ where non gauge invariant charges are introduced, have as a consequences that they induce boundary condition and these boundary condition imply the vanishing of the spoiling terms, and in the special example choose that the universe sits in the false vacuum in a certain time slice

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8 Neutrino Masses and Mixing Within a SU(3) Family Symmetry Model With One or Two Light Sterile Neutrinos

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Abstract. We report a global fit of parameters for fermion masses and mixing, including light sterile neutrinos, within a local vector $SU(3)$ family symmetry model. In this scenario, ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from **Dirac See-saw** mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlet vector-like fermions, U, D, E, N, with N a sterile neutrino. The $N_{L,R}$ sterile neutrinos allow the implementation of a 8×8 general tree level See-saw Majorana neutrino mass matrix with four massless eigenvalues. Hence, light fermions, including light neutrinos obtain masses from one loop radiative corrections mediated by the massive $SU(3)$ gauge bosons. This BSM model is able to accommodate the known spectrum of quark masses and mixing in a 4×4 non-unitary V_{CKM} as well as the charged lepton masses. The explored parameter space region provide the vector-like fermion masses: $M_D \approx 914.365$ GeV, $M_U \approx 1.5$ TeV, $M_E \approx 5.97$ TeV, $SU(3)$ family gauge boson masses of $\mathcal{O}(1-10)$TeV, the neutrino masses $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0.0085, 0.049, 0.22, 3.21, 1749.96, 1 \times 10^8, 1 \times 10^9)$ eV, with the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 7.23 \times 10^{-5}$ eV^2, $m_3^2 - m_1^2 \approx 2.4 \times 10^{-3}$ eV^2, $m_4^2 - m_1^2 \approx 0.049$ eV^2, $m_5^2 - m_1^2 \approx 10.3$ eV^2. We also show the corresponding U_{PMNS} lepton mixing matrix. However, the neutrino mixing angles are extremely sensitive to parameter space region, and an improved and detailed analysis is in progress.

Povzetek. V modelu, kjer družinsko kvantno število določa grupa $SU[3]$, poskrbi za maso obeh težkih kvarkov (t in b) ter za maso težkega leptona (tau) tako imenovani „Diracov seasaw" mehanizem že na drevesnem nivoju. To dosežem tako, da predpostavim eksistenco novih kvarkov U in D ter novih leptonov E in N. Vsi so levoročni in brez šibkega naboja ter nosijo tripletni naboj $SU(2)$. Nevtrini $N_{L,R}$, ki ne nosijo nobenega naboja, določajo na drevesnem nivoju masno matriko 8×8 Majoraninega tipa, ki ima štiri lastne vrednosti enake nič. Za maso lahkih fermionov, vključno z nevtrini, poskrbijo v popravkih z eno zanko masivni umeritveni bozoni družinskega kvantnega števila grupe $SU(3)$. S primerno izbiro parametrov dosežem, da se lastnosti kvarkov in (z elektromagnetnim nabojem) nabitih leptonov ujemajo z izmerjenimi. Kvarkovska mešalna matrika 4×4 ni unitarna. Nova družina kvarkov in nabitih leptonov ima mase okoli TeV ali več ($M_D \approx 914.365$ GeV, $M_U \approx 1.5$ TeV, $M_E \approx 5.97$ TeV), mase umeritvenih bozonov z družinskim kvantnimi števili grupe $SU(3)$ pa so $\mathcal{O}(1-10)$TeV. Za mase nevtrinov najdem $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0.0085, 0.049, 0.22, 3.21, 1749.96, 1 \times 10^8, 1 \times 10^9)$ eV, za kvadrate razlik njihovih mas

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pa: $m_2^2 - m_1^2 \approx 7.23 \times 10^{-5}$ eV^2, $m_3^2 - m_1^2 \approx 2.4 \times 10^{-3}$ eV^2, $m_4^2 - m_1^2 \approx 0.049$ eV^2, $m_5^2 - m_1^2 \approx 10.3$ eV^2. Izračunam tudi leptonsko mešalno matriko. Ker je le-ta močno odvisna od izbire parametrov, bo vanjo potrebno vložiti še nekaj truda. Delo je v teku.

8.1 Introduction

The standard picture of three flavor neutrinos has been successful to account for most of the neutrino oscillation data. However, several experiments have reported new experimental results, on neutrino mixing[1], on large θ_{13} mixing from Daya Bay[2], T2K[3], MINOS[4], DOUBLE CHOOZ[5], and RENO[6], implying a deviation from TBM[7] scenario. In addition, the recent experimental results from the LSND and MiniBooNe short-baseline neutrino oscillation experiments, provide indications in favor of the existence of light sterile neutrinos in the eV-scale, in order to explain the tension in the interpretation of these data[8,9].

The strong hierarchy of quark and charged lepton masses and quark mixing have suggested to many model building theorists that light fermion masses could be generated from radiative corrections[10], while those of the top and bottom quarks and the tau lepton are generated at tree level. This may be understood as the breaking of a symmetry among families , a horizontal symmetry . This symmetry may be discrete [11], or continuous, [12]. The radiative generation of the light fermions may be mediated by scalar particles as it is proposed, for instance, in references [13,14] and the author in [15], or also through vectorial bosons as it happens for instance in "Dynamical Symmetry Breaking" (DSB) and theories like " Extended Technicolor " [16].

In this report, we address the problem of fermion masses and quark mixing within an extension of the SM introduced by the author in [17], which includes a vector gauged $SU(3)$[18] family symmetry commuting with the SM group. In previous reports[19] we showed that this model has the properties to accommodate a realistic spectrum of charged fermion masses and quark mixing. We introduce a hierarchical mass generation mechanism in which the light fermions obtain masses through one loop radiative corrections, mediated by the massive bosons associated to the $SU(3)$ family symmetry that is spontaneously broken, while the masses for the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw"[20] mechanisms implemented by the introduction of a new generation of $SU(2)_L$ weak singlets vector-like fermions.

Recently, some authors have pointed out interesting features regarding the possibility of the existence of vector-like matter, both from theory and current experiments[23]. From the fact that the vector-like quarks do not couple to the W boson, the mixing of U and D vector-like quarks with the SM quarks gives rise to an extended 4×4 non-unitary CKM quark mixing matrix. It has pointed out for some authors that these vector-like fermions are weakly constrained from Electroweak Precision Data (EWPD) because they do not break directly the custodial symmetry, then main experimental constraints on the vector-like matter come from the direct production bounds, and their implications on flavor physics. See the ref. [23]

for further details on constraints for vector-like fermions. Theories and models with extra matter may also provide attractive scenarios for present cosmological problems, such as candidates for the nature of the Dark Matter ([21],[22]).

In this article, we report for the first time a global fit of the free parameters of the $SU(3)$ *family symmetry model to accommodate quark and lepton masses and mixing, including light sterile neutrinos.*

8.2 Model with $SU(3)$ flavor symmetry

8.2.1 Fermion content

We define the gauge group symmetry $G \equiv SU(3) \otimes G_{SM}$, where $SU(3)$ is a flavor symmetry among families and $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" gauge group, with g_s, g and g' the corresponding coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

$$\psi_q^o = (3,3,2,\frac{1}{3})_L \quad , \quad \psi_u^o = (3,3,1,\frac{4}{3})_R \quad , \quad \psi_d^o = (3,3,1,-\frac{2}{3})_R$$

$$\psi_l^o = (3,1,2,-1)_L \quad , \quad \psi_e^o = (3,1,1,-2)_R \ ,$$

where the last entry corresponds to the hypercharge Y, and the electric charge is defined by $Q = T_{3L} + \frac{1}{2}Y$. The model also includes two types of extra fermions: Right handed neutrinos $\Psi_\nu^o = (3,1,1,0)_R$, and the $SU(2)_L$ singlet vector-like fermions

$$U_{L,R}^o = (1,3,1,\frac{4}{3}) \quad , \quad D_{L,R}^o = (1,3,1,-\frac{2}{3}) \tag{8.1}$$

$$N_{L,R}^o = (1,1,1,0) \quad , \quad E_{L,R}^o = (1,1,1,-2) \tag{8.2}$$

The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U \, \bar{U}_L^o \, U_R^o \, + \, M_D \, \bar{D}_L^o \, D_R^o \, + \, M_E \, \bar{E}_L^o \, E_R^o + h.c. \ , \tag{8.3}$$

and

$$m_D \, \bar{N}_L^o \, N_R^o \, + \, m_L \, \bar{N}_L^o \, (N_L^o)^c \, + \, m_R \, \bar{N}_R^o \, (N_R^o)^c \, + \, h.c \tag{8.4}$$

The above fermion content make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the usual form under the standard model group and in the fundamental 3-representation under the $SU(3)$ family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies[24]. The $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. These vector like fermions play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses, together with the radiative corrections.

8.3 SU(3) family symmetry breaking

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of SU(3), we introduce the flavon scalar fields: $\eta_i,\ i = 2,3$, transforming under the gauge group as $(3,1,1,0)$ and taking the "Vacuum Expectation Values" (VEV's):

$$\langle\eta_3\rangle^T = (0,0,\Lambda_3) \quad , \quad \langle\eta_2\rangle^T = (0,\Lambda_2,0) \, . \tag{8.5}$$

The above scalar fields and VEV's break completely the SU(3) flavor symmetry. The corresponding SU(3) gauge bosons are defined in Eq.(8.29) through their couplings to fermions. Thus, the contribution to the horizontal gauge boson masses from Eq.(8.5) read

- η_3 : $\frac{g_{H_3}^2\Lambda_3^2}{2}(Y_2^+Y_2^- + Y_3^+Y_3^-) + g_{H_3}^2\Lambda_3^2\frac{Z_2^2}{3}$
- η_2 : $\frac{g_{H_2}^2\Lambda_2^2}{2}(Y_1^+Y_1^- + Y_3^+Y_3^-) + \frac{g_{H_2}^2\Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2Z_1\frac{Z_2}{\sqrt{3}})$

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms

$$M_1^2\,Y_1^+Y_1^- + \frac{M_1^2}{2}\,Z_1^2 + (\frac{4}{3}\,M_2^2 + \frac{1}{3}\,M_1^2)\frac{Z_2^2}{2} - \frac{M_1^2}{\sqrt{3}}\,Z_1\,Z_2 + M_2^2\,Y_2^+Y_2^- + (M_1^2 + M_2^2)\,Y_3^+Y_3^- \tag{8.6}$$

$$M_1^2 = \frac{g_{H_2}^2\Lambda_2^2}{2} \quad , \quad M_2^2 = \frac{g_{H_3}^2\Lambda_3^2}{2} \quad , \quad M_3^2 = M_1^2 + M_2^2 \tag{8.7}$$

From the diagonalization of the $Z_1 - Z_2$ squared mass matrix, we obtain the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_1^2 + M_2^2 - \sqrt{(M_2^2 - M_1^2)^2 + M_1^2M_2^2}\right) ,$$
$$M_+^2 = \frac{2}{3}\left(M_1^2 + M_2^2 + \sqrt{(M_2^2 - M_1^2)^2 + M_1^2M_2^2}\right) \tag{8.8}$$

$$M_1^2\,Y_1^+Y_1^- + M_-^2\,\frac{Z_-^2}{2} + M_+^2\,\frac{Z_+^2}{2} + M_2^2\,Y_2^+Y_2^- + (M_1^2 + M_2^2)\,Y_3^+Y_3^- \tag{8.9}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \tag{8.10}$$

$$\cos\phi\ \sin\phi = \frac{\sqrt{3}}{4}\,\frac{1}{\sqrt{(\frac{M_2^2}{M_1^2} - 1)^2 + \frac{M_2^2}{M_1^2}}}$$

with the hierarchy $M_1, M_2 \gg M_W$.

	Z_1	Z_2
Z_1	M_1^2	$-\frac{M_1^2}{\sqrt{3}}$
Z_2	$-\frac{M_1^2}{\sqrt{3}}$	$(\frac{4}{3}M_2^2 + \frac{1}{3}M_1^2)$

8.4 Electroweak symmetry breaking

Recently ATLAS[25] and CMS[26] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. The electroweak symmetry breaking in the $SU(3)$ family symmetry model involves the introduction of two triplets of $SU(2)_L$ Higgs doublets.

To achieve the spontaneous breaking of the electroweak symmetry to $U(1)_Q$, we introduce the scalars: $\Phi^u = (3, 1, 2, -1)$ and $\Phi^d = (3, 1, 2, +1)$, with the VEVs: $\langle\Phi^u\rangle^T = (\langle\Phi_1^u\rangle, \langle\Phi_2^u\rangle, \langle\Phi_3^u\rangle)$, $\langle\Phi^d\rangle^T = (\langle\Phi_1^d\rangle, \langle\Phi_2^d\rangle, \langle\Phi_3^d\rangle)$, where T means transpose, and

$$\langle\Phi_i^u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_i \\ 0 \end{pmatrix} \quad , \quad \langle\Phi_i^d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ V_i \end{pmatrix} . \tag{8.11}$$

The contributions from $\langle\Phi^u\rangle$ and $\langle\Phi^d\rangle$ generate the W and Z gauge boson masses

$$\frac{g^2}{4}(v_u^2 + v_d^2)\, W^+W^- + \frac{(g^2 + g'^2)}{8}(v_u^2 + v_d^2)\, Z_o^2 \tag{8.12}$$

$v_u^2 = v_1^2 + v_2^2 + v_3^2$, $v_d^2 = V_1^2 + V_2^2 + V_3^2$. Hence, if we define as usual $M_W = \frac{1}{2}gv$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

8.5 Tree level neutrino masses

Now we describe briefly the procedure to get the masses for ordinary fermions. The analysis for quarks and charged leptons has already discussed in [19]. Here, we introduce the procedure for neutrinos.

Before "Electroweak Symmetry Breaking"(EWSB) all ordinary, "Standard Model"(SM) fermions remain massless, and the quarks and leptons global symmetry is:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R} \tag{8.13}$$

8.5.1 Tree level Dirac neutrino masses

With the fields of particles introduced in the model, we may write the Dirac type gauge invariant Yukawa couplings

$$h_D\,\bar{\Psi}^o_l\,\Phi^u\,N^o_R \;+\; h_2\,\bar{\Psi}^o_\nu\,\eta_2\,N^o_L \;+\; h_3\,\bar{\Psi}^o_\nu\,\eta_3\,N^o_L \;+\; M_D\,\bar{N}^o_L\,N^o_R \;+h.c \quad (8.14)$$

h_D, h_2 and h_3 are Yukawa couplings, and M_D a Dirac type, invariant neutrino mass for the sterile neutrinos $N^o_{L,R}$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi^{o\,T}_{\nu L,R} = (\nu^o_e, \nu^o_\mu, \nu^o_\tau, N^o)_{L,R}$, the mass terms

$$h_D\,\left[v_1\,\bar{\nu}^o_{eL} + v_2\,\bar{\nu}^o_{\mu L} + v_3\,\bar{\nu}^o_{\tau L}\right]\,N^o_R + \left[h_2\,\Lambda_2\,\bar{\nu}^o_{\mu R} + h_3\,\Lambda_3\,\bar{\nu}^o_{\tau R}\right]\,N^o_L + M_D\,\bar{N}^o_L\,N^o_R + h.c.\,, \quad (8.15)$$

8.5.2 Tree level Majorana masses:

Since $N^o_{L,R}$, Eq.(8.2), are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L\,\bar{\Psi}^o_l\,\Phi^u (N^o_L)^c \quad + \quad m_L\,\bar{N}^o_L\,(N^o_L)^c \quad (8.16)$$

and

$$h_{2R}\,\bar{\Psi}^o_\nu\,\eta_2\,(N^o_R)^c \;+\; h_{3R}\,\bar{\Psi}^o_\nu\,\eta_3\,(N^o_R)^c \;+\; m_R\,\bar{N}^o_R\,(N^o_R)^c + h.c\,, \quad (8.17)$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L\,\left[v_1\,\bar{\nu}^o_{eL} + v_2\,\bar{\nu}^o_{\mu L} + v_3\,\bar{\nu}^o_{\tau L}\right]\,(N^o_L)^c \quad + \quad m_L\,\bar{N}^o_L\,(N^o_L)^c + h.c.\,, \quad (8.18)$$

$$+\left[h_{2R}\,\Lambda_2\,\bar{\nu}^o_{\mu R} + h_{3R}\,\Lambda_3\,\bar{\nu}^o_{\tau R}\right]\,(N^o_R)^c \;+\; m_R\,\bar{N}^o_R\,(N^o_R)^c + h.c.\,, \quad (8.19)$$

Thus, in the basis $\Psi^{o\,T}_\nu = \left(\nu^o_{eL}\,, \nu^o_{\mu L}\,, \nu^o_{\tau L}\,, (\nu^o_{eR})^c\,, (\nu^o_{\mu R})^c\,, (\nu^o_{\tau R})^c\,, N^o_L\,, (N^o_R)^c\right)$, the Generic 8×8 tree level Majorana mass matrix for neutrinos $\mathcal{M}^o_\nu$, from Table 8.1, $\bar{\Psi^o_\nu}\;\mathcal{M}^o_\nu\;(\Psi^o_\nu)^c\;+h.c.$, read

$$\mathcal{M}^o_\nu = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \alpha_1 & a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_2 & a_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_2 & \beta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_3 & \beta_3 \\ \alpha_1 & \alpha_2 & \alpha_3 & 0 & b_2 & b_3 & m_L & m_D \\ a_1 & a_2 & a_3 & 0 & \beta_2 & \beta_3 & m_D & m_R \end{pmatrix} \quad (8.20)$$

	$(\nu_{eL}^o)^c$	$(\nu_{\mu L}^o)^c$	$(\nu_{\tau L}^o)^c$	ν_{eR}^o	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	$(N_L^o)^c$	N_R^o
$\overline{\nu_{eL}^o}$	0	0	0	0	0	0	$h_L v_1$	$h_D v_1$
$\overline{\nu_{\mu L}^o}$	0	0	0	0	0	0	$h_L v_2$	$h_D v_2$
$\overline{\nu_{\tau L}^o}$	0	0	0	0	0	0	$h_L v_3$	$h_D v_3$
$\overline{(\nu_{eR}^o)^c}$	0	0	0	0	0	0	0	0
$\overline{(\nu_{\mu R}^o)^c}$	0	0	0	0	0	0	$h_2\Lambda_2$	$h_{2R}\Lambda_2$
$\overline{(\nu_{\tau R}^o)^c}$	0	0	0	0	0	0	$h_3\Lambda_3$	$h_{3R}\Lambda_3$
$\overline{N_L^o}$	$h_L v_1$	$h_L v_2$	$h_L v_3$	0	$h_2\Lambda_2$	$h_3\Lambda_3$	m_L	M_D
$\overline{(N_R^o)^c}$	$h_D v_1$	$h_D v_2$	$h_D v_3$	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	M_D	m_R

Table 8.1. Tree Level Majorana masses

Diagonalization of the $\mathcal{M}_\nu^{(o)}$, Eq.(8.20), yields four zero eigenvalues, associated to the neutrino fields: $ap = \sqrt{a_1^2 + a_2^2}$

$$\frac{a_2}{ap}\,\nu_{eL}^o - \frac{a_1}{ap}\,\nu_{\mu L}^o \quad , \quad \frac{a_1\,a_3}{ap\,a}\,\nu_{eL}^o + \frac{a_2\,a_3}{ap\,a}\,\nu_{\mu L}^o - \frac{a_p}{a}\,\nu_{\tau L}^o ,$$

$$\nu_{eR}^o \quad , \quad \frac{b_3}{b}\,\nu_{\mu R}^o - \frac{b_2}{b}\,\nu_{\tau R}^o$$

Assuming for simplicity, $\frac{h_2}{h_{2R}} = \frac{h_3}{h_{3R}}$, the Characteristic Polynomial for the nonzero eigenvalues of $\mathcal{M}_\nu^o$ reduce to the one of the matrix m_4, Eq.(8.21), where

$$m_4 = \begin{pmatrix} 0 & 0 & \alpha & a \\ 0 & 0 & b & \beta \\ \alpha & b & m_L & m_D \\ a & \beta & m_D & m_R \end{pmatrix} \quad , \quad U_4 = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \tag{8.21}$$

$$a = \sqrt{a_1^2 + a_2^2 + a_3^2},\ \alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}\ ,$$
$$b = \sqrt{b_2^2 + b_3^2}\ , \qquad \beta = \sqrt{\beta_2^2 + \beta_3^2}$$

$$U_4^T\, m_4\, U_4 = Diag(m_5^o, m_6^o, m_7^o, m_8^o) \equiv d_4 \quad , \quad m_4 = U_4\, d_4\, U_4^T \tag{8.22}$$

Eq.(8.22) impose the constrains

$$u_{11}^2\, m_5^o + u_{12}^2\, m_6^o + u_{13}^2\, m_7^o + u_{14}^2\, m_8^o = 0 \tag{8.23}$$

$$u_{21}^2\, m_5^o + u_{22}^2\, m_6^o + u_{23}^2\, m_7^o + u_{24}^2\, m_8^o = 0 \tag{8.24}$$

$$u_{11}u_{21}\, m_5^o + u_{12}u_{22}\, m_6^o + u_{13}u_{23}\, m_7^o + u_{14}u_{24}\, m_8^o = 0\,, \tag{8.25}$$

corresponding to the $(m_4)_{11} = (m_4)_{22} = (m_4)_{12} = 0$ zero entries, respectively.

In this form, we diagonalize $\mathcal{M}_\nu^o$ by using the orthogonal matrix

$$U_\nu^o = \begin{pmatrix} \frac{a_2}{a_p} & \frac{a_1\, a_3}{a\, a_p} & 0 & 0 & \frac{a_1}{a}u_{11} & \frac{a_1}{a}u_{12} & \frac{a_1}{a}u_{13} & \frac{a_1}{a}u_{14} \\ -\frac{a_1}{a_p} & \frac{a_2\, a_3}{a\, a_p} & 0 & 0 & \frac{a_2}{a}u_{11} & \frac{a_2}{a}u_{12} & \frac{a_2}{a}u_{13} & \frac{a_2}{a}u_{14} \\ 0 & -\frac{a_p}{a} & 0 & 0 & \frac{a_3}{a}u_{11} & \frac{a_3}{a}u_{12} & \frac{a_3}{a}u_{13} & \frac{a_3}{a}u_{14} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{b_3}{b} & \frac{b_2}{b}u_{21} & \frac{b_2}{b}u_{22} & \frac{b_2}{b}u_{23} & \frac{b_2}{b}u_{24} \\ 0 & 0 & 0 & -\frac{b_2}{b} & \frac{b_3}{b}u_{21} & \frac{b_3}{b}u_{22} & \frac{b_3}{b}u_{23} & \frac{b_3}{b}u_{24} \\ 0 & 0 & 0 & 0 & u_{31} & u_{32} & u_{33} & u_{34} \\ 0 & 0 & 0 & 0 & u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \tag{8.26}$$

$$(U_\nu^o)^T\, \mathcal{M}_\nu^o\, U_\nu^o = \mathrm{Diag}(0, 0, 0, 0, m_5^o, m_6^o, m_7^o, m_8^o) \tag{8.27}$$

Notice that the first four columns in U_ν^o correspond to the four massless eigenvectors. Hence, the tree level mixing, U_ν^o, depends on the ordering we define for these four degenerated massless eigenvectors. However, it turns out that the final mixing product $U_\nu^o\, U_\nu$, as well as the final mass eigenvalues are independent of the choice of this ordering.

8.6 One loop neutrino masses

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R} \tag{8.28}$$

Therefore, in this scenario light neutrinos may get extremely small masses from radiative corrections mediated by the SU(3) heavy gauge bosons.

8.6.1 One loop Dirac Neutrino masses

After the breakdown of the electroweak symmetry, neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 8.1, The internal fermion line in this diagram represent the tree level see-saw mechanism, Eq.(8.15). The vertices read from the $SU(3)$ flavor symmetry interaction Lagrangian

$$i\mathcal{L}_{int} = \frac{g_H}{2}\left(\bar{e^o}\gamma_\mu e^o - \bar{\mu^o}\gamma_\mu \mu^o\right) Z_1^\mu + \frac{g_H}{2\sqrt{3}}\left(\bar{e^o}\gamma_\mu e^o + \bar{\mu^o}\gamma_\mu \mu^o - 2\bar{\tau^o}\gamma_\mu \tau^o\right) Z_2^\mu + \frac{g_H}{\sqrt{2}}\left(\bar{e^o}\gamma_\mu \mu^o Y_1^+ + \bar{e^o}\gamma_\mu \tau^o Y_2^+ + \bar{\mu^o}\gamma_\mu \tau^o Y_3^+ + h.c.\right) , \quad (8.29)$$

The contribution from these diagrams may be written as

$$c_Y \frac{\alpha_H}{\pi} m_\nu(M_Y)_{ij} \quad , \quad \alpha_H = \frac{g_H^2}{4\pi} , \quad (8.30)$$

$$m_\nu(M_Y)_{ij} \equiv \sum_{k=5,6,7,8} m_k^o\, U_{ik}^o U_{jk}^o\, f(M_Y, m_k^o) \quad (8.31)$$

and $f_{Y_k} = \frac{M_Y^2}{M_Y^2 - m_k^{o\,2}} \ln\frac{M_Y^2}{m_k^{o\,2}} \approx \ln\frac{M_Y^2}{m_k^{o\,2}}$

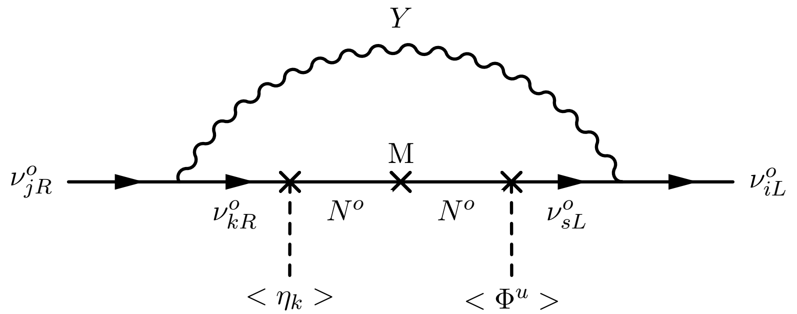

Fig. 8.1. Generic one loop diagram contribution to the Dirac mass term $m_{ij}\ \bar{\nu}_{iL}^o \nu_{jR}^o$. $M = M_D, m_L, m_R$

	ν_{eR}^o	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	N_R^o
$\bar{\nu}_{eL}^o$	$D_{\nu\,14}$	$D_{\nu\,15}$	$D_{\nu\,16}$	0
$\bar{\nu}_{\mu L}^o$	0	$D_{\nu\,25}$	$D_{\nu\,26}$	0
$\bar{\nu}_{\tau L}^o$	0	$D_{\nu\,35}$	$D_{\nu\,36}$	0
$\bar{N}_L^o$	0	0	0	0

Table 8.2. One loop Dirac mass terms $m_{ij}\ \bar{\nu}_{iL}^o\ \nu_{jR}^o$

$$m_\nu(M_{Z_1})_{ij} = \cos\phi\, m_\nu(M_-)_{ij} - \sin\phi\, m_\nu(M_+)_{ij}$$

$$m_\nu(M_{Z_2})_{ij} = \sin\phi\, m_\nu(M_-)_{ij} + \cos\phi\, m_\nu(M_+)_{ij}$$

$$G_{\nu, m\, ij} = \frac{\sqrt{\alpha_2\alpha_3}}{\pi}\, \frac{1}{2\sqrt{3}}\, \cos\phi\, \sin\phi\, [m_\nu(M_-)_{ij} - m_\nu(M_+)_{ij}]$$

$$\mathcal{F}(M_Y) = m_5^o\, u_{11}^2\, f_{Y_5} + m_6^o\, u_{12}^2\, f_{Y_6} + m_7^o\, u_{13}^2\, f_{Y_7} + m_8^o\, u_{14}^2\, f_{Y_8} \tag{8.32}$$

$$\mathcal{G}(M_Y) = m_5^o\, u_{21}^2\, f_{Y_5} + m_6^o\, u_{22}^2\, f_{Y_6} + m_7^o\, u_{23}^2\, f_{Y_7} + m_8^o\, u_{24}^2\, f_{Y_8} \tag{8.33}$$

$$\mathcal{H}(M_Y) = m_5^o\, u_{11}u_{21}\, f_{Y_5} + m_6^o\, u_{12}u_{22}\, f_{Y_6} + m_7^o\, u_{13}u_{23}\, f_{Y_7} + m_8^o\, u_{14}u_{24}\, f_{Y_8} \tag{8.34}$$

$$m_\nu(M_Y)_{15} = \frac{a_1 b_2}{ab}\mathcal{H}(M_Y)\ ; \quad m_\nu(M_Y)_{16} = \frac{a_1 b_3}{ab}\mathcal{H}(M_Y)$$

$$m_\nu(M_Y)_{25} = \frac{a_2 b_2}{ab}\mathcal{H}(M_Y)\ ; \quad m_\nu(M_Y)_{26} = \frac{a_2 b_3}{ab}\mathcal{H}(M_Y)$$

$$m_\nu(M_Y)_{35} = \frac{a_3 b_2}{ab}\mathcal{H}(M_Y)\ ; \quad m_\nu(M_Y)_{36} = \frac{a_3 b_3}{ab}\mathcal{H}(M_Y)$$

$$D_{\nu\,14} = \frac{1}{2}\left[\frac{a_2 b_2}{ab}\mathcal{H}(M_1) + \frac{a_3 b_3}{ab}\mathcal{H}(M_2)\right] \,,$$

$$D_{\nu\,15} = \frac{a_1 b_2}{ab}\,\left[-\frac{1}{4}\mathcal{H}(M_{Z_1}) + \frac{1}{12}\mathcal{H}(M_{Z_2})\right] \,,$$

$$D_{\nu\,25} = \frac{a_2 b_2}{ab}\,\left[\frac{1}{4}\mathcal{H}(M_{Z_1}) + \frac{1}{12}\mathcal{H}(M_{Z_2}) - \mathcal{H}(G_{\nu,m})\right] + \frac{1}{2}\frac{a_3 b_3}{ab}\mathcal{H}(M_3) \,,$$

$$D_{\nu\,36} = \frac{1}{2}\frac{a_2 b_2}{ab}\mathcal{H}(M_3) + \frac{1}{3}\frac{a_3 b_3}{ab}\mathcal{H}(M_{Z_2}) \,,$$

$$D_{\nu\,16} = \frac{a_1 b_3}{ab}\left[-\frac{1}{6}\mathcal{H}(M_{Z_2}) - \mathcal{H}(G_{\nu,m})\right] \,,$$

$$D_{\nu\,26} = \frac{a_2 b_3}{ab}\left[-\frac{1}{6}\mathcal{H}(M_{Z_2}) + \mathcal{H}(G_{\nu,m})\right] \,,$$

$$D_{\nu\,35} = \frac{a_3 b_2}{ab}\left[-\frac{1}{6}\mathcal{H}(M_{Z_2}) + \mathcal{H}(G_{\nu,m})\right] \,,$$

$$\mathcal{H}(G_{\nu,m}) = \frac{\sqrt{\alpha_2\alpha_3}}{\pi}\,\frac{1}{2\sqrt{3}}\,\cos\phi\,\sin\phi\,[\mathcal{H}(M_-) - \mathcal{H}(M_+)]$$

8.6.2 One loop L-handed Majorana masses

Neutrinos also obtain one loop corrections to L-handed and R-handed Majorana masses from the diagrams of Fig. 8.2 and Fig. 8.3, respectively.

A similar procedure as for Dirac Neutrino masses, leads to the one loop Majorana mass terms

$$m_\nu(M_Y)_{11} = \frac{a_1^2}{a^2}\mathcal{F}(M_Y)\,; \quad m_\nu(M_Y)_{12} = \tfrac{a_1 a_2}{a^2}\mathcal{F}(M_Y)$$

$$m_\nu(M_Y)_{13} = \frac{a_1 a_3}{a^2}\mathcal{F}(M_Y) \quad ; \quad m_\nu(M_Y)_{22} = \tfrac{a_2^2}{a^2}\mathcal{F}(M_Y)$$

$$m_\nu(M_Y)_{23} = \frac{a_2 a_3}{a^2}\mathcal{F}(M_Y) \quad ; \quad m_\nu(M_Y)_{33} = \tfrac{a_3^2}{a^2}\mathcal{F}(M_Y)$$

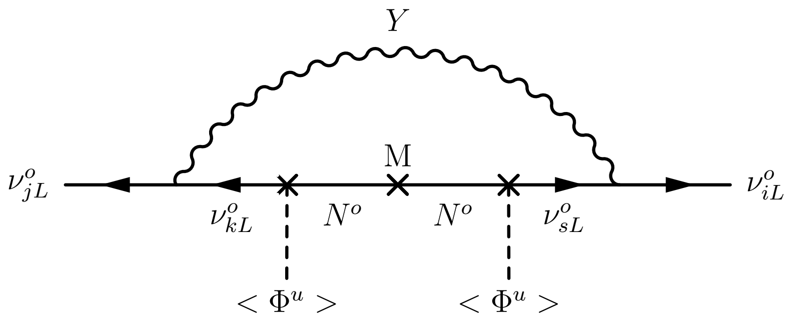

Fig. 8.2. Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij}\ \bar{\nu}^o_{iL}(\nu^o_{jL})^T$. $M = M_D, m_L, m_R$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	0
$\nu^o_{\mu L}$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0
$\nu^o_{\tau L}$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0
N^o_L	0	0	0	0

Table 8.3. One loop L-handed Majorana mass terms $m_{ij}\ \bar{\nu}^o_{iL}\ (\nu^o_{jL})^T$

$$L_{\nu\,11} = \frac{a_1^2}{a^2}\left[\frac{1}{4}\mathcal{F}(M_{Z_1}) + \frac{1}{12}\mathcal{F}(M_{z_2}) + \mathcal{F}(G_{\nu,m})\right]\ ,$$

$$L_{\nu\,22} = \frac{a_2^2}{a^2}\left[\frac{1}{4}\mathcal{F}(M_{Z_1}) + \frac{1}{12}\mathcal{F}(M_{z_2}) - \mathcal{F}(G_{\nu,m})\right]\ ,$$

$$L_{\nu\,33} = \frac{1}{3}\frac{a_3^2}{a^2}\mathcal{F}(M_{z_2})\ ,$$

$$L_{\nu\,12} = \frac{a_1 a_2}{a^2}\left[-\frac{1}{4}\mathcal{F}(M_{Z_1}) + \frac{1}{2}\mathcal{F}(M_1) + \frac{1}{12}\mathcal{F}(M_{z_2})\right]\ ,$$

$$L_{\nu\,13} = \frac{a_1 a_3}{a^2}\left[-\frac{1}{6}\mathcal{F}(M_{Z_2}) + \frac{1}{2}\mathcal{F}(M_2) - \mathcal{F}(G_{\nu,m})\right]\ ,$$

$$L_{\nu\,23} = \frac{a_2 a_3}{a^2}\left[-\frac{1}{6}\mathcal{F}(M_{Z_2}) + \frac{1}{2}\mathcal{F}(M_3) + \mathcal{F}(G_{\nu,m})\right]$$

$$\mathcal{F}(G_{\nu,m}) = \frac{\sqrt{\alpha_2\alpha_3}}{\pi}\,\frac{1}{2\sqrt{3}}\,\cos\phi\,\sin\phi\,[\mathcal{F}(M_-) - \mathcal{F}(M_+)] \tag{8.35}$$

8.6.3 One loop R-handed Majorana masses

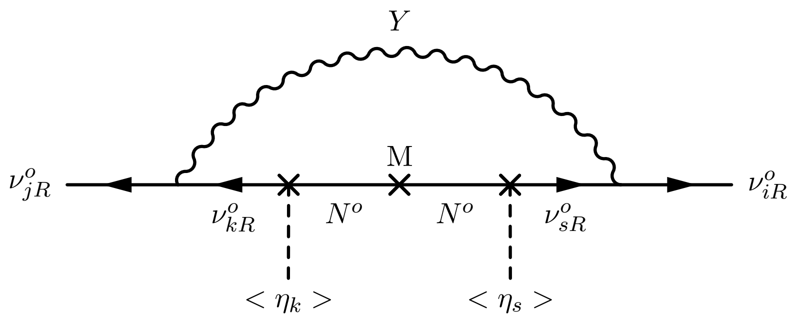

Fig. 8.3. Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij}\ \bar{\nu}^o_{iR}(\nu^o_{jR})^T$. $M = M_D, m_L, m_R$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	$R_{\nu\,55}$	$R_{\nu\,56}$	0
$\nu^o_{\tau R}$	0	$R_{\nu\,56}$	$R_{\nu\,66}$	0
N^o_R	0	0	0	0

Table 8.4. One loop R-handed Majorana mass terms $m_{ij}\ \bar{\nu}^o_{iR}\ (\nu^o_{jR})^T$

$$m_\nu(M_Y)_{55} = \frac{b_2^2}{b^2}\mathcal{G}(M_Y)\ ; \quad m_\nu(M_Y)_{66} = \frac{b_3^2}{b^2}\mathcal{G}(M_Y)$$

$$m_\nu(M_Y)_{56} = \frac{b_2 b_3}{b^2}\mathcal{G}(M_Y)$$

$$R_{\nu\,55} = \frac{b_2^2}{b^2}\left[\frac{1}{4}\mathcal{G}(M_{Z_1}) + \frac{1}{12}\mathcal{G}(M_{z_2}) - \mathcal{G}(G_{\nu,m})\right] ,$$

$$R_{\nu\,66} = \frac{1}{3}\frac{b_3^2}{b^2}\mathcal{G}(M_{z_2}) ,$$

$$R_{\nu\,56} = \frac{b_2 b_3}{b^2}\left[-\frac{1}{6}\mathcal{G}(M_{Z_2}) + \frac{1}{2}\mathcal{G}(M_3) + \mathcal{G}(G_{\nu,m})\right]$$

$$\mathcal{G}(G_{\nu,m}) = \frac{\sqrt{\alpha_2\alpha_3}}{\pi}\frac{1}{2\sqrt{3}}\cos\phi\,\sin\phi\,[\mathcal{G}(M_-) - \mathcal{G}(M_+)]$$

Thus, in the Ψ_ν^o basis, we may write the one loop contribution for neutrinos as

$$\mathcal{M}^o_{1\,\nu} =$$

$$\begin{pmatrix} L_{\nu\,11} & L_{\nu\,12} & L_{\nu\,13} & D_{\nu\,14} & D_{\nu\,15} & D_{\nu\,16} & 0 & 0 \\ L_{\nu\,12} & L_{\nu\,22} & L_{\nu\,23} & 0 & D_{\nu\,25} & D_{\nu\,26} & 0 & 0 \\ L_{\nu\,13} & L_{\nu\,23} & L_{\nu\,33} & 0 & D_{\nu\,35} & D_{\nu\,36} & 0 & 0 \\ D_{\nu\,14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ D_{\nu\,15} & D_{\nu\,25} & D_{\nu\,35} & 0 & R_{\nu\,55} & R_{\nu\,56} & 0 & 0 \\ D_{\nu\,16} & D_{\nu\,26} & D_{\nu\,36} & 0 & R_{\nu\,56} & R_{\nu\,66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{8.36}$$

8.6.4 Neutrino mass matrix up to one loop

Finally, we obtain the Majorana mass matrix for neutrinos up to one loop

$$\mathcal{M}_\nu = (U_\nu^o)^T \mathcal{M}^o_{1\,\nu} U_\nu^o + Diag(0,0,0,0,m_5^o,m_6^o,m_7^o,m_8^o) , \tag{8.37}$$

where explicitly

$$\mathcal{M}_\nu =$$

$$\begin{pmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\ N_{12} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\ N_{13} & N_{23} & 0 & 0 & N_{35} & N_{36} & N_{37} & N_{38} \\ N_{14} & N_{24} & 0 & N_{44} & N_{45} & N_{46} & N_{47} & N_{48} \\ N_{15} & N_{25} & N_{35} & N_{45} & N_{55}+m_5^o & N_{56} & N_{57} & N_{58} \\ N_{16} & N_{26} & N_{36} & N_{46} & N_{56} & N_{66}+m_6^o & N_{67} & N_{68} \\ N_{17} & N_{27} & N_{37} & N_{47} & N_{57} & N_{67} & N_{77}+m_7^o & N_{78} \\ N_{18} & N_{28} & N_{38} & N_{48} & N_{58} & N_{68} & N_{78} & N_{88}+m_8^o \end{pmatrix} \tag{8.38}$$

Majorana L-handed:

$$N_{11} = \frac{a_1^2 a_2^2}{a_p^2 a^2}(F_{Z_1} - F_1) \tag{8.39}$$

$$N_{12} = -\frac{a_1 a_2 a_3}{2a^3}[\frac{a_2^2 - a_1^2}{a_p^2}(F_{Z_1} - F_1) + F_2 - F_3 - 6F_m] \tag{8.40}$$

$$N_{22} = \frac{a_3^2}{a^2}\left[\frac{1}{4}\frac{(a_2^2 - a_1^2)^2}{a_p^2\, a^2}(F_{Z_1} - F_1) + \frac{a_2^2}{a^2}(F_2 - F_3) + \frac{a_p^2}{4\,a^2}(F_1 + 3F_{Z_2} - 4F_2) - 3\frac{a_2^2 - a_1^2}{a^2}F_m\right] \tag{8.41}$$

Dirac:

$$N_{13} = \frac{a_2}{2\,a_p}\,(\frac{a_2\,b_2}{a\,b}H_1 + \frac{a_3\,b_3}{a\,b}H_2) = q_{11} \tag{8.42}$$

$$N_{14} = -\frac{a_1\,b_3}{2\,a_p\,b}\,(\frac{a_2\,b_2}{a\,b}H_{Z_1} + \frac{a_3\,b_3}{a\,b}H_3 - 6\frac{a_2\,b_2}{a\,b}H_m) = q_{12} \tag{8.43}$$

$$N_{23} = \frac{a_1\,a_3}{2\,a_p\,a}\,(\frac{a_2\,b_2}{a\,b}H_1 + \frac{a_3\,b_3}{a\,b}H_2) = q_{21} \tag{8.44}$$

$$N_{24} = \frac{a_2\,(a_p^2\,b_2^2 + a_3^2\,b_3^2)}{2\,a_p\,a^2\,b^2}H_3 + \frac{(a_2^2 - a_1^2)a_3\,b_2\,b_3}{4\,a_p\,a^2\,b^2}H_{Z_1} + \frac{3\,a_p\,a_3\,b_2\,b_3}{4\,a^2\,b^2}H_{Z_2} - \frac{3\,a_2^2\,a_3\,b_2\,b_3}{a_p\,a^2\,b^2}H_m = q_{22} \tag{8.45}$$

Majorana R-handed:

$$N_{44} = \frac{b_2^2\, b_3^2}{4\, b^4}\,(\, G_{Z_1} + 3G_{Z_2} - 4G_3 - 12G_m\,) \tag{8.46}$$

Majorana L-handed and Dirac:

$$N_{15} = -F_{15}\, u_{11} + q_{13}\, u_{21} \quad ; \quad N_{16} = -F_{15}\, u_{12} + q_{13}\, u_{22} \tag{8.47}$$

$$N_{17} = -F_{15}\, u_{13} + q_{13}\, u_{23} \quad ; \quad N_{18} = -F_{15}\, u_{14} + q_{13}\, u_{24} \tag{8.48}$$

$$F_{15} = \frac{a_1\, a_2}{2\, a_p\, a} \left[\frac{a_2^2 - a_1^2}{a^2}(F_{Z_1} - F_1) + \frac{a_3^2}{a^2}(F_3 - F_2) + 2\,\frac{(2\, a_3^2 - a_p^2)}{a^2}\, F_m \right]$$

$$q_{13} = -\frac{a_1\, b_2}{2a_p\, b} \left[\frac{a_2\, b_2}{a\, b} H_{Z_1} + \frac{a_3\, b_3}{a\, b} H_3 - 2\frac{a_2}{a}\, \frac{b_2^2 - 2b_3^2}{b_2\, b}\, H_m \right]$$

$$N_{25} = F_{25}\, u_{11} + q_{23}\, u_{21} \quad ; \quad N_{26} = F_{25}\, u_{12} + q_{23}\, u_{22} \tag{8.49}$$

$$N_{27} = F_{25}\, u_{13} + q_{23}\, u_{23} \quad ; \quad N_{28} = F_{25}\, u_{14} + q_{23}\, u_{24} \tag{8.50}$$

$$F_{25} = \frac{a_3}{4\, a_p\, a^4}\, \left[\, (a_2^2 - a_1^2)^2\, (F_{Z_1} - F_1) + 2\, a_2^2 (a_3^2 - a_p^2)\, (F_3 - F_2) - a_p^4\, (F_{Z_2} - F_1) - 2\, a_p^2 (a_3^2 - a_p^2)\, (F_{Z_2} - F_2) + 4\, (a_2^2 - a_1^2)\, (a_3^2 - 2a_p^2)\, F_m \,\right]$$

$$q_{23} = \frac{a_2\, (a_3^2 - a_p^2\,) b_2\, b_3}{2\, a_p\, a^2\, b^2} H_3 + \frac{(a_2^2 - a_1^2) a_3\, b_2^2}{4\, a_p\, a^2\, b^2} H_{Z_1} + \frac{a_p\, a_3\, (b_2^2 - 2\, b_3^2)}{4\, a^2\, b^2} H_{Z_2} - \frac{a_3\, [a_p^2\, b^2 + a_2^2 (b_2^2 - 2\, b_3^2)]}{a_p\, a^2\, b^2} H_m$$

Dirac:

$$N_{35} = q_{31}\, u_{11}\;,\; N_{36} = q_{31}\, u_{12}\;,\; N_{37} = q_{31}\, u_{13}\;,\; N_{38} = q_{31}\, u_{14} \tag{8.51}$$

$$q_{31} = \frac{a_1}{2a}\, (\frac{a_2\, b_2}{a\, b} H_1 + \frac{a_3\, b_3}{a\, b} H_2)$$

Dirac and Majorana R-handed:

$$N_{45} = q_{32}\, u_{11} + G_{45}\, u_{21} \quad , \quad N_{46} = q_{32}\, u_{12} + G_{45}\, u_{22} \tag{8.52}$$

$$N_{47} = q_{32}\, u_{13} + G_{45}\, u_{23} \quad , \quad N_{48} = q_{32}\, u_{14} + G_{45}\, u_{24} \tag{8.53}$$

$$q_{32} = -\frac{a_2\, a_3\, (b_2^2 - b_3^2)}{2\, a^2\, b^2}\, H_3 + \frac{(a_2^2 - a_1^2)\, b_2\, b_3}{4\, a^2\, b^2}\, H_{Z_1} - \frac{(2a_3^2 - a_p^2) b_2\, b_3}{4\, a^2\, b^2}\, H_{Z_2} + \frac{(a_3^2 + a_1^2 - 2a_2^2)\, b_2\, b_3}{a^2\, b^2}\, H_m$$

$$G_{45} = \frac{b_2\, b_3}{4\, b^2} \left[\frac{b_2^2 - 2b_3^2}{b^2}\, (G_{Z_2} - G_3) + \frac{b_2^2}{b^2}\, (G_{Z_1} - G_3) - 4\frac{(2\, b_2^2 - b_3^2)}{b^2}\, G_m \right]$$

Majorana L-handed, Dirac and Majorana R-handed:

$$N_{55} = F_{55}\, u_{11}^2 + 2\, q_{33}\, u_{11}\, u_{21} + G_{55}\, u_{21}^2 \tag{8.54}$$

$$N_{56} = F_{55}\, u_{11}\, u_{12} + q_{33}\, (u_{11}\, u_{22} + u_{12}\, u_{21}) + G_{55}\, u_{21}\, u_{22} \tag{8.55}$$

$$N_{57} = F_{55}\, u_{11}\, u_{13} + q_{33}\, (u_{11}\, u_{23} + u_{13}\, u_{21}) + G_{55}\, u_{21}\, u_{23} \tag{8.56}$$

$$N_{58} = F_{55}\, u_{11}\, u_{14} + q_{33}\, (u_{11}\, u_{24} + u_{14}\, u_{21}) + G_{55}\, u_{21}\, u_{24} \tag{8.57}$$

$$N_{66} = F_{55}\, u_{12}^2 + 2\, q_{33}\, u_{12}\, u_{22} + G_{55}\, u_{22}^2 \tag{8.58}$$

$$N_{67} = F_{55}\, u_{12}\, u_{13} + q_{33}\, (u_{13}\, u_{22} + u_{12}\, u_{23}) + G_{55}\, u_{22}\, u_{23} \tag{8.59}$$

$$N_{68} = F_{55}\, u_{12}\, u_{14} + q_{33}\, (u_{14}\, u_{22} + u_{12}\, u_{24}) + G_{55}\, u_{22}\, u_{24} \tag{8.60}$$

$$N_{77} = F_{55}\, u_{13}^2 + 2\, q_{33}\, u_{13}\, u_{23} + G_{55}\, u_{23}^2 \tag{8.61}$$

$$N_{78} = F_{55}\, u_{13}\, u_{14} + q_{33}\, (u_{14}\, u_{23} + u_{13}\, u_{24}) + G_{55}\, u_{23}\, u_{24} \tag{8.62}$$

$$N_{88} = F_{55}\, u_{14}^2 + 2\, q_{33}\, u_{14}\, u_{24} + G_{55}\, u_{24}^2 \tag{8.63}$$

$$F_{55} = \frac{a_1^2\, a_2^2}{a^4}\, F_1 + \frac{a_1^2\, a_3^2}{a^4}\, F_2 + \frac{a_2^2\, a_3^2}{a^4}\, F_3 + \frac{(a_2^2 - a_1^2)^2}{4\, a^4}\, F_{Z_1} + \frac{(2a_3^2 - a_p^2)^2}{12\, a^4}\, F_{Z_2} + \frac{(a_2^2 - a_1^2)\,(2\, a_3^2 - a_p^2)}{a^4}\, F_m$$

$$q_{33} = \frac{a_2\, a_3\, b_2\, b_3}{a^2\, b^2}\, H_3 + \frac{(a_2^2 - a_1^2)\, b_2^2}{4\, a^2\, b^2}\, H_{Z_1} - \frac{(2a_3^2 - a_p^2)\,(b_2^2 - 2b_3^2)}{12\, a^2\, b^2}\, H_{Z_2} + \frac{a_3^2\, b_2^2 - a_p^2\, b_3^2 - a_2^2\,(b_2^2 - 2\, b_3^2)}{a^2\, b^2}\, H_m$$

$$G_{55} = \frac{b_2^2\, b_3^2}{b^4}\, G_3 + \frac{b_2^4}{4\, b^4}\, G_{Z_1} + \frac{(b_2^2 - 2b_3^2)^2}{12\, b^4}\, G_{Z_2} - \frac{b_2^2\,(b_2^2 - 2b_3^2)}{b^4}\, G_m$$

8.6.5 $(V_{CKM})_{4\times 4}$ and $(V_{PMNS})_{4\times 8}$ mixing matrices

Within this SU(3) family symmetry model, the transformation from massless to physical mass fermions eigenfields for quarks and charged leptons is

$$\psi_L^o = V_L^o\, V_L^{(1)}\, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\, \Psi_R\,,$$

and for neutrinos $\Psi_\nu^o = U_\nu^o\, U_\nu\, \Psi_\nu$. Recall now that vector like quarks, Eq.(8.1), are $SU(2)_L$ weak singlets, and hence, they do not couple to W boson in the interaction basis. In this way, the interaction of L-handed up and down quarks; ${f_{uL}^o}^T = (u^o, c^o, t^o)_L$ and ${f_{dL}^o}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}}\, \bar{f^o}_{uL}\gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\, [(V_{uL}^o\, V_{uL}^{(1)})_{3\times 4}]^T\, (V_{dL}^o\, V_{dL}^{(1)})_{3\times 4}\, \gamma_\mu \Psi_{dL}\, W^{+\mu}\,, \tag{8.64}$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times 4} = [(V_{uL}^o\, V_{uL}^{(1)})_{3\times 4}]^T\, (V_{dL}^o\, V_{dL}^{(1)})_{3\times 4} \tag{8.65}$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4\times 8} = [(V_{eL}^o\, V_{eL}^{(1)})_{3\times 4}]^T\, (U_\nu^o\, U_\nu)_{3\times 8} \tag{8.66}$$

8.7 Numerical results

To illustrate the spectrum of masses and mixing, let us consider the following fit of space parameters at the M_Z scale [27]

Using the strong hierarchy for quarks and charged leptons masses[15], here we report the fermion masses and mixing, coming out from a global fit of the parameter space.

In the approach $\alpha_2 \approx \alpha_3 = \alpha_H$, we take the input values

$$M_1 = 10\,\text{TeV} \quad , \quad M_2 = 1\,\text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.05$$

for the M_1, M_2 horizontal boson masses, Eq.(8.7), and the $SU(3)$ coupling constant, respectively, and the ratio of electroweak VEV's: V_i from Φ^d, and v_i from Φ^u

$$\frac{V_1}{V_2} = 0.09981 \quad , \quad \frac{\sqrt{V_1^2 + V_2^2}}{V_3} = 0.54326$$

$$\frac{v_1}{v_2} = 0.1 \quad , \quad \frac{\sqrt{v_1^2 + v_2^2}}{v_3} = 0.5$$

8.7.1 Quark masses and mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 7933.76 \\ 0 & 0 & 0 & 79337.6 \\ 0 & 0 & 0 & 159467. \\ 0 & 1.18613\times 10^6 & -841128. & 374542. \end{pmatrix} \mathrm{MeV}\,, \tag{8.67}$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} -1.40509 & 187.442 & -66.8139 & -255.74 \\ -0.125675 & -609.844 & 408.793 & 1564.71 \\ -0.062809 & -1197.67 & -172100. & 1825.79 \\ -0.001885 & -35.9461 & 14.3165 & 1.502\times 10^6 \end{pmatrix} \mathrm{MeV} \tag{8.68}$$

and the u-quark masses

$$(\, m_u \,,\, m_c \,,\, m_t \,,\, M_U \,) = (\, 1.3802 \,,\, 640.801 \,,\, 172105 \,,\, 1.502\times 10^6 \,)\, \mathrm{MeV} \tag{8.69}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 1740.94 \\ 0 & 0 & 0 & 17442.3 \\ 0 & 0 & 0 & 32265.8 \\ 0 & 70019.9 & -41383.4 & 910004 \end{pmatrix} \mathrm{MeV} \tag{8.70}$$

$$\mathcal{M}_d = \begin{pmatrix} 3.09609 & 28.1593 & -47.4565 & -4.23475 \\ 0.271539 & -40.5966 & 215.617 & 19.2404 \\ 0.147401 & -176.235 & -2846.26 & 37.484 \\ 0.005900 & -7.05504 & 16.8159 & 914365. \end{pmatrix} \mathrm{MeV} \tag{8.71}$$

$$(\, m_d \,,\, m_s \,,\, m_b \,,\, M_D \,) = (\, 2.82 \,,\, 61.9998 \,,\, 2860 \,,\, 914365 \,)\, \mathrm{MeV} \tag{8.72}$$

and the quark mixing

$$V_{CKM} = \begin{pmatrix} 0.974352 & 0.225001 & 0.003647 & 0.000410 \\ -0.224958 & 0.973502 & 0.041031 & -0.001417 \\ -0.005632 & 0.040662 & -0.997868 & -0.039994 \\ 0.000576 & -0.002325 & 0.031130 & 0.001251 \end{pmatrix} \tag{8.73}$$

8.7.2 Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 28340.3 \\ 0 & 0 & 0 & 283940. \\ 0 & 0 & 0 & 525249. \\ 0 & 17105.4 & -11570.9 & 5.94752 \times 10^6 \end{pmatrix} \text{MeV} \tag{8.74}$$

$$\mathcal{M}_e = \begin{pmatrix} -0.499137 & 29.7086 & -43.9181 & -0.15097 \\ -0.043776 & -72.8148 & 238.953 & 0.821414 \\ -0.023663 & -183.913 & -1720.65 & 1.18425 \\ -0.002378 & -18.4839 & 34.6241 & 5.977 \times 10^6 \end{pmatrix} \text{MeV} \tag{8.75}$$

fit the charged lepton masses:

$$(m_e\,,\, m_\mu\,,\, m_\tau\,,\, M_E) = (0.486\,,\, 102.7\,,\, 1746.17\,,\, 5.977 \times 10^6\,)\,\text{MeV}$$

and the mixing

$$V_{e\,L}^o\, V_{e\,L}^{(1)} =$$

$$\begin{pmatrix} 0.968866 & 0.24054 & -0.0584594 & 0.00474112 \\ 0.205175 & -0.912554 & -0.350561 & 0.0475013 \\ -0.138557 & 0.330471 & -0.929446 & 0.0878703 \\ -0.00217545 & 0.0132348 & 0.0990967 & 0.994987 \end{pmatrix} \tag{8.76}$$

8.7.3 Neutrinos:

$$\mathcal{M}_\nu^o = \text{eV}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 53594.6 & 44137.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 535946. & 441372. \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.07 \times 10^6 & 887147. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.80 \times 10^6 & 1.49 \times 10^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & -886604. & -730152. \\ 53594.6 & 535946. & 1.07 \times 10^6 & 0 & 1.8097 \times 10^6 & -886604. & 1.97 \times 10^8 & 4.88 \times 10^8 \\ 44137.2 & 441372. & 887147. & 0 & 1.49 \times 10^6 & -730152. & 4.88 \times 10^8 & 7.02 \times 10^8 \end{pmatrix} \tag{8.77}$$

$$\mathcal{M}_\nu = \text{eV}$$

$$\begin{pmatrix} -0.0119 & 0.0527 & 0.0227 & -0.0878 & -0.0693 & 0.1674 & -0.0016 & 0.0004 \\ 0.0527 & -0.036 & 0.002 & 0.068 & 0.043 & -0.748 & 0.007 & -0.002 \\ 0.0227 & 0.002 & 0. & 0. & 0.0008 & 0.0005 & -5.2\times 10^{-6} & 1.5\times 10^{-6} \\ -0.0878 & 0.068 & 0. & -0.125 & -0.1218 & 1.282 & -0.012 & 0.003 \\ -0.0693 & 0.043 & 0.0008 & -0.121 & 3.206 & -0.7430 & 0.0074 & -0.0021 \\ 0.1674 & -0.748 & 0.0005 & 1.282 & -0.7430 & 1749.96 & 0.0003 & -0.0001 \\ -0.0016 & 0.007 & -5.2\times 10^{-6} & -0.012 & 0.0074 & 0.0003 & -1.\times 10^{8} & 1.1\times 10^{-6} \\ 0.0004 & -0.002 & 1.5\times 10^{-6} & 0.003 & -0.0021 & -0.0001 & 1.1\times 10^{-6} & 1.\times 10^{9} \end{pmatrix} \tag{8.78}$$

generates the neutrino mass eigenvalues

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = \text{eV} \\ (\,0,\,-0.0085\,,\,0.049\,,\,-0.22\,,\,3.21\,,\,1749.96\,,\,-1\times 10^{8}\,,\,1\times 10^{9}\,) \tag{8.79}$$

the squared mass differences

$$m_2^2 - m_1^2 \approx 0.0000723\,\text{eV}^2 \quad , \quad m_3^2 - m_1^2 \approx 0.0024\,\text{eV}^2 \tag{8.80}$$

$$m_4^2 - m_1^2 \approx 0.0492\,\text{eV}^2 \quad , \quad m_5^2 - m_1^2 \approx 10.3182\,\text{eV}^2 \tag{8.81}$$

and the lepton mixing matrix

$$U_{PMNS} =$$

$$\begin{pmatrix} 0.2104 & 0.3520 & 0.8658 & -0.2861 & 0.0060 & 0.0053 & 0.00005 & 0.00001 \\ -0.8282 & 0.0030 & 0.0186 & -0.5478 & 0.1038 & -0.0507 & -0.0005 & -0.0001 \\ 0.0807 & 0.0041 & 0.0052 & 0.0881 & 0.8475 & -0.5074 & -0.0050 & -0.0014 \\ 0.0034 & 0.0003 & 0.0011 & -0.0021 & -0.0857 & 0.0512 & 0.0005 & 0.0001 \end{pmatrix} \tag{8.82}$$

8.8 Conclusions

We have reported a low energy parameter space, within a local $SU(3)$ Family symmetry model, which combines tree level "Dirac See-saw" mechanisms and radiative corrections to implement a successful hierarchical spectrum, for charged fermion masses and quark mixing. In section 8.7 we illustrated the predicted values for quark and charged lepton masses at the the M_Z scale[27], and a non-unitary quark mixing matrix $(V_{CKM})_{4\times 4}$ within allowed values reported in PDG 2012 [28], coming from a parameter space with the horizontal gauge boson masses within (1-10) TeV, the $SU(2)_L$ weak singlet vector-like fermion masses $M_D \approx$

914.365 GeV, $M_U \approx 1.5$ TeV, $M_E \approx 5.97$ TeV, the neutrino masses in Eq.(8.79), including two light sterile neutrinos, and the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 7.23 \times 10^{-5}\,\text{eV}^2$, $m_3^2 - m_1^2 \approx 2.4 \times 10^{-3}\,\text{eV}^2$, $m_4^2 - m_1^2 \approx 0.049\,\text{eV}^2$, $m_5^2 - m_1^2 \approx 10.3\,\text{eV}^2$.

Hence the new particles introduced in this model are within reach at the current LHC and neutrino oscillation experiments.

It is worth to comment here that the symmetries and the transformation of the fermion and scalar fields, all together, forbid tree level Yukawa couplings between ordinary standard model fermions. Consequently, the flavon scalar fields introduced to break the symmetries: Φ^u, Φ^d, η_2 *and* η_3, *couple only ordinary fermions to their corresponding vector like fermion at tree level. Thus, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which as is shown in* $(V_{CKM})_{4\times4}$, *Eq.(8.73), may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.*

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9 Primordial Black Hole Clusters and Their Evolution

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Abstract. A possibility of pregalactic seeds of the Active Galactic Nuclei can be a nontrivial cosmological consequence of particle theory. Such seeds can appear as Primordial Black Hole (PBH) clusters, formed in the succession of phase transitions with spontaneous and then manifest breaking of the global U(1) symmetry. If the first phase transition takes place at the inflationary stage, a set of massive closed walls may be formed at the second phase transition and the collapse of these closed walls can result in formation of PBH clusters. We present the results of our studies of the evolution of such PBH Clusters.

Povzetek. Prvinske črne luknje, ki se oblikujejo v zaporedju faznih prehodov s spontano in nato neposredno zlomitvijo globalne simetrije U(1), lahko pomenijo zasnovo aktivnih jeder galaksij. Če poteka prvi fazni prehod v času, ko se vesolje eksponentno širi (inflacijsko širjenje vesolja), se lahko ob drugem faznem prehodu tvori množica zaprtih sten, kolaps le teh pa lahko vodi k tvorbi gruč prvinskih črnih lukenj. Predstavimo rezultate našega študija razvoja gruč prvinskih črnih lukenj.

9.1 Introduction

Primordial Black Holes (PBHs) are a very sensitive cosmological probe for physics phenomena occurring in the early Universe. They could be formed by many different mechanisms, reflecting the fundamental structure of particle theory and nonhomogeneity of very early Universe. Here after a brief review of mechanisms of PBH formation we consider a nontrivial possibility of clusters of massive PBHs. The evolution of such clusters can provide pregalactic seeds of Active Galactic Nuclei (AGN) and we discuss various aspects of this evolution in the present paper.

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9.2 PBH Formation

Primordial Black Holes can be formed in many different ways [1,2], such as initial density inhomogeneities, first order and non-equilibrium second order phase transitions, etc. Let us give a brief review of these possibilities.

9.2.1 PHB formation in initial density inhomogeneities

A probability for fluctuation of 1 for metric fluctuations distributed according to Gaussian law with dispersion $\langle \delta^2 \rangle \ll 1$ is determined by exponentially small tail of high amplitude part of this distribution. In non-Gaussian fluctuations, this process can be more suppressed [3]. In the space, described by

$$p = \gamma \varepsilon, 0 \leq \gamma \leq 1 \tag{9.1}$$

equation of state a probability to form black hole from fluctuation within cosmological horizon is given by [4,5]

$$W_{PBH} = e^{-\frac{\gamma^2}{2\langle \delta^2 \rangle}} \tag{9.2}$$

It provides exponential sensitivity of PBH spectrum to softening of equation of state in early Universe ($\gamma \to 0$) or to increase of ultraviolet part of spectrum of density fluctuations ($\delta^2 \to 1$). These phenomena can appear as cosmological consequence of particle theory (see [4,5] for review of this and some other mechanisms of PBH formation and for references).

9.2.2 PBH from non-equilibrium second order phase transition

The mechanism of PBH formation in the non-equilibrium second order phase transition is of special interest, since it can provide formation of massive and even Supermassive PBHs. PBHs are produced in this mechanism by self-collapsing of closed domain walls. If there are two vacuum states of a system, there are two possibilities to populate that states in the early Universe: under the usual circumstances of thermal phase transition the Universe contains both states populated with equal probability. The other possibility is beyond the pure thermodynamical equilibrium condition, when the two vacuum states are populated with islands of the less probable vacuum, surrounded by the sea of another, more preferable, vacuum.

It is necessary to redefine effectively the correlation length of the scalar field that drives a phase transition and consequently the formation of topological defects and the only necessary ingredient for that is the existence of an effectively flat direction(s), along which the scalar potential vanishes during inflation.

The background deSitter fluctuations of such effectively massless scalar field could provide non-equilibrium redefinition of correlation length and give rise to the islands of one vacuum in the sea of another one. In spite of such redefinition the phase transition itself takes place deeply in the Friedman-Robertson-Walker (FRW) epoch. After the phase transition two vacua are separated by a wall, and such a closed wall, separating the island with the less probable vacuum, can be very large.

At some moment after crossing horizon the walls start shrinking due to surface tension. As a result, if the wall does not release the significant fraction of its energy in the form of outward scalar waves, almost the whole energy of such closed wall can be concentrated in a small volume within its gravitational radius what is the necessary condition for PBH formation.

The mass spectrum of the PBHs which can be created by such a way depends on the scalar field potential which parameterizes the flat direction during inflation and triggers the phase transition at the FRW stage.

We consider the Universe that, due to the existence of an inflaton, goes through a period of inflation and then settles down to the standard FRW geometry. Then we introduce a complex scalar field φ, not the inflaton, with a large radial mass $\sqrt{\lambda f} > H_i$ that has got Mexican hat potential

$$V(\varphi) = \lambda \left(|\varphi|^2 - \frac{f^2}{2} \right), \tag{9.3}$$

which provides the U(1) symmetry spontaneous breaking in the period of inflation, corresponding to the scales of the modern cosmological horizon. Therefore we deal only with the phase of that complex field $\theta = \frac{\varphi}{f}$, which parameterizes potential

$$V = \Lambda^4 \left(1 - \cos\frac{\phi}{f} \right) \tag{9.4}$$

Under this condition we come to the conclusion, that the correlation length of second order phase transition with spontaneously broken U(1) symmetry exceeds the present cosmological horizon, and all global U(1) strings are beyond our horizon. If we assume $m \ll H_i$ then this implies that during inflation the potential energy of field φ is much smaller than the cosmological friction term what justifies neglecting the potential until the Universe goes deeply into the FRW phase. During inflation and long afterward, H_i is very large (by assumption) compared to the potential (2). It follows that we can drop the gradient term in the equation of motion [6]

$$\ddot{\theta} + 3H\dot{\theta} + \frac{dV}{d\theta} = 0 \tag{9.5}$$

and resulting equation is solved by $\theta_0 = \theta_{N_{max}}$, where $\theta_{N_{max}}$ is an arbitrary constant. In the standard assumption, our present horizon has been nucleated at the N_{max} e-folds before the end of inflationary epoch, being embedded in an enormous inflation horizon, created by exponential blow up of a single casual horizon. It follows that $\theta_{N_{max}}$ will be the same over the inter inflationary horizon. Without loss of generality, we put $\theta_{N_{max}} < \pi$ and considering the quantum fluctuations of the phase θ at the deSitter background. There are quantum fluctuations produced on the vacuum state of θ due to the boundary conditions of deSitter space. These fluctuations are sometimes referred to as contribution to the "Hawking temperature" of deSitter space but, there are no true thermal effects. It makes the dynamics of phase θ strongly non-equilibrium leading to the non-thermal distribution of scales populated with different vacuums in the postinflationary Universe. The average amplitude of such fluctuations for massless field generated during each time interval H_i^{-1} is $\delta\theta = \frac{H_i}{2\pi f}$. The total number of steps during time interval Δt is given by $N = H_i \Delta t$ - looks like the one-dimensional Brownian motion. Each domain is characterized by average phase value $\theta_{Nmax-1} = \theta_{Nmax} \pm \delta\theta$.

In the half of these domains the phases evolve toward π while in the other domains they move toward zero. This process is duplicated in each volume of size H^{-1} during next e-fold. Now at any given scale $l = k^{-1}$ the size of distribution of the phase value θ can be described by Gaussian law [7]

$$P(\theta_l, l) = \frac{1}{\sqrt{2\pi}\sigma_l} \exp\left(-\frac{(\theta_{Nmax} - \theta_l)^2}{2\sigma_l^2}\right) \tag{9.6}$$

It is recommended for more information to address the papers [1,6,8,9].

9.2.3 Initial PBH Mass spectrum

Initial mass spectrum $n\,(m, t = 0)$ depends of parameter f and Λ. [6] In addition, is a numerical solution. There is another way: one can describe this system by Ito's equation. One-dimensional Brownian motion in the terms of stochastic equations is an Ornstein-Uhlenbeck process. [10,11] Using this mathematical framework, one can find the analytical solution of Ito's equation and obtain initial mass spectrum as an analytical formula $n_0 = n_{f,\Lambda}\,(m, t = 0)$. This method is in the process of development.

9.3 Clusters of PHBs

According to the 2^{nd} order phase transitions mechanism, PBH appears as a sufficiently large cluster, which could collapse into one large Super Massive Black Hole (SMBH) - the Active Galactic Nucleus (AGN) of the future galaxy.

9.3.1 PBH Cluster dynamics

By analogy with the star cluster [12] with the difference that the black holes can merge into one, the following processes are significant:

- BH collisions $\rightarrow$ BH merging and as a result $N_{BH} \rightarrow 1$
- Flying-out BH from cluster $\rightarrow$ reducing the mass of the cluster [13,14]
- Dynamical friction $\rightarrow$ lower Maxwell distribution [15]

Dynamical friction mostly contributes into the $\langle\sigma v\rangle$ of the collision process. [15] The equation describing the dynamics of the BH Cluster is a modification of Smoluchowski (or Kolmogorov-Feller) equation and runs as follows

$$\begin{aligned} n' = & \int_0^M n\,(m-\mu, t)\,\langle\sigma v\rangle_{m-\mu,m}\,d\mu \\ -n\,(m,t) & \left(\int_0^\infty n\,(\mu,t)\,\langle\sigma v\rangle_{\mu,m}\,d\mu + \int_0^\infty n\,(\mu,t)\,\Lambda\,(m,\mu)\,\mu^2 d\mu\right), \end{aligned} \tag{9.7}$$

where M is total initial cluster mass, $\Lambda\,(m,\mu)$ one can find in [14]. Let us consider numerical solution of that equation for arbitrary initial parameters:

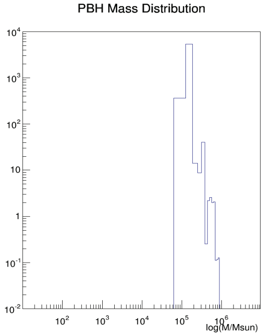

Fig. 1 PBH mass spectrum, only merge effect

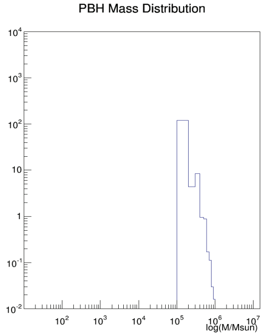

Fig. 2 PBH mass spectrum, merge and fly-out effect

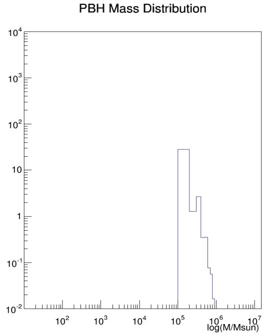

Fig 3. PBH mass spectrum, merge,fly-out and dynamical fricion effect

Numerical solution of that equation shows, that there is the “trend” to the decrease of BHs with larger masses. However, the numerical solution is indispensable because of the unknown initial parameters f and Λ of the model. Exact analytical solution of that equation is overly precise and is a very nontrivial exercise. If a single SMBH is supposed to be the result, one needs to get a solution of that equation as

$$n(m,t) = \delta(m - m_{SMBH})\chi(t - t_{gen}) \tag{9.8}$$

Substituting this partial solution into the equation of the PBH cluster dynamics, one can obtain the timescale of the process and the mass of the resulting SMBH as a functionals of initial conditions:

$$m_{SMBH} = F\,[n\,(m, t = 0)] \tag{9.9}$$

$$t_{gen} = G\,[n(m, t = 0)] \tag{9.10}$$

The calculated values of t_{gen} and m_{SMBH} can be confronted with the observational data, putting constraints on the fundamental physical scales f and Λ.

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10 *Spin-Charge-Family* Theory is Explaining Appearance of Families of Quarks and Leptons, of Higgs and Yukawa Couplings

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Abstract. The so far observed three families of quarks and leptons, the vector gauge fields of the fermions charges and the scalar Higgs responsible for masses of fermions and weak bosons, all these confirming the *standard model*, make most of physicists to declare that the Higgs was the last missing particle to be confirmed. But can this at all be true? Is it not self evident that there must be additional scalar fields which manifest effectively the appearance of the Yukawa couplings and that the Yukawa couplings can only be understood if we understand the origin of families? The *spin-charge-family* theory [1–4] is offering a possible explanation for the origin of families and also for several scalar fields, which are responsible for masses of fermions and weak vector boson fields. The theory is offering the explanation also for other assumptions of the *standard model*. The theory predicts at the observable regime two decoupled groups of four families. The fourth family, coupled to the measured three, will be observed at the LHC. The fifth family is the candidate for the dark matter. Masses of each group of the four families and of each of the two corresponding vector bosons are triggered by a different group of condensates. The theory explains why the scalar fields are doublets with respect to the weak charge, while they are triplets with respect to the family groups. The accuracy with which the fourth family masses can be predicted in this theory depends strongly on the accuracy with which the two mixing matrices will be measured. Correspondingly might the properties of the scalar fields (the low energy effective representation of which is the observed Higgs) be estimated also from the mass matrices of quarks and leptons. The main progress this year in the *spin-charge-family* theory is that I can "pedagogically" explain: i. Why the scalar fields are doublets with respect to the weak charge, carrying in addition the appropriate hyper charge. ii. Why the two groups of four families have so different masses although both groups of the scalar fields contributing to masses of the upper and lower four families, contribute also to masses of the weak bosons, while the second (not yet observed) $SU(2)$ gauge vector field have much higher masses. iii. The numerical calculations have improved so that we shall hopefully soon be able to say more about the intervals of masses of the fourth to the so far observed three families.

Povzetek. Doslej smo izmerili tri družine kvarkov in leptonov, tri vrste vektorskih polj, s katerimi so kvarki in leptoni sklopljeni ter skalarni Higgsov delec, ki je odgovoren za mase fermionov in šibkih bozonov. Vsa ta fermionska in bozonska polja so v skladu s *standardnim modelom*. Večina fizikov meni, da je Higgs zadnji delec, ki ga je bilo treba potrditi. Ali je to sploh lahko res? Ali ni očitno, da je skalarnih polj več, ki se efektivno kažejo kot Yukawine sklopitve in da lahko Yukawine sklopitve razumemo le, če razumemo izvor družin? Teorija

spinov-nabojev-družin [1–4] ponuja razlago za izvor družin in napoveduje, da določajo mase fermionov in šibkih vektorskih bozonov dva tripleta skalarnih polj, ki nosijo družinska kvantna števila in trije singleti, ki se sklapljajo z vsakim družinskim članom drugače. Teorija pojasni, zakaj so vsa skalarna polja šibki dubleti in zakaj nosijo tudi hyper naboj. Teorija razloži tudi ostale predpostavke *standardnega modela*. Teorija napove dve skupini štirih družin, ki nista sklopljeni in se razlikujeta po masah, ker sodelujejo pri nastanku mas vsake od skupin drugačna skalarna polja in pri eni od obeh tudi kondenzat desnoročnih nevtrinov. Četrto družino, sklopljeno s prvimi tremi že izmerjenimi, bodo opazili na LHC. Peta družina pojasni temno snov.

Natančnost, s katero lahko v tej teoriji izračunamo masne matrike in napovemo mase četrte družine, je odvisna od natačnosti meritev mas in matričnih elementov mešalnih matrik za tri poznane družine. Iz masnih matrik pa lahko sklepamo tudi na nekatere lastnosti skalarnih polj, ki smo jih doslej opazili kot Higgsovo skalarno polje in Yukavine sklopitve. Od lanskega zbornika je napredek teorije *spinov-nabojev-družin* predvsem v tem: i. Da lahko "pedagoško" razložim: i. Zakaj so skalarna polja dubleti glede na šibki naboj, in nosijo hipernaboj, da „obleęjo" desnoročne družinske člane v prava kvantna števila? ii. Zakaj imata dve skupini štirih družin tako različne mase, in zakaj sta tako zelo različnih mas tudi obe umeritveni polji, vsaka s svojo grupo $SU(2)$ (šibke bozone poznamo, druge vrste pa še ne), čeprav obe skupini skalarnih polj, ki sicer prispevata vsaka k masam svoje skupine štirih družin, prispevata k masi šibkih bozonov? II. Numerični izračuni so napredovali, tako da bo kmalu lahko podrobneje določiti intervale za mase četrte družine in njihove sklopitve s poznanimi tremi.

10.1 Introduction

The (extremely) efficient *standard model* is built on several assumptions, chosen to be in agreement with the data: i. There exist before the electroweak break massless coloured quarks and colourless leptons, left handed weak charged and right handed weak chargeless. ii. There exist families of fermions. iii. There exist the gauge fields to the observed charges of the family members. iv. There exists the boson – the scalar field and the anti-scalar field with the non zero vacuum expectation values after the elecroweak break and the properties to successfully "dress" the right handed fermions, giving them properties of the left handed ones and manifesting as doublets when interacting with the weak bosons. v. There exist the Yukawa couplings, distinguishing among the family members, to ensure right properties of families of fermions.

The questions are: a. Where do the families originate from and how many of them might be observable at the low energy regime? b. Where do the scalar fields and the Yukawa couplings originate from? c. Why is the Higgs a scalar boson manifesting as a doublet in the weak charge, while all the other bosons are in the vector representations with respect to all the charges, if they are not singlets [4]? d. Do we understand the appearance of the charges?

There are many other open questions, but the most urgent ones are to my understanding the first two, if we want to make a step towards understanding the *standard model* assumptions.

We should be able to predict what will the extremely expensive experiments measure in the near future.

There are several inventive proposals in the literature [6–14] extending the *standard model*. No one explains, to my knowledge, the origin of families. There are several proposals in the literature trying to explain the mass spectrum and mixing matrices of quarks and leptons [15] and properties of the scalar fields [16–19]. All of them just assuming on one or another way the number of families.

I am proposing the *spin-charge-family* theory [1–3,20–23], which does offer the explanation for the assumptions of the *standard model*:

- For the origin of massless families, explaining also the appearance of the family members with their charges.
- For the origin of the vector gauge fields.
- For the origin of several scalar fields which manifest effectively in the low energy regime as the Higgs and Yukawa couplings, explaining, why do the scalar fields and consequently the Higgs manifest as doublets with respect to the weak charge and carry the appropriate hyper charge and why do the family members manifest so different properties.

The theory is consequently able to make the *prediction* for the *number of families* and their *properties* and for the *number of scalar fields* and their *properties*, measurable in the today experiments. It is explaining also the *appearance of the dark matter*.

My starting assumption is a simple action in $d > (3+1)$ which leads to:

1. The Weyl equation for massless fermions couple to vielbeins and the spin connections of two kinds: The ones which are the gauge fields of $S^{ab} = \frac{i}{4}\,(\gamma^a\gamma^b - \gamma^b\gamma^a)$, where γ^a are the Dirac γ^a's defined in any d, and the ones which are the gauge fields of $\tilde{S}^{ab} = \frac{i}{4}\,(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$, where $\tilde{\gamma}^a$ are the second kind of the Clifford algebra objects, anti-commuting with the Dirac ones, again defined in any d.
2. The first kind of the Clifford algebra objects, γ^a, describes the spin in any d and after the break of the starting symmetry the spin in $d = (3+1)$ and all the so far observed charges, conserved and non-conserved.
3. The second kind of the Clifford algebra objects, since defining the equivalent representations with respect to the Dirac one, while there are only two kinds of the Clifford algebra objects (connected with the left - the Dirac one - and the right - my $\tilde{\gamma}^a$ - multiplication of any Clifford algebra object, which is a polynomial of powers of γ^a), the second kind must be used to describe families, which form the equivalent representations with respect to spin and charges.
4. The equations for boson fields, the vielbeins and spin connections of both kinds, are linear in the curvature.
5. d is chosen to be $(13+1)$ since one massless Weyl representation in $d = (13+1)$ contains, if analysed with respect to the *standard model* spin and charge groups, all the members of one family and their antiparticles: The left handed weak charged and the right handed weak chargeless coloured quarks of by the *standard model* required hyper charges and the left handed weak charged and the right handed weak chargeless colourless leptons - neutrinos and electrons - with by the *standard model* required hyper charges and their antiparticles according to the requirements of the ref. [5]. There are $2^{\frac{d}{2}-1}/2$

of massless particle plus antiparticle states if we pay attention to states of particular handedness and helicity only once.

6. The break of the starting $SO(13+1)$ symmetry first to **i.** $SO(7,1) \times U(1)_{II} \times SU(3)$, when (still massless) left handed weak charged and right handed weakless fermions and left handed weakless and right handed weak charged antifermions, differ further in the baryon quantum number ($U(1)_{II}$ ($\pm\frac{1}{6}$, for quarks $(+)$ and for antiquarks $(-)$ and $\mp\frac{1}{2}$, for leptons $(-)$ and antileptons $(+)$) while quarks and leptons differ further in the colour (quarks are triplets, antiquarks antitriplets, leptons are colourless singlets and antileptons anticolourless singlets), leaves these family members in $2^{\frac{7}{2}-1} = 8$ massless families, which stay massless also in the further breaks to **ii.** $SO(3,1)\times SU(2)_{I} \times SU(2)_{II}\times U(1)_{II} \times SU(3)$.
7. At the further two breaks, to $SO(3,1)\times SU(2)_{I} \times U(1)_{I} \times SU(3)$, when a weakless and hyper chargeless condensate of the right handed neutrinos carrying the quantum numbers of the upper four families brings masses to the $SU(2)_{II}$ gauge vector bosons, and to the electroweak break to $SO(3,1)\times U(1) \times SU(3)$, fermions, coupled to particular gauge scalar fields, which are vielbeins and spin connections with the scalar index with respect to $(3+1)$ and gain nonzero vacuum expectation values, become massive.
8. At the breaks some of the gauge fields stay massless (the colour vector bosons) and the final ($U(1)$) vector gauge field - the electromagnetic field - while the two $SU(2)$ vector gauge bosons become massive when the corresponding symmetry is broken.
9. The *standard model* can be interpreted as a low energy manifestation of the *spin-charge-family* theory.

In this talk I briefly present the *spin-charge-family* theory (already presented in several talks and papers): The fermions and gauge bosons starting action and the action after breaks, sect. 10.2, the fermion representations, sect. 10.2.1, and the scalar and vector representations, sect. 10.2.2. I answer the question why do scalar gauge bosons, carrying the family quantum numbers, manifest as weak (fermion) doublets, while they behave as triplets with respect to the family groups 10.2.2. I discuss a possible answer to the question: Why do the two gauge fields appearing in this theory, the gauge fields of the two kinds of the charges, $SU(2)_{II}$ and $SU(2)_{I}$, distinguish so much in their masses (the $SU(2)_{II}$ gauge vector boson has not yet been observed), although the two groups of the scalar fields, one responsible for the masses of the upper four families and another for the masses of the lower four families, are all weak ($SU(2)_{I}$) doublets and the hyper charge singlets 10.2.3.

I discuss predictions of the *spin-charge-family* theory: The properties of the fourth family coupled to the observed three [21,25], of the stable fifth family, of the scalar fields and of the accuracy of measurements needed that predictions will be more accurate, sect. 10.3, 10.2.2. To predict the fourth family properties (masses of the family members and the mixing matrix elements coupling the fourth family members to the observed three ones) accurately enough the two 3×3 mixing (sub)matrices should be measured pretty much more accurately. Properties of several scalar fields, leading effectively in the low energy regime to the scalar

Higgs and the Yukawa couplings, manifest in the mass matrices and can therefore some of their properties be evaluated by analysing properties of mass matrices.

The *spin-charge-family* theory opens several questions like: How many dimensions does the space have? Are there non-observable dimensions curled into compact or non-compact spaces? And many others.

10.2 Brief presentation of the *spin-charge-family* theory

In this section the *spin-charge-family* theory is briefly presented, first the simple starting action for massless fermions and massless gauge fields, with which I start and which includes families of fermions. I follow in this part to high extent the ref. [4]. In subsect. 10.2.1 the fermion representations are discussed, leading to mass matrices of family members.

The explanation is presented for why does the starting action manifest effectively, after several breaks up to the electroweak one, two decoupled groups of massive four families of quarks and leptons, three of the lower four already observed, and to the known gauge fields, the scalar Higgs and the Yukawa couplings. Each group of four families are coupled to their own kind of the scalar fields, the gauge fields with the scalar index with respect to $d = (3+1)$ of the two kinds of the Clifford algebra objects. Both groups of scalar fields gain nonzero vacuum expectation values. There is also the condensate, ref. 10.2.3, of the right handed neutrinos with the family quantum numbers of the upper four families, with the $SU(2)_{II}$ charge equal to 1, weakless and with the hyper charge equal to zero, bringing mass to the $SU(2)_{II}$ vector gauge fields. The scalars interacting with the lower four families determine, in loop corrections in all orders together with other fields, mass matrices of quarks and leptons, the three of which are the known ones. Mass matrices of all the family members, quarks and leptons, belonging to the lower four families, manifest the same symmetry 10.2.2. All these scalars are doublets with respect to the weak charge, while they carry appropriate hyper charge Y, 10.2.2, and manifest effectively at low energies as the Higgs and the Yukawa couplings.

The theory assumes that the spinor carries in $d(= (13+1))$-dimensional space two kinds of the spin, no charges [1,2,4,3]: **i.** The Dirac spin, described by $\gamma^{\mathbf{a}}$'s, defines the spinor representations in $d = (13+1)$ $(SO(13,1))$, and correspondingly in the low energy regime, after several breaks of symmetries and before the electroweak break, the spin $(SO(3,1))$ and all the charges (the colour $SU(3)$, the weak $SU(2)$, the hyper charge Y and the non conserved hyper charge Y') of quarks and leptons. There are the left handed weak charged and the right handed weak chargeless quarks and leptons. Handedness is determined by the spin properties in $d = (3+1)$, in agreement with the *standard model*. **ii.** The second kind of the spin [27,28,26], described by $\tilde{\gamma}^{\mathbf{a}}$'s ($\{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}$) and anticommuting with the Dirac γ^a ($\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$), defines the families of spinors, which at the symmetries of $SO(3,1) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ manifests two groups of four massless families, each belonging to different $SU(2) \times SU(2)$ symmetry, namely: $(\tilde{SU}(2)_R \times \tilde{SU}(2)_{II}) \times (\tilde{SU}(2)_L \times \tilde{SU}(2)_I)$, the first one determines the symmetries of one of the four families and the second one of the second one of four families.

One can understand the appearance of the (only) two kinds of the Clifford algebra objects as follows: If the Dirac one corresponds to the multiplication of any spinor object B (any product of the Dirac γ^a's, which represents a spinor state when being applied on a spinor vacuum state $|\psi_0>$) from the left hand side, can the second kind of the Clifford objects be understood (up to a factor, determining the Clifford evenness ($n_B = 2k$) or oddness ($n_B = 2k+1$) of the object B) as the multiplication of the object from the right hand side

$$\tilde{\gamma}^a B\,|\psi_0>:= i(-)^{n_B} B\gamma^a\,|\psi_0>_{fam}, \tag{10.1}$$

with $|\psi_0>_{fam}$ determining the vacuum state on which B applies. Accordingly we have

$$\begin{aligned}
&\{\gamma^a,\gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a,\tilde{\gamma}^b\}_+,\quad \{\gamma^a,\tilde{\gamma}^b\}_+ = 0,\\
&S^{ab} := (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a),\quad \tilde{S}^{ab} := (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a),\quad \{S^{ab},\tilde{S}^{cd}\}_- = 0.
\end{aligned} \tag{10.2}$$

More detailed explanation can be found, for example in appendix of the ref. [4] and in the refs [3,28,27].

The *spin-charge-family* theory proposes in $d = (13+1)$ a simple action for a Weyl spinor and for the corresponding gauge fields

$$\begin{aligned}
S\;=&\int d^dx\;E\;\mathcal{L}_f +\\
&\int d^dx\;E\;(\alpha\,R + \tilde{\alpha}\,\tilde{R}),
\end{aligned} \tag{10.3}$$

$$\begin{aligned}
\mathcal{L}_f &= \frac{1}{2}\,(\bar{\psi}\,\gamma^a p_{0a}\psi) + h.c.,\\
p_{0a} &= f^\alpha{}_a p_{0\alpha} + \frac{1}{2E}\,\{p_\alpha, Ef^\alpha{}_a\}_-,\\
p_{0\alpha} &= p_\alpha - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha},\\
R &= \frac{1}{2}\,\{f^{\alpha[a}f^{\beta b]}\;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^c{}_{b\beta})\} + h.c.\;,\\
\tilde{R} &= \frac{1}{2}\,f^{\alpha[a}f^{\beta b]}\;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\tilde{\omega}^c{}_{b\beta}) + h.c.\;.
\end{aligned} \tag{10.4}$$

Here [1] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$. To see that the action (Eq.(10.3)) manifests after the breaks of symmetries [2,4,3] all the known gauge fields and the scalar

[1] $f^\alpha{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b,\; e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

fields and the mass matrices of the observed families, let us rewrite formally the action for a Weyl spinor of (Eq.(10.3)) as follows

$$\begin{aligned}\mathcal{L}_f &= \bar{\psi}\gamma^n(p_n - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_n)\psi + \\ &\{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\; \psi\} + \text{the rest}\,, \\ p_{0s} &= p_s - \frac{1}{2}S^{tt'}\omega_{tt's} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}\,, \end{aligned} \tag{10.5}$$

where $n = 0, 1, 2, 3$ with

$$\begin{aligned}\tau^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab}\,, \\ \{\tau^{Ai}, \tau^{Bj}\}_- &= i\delta^{AB} f^{Aijk}\tau^{Ak}\,.\end{aligned} \tag{10.6}$$

All the charges (τ^{Ai}, Eqs. (10.6), (10.8), (10.9)) and the spin (Eq. (10.7)) operators are expressible with S^{ab}, which determine all the internal degrees of freedom of one family: the spin and the charges.

$$\vec{N}_\pm (= \vec{N}_{(L,R)}) : = \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03})\,, \tag{10.7}$$

determine representations of the two $SU(2)$ subgroups of $SO(3,1)$, while

$$\begin{aligned}\vec{\tau}^1 &: = \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78})\,, \\ \vec{\tau}^2 &: = \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78})\,,\end{aligned} \tag{10.8}$$

determine representations of $SU(2)_I \times SU(2)_{II}$ of $SO(4)$, which is the subgroup of $SO(7,1)$ and

$$\begin{aligned}\vec{\tau}^3 := &\frac{1}{2}\{S^{9\,12} - S^{10\,11}\,, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, \\ &S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}\,, S^{11\,14} - S^{12\,13}\,, \\ &S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}, \\ \tau^4 := &-\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})\,,\end{aligned} \tag{10.9}$$

determine representations of $SU(3) \times U(1)$, originating in $SO(6)$.

Family quantum numbers, expressible with $\tilde{S}^{ab}$,

$$\vec{\tilde{N}}_\pm (= \vec{\tilde{N}}_{(L,R)}) : = \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03})\,, \tag{10.10}$$

determine representations of the two $SU(2)$ subgroups of $SO(3,1)$ in the $\tilde{S}^{ab}$ sector, while

$$\begin{aligned}\vec{\tilde{\tau}}^1 &: = \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78})\,, \\ \vec{\tilde{\tau}}^2 &: = \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78}),\end{aligned} \tag{10.11}$$

determine representations of $SU(2)_{I}\times SU(2)_{II}$ of $SO(4)$, which is the subgroup of $SO(7,1)$ again in the $\tilde{S}^{ab}$ sector.

Families gain masses through the interaction with the scalar fields $\frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$, the gauge fields of the family charges ($\tilde{SU}(2)_{R}\times \tilde{SU}(2)_{II}$ the upper four families and $\tilde{SU}(2)_{L}\times \tilde{SU}(2)_{I}$ the lower four families), where we assume that after the breaks we end up with $(a,b)\in\{n,s\}$, $n=(0,1,2,3)$ and $s=(7,8)$. The upper four families and the vector gauge fields of the group $SU(2)_{II}$ gain masses also through the interaction with the right handed neutrinos condensate (sect. 10.2.3, Table 10.6), which is weakless, hyper chargeless and the electromagnetic chargeless, belonging to the $SU(2)_{II}$ triplet, carrying τ^{23} equal 1 and $\tau^{4}=-1$.

At the electroweak break the scalar fields which are the gauge fields of $\tilde{SU}(2)_{L}\times \tilde{SU}(2)_{I}$ contribute to masses of the lower four families, while the scalars, the gauge fields of Q, Q' and Y' contribute to masses of all the eight families, distinguishing among the family members (sect. 10.2.2, 10.2.2). All these scalar gauge fields, since they are doublets with respect to the weak charge, carrying also the hyper charge, contribute to the masses of the weak bosons.

Correspondingly index A in Eq. (10.6) enumerates all possible spinor charges and g^{A} is the coupling constant to a particular gauge vector field A^{Ai}_{n}. τ^{3i} describe the colour charge ($SU(3)$), τ^{1i} the weak charge ($SU(2)_{I}$), τ^{2i} the second $SU(2)_{II}$ charge, τ^{4} determines the $U(1)_{II}$ charge and $\tau^{23}=Y$ describes also the hyper charge, $Q=Y+\tau^{13}=S^{56}+\tau^{4}$ is the electromagnetic charge, $Q'=\tau^{13}-Y\tan^{2}\theta$ and $\tau^{\pm}=\tau^{11}\pm i\tau^{12}$.

The theory starts with one (massless, left handed) Weyl representation of $SO(13,1)$ spinors in $2^{d/2-1}$ families. In the breaks of the starting symmetry to the symmetry of $SO(7,1)\times SU(3)\times U(1)_{II}$ only eight ($2^{(7+1)/2-1}$) of them stay massless [2]. Families stay massless also after breaks to $SO(3,1)\times SU(2)_{I}\times SU(2)_{II}\times U(1)_{II}\times SU(3)$.

In the further two breaks, the first to $SO(3,1)\times SU(2)_{I}\times U(1)_{I}\times SU(3)$, triggered by the right handed neutrino condensate, carrying the family quantum numbers of $\vec{\tilde{N}}_{R}$ and $\vec{\tilde{\tau}}^{2}$, and belonging to the $SU(2)_{II}$ triplet with $\tau^{23}=1$ and $\tau^{4}=-1$, and correspondingly with zero electromagnetic, weak and hyper charges, and the electroweak break caused by the scalar fields which gain nonzero vacuum expectation values, all the fermions become massive. All the scalar fields, which contribute in the breaks, are doublets with respect to the weak charge carrying also the hyper charge Y(sect. 10.2.2).

In Eq. (10.12) the effective action for fermions at the electroweak is presented. The second line manifests the covariant momentum for fermions as seen by the *standard model* in agreement with the so far observed fermion and vector boson fields. The third line presents the contribution to the covariant momentum of the massive $SU(2)_{II}$ gauge fields, coupled through Y' and $\tau^{2\pm}$ to fermions. To masses of these vector gauge bosons mostly the condensate of the right handed neutrinos contributes. The fourth line determines the mass term for both groups of four

[2] We proved that it is possible to have massless fermions after a break if one starts with massless fermions and assume particular boundary conditions or particular vielbeins and spin connections causing the breaks [23,24] and taking care of massless and mass protected families after the break.

families on the tree level. It is assumed that the symmetries in the $\tilde{S}^{ab}\,\tilde{\omega}_{abc}$ and $S^{ab}\omega_{abc}$ part break in a correlated way. The generators $\tilde{S}^{ab}$ (Eqs. (10.10), (10.11)) transform each member of one family into the same family member of another family, due to the fact that $\{S^{ab},\tilde{S}^{cd}\}_- = 0$. The generators S^{ab} transform the family member into another one, keeping family quantum number unchanged.

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\,(\gamma^m\, p_{0m} - M)\psi\,,\\
p_{0m} &= p_m - \{e\,Q\,A_m + g^{Q'}\,Q'\,Z^{Q'}_m + \frac{g^1}{\sqrt{2}}\,(\tau^{1+}\,W^{1+}_m + \tau^{1-}\,W^{1-}_m) + \\
&\quad + g^{Y'}\,Y'\,A^{Y'}_m + \frac{g^2}{\sqrt{2}}\,(\tau^{2+}\,A^{2+}_m + \tau^{2-}\,A^{2-}_m)\,,\\
\bar{\psi}\,M\,\psi &= \bar{\psi}\,\gamma^s\,p_{0s}\psi\\
p_{0s} &= p_s - \{\tilde{g}^{\tilde{N}_R}\,\vec{\tilde{N}}_R\,\vec{\tilde{A}}^{\tilde{N}_R}_s + \tilde{g}^{\tilde{Y}'}\,\tilde{Y}'\,\tilde{A}^{\tilde{Y}'}_s + \frac{\tilde{g}^2}{\sqrt{2}}\,(\tilde{\tau}^{2+}\,\tilde{A}^{2+}_s + \tilde{\tau}^{2-}\,\tilde{A}^{2-}_s)\\
&\quad + \tilde{g}^{\tilde{N}_L}\,\vec{\tilde{N}}_L\,\vec{\tilde{A}}^{\tilde{N}_L}_s + \tilde{g}^{\tilde{Q}'}\,\tilde{Q}'\,\tilde{A}^{\tilde{Q}'}_s + \frac{\tilde{g}^1}{\sqrt{2}}\,(\tilde{\tau}^{1+}\,\tilde{A}^{1+}_s + \tilde{\tau}^{1-}\,\tilde{A}^{1-}_s)\\
&\quad + e\,Q\,A^Q_s + g^{Q'}\,Q'\,Z^{Q'}_s + g^{Y'}\,Y'\,A^{Y'}_s\,\},\ s\in\{7,8\}\,. \qquad (10.12)
\end{aligned}
$$

The term $\bar{\psi}\,M\,\psi$ determines the tree level mass matrices of quarks and leptons. The two groups of four families are decoupled due to different family quantum numbers: One group carries the quantum numbers of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, the other of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Since the condensate contributes in loop corrections only to one of the two groups, the first one, the mass matrices are expected to appear at two different energy scales. Also the scalar fields couple to either the upper or to the lower four families.

Since all the scalar fields, which gain nonzero vacuum expectation values - those with the quantum numbers of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, with $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, and those with Q, Q', Y' - are doublets with respect to the weak charge carrying also the hyper charge (10.2.2), all contribute to masses of the vector bosons W and Z. It is, namely, $-2iS^{0s}$, $s = 7, 8$, which transform the right handed weak chargeless quarks and leptons into the corresponding left handed weak charged partners, transforming at the same time the hyper charge Y. The gauge scalar fields have correspondingly the weak and hyper charges.

10.2.1 Fermions through breaks

I discuss properties of quarks, u and d, and leptons, ν and e, all left and right handed, for two decoupled groups of four families, before and after they gain masses, triggered by the vacuum expectation values of the scalar fields with which each of the two groups couples.

At the stage of the symmetry

$$
\begin{aligned}
&SO(3,1)_\gamma \times SO(3,1)_{\tilde{\gamma}} \times SU(2)_{I\,\gamma} \times SU(2)_{I\,\tilde{\gamma}}\\
&\times SU(2)_{II\,\gamma} \times SU(2)_{II\,\tilde{\gamma}} \times U(1)_{II\,\gamma} \times U(1)_{II\,\tilde{\gamma}}\\
&\times SU(3)_\gamma, \qquad (10.13)
\end{aligned}
$$

the eight families are assumed to be massless. The two indices γ and $\tilde{\gamma}$ are to point out that there are two kinds of subgroups of $SO(7,1)$, those defined by S^{ab} responsible for properties (spin and charges) of family members and those defined by $\tilde{S}^{ab}$ responsible for the appearance of families.

To manifest how do the operators presented in Eqs. (10.7, 10.8, 10.9) transform one family member into another one of the same family, in Table 10.1 quarks of a particular colour charge are presented in the spinor technique [28]. A brief introduction into the technique can be found also in Appendix of this talk. Spinor states are defined as products of nilpotents ($[\stackrel{ab}{(k)}]^2 = 0$) and projectors ($[\stackrel{ab}{[k]}]^2 = \stackrel{ab}{[k]}$) (Eq. (10.37) in Appendix 10.4)

$$
\begin{aligned}
\stackrel{ab}{(\pm i)}: &= \frac{1}{2}(\gamma^a \mp \gamma^b),\; \stackrel{ab}{[\pm i]}:= \frac{1}{2}(1 \pm \gamma^a\gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = -1,\\
\stackrel{ab}{(\pm)}: &= \frac{1}{2}(\gamma^a \pm i\gamma^b),\; \stackrel{ab}{[\pm]}:= \frac{1}{2}(1 \pm i\gamma^a\gamma^b), \quad \text{for } \eta^{aa}\eta^{bb} = 1\,, \qquad (10.14)
\end{aligned}
$$

chosen to be eigen states of S^{ab}. They are at the same time also the eigenstates of $\tilde{S}^{ab}$ (Eq. (10.38) in Appendix 10.4).

$$
S^{ab}\stackrel{ab}{(k)}= \frac{k}{2}\stackrel{ab}{(k)}, \quad S^{ab}\stackrel{ab}{[k]}= \frac{k}{2}\stackrel{ab}{[k]}, \quad \tilde{S}^{ab}\stackrel{ab}{(k)}= \frac{k}{2}\stackrel{ab}{(k)}, \quad \tilde{S}^{ab}\stackrel{ab}{[k]}= -\frac{k}{2}\stackrel{ab}{[k]}\,. \qquad (10.15)
$$

The choice of the Cartan subalgebra of the commuting operators is made as follows:

$$
\begin{aligned}
&S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}\,. \qquad (10.16)
\end{aligned}
$$

Let the reader note that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$ (Eq. (10.39) in Appendix 10.4)

$$
\gamma^a \stackrel{ab}{(k)}= \eta^{aa}\stackrel{ab}{[-k]},\; \gamma^b \stackrel{ab}{(k)}= -ik\stackrel{ab}{[-k]},\; \gamma^a \stackrel{ab}{[k]}=\stackrel{ab}{(-k)},\; \gamma^b \stackrel{ab}{[k]}= -ik\eta^{aa}\stackrel{ab}{(-k)}, \qquad (10.17)
$$

$$
\tilde{\gamma}^a \stackrel{ab}{(k)}= -i\eta^{aa}\stackrel{ab}{[k]},\; \tilde{\gamma}^b \stackrel{ab}{(k)}= -k\stackrel{ab}{[k]},\; \tilde{\gamma}^a \stackrel{ab}{[k]}= i\stackrel{ab}{(k)},\; \tilde{\gamma}^b \stackrel{ab}{[k]}= -k\eta^{aa}\stackrel{ab}{(k)} \qquad (10.18)
$$

The nilpotents and projectors of Table 10.1 operate on a vacuum state, not presented in the table. The states solve the Weyl equation Eq.(10.19)

$$
\begin{aligned}
\gamma^0\gamma^a p_a\psi = 0 &= \gamma^0\,(\gamma^m p_m + \sum_{s=7,8} \bar{\psi}\gamma^s\, p_s\,)\psi\\
&= \gamma^0\,(\stackrel{78}{(-)}\; p_- + \stackrel{78}{(+)}\; p_+)\psi\,,\\
\stackrel{78}{(\pm)} &= \frac{1}{2}\,(\gamma^7 \pm i\,\gamma^8)\,,\\
p_\pm &= (p_7 \mp i\,p_8)\,, \qquad (10.19)
\end{aligned}
$$

for free massless spinors in the coordinate system where $p^a = (p^0, 0, 0, p^3, \vec{0})$, $\vec{0}$ stays for all the components in $d > 4$.

There are $2^{\frac{d}{2}-1} = 64$ basic spinor states of one family representation in $d = (13+1)$, defining the spinors (colour triplets quarks and antitriplets antiquarks and colourless leptons and anticolourless antileptons). Family members of a particular colour or the colourless ones form $2^{(7+1)/2=8}$ states and so do anticoloured and colourless spinors. One easily sees that the operator $\gamma^0 \stackrel{78}{(\pm)} I_{\vec{x}_3}$, $I_{\vec{x}_3}$ reflecting (x^1, x^2, x^3) into $(-x^1, -x^2, -x^3)$, transforms the state u_{1R} from the first line into the state u_{1L} from the seventh line, while $\vec{\tau}^3$ transforms any of the quark states of the starting colour charge into otherwise the same states but in general of of another colour charges.

$S^{9\,10}$, for example, transforms the u_{1R} quark from the first line into the ν_{1R} lepton from the first line in Table 10.2. Such transformations are after the breaks not allowed. Following the proposal from the ref. [5] for the definition of the discrete symmetries in cases of the Kaluza-Klein kind for d even

$$\begin{aligned}
\mathcal{C}_{\mathcal{N}}\,\psi(x^0,\vec{x}) &= \Gamma^{(3+1)}\,\gamma^2\,K\,\psi(x^0,x^1,x^2,x^3,x^5,-x^6,x^7,-x^8,\dots,x^{d-1},-x^d)\\
&= \Gamma^{(3+1)}\,\gamma^2\,K\,I_{6,8,\cdots,d}\,\psi(x^0,\vec{x})\,,\\
\mathcal{T}_{\mathcal{N}}\,\psi(x^0,\vec{x}) &= \Gamma^{(3+1)}\,\gamma^1\,\gamma^3\,K\,\psi(-x^0,x^1,x^2,x^3,-x^5,x^6,-x^7,\dots,-x^{d-1},x^d)\\
&= \Gamma^{(3+1)}\,\gamma^1\,\gamma^3\,K\,I_{x^0}\,I_{5,7,\cdots,d-1}\,\psi(x^0,\vec{x})\,,\\
\mathcal{P}^{d-1}_{\mathcal{N}}\,\psi(x^0,\vec{x}) &= \gamma^0\,\Gamma^{(3+1)}\,\Gamma^{(d)}\,\psi(x^0,-x^1,-x^2,-x^3,x^5,x^6,\dots,x^{d-1},x^d)\\
&= \gamma^0\,\Gamma^{(3+1)}\,\Gamma^{(d)}\,I_{\vec{x}_3}\,\psi(x^0,\vec{x})\,,
\end{aligned} \tag{10.20}$$

where $I_{\vec{x}_3}$ reflects (x^1, x^2, x^3), $I_{6,8,\cdots,d}$ reflects $(x^6, x^8, \cdots, x^d)$, I_{x^0} reflects the time component x^0 and $I_{5,7,\cdots,d-1}$ reflects $(x^5, x^7, \cdots, x^{d-1})$, it is $\mathcal{C}_{\mathcal{N}} \cdot \mathcal{P}^{d-1}_{\mathcal{N}}$, which transforms the positive energy states into the corresponding negative energy states, staying within the same Weyl, while either $\mathcal{C}_{\mathcal{N}}$ or $\mathcal{P}^{d-1}_{\mathcal{N}}$ jumps out of the starting Weyl representation.

Emptying the negative energy state obtained by the application of the $\mathcal{C}_{\mathcal{N}} \cdot \mathcal{P}^{d-1}_{\mathcal{N}}$ on the single particle state put on the top of the Dirac sea, one creates the corresponding antiparticle state with the positive energy and put on the top of the Dirac sea, carrying all the properties of the starting particle, except the S^{03} value and the charges [5].

The above requirements can be expressed as follows.

Statement: *The antiparticle state put on the top of the corresponding Dirac sea follows from the particle state put on the top of this Dirac sea by applying on the particle state the operator* $\mathbb{O}_{\mathcal{N}}$

$$\begin{aligned}
\{\mathbb{O}_{\mathcal{N}} &= \mathrm{emptying} \times \mathcal{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}\\
&= \gamma^0 \prod_{\gamma^a \in \Im, a \neq 2} \gamma^a\,\Gamma^{(3+1)}\,I_{\vec{x}_3}\,I_{6,8,\dots,d}\Gamma^{(d)}\,\}\ \text{particle state.}
\end{aligned} \tag{10.21}$$

The corresponding antiparticle state on the top of the Dirac sea also solves the Weyl equation (10.19).

Using Eq. (10.21) it is easy to find the antiparticle state of positive energy (which are put on the top of the Dirac sea) to the particle states (which are put on the top of the Dirac sea), presented in Tables (10.1, 10.2). The corresponding two tables are presented in Tables (10.3, 10.4).

ψ_i^{pos}	positive energy state	$\frac{p^0}{\vert p^0\vert}$	$\frac{p^3}{\vert p^3\vert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	τ^{13}	τ^{23}	τ^4	Y	Q
$u_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\;\;e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	$+1$	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
$u_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\;\;e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	$+1$	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
$d_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\;\;e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	$+1$	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$d_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\;\;e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	$+1$	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$d_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\;\;e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
$d_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\;\;e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
$u_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\;\;e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
$u_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\;\;\stackrel{11\,12}{(-)}\;\;\stackrel{13\,14}{(-)}\;\;e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$

Table 10.1. One $SO(7,1)$ sub representation [5] of the representation of $SO(13,1)$, the one representing quarks, which carry the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$). All members have $\Gamma^{(13+1)} = -1$. The states representing particles are put on the top of the Dirac sea. All states are the eigenstates of the Cartan subalgebra ($S^{03}, S^{12}, S^{56}, S^{7,8}, S^{9\,10}, S^{11\,12}, S^{13\,14}$) and solve the Weyl equation (10.19) for the choice of the coordinate system $p^a = (p^0, 0, 0, p^3, 0, \ldots, 0)$ for free massless fermions. The infinitesimal generators of the weak charge $SU(2)_I$ group ($\vec{\tau}^1$) and of another $SU(2)_{II}$ group ($\vec{\tau}^2$) are defined in Eq. (10.8) and of the τ^4 charge and the colour charge group ($\vec{\tau}^3$) in Eq. (10.9). $Y = \tau^{23} + \tau^4$, $Q = \tau^{13} + Y$. Nilpotents $\stackrel{ab}{(k)}$ and projectors $\stackrel{ab}{[k]}$ operate on the vacuum state $|vac>_{fam}$ not written in the table.

ψ_i^{pos}	positive energy state	$\frac{p^0}{\lvert p^0\rvert}$	$\frac{p^3}{\lvert p^3\rvert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	τ^{13}	τ^{23}	τ^4	Y	Q
$\nu_{1\,R}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{(+)}\mid\mid\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	$+1$	$+1$	$+1$	$+1$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
$\nu_{2\,R}$	$\overset{03}{[-i]}\overset{12}{[-]}\mid\overset{56}{(+)}\overset{78}{(+)}\mid\mid\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	$+1$	-1	-1	$+1$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
$e_{1\,R}$	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{[-]}\overset{78}{[-]}\mid\mid\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	$+1$	$+1$	$+1$	$+1$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
$e_{2\,R}$	$\overset{03}{[-i]}\overset{12}{[-]}\mid\overset{56}{[-]}\overset{78}{[-]}\mid\mid\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	$+1$	-1	-1	$+1$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
$e_{1\,L}$	$\overset{03}{[-i]}\overset{12}{(+)}\mid\overset{56}{[-]}\overset{78}{(+)}\mid\mid\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	$+1$	-1	-1	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
$e_{2\,L}$	$\overset{03}{(+i)}\overset{12}{[-]}\mid\overset{56}{[-]}\overset{78}{(+)}\mid\mid\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	$+1$	$+1$	$+1$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
$\nu_{1\,L}$	$\overset{03}{[-i]}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{[-]}\mid\mid\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	$+1$	-1	-1	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
$\nu_{2\,L}$	$\overset{03}{(+i)}\overset{12}{[-]}\mid\overset{56}{(+)}\overset{78}{[-]}\mid\mid\overset{9\,10}{(+)}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}\;e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	$+1$	$+1$	$+1$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0

Table 10.2. One $SO(7,1)$ sub-representation [5] of the representation of $SO(13,1)$, the one representing the colourless leptons, when the state is put on the top of the Dirac sea. The rest of definitions is the same as in Table 10.1.

$\bar{\psi}^{pos}_i$	positive energy state	$\frac{p^0}{\vert p^0\vert}$	$\frac{p^3}{\vert p^3\vert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	τ^{13}	τ^{23}	τ^4	Y	Q
$\bar{u}_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	-1	0	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
$\bar{u}_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	-1	0	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
$\bar{d}_{1\,L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	-1	0	$+\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
$\bar{d}_{2\,L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	-1	0	$+\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
$\bar{d}_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	$+1$	$+\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
$\bar{d}_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	$+1$	$+\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
$\bar{u}_{1\,R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	$+1$	$+1$	$+1$	$+1$	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
$\bar{u}_{2\,R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	$+1$	-1	-1	$+1$	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$

Table 10.3. One $SO(7,1)$ sub representation [5] of the representation of $SO(13,1)$, the one representing antiquarks, which carry the anticolour charge ($\tau^{33} = -1/2$, $\tau^{38} = -1/(2\sqrt{3})$) and it is put on the top of the Dirac sea. All the other comments are the same as in Table 10.1.

Let us find now, according to Eq. (10.21), the antilepton states (to be put on the top of the Dirac sea) to the states, presented in Table 10.2. One finds the states (to be put on the top of the Dirac sea), presented in Table 10.4

One can easily check that $\gamma^0 \stackrel{78}{(+)} I_{\vec{x}_3}$ transforms the weakless antiparticle state put on the top of the Dirac sea $\bar{u}_L$ with the hyper charge $Y = -\frac{2}{3}$ from the first line in Table 10.3 into the weak charged antiparticle state $\bar{u}_R$, put on the top of the Dirac sea from the seventh line in the same table. $\bar{u}_R$ has $Y = -\frac{1}{6.}$. Similarly does $\gamma^0 \stackrel{78}{(+)} I_{\vec{x}_3}$ transform the weakless $\bar{e}_{1L}$ from the third line in Table 10.4 with $Y = 1$ into the weak charged $\bar{e}_R$ from the fifth line in the same table, with $Y = \frac{1}{2}$, both antiparticle states put on the top of the Dirac sea.

One sees that the term $\gamma^0 \sum_{s=7,8} \gamma^s p_{0s}$ determines the mass term as soon as a superposition of the fields $\tilde{\omega}_{abs}$ or of the fields ω_{abs}, or both superposition, gain nonzero vacuum expectation values. I shall demonstrate this in the next subsection.

Families of fermions Here I again follow a lot the ref.[4]. The generators $\tilde{N}^{\pm}_{R,L}$ and $\tilde{\tau}^{(2,1)\pm}$ (Appendix 10.4, Eq. (10.50)), which are superposition of $\tilde{S}^{ab}$, transform each member of one family into the same member of another family, due to the fact that $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$ (Eq.(10.2)).

The eight families of the first member of the eight-plet of quarks from Table 10.1, for example, that is of the right handed u_{1R} quark, are presented in the left column of Table 10.5 [4]. In the right column of the same table the equivalent eight-plet of the right handed neutrinos ν_{1R} are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

Masses of fermions We saw in subsect. 10.2.1 that the term $\psi^\dagger\gamma^0 M\psi$ in Eq. (10.12) causes the appearance of masses of fermions as soon as the corresponding scalar fields, presented in the covariant momentum in the fifth, sixth and seventh line of the same equation gain nonzero expectation values.

$\bar{\psi}^{pos}_i$	positive energy state	$\frac{p^0}{\vert p^0\vert}$	$\frac{p^3}{\vert p^3\vert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	τ^{13}	τ^{23}	τ^4	Y	Q
$\bar{\nu}_{1\,L}$	$\overset{03}{[-i]}\overset{12}{(+)}\vert\overset{56}{[-]}\overset{78}{[-]}\Vert\overset{9\,10}{[-]}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	+1	−1	−1	−1	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
$\bar{\nu}_{2\,L}$	$\overset{03}{(+i)}\overset{12}{[-]}\vert\overset{56}{[-]}\overset{78}{[-]}\Vert\overset{9\,10}{[-]}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	+1	+1	+1	−1	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
$\bar{e}_{1\,L}$	$\overset{03}{[-i]}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{[-]}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	+1	−1	−1	−1	0	$+\frac{1}{2}$	$+\frac{1}{2}$	+1	+1
$\bar{e}_{2\,L}$	$\overset{03}{(+i)}\overset{12}{[-]}\vert\overset{56}{(+)}\overset{78}{(+)}\Vert\overset{9\,10}{[-]}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	+1	+1	+1	−1	0	$+\frac{1}{2}$	$+\frac{1}{2}$	+1	+1
$\bar{e}_{1\,R}$	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{(+)}\overset{78}{[-]}\Vert\overset{9\,10}{[-]}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	+1	+1	+1	+1	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	+1
$\bar{e}_{2\,R}$	$\overset{03}{[-i]}\overset{12}{[-]}\vert\overset{56}{(+)}\overset{78}{[-]}\Vert\overset{9\,10}{[-]}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	+1	−1	1	+1	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	+1
$\bar{\nu}_{1\,R}$	$\overset{03}{(+i)}\overset{12}{(+)}\vert\overset{56}{[-]}\overset{78}{(+)}\Vert\overset{9\,10}{[-]}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0+i\vert p^3\vert x^3}$	+1	+1	+1	+1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0
$\bar{\nu}_{2\,L}$	$\overset{03}{[-i]}\overset{12}{[-]}\vert\overset{56}{[-]}\overset{78}{(+)}\Vert\overset{9\,10}{[-]}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}\; e^{-i\vert p^0\vert x^0-i\vert p^3\vert x^3}$	+1	−1	−1	+1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0

Table 10.4. One $SO(7,1)$ sub representation [5] of the representation of $SO(13,1)$, the one representing the colourless antileptons, when the state is put on the top of the Dirac sea. The rest of definitions is the same as in Table 10.1.

I	$u_{I\,1R}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{I\,1R}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
I	$u_{II\,1R}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{II\,1R}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
I	$u_{III\,1R}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{III\,1R}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
I	$u_{IV\,1R}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{IV\,1R}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
II	$u_{I\,1R}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{I\,1R}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
II	$u_{II\,1R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{II\,1R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[+]}\stackrel{78}{[+]}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
II	$u_{III\,1R}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{III\,1R}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$
II	$u_{IV\,1R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{IV\,1R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$

Table 10.5. Eight families of the right handed u_{1R} (10.1) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and of the colourless right handed neutrino ν_{1R} of spin $\frac{1}{2}$ (10.2) are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$) and a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$), the other (II) is a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$) and a doublet with with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (10.50). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (10.50)) transform u_{1R} to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino ν_{1R} to all the colourless members of the same family.

If the operators γ^7 and γ^8 in Eq. (10.12) are expressed in terms of the nilpotents $\stackrel{78}{(\pm)}$, the mass term can be rewritten as follows

$$
\begin{aligned}
\bar{\psi} M \psi &= \sum_{s=7,8} \bar{\psi}\gamma^s\, p_{0s}\, \psi = \psi^{\dagger}\gamma^0\, (\stackrel{78}{(-)}\; p_{0-} + \stackrel{78}{(+)}\; p_{0+})\psi\,, \\
\stackrel{78}{(\pm)} &= \frac{1}{2}\,(\gamma^7\; \pm i\gamma^8)\,, \\
p_{0\pm} &= (p_{07} \mp i\, p_{08}) = (p_7 \mp i\, p_8) - (\Phi^{Ai}_7 \mp i\,\Phi^{Ai}_8)\,, \\
\Phi^{Ai}_{\mp} &= \{\vec{\tilde{A}}^{\tilde{N}_R}_{\mp}, \vec{\tilde{A}}^{2}_{\mp}, \vec{\tilde{A}}^{\tilde{N}_L}_{\mp}, \vec{\tilde{A}}^{1}_{\mp}, A^{Q}_{\mp}, Z^{Q'}_{\mp}, A^{Y'}_{\mp}\}\,.
\end{aligned} \tag{10.22}
$$

We clearly see that all the scalars $\Phi^{Ai}_{\mp}$ are doublets with respect to the weak charge, carrying also the hyper charge, $(\tau^{13}, Y)\,\Phi^{Ai}_{-} = (\frac{1}{2}, -\frac{1}{2})\;\Phi^{Ai}_{-}$, $(\tau^{13}, Y)\,\Phi^{Ai}_{+} = (-\frac{1}{2}, \frac{1}{2})\;\Phi^{Ai}_{+}$, since they obviously bring the right quantum numbers to the right handed partners, to (u_R, ν_R) the scalars Φ^{Ai}_{-}, and to (d_R, e_R) the scalars Φ^{Ai}_{+}, as we have checked in Tables 10.1 and 10.2, manifesting that $\gamma^0\;\stackrel{78}{(-)}$ transforms (u_R, ν_R) into (u_L, ν_L), and equivalently for other quarks and leptons. We shall discuss properties of scalar fields also in subsect. 10.2.2, 10.2.2.

To masses of one of the two groups of four families only the scalar fields, which are the gauge fields of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ contribute, to masses of the other group of four families only the gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ contribute.

The scalars A^Q_s, $Z^{Q'}_s$ and $A^{Y'}_s$ from the last line in Eq. (10.12) contribute to all eight families, distinguishing among the family members and not among the families.

In loop corrections also all the gauge fields which couple to fermions contribute. To the upper four families contributes in addition the (assumed to be) condensate of the right handed neutrinos (10.2.3), carrying the spin equal zero, $Q = 0 = Y$, $\tau^{13} = 0$, $\tau^{23} = 1$ and $\tau^4 = -1$. It also carries the $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R = 1$ charges.

The mass matrix of any family member belonging to any of the two groups of four families manifests, due to the $\widetilde{SU}(2)_{(R,L)} \times \widetilde{SU}(2)_{(II,I)}$ (either (R, II) or (L, I)) structure of the quantum numbers of the scalar fields which are the gauge fields of the $\vec{\tilde{N}}_{R,L}$ and $\vec{\tilde{\tau}}^{2,1}$, the symmetry presented in Eq. (10.23)

$$\mathcal{M}^\alpha = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^\alpha , \tag{10.23}$$

the same for all the family members $\alpha \in \{u, d, \nu, e\}$. The properties of the mass matrices and the procedure how to extract from the observed properties of the lower three families of the lower group of four families the masses and mixing matrix elements is discussed in the contribution to this proceedings [25] and in the refs. [3,4]. All the parameters of the mass matrix are determined by the tree level contributions and the loop corrections in all orders of all the fields, which couple to particular family member of one of the two groups of four families.

If assuming that the mass matrix elements are real then there are 6 free parameters for each family member. The mixing matrix for quarks has then 6 free parameters and so has the corresponding one for leptons. Since any $(n-1)\times(n-1)$ sub-matrix of the $n \times n$ unitary matrix determines for $n \geq 4$ the unitary matrix uniquely, we would be able to calculate from two times three masses and the mixing matrix elements of the 3×3 sub-matrix the fourth family members masses for the accurately enough experimental data.

We have not yet started to study the CP violation.

Let us learn [4,3] how do fermions interact with the scalar fields. Let $\psi^\alpha_{(L,R)}$ denote massless and $\Psi^\alpha_{(L,R)}$ massive four vectors for each family member $\alpha = (u_{L,R}, d_{L,R}, \nu_{L,R}, e_{L,R})$ after taking into account loop corrections in all orders [3,22], for any of the two groups of four families. $\psi^\alpha_{(L,R)} = V^\alpha_{(L,R)}\, \Psi^\alpha_{(L,R)}$,

$$\begin{aligned} \psi^\alpha_{(L,R)} &= V^\alpha\, \Psi^\alpha_{(L,R)} , \\ V^\alpha &= V^\alpha_{(o)}\, V^\alpha_{(1)} \cdots V^\alpha_{(k)} \cdots . \end{aligned} \tag{10.24}$$

It then follows

$$\begin{aligned} < \psi^\alpha_L|\gamma^0\, M^\alpha\, |\psi^\alpha_R > &= < \Psi^\alpha_L|\gamma^0\, (V^\alpha)^\dagger\, M^\alpha\, V^\alpha\, |\Psi^\alpha_R >= \\ & < \Psi^\alpha_L|\gamma^0\, diag(m^\alpha_1, \cdots, m^\alpha_4)|\Psi^\alpha_R > . \end{aligned} \tag{10.25}$$

It follows then that $V^{\alpha\dagger}\, \mathcal{M}^\alpha\, V^\alpha = \Phi^\alpha_\Psi$ determines the superposition of the scalar dynamical fields which couple with the coupling constants m^α_k (in some units) to the family member belonging to the k^{th} family

$$(\Phi^{\alpha}_{\Psi})_{k\,k'}\,\Psi^{\alpha\,k'} = \delta_{k\,k'}\,m^{\alpha}_{k}\,\Psi^{\alpha\,k}\,. \qquad (10.26)$$

Let us repeat that to loop corrections two kinds of scalar dynamical fields contribute, those originating in $\tilde{\omega}_{abs}$ ($\tilde{g}^{\tilde{N}_R}\,\vec{\tilde{N}}_R\vec{\tilde{A}}^{\tilde{N}_R}_s$, $\tilde{g}^2\,\vec{\tilde{\tau}}^2\,\vec{\tilde{A}}^2_s$ to the upper four families and $\tilde{g}^{\tilde{N}_L}\vec{\tilde{N}}_L\,\vec{\tilde{A}}^{\tilde{N}_L}_s$ to the lower four families) those originating in ω_{abs} ($e\,Q\,A^{Q}_s$, $g^1\,Q'\,Z^{Q'}_s$ and $g^{Y'}Y'\,A^{Y'}_s$ to all eight families), the vector gauge fields from Eq.(10.12), the fermion fields and to the upper four families also the condensate.

Even if we are able to reproduce the mass matrices, as we are trying in the ref. [25], it is not easy to extract some properties of the scalar fields from the known mass matrices.

10.2.2 Scalars and gauge fields through breaks

In the *spin-charge-family* theory there are the vielbeins $e^s{}_\sigma$

$$e^a{}_\alpha = \begin{pmatrix} \delta^m{}_\mu & 0 \\ 0 & e^s{}_\sigma \end{pmatrix}$$

in a strong correlation with the spin connection fields of both kinds, $\tilde{\omega}_{ab\sigma}$ $((a,b) \in \{0,\dots,3,5,\dots,8\}$, $\sigma \in \{7,8\}$) and with $\omega_{st\sigma}$ $((s,t) \in \{5,6,7,8\}, \sigma \in \{5,6,7,8\})$, which manifest in $d=(3+1)$-dimensional space as scalar fields after particular breaks of the starting symmetry. Phase transitions are (assumed to be) triggered by nonzero vacuum expectation values of the fields $f^{\alpha}{}_s\,\tilde{\omega}_{ab\alpha}$ and $f^{\alpha}{}_s\,\omega_{ab\alpha}$ [4] and the fermion (the right handed neutrinos from the upper four families) condensate.

The gauge fields then correspondingly appear as

$$e^a{}_\alpha = \begin{pmatrix} \delta^m{}_\mu & 0 \\ e^s{}_\mu = e^s{}_\sigma E^\sigma{}_{Ai} A^{Ai}_\mu & e^s{}_\sigma \end{pmatrix},$$

with $E^{\sigma Ai} = \tau^{Ai}\,x^\sigma$, where A^{Ai}_μ are the gauge fields, corresponding to (all possible) Kaluza-Klein charges τ^{Ai}, manifesting in $d=(3+1)$. Since the space symmetries include only S^{ab} ($M^{ab} = L^{ab} + S^{ab}$) and not $\tilde{S}^{ab}$, there are no vector gauge fields of the type $e^s{}_\sigma \tilde{E}^\sigma{}_{Ai}\tilde{A}^{Ai}_\mu$, with $\tilde{E}^\sigma{}_{Ai} = \tilde{\tau}_{Ai}\,x^\sigma$. The gauge fields of $\tilde{S}_{ab}$ manifest in $d=(3+1)$ only as scalar fields.

There occurs two successive breaks from $SO(3,1)\times SU(2)_{I}\times SU(2)_{II}\times U(1)_{II}\times SU(3)$ to $SO(3,1)\times U(1)\times SU(3)$.

I assume that the first break, that is to $SO(3,1)\times SU(2)_I \times U(1)_I \times SU(3)$, is triggered by the right handed neutrinos belonging of the upper four families forming a condensate, 10.2.3, with the quantum numbers (spin equal zero, $Q = 0 = Y$, $\tau^{13} = 0$, $\tau^{23} = 1$, $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$, or any other $\tilde{\tau}^{23}$ and $\tilde{N}^3_R$ values). It couples correspondingly to the gauge fields $\vec{A}^2_m$, bringing them masses (leaving the weak bosons massless). The condensate, carrying the family quantum numbers $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$ of the upper four families, couples also to the upper four families.

At the electroweak break, when all the scalar fields gain nonzero vacuum expectation values, all the family members of both groups of four families become massive. Since all the scalar fields are doublets with respect to the weak charge and carry also the hyper charge, their nonzero vacuum expectation values contribute on the tree level to the masses of Z_m and $W^{\pm}_m$ according to

$$(\frac{1}{2})^2\,(g^1)^2\,\nu_I^2(\frac{1}{(\cos\theta_1)^2}\,Z^{Q'}_m Z^{Q'\,m} + 2\,W^{+}_m W^{-\,m})\,, \tag{10.27}$$

where ν_I are the contribution to the vacuum expectation value of all the scalar fields $\Phi^{I\,Ai}_{\mp}$. Eq. (10.27) is in agreement with the *standard model*.

To know the properties of the scalar fields one should study in details breaks, in which the condensate of the right handed neutrinos, and the scalar fields carrying the weak and hyper charges and the family quantum numbers participate, which is not an easy job.

However, from the mass matrices and the interactions of the scalar fields with fermions we can still learn something about properties of the scalar fields.

I demonstrate in subsect. 10.2.2 that all the scalar fields are doublets with respect to the weak charge and that they carry a hyper charge. I comment in subsect. 10.2.2 that the symmetry of mass matrices are the same for all the family members and that loop corrections keep this symmetry. I demonstrate the properties of the condensate in subsect. 10.2.3 and comment on why do the two groups of four families differ in masses, and why do the two gauge vector fields, carrying the $SU2_{II}$ and $SU(2)_I$ quantum numbers, respectively, differ in masses.

Scalar fields - doublets with respect to weak charge and carrying hyper charge

We saw in sect. 10.2.1, Eqs. (10.12, 10.22) that the operators $\overset{78}{(-)}\,\Phi^{Ai}_{-}$ and $\overset{78}{(+)}\,\Phi^{Ai}_{+}$ transform the right handed u_R-quarks and ν_R-leptons and the right handed d_R-quarks and e_R-leptons, respectively, into the corresponding left handed partners for all the scalar fields, independent of the family quantum numbers. Scalar fields $\Phi^{Ai}_{\mp}$ ($\Phi^{Ai}_{\mp}$ stay for $\{\vec{\tilde{A}}^{\tilde{N}_R}_{\mp}, \vec{\tilde{A}}^{2}_{\mp}, \vec{\tilde{A}}^{\tilde{N}_L}_{\mp}, \vec{\tilde{A}}^{1}_{\mp}, A^{Q}_{\mp}, Z^{Q'}_{\mp}, A^{Y'}_{\mp}\}$ (Eq. (10.22)) with nonzero vacuum expectation values must accordingly carry the appropriate quantum numbers. All these scalar fields appear in Eqs. (10.12, 10.22) as follows

$$\begin{aligned}&\psi^{\dagger}\gamma^0\,\sum_{Ai}(\overset{78}{(-)}\;\Phi_{-} + \overset{78}{(+)}\;\Phi_{+})\psi\,,\\ &\qquad\qquad \Phi_{\mp} = \Phi_7 \pm i\Phi_7\,.\end{aligned} \tag{10.28}$$

Let us analyse their properties. Eqs. (10.8, 10.9) and Table 10.1 require [4] that

$$\begin{aligned}\vec{\tau}^1 &= \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{6,8}, S^{56} - S^{7,8})\,,\\ \vec{\tau}^2 &= \frac{1}{2}(S^{58} + S^{6,7}, S^{57} - S^{6,8}, S^{56} + S^{7,8})\,,\\ Y = \tau^{23} + \tau^4\,, &\quad \tau^4 = -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})\,.\end{aligned} \tag{10.29}$$

Any vector A^d has the transformation property

$$(S^{ab})^c{}_d A^d = i(\eta^{ac}\,\delta^b_d - \eta^{bc}\,\delta^a_d)\,A^d\,. \tag{10.30}$$

Correspondingly one finds the following properties of the fields

$$\begin{aligned}
&\tau^4\,(\Phi^7 \pm i\Phi^8) = 0\,, \quad Y\,(\Phi^7 \pm i\Phi^8) = \mp\frac{1}{2}(\Phi^7 \pm i\Phi^8)\,,\\
&\tau^{13}\,(\Phi^7 \pm i\Phi^8) = \pm\frac{1}{2}(\Phi^7 \pm i\Phi^8)\,,\\
&\tau^{1+}\,(\Phi^7 + i\Phi^8) = -(\Phi^5 + i\Phi^6)\,, \quad \tau^{1-}\,(\Phi^7 + i\Phi^8) = 0\,,\\
&\tau^{1-}\,(\Phi^7 - i\Phi^8) = (\Phi^5 - i\Phi^6)\,, \quad \tau^{1+}\,(\Phi^7 - i\Phi^8) = 0\,,\\
&\tau^{1+}\,(\Phi^5 + i\Phi^6) = 0\,, \quad \tau^{1-}\,(\Phi^5 + i\Phi^6) = -(\Phi^7 + i\Phi^8)\,,\\
&\tau^{1+}\,(\Phi^5 - i\Phi^6) = (\Phi^7 - i\Phi^8)\,, \quad \tau^{1-}\,(\Phi^5 - i\Phi^6) = 0\,,\\
&\tau^4\,(\Phi^5 \pm i\Phi^6) = 0\,, \quad \tau^{13}\,(\Phi^5 \pm i\Phi^6) = \mp\frac{1}{2}(\Phi^5 \pm i\Phi^6)\,,\\
&Y(\Phi^5 \pm i\Phi^6) = \mp\frac{1}{2}(\Phi^7 \pm i\Phi^8)\,.
\end{aligned} \tag{10.31}$$

In Eq. (10.31) the fields $(\Phi^7 \pm i\Phi^8) = \Phi_\mp$ stay for all $\Phi^{Ai}_\mp$.

It is, therefore, just proved that the scalar fields $\Phi^{Ai}_\mp$ with nonzero vacuum expectation values contribute on the tree level to the mass term of fermions with which they interact, "dressing" at the same time the right handed u_R-quarks and ν_R-leptons with the weak charge $\tau^{13} = \frac{1}{2}$ and the hyper charge $Y = -\frac{1}{2}$, while they "dress" the right handed d_R-quarks and e_R-leptons with the weak charge $\tau^{13} = -\frac{1}{2}$ and the hyper charge $Y = \frac{1}{2}$.

Why are symmetries of mass matrices kept in all orders of loop corrections? I have checked, together with the coauthor [31], that the symmetry of the mass matrix, Eq. 10.23, suggested by the *spin-charge-family* theory, stays unchanged in all orders of loop corrections, for several types of loop contributions. The evaluations were done in the massless basis. The final proof is under investigations and looks promising.

10.2.3 Do we understand why do two groups of four families distinguish in masses and why do two vector boson $SU(2)$ fields distinguish in masses?

All the scalar fields, which gain nonzero vacuum expectation values, are doublets with respect to the weak charge carrying also the hyper charge, as we have seen in the above discussions. This is true independently of what family quantum numbers the scalar fields carry. Correspondingly all the scalar fields contribute to the masses of Z_m and $W^\pm_m$ vector bosons. Each of the two groups of four families carry different family charges, coupling correspondingly only to those scalars, which are the gauge fields of their family groups.

How can then the two groups of families have so different masses? And why are the masses of the vector gauge fields of the group $SU(2)_{II}$ so much larger than those of the vector bosons Z_m and $W^\pm_m$?

The right handed neutrinos with the family quantum numbers of the upper group of four families are solving this problem, provided that they form a condensate with quantum numbers $Q = 0 = Y$, $\tau^{13} = 0$, $\tau^{23} = 1$, $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$, different values of $\tilde{\tau}^{23}$, $\tilde{N}^3_R$ are also acceptable. Such a condensate couples to the gauge fields $\vec{A}^2_m$ and, in loop corrections, to the upper four families. It does not couple to the lower four families and also not to the vector bosons Z_m and $W^{\pm}_m$. The condensate causes a non conservation of the fermion quantum number, keeping (3× quark minus lepton) quantum number unbroken, as long as Y is a conserved quantity.

In Table 10.6 a triplet of the group $SU(2)_{II}$ with the generators τ^{2i} is presented: The condensate of the right handed neutrinos and the two partners, all carrying τ^4 equal to -1. The family quantum numbers $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R$ are chosen. Any of the rest possibilities for these two family quantum numbers values, or all of them are acceptable as well.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{23}$	$\tilde{N}^3_R$
$(\|\nu_{1R}>_1 \|\nu_{2R}>_2)_{\mathcal{A}}$	0	0	0	1	−1	0	0	1	1
$(\|\nu_{1R}>_1 \|e_{2R}>_2)_{\mathcal{A}}$	0	0	0	0	−1	−1	−1	1	1
$(\|e_{1R}>_1 \|e_{2R}>_2)_{\mathcal{A}}$	0	0	0	−1	−1	−2	−2	1	1

Table 10.6. The condensate of two right handed neutrinos ν_R, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, together with its two partners, is presented. The condensate has $Q = 0 = Y$. The triplet carries $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R = 1$ (All belong to the family IVR of the group II from Table 10.5). The family quantum numbers IVR are not noted on the states. Index $\mathcal{A}$ stays for anti symmetrization.

There could be condensates also from the anti-neutrinos, right handed and belonging to the upper four families with the same family quantum numbers, or with other possible family quantum numbers of the same group. The corresponding condensate of two anti-neutrinos to the neutrinos presented in Table 10.6 would carry $\tau^{23} = -1$ and $\tau^4 = -1$.

It stays an open question, what does make the right handed neutrinos (or antineutrinos), belonging to the upper four families, to form such a condensate.

10.3 Conclusions and predictions of *spin-charge-family* theory

I demonstrate in this talk that the *spin-charge-family* theory is offering the explanation for the appearance of families, explaining as well the appearance of several scalar fields and of so far observed charges of fermions and the corresponding gauge fields. I demonstrate why are these scalar fields doublets with respect to the weak charge and singlets with respect to the hyper charge. I also offers predictions of the theory.

The theory predicts that there are two decoupled massless four families at some low energy scale, which stay massless also after they become massive, since each of the two groups carries different family quantum numbers.

There are two kinds of triggers responsible for the appearance of fermion masses: **i.** The condensate of the right handed neutrinos, carrying the family quantum numbers of the upper four families. Carrying the quantum numbers of the $SU(2)_{II}$ gauge vector field, the condensate makes this gauge field massive. Carrying the family quantum numbers of only one of the two groups, the condensate contribute to masses of the upper four families. **ii.** The scalar fields after they gain nonzero vacuum expectation values. The scalar fields belong to three groups: *ii.a.* The two scalar triplets with respect to the family quantum numbers of the upper four families bring masses to the upper four families. *ii.b.* The two scalar triplets with respect to the family quantum numbers of the lower four families bring masses to the lower four families. These two kinds of scalar fields do not distinguish among family members. *ii.c.* The third kind of the scalars are singlets which carry the quantum numbers (Q, Q', Y') of the family members, distinguishing correspondingly among the family members and not among families. They contribute to masses of all the eight families.

I demonstrate that all the scalar fields are doublets with respect to the weak charge carrying also the hyper charge, just as the so far observed Higgs is. They "dress" correspondingly the right handed u_R-quark and ν_R- lepton with the weak $\tau^{13} = \frac{1}{2}$ and the hyper charge $Y = -\frac{1}{2}$, while they "dress" the right handed d_R-quark and e_R- lepton with the weak $\tau^{13} = -\frac{1}{2}$ and the hyper charge $Y = \frac{1}{2}$. Correspondingly all the scalar fields contribute to masses of Z_m and $W^{\pm}_m$.

I demonstrate properties of one representation of the $SO(13,1)$, which includes all the family members, left and right handed, coloured and colourless, as well as their antiparticles, and the properties of families of all these quarks and leptons and the antiquarks and antileptons, using our spinor technique.

The appearance of several scalar fields manifest at the low energy regime as the Higgs, explaining the Yukawa couplings.

I offer the answer to the question: Why are the two $SU(2)$ gauge fields, $SU(2)_{II}$, which is not yet observed, and the weak $SU(2)_{I}$ so different in masses and why are also the two groups of four families so different in masses. The condensate of the right handed neutrinos with the family quantum numbers of the upper four families resolves this problem, since it couple only to the $SU(2)_{II}$ gauge bosons and to the upper four families.

The theory predicts that there are two times decoupled four families at the low energy.

The lowest of the upper four families is stable and is the candidate to form the dark matter [21]. The fourth of the lower four families will be observed at the LHC. Accurately enough measured mixing 3×3 sub matrices of quarks and leptons will enable to determine the masses of the fourth family members accurately. The ref. [25] is reporting on this calculations.

The *spin-charge-family* theory is treating all the family members, quarks and leptons, equivalently. I report on the trial to prove that the symmetry of mass matrices predicted by the theory, the same one for all the family members, is kept in all loop corrections. Loop corrections in all orders are needed to understand why are mass matrices so different in values for different family members, while they all demonstrate the same symmetry.

10.4 APPENDIX: Short presentation of technique [27,28]

I make in this appendix a short review of the technique [28], initiated and developed by me when proposing the *spin-charge-family* theory [1–4,20,21] assuming that all the internal degrees of freedom of spinors, with family quantum number included, are describable in the space of d-anti-commuting (Grassmann) coordinates [27], if the dimension of ordinary space is also d. There are two kinds of operators in the Grassmann space, fulfilling the Clifford algebra which anti-commute with one another. The technique was further developed in the present shape together with H.B. Nielsen [28] by identifying one kind of the Clifford objects with γ^s's and another kind with $\tilde{\gamma}^a$'s. In this last stage we constructed a spinor basis as products of nilpotents and projections formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s. The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

The objects γ^a and $\tilde{\gamma}^a$ have properties

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,, \tag{10.32}$$

for any d, even or odd. I is the unit element in the Clifford algebra.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned}
S^{ab} &:= (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,,\\
\tilde{S}^{ab} &:= (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,,\\
\{S^{ab}, \tilde{S}^{cd}\}_- &= 0\,,\\
\{S^{ab}, S^{cd}\}_- &= i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac})\,,\\
\{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- &= i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})\,,
\end{aligned} \tag{10.33}$$

We assume the "Hermiticity" property for γ^a's and $\tilde{\gamma}^a$'s

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a\,, \qquad \tilde{\gamma}^{a\dagger} = \eta^{aa}\tilde{\gamma}^a\,, \tag{10.34}$$

in order that γ^a and $\tilde{\gamma}^a$ are compatible with (10.32) and formally unitary, i.e. $\gamma^{a\,\dagger}\gamma^a = I$ and $\tilde{\gamma}^{a\,\dagger}\tilde{\gamma}^a = I$.

One finds from Eq.(10.34) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(10.33) that two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$\begin{aligned}
S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, &\quad \text{if} \quad d = 2n \ge 4,\\
S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,,\\
\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, &\quad \text{if} \quad d = 2n \ge 4\,,\\
\tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,.
\end{aligned} \tag{10.35}$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$\Gamma^{(d)} := (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n,$$
$$\Gamma^{(d)} := (i)^{(d-1)/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n + 1\,. \tag{10.36}$$

One can proceed equivalently for $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1 \cdots \gamma^d$. It follows from Eq.(10.34) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma,\ \Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$), while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows (Eq. (10.14))

$$\stackrel{ab}{(k)}: = \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \stackrel{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,,$$
$$\stackrel{+}{\circ}: = \frac{1}{2}(1 + \Gamma)\,, \qquad \stackrel{-}{\bullet}:= \frac{1}{2}(1 - \Gamma), \tag{10.37}$$

where $k^2 = \eta^{aa}\eta^{bb}$. One can easily check by taking into account the Clifford algebra relation (Eq.10.32) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq.10.33) that if one multiplies from the left hand side by S^{ab} or $\tilde{S}^{ab}$ the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, it follows that

$$S^{ab} \stackrel{ab}{(k)}= \frac{1}{2}\, k \stackrel{ab}{(k)}\,, \qquad S^{ab} \stackrel{ab}{[k]}= \frac{1}{2}\, k \stackrel{ab}{[k]}\,,$$
$$\tilde{S}^{ab} \stackrel{ab}{(k)}= \frac{1}{2}\, k \stackrel{ab}{(k)}\,, \qquad \tilde{S}^{ab} \stackrel{ab}{[k]}= -\frac{1}{2}\, k \stackrel{ab}{[k]}\,, \tag{10.38}$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognize (Eq. 10.17,10.18) that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$\gamma^a \stackrel{ab}{(k)}= \eta^{aa} \stackrel{ab}{[-k]},\ \gamma^b \stackrel{ab}{(k)}= -ik \stackrel{ab}{[-k]},\ \gamma^a \stackrel{ab}{[k]}=\stackrel{ab}{(-k)},\ \gamma^b \stackrel{ab}{[k]}= -ik\eta^{aa} \stackrel{ab}{(-k)}\,,$$
$$\tilde{\gamma^a} \stackrel{ab}{(k)}= -i\eta^{aa} \stackrel{ab}{[k]},\ \tilde{\gamma^b} \stackrel{ab}{(k)}= -k \stackrel{ab}{[k]},\ \tilde{\gamma^a} \stackrel{ab}{[k]}=\ i \stackrel{ab}{(k)},\ \tilde{\gamma^b} \stackrel{ab}{[k]}= -k\eta^{aa} \stackrel{ab}{(k)} \tag{10.39}$$

From Eq.(10.39) it follows

$$
\begin{aligned}
S^{ac}\, \stackrel{ab}{(k)}\stackrel{cd}{(k)} &= -\frac{i}{2}\eta^{aa}\eta^{cc}\, \stackrel{ab}{[-k]}\stackrel{cd}{[-k]}\,, &\quad \tilde{S}^{ac}\, \stackrel{ab}{(k)}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{aa}\eta^{cc}\, \stackrel{ab}{[k]}\stackrel{cd}{[k]}\,,\\
S^{ac}\, \stackrel{ab}{[k]}\stackrel{cd}{[k]} &= \frac{i}{2}\, \stackrel{ab}{(-k)}\stackrel{cd}{(-k)}\,, &\quad \tilde{S}^{ac}\, \stackrel{ab}{[k]}\stackrel{cd}{[k]} &= -\frac{i}{2}\, \stackrel{ab}{(k)}\stackrel{cd}{(k)}\,,\\
S^{ac}\, \stackrel{ab}{(k)}\stackrel{cd}{[k]} &= -\frac{i}{2}\eta^{aa}\, \stackrel{ab}{[-k]}\stackrel{cd}{(-k)}\,, &\quad \tilde{S}^{ac}\, \stackrel{ab}{(k)}\stackrel{cd}{[k]} &= -\frac{i}{2}\eta^{aa}\, \stackrel{ab}{[k]}\stackrel{cd}{(k)}\,,\\
S^{ac}\, \stackrel{ab}{[k]}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{cc}\, \stackrel{ab}{(-k)}\stackrel{cd}{[-k]}\,, &\quad \tilde{S}^{ac}\, \stackrel{ab}{[k]}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{cc}\, \stackrel{ab}{(k)}\stackrel{cd}{[k]}\,.
\end{aligned}
\qquad (10.40)
$$

From Eqs. (10.40) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$
\begin{aligned}
&\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0\,, && \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa}\, \stackrel{ab}{[k]}\,, && \stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa}\, \stackrel{ab}{[-k]}\,, && \stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0\,,\\
&\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}\,, && \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0\,, && \stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0\,, && \stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]}\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0\,, && \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,, && \stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}\,, && \stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0\,,\\
&\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}\,, && \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, && \stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0\,, && \stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}\,.
\end{aligned}
\qquad (10.41)
$$

We recognize in the first equation of the first line and the first and the second equation of the second line the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively. Defining

$$
\stackrel{ab}{(\tilde{\pm} i)} = \frac{1}{2}\,(\tilde{\gamma}^a \mp \tilde{\gamma}^b)\,, \quad \stackrel{ab}{(\tilde{\pm} 1)} = \frac{1}{2}\,(\tilde{\gamma}^a \pm i\tilde{\gamma}^b)\,, \qquad (10.42)
$$

one recognizes that

$$
\stackrel{ab}{(\tilde{k})}\, \stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{(\tilde{-k})}\, \stackrel{ab}{(k)} = -i\eta^{aa}\, \stackrel{ab}{[k]}\,, \quad \stackrel{ab}{(\tilde{k})}\, \stackrel{ab}{[k]} = i\, \stackrel{ab}{(k)}\,, \quad \stackrel{ab}{(\tilde{k})}\, \stackrel{ab}{[-k]} = 0\,. \qquad (10.43)
$$

Recognizing that

$$
\stackrel{ab}{(k)}^{\dagger} = \eta^{aa}\, \stackrel{ab}{(-k)}\,, \quad \stackrel{ab}{[k]}^{\dagger} = \stackrel{ab}{[k]}\,, \qquad (10.44)
$$

we define a vacuum state $|\psi_0>$ so that one finds

$$
\begin{aligned}
&< \stackrel{ab}{(k)}^{\dagger}\, \stackrel{ab}{(k)} >= 1\,,\\
&< \stackrel{ab}{[k]}^{\dagger}\, \stackrel{ab}{[k]} >= 1\,.
\end{aligned}
\qquad (10.45)
$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of $d/2$, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(10.35)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$
\begin{array}{r}
\stackrel{0d}{(k_{0d})}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\,\psi_0 \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{[-k_{12}]}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\,\psi_0 \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\,\psi_0 \\
\vdots \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\;d-2}{[-k_{d-1\;d-2}]}\,\psi_0 \\
\stackrel{od}{(k_{0d})}\stackrel{12}{[-k_{12}]}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\;d-2}{(k_{d-1\;d-2})}\,\psi_0 \\
\vdots
\end{array}
\tag{10.46}
$$

All the states have the handedness Γ, since $\{\Gamma, S^{ab}\} = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrate that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

For d odd a Weyl spinor has besides a product of $(d-1)/2$ nilpotents or projectors also either the factor $\stackrel{+}{\circ} := \frac{1}{2}(1+\Gamma)$ or the factor $\stackrel{-}{\bullet} := \frac{1}{2}(1-\Gamma)$. As in the case of d even, all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of $(1+\Gamma)$ and $(d-1)/2$ nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. But γ^a's, being applied from the left hand side, do not change the handedness of the Weyl spinor, since $\{\Gamma, \gamma^a\}_- = 0$ for d odd. A Dirac and a Weyl spinor are for d odd identical and a "family" has accordingly $2^{(d-1)/2}$ members of basic states of a definite handedness.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (10.35)) generate all the states of one representation, generate $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra(Eq. (10.35)) the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set of the algebra S^{ab} and $\tilde{S}^{ab}$

$$\begin{aligned} &S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}, \\ &\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}, \end{aligned} \tag{10.47}$$

a left handed ($\Gamma^{(13,1)} = -1$) eigen state of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,, 1/(2\sqrt{3}))$, for example, can be written as

$$\begin{aligned} &\stackrel{03}{(+i)}\stackrel{12}{(+)} \,|\, \stackrel{56}{(+)}\stackrel{78}{(+)} \,||\, \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\; |\psi\rangle = \\ &\frac{1}{2^7}(\gamma^0-\gamma^3)(\gamma^1+i\gamma^2)|(\gamma^5+i\gamma^6)(\gamma^7+i\gamma^8)|| \\ &(\gamma^9+i\gamma^{10})(\gamma^{11}-i\gamma^{12})(\gamma^{13}-i\gamma^{14})|\psi\rangle \,. \end{aligned} \tag{10.48}$$

This state is an eigen state of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (10.16)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (10.16)), generate families from the starting u_R quark, transforming u_R quark from Eq. (10.48) to the u_R of another family, keeping all the properties with respect to S^{ab} unchanged. In particular $\tilde{S}^{01}$ applied on a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$ from Eq. (10.48) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$

$$\tilde{S}^{01}\; \stackrel{03}{(+i)}\stackrel{12}{(+)} \,|\, \stackrel{56}{(+)}\stackrel{78}{(+)} \,||\, \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} = -\frac{i}{2}\; \stackrel{03}{[+i]}\stackrel{12}{[+]} \,|\, \stackrel{56}{(+)}\stackrel{78}{(+)} \,||\, \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}\;. \tag{10.49}$$

Below some useful relations [2] are presented

$$\begin{aligned} &N^{\pm}_{+} = N^1_{+} \pm i\, N^2_{+} = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}, \quad N^{\pm}_{-} = N^1_{-} \pm i\, N^2_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}, \\ &\tilde{N}^{\pm}_{+} = -\stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}, \quad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}}, \\ &\tau^{1\pm} = (\mp)\; \stackrel{56}{(\pm)}\stackrel{78}{(\mp)}, \quad \tau^{2\mp} = (\mp)\; \stackrel{56}{(\mp)}\stackrel{78}{(\mp)}, \\ &\tilde{\tau}^{1\pm} = (\mp)\; \stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}, \quad \tilde{\tau}^{2\mp} = (\mp)\; \stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}\;. \end{aligned} \tag{10.50}$$

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11 Small Representation Principle

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Abstract. In a previous article [2] Don Bennett and I looked for, found and proposed a game in which the Standard Model Gauge *Group* $S(U(2) \times U(3))$ gets singled out as the "winner". This "game" means that the by Nature chosen gauge group should be just that one, which has the maximal value for a quantity, which is a modification of the ratio of the quadratic Casimir for the adjoint representation and that for a "smallest" faithful representation. In a recent article [1] I proposed to extend this "game" to construct a corresponding game between different potential dimensions for space-time. The idea is to formulate, how the same competition as the one between the potential gauge groups would run out, if restricted to the potential Lorentz or Poincare groups achievable for different dimensions of space-time d. The remarkable point is, that it is the experimental space-time *dimension 4, which wins*.

Our "goal quantity" to be maximized has roughly the favouring meaning that the Lie-group in question can have the "smallest" possible faithful representations. This idea then suggests that the representations of the Standard Model group to be found on the (Weyl)Fermions and the Higgs Boson should be in the detailed way measured by our "goal quantity" be the smallest possible. The Higgs in the Standard Model belongs remarkably enough just to the in such a way "smallest" representation. For the chiral Fermions there are needed restriction so as to avoid anomalies for the gauge symmetries, and in an earlier work[14,12] we have already suggested that the Standard Model Fermion representations could be considered being the smallest possible. We hope in the future to show that also taking smallness in the specific sense suggested here would lead to the correct Standard Model representation system.

So with the suggestion here the whole Standard Model is specified by requiring SMALLEST REPRESENTATIONS! Speculatively we even argue that our principle found suggests the group of gauge transformations and some manifold(suggestive of say general relativity).

Povzetek. V prejšnjem članku [2] sva z Donom Bennettom iskala, našla in predlagala igro, v kateri se umeritvena *grupa* $S(U(2) \times U(3))$ standardnega modela izkaže kot "zmagovalka". Ta "igra" pomeni, da je Narava izbrala umeritveno grupo z maksimalno vrednostjo količine, ki je nekoliko spremenjeno razmerje med kvadratom Casimirja za adjungirano upodobitev in le tega za "najmanšo" zvesto upodobitev. V nedavnem članku [1] sem predlagal razširitev te "igre" s konstrukcijo ustrezne igre med različnimi možnimi razsežnostmi prostor-časa. Tokrat bi tekmovale Lorentzove in Poincarejeve grupe za različne razsežnosti d prostor-časa. Izkaže se, da v tej igri zmaga prav opažena *razsežnost* 4 . Naša "ciljna količina", ki jo želimo maksimizirati, v grobem pomeni, da ima iskana Liejeva grupa "najmanšo" možno zvesto upodobitev. Ali, da ima v primeru upodobitev grup standardnega modela za

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(Weylove) fermione in Higgsov bozon “ciljna količina” najmanjšo vrednost. Higgsov delec v standardnem modelu pripada prav upodobitvi, ki je v zgornjem smislu “najmanjša”. Za kiralne fermione moramo zahtevati omejitve, ki poskrbijo, da se izognemo anomalijam umeritvenih simetrij. V prejšnjem delu [14,12] smo že predlagali “ciljno količino”, ki zagotovi zmago upodobitvam za fermione standardnega modela. Upati je, da v tem prispevku definirana “ciljna količina” prav tako zagotovi zmago upodobitvam standardnega modela. V skladu z našim predlogom bi torej lahko izpeljali grupe standardnega modela iz zahteve za NAJMANJŠO UPODOBITEV! Postavljamo tedaj spekulativno trditev, da naše načelo predlaga grupo umeritvenih transformacij in mnogoterost (kar namiguje na splošno teorijo relativnosti).

11.1 Introduction

In two earlier articles[2,1] Don Bennett and I proposed a quantity depending on a group - thought of as the gauge *group*(group in the sense of O’Raifeartaigh [3] - which were found to take its largest value on just the Standard Model gauge *group* $S(U(2) \times U(3))$. My article [1] were to tell that the same quantity applied to the Lorentz or by some crude technology to essentially the Poincare group selected as the number of dimensions winning the highest quantity just the experimental number of dimensions 4 for space time. (The prediction of the d=4 dimensions from various reasons have been considered in e.g. [4–7]. N.Brene and I have earlier proposed another quantity to be extremized to select the Standard Model group, namely that it is the most “skew”[8] (i.e. it has the smallest number of automorphisms, appropriately counted). But in this article we shall discuss a “goal quantity”that being maximized as it shall, rather may mean crudely that the group can have as small representations as possible.

To define this wonderful group dependent quantity, which can in this way select as the highest scoring group the by Nature chosen Standard Model *group*, and the by Nature chosen space time dimension 4, let us think of a general Lie group written by means of a cross product of a series of simple Lie groups H_i (take the H_i’s to be the covering groups at first) and a series of real number **R** factors in this cross product

$$G_{cover} = \left(\bigtimes_i^I H_i \right) \times \mathbf{R}^J. \tag{11.1}$$

Here the I is the number of, different or identical, as it may be, H_i-groups, which are supposed to be simple Lie groups, while **R** denotes the Abelian group of real numbers under addition.The number of Abelian dimensions in the Lie algebra is called J. A very general group is obtained by dividing an invariant discrete subgroup D of the center out of this group G_{cover}. Denoting this general - though assumed connected - group as G we can indeed write it as

$$G = G_{cover}/D. \tag{11.2}$$

Of course G_{cover} is the covering group of G and the groups H_i (i = 1,2, ... I-1,I) are its invariant simple Lie groups.

The main ingredient in defining our goal quantity is the ratio of the quadratic Casimirs[10] C_A/C_F of the quadratic Casimir C_A for the adjoint representation divided by the quadratic Casimir C_F a representation chosen, so as to make the quadratic Casimir C_F of F so small as possible though still requiring the representation F to be faithful or basically to be non-trivial. Here I now ought to remind the reader of the concept of a quadratic Casimir operator:

The easiest may be to remember the concept of quadratic Casimir first for the most well known example of a nonabelian Lie group, namely the group of rotations in 3 dimensions $SO(3)$ (when you do not include reflection in a point but only true rotations) for which the covering group is $SU(2)$. In this case the quadratic Casimir operator is the well known square of the angular momentum operator

$$\vec{J}^2 = J_x^2 + J_y^2 + J_z^2 \tag{11.3}$$

Now our goal quantity, which so nicely points to both the Standard Model group and the dimension of space time, is given as the d_G'th root of the product with one factor from each invariant simple group H_i, namely $(C_A/C_F)^{d_i}$ (C_F/C_A is related to the Dynkin-index [11]) and some factors e_A^2/e_F^2 for each of the J Abelian factors. (Here the dimension of the simple groups H_i are denoted d_i, while the dimension of the total group G or of G_{cover} is denoted d_G.) Our goal quantity in fact becomes

$$\text{“goal quantity”} \tag{11.4}$$

$$= \left(\prod_{\text{simple groups } i} \left(\frac{C_A}{C_F}\right)_i^{d_i} * \prod_{\text{Abelian factors } j}^{J} \left(\frac{e_A^2}{e_F^2}\right)_j \right)^{1/d_G} . \tag{11.5}$$

To fully explain this expression I need to explain what means the “charges” e_F for the “small” representation (essentially F) and e_A for the analogon [1] to the adjoint representation: Of course the reader should have in mind that the Abelian groups, the **R** subgroups, have of course no adjoint representations in as far as the basis in the Lie algebra of an Abelian group is only transformed trivially. In stead of defining these “charges” – as we shall do below – by first defining a replacement for the adjoint we shall define these factors $\prod_{\text{Abelian factors } j}^{J} (\frac{e_A^2}{e_F^2})_j$ from the Abelian factors in the Lie algebra by means of the system of *allowed and not allowed representations* of the *group* $G = G_{cover}/D = \left(\left(\bigtimes_i^I H_i\right) \times \mathbf{R}^J\right)/D$. Each irreducible representation of this G is characterized in addition to its representations under the simple Lie groups H_i also by a “vector” of “charges” representing the phase factors $\exp(i\delta_1 e_{r1} + i\delta_2 e_{r2} + ... + i\delta_J e_{rJ})$, which multiply the representation vector under an element $(\delta_1, \delta_2, ..., \delta_J) \in \mathbf{R}^J$, i.e. in the Abelian factor of G. The easiest may be to say that we consider the whole lattice system of allowed “vectors”

[1] We might define an analogon of the adjoint representation for also a set of J properly chosen **R**-factors of G by assigning the notion of “analogon to the adjoint representation” to that representation of one of the **R**-factors, which has the smallest charge, e_A called, allowed for a representation of $\mathbf{R}/(\mathbf{R} \cap D)$, where **R** stands for the **R**-factor considered, and $\mathbf{R} \cap D$ for the intersection of the to be divided out discrete group D with this Abelian factor **R**.

$\{(e_{r1}, e_{r2}, ..., e_{rJ})|r \text{allowed by } G\}$ of sets "charges" allowed by the *group* G, and then compare with corresponding set in which we *only consider those representations* r*, which represent the simple non-abelian groups only trivially*:

$$\{(e_{r1}, e_{r2}, ..., e_{rJ})|r \text{ allowed by } G, \text{ and with the representation of the } H_i\text{'s being only trivial}\}.$$

In this comparison you ask for a going to an infinitely big region in the J-dimensional lattice after the ratio of the number of "charge vectors" in the first lattice

$$\{(e_{r1}, e_{r2}, ..., e_{rJ})|r \text{allowed by } G\}$$

relative to that in the second

$$\{(e_{r1}, e_{r2}, ..., e_{rJ})|r \text{ G-allowed with the } H_i\text{'s represented trivially}\}.$$

Then the whole factor under the d_G'th root sign is the product of the factor comming from the semisimple part of the group G

$$\text{"Semisimple factor"} = \prod_{\text{simple groups } i} (\frac{C_A}{C_F})_i^{d_i} \tag{11.6}$$

and the ratio of the number of charge combinations at all allowed by the *group* G to the number of charge combinations, when the semisimple groups are restricted to be represented trivially - in the representation of the whole G representing the Abelian part by the charge combination in question :

$$\text{"Abelian factor"} = \left(\frac{\#\{(e_{r1}, e_{r2}, ..., e_{rJ})|r \text{ allowed by } G\}}{\#\{(e_{r1}, e_{r2}, ..., e_{rJ})|r \text{ G-allowed } \textit{with the } H_i\textit{'s represented trivially}\}}\right)^2. \tag{11.7}$$

Here # stands for the number of elements in the following set, i.e. the cardinal number; but it must be admitted that the numbers of these charge combinations are infinite, and that to make the finite result, which we shall use, we have to take a cut off and take the limit of the ratio for that cut off going to be a bigger and bigger sphere finally covering the whole J-dimensional space with the charge combinations embedded. So strictly speaking we define rather

$$\text{"Abelian factor"} = \left(\lim_{S\to\infty} \frac{\#\{(e_{r1}, e_{r2}, ..., e_{rJ})|r \text{ allowed by } G\}_{\text{cut off by } S}}{\#\{(e_{r1}, e_{r2}, ..., e_{rJ})|r \text{ G-allowed } \textit{with the } H_i\textit{'s represented trivially}\}_{\text{cut off by } S}}\right)^2.$$

where S is some large "sphere say" in the J-dimensional space of charge combinations. The symbol $S \to \infty$ shall be understood to mean that the region S is taken to be larger and larger in all directions so as to in the limit cover the whole space.

Then our goal quantity to be maximized so as to select the gauge group supposed to be chosen by nature can be written

$$\text{"goal quantity"} = (\text{"Semisimple factor"} * \text{"Abelian factor"})^{1/d_G}. \tag{11.8}$$

Really it is nice to express the quantity "Abelian factor" by means of the representations allowed by the group, because after all the phenomenological determination of the *Lie-group* rather than only the Lie algebra[3] is based on such a system of allowed representations.

11.1.1 Motivation

Before illustrating the calculation of our "goal quantity" with Standard Model as the example, let me stress the motivation or interest in looking for such a function defined on gauge groups or more abstractly somehow on theories and can be used to single out the by Nature chosen model. A major reason making such a singling out especially called for is that the Standard Model and e.g. its group is not in an obvious way anything special! It is a combination of several subgroups like $SU(2)$, $SU(3)$, and $U(1)$ of groups that cannot all be the obvious *one*, since we already use 3. There exist both several groups with lower rank, say than the 4 of the Standard Model group, and of cause infinitely many with higher rank. That it truly has been felt, not only by us, but by many physicists that the Standard Model is *a priori* not anything obviously special - except for the fact, that it is the model that agrees with experiment - can be seen from the great interest in - and even belief in - grand unification theories[19] seeking to find e.g. an extended gauge group, of which the Standard Model gauge group is then only the small part, which survived some series of (spontaneous) break downs of part of the larger group. Let me put some of the predictions of the typical grand unification model as $SU(5)$ in the perspective: When they are concerned with representations possible for say the $SU(5)$, there are restrictions for what they can be for the Standard Model "$SU(2) \times SU(3) \times U(1)$"- and they agree with experiment -;but then these restrictions are truly a consequence of that *the sub***group** *of* $SU(5)$ *having the Lie-algebra (of)* $SU(2) \times SU(3) \times U(1)$ *is precisely the* **group** $S(U(2) \times U(3))$. Indeed the condition on the possible representations, when there is an $SU(5)$ GUT theory beyond the Standard Model, is the same condition (11.10) as comes from $S(U(2) \times U(3))$. There is of course more information in specifying the group than only the Lie-algebra; but that of course only implies that an a priori not special group is even less special than an algebra, because there are even more *groups* among which to choose than there are *algebras*. (Of course there are truly infinitely many both groups and algebras, but for a given range of ranks, say, there are more Lie-*groups* than Lie-*algebras*).

Another hope of explaining, why the Standard Model including its gauge group is chosen by Nature, is the superstring theories, which predict at the fundamental or string level the gauge groups $E_8 \times E_8$ or $SO(32)$. But from the point of view of our "goal quantity" - as can be seen below form our tables - especially E_8 and consequently also $E_8 \times E_8$ (since our "goal quantity" has the property of being the same for a group G and its cross products with itself any number of times) is the worst group from the point of view of our "goal quantity": In fact the nature of our "goal quantity" construction is so, that we always must have

$$\text{"goal quantity"} \geq 1. \tag{11.9}$$

But E_8 according to the table below gives just this 1 for its "goal quantity"

$$\text{"goal quantity"}_{E_8 \times E_8} = \text{"goal quantity"}_{E_8} = 1$$

actually because E_8 has no smaller representation than its adjoint representation.

The connection to my personal pet-theory (or dream, or program) of Random Dynamics [13,15,17,16,18,20] is that a priori the present work is ideally phenomenologically - as to be explained in subsection 11.3.3-,i. e. the spirit is to ask nature and just seek to find what is characteristic for the Standard Model group without theoretical guesses behind a priori. However, it(= our phenomenological result) leads to the suggestion that the (gauge)group that wins - gets highest "goal quantity" - is the one that most likely would become approximately a good symmetry by accident. This would then mean, that in a random model, as is the picture in Random Dynamics, the group, that is selected by our game, is just the one most likely to be realized as an approximately good symmetry by accident. So indeed Random Dynamics could be a background theory for the present work. So in this sense random dynamics ends up being favoured by the present article, although we in principle started out purely phenomenologically. (It must be admitted though, that historically the idea appeared as an *extract* from a long Random Dynamics inspired calculation - which has so far not been published - by Don Bennett and myself.) Having approximately gauge symmetries, there is according to some earlier works of ours and others [16,25,29,30] the possibility that the gauge symmetry may become exact by quantum fluctuations; really one first writes it formally as if the remaining small breaking were a Higgsing, and then argue that quantum fluctuations wash away this "Higgs"effect.

11.1.2 Plan of Article

In the next section 11.2 we shall with the Standard Model group as an example tell how to calculate the goal quantity, and we deliver in this section 11.2 also some *tables* to use for such computations. Then in section 11.3 we discuss the attempt to also postdict the dimension of space time; for that several slight modifications are used to in an approximate sense construct a goal quantity like quantity for even the Poincare group in an arbitrary dimension d for space time. Successively in section 11.4 we consider, how we can extend our ideas to measure the *size of a representation* of the Standard Model group, and then the wonderful result is that the representation, under which the *Higgs* fields transform, remarkably enough turns out to be just the *smallest (non- trivial) representation*!

The following sections are about work still under development, and in section 11.5 we review an old work making more precise, what is already rather intuitively obvious: That the fermion representations in the Standard Model are rather "small" and that that together with anomaly conditions settles what they can be *assuming mass protected fermions only*. In the next section 11.6 we point to a way of changing the point of view so as to say, that, what we predict, is rather than the gauge group the group of gauge transformations. This may be the beginning to predict also a manifold structure for the whole gauge theory. Are we on the way to general relativity? We conclude and resume in section 11.7.

11.2 Calculation of "Goal quantity" Illustrated with the Standard Model Group $S(U(2) \times U(3))$

Rather than going into using the structure as a *group* rather than only the Lie-*algebra* structure we just above remarked that we can determine the "Abelian factor" (see 11.8) by studying the system of representation allowed as representations of the *group* rather than being just allowed by the Lie-algebra.

For example the phenomenological feature of the Standard Model, that gives rise to, that the Standard Model Group indeed must be taken as $S(U(2) \times U(3))$ [3], is the restriction on the weak hypercharge y quantization (or rather we prefer to use the half weak hypercharge $y/2$) realizing the usual assumption in the Standard Model about electric charge quantization (Milikan quantization extended with the well known rules for quarks). This rule become written for the Standard Model:

$$y/2 + I_W + \text{“triality”}/3 = 0(\text{mod} 1). \tag{11.10}$$

According to the rule to calculate the Abelian factor we shall in the limit of a going to infinity big range of $y/2$-values ask for what fraction of the number of values possible with the rule (11.10) imposed and the same but only including representations with the simple groups $SU(2)$ and $SU(3)$ in the Lie algebra of the Standard Model represented trivially. If we only allowed the adjoint or the trivial representations of these simple groups, so that $I_W = 0(\text{mod}\, 1)$ and "triality" $= 0$, it is quite obvious in our Standard Model example, that the Standard Model rule (11.10) allows, when the simple representations can be adjusted, all $y/2$ being an integer multiplum of $1/6$. If we, however, limit the simple groups to have trivial (or adjoint) representations only, then we can only have $y/2$ being integer. It is clear that this means in the limit of the large range S that there are 6 times as many $y/2$ values allowed, when the representations of the simple groups are free, as when it is restricted to be trivial (or adjoint). We therefore immediately find for the Standard Model Group

$$\text{“Abelian factor”}_{S(U(2)\times U(3))} = 6^2 = 36. \tag{11.11}$$

In order to calculate the factor "Semisimple factor " (11.6) we must look up the table for the C_A/C_F for the simple groups involved, then raise these factors to the power of the dimension of the Lie-algebras in question, and very finally after having multiplied also by the "Abelian factor" we must take the root of the total dimension of the whole group.

11.2.1 Useful Table

Here we give the table to use, our (essentially inverse Dynkin index [11]) ratios for the *simple* Lie groups, with the representation F selected so as to provide the biggest possible ratio C_A/C_F still keeping F non trivial, or let us say faithful (in a few cases the choice of this F is not clear at the outset and the user of the table has to choose the largest number among "vector" and "spinor" after he has provided the rank n he wants to use) :

Our Ratio of Adjoint to "Simplest" (or smallest) Quadratic Casimirs C_A/C_F

$$\frac{C_A}{C_F}|_{A_n} = \frac{2(n+1)^2}{n(n+2)} = \frac{2(n+1)^2}{(n+1)^2-1} = \frac{2}{1-\frac{1}{(n+1)^2}} \tag{11.12}$$

$$\frac{C_A}{C_{F\ vector}}|_{B_n} = \frac{2n-1}{n} = 2-\frac{1}{n} \tag{11.13}$$

$$\frac{C_A}{C_{F\ spinor}}|_{B_n} = \frac{2n-1}{\frac{2n^2+n}{8}} = \frac{16n-8}{n(2n+1)} \tag{11.14}$$

$$\frac{C_A}{C_F}|_{C_n} = \frac{n+1}{n/2+1/4} = \frac{4(n+1)}{2n+1} \tag{11.15}$$

$$\frac{C_A}{C_{F\ vector}}|_{D_n} = \frac{2(n-1)}{n-1/2} = \frac{4(n-1)}{2n-1} \tag{11.16}$$

$$\frac{C_A}{C_{F\ spinor}}|_{D_n} = \frac{2(n-1)}{\frac{2n^2-n}{8}} = \frac{16(n-1)}{n(2n-1)} \tag{11.17}$$

$$\frac{C_A}{C_F}|_{G_2} = \frac{4}{2} = 2 \tag{11.18}$$

$$\frac{C_A}{C_F}|_{F_4} = \frac{9}{6} = \frac{3}{2} \tag{11.19}$$

$$\frac{C_A}{C_F}|_{E_6} = \frac{12}{\frac{26}{3}} = \frac{18}{13} \tag{11.20}$$

$$\frac{C_A}{C_F}|_{E_7} = \frac{18}{\frac{57}{4}} = \frac{72}{57} = \frac{24}{19} \tag{11.21}$$

$$\frac{C_A}{C_F}|_{E_8} = \frac{30}{30} = 1 \tag{11.22}$$

For calculation of this table seek help in[27,26].

In the just above table we have of course used the conventional notation for the classification of Lie algebras, wherein the index n on the capital letter denotes the rank (the rank n is the maximal number of mutually commuting basis-vectors in the Lie algebra) of the Lie algebra, and:

- A_n is $SU(n+1)$,
- B_n is the odd dimension orthogonal group Lie algebra i.e. for $SO(2n+1)$ or for its covering group $Spin(2n+1)$,
- C_n are the symplectic Lie algebras.
- D_n is the even dimension orthogonal Lie algebra i.e. for $SO(2n)$ or its covering group $Spin(2n)$,
- while F_4, G_2, and E_n for $n = 6, 7, 8$ are the exceptional Lie algebras.

The words $spinor$ or $vector$ following in the index the letter F, which itself denotes the "small" representation - i.e. most promising for giving a small quadratic Casimir C_F - means that we have used for F respectively the smallest spinor and the smallest vector representation.

11.2.2 End of calculation of the "goal quantity" for the Standard Model Group

Since the Lie-algebra in addition to the Abelian part ($U(1)$ usually called) consists of $SU(2)$ and $SU(3)$ we must look these two simple Lie algebras up in the table above, finding respectively for the C_A/C_F ratios 8/3 and 9/4, which must be taken to respectively the powers 3 and 8, since the dimensions of the $A_n = SU(n+1)$ Lie-groups are "$dimension'' = (n+1)^2 - 1$, leading to

$$\text{“Semisimple factor”}_{S(U(")\times U(3))} = (\frac{8}{3})^3 \cdot (\frac{9}{4})^8 = 3^{13} \cdot 2^{-7} = 1594323/128$$
$$= 12455.6484375. \qquad (11.23)$$

Remembering that we got $6 = 3 \cdot 2$ for the ratio of numbers of $y/2$-values, when all representation obeying (11.10) were counted relative to this number for only the representations with trivial representations of $SU(2)$ and $SU(3)$, the "Abelian factor"$= 6^2 = 3^2 \cdot 2^2$. Then the whole factor, of which to next take the 12th root (since the total dimensionality of the Standard Model group is 12) becomes

$$\text{“}Semisimplefactor'' \cdot \text{“}Abelianfactor'' = (\frac{8}{3})^3(\frac{9}{4})^8 \cdot 36 = 2^{-5} \cdot 3^{21}$$
$$= 448403.34375. \qquad (11.24)$$

Thus we just have to take the 12th root of this quantity to obtain the score or "goal quantity" for the Standard Model *group* $S(U(2) \times U(3))$

$$\text{“}goalquantity''_{S(U(2)\times U(3))} = (2^{-5} \cdot 3^{15})^{1/12} = 3 \cdot (\frac{27}{32})^{1/12} = 3 \cdot 0.985941504$$
$$= \mathbf{2.957824511.} \qquad (11.25)$$

Similar calculations give the "goal quantity" for other groups. But it requires of course either a lot of work or some rules and experiences with calculating such goal quantities in order to see, which alternative groups are the severe competitors of the Standard Model *group* $S(U(2)\times U(3))$ that have to have their "goal quantities" computed in order to establish that the by Nature selected Standard Model group $S(U(2) \times U(3))$ is indeed the winner in obtaining the highest "goal quantity" (except for groups being higher powers of the Standard Model group itself).

For example a very near competitor is the group $U(2)$, for which one easily calculates

$$\text{“}Semisimplefactor''_{U(2)} = (\frac{8}{3})^3 = 2^9/3^3 = 18.962962963 \qquad (11.26)$$

$$\text{“}Abelianfactor''_{U(2)} = 2^2 = 4 \qquad (11.27)$$

$$\text{“}goalquantity''_{U(2)} = (2^{11}/3^3)^{1/4} = 2^3/3 \cdot (\frac{3}{2})^{1/4}$$
$$= 2^3/3 \cdot 1.10668192 = \mathbf{2.951151786} \qquad (11.28)$$

On the Fig. 11.1 we illustrate the three groups getting the three highest "goal quantities". The third group winning so to speak the bronze medal in this competition is $Spin(5) \times SU(3) \times U(1)/\text{“}Z_6''$ (where "Z_6'' stands for a certain with the integers modulo 6 isomorphic subgroup of the center of the cross product group; it arranges a quantization rule for the allowed representations quite analogous to that of the Standard Model group except, that the weak Lie algebra $SU(2)$ has been replaced by $Spin(5)$ (which is the covering group of $SO(5)$), which is very analogous to the Standard Model group just with SO(5) or rather Spin(5) which is its covering group replacing the SU(2) in the Standard Model:

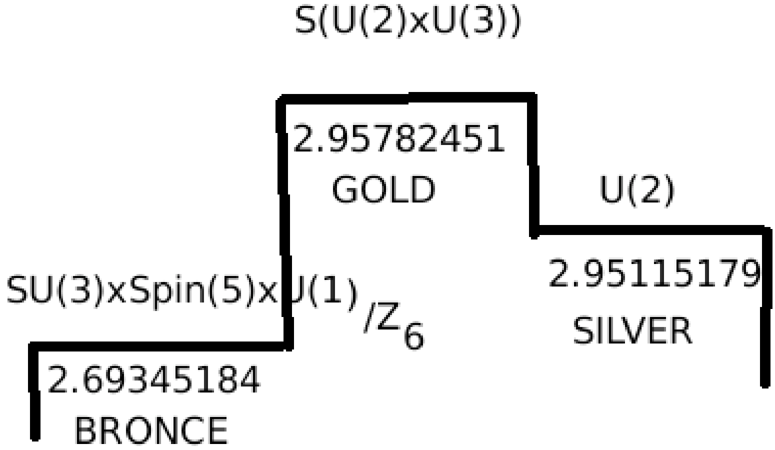

Fig. 11.1. This figure illustrates the three Lie groups getting in our game the highest scores for our "goal quantity" as were the sportsmen winning gold silver and bronze medals.

11.3 Dimension of Space-time Also

The main point of my progress since last year [2] is to say:

The choice of dimensionality of space time, that nature have made, - at least 3+1 for practical purpose - **can be considered also a choice of a group,** - and even a gauge group, if we invoke general relativity -**namely say the Lorentz group or the Poincare group.** So if we have a "game" or a "goal quantity" selecting by letting it be maximal the gauge group of the Standard Model, it is in principle possible to ask:

Which among the as Lorentz or Poincare group applicable groups get the highest "goal quantity" score? Which dimension wins the competition among Lorentz or Poincare groups?.

We would of course by extrapolation from the gauge group story (= previous work(with Don)[2]) expect that Nature should again have chosen the "winner".

It is my point now that - with only very little "cheat" - I can claim that indeed **Nature has chosen that dimension** $d = 4$ (presumably meant to be the practical one, we see, and not necessarily the fundamental dimension, since our quantity could represent some stability against collapsing the dimension)**that gives the biggest score for the Poincare group!** (for the Lorentz groups $d = 4$ and $d = 3$ share the winner place !)

11.3.1 Development of Goal Quantities for dimension fitting.

In the present article we shall ignore anthropic principle arguments for what space time dimension should be, and seek to get a statement, that the experimental number of dimensions (4 if you count the truly observed one and take the convention to include time as one dimension) just maximizes some quantity, that is a relatively simple function of the group structure of, say, the Lorentz group, and which we then call a "goal quantity".

Making a "goal quantity" for Dimension is a Two step Procedure:

- 1) We first use the proposals in my work with Don Bennett to give a number - a goal quantity - for any Lie group.
- 2) We have to specify on which group we shall take and use the procedure of the previous work; shall it be the Lorentz group?, its covering group ? or somehow an attempt with the Poincare group ? :

Developing a "Goal quantity" for "predicting"(fitting) the Space Time dimension

A series of four proposals:

- a. Just take the Lorentz group and calculate for that the inverse Dynkin index or rather the quantity which we already used as "goal quantity" in the previous work and above (11.5) C_A/C_F. (Semi-simple Lorentz groups except for dimension $d = 2$ or smaller and in fact simple for 3 , 5 and higher).
- b. We supplement in a somewhat *ad hoc way* the above *a.*, i.e. C_A/C_F by taking its $\frac{d+1}{d-1}$th power. The idea behind this proposal is that we think of the *Poincare group* instead of as under *a.* only on the Lorentz group part, though still *in a crude way*. This means we think of a group, which is the Poincare group, except that we for simplicity *ignore that the translation generators do not commute with the Lorentz group part*. Then we assign in accordance with the ad hoc rule used for the gauge group the Abelian sub-Lie-algebra a formal replacement 1 for the ratio of the quadratic Casimirs C_A/C_f - because there is no limit to how small momenta can be quantized and no natural way to obtain the charges e_r for restricted representations, since we have essentially **R** as the Abelian group rather than $U(1)$ or complicated discrete subgroups D being divided out -: I.e. we put "e_A^2/e_F^2" = "$C_A/C_F|''_{Abelean\ formal} = 1$. Next we construct an "average" averaged *in a logarithmic way* (meaning that we average the logarithms and then exponentiate again) weighted with the dimension of the Lie groups over all the dimensions of the Poincare Lie group. Since

the dimension of the Lorentz group for d dimensional space-time is $\frac{d(d-1)}{2}$ while the Poincare group has dimension $\frac{d(d-1)}{2}+d=\frac{d(d+1)}{2}$ the logarithmic averaging means that we get

$$exp(\frac{\frac{d(d-1)}{2}\ln(C_A/C_F)|_{Lorentz}+\ln(1)*d}{d(d+1)/2}) = (C_A/C_F)|_{Lorentz}^{\frac{d(d-1)}{2}/\frac{d(d+1)}{2}}$$
$$= (C_A/C_F)|_{Lorentz}^{\frac{d-1}{d+1}} \quad (11.29)$$

That is to say we shall make a certain ad hoc partial inclusion of the Abelian dimensions in the Poincare groups.
To be concrete we here propose to say crudely: Let the Poincare group have of course d "Abelian" generators or dimensions. Let the dimension of the Lorentz group be $d_{Lor}=d(d-1)/2$; then the total dimension of the Poincare group is $d_{Poi}=d+d_{Lor}=d(d+1)/2$. If we crudely followed the idea of weighting proposed in the previous article [2] or above (11.5 as if the d "abelian" generators were just simple cross product factors - and not as they really are: not quite usual, because they do not commute with the Lorentz generators - then since we formally are from this previous article suggested to use the *as if number 1 for the Abelian groups*, we should use the quantity

$$(C_A/C_F)|_{Lor}^{\frac{d_{Lor}}{d_{Poi}}} = (C_A/C_F)|_{Lor}^{\frac{d-1}{d+1}} \quad (11.30)$$

as goal quantity.
Really you can simply say: we put the "Abelian factor " =1, but still take the $d_{Poi}=d(d+1)/2$th root at the end, by using the total dimension of the Poincare group d_{Poi}. The crux of taking this "1" is that we do not have anything corresponding to the division out of a discrete group giving the restriction like (11.10 in the Poincare case.

- c. We could improve the above proposals for goal quantities *a.* or *b.* by including into the quadratic Casimir C_A for the adjoint representation also contributions from the translation generating generators, so as to define a quadratic Casimir for the whole Poincare group. This would mean, that we for calculating our goal quantity would do as above but

$$\textbf{Replace}: C_A \to C_A + C_V, \quad (11.31)$$

where C_V is the vector representation quadratic Casimir, meaning the representation under which the translation generators transform under the Lorentz group. Since in the below table we in the lines denoted "no fermions" have taken the "small representation" F to be this vector representation V, this replacement means, that we replace the goal quantity ratio C_A/C_F like this:

$$\textbf{(S)O(d), "no spinors"}:$$
$$C_A/C_F = C_A/C_V \to (C_A+C_V)/C_F = C_A/C_F + 1 \quad (11.32)$$
$$\textbf{Spin(d), "with spinors"}:$$
$$C_A/C_F \to (C_A+C_V)/C_F$$
$$= C_A/C_F + (C_A/C_V)^{-1}(C_A/C_F)$$
$$= (1+(C_A/C_F)|_{\text{no spinors}}^{-1})C_A/C_F. \quad (11.33)$$

Let me stress though that this proposal c. is not quite "fair" in as far as it is based on the Poincare group, while the representations considered are not faithful w.r.t. to the whole Poincare group, but only w.r.t. the Lorentz group

- d. To make the proposal *c.* a bit more "fair" we should at least say: Since we in *c.* considered a representation which were only faithful w.r.t. the Lorentz subgroup of the Poincare group we should at least correct the quadratic Casimir - expected crudely to be "proportional" to the number of dimensions of the (Lie)group - by a factor $\frac{d+1}{d-1}$ being the ratio of the dimension of the Poincare (Lie)group, $d + d(d-1)/2$ to that of actually faithfully represented Lorentz group $d(d-1)/2$. That is to say we should before forming the ratio of the improved C_A meaning $C_A + C_V$ (as calculated under *c.*) to C_F replace this C_F by $\frac{d+1}{d-1} * C_F$, i.e. we perform the replacement:

$$C_F \rightarrow C_F * \frac{d(d-1)/2 + d}{d(d-2)/2} = C_F * \frac{d+1}{d-1}. \tag{11.34}$$

Inserted into $(C_A + C_V)/C_F$ from *c.* we obtain for the in this way made more "fair" approximate "goal quantity"

$$\text{"goal quantity"}|_{\text{no spinor}} = (C_A/C_F + 1) * \frac{d-1}{d+1} \tag{11.35}$$

$$\text{"goal quantity"}|_{\text{w. spinor}} = (1 + (C_A/C_F)|^{-1}_{\text{no spinor}}) * C_A/C_F * \frac{d-1}{d+1} \tag{11.36}$$

This proposal d. *should then at least be crudely balanced with respect to how many dimensions that are represented faithfully.*

11.3.2 Philosophy of the goal quantity construction/development

The reader should consider these different proposals for a quantity to maximize (= use as goal quantity) as rather closely related versions of a quantity suggested by a perhaps a bit vague ideas being improved successively by treating the from our point of view a bit more difficult to treat Abelian part (=the translation part of the Poincare group) at least in an approximate way.

One should have in mind, that this somewhat vague basic idea behind is: The group selected by nature is the one that counted in a "normalization determined from the Lie algebra of the group" can be said to have a faithful representation (F) the matrices of which move as little as possible, when the group element being represented move around in the group.

Let me at least clarify a bit, what is meant by this statement:

We think by representations as usual on linear representations, and thus it really means representation of the group by means of a homomorphism of the group into a group of matrices. The requirement of the representation being faithful then means, that this group of matrices shall actually be an isomorphic image of the original group. Now on a system of matrices we have a natural metric, namely the metric in which the distance between two matrices **A** and **B** is given by the square root of the trace of the numerical square of the difference

$$\mathrm{dist} = \sqrt{\mathrm{tr}((\mathbf{A}-\mathbf{B})(\mathbf{A}-\mathbf{B})^{+})}. \quad (11.37)$$

To make a comparison of one group and some representation of it with another group and its representation w.r.t. to, how fast the representation matrices move for a given motion of the group elements, we need a normalization giving us a well-defined metric on the groups, w.r.t. which we can ask for the rate of variation of the representations. In my short statement I suggested that this "normalization should be determined from the Lie algebra of the group". This is to be taken to mean more precisely, that one shall consider the *adjoint* representation, which is in fact completely given by the Lie algebra, and then use the same distance concept as we just proposed for the matrix representation $\sqrt{\mathrm{tr}((\mathbf{A}-\mathbf{B})(\mathbf{A}-\mathbf{B})^{+})}$. In this way the quantity to minimize would be the ratio of the motion-distance in the representation - F say - and in the Lie algebra representation - i.e. the adjoint representation. But that ratio is just for infinitesimal motions $\sqrt{C_F/C_A}$. So if we instead of talking about what to minimize, inverted it and claimed we should maximize we would get $\sqrt{C_A/C_F}$ to be *maximized*. Of course the square root does not matter, and we thus obtain in this way a means to look at the ratio C_A/C_F as a measure for the motion of an element in the group compared to the same element motion on the representation.

It might not really be so wild to think that a group which can be represented in a way so that the representation varies little when the group element moves around would be easier to get realized in nature than one that varies more. If one imagine that the potential groups become good symmetries by accident, then at least it would be less of an accident required the less the degrees of freedom moves around under the to the group corresponding symmetry (approximately). It is really such a philosophy of it being easier to get some groups approximately being good symmetries than other, and those with biggest C_A/C_F should be the easiest to become good symmetries by accident, we argue for. That is indeed the speculation behind the present article as well as the previous one [2] that symmetries may appear by accident(then perhaps being strengthened to be exact by some means [16,25]).

11.3.3 Phenomenological Philosophy

But let us stress that you can also look at the present work and the previous one in the following phenomenological philosophy:

We wonder, why Nature has chosen just 4 (=3+1) dimensions and why Nature - at the present experimentally accessible scale at least - has chosen just the Standard Model group $S(U(2)\times U(3))$? Then we speculate that there might be some quantity characterizing groups, which measures how well they "are suited " to be the groups for Nature. And then we begin to *seek* that quantity as being some function defined on the class of abstract groups - i.e. giving a number for each abstract (Lie?) group - of course by proposing for ourselves at least various versions or ideas for what such a *relatively simple* function defined on the abstract Lie groups could be. Then the present works - this paper and the previous ones[2] and [1] - represents the present status of the search: We found that with small variations the types of such functions representing the spirit of the *little motion of the "best" faithful representation*,i.e. essentially the largest C_A/C_F, turned out truly to bring Natures choices to be (essentially) the winners.

In this sense we may then claim that we have found by phenomenology, that at least the "direction" of a quantity like C_A/C_F or light modifications of it is a very good quantity to make up a "theory" for, why we have got the groups we got!

Here we bring the table in which we present the calculations of our for the space-time dimension relevant various "goal quantities":

Dimension	Lorentz group, covering	Ratio C_A/C_F for spinor	Ratio C_A/C_F as no spinor	*c.*-quantity max c)	$\frac{d-1}{d+1}$	*d.*-quantity max d)
2^2	U(1)	-(formally 2)	-(formally 1)	4	1/3	4/3 =1.33
3	spin(3)	$\frac{8}{3} = 2.67$	1	$\frac{16}{3} = 5.3$	$\frac{2}{4}$	$\frac{8}{3} = 2.67$
4	$Spin(4)$ $SU(2)\times$ $SU(2)$	$\frac{8}{3}$ =2.67	$\frac{4}{3}$	$\frac{14}{3}$ =4.67	$\frac{3}{5}$	$\frac{14}{5}$ =2.8
5	Spin(5)	$\frac{12}{5} = 2.4$	$\frac{3}{2} = 1.5$	4	$\frac{4}{6}$	$\frac{8}{3} = 2.67$
6	$Spin(6)$	$\frac{32}{15}$	$\frac{8}{5} = 1.6$	$\frac{52}{15} = 3.5$	$\frac{5}{7}$	$\frac{52}{21} = 2.5$
d odd	Spin(d)	$\frac{8(2n-1)}{n(2n+1)} = \frac{16(d-2)}{d(d-1)}$	$2 - 1/n = 2 - \frac{2}{d-1}$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{d-1}{d+1}$	$\frac{8(3d-5)}{d(d+1)}$
d even	$Spin(d)$	$\frac{16(d-2)}{d(d-1)}$	$\frac{4(n-1)}{2n-1} = \frac{2d-4}{d-1}$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{d-1}{d+1}$	$\frac{8(3d-5)}{d(d+1)}$
d ∞	Spin(d)	$\approx 16/d$	$\to 2$	$\approx 24/d$	$\to 1$	$\approx 24/d$ $\to 0$
d ∞	Spin(d)	$\approx 16/d$	$\to 2$	$\approx 24/d$	$\to 1$	$\approx 24/d$ $\to 0$

Caption: We have put the goal-numbers for the third proposal *c* in which I (a bit more in detail) seek to make an analogon to the number used in the reference [2] in which we studied the gauge group of the Standard Model. The purpose of *c.* is to approximate using the *Poincare group* a bit more detailed, but still not by making a true representation of the Poincare group. I.e. it is still not truly the Poincare group we represent faithfully, but only the Lorentz group, or here in the table only the covering group $Spin(d)$ of the Lorentz group. However, I include in the column marked "*c.*, max c)" in the quadratic Casimir C_A of the Lorentz group an extra term coming from the structure constants describing the non-commutativity of the Lorentz group generators with the translation generators C_V so as to replace C_A in the starting expression of ours C_A/C_F by $C_A + C_V$. In the column marked "*d.*, max d) " we correct the ratio to be more "fair" by counting at least that because of truly faithfully represented part of the Poincare group in the representations, I use, has only dimension $d(d-1)/2$ (it is namely only the Lorentz group) while the full Poincare group - which were already in *c.* but also in *d.* used in the improved C_A being $C_A + C_V$ - is $d(d-1)/2 + d = d(d+1)/2$. The correction is crudely made by the dimension ratio $dim(Lorentz)/dim(Poincare) = (d-1)/(d+1)$ given in the next to last column.

Dimension	Lorentz group (covering)	Ratio C_F/C_A for spinor	Ratio C_A/C_F "no spinor"	*c.*-quantity max c)	*d.*-quantity max d)
2^3	U(1)	-(f.: 2)	-(f.: 1)	4	4/3=1.33
3	spin(3)	$\frac{8}{3} = 2.67$	1	$\frac{16}{3} = 5.33$	$\frac{8}{3} = 2.67$
4	$Spin(4) = SU(2) \times SU(2)$	$\frac{8}{3} = 2.67$	$\frac{4}{3}$	$\frac{14}{3} = 4.67$	$\frac{14}{5} = 2.8$
5	Spin(5)	$\frac{12}{5} = 2.4$	$\frac{3}{2} = 1.5$	4	$\frac{8}{3} = 2.6667$
6	$Spin(6)$	$\frac{32}{15}$	$\frac{8}{5} = 1.6$	$\frac{52}{15} = 3.47$	$\frac{52}{21} = 2.4762$
d odd	Spin(d)	$\frac{8(2n-1)}{n(2n+1)} = \frac{16(d-2)}{d(d-1)}$	$2 - 1/n = 2 - 2/(d-1)$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{8(3d-5)}{d(d+1)}$
d even	$Spin(d)$	$\frac{16(d-2)}{d(d-1)}$	$\frac{4(n-1)}{2n-1} = \frac{2d-4}{d-1}$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{8(3d-5)}{d(d+1)}$
d odd $\to \infty$	Spin(d)	$\approx 16/d$	$\to 2$	$\approx 24/d$	$\approx 24/d \to 0$
d even $\to \infty$	Spin(d)	$\approx 16/d$	$\to 2$	$\approx 24/d$	$\approx 24/d \to 0$

11.4 The Higgs Representation

A rather simple and successful application of our ideas is to seek the answer to the question: Why has the Higgs field just got the representation $(\mathbf{2}, \mathbf{1}, y/2 = 1/2)$ under the Standard Model group with the Lie algebra factors written in the order $SU(2) \times SU(3) \times U(1)$?

Note that the selection of the gauge group by our "goal quantity" had the character of being obtained as a ratio - of the quadratic Casimirs C_A for adjoint and C_F for another faithful representation or some "replacements" for them in the Abelian cases - of an adjoint representation parameter to one for *another representation*F. Also this other representation F gets basically selected by the same principle as the selection of the whole gauge group by maximizing our "goal quantity", because we also *select the representation* F *from the requirement that our "goal quantity " be maximized.*

Thus in reality we have hit on a quantity that tends to select both a *group* and a smallest C_F representation.

Now strictly speaking most irreducible representations of say the Standard Model group $S(U(2) \times U(3))$ will not usually be completely faithful. It is rather so that the various representations F appearing as representations of the simple subgroups will not be truly faithful, but rather *only be faithful for some subgroup* of the $S(U(2) \times U(3))$ group say. If we therefore now shall make some numbers assigned to the various*not completely faithful* representations which are allowed as representations of the Standard Model *group* $S(U(2) \times U(3))$, it would be most "fair" to count the ratio of the quadratic Casimir in the "Adjoint" representation - or better in the group itself - *by not using the full say Standard Model Group, but rather only that part of the group* $S(U(2) \times U(3))$ *that is indeed faithfully represented on the representation, which is up to be tested, with a number to specify which representation should be favoured*.

So let us say we have some representation R of say the Standard Model *group*, i.e. an allowed one, which of course then also obeys the quantization rule (such as) (11.10).

Now there is always a kernel K consisting of the elements in the group $S(U(2) \times U(3))$ or more generally the Lie group G, with which we work, for which the elements in R are transformed trivialy, it means not shifting to another element, but only to itself. This kernel K is of course an invariant subgroup of the full group G. This means that G/K is a well defined factor group in G. Then we should naturally suggest the "fair" rule that we construct the number according to which the representation R should be selected *as the number we would get by calculating the "goal quantity " of ours for the group* G/R *with though the restriction that the* F *should correspond to* R.

Let us illustrate this rule proposed by looking at a couple of examples:

If we want to consider one of the representations F giving the maximal G_A/G_F for one of the simple subgroups, which in Standard Model can only be $SU(2)$ or $SU(3)$, then for these two groups the F-representations are respectively **2** and **3** (or one could take the equivalent $\bar{\mathbf{3}}$ for the $SU(3)$). But of course say **2** alone without any $y/2$ charge would not be allowed by the Standard Model group $S(U(2) \times U(3))$. Thus we are forced to include an appropriate $y/2$. Doing that you can easily find that the relevant factor groups $S(U(2) \times U(3))/K$ becomes in the two cases respectively $U(2)$ and $U(3)$. Actually with smallest $y/2$ values allowed in the two cases $y/2 = 1/2$ (or $-1/2$) for $SU(2)$ and $y/2 = -1/3$ for $SU(3)$ with **3** we get just the same F as is used in our calculations of our "goal quantity". This means that quantities to select the representation happens to be in our two cases just the "goal quantities" for the two groups $U(2)$ and $U(3)$, namely just the factor groups. We already know that $U(2)$ were the "silver medal winner" and thus that it should be trivially $U(2)$ related to measuring the size of the representation $\mathbf{2}, \mathbf{1}, y/2 = 1/2)$ which gets selected. This means the winning - and that means "smallest" representation of the Standard Model (measured by using the associated factor group for which it is faithful) - representation of the Standard Model group became this $\mathbf{2}, \mathbf{1}, y/2 = 1/2)$. This is just the representation of the Higgs. So the Higgs representation is predicted this way (as the "smallest" in our way of counting, closely related to the game we used to tell the gauge group with) !

11.5 The (chiral) Fermion Representations

It is now the idea to use the very same "goal quantity" as the one, with which we exercised in deriving the Higgs representation above, to argue for the Fermion representations in the Standard Model - or rather what we in the present philosophy expect for the choice of Nature - as to what they should be.

Here the situation is somewhat more complicated because the requirement that there be no gauge- nor gauge gravity anomalies imposes restrictions on the whole system of representations for the chiral fermions. Assuming that we work with 3+1 dimensions we can take it as our convention to work with only left handed spin 1/2 Fermions, because we can let the right handed ones simply be represented by their CP-analogue left handed ones.

We must therefore first write down the non-anomaly conditions for having various thinkable numbers of families for the various representations of the Standard Model.

Now the use of anomaly conditions together with the assumption of "small representations" (in some meaning or the other) we already used in some articles years ago. For instance in "Why do we have parity violation?"[12] Colin Froggatt and I sought to answer this question by using the principle of small representations to derive the representations that the Standard Model should have and thus why they would give parity violation the well known way. Also in [14] we allude to the principle of small representations (here in the last section). In fact in the section XIII, called "Hahn-Nambu-like Charges" we sought to derive the system of the representations of Weyl Fermions (we use a notation there of only counting the left handed spin 1/2 fermions, letting it be understood that the right handed components achieved by CP i.e. of the anti particles of course exists but are just not listed in the way we keep track of the particles in this notation; that is to say that normally considered right handed Weyl particles are just counted by their CP-antiparticle, which if left handed). We sought to derive it from the no-anomaly-conditions and a principle of "small representations". The latter were not exactly the same as we seek to develop in the present more recent but in some approximate sense it were very close to the present idea of a small representation principle, as we claim the choice of nature of the Standard Model group $S(U(2) \times U(3))$ indicates. Nevertheless the two ideas of a "small representation principle" are so close that at least I give/gave them the same name "small representations".

In the section XIII of the Puerto-Rico conference proceedings [14] we use somewhat special technology to argue that*imposing the conditions for:'*

- 1. no chiral anomalies and no mixed anomalies for the gauge charge conservations,
- 2. together with a *small representation principle* (formulated using the concept to be explained of "Hahn Nambu charges")
- 3. and that the fermions shall be *mass protected* (i.e. get zero mass due to gauge (charge) conservation, were it not for the "Higgsing"),

lead to the Standard Model representations spectrum basically (i.e. we get that there should be a number of families of the type we know, but how many we do not predict from these assumptions).

The technology used in [14] was to consider only a certain subset of charges - called there "Hahn-Nambu charges" - of the Cartan algebra of the Standard Model Lie algebra, or of the Lie algebra for any other gauge grouop being discussed.

Since the rank of the Standard model (gauge)group is 4, there are of course 4 linearly independent Cartan algebra charges. But now we used in the reference [14] not linearly independent charges, but rather linear combinations of the Cartan algebra charges selected to have the special property, that for representations allowed for the Standard Model *group* these specially selected Cartan algebra charges had only the integer values and even in the usual Standard Model system of representaions took only the values $-1, 0, \text{or} 1$.

Let me explain the technique of our Costa Rica proceedings paper [14] a bit more:

Starting from assuming a gauge group with rank four say (but we really have in mind using a similar discussion on any potential gauge group, so that we also with those considerations could hope for approaching a derivation of an answer to why just the Standard Model) and deciding to consider only the Cartan algebra part, we would basically have assumed effectively an $\mathbf{R}^4$ gauge Lie algebra. But as a rudiment of as well the explicite charge quantization rule resulting from the *group* structure as from the charge quantizations caused by the non-abelian Lie algebra structure present before we threw the non-abelian parts away - only keeping the Cartan algebra - we would have quatization rules for the Cartan algebra charges. Indeed we would rather obtain an effective gauge *group* after this keeping nothing but the Cartan algebra being $U(1)^4$ than the here first mentioned $\mathbf{R}^4$. This would mean that in the appropriate basis choice for these Cartan algebra charges they would all be restricted by the group structure to be integers. Making sums and/or differences of such "basis" charges restricted to be integers one can easily write down combinations which again would be restricted to have only integer charges.

But now the main question of interest in our earlier quantization of certain Cartan algebra charges were to implement the requirement/assumption of "small representations" or for Abelian equivalently "small charges".

We formulated the requirement of such "small charges" via defining a concept of a "Hahn Nambu charge". Such a type of charge, which we would denote as "Hahn Nambu charge", were by our definition assumed to obey:

- A "Hahn Nambu charge" should be one of the combinations of the Cartan algebra charges, which precisely were allowed to take on integers - no more no less - (due to the group structure of the $U(1)^4$ say for rank 4).
- But in the actual detailed model one should for the Hahn Nambu charge *only find the charge eigenvalues* $-1, 0, or 1$. (This assumption is, one may say, an assumption of small charge values -for the Hahn Nambu charge type - in as far as the charge value numerically less than or equal to 1 is "small" compared to the quantization interval assumed just above to be 1).

Then instead of assuming in some other way, that we seek a model with the smallest possible charge values, we used then in [14] and [12] to say in stead - and crudely equvalently - that we should arrange so many "Hahn Nambu charges" to exist in the model to be sought as possible.

In order that the reader shall get an idea what type of charges these "Hahn Nambu charges" are, let me mention the Hahn Nambu charges of the Standard

Model:

$$\text{“HNred”} = y/2 + I_{W3} + \sqrt{3}\lambda_{8red} \quad (11.38)$$
$$\text{“HNblue”} = y/2 + I_{W3} + \sqrt{3}\lambda_{8blue} \quad (11.39)$$
$$\text{“HNyellow”} = y/2 + I_{W3} + \sqrt{3}\lambda_{8yellow} \quad (11.40)$$
$$\lambda_{2yellow} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (11.41)$$
$$\lambda_{2yellow} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (11.42)$$
$$\lambda_{2red} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (11.43)$$
$$\lambda_{2blue} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (11.44)$$
$$\text{“Twise weak isospin } I_{W3}\text{”} = 2I_{W3} \quad (11.45)$$

Here we have used a notation, wherein the colors are listed in the series (“red”, “blue”, “yellow”)in columns and rows and defined the variously colordefined λ_8-matrices:

$$\sqrt{3}\lambda_{8red} = \begin{pmatrix} -2/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix} \quad (11.46)$$
$$\sqrt{3}\lambda_{8blue} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & -2/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix} \quad (11.47)$$
$$\sqrt{3}\lambda_{8yellow} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & -2/3 \end{pmatrix} \quad (11.48)$$

It is easy to check that these 7 “Hahn Nambu charges” are related to each other by being sums or differences of each other, and also that they are indeed according to our definition indeed “Hahn Nambu charges” in the wellknown Standard Model. Indeed you should also see that the first three of them, “HNred”, “HNblue”, and “HNyellow” are indeed three color choices for what historically Hahn and Nambu proposed as the electric charge to be used in a QCD including model. Nowadays we know, that quarks only have electric charges $2/3$ or $-1/3$ fundamental charges, but the original Hahn Nambu charge were precisely constructed to have only the integer Millikan charge *even for the quarks*.

The crux of the calculation, we want to extract from the study of Hahn Nambu charges in our old works [14,12], is that imposing the no gauge anomaly conditions for the Cartan subalgebra, using the the assumption that we have as many "Hahn Nambu charges" as possible still having a mass protected system of (Weyl)fermions, we are led to a system of representations which *is* indeed the usual one when extended to get the non-abelian charges too.

The technique we used in the old paper(s) [14] were in fact to study the no-anomaly constraint equations moulo 2, which for Hahn Nambu charges, that never take by assumption/definiton charge values bigger than 1 numerically, close to be enough.

Actually it turned out that we could first find a system of mass-protected Weyl-fermions, when the dimension of the Cartan algebra (= the number of linearly independent Hahn Nambu charges) became at least 4. In that case then we had indeed to have a system of Weyl fermions, which modulo some trivial symmetries, had to be the one found experimentally w.r.t. these "Hahn Nambu charges".

This should be interpreted to say, that requiring maximal numbers of "Hahn Nambu Charges" in our sense, which is a slightly special way of requiring small representations together with the assumtions of mass protection and no anomalies, leads to the Standard Model fermion system.

That is to say we should consider the structure of a family in the Standard Model to essentially come out of such requirements. In this way we can count the fermion system/spectrum as largely being a successful result comming out from a "Small representation principle"!

11.6 Speculations on the Full Group of Gauge Transformations and Diffeomorphism Symmetry

In the above discussion and in the previous articles in the present series of papers [2,1] we sought to find a game leading to the "gauge group". But now we want to have in mind that the "gauge group" is not truly the most physical and simple concept in as far as the true symmetry in a gauge theory with "gauge group"G is really not truly G, but rather a cross product of one copy of G, say $G(x)$ for every point x in space time. That is to say the true symmetry group of the gauge theory having the "gauge group" G is rather $\bigtimes_x G(x) = G \times G \times \cdots \times G$, where in the cross product it is supposed that we have one factor for every space time point x.

Above we saw that the goal quantity for a group were suggested to be of a type, that is balanced in such a way, that the score or goal quantity is the *same for a group* G *and for the cross product of this group with itself* $G \times G \times \cdots \times G \times G$ *any number of times.*

This means of course, if as we found the Standard Model group $S(U(2) \times U(3))$ wins our game, then in fact any product of this group with itself any number of times can also be said to get just the same score, and thus it will also win! That it to say that we might reinterpret our work by saying: It is not truly the gauge group for the realized gauge theory we predict to be the winner. Rather we could say that the group that wins is the whole symmetry group of the full quantum field theory supposed to be realized. The concept of the full gauge symmetry (or we could say reformulation symmetry) is – we would say – a simpler concept than the concept of the "gauge group" for which it would have to be specified how this gauge group would have to be applied, namely one should construct a group of all gauge transformations $\bigtimes_x G(x) = G \times G \times \cdots \times G$.

But since this full group gets just the same score as the more complicatedly defined "gauge group" we could claim that our prediction is, that it is this group of all the gauge transformations that gets the maximal score.

This would mean in some sense a slight simplification of our assumption.

11.6.1 Could we even predict the manifold?

Very speculatively - and with the success of predicting the dimension in mind - we could seek to argue that the group of gauge transformations $\bigtimes_x G(x) = G \times G \times \cdots \times G$ in some way could be claimed to represent a somewhat larger group than just this $\bigtimes_x G(x) = G \times G \times \cdots \times G$ in as far as we even on the same representation space of a direct sum of the representations F for the different points in space time could claim to represent *also a diffeomorphism group*. Since this diffeomorphism group shuffles around the direct sum of the F-type representations we could claim, that we managed to represent a group which is really the combination of the diffeomorphism group and the group of gauge transformations on just the same space of linear representations as the group of gauge transformations alone gets represented on as its "record (in our game) representation". Intuitively this means that we have got an even bigger group relative to the representation than if we just represent the Standard Model group $S(U(2) \times U(3))$ on its F's. Thus including such a diffeomorphism extension sounds like providing a superwinner superseding the formal winner itself the $S(U(2) \times U(3))$ (or its cross products with itself). So there is the hope that formulating the details appropriately we could arrange to get our true prediction become *the group of gauge transformations with the gauge group* $S(U(2) \times U(3))$ *extended with a diffeomorphism group*. If indeed in addition the dimension $d = 4$ for space time favoured by our game, because of its gauge group for general relativity, and thus hopefully the group of diffeomorphisms for just a four dimensional manifold would get exceptionally high score, it becomes very reasonable to expect that our game could predict just the right dimension of the manifold, on which the cross product of the standard model group with itself gets extended by the diffeomorphism symmetry.

This means that we are very close to have an argument that the most favoured symmetry group would precisely be the group of Standard Model gauge transformations extended by just a four dimensional diffeomorphism symmetry.

But if so, it would mean, that we had found a principle, a game, favouring precisely the group of gauge transformations found empirically.

Well, it must here be admitted a little caveat: The groups we considered to derive the dimension were the group of Lorentz transformation or Poincare transformations, *and not the full group of linear d-dimensional maps as would locally correspond to the diffeomorphism symmetry*. Thus one should presumably rather hope for our scheme to lead not to the full diffeomorphism symmetry as part of the winning symmetry group, but rather only that part of the diffeomorphism group, which does not shift the metric tensor $g_{\mu\nu}$. It would namely rather be this subgroup of the diffeomorphism group that would locally be like the $\mathtt{Spin}(4)$ or SO(4) as we discussed in the dimension fitting.

But somehow this is presumably also rather what we should hope for to have a successful theory of ours.

"Going for" the Standard Model as were our starting point means that we really concentrated on only looking for the long wave length or practically accessible part of whatever the true theory for physics might be. This long wave length practical section should presumably be defined as what we can learn from few particle collisions with energies only up to about a few TeV. But in such few particle practical experiments we should not discover gravity and general relativity. We should only "see" the flat Minkowski space time and the Standard Model. But that should then mean that we should not truly "see" diffeomorphism group, but only some rudiments associated with the metric tensor leaving part of this group.

The ideal picture which we should hope to become the prediction in this low energy section philosophy should rather be that the geometrical symmetries are only the flat Poincare group combined with the full gauge group for the Standard Model.

11.7 Conclusion

The main point of the present article is the suggestion that in a way - that may have to be made a bit precise in the future/coming further work - a principle of "small representations" should be sufficient to imply a significant part of the details of the Standard Model. The real recently most important progress in the work with Don Bennett [2] is that it seems that even for the selection of the gauge group itself this selection of "small representations" is so important *that the very group is selected so as to in the appropriate way of counting have the smallest faithful representations.* That is to say the Standard Model gauge group should have been selected to be the model of Nature precisely, because it could cope with smaller representations, measured in our slightly specific way, than any other proposal for the gauge group (except for cross products of the Standard Model group with itself a number of times). This so successful specific way of measuring the "smallness" of the representations takes its outset from the (inverted) Dynkin index in the case of simple Lie groups: C_A/C_F. This is then averaged actually in the way that the logarithms of it is averaged weighted with the dimensions of the various simple groups in the cross product (and then we may of course reexponentiate if we want) and extended to the most natural analogue for the Abelian Lie-algebra parts, essentially replacing the C_A/C_F by e_A^2/e_F^2 meaning the charge square ratio for two representations analogous to the adjoint and the F ones.

The philosophy that taking outset in C_A/C_F with F, as we did, being chosen so as to maximize this ratio C_A/C_F can be considered assuming a principle of "small representations" is obvious. If we consider the adjoint representation quadratic Casimir C_A for the simple group under investigation as just a normalization - to have something to compare quadratic Casimirs of other representations to - maximizing our starting quantity C_A/C_F means really selecting a (simple) group according to how small faithful representations F one can find for it. So it is really selecting the group with the smallest representations. Here of course then the concept of the size of the representation has been identified with the size of the quadratic Casimir, but that is at first a very natural identification and secondly, that were the one with which we had the success. It is also the quadratic Casimir, which is connected with natural metric on the space of unitary matrices in the representations. In fact our outset quantity C_A/C_F becomes the square of the ratio of the distance the unitary representation matrix moves for an infinitesimal motion of the group element in the adjoint and in the representation F, wherein by choice of F this distance is minimal. So our "goal quantity" which is the appropriate average of the ratio C_A/C_F and its extension to the Abelian parts becomes (essentially) the square of the volume of the volume of the representation space - in representations of the F's - and the corresponding representation space using the adjoint representation or an analogue of adjoint space representation, if Abelian parts are present. But the crux of the matter is *a surprisingly large amount of details of the Standard Model including its Gauge group is determined from a requirement of essentially minimizing the quadratic Casimirs of the representations*:

- First the **gauge group** - and here we stress *group* - $S(U(2) \times U(3))$ of the Standard Model is *selected* by for our "goal quantity" (11.5) obtaining the highest score 2.95782451 which is rather tinnily,0.0067,above the next (silver medal) (not being just a trivial cross product including the Standard Model itself), namely $U(2)$ (= standard model missing the strong interactions QCD) 2.95115179.
- **The dimension 4 for space time is also *selected*** by the Poincare group getting the highest score for approximately the same "goal quantity", which we used for the gauge group. It must be admitted though that we did not treat the Poincare group exactly - because it does not have the nice finite dimensional representations we would like to keep to have as strong similarity with the gauge group as possible - but instead made the trick of making some crude corrections starting from the Lorentz group. When using the Lorentz group dimension d=3 and d=4 stand equal. When we correct in reasonably "fair" ways the dimension d=4 (the experimental one for practical purposes in our notation that include the time) wins by having the highest corrected "goal quantity" for the Lorentz group, corrected to simulate the Poincare group. In this sense our principle, which is at the end a principle of small representations, point to the experimentally observed number of dimensions d=4.
- **The representation of the Higgs field** is when we use our "goal quantity" inspired way of defining in a very precise way numerically the smallest of the possible various irreducible representations to be the inverse of the this "goal quantity" for the factor group $G/K = S(U(2) \times U(3))/K$, for which the thought upon representation R is faithful. By this we just mean that we define K as the (invariant) subgroup, the elements of which are represented just by the unit matrix in the representation R. This we then in principle go through for all irreducible representations R for the Standard Model and ask for each possible R: what is the "goal quantity" for the corresponding $S(U(2)\times U(3))/K$ (here K depends on R of course) group. For $R = (\mathbf{2}, \mathbf{1}, y/2 = 1/2)$ this factor group $S(U(2) \times U(3))/K$ turns out to be just $U(2)$ and score "goal quantity"

for the representation $R = (\mathbf{2}, \mathbf{1}, y/2 = 1/2)$ is just that of the group $U(2)$ because it happens that the F for the $SU(2)$ inside $U(2)$ is just the **2**. Thus the quantity to determine to decide on the representation $R = (\mathbf{2}, \mathbf{1}, y/2 = 1/2)$ becomes exactly the "goal quantity"$_{U(2)}$ which we knew already were unbeatable (except if there should have been an irreducible representation faithful for the whole Standard Model group, but there is not). Thus assuming that the representation is smallest meaning, since "size" = 1/"goal quantity" for representations using our scheme, for predict representations the Higgs which is scalar and has no anomaly problems should be that representation that won $R = (\mathbf{2}, \mathbf{1}, y/2 = 1/2)$, and that is precisely the representation of the Higgs!

- The Fermion representations all for mass protected Fermions (meaning that gauge symmetry would have to be broken, spontaneously by a Higgs presumably) in order for the Fermions to obtain nonzero masses. This makes them easily make anomalies in the gauge symmetries (charge conservations). In order that no anomalies really occur relations between the number of species of Fermions in various representations get severely restricted. Together with some requirement of "small representations" it looks rather suggestive, that the Standard Model system of particles in a family comes out just intuitively. In our article [14] we did an attempt to make the requirement of small representations precise in a quite different way than in the present article- but it were an attempt to assume small representations in some way at least -and we mainly worked with the Cartan algebra only. But the result was, that the Standard Model representations came out/were postdicted for the Cartan algebra at least.
- At the end we sought to change the point of view as to what group should be the one, that shall win the game of getting the largest "goal quantity" from being *the gauge group to be the group of all the gauge transformations*. Since it happens that we had balanced our "goal quantity"so much in order to avoid making the dimension of the group of much influence the value of this "goal quantity" had turned out to be exactly the same for a group and its cross product with itself, ever so many times. Since now the group of all gauge transformations is basically an infinite cross product of what we usually call the gauge group, it means that w.r.t. our competition selecting the gauge group or the group of gauge transformations makes no difference. So if we e.g. should think that the group of all the gauge transformations is a more fundamental and well defined concept, we are free to choose our scheme to select that group of gauge transformations rather than the gauge group.
  But if we are very speculatively optimistic we might find some argument that many cross product factors would occur and hope in the long run to get a kind of understanding of the gauge symmetry on a whole manifold to optimistically come out of our game.
  Perhaps extension of this point of view to the Lorentz (or crudely Poincare) group as gauge symmetry should in later work give a better way of arguing for the dimension of space time d=4, at the same time getting close to general relativity.

This series of ideas for points resulting from some principle or another, but presumably best by using our "goal quantity" (11.5), shows that such a type of principle is close to deriving a lot of the structure of the Standard Model: The gauge group, in the "group" included some quantization rule (11.10), the space time dimension, the Higgs representation, the fermion representations, and more doubtfully some argument that we have gauge symmetry at all.

In conclusion I think that this kind of principle - a precise making of a principle of "small representations" - could have a very good chance to explain a lot of the structure of the Standard Model and thereby of the physics structure, we see today!

11.7.1 Outlook and speculation on finestructure constants

If we take the above results of having success with "goal quantity" related to the representations F being in fact the representations of the Standard Model group $(y/2 = 1/2, \underline{2}, \underline{1})$ and $(y/2 = -1/3, \underline{1}, \underline{3})$ to mean that these two representations represent the dominant fields (for the gauge field on a lattice say), then it *happens* that we got an "important representation" being the direct sum of these two representations. This sum corresponds to the $\underline{5}$ of the SU(5) in grand unification [19]. If we also took it, that the involvement of the natural measure on the representation space of unitary matrices in the definition of our successful goal quantity to mean, that we should use this distance measure on the representations to suggest the strength of the gauge couplings, we would end up with a simulated SU(5)-unification prediction!

We hope that our scheme might suggest an *approximate* SU(5)-relation between the couplings only, because we presumably even would if this should work at all for our kind of thinking rather at some fundammental/Planck scale than at an adjustable scale like in conventional Grand Unification. (We hope to return to our hopes of obtaining approximate $SU(5)$ coupling relations at the Planck scale in later works in which we should then take into account that there are also secondary representations in the series of our smallness and that how much they shuld contribute might be something we at least at first could start fitting and playing with).

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12 Towards a Derivation of Space

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Abstract. This attempt to "derive" space is part of the Random Dynamics project [1]. The Random Dynamics philosophy is that what we observe at our low energy level can be interpreted as some Taylor tail of the physics taking place at a higher energy level, and all the concepts like numbers, space, symmetry, as well as the known physical laws, emerge from a "fundamental world machinery" being a most general, random mathematical structure. Here we concentrate on obtaining spacetime in such a Random Dynamics way. Because of quantum mechanics, we get space identified with about half the dimension of the phase space of a very extended wave packet, which we call "the Snake". In the last section we also explain locality from diffeomorphism symmetry.

Povzetek. Ta poskus "izpeljave" prostora je del projekta Naključne dinamike [1]. Folozofija je, da vse kar opazimo pri nizkih energijah lahko razložimo kot Taylorjev „rep" dogajanja pri višjih energijah; vsi poljmi, kot so števila, prostor, simetrija, pa tudi vsi znani fizikalni zakoni, se porajajo iz "osnovnega stroja sveta", ki je najbolj splošna, naključna matematična struktura. V tem prispevku izpeljujemo iz Naključne dinamike prostor-čas. Zaradi kvantne mehanike idemntificiramo prostor s približno polovično razsežnim faznim prostorom za zelo razsežen valovni paket, ki mu pravimo "Kača". V zadnjem razdelku razložimo lokalnost iz difeomorfnostne simetrije.

12.1 The space manifold

This is an attempt to "derive" space from very general assumptions:

1) First we postulate the existence of a phase space or state space, which is quite general and abstract. It is so to speak an "existence space", with very general properties, and to postulate it is close to assume nothing.

So we start with the quantized phase space of very general analytical mechanics:

$$\begin{cases} q_1, q_2, ..., q_N \\ p_1, p_2, ..., p_N = i\frac{\partial}{\partial q_1}, ..., i\frac{\partial}{\partial q_N} \\ H(\vec{q}, \vec{p}) \end{cases}$$

where N is huge. This is (almost) only quantum mechanics of a system with a classical analogue, which is a very mild assumption.

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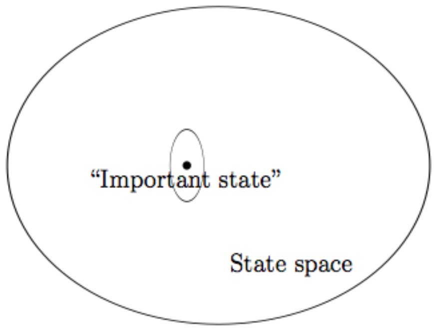
2) For the Hamiltonian H we then examine the statistically expected "random $H(\vec{q}, \vec{p})$" functional form (random and generic).

3) In the phase space we single out an "important state" and its neighbourhood - the "important state" supposedly being the ground state of the system.

The guess is that the "important state" is such that the state of the Universe is in the neighbourhood of this "important state" - which presumably is the vacuum.



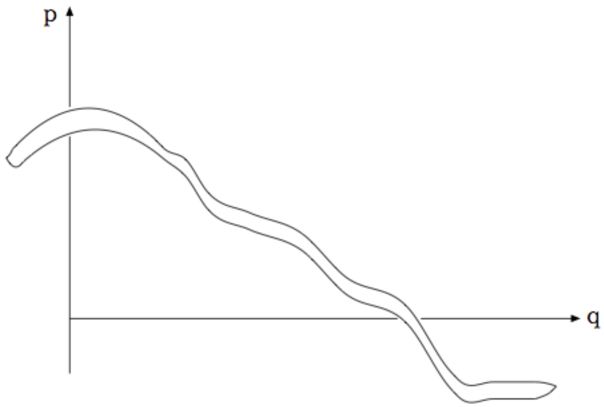
The state we know from astronomical observations is very close to vacuum. According to quantum field theory this means a state which mainly consists of filled Dirac seas, with only very few true particles above the Dirac seas, and very few holes. This vacuum is our "important state", supposedly given by a wave packet. If the system considered is the whole Universe, each point in the phase space is a state of the world.

Classically, a state is represented as a point in phase space, but quantum mechanically, due to Heisenberg, this phase space point extends to a volume h^N. Now assume that this volume is not nicely rounded, but stretched out in some phase space directions, and compressed in others.

The phase space has 2N dimensions, so a wave packet apriori fills a 2N-dimensional region. Our assuption is that the vacuum wave packet is narrow in roughly N of these dimensions. The vacuum state is thus extended to a very long and narrow surface of dimension N in the phase space (where N is half the phase space dimension).

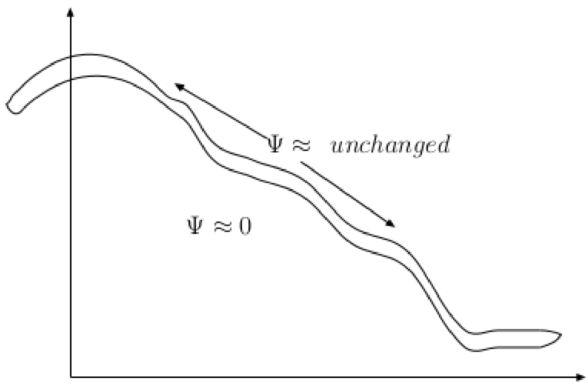
The really non-empty information in this assumption is that some of the widths are much smaller than others. N is moreover enormous, equal to the number of degrees of freedom of the Universe, so our model is really like a particle in N dimensions, $(q_1, q_2, ..., q_N)$. The "important state" is one where "the particle" is in a superposition of being in enormously many places (and velocities).

We envisage the points along the narrow, infinitely thin wave packet as embedded in the phase space, and that they in reality are our space points. In relation to this infinitely narrow "snake", these points are seemingly "big" (one can imagine the points as almost 'filling up' the Snake volume in the transversal direction). In the simplest scheme half of the phase space dimensions are narrow on this Snake, and the other half are very extended, long dimensions on the Snake. Along the Snake surface, the "important state" vacuum wave packet, i.e. the



The "very important state", for N=1

wave function $\Psi(q_1, q_2, ..., q_N)$ of the Universe, is supposed to be approximately constant. With $\Psi \approx$ constant, reparametrization (once it has been defined) under continuous reshuffling of the "points" along the long directions of the wave packet, is a symmetry of the "important state". The idea is to first parametrize the



The wave function Ψ (in configuration space)

N "longitudinal" dimensions so Ψ gets normalized to be 1 all along the Snake. It is however not Ψ we are most interested in, but the probability of the Universe to be at x, corresponding to

$$\int_t |\Psi(x, y)|^2 d_t^N y \tag{12.1}$$

where t stands for transverse.

With some smoothness assumptions, the longitudinal dimensions will be like a manifold, i.e. the points given by the longitudinal dimensions constitute a "space manifold". Since N is huge, the wave packet extension is probably also huge. And since there is a huge number of possibilities in phase space, the Snake is most certainly also very curled.

A wave packet can be perceived as easily excitable displacements of the transversal directions of the N-dimensional Snake (approximate) manifold. There are presumably different q_i and p_i at different points on the manifold, and states neighbouring to the vacuum ("the important state") correspond to wave packets just a tiny bit displaced from the vacuum. Thus the true state is only somewhat different from the vacuum (there is a topology on the phase space, so "sameness" and "near sameness" can be meaningfully defined). Corresponding to different points on the long directions of the wave packet (manifold), "easy" excitations can then be represented as some combinations $\sum_i(\alpha_i \Delta q_i + \beta_i \Delta p_i)$ of the ordered set $(\Delta q_1, ..., \Delta q_N, \Delta p_1, ..., \Delta p_N)$, where q_i and p_i are different phase space points of the N-dimensional manifold. The "easy" degrees of freedom are thus assigned to points on the manifold, so an "easy" displacement on the Snake is extended over some region along the Snake, that is, in x. In that sense the "easy" degrees of freedom can be interpreted as functions of x, $\phi_1(x)$, $\phi_2(x)$,...., which actually look like fields on the manifold (this is just notation, but in some limit it is justified). The wave packet Ψ consisting of easily excitable displacements, can then be perceived as superpositions of the $\phi_i(x)$. A field is just degrees of freedom expressed as a function of x (a field actually has to be a degree of freedom, in the sense that it is among parameters describing the state of the Universe), and these superpositions really seem to be fields.

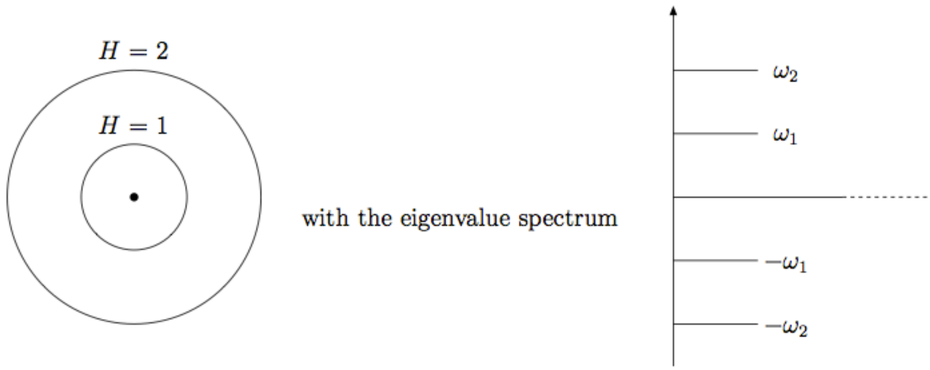
Now, let us make superpositions of such "easy" displacements to form one only non-zero displacement very locally, this is certainly legitimate. But with the identification of the Snake with space (or the space manifold), we should require that changing a field $\phi(x)$ only at x_0 corresponds to keeping the Snake unchanged, except at x_0.

So far we have identified the "important state" as the "ground state", i.e. the classical ground state $\approx$ Snake. Now consider the classical approximation for directions transverse to the Snake: In the transverse directions ($\sim y$), taking H as function of y at the minimum of the crossing point with the Snake (chosen to be the origin), the Taylor expansion of H with regard to y near the Snake is given by (discarding unimportant constants) second order expansions

$$H \approx \frac{1}{2} \frac{\partial^2}{\partial y_i \partial y_j} H(y, x)|_{y=0} \cdot y_i y_j \tag{12.2}$$

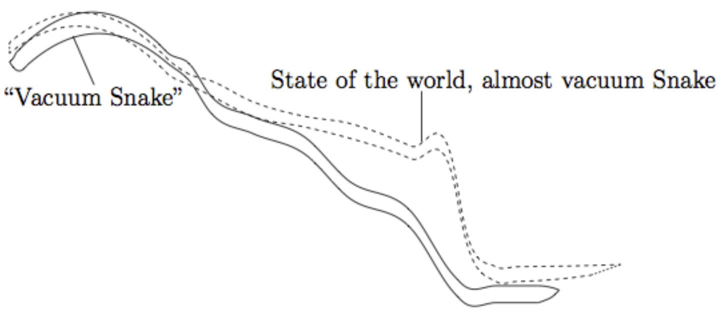
We now diagonalize, i.e. look for eigenvalues of the matrix $(\partial^2 H/\partial y_i \partial y_j)_{ij}$, where the "easy modes" correspond to the lowest eigenvalues.

From smoothness considerations these eigenvalues $\omega_1, \omega_2, ...$ can be defined as continuous and differentiable as functions of x, where x are the coordinates along the Snake. So, if N>3, we could strictly speaking identify these eigenvalues by enumeration: The lowest, next lowest, etc., except for crossings. As an example take a very specific Hamiltonian giving cotecurves of H by choice of coordinates y, so $H \sim \bar{y}^2$, and the commutator $[y_i, y_j]$ being very complicated.



12.1.1 The vacuum Snake

Until now, our main assumption is that the world is in a state in the neighbourhood of "the vacuum Snake". The true Snake is in reality a state that can be considered a superposition of a huge number of states that are all needed to be there in the ground state because there are terms in the Hamiltonian with matrix elements between these states (of which it is superposed). We could think of these terms enforcing the superposition for the ground state as some kind of "generalized exchange forces." To go far away from the Snake would be so rare and so expensive that it in principle doesn't occur, except at the Big Bang. It is also possible that the Snake is the result of some Hubble expansion-like development just shortly after Big Bang. It must in reality be the expansion that has somehow brought the Universe to be near an effective ground state or vacuum, because we know phenomenologically from usual cosmological models that the very low energy density reached is due to the Hubble expansion. Thinking of some region following the Hubble expansion, its space expands but we can nevertheless consider analytical mechanical systems. Starting with a high energy density state, i.e. rather far from



vacuum, the part of the Snake neighbourhood which is used gets smaller and smaller after Big Bang. Already very close to the singularity - if there were one - the only states were near the Snake. We may get away from the "Snake valley", but only at Planck scale energies. And we will probably never have accelerators bringing the state very far away from the Snake. So far, we have identified "the Snake" in the phase space of the very general and very complicated analytical mechanics system quantized.

Aiming at deriving a three-dimensional space, we must have in mind that this manifold, which is the protospace, has a very high dimension of order N which is the number of degrees of freedom of the whole universe. If that were what really showed up as the dimension of space predicted by our picture, then of course our picture would be immediately killed by comparison with experiment. If there shall be any hope for ever getting our ideas to fit experiment, then we must at least be able to speculate or dream that somehow the effective spatial dimension could be reduced to become 3.

For many different reasons, it seems justified to believe that 3 is the dimension of space. The naive argument is that we experience space as 3-dimensional, the number of dimensions is however not to be taken for granted, as we know from e. g. Kaluza-Klein, and string theory. We shall in the following at least refer to some older ideas that could make such a reduction possible. For instance one can have that in some generic equations of motion one gets for the particle only non-zero velocity in three of the a priori possibly many dimensions.

12.2 The number of space dimensions

In the 1920-ies Paul Ehrenfest [2] argued that for a $d = D+1$-dimensional spacetime with $D > 3$, a planet's orbit around its sun cannot remain stable, and likewise for a star's orbit around the center of its galaxy. About the same time, in 1922, Hermann Weyl [3] stated that Maxwell's theory of electromagnetism only works for $d = 3 + 1$, and this fact *"...not only leads to a deeper understanding of Maxwell's theory, but also of the fact that the world is four dimensional, which has hitherto always been accepted as merely 'accidental,' become intelligible through it."*

The intuition that four dimensions are 'special' is also supported by mathematician Simon Donaldson [4], whose work from the early 1980-ies on the classification topological four-manifolds indicates that the most complex geometry and topology is found in four dimensions, in that only in four dimensions do exotic manifolds exist, i.e. 4-dimensional differentiable manifolds which are topologically but not differentiably equivalent to the standard Euclidean R^4.

The existence of such wealth in 4-dimensional complexity is reminiscent of Leibniz' idea [5] that God maximizes the variety, diversity and richness of the world, at the same time as he minimizes the complexity of the set of ideas that determine the world, namely the laws of nature. Only, Leibniz never told in what dimensions this should be the case, but according to Donaldson, this wealth of structure is maximal precisely in a 4-dimensional spacetime manifold.

12.2.1 3+1 dimensions and the Weyl equation

Another way to "derive" $3 + 1$ dimensions, is by assigning primacy to the Weyl equation [6]. The argument is that in a non-Lorentz invariant world, the Weyl equation in $d = 3+1$ dimensions requires less finetuning than other equations. This means that in $3+1$ dimensions the Weyl equation is especially stable, in the sense that even if general, non-Lorentz invariant terms are added, the Weyl equation is regained. So in this scheme both $3+1$ dimensions and Lorentz invariance eventually emerge.

Before $3+1$ dimensions there is no geometry. Starting with an abstract mathematical space with hermitian operators $\bar{\sigma}$ and $\bar{p}\psi$, and a wave function ψ in a world without geometry, choose a two-component wave function,

$$\bar{\sigma}\bar{p}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = p_0\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

where p_0 is the energy. In vielbein formulation this is $V_a^{\mu}\sigma^a p_{\mu}\psi = 0$, which is the Weyl equation with hermitian matrices σ^a that are the Pauli matrices σ^1, σ^2, σ^3. The vielbeins are really just coefficients coming about because we write the most general equation. The Weyl equation is Lorentz invariant and the most general stable equation with a given number of ψ-components, and as a general linear equation with 2x2 hermitian matrices, it points to $3+1$.

In d dimensions the Weyl equation reads

$$\sigma^a e_a^{\mu}\frac{\partial\psi}{\partial x^{\mu}} = 0, \tag{12.3}$$

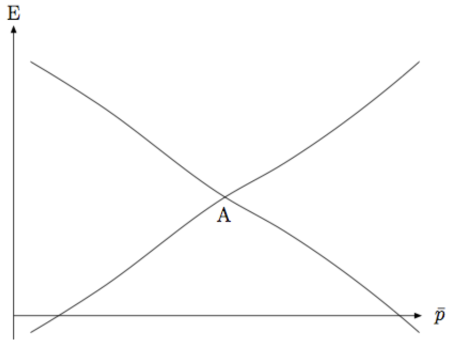
a=(0,1,2,3), and the metric $g^{\mu\nu} = \sum_a \eta_{aa}e_a^{\mu}e_a^{\nu}$ is of rank=4. If the dimension $d > 4$, there is however degeneracy.

For each fermion, there are generically two Weyl components. If we had a generic equation with a 3-component ψ, we would in the neighbourhood of a degeneracy point in momentum space, have infinitely many points with two of the three being degenerate.

Assume that ψ has N components, $\psi = (\psi_1, ..., \psi_N)$. Consider a C-dimensional subspace of the ψ-space spanned by the ψ-components $\psi_1, ..., \psi_C$, with $N \geq C$, and at the "C-degenerate point", there is a C-dimensional subspace in ψ-space (N-dim) for which $H\psi = \omega\psi$, with only one ω for the whole C-dimensional subspace (degenerate eigenvalue ω with degeneracy C - the eigenvalue ω is constant in the entire C-dimensional subspace). In the neighbourhood we generically have $\bar{p}\bar{\gamma}$ extra in H, where

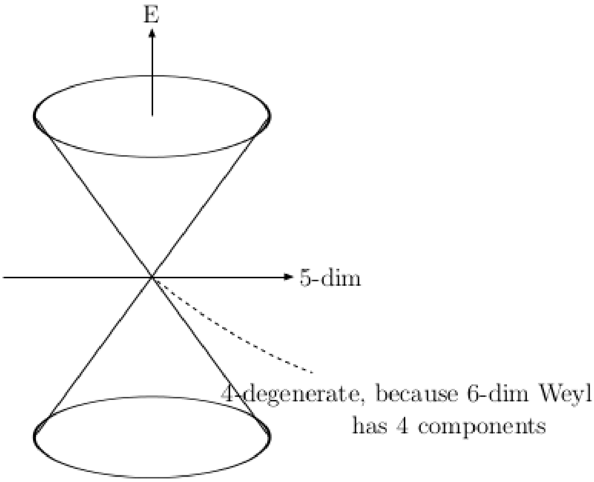
$$H(\bar{p}) = H(\bar{p}_{degenerate}) + \bar{p}_0\bar{\gamma} \tag{12.4}$$

for which $H\psi = \omega\psi$. There are lower degeneracy points in the neighbourhood (meaning p^{μ}-combinations with more than one polarization), where in the situation with two polarizations. In the above figure A represents the 2-generate point and the curves outside of A represent the situation where only one eigenvector in ψ-space is not degenerate. In the neighbourhood of a "generic" 3-degenerate (or more) point there are also 2-degenerate points. But the crux is the filling of the Dirac-sea. Think of the dispersion relation as a topological space: Can we divide this topological space into two pieces, one "filled" and one "unfilled" so that the border surface ∂"unfilled" $=$ ∂"filled" only consists of degenerate states/dispersion points? If not, we have a "metal".



The question is whether there is a no-metal theorem. To begin with, we can formulate one almost trivial theorem: If the border ∂"unfilled" contains a more than 3-degenerate point, we generally either have a metal or else 2-degenerate points on this border. There is also the disconnected dispersion relation, corresponding to an insulator.

Counter example: Imagine a 6-dimensional Weyl equation. In this case, the border ∂"filled" has only one point in the 6-dimensional Weyl, so there is only a 4-degenerate point and no 2-degenerate points on the border. The statement about



the stability of the Weyl equation in $3 + 1$ dimensions would thus be false if the 6-dim Weyl were "generic". But it is not, so there is no problem.

In d dimensions the number of $(1+\gamma^5)\gamma^\mu$ matrices is d (where $(1+\gamma^5)$ project to Weyl, i.e. the handedness), and the Weyl ψ has $2^{d/2-1}$ components. That means that there are 2^{d-2} matrix elements in each $(1+\gamma^5)$ projected γ^μ. Assuming that the dimension d is even, normal matrices γ^μ (i.e. Dirac gamma matrices) have $2^{\frac{d}{2}}$ matrix elements in each γ^μ.

Now, for $2^{d-2} > d$, one can form matrices which on the one hand act on the Weyl field ψ (with its $2^{\frac{d}{2}-1}$ components), but on the other hand are not in the space spanned by the projected γ^μ-matrices. One could in other words change the Weyl equation by adding some of these matrices, thus for $2^{d-2} > d$ the Weyl equation is not stable under addition of further terms. So the Weyl equation is not "generic" for $2^{d-2} > d$, i. e. it so to speak has zero measure (in the sense that if you write down a random equation of the form $[\sum_a p_a M^a(nxn)]\psi = 0$ in d dimensions, where n is the number of ψ-components and $2^{d-2} > d$, the probability that it is the Weyl equation is zero). It is on the other hand impossible to have d linearly independent projected γ^μ-matrices if $2^{d-2} < d$, for even dimension d.

Looking at different number of dimensions d, we conclude that for $d = 4$, $2^{d-2} = d$, seemingly confirming the "experimental" number of dimensions $4 = 3+1$, i.e. there is genericness: It seems like the 4-dimensional Weyl equation is just the most general stable equation with a given number of ψ-components.

d	2^{d-2}
0	1/4
1	1/2
2	1
3	2
4	4 - equality!
5	8
6	16

So on the one hand the experienced number of dimension is $4 = 3+1$, and on the other hand, in $d = 4$ the Weyl equation is stable under small modifications (so here the Weyl equation is "generic").

12.2.2 Bosons and fermions

Arguing that space has $3+1$ dimensions, we however run into the old story that we get $3+1$ dimensions and Lorentz invariance separately for each type of particle.

From one perspective, fermions should however not exist at a fundamental level, since they violate locality,

$$[\psi(\bar{x}), \psi(\bar{y})] \neq 0 \tag{12.5}$$

One way out could be to get effective fermions from bosons, à la the relation in 1+1 dimensions,

$$\psi \sim_{def} e^{i\phi} \tag{12.6}$$

where ϕ is a boson field. If there are N_f fermion components and N_b boson components, then moreover [7]

$$\frac{N_f}{N_b} \approx \frac{2^{d-1}}{2^{d-1}-1} \tag{12.7}$$

A bosonic counterpart to the Weyl equation would be of the form

$$K^{\mu}_{ba}\partial_{\mu}\psi_a = 0, g^{\mu\nu} = K^{\mu}_{ba}K^{\nu}_{cd}\Pi^{bacd} \tag{12.8}$$

where e.g. Π^{bacd}=$\delta^{ba}\delta^{cd}$, and $K^0 = \delta^{ab}$ for $a = b$, and $K^i_{ba} = i\epsilon^i_{ab}$, $H = 1/2\sum\tilde{\psi}^2_a(\bar{p}) \to \delta_{ab}\tilde{\psi}_a\tilde{\psi}_b$.

In the game for gauge bosons or Weyl fermions, we look for a mechanism of aligning the metrics for the different species of particles. We want to generalize the coherent state concept and show that the states on the manifold can be called generalized coherent state. Coherent states are usually given from harmonic oscillators with $q's$ and $p's$. So we must locally (in the phase space) approximate the system by harmonic oscillators, then seek to extract $q's$ and $p's$ as operators, and so we might have proven the quantized analytical mechanics model.

Define a generalized coherent state $A(q,p)q_{op} + iB(q,p)p_{op}$, such states are given by points on a manifold. Differentiating with respect to a coordinate on the manifold should give p or q acting on the state,

$$(Aq_{op} + iBp_{op})|q', p' >= (Aq' + iBp')|q', p' > \tag{12.9}$$

One thing is to have a manifold of rays, another is to have one of state vectors (in the Hilbert space) $|\lambda >= e^{\lambda a^{\dagger}}|0 >$,

$$\frac{d}{d\lambda}|\lambda > \approx a^{\dagger}|\lambda > \tag{12.10}$$

$a^{\dagger} = \alpha q + ip$

As point of departure, we use gauge particles at low energy. There come metrics out of it, one for each gauge boson. The equation of motion we get is

$$\partial_t \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = i \begin{pmatrix} 0 & A_{12} & A_{13} \\ -A_{12} & 0 & A_{23} \\ -A_{13} & -A_{23} & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \tag{12.11}$$

where

$\mathbf{A} \approx \bar{p}$ and $\phi_i = B_i + iE_i \simeq F_{jk}\epsilon_i^{jk} + iF_{0i}$.

Together with C. Froggatt, one of us has shown [8] that looking at the very low energy behavior of a (rather) generic system of bosons, one may arrive at an approximate equation of motion for three of the fields of the form (12.11). However, typically for Random Dynamics, we should argue that the coefficients the A's here are dynamical. These A's are (essentially) the same as the K's in equation (12.8) and we have already written that a metric tensor comes out of them. Of course all fields are basically of the form of some combination of the $\phi_i(x)$'s, since they make up at least all the "important" degrees of freedom. This is also true for the A's, or equivalently the K's, thus in the end the metric tensor comes to depend on the ϕ's.

12.3 Reparametrization

If a space has N dimensions, the phase space dimension is 2N, and the Hilbert space can be perceived as a sum $\mathcal{H} = \sum_{\oplus}\mathcal{H}_N$. N is not a constant of the motion, so we need some term in the Hamiltonian going from one N to another. So let us imagine an only quantum mechanically describable term with matrix elements between wave packets connected to the phase space for one N, and the wave functions connected to another of the N values (another phase space so to speak).

The full Snake must then be imagined as really a superposition of one (or more) snakes in each or at least several of the phase spaces corresponding to the various N values. Hereby the snakes in the different N-value phase spaces get locked together, but they will somehow be locked so as to follow each other - due to the quantum matrix elements connecting the different N-value phase spaces - and we effectively have only one snake.

We let x enumerate the points along the Snake, i.e. in the "longitudinal direction", x is chosen by convention. We can just as well choose again, now choosing it to be x′=x′(x), it should not matter. The crux is whether the action is independent on these choices, i.e. whether $S(\psi_i(x), ...)$ and $S'(\psi_i(x'), ...)$ are of the same form, supposedly something like

$$S = \int (\sum_i \dot{q}_i p_i - H) dt, \tag{12.12}$$

presumably they are not. That means that reparametrization invariance is not automatically given, but must be derived.

In General Relativity we have $S = \int R\sqrt{g} d^4x$. If we put $x'^{\mu} = x'^{\mu}(x^{\rho})$ into S, and transform $g_{\mu\nu}$the conventional way, $g'_{\mu\nu}(x')$=$g'_{\mu\nu}(x')(..)$, we get $S = S'$ from the constructed form (of Einstein and Hilbert). But since we have no a priori reparametrization invariance, we cannot state that the action is independent in this way. So far, our Snake model doesn't even have translational invariance. It needs to be derived, and we also need to derive diffeomorphism invariance.

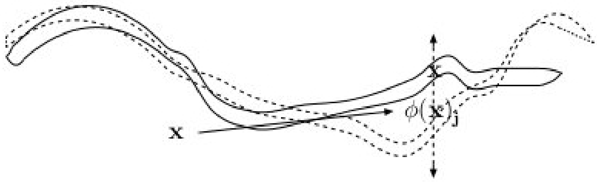
Following the scheme of Lehto-Nielsen-Ninomiya [9], the diffeomorphism invariance should be achieved by quantum fluctuations, in the sense that quantum fluctuations should produce translational invariance and in the end even reparametrization invariance.

We do this by relating points on the Snake to points 'on' the metric (assuming that the effects of going along the string on the effective parameters that are being averaged are bounded, so that the average at least converge): Consider a point given by computation using the $g^{\mu\nu}$, which quantum fluctuates. These fluctuations so to speak smear out the differences between points chosen on the Snake, thus ensuring translational invariance (and diffeomorphism invariance).

In this way we can always formally get diffeomorphism invariance, but we risk to have some absolute coordinates functioning as "Guendelman variables" [10]. To show practical reparametrization invariance then depends on how we get rid of these absolute coordinates, or rather how their effects are washed away.

12.3.1 Procedure

1. We have the Snake in the phase space of the very general and very complicated analytical mechanics system quantized. We get the fields ϕ_j corresponding to the small displacements in transverse directions in which the frequencies of vibrations



are "small" at the position x, telling where in phase space we are in the longitudinal directions of the Snake.

2. We assume (or show) that there are some fields (essentially among the $\phi_j(x)$'s or related to their development), a set of "upper index metric fields" $g^{\mu\nu}(x)$.

As a matrix, this metric should have rank 4, and we expect to find one $g^{\mu\nu}$ for each species of particles. Here we first think of gauge particles, postponing the fermions.

That is to say, we get some equations of motion for three effectively relevant fields ϕ_m (m=1,2,3) for each gauge particle species.

With equation (12.8) in mind, we consider the form

$$K^0_{mn}\tilde{\phi}_n - K^j_{mn}p_j\tilde{\phi}_n = 0 \tag{12.13}$$

or just $K^\mu_{mn}p_\mu\tilde{\phi}_n = 0$, where the p^μ stands for $p^\mu - p^\mu_0$, and $p^\mu = i\partial/\partial x^\mu$.

$$K^0_{mn} = \delta_{mn} \quad \text{and} \quad K^i_{mn} = i\epsilon^{imn} \tag{12.14}$$

But at first we only have

$$K^\mu_{mn} = K^{\mu *}_{nm} \quad (\text{hermiticity}) \quad \text{and} \quad K^0_{mn} = \delta_{mn} \quad (\text{essentially definition}) \tag{12.15}$$

because we have chosen the simple Hamiltonian

$$H = \int (\sum_m \tilde{\phi}^2_m(\vec{p}))d^{d-1}\vec{p} \tag{12.16}$$

to be $\delta^{mn}\tilde{\phi}_n\tilde{\phi}_m$, and $K^i_{mn} = -K^i_{nm}$, because we let all the K^i_{mn} come from the Poisson bracket (commutator)

$$[\tilde{\phi}_m(\vec{p}), \tilde{\phi}_n(\vec{p})] = K^i_{mn}(p'_i - p_{i0}) \tag{12.17}$$

near the zero point in $\vec{p}$-space. From this K^μ_{mn} one then constructs the defining relation

$$g^{\mu\nu} = K^\mu_{mn}K^\nu_{op}\delta^{mo}\delta^{np} \tag{12.18}$$

for the rank 4 metric with upper indices $g^{\mu\nu}$.

3. Assume (this must be true) that what we conceive as a point in space is calculated by using a metric $g^{\mu\nu}$ (we may have the problem of getting too many matrices $g^{\mu\nu}$, i. e. $g^{\mu\nu}_1(x), g^{\mu\nu}_2(x), g^{\mu\nu}_3(x), ...$) integrating it roughly up to calculate where we have a point with given coordinates.

4. The formulation we shall use is by construction diffeomorphism invariant for the coordinate set x enumerating the points along the Snake. But that does not mean that we have a diffeomorphic symmetric Hamiltonian H or action S. We can namely have an underlying absolute coordinate system - or "Guendelman variables". We could indeed imagine that we at first describe the longitudinal manifold along the Snake by a set of coordinates ξ, as many ξ as there are x-coordinates, of course. When introducing the diffeomorphism transformable x, we perceive $\xi(x)$ as some (scalar) fields which are functions of x. But all the special structure of the phase space or analytical mechanics system as it varies along the Snake, appears as explicitly dependent on H, or S on the ξ's taking specific values. There is so to speak no translational invariance in ξ, but there is trivially in x, since translation is (apart from boundary problem) just a special diffeomorphism. Since in the "vacuum" it could at first seem that the ξ's have in x varying values as one goes along in x, the presence of these ξ (expectation) values even in "vacuum" means a spontaneous breakdown of translational invariance, and even more a spontaneous breakdown of diffeomorphism symmetry.

At first glance, it thus looks like the "Guendelman" ξ-fields imply a spontaneous breakdown of translational and diffeomorphism invariance. So to prove that we do indeed have diffeomorphism invariance for say the Hamiltonian H, we must show that the practical effects of the "Guendelman fields" or original absolute coordinates ξ, wash out. Under the conditions which we shall consider, the ξ-dependent effects in practice average out. We shall argue that if we (as humans or physicists) count our position by integrating up some of the $g_{\mu\nu}$ obtained from $g^{\mu\nu}$ (or some average of $g_1^{\mu\nu}, g_2^{\mu\nu}, g_3^{\mu\nu}, ..$), we fluctuate around relative to the ξ-coordinates (which are fixed in phase space along the Snake). These fluctuations were assumed under 3.

5. Now we need the assumption that the potentials, or more generally the Hamiltonian contributions depending on ξ (and thus via the spontaneous breakdown violating the translational invariance), are bounded or at least as effectively bounded as fluctuations of ξ, so the averages over large regions in ξ become (approximate) constants.

By taking this boundedness of the ξ-dependent part of the Hamiltonian as a reasonable assumption, the ξ-dependent contributions to the Hamiltonian wash completely out to nothing, the reason being the integrated up metric becomes integrated up over regions in x-space of the order of the size of the Universe, whereby the fluctuations become enormous.

If that is so, we have shown that for "us" situated in a place determined from the metric tensor fields $g_i^{\mu\nu}$ or rather their inverse $g_{\mu\nu}$ by long distance integration, the diffeomorphism invariance has been (effectively) (re)stored. In this way the formally introduced diffeomorphism invariance - just by thinking of x as an arbitrary set of variables - has become a good symmetry because of the ξ's representing the lack of diffeomorphism symmetry by spontaneously breaking it, have gone practically out of the game.

It should be noticed that by this argumentation we have argued for diffeomorphism symmetry in the whole x-space of dimensions suspected to be 3, even if the metric tensor only has (because, say, of inheriting from K^{μ}_{mn}) rank 4, thus delivering an effective spacetime of dimension $3+1$.

The point is that even though the single $g^{\mu\nu}(x)$ has only rank $4 = 3+1$, it can fluctuate so all the fluctuation values of $g^{\mu\nu}(x)$ are included, and all directions in x-space covered. One may imagine the 3-dimensional space as a 3-dimensional submanifold embedded in the much higher dimensional x-space (the longitudinal space on the Snake). Then this submanifold not only fluctuates by extending and contracting in its own 3-dimensional directions, but also fluctuates around its transverse directions inside the x-space. Thus by quantum fluctuations (integrated up), the 3-space submanifold floats around (almost) all over the Snake in its longitudinal space.

For each fixed configuration of $g^{\mu\nu}(x)$ one has a whole "fibration" of 3-spaces lying parallel to each other in the x-space. Then the whole fibration fluctuates around in x-space. Accepting the above, we arrive at an approximate Hamiltonian (or an approximate action S) being exactly diffeomorphism invariant, whereby we can deduce locality. After having derived locality that way, we get a picture very close to a model with gauge bosons and a dynamical metric, seemingly with 3 space dimensions. It looks rather like what we see phenomenologically, but there are a few weak points:

- The problem of each particle species, here each gauge particle species, having its own $K^{\mu}_{mn}(x)$, and thus it own $g^{\mu\nu}(x)$.
- We have in some sense much more than 3 spatial dimensions because we have as many as has the longitudinal direction on the Snake.

These problems may not be very severe: Calculating our position from the $g^{\mu\nu}$ as if space were 3-dimensional, we obtain what we use as position. Then it does not matter so much that relative to the Snake, the ξ-absolute coordinates fluctuate both in the 3 and the many other coordinates. Since the ξ's are supposedly bounded - and thus relatively easy to average out to a constant - it will just become even easier to get them averaged out over the bigger region where the position of "us" fluctuates.

We should however have in mind that signals going along the 3-dimensional surfaces along which the quanta can move, for every fixed imagined position of the 3-manifold inside the much higher dimensional x-space, will only be able to move along that 3-surface. However, when this surface fluctuates wildly, also the signals running on it get swept along in much more than three directions. That will however not be noticed by the physicist using the point-concept resulting from integrating up the metric $g_{\mu\nu}$, or what we consider the more genuinely existing (∴ a bit more fundamental) $g^{\mu\nu}$. The physicist can only get motions in the three dimensions, simply because he only evaluates three coordinates in his position calculations.

So this problem is not so severe.

We however need to resolve the problem that each particle species has its own metric. A plausible solution goes in the direction that the metrics are in some way "dynamical", and interact with each other in such a way that they finally align, thus behaving as if they were all proportional to each other. We would hope that e.g. the metric determining the gluon propagation would by interaction with the metric tensor (similarly related to say the W's and determining their propagation) bring them in the lowest energy situation to become aligned, where this aligning then really should stand for that they become proportional to each other.

It should be noted that our theory is a priori not Lorentz invariant, at least not in the metric degrees of freedom, the Lorentz invariance supposed to be derived subsequently. Considering that our Snake is in its ground state, there are no ghosts, the question is how the different metrics behave. To begin with, we ask how *one* metric $g_{\mu\nu}$ can avoid having ghosts.

12.3.2 Idea of Attracting Metric Tensors

The basic idea in getting dynamical metrics which are adjusted to be parallel/proportional is not so difficult. Multi-metric gravity is however complicated by the (Boulware-Deser) ghosts [11] that threaten to appear as one of the gravitons becomes massive. Indeed lets us give the main hope:

1. For each type of particle, initially meaning each type of gauge particle (but if we add fermions we could also have a metric tensor for each type of Weyl particle) there is a characteristic metric tensor $g^{\mu\nu}$ (with upper indices, prepared for being contracted with a derivative ∂_μ w.r.t. to the coordinates x^μ). So we shall strictly speaking attach a particle species name to each of these metrics, e.g. $g^{\mu\nu}_{(W)}$ for the metric assigned to the W gauge boson.

2. we argue that this metric is "dynamical" and even a field. Thus, it is not just a constant metric, but such that it

- can vary with initial conditions and fluctuate quantum mechanically,
- can vary in time,
- and even in space, since we take it as a field (we anyway have no translational invariance yet). The coefficients in the time development of the fields which are going to be interpreted as the gauge boson fields, will nevertheless depend on the precise position of the Snake near the place to which the fields in question are assigned. The point of view that the coefficients which give rise to the metric tensor are fields should be unavoidable.

3. Taking seriously the Random Dynamics assumption that everything that is allowed to interact also *does* interact, we deduce that the different metric tensors associated with different particle species will indeed interact.

4. We introduce the symmetries restricting the interactions between the various fields, paying attention to the metric tensor fields associated with the different particle species. At some point we get reparametrization from diffeomorphism invariance, which then restricts the way these metrics (which transform as upper index tensors) interact. Do not forget that by taking the inverse of the upper index matrix we can get one with lower indices instead (were it not for the problem that the metric only has rank 3 +1 and thus canot be inverted).

5. These restrictions from diffeomorphism or other symmetries, also mean that the equations between the fields (resulting from the minimum energy state for the system w.r.t. to, say, the metric tensors) also share these symmetries. This

gives hope that the metric tensors will come to be proportional (or even equal) to each other.

6. Now, if the metric tensors for the different species of particles indeed get proportional, it really means that the Lagrangian terms or equations of motions for the different particle species can be written with the *same* metric and just some overall factors in addition. This in its turn means that in the end, there are no effectively different metrics.

If you have several different metrics, this is what supposedly happens:

- You get bigravity or multigravity, meaning that you get a model with several spin=2 particles [12] [13].
- We can (after some partial gauge fixing) interpret the massless graviton as a Nambu-Goldstone particle for diffeomorphism symmetry, and we expect that even after getting several metric tensors we should only have one massless graviton if the diffeomorphism symmetry remains [14]. So we expect one massless graviton and several massive spin 2 particles, namely the number of metric tensors minus one.
  The graviton becomes a real Nambu-Goldstone particle due to a linearly varying gauge function. Simple shift by adding a constant to a coordinate, perceived as a reparametrization/gauge transformation, is not spontaneously broken in Einstein gravity. It's only the linear variation of ϵ with x, that makes the metric tensor field spontaneously breaking the transformation.
- Then our "poor physicist thinking" means that we guess that all particle species which don't have a reason for being massless (or almost massless), have so big masses that they are in practice not present (it is so to speak the Universe after the very first singularity (supposing there was one), which is so cold that massive particles do not occur even if they exist in the sense that they could in principle be produced in some enormously expensive accelerator).
  This means that all the heavy graviton field degrees of freedom are in their no-excitation state. If these fields are the metrics, or better some linear combination of metrics for the different particles, the non-excitation of the majority of these linear combinations leaves only one excitable combination $\sum a_i g_i^{\mu\nu} = a_W g_W^{\mu\nu} + a_{gluon} g_{gluon}^{\mu\nu} + ...$ of the various metrics, namely the massless combination. This means that the various metric fields are forced to follow each other. They will namely all follow the massless graviton field, simply being equal to this massless metric multiplied by some constant.

If indeed a massive spin two graviton would appear, there will no longer be any proportional metrics. But that would be rare, and we would interpret the effect of having different metrics for different species as effects of interaction with this heavy graviton.

So once we have argued that the metric tenors are dynamical and interacting, there is really good hope for getting rid of the old problem in Random Dynamics, that different species have different metric tensors. The crux of the matter is that the different metrics have the chance to dynamically influence each other, and thereby for symmetry reason become (apart from some extra factors) the same metric.

12.3.3 General Ghost Problems

Making theories with one or several massive gravitons, i.e. bigravity, is highly non-trivial due to the ghost-problem of Boulware and Deser. The problem is that if you essentially randomly create theories for spin 2 particles, you are very likely to run into the problem of unstable modes of vibration. We here think of classical fields, and

for the theory to be stable - i.e. have a bottom in the Hamiltonian - all modes of vibration should be like harmonic oscillators rather than like $inverted$ harmonic oscillators. It is, however, rather an art to avoid getting such ghosts or unstable vibrations, if one seeks a massive spin two. Thereby it becomes a problem also for making an interacting bigravity or multigravity. We argued that we expected only one massless graviton. If we have several, it is most likely that one or more are heavy gravitons, which then in turn brings their ghost-problem.

Hassan and Rosen [15] argue that they have got the only bigravity without ghosts. A characteristic of this two metric theory (= bigravity) is that the interaction, apart from the usual factor $\sqrt{-detg}$, is a function only of a kind of ratio of the interacting metrics $f_{\mu\nu}$ and $g_{\mu\nu}$, formally written $\sqrt{g^{-1}f}$. This means that it depends on a constructed metric γ^{μ}_{ν} defined by the equation

$$\gamma^{\mu}_{\nu}\gamma^{\nu}_{\rho} = g^{\mu\rho}f_{\rho\nu}. \tag{12.19}$$

In fact the interaction part of the Lagrangian density is written as a sum with coefficients β_n of symmetrized products of eigenvalues of the matrix γ^{μ}_{ν}.

There is as a side remark for us who have a theory in which the metric tensor appears as a product of two matrices: We may construct the square root matrix γ^{μ}_{ν} directly from the matrices that must essentially be squared to obtain the metric, i.e. our original variables from which we construct the metric are already a kind of square roots of the metric.

Concerning the Bouleware-Deser ghosts or unstable modes, for the purpose of our machinery for obtaining relativity and space, we may think as follows:

If we have chosen to consider states around a ground state which has the lowest possible energy, there cannot be any vibration modes unless the vibration leads to positive or at least non-negative energy. That means that all the vibrations around our ground state - the ground state of the Snake - must be of the type of a positive frequency and energy, i.e. ordinary rather than inverted harmonic oscillator. So from our a priori very general model one deduces a good behavior of the resulting particle field equations. There shall be no unstable modes of vibration in the effective field theory resulting from our Snake model. We logically allow a type of bigravity or massive gravity which avoids the ghosts, and if it is claimed that there is no alternative to a certain special type of models to avoid the instabilities (that would mean that the bottom falls out of the Hamiltonian, so some states would have energy less than the state assumed to have the lowest energy around which we expand) we should be formally allowed to conclude that this type of model is effectively realized in our Snake model. It's only once we manage to get dynamical metrics that the discussion of bigravity type theories becomes relevant, but we at least get some coefficient-fields which we strongly expect to become dynamical variables. Surely there will to these fields, which if dynamical, formally correspond to some “metric tensors”.

In the spirit that all allowed terms should be there, the speculation that these metric fields must obtain some kind of kinetic term in our very general model, seems very well supported. This is essentially just the Random Dynamics assumption that the coupling parameters can be considered random, so they cannot be in any (simple) special value system that would have measure zero. Thus the possible kinetic terms must be allowed, and the sign(s) can only be as needed for the already discussed ground state to indeed be the ground state.

We take this argumentation to mean that we must expect our very general analytical mechanical system treated as the Snake to be approximated by the matter gauge fields (and Weyl fermions if we allowed), in addition to a say in the two gauge boson case (for simplicity) the bi-gravity of Hassan and Rosen, cleverly adjusted to have no instabilities (∴ no ghosts). This Hassan Rosen model should apart from possible modifications of the kinetic energy have an action like

$$S = M_p^2 \int d^4x \left[\sqrt{-g} \left(R + 2m^2 \sum_{n=0}^{4} \beta_n e_n(\sqrt{g^{-1}f}) \right) + M_{pf}^2 \sqrt{-f} \right] \tag{12.20}$$

which is equation (2.1) in [15] with a kinetic term $\propto R_f$ for the $f_{\mu\nu}$. This equation looks a bit less symmetric than it will be in the end. The notation is that we have two metric tensors $g_{\mu\nu}$ and $f_{\mu\nu}$ and R denotes the usual Einstein Hilbert action scalar curvature calculated from $g_{\mu\nu}$ in the usual way. The symbols g and f are of course the determinants of the two metric fields, but the symbol $\sqrt{g^{-1}f}$ is *not* related to the determinants but rather it means a *matrix* γ^{μ}_{ν} determined as the square root from the condition:

$$\gamma^{\mu}_{\nu}\gamma^{\nu}_{\rho} = g^{\mu\nu}f_{\nu\rho} \tag{12.21}$$

Notice the natural use of g^{-1} for $g^{\mu\nu}$ which is of course the inverse of the g matrix $g_{\mu\nu}$ as the metric with upper indices always is.

The symbols $e_n(\gamma^{\mu\nu})$ for n running from 0 to 4 are the symmetrized eigenvalues of the matrix $\gamma_{\mu\nu}$. That is to say

$$\begin{aligned} e_0(\sqrt{g^{-1}f}) &= 1 \\ e_1(\sqrt{g^{-1}f}) &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 && (12.22) \\ e_2(\sqrt{g^{-1}f}) &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 \\ &\cdots\cdots && (12.23) \end{aligned}$$

12.4 Locality and nonlocality

Once we have established the diffeomorphism symmetry of our model, the next step is to derive locality.

According the Random Dynamics philosophy nature is inherently nonlocal, in field theory locality is however taken for granted, meaning that every degree of freedom is assigned a spatio-temporal site, i.e. that all interactions take place in one spacetime point. This implies that there is a system for assigning one site to each degree of freedom, and in a local theory the action can then be factorized. The partition function of the Universe then has the form

$$Z = \int \mathcal{D}\psi e^{(iS+\text{sources})}$$

where $S = S_1 + S_2 + ...$, and each contribution only depends on the fields in limited regions of spacetime, corresponding to $S = \int \mathcal{L}(x)d^4x$ in the continuum limit.

Nonlocality would then mean that a degree of freedom is a function of more than one spacetime point. An example of nonlocality is microcanonical ensemble, which in a formal sense is nonlocal - to approximate it to a canonical ensemble would from this perspective be analogous to approximating nonlocality with locality. In the microcanonical ensemble it is a constraint that gives rise to nonlocality, and this (omnipresent) nonlocality can be viewed as due to the presence of fixed extensive quantities, in a manner reminiscent of a microcanonical ensemble. This would then be a nonlocality inherent in nature, as opposed to one emerging from dynamical effects, i.e. not to the same as the "nonlocality" which refers to quantum nonlocality in the sense of non-separability, which occurs as nonlocal correlations which occur in settings such as the one discussed by Einstein, Podolsky and Rosen.

12.4.1 Fundamental nonlocality

Since there are no instances in quantum mechanics of signals propagating faster than light, from the Random Dynamics point of view, quantum mechanics is not really nonlocal. In the Random Dynamics scenario it is nonlocality that is taken for granted, locality appearing as a result of reparametrization invariance, i.e. as a result of diffeormorphism symmetry.

Our basic assumptions are as follows:

- Locality only makes sense when you have a spacetime, or at least a manifold, so our starting point is a fundamental, differentiable manifold $\mathcal{M}$. To grant reparametrization invariance, we cannot do with simple Minkowski space, we also need general relativity. A reparametrization invariant formulation demands that also $g^{\mu\nu}$ gets transformed, since $g^{\mu\nu} = \eta^{\mu\nu}$ would violate reparametrization invariance. So if $g^{\mu\nu}$ is perceived as nothing but a field (i.e. in reality 10 fields), there is only a manifold. Our manifold is moreover 4-dimensional, and it is only $g^{\mu\nu}$ that determines whether this means 4+0-dimensional, or $3 + 1$ - or $2 + 2$-dimensional.
- Some fundamental fields $\psi^k(x), A^{k\mu}(x), ..., K^{k\mu\nu}(x),...$ defined on the manifold $\mathcal{M}$. We also want to have a $g^{\mu\nu}$ with contravariant, upper indices. Indices are important since upper and lower indices transform differently under reparametrization mappings, and if we were to include fermions, we should have vierbeins as well, presumeably with upper curved index. Assume that the chiral theory is formulated in terms of the Weyl equation, then we need vierbeins e^{μ}_{a} which transform as four-vectors with upper index, while ψ transforms as a scalar under the curved index and thus reparametrization. In addition

there is flat index transformation under which ψ transforms as a spinor, e^μ_a as a four-vector, and $g^{\mu\nu}$ as a scalar.
In higher dimensional theories you usually assume locality in the high dimensional space, for example in the case D=14, $\int \mathcal{L}_4 d^4x$ is local in higher dimensions. In an apriori arbitrary parametrization of the form $\mathcal{R}^4 x \mathcal{R}^{14-4}$, we get $\int \mathcal{L}_4 d^4x$, where

$$\mathcal{L}_4 d^4x = \int \mathcal{L}(x,y) d^{14-4}y \tag{12.24}$$

and $\mathcal{L}_4 d^4x$ only depends on x, while $\int \mathcal{L}(x,y)d^{14-4}y$ only depends on "infinitesimal" neighbourhood in (x, y); and in this sense the lower dimensions 'inherit' locality from the higher dimensions.
Even if $y \to \infty$ is non-compact far away, this argument is valid. That is, even in the case of non-compact extra dimensions, 4-locality is there.

- Diffeomorphism symmetry, i.e. invariance under reparametrization mappings. Initially we however have a somewhat weaker assumption, demanding invariance only under $x \Rightarrow x'(x) = x + \epsilon(x)$, for det $(\partial x'^\mu/\partial x^\nu) = 1$.
- We need some "smoothness assumptions", expecting Taylor expandability. When deriving locality we obviously don't start with a local action, so our starting function is just some generic action $S[g^{\mu\nu}, \psi, \phi]$, where $\psi(x), \phi(x)$ are defined in four-dimensional spacetime represented by x, the reparametrization invariance implying that $S[\psi'] = S[\psi]$.

For this action $S[g^{\mu\nu}, \psi, ...]$ we formulate some theorems:

<u>Theorem I :</u>
With our assumptions, the "action" $S[g^{\mu\nu}, \psi, ...]$ becomes a function of a basis for all the integrals you can form in a reparametrization invariant way from polynomials and mononomials in the fields and the derivatives at a single point x integrated over $\int ...d^4x$ (i. e. the whole manifold).

We assume the manifold to be finite (compact), as a kind of infrared cutoff. Note that theorem I only implies a mild locality, i. e. an action of the form

$$S = S(\int \mathcal{L}_1 d^4x, \int \mathcal{L}_2 d^4x, ...). \tag{12.25}$$

We derive something like a Lagrangian form, because we have many $\mathcal{L}_j$, and a complicated functional form.

<u>Theorem II :</u>
When an action is of the form $S(\int \mathcal{L}_1 d^4x, \int \mathcal{L}_2 d^4x, ...)$, called "mild" locality, then inside a small region of the manifold (a neighbourhood), and for a single field development, $g^{\mu\nu}_{actual}, \psi_{actual}$, the "Euler-Lagrange equations"

$$\frac{\delta S}{\delta \psi(y)}|_{\psi=\psi_{actual}, g^{\mu\nu}=g^{\mu\nu}_{actual}} = 0 \tag{12.26}$$

are <u>as if</u> the action were of the form $S = \int \mathcal{L}(x) d^4x$ where $\mathcal{L}(x)$ is a linear combination of the $\mathcal{L}_j(x)'s$ with coefficients only depending on $g^{\mu\nu}_{actual}$ and ψ_{actual}, but in such a way that these coefficients depend only very little on $g^{\mu\nu}_{actual}, \psi_{actual}$ in the small local region considered.

According to Theorem II these "coefficients" do indeed exist, but it is apriori not certain that they are Taylor expandable. Actually there is a function-Taylor expansion for the function coming out of Theorem I.

$$\psi^k(x) \to \psi^k(x)_{new} = \psi^k(x) \circ x' \tag{12.27}$$

for each fixed k, i.e. $\psi^k(x)_{new} = \psi^k(x'(x)) = \psi^k(x)$, and

$$A^{k\mu}_{new}(x'(x)) = A^{k\nu}(x)\frac{\partial x'^{\mu}}{\partial x^{\nu}} \quad \text{and} \quad K^{k\rho\sigma}_{new}(x'(x)) = K^{k\mu\nu}(x)\frac{\partial x'^{\rho}}{\partial x^{\mu}}\frac{\partial x'^{\sigma}}{\partial x^{\nu}}. \tag{12.28}$$

Proof of theorem I: When we want to derive locality, we have to consider the "locality postulates". The first locality postulate is that the Lagrangian $\mathcal{L}$ depends on an infinitesimal neighbourhood, i.e. $\int \mathcal{L} d^4x$ is used for minimizing. An evidently local action is then $S = \int \mathcal{L} d^4x$, with $\mathcal{L} = \mathcal{L}(\psi, \partial\psi/\partial x, ...)$; the goal being to formulate an action such that the reparametrized action is a functional of the type

$$S(\psi') = \mathcal{F}(\int \mathcal{L}_1(x)d^4x, \int \mathcal{L}_2(x)d^4x, ..., \int \mathcal{L}_n(x)d^4x) \tag{12.29}$$

We also make the "weak assumption" that S is functional expandable,

$$S[\psi] = \sum_{k=0}^{\infty} \int\int\int ...\psi(x^{(1)})\psi(x^{(2)})...\psi(x^{(k)})\frac{\delta S}{\delta\psi(x^{(1)})\delta\psi(x^{(2)})...\delta\psi(x^{(k)})} d^4x^{(1)}...d^4x^{(k)} \tag{12.30}$$

The diffeomorphism symmetry implies that $S[\psi \circ x'] = S[\psi]$, where $\psi' = \psi \circ x'$, $\psi'(x) = \psi(x') = \psi(x'(x))$, the invariance meaning that $S[\psi'] = S[\psi]$ In the Taylor expansion, one has to pay attention to that

$$\frac{\delta\psi'(x)}{\delta\psi(y)} = \delta(x'(x) - y), \tag{12.31}$$

thus

$$\frac{\delta S[\psi']}{\delta\psi(y)} = \frac{\delta S[\psi(x'(x))]}{\delta\psi(y)} = \int \frac{S[\psi]}{\delta\psi(y)}\delta(x' - y)d^4x = \det()\frac{\delta S[\psi]}{\delta\psi(x'^{-1}(y))}$$

where we in the first round choose $\det() = 1$. Generalized:

$$\frac{\delta S[\psi']}{\delta\psi(y^{(1)})...\psi(y^{(k)})} = \det()\frac{\delta S[\psi]}{\delta\psi(x'^{-1}(y^{(1)}))...\delta\psi(x'^{-1}(y^{(k)}))} \tag{12.32}$$

We want to choose x' in such a way that $x'^{-1}(y^{(1)}) = z^{(1)}, x'^{-1}(y^{(2)}) = z^{(2)}, ..,$ but with the demand that $z^{(j)} \neq y^{(k)}$ for all $j \neq k$. If all $z^{(j)}$ are all different among themselves, and likewise the $y^{(j)}$ are all different among themselves, the functional derivative is a constant, but if we have a situation where some points are the same, e.g. $z_3 = z_4 = z_5$, the functional derivative will depend precisely on which points are not identical (under the reparametrizarion mapping that brings $z_3 = z_4 = z_5$ onto the points y_3, y_4, y_5, implying that $y_3 = y_4 = y_5$), i.e. $\delta S[\psi]/\delta\psi(y^{(1)})...\delta\psi(y^{(k)})$ only depends on how many in each group are identical. All aberrances belong to a null set, and if we ignore this null set, we have

$$\frac{\delta S[\psi]}{\delta\psi(y^{(1)})...\delta\psi(y^{(k)})} = f_k \tag{12.33}$$

which is independent of the $y^{(j)}$'s. We then have

$$S[\psi] = \sum_{k=0}^{\infty} \frac{1}{k!} \int \cdots \int \frac{\delta^k S}{\delta\psi(y^{(1)} \cdots \delta\psi(y^{(k)})} \psi(y^{(1)} \cdots \psi(y^{(k)} d^4y^{(1)} \cdots d^4y^{(k)} =$$
$$= \sum \frac{f_k}{k!} \int \cdots \int \psi(y^{(1)} \cdots \psi(y^{(k)} d^4y^{(1)} \cdots d^4y^{(k)} \tag{12.34}$$

and

$$\sum_{k=0}^{\infty} \frac{f(k)}{k!} (\int \psi(y) d^4y)^k = F(\int \psi(y) d^4y), \tag{12.35}$$

so we got "mild" locality of the form (12.25), i.e. some function of usual action-like terms (in reality "mild" super local where super stands for no derivatives).
Now, if the null set argument is incorrect, consider that

$$\frac{\delta S}{\delta\psi(y^{(1)})\delta\psi(y^{(2)})} = \text{const.} + \delta^4(y^{(1)} - y^{(2)}) \tag{12.36}$$

and

$$\frac{\delta S}{\delta\psi(y^{(1)})\delta\psi(y^{(2)})...\delta\psi(y^{(k)})} =$$
$$C_1 + C_2 \sum_{j,l}^{k} \delta(y^{(j)} - y^{(k)}) + C_3 \sum \delta(y^{(j)} - y^{(k)}) \sum \delta(y^{(i)} - y^{(l)}) \tag{12.37}$$

where C_j are constants. Here we integrate over all points, whereby the same points might reappear several times. The resulting action is of the form

$$S = F(\int \psi(x) d^4x, \int \psi(x)^2 d^4x, \int \psi(x)^3 d^4x, ...) \tag{12.38}$$

Now, what does such an action look like locally?

We can Taylor expand S:

$$\frac{\delta S[\psi]}{\delta\psi(y)}|_{\psi=\psi_a} = \sum_{\chi=1} \frac{\partial F}{\partial(\int \psi(x)^\chi d^4x)} \chi\psi(x)^{\chi-1} = f(\psi(x))$$

where $\partial F/\partial(\int \psi(x)^\chi d^4x)\chi\psi(x)^{\chi-1}$ can be locally approximated with a constant, and $f(\psi(x))$ depends on what happens in the entire universe.

We now have a situation where $S \approx \int h(\psi(x)) d^4x$ (where the function h is defined so that $h'(\psi) = f(\psi)$ i.e. it is the stem function of f), corresponding to a super local Lagrangian.

12.4.2 An exercise

As an exercise we will consider a theory with ψ and A^μ (a contravariant vector field), keeping in mind that A^μ and A_μ transform differently under diffeomorphisms.

Taylor expanding the functional $S[\psi, A^\mu]$:

$$S[\psi, A^\mu] = \sum_{k=0}^{\infty} \int\int \frac{\delta^k S}{\delta\psi(y^{(1)})} \delta\psi(y^{(2)})...\delta A^{\mu k}(y^{(k)})\psi(y^{(1)})...A^{\mu k}(y^{(k)})\frac{1}{k!}d^4y^{(1)}...d^4y^{(k)} \tag{12.39}$$

and consider

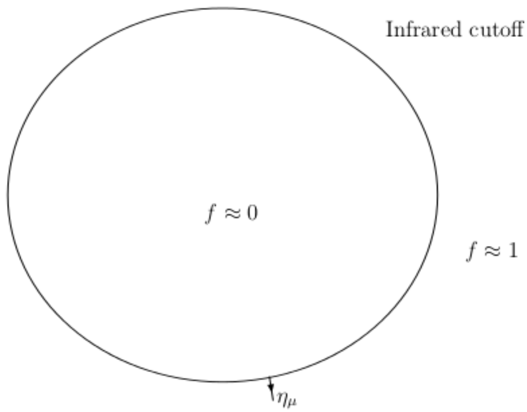
$$\frac{1}{1!}\int \frac{\delta S}{\delta A^\mu}(y^{(1)})d^4y^{(1)} \tag{12.40}$$

where $\delta S/\delta A^\mu(y^{(1)})$ is forced to be zero under reperametrization transformations. But if we only include boundary terms,

$$\frac{\delta S}{\delta A^\mu(y^{(1)})} \sim \int \partial_\mu \delta(y^{(1)} - x)d^4x \approx \text{only boundary terms} \tag{12.41}$$

where the normal to the boundary $\eta_\mu \sim \partial f/\partial x^\mu$, and

$$\int_{\partial V_4} \eta_\mu A^\mu d^3x = \int A^\mu [dx]_\mu, \tag{12.42}$$



the reparametrization invariance implies that

$$\frac{\delta S}{\delta A^\mu} = \eta_\mu \text{ on the boundary, and } 0 \text{ on the inside of } V_4. \tag{12.43}$$

We want to have

$$\int \frac{\delta S}{\delta A^{\mu}} A^{\mu}(y) d^4 y = \int_{\partial S} const. A^{\mu} \eta_{\mu} d^3 y|_{boundary} \tag{12.44}$$

This is integrated with A^{μ} as a variable, to

$$\mathbf{C}\delta_{\mu} A^{\mu} d^4 x = \mathbf{C} \int A_{\mu} \eta d^3 x \tag{12.45}$$

where **C** is a constant, and $\eta d^3 x$ represents the boundary. Now the action is

$$S = F(\int \psi(x) d^4 x, \int \psi(x)^2 d^4 x, ..., \int \partial_{\mu} A^{\mu} d^4 x, \int \psi(x) \partial_{\mu} A^{\mu} d^4 x, ...) \tag{12.46}$$

We now take all reparametrization invariant Lagrange density suggestions and let

$$S = F(\int \mathcal{L}_1 d^4 x, \int \mathcal{L}_2 d^4 x, ...) \tag{12.47}$$

where we have remarked that the various integrands occuring (49), i.e. $\psi(x)$, $\psi(x)^2$, ..., $\partial_{\mu} A^{\mu}(x)$, $\psi(x) \partial \mu A^{\mu}(x)$,... are easily seen to be just those integrands which ensures reparametrization invariance (under our (simplifying) assumption of the determinant in the reparametrization $x'(x)$ being unity.). We have therefore hereby finished the proof (or at least argument for) our above theorem I.

The theorem II is shown by arguing that, if we think of only investigating say the equations of motion in a small subregion of the whole spacetime region in which the universe have existed and will exist, then the integrals occurring in the function $F(\int \mathcal{L}_1(x) d^4 x, (\int \mathcal{L}_2(x) d^4 x, ...)$ will only obtain a relatively very little part of their contribution for this very small local region. Thus these integrals as a whole will practically independent of the fields $\psi(x)$ etc. in the small region (where we live, and which is considered of interest). So indeed the statement of theorem II is true and we consider theorem II proven.

The final point is that we hereby have argue for that we *for practical purposes* got locality from assuming mainly diffeomorphism or reparametrization invariance for practical purposes, in the sense that we only investigate it in an in space and time relative to the spacetime volume of the full existence of the universe small region. Further it were based on Taylor expandability of the very general a priori non-local action $S[\psi, A^{\mu}, ...]$.

This "derivation" of locality were initiated in collaboration with Don Bennett.

12.5 Conclusion

We have in this article sought to provide some - perhaps a bit speculative - ideas for how to "derive" spacetime from very general starting conditions, namely a quantized analytical mechanical system. From a few and very reasonable assumptions, spacetime almost unavoidably appears, with the empirical properties of 3+1 dimensionality, reparametrization symmetry - and thereby translational invariance, existence of fields, and practical locality (though not avoiding the nonlocalities due to quantum mechanics). Our initial assumption was that the states of the world were very close to a ground state, which in the phase space was argued to typically extend very far in N dimensions, while only very shortly in the N other dimensions. Here the number of degrees of freedom were called N and thus the dimension of the phase 2N. This picture of the ground state in the phase space we called the Snake, because of its elongation in some, but not all directions. The long directions of the Snake becomes the protospace in our picture. The translation and diffeomorphism symmetry are supposed to come about by first being formally introduced, but spontaneously broken by some "Guendelmann fields ξ". It is then argued that this spontaneous breaking is "fluctuated away" by quantum fluctuations, so that the symmetry truly appears, in the spirit of Lehto-Ninomiya-Nielsen. At the end we argued that once having gotten diffeomorphism symmetry, locality follows from simple Taylor expansion of the action and the diffeomorphism symmetry.

We consider this article as a very significant guide for how the project of Random Dynamics - of deriving all the known physical laws - could be performed in the range from having quantum mechanics and some smoothness assumptions to obtaining a useful spacetime manifold.

Acknowledgement

First we would like to thank Don Bennett who initially was a coauthor of the last piece of this work: the derivation of locality; but then fell ill and could not finish and continue.

One of us (HBN) wants to thank the Niels Bohr Institute for support as an professor emeritus with office and support of the travels for the present work most importantly to Bled where this conference were held. But also Dr. Breskvar is thanked for economical support to this travel.

We also thank for the discussions in Bled with our colleagues.

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Discussion Section

The discussion section is meant to present in the workshop discussed open problems, which might start a collaboration among participants or at least stimulate them to start to think about possible solutions in a different way. Since the time between the workshop and the deadline for the contributions for the proceedings is very short and includes for most of participants also their holidays, it is not so easy to prepare besides their presentation at the workshop also the common contributions to the discussion section. However, the discussions, even if not presented as a contribution to this section, influenced participants' contributions, published in the main section.

This year discussions stressed out several topics, all connected with proposals how to understand and explain the *standard model* assumptions, which were experimentally confirmed so accurately that no help to make a decision for the next step is available. Also talks concerned several proposals for answering the question: What is the origin of families? How many families do we have? How are Yukawa couplings connected with the origin of families? How many dimensions does the space-time manifest? Why is Nature making a choice of the so far observed fermion and boson fields? How is this connected with smallness of the group representations? But the covering discussion topic was: What will the LHC measure in the coming experiments up to 14 TeV? What predictions is common to all the proposals? What do different proposals have in common?

Some answers to these (and other) questions can be found in the talks section. This year discussion section has three contributions only. The one of Valeri Dvoeglazov, who was several times at Bled, but not this year, discussing the connected topic. His contribution concerns the open problem, to which the contribution of this year participants are answering, namely the question of the Weyl, Dirac and Majorana representations. The third contribution concerns the appearance of the dimension of space-time.

All the rest topics, discussed at the workshop but not appearing in this section, will hopefully appear as next year contributions or as talks.

All discussion contributions are arranged alphabetically with respect to the authors' names.

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13 Dirac and Higher-Spin Equations of Negative Energies

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Abstract. It is easy to check that both algebraic equation $\det(\hat{p}-m)=0$ and $\det(\hat{p}+m)=0$ for 4-spinors $u-$ and $v-$ have solutions with $p_0=\pm E_p=\pm\sqrt{\mathbf{p}^2+m^2}$. The same is true for higher-spin equations. Meanwhile, every book considers the $p_0=E_p$ only for both $u-$ and $v-$ spinors of the $(1/2,0)\oplus(0,1/2))$ representation, thus applying the Dirac-Feynman-Stueckelberg procedure for elimination of negative-energy solutions. Recent works of Ziino (and, independently, of several others) show that the Fock space can be doubled. We reconsider this possibility on the quantum field level for both $s=1/2$ and higher spins particles.

Povzetek. Zlahka preverimo, da imata algebrajski enačbi $\det(\hat{p}-m)=0$ in $\det(\hat{p}+m)=0$ za 4-spinorja $u-$ in $v-$ rešitvi za $p_0=\pm E_p=\pm\sqrt{\mathbf{p}^2+m^2}$. Enako velja za enačbe za spinorje z višjimi spini. Vseeno učbeniki obravnavajo samo $p_0=E_p$ za oba spinorja $u-$ in $v-$ upodobitve $(1/2,0)\oplus(0,1/2))$, torej uporabijo postopek Diraca, Feynmana in Stueckelberga za izločitev rešitev z negativnimi energijami. Nedavni članki Ziina (in, neodvisno, nekaterih drugih) kažejo, da lahko Fockov prostor podvojimo. Ponovno obravnavamo to možnost na nivoju kvantnih polj, tako za delce s spinom $s=1/2$ kot za tiste z višjimi spini.

The Dirac equation is:

$$[i\gamma^\mu\partial_\mu - m]\Psi(x)=0\,. \tag{13.1}$$

At least, 3 methods of its derivation exist [1–3]:

- the Dirac one (the Hamiltonian should be linear in $\partial/\partial x^i$, and be compatible with $E_p^2-\mathbf{p}^2c^2=m^2c^4$);
- the Sakurai one (based on the equation $(E_p-\sigma\cdot\mathbf{p})(E_p+\sigma\cdot\mathbf{p})\phi=m^2\phi$);
- the Ryder one (the relation between 2-spinors at rest is $\phi_R(\mathbf{0})=\pm\phi_L(\mathbf{0})$).

The γ^μ are the Clifford algebra matrices

$$\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2g^{\mu\nu}\,. \tag{13.2}$$

Usually, everybody uses the following definition of the field operator [4]:

$$\Psi(x)=\frac{1}{(2\pi)^3}\sum_h\int\frac{d^3\mathbf{p}}{2E_p}[u_h(\mathbf{p})a_h(\mathbf{p})e^{-ip\cdot x}+v_h(\mathbf{p})b_h^\dagger(\mathbf{p})e^{+ip\cdot x}]\,, \tag{13.3}$$

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as given *ab initio*. After introducing $\exp(\mp i p_\mu x^\mu)$ the 4-spinors ($u-$ and $v-$) satisfy the momentum-space equations: $(\hat{p} - m)u_h(p) = 0$ and $(\hat{p} + m)v_h(p) = 0$, respectively; the h is the polarization index. It is easy to prove from the characteristic equations $\det(\hat{p} \mp m) = (p_0^2 - \mathbf{p}^2 - m^2)^2 = 0$ that the solutions should satisfy the energy-momentum relation $p_0 = \pm E_p = \pm\sqrt{\mathbf{p}^2 + m^2}$.

The general scheme of construction of the field operator has been presented in [5]. In the case of the $(1/2, 0) \oplus (0, 1/2)$ representation we have:

$$
\begin{aligned}
\Psi(x) &= \frac{1}{(2\pi)^3}\int d^4p\,\delta(p^2 - m^2)e^{-ip\cdot x}\Psi(p) = \\
&= \frac{1}{(2\pi)^3}\sum_h \int d^4p\,\delta(p_0^2 - E_p^2)e^{-ip\cdot x}u_h(p_0, \mathbf{p})a_h(p_0, \mathbf{p}) = \qquad (13.4)\\
&= \frac{1}{(2\pi)^3}\int \frac{d^4p}{2E_p}[\delta(p_0 - E_p) + \delta(p_0 + E_p)][\theta(p_0) + \theta(-p_0)]e^{-ip\cdot x}\sum_h u_h(p)a_h(p)\\
&= \frac{1}{(2\pi)^3}\sum_h \int \frac{d^4p}{2E_p}[\delta(p_0 - E_p) + \delta(p_0 + E_p)]\Big[\theta(p_0)u_h(p)a_h(p)e^{-ip\cdot x}\\
&\qquad + \theta(p_0)u_h(-p)a_h(-p)e^{+ip\cdot x}\Big]\\
&= \frac{1}{(2\pi)^3}\sum_h \int \frac{d^3\mathbf{p}}{2E_p}\theta(p_0)\Big[u_h(p)a_h(p)|_{p_0=E_p}e^{-i(E_pt - \mathbf{p}\cdot\mathbf{x})}\\
&\qquad + u_h(-p)a_h(-p)|_{p_0=E_p}e^{+i(E_pt - \mathbf{p}\cdot\mathbf{x})}\Big]
\end{aligned}
$$

During the calculations above we had to represent $1 = \theta(p_0) + \theta(-p_0)$ in order to get positive- and negative-frequency parts.[1] Moreover, during these calculations we did not yet assumed, which equation this field operator (namely, the $u(p)$ spinor) satisfies, with negative- or positive- mass?

In general we should transform $u_h(-p)$ to the $v(p)$. The procedure is the following one [7]. In the Dirac case we should assume the following relation in the field operator:

$$\sum_h v_h(p)b_h^\dagger(p) = \sum_h u_h(-p)a_h(-p)\,. \qquad (13.5)$$

We know that [3]

$$\bar{u}_\mu(p)u_\lambda(p) = +m\delta_{\mu\lambda}\,, \qquad (13.6)$$
$$\bar{u}_\mu(p)u_\lambda(-p) = 0\,, \qquad (13.7)$$
$$\bar{v}_\mu(p)v_\lambda(p) = -m\delta_{\mu\lambda}\,, \qquad (13.8)$$
$$\bar{v}_\mu(p)u_\lambda(p) = 0\,, \qquad (13.9)$$

but we need $\Lambda_{\mu\lambda}(p) = \bar{v}_\mu(p)u_\lambda(-p)$. By direct calculations, we find

$$-mb_\mu^\dagger(p) = \sum_\lambda \Lambda_{\mu\lambda}(p)a_\lambda(-p)\,. \qquad (13.10)$$

[1] See [6] for some discussion.

Hence, $\Lambda_{\mu\lambda} = -im(\sigma \cdot \mathbf{n})_{\mu\lambda}$ and

$$b^\dagger_\mu(p) = i \sum_\lambda (\sigma \cdot \mathbf{n})_{\mu\lambda} a_\lambda(-p)\,. \tag{13.11}$$

Multiplying (13.5) by $\bar{u}_\mu(-p)$ we obtain

$$a_\mu(-p) = -i \sum_\lambda (\sigma \cdot \mathbf{n})_{\mu\lambda} b^\dagger_\lambda(p)\,. \tag{13.12}$$

The equations are self-consistent.[2]

However, other ways of thinking are possible. First of all to mention, we have, in fact, $u_h(E_p, \mathbf{p})$ and $u_h(-E_p, \mathbf{p})$ originally, which satisfy the equations:[3]

$$\left[E_p(\pm\gamma^0) - \gamma \cdot \mathbf{p} - m\right] u_h(\pm E_p, \mathbf{p}) = 0\,. \tag{13.14}$$

Due to the properties $U^\dagger \gamma^0 U = -\gamma^0$, $U^\dagger \gamma^i U = +\gamma^i$ with the unitary matrix $U = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \gamma^0\gamma^5$ in the Weyl basis,[4] we have

$$\left[E_p\gamma^0 - \gamma \cdot \mathbf{p} - m\right] U^\dagger u_h(-E_p, \mathbf{p}) = 0\,. \tag{13.15}$$

Thus, unless the unitary transformations do not change the physical content, we have that the negative-energy spinors $\gamma^5\gamma^0 u^-$ (see (13.15)) satisfy the accustomed "positive-energy" Dirac equation. Their explicite forms $\gamma^5\gamma^0 u^-$ are different from the textbook "positive-energy" Dirac spinors. They are the following ones:[5]

$$\tilde{u}(p) = \frac{N}{\sqrt{2m(-E_p + m)}} \begin{pmatrix} -p^+ + m \\ -p_r \\ p^- - m \\ -p_r \end{pmatrix}, \tag{13.16}$$

$$\tilde{\tilde{u}}(p) = \frac{N}{\sqrt{2m(-E_p + m)}} \begin{pmatrix} -p_l \\ -p^- + m \\ -p_l \\ p^+ - m \end{pmatrix}. \tag{13.17}$$

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[2] In the $(1,0) \oplus (0,1)$ representation the similar procedure leads to somewhat different situation:

$$a_\mu(p) = [1 - 2(\mathbf{S} \cdot \mathbf{n})^2]_{\mu\lambda} a_\lambda(-p)\,. \tag{13.13}$$

This signifies that in order to construct the Sankaranarayanan-Good field operator (which was used by Ahluwalia, Johnson and Goldman [Phys. Lett. B (1993)], it satisfies $[\gamma_{\mu\nu}\partial_\mu\partial_\nu - \frac{(i\partial/\partial t)}{E} m^2]\Psi(x) = 0$, we need additional postulates. For instance, one can try to construct the left- and the right-hand side of the field operator separately each other [6].

[3] Remember that, as before, we can always make the substitution $\mathbf{p} \to -\mathbf{p}$ in any of the integrands of (13.4).

[4] The properties of the $U-$ matrix are opposite to those of $P^\dagger\gamma^0 P = +\gamma^0$, $P^\dagger\gamma^i P = -\gamma^i$ with the usual $P = \gamma^0$, thus giving $\left[-E_p\gamma^0 + \gamma \cdot \mathbf{p} - m\right] Pu_h(-E_p, \mathbf{p}) = -\left[\hat{p} + m\right] \tilde{v}_?(E_p, \mathbf{p}) = 0$. While, the relations of the spinors $v_h(E_p, \mathbf{p}) = \gamma_5 u_h(E_p, \mathbf{p})$ are well-known, it seems that the relations of the $v-$ spinors of the positive energy to $u-$ spinors of the negative energy are frequently forgotten, $\tilde{v}_?(E_p, \mathbf{p}) = \gamma^0 u_h(-E_p, \mathbf{p})$.

[5] We use tildes because we do not yet know their polarization properties.

$E_p = \sqrt{\mathbf{p}^2 + m^2} > 0$, $p_0 = \pm E_p$, $p^{\pm} = E \pm p_z$, $p_{r,l} = p_x \pm i p_y$. Their normalization is to $-2N^2$.

What about the $\tilde{v}(p) = \gamma^0 u^-$ transformed with the γ_0 matrix? Are they equal to $v_h(p) = \gamma^5 u_h(p)$? The answer is NO. Obviously, they also do not have well-known forms of the usual $v-$ spinors in the Weyl basis differing by phase factor and in the sign at the mass term (!)

Next, one can prove that the matrix

$$P = e^{i\theta}\gamma^0 = e^{i\theta}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{13.18}$$

can be used in the parity operator as well as in the original Weyl basis. The parity-transformed function $\Psi'(t, -\mathbf{x}) = P\Psi(t, \mathbf{x})$ must satisfy

$$[i\gamma^\mu \partial'_\mu - m]\Psi'(t, -\mathbf{x}) = 0\,, \tag{13.19}$$

with $\partial'_\mu = (\partial/\partial t, -\nabla_i)$. This is possible when $P^{-1}\gamma^0 P = \gamma^0$ and $P^{-1}\gamma^i P = -\gamma^i$. The matrix (13.18) satisfies these requirements, as in the textbook case. However, if we would take the phase factor to be zero we obtain that while $u_h(p)$ have the eigenvalue $+1$, but

$$PR\tilde{u}(p) = PR\gamma^5\gamma^0 u(-E_p, \mathbf{p}) = -\tilde{u}(p)\,, \quad PR\tilde{\tilde{u}}(p) = PR\gamma^5\gamma^0 u(-E_p, \mathbf{p}) = -\tilde{\tilde{u}}(p)\,. \tag{13.20}$$

Perhaps, one should choose the phase factor $\theta = \pi$. Thus, we again confirmed that the relative (particle-antiparticle) intrinsic parity has physical significance only.

Similar formulations have been presented by [8], and [9]. The group-theoretical basis for such doubling has been given in the papers by Gelfand, Tsetlin and Sokolik [10], who first presented the theory in the 2-dimensional representation of the inversion group in 1956 (later called as "the Bargmann-Wightman-Wigner-type quantum field theory" in 1993).

M. Markov wrote long ago *two* Dirac equations with the opposite signs at the mass term [8].

$$[i\gamma^\mu \partial_\mu - m]\,\Psi_1(x) = 0\,, \tag{13.21}$$

$$[i\gamma^\mu \partial_\mu + m]\,\Psi_2(x) = 0\,. \tag{13.22}$$

In fact, he studied all properties of this relativistic quantum model (while he did not know yet the quantum field theory in 1937). Next, he added and subtracted these equations. What did he obtain?

$$i\gamma^\mu \partial_\mu \varphi(x) - m\chi(x) = 0\,, \tag{13.23}$$

$$i\gamma^\mu \partial_\mu \chi(x) - m\varphi(x) = 0\,, \tag{13.24}$$

thus, φ and χ solutions can be presented as some superpositions of the Dirac 4-spinors $u-$ and $v-$. These equations, of course, can be identified with the equations for the Majorana-like $\lambda-$ and $\rho-$ we presented in ref. [11].[6]

$$i\gamma^\mu \partial_\mu \lambda^S(x) - m\rho^A(x) = 0\,, \tag{13.25}$$
$$i\gamma^\mu \partial_\mu \rho^A(x) - m\lambda^S(x) = 0\,, \tag{13.26}$$
$$i\gamma^\mu \partial_\mu \lambda^A(x) + m\rho^S(x) = 0\,, \tag{13.27}$$
$$i\gamma^\mu \partial_\mu \rho^S(x) + m\lambda^A(x) = 0\,. \tag{13.28}$$

Neither of them can be regarded as the Dirac equation. However, they can be written in the 8-component form as follows:

$$[i\Gamma^\mu \partial_\mu - m]\,\Psi_{(+)}(x) = 0\,, \tag{13.29}$$
$$[i\Gamma^\mu \partial_\mu + m]\,\Psi_{(-)}(x) = 0\,, \tag{13.30}$$

with

$$\Psi_{(+)}(x) = \begin{pmatrix} \rho^A(x) \\ \lambda^S(x) \end{pmatrix}, \Psi_{(-)}(x) = \begin{pmatrix} \rho^S(x) \\ \lambda^A(x) \end{pmatrix}, \text{and } \Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix} \tag{13.31}$$

You may say that all this is just related to the basis rotation (unitary transformations). However, in the previous papers I explained: The connection with the Dirac spinors has been found [11,13].[7] For instance,

$$\begin{pmatrix} \lambda^S_\uparrow(\mathbf{p}) \\ \lambda^S_\downarrow(\mathbf{p}) \\ \lambda^A_\uparrow(\mathbf{p}) \\ \lambda^A_\downarrow(\mathbf{p}) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} & i & -1 & i \\ -i & 1 & -i & -1 \\ 1 & -i & -1 & -i \\ i & 1 & i & -1 \end{pmatrix} \begin{pmatrix} u_{+1/2}(\mathbf{p}) \\ u_{-1/2}(\mathbf{p}) \\ v_{+1/2}(\mathbf{p}) \\ v_{-1/2}(\mathbf{p}) \end{pmatrix}. \tag{13.32}$$

Thus, we can see that the two 4-spinor systems are connected by the unitary transformations, and this represents itself the rotation of the spin-parity basis. However, the $\lambda-$ and $\rho-$ spinors describe the neutral particles, meanwhile $u-$ and $v-$ spinors describe the charged particles. Kirchbach [13] found the amplitudes for neutrinoless double beta decay $00\nu\beta$ in this scheme. It is obvious from (13.32) that there are some additional terms comparing with the standard formulation.

One can also re-write the above equations into the two-component form. Thus, one obtains the Feynman-Gell-Mann [12] equations. As Markov wrote himself, he was expecting “new physics” from these equations.

Barut and Ziino [9] proposed yet another model. They considered γ^5 operator as the operator of the charge conjugation. Thus, the charge-conjugated Dirac equation has the different sign comparing with the ordinary formulation:

$$[i\gamma^\mu \partial_\mu + m]\Psi^c_{BZ} = 0\,, \tag{13.33}$$

and the so-defined charge conjugation applies to the whole system, fermion+electromagnetic field, $e \to -e$ in the covariant derivative. The superpositions of the Ψ_{BZ}

[6] Of course, the signs at the mass terms depend on, how do we associate the positive- or negative- frequency solutions with λ and ρ.

[7] I also acknowledge personal communications from D. V. Ahluwalia on these matters.

and Ψ^c_{BZ} also give us the "doubled Dirac equation", as the equations for $\lambda-$ and $\rho-$ spinors. The concept of the doubling of the Fock space has been developed in Ziino works (cf. [10,14]) in the framework of the quantum field theory. In their case the charge conjugate states are simultaneously the eigenstates of the chirality. Next, it is interesting to note that for the Majorana-like field operators we have

$$\left[\nu^{ML}(x^\mu) + \mathcal{C}\nu^{ML\,\dagger}(x^\mu)\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}\sum_\eta \left[\begin{pmatrix} i\Theta\phi^{*\,\eta}_L(p^\mu)\\ 0\end{pmatrix} a_\eta(p^\mu)e^{-ip\cdot x} + \begin{pmatrix} 0\\ \phi^\eta_L(p^\mu)\end{pmatrix} a^\dagger_\eta(p^\mu)e^{ip\cdot x}\right], \tag{13.34}$$

$$\left[\nu^{ML}(x^\mu) - \mathcal{C}\nu^{ML\,\dagger}(x^\mu)\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}\sum_\eta \left[\begin{pmatrix} 0\\ \phi^\eta_L(p^\mu)\end{pmatrix} a_\eta(p^\mu)e^{-ip\cdot x} + \begin{pmatrix} -i\Theta\phi^{*\,\eta}_L(p^\mu)\\ 0\end{pmatrix} a^\dagger_\eta(p^\mu)e^{ip\cdot x}\right], \tag{13.35}$$

which, thus, naturally lead to the Ziino-Barut scheme of massive chiral fields, ref. [9].

Finally, I would like to mention that, in general, in the Weyl basis the $\gamma-$ matrices are *not* Hermitian, $\gamma^{\mu^\dagger} = \gamma^0\gamma^\mu\gamma^0$. The energy-momentum operator $i\partial_\mu$ is obviously Hermitian. So, the question, if the eigenvalues of the Dirac operator (the mass, in fact) would be always real, and the question of the complete system of the eigenvectors of the *non*-Hermitian operator deserve careful consideration [15]. Bogoliubov and Shirkov [5, p.55-56] used the scheme to construct the complete set of solutions of the relativistic equations, fixing the sign of $p_0 = +E_p$.

The conclusion is: the doubling of the Fock space and the corresponding solutions of the Dirac equation got additional mathematical bases in this talk presentation. Similar conclusion can be deduced for the higher-spin equations. I appreciate the discussions with participants of several recent Conferences.

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14 Some Thoughts on the Question of Dimensions

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Abstract. Some issues related to the question of space dimensions are discussed.

Povzetek. Obravnavamo nekatere probleme povezane z vprašanjem razsežnosti prostora.

The question of the number of space dimensions is age-old. The naive assumption, that space has three dimensions, seems justified for many different reasons. One argument is the mere experience of space as 3-dimensional, while a stronger argument is for example that the 4-dimensionality of spacetime is a cornerstone in general relativity, where spacetime is modeled as a 4-manifold. The number of dimensions is however not to be taken for granted, as we know e.g. from Kaluza-Klein, and string theory.

Already in the 1780-ies Immanuel Kant [1] reasoned that there must be three dimensions of space, since 3-dimensional space is due to the inverse square law of universal gravitation. His argument was later inverted, as it is the inverse square law that is explained by 3-dimensional space and Gauss' law, since in a D-dimensional space gravitational or electrostatic force varies like R^{1-D}, where R is the distance between the two bodies/charges.

Some 140 years later, Paul Ehrenfest [2] argued that in a $d = D+1$-dimensional spacetime with $D > 3$, the orbit of a planet around its sun does not remain stable, and likewise for star's orbit around the center of its galaxy. This is because in an even-dimensional space the different parts of a wave travel at different speeds, while for $D > 3$ and odd, the wave impulses become distorted.

About the same time, in 1922, Hermann Weyl [3] stated that Maxwell's theory of electromagnetism only works for $d = 3 + 1$, and this fact *"...not only leads to a deeper understanding of Maxwell's theory, but also of the fact that the world is four dimensional, which has hitherto always been accepted as merely 'accidental,' become intelligible through it."*

In the end of the 1990-ies Max Tegmark [4] pushed the anthropic principle by arguing that spaces with dimension $D < 3$ are too poor in complexity to allow for intelligent beings like ourselves. Like Ehrenfest, he reasoned that for $D > 3$ neither atoms nor planetary systems can be stable, since for $D > 3$, the two-body problem has no stable orbit solution. Similarly, for $D > 3$ the H-atom has no bound states.

The intuition that four dimensions are 'special' is moreover supported by group theory, as many different families of symmetry groups are distinct in 4-dimensional spaces, while in fewer dimensions they are indistinguishable, and in spaces of higher dimensional spaces they do not exist.

$d = 4$ is also singled out in mathematician Simon Donaldson's work on the classification topological four-manifolds from from the early 1980-ies [5], where it was demonstrated that the most complex geometry and topology occur precisely in four dimensions. Donaldson's work focused on 4-manifolds admitting a differentiable structure, using instantons, i.e. self-dual solutions of the Yang-Mills equations. From gauge theory he derived polynomial invariants, new topological invariants sensitive to the underlying smooth structure of a 4-manifold. This made it possible to deduce the existence of exotic 4-manifolds, differentiable 4-dimensional manifolds which are topologically but not differentiably equivalent to the standard Euclidean R^4. This is unique for $d = 4$, in the sense that for $d \neq 4$, there are no exotic smooth structures on R^d.

So within the realm of pure mathematics, Donaldson's work singled out 4-dimensionality, echoing Leibniz' claim [6] that our (3+1-dimensional) world is *"the one which is at the same time the simplest in hypothesis and the richest in phenomena."*

A different type of arguments for $d = 3 + 1$, is based on assigning primacy to the Weyl equation and the stability it displays in four dimensions [7]. The dimensions $4 = 3+1$ have the special property that in these dimensions the number of linearly independent matrices that appear in the Weyl equation is exactly equal to the dimension of spacetime. This is relevant for actual, physical spacetime, because the fermions of our world are basically Weyl particles, in the sense that each Dirac particle is to be considered as (composed of) two Weyl particles. The stability of the Weyl equation moreover means that in four dimensions the equation is stable under addition of extra terms. The Weyl equation has the form

$$i\sigma^{\mu} D_{\mu} \psi = 0 \tag{14.1}$$

and that it is stable means that one can add any smooth, Hermitian operator $\mathcal{O}$ to the operator $i\sigma^{\mu} D_{\mu}$,

$$(i\sigma^{\mu} D_{\mu} + \mathcal{O})\psi = 0 \tag{14.2}$$

and again obtain the Weyl equation in the low energy limit of a Taylor expansion.

Since $4 = 3 + 1$ is the "experimental" number of dimensions, and in 4 dimensions the Weyl equation is stable under small modifications, there seems to be genericness.

14.1 Manifolds

We regard manifolds as fundamental, the ultimate manifold being "experienced space" itself. A manifold is a topological space that is locally Euclidean, while globally it can have a completely different structure.

A differentiable manifold of dimension D locally looks like R^D, so it has open sets, continuous functions and differentiable functions. The structure of a manifold M is studied by mapping M into R^n, and then back to M. So if f_1 and f_2 are two different overlapping mappings from M to R^n, $f_j : M \to R^n, j = 1, 2$, and a transition map is a mapping relating these maps, $T(p) = f_1[f_2^{-1}(p)]$.

The manifold structure is defined by the properties of the transition functions, e.g. if each transition function is a smooth map, the manifold itself is smooth.

The transition map is not well-defined unless both charts are restricted to the intersection of their domains of definition. With differentiable transition functions T, the manifold is differentiable, and functions on the manifold can be differentiated.

In this way a manifold's structure is established by relating it to something well-known, namely real numbers. They at least seem to be well-known, but in a certain sense the reals are just as abstract as any other abstract notions; mathematician Gregory Chaitin [8] goes so far as to claim that *"most individual real numbers are like mythical beasts, like Pegasus or unicorns"*.

His reasoning is that a real number is a number which is measured with arbitrary - infinite - precision. Each point on the number line is a real number, which from a geometrical point of view is seemingly uncomplicated, but arithmetically it is more problematic.

If you want to compute a number, you use an algorithm that you run on a computer, say. The number of computers and algorithms is however countable. Thereby, the number of computable reals is countable.

- The number of computable reals = The number of computer programs = The number of natural numbers = $\aleph_0$
- The number of uncomputable reals = The number of all reals = $\aleph_1$

Since $\aleph_1 \gg \aleph_0$, most reals are not computable. We can refer to them, but not compute them, and in this sense most reals do not 'exist'.

In spite of the fact that we in reality *handle* only a small subset of the real numbers, the real numbers are still well-known, and it is natural to study the structure of some abstract space by mapping it into the realm of reals.

14.1.1 Classification of manifolds

Up to four dimensions, manifolds are classified by geometric structure, while manifolds of higher dimensions are classified algebraically.

Generically, manifolds are classified by their invariants, the most familiar being the manifold *orientability* and the manifold *genus* or the related Euler characteristic χ. That a manifold is orientable basically establishes that it is not a Möbius band, ensuring that helicity or rotation can be unambiguously defined on the manifold.

The intuitively compelling genus g corresponds, loosely speaking, to the number of handles or holes in a manifold, and a compact 2-dimensional manifold is completely characterized by its genus and orientability.

In dimensions $2 < d \leq 4$, the characterizing invariants are the manifold's orientability and its Euler characteristic, closely related to the genus. In 3 dimensions, the Euler characteristic χ of a (convex) polyhedron is the relation

$$\chi = V - E + F = 2, \tag{14.3}$$

where V, E, F are the vertices, edges and faces of the polyhedron. Since a (closed) convex polyhedron is homeomorphic to the sphere S^2, the sphere also has the

Euler characteristic 2, and for a compact, orientable surface the Euler characteristic χ and the genus g are related by

$$\chi = 2 - 2g. \tag{14.4}$$

So while S^2 has Euler characteristic 2, the Euler characteristic of the torus, which has genus $g = 1$, is $\chi = 2 - 2 = 0$.

The classification of manifolds basically concerns whether two manifolds are homeomorphic or not: if the manifold X is homeomorphic to the manifold Y, then $\chi(X) = \chi(Y)$. The classification of smooth (differentiable) closed manifolds is well understood, even though the methods of classification differ for $d < 4$ and $d > 4$. In four dimensions, $d = 4$, the classification is however not possible, except for simply connected manifolds, the reason being that in 4-dimensions manifolds display a much higher degree of complexity than in other dimensions. What Simon Donaldson showed in 1982 was that there is a large class of 4-manifolds which admit no smooth structure at all, and even if there exists a smooth structure it need not be unique. The exotic manifolds, which are homeomorphic but not diffeomorphic to Euclidean R^4, thus single out 4 dimensions as encompassing a maximum of complexity.

14.2 Four dimensions

Locally a manifold resembles 'experienced space', and is naturally easy to intuit. But there are subtleties to keep in mind, for example that a manifold's dimension is actually locally defined. This can be illustrated by a sheet of paper, where the surface of the sheet is 2-dimensional, while the border of the sheet is 1-dimensional. Each connected part of the manifold however has a fixed dimension, i.e. all the points in a connected manifold have the same dimension.

Manifolds moreover display different properties in different dimensions. One way of 'probing' the different dimensions is to study the hypercube in N dimensions.

The 3D cube is characterized by its Euler characteristic $\chi = V - E + F = 2$, where V, F and E are the vertices, edges and faces, thus the 0-dimensional, 1-dimensional and 2-dimensional simplices constituting the cube.

In N dimensions the Euler characteristic of the N-dimensional cube can be expressed as $\chi_N = \sum_{k=0}^{N-1} (-1)^k S_k$, where S_k is the number of k-dimensional simplices, which constitute the N-dimensional cube, $k = 0, \dots, N-1$.

In order to calculate the number of simplices S_k for each dimension k, we start by considering the 3D cube. The 3-cube has 2^3 corners (vertices) V. At each corner (vertex) of the cube there is one vertex and 3 convening edges. Each edge "shares" two vertices with other edges, so the total number of egdes on the cube is $E = 3V/2$.

At each corner of the cube there are moreover 3 convening faces, each face "sharing" four vertices. Thus the total number of faces is $F = 3V/4$. Therefore the Euler characteristic is

$$\chi_{3D} = V[1 - 3/2 + 3/4] = V/4 = 2, \tag{14.5}$$

since V = 2^3 = 8.

We repeat the reasoning for the 4-dimensional cube, the tesseract. At each corner there are 4 orthogonal convening edges, and since each 2-dimensional face is spanned by two orthogonal egdes, the 4 edges span $4x3/2! = 6$ faces, so there are 6 faces at each corner. Each edge "shares" 2 vertices with other edges, and each face "shares" 4 vertices with other faces, so the total number of edges and faces of the tesseract is $4V/2$ and $6V/4$, respectively. In addition, there are the 3D cubes, and each 3-simplex is spanned by 3 orthogonal egdes. From the 4 orthogonal edges that convene at each of the tesseract's corners, we thus get $4x3x2/3! = 4$ three-dimensional cubes, and since each cube "shares" 8 vertices with other 3-cubes, the total number of 3-cubes on the tesseract is 4V/8. The Euler characteristic is then

$$\chi_{4D} = V[1 - 4/2 + 6/4 - 4/8] = 0, \tag{14.6}$$

in agreement with the Euler characteristic being 0 for *any odd* dimensional manifold, since the manifold we are considering is the *surface* of the 4D tesseract, which is 3-dimensional, i.e. odd, just like the surface of the 3D cube is 2-dimensional, and thus has Euler characteristic 2.

Generally speaking, at each corner, or vertex, of the N-cube, there are N convening edges. A k-dimensional simplex is spanned by k of these edges, just like a (2-dimensional) face on the 3D cube is spanned by 2 of the 3 edges that meet at each of the cube's vertices.

Each k-dimensional simplex of the cube moreover has 2^k vertices (to be "shared"), so at each corner of the hypercube, there are

$$\frac{N(N-1)...(N-(k-1))}{k!2^k} = \frac{N!}{(N-k)!k!2^k} \tag{14.7}$$

k-dimensional simplices, which gives the Euler characteristic of a N-cube

$$\chi = 2^N \sum_{k=0}^{N-1} (-1)^k \frac{N!}{(N-k)!k!2^k} \tag{14.8}$$

oscillating between 0 and 2, corresponding to surfaces of odd and even dimension, respectively.

That there are generally valid formulae like the one for the Euler characteristic does however not mean that we can transcribe everything from one dimension to an other. It is for example hard to carry the concept of volume between different dimensions.

One way of looking at it, is to consider the longest straight line that we can draw within a N-dimensional hypercube with edge length l, namely the line which connects two opposite corners and has the length $L = \sqrt{l^2 + l^2 + l^2 + ...} = \sqrt{N}l$.

With the normalization $L = 1$ we get $l = 1/\sqrt{N}$, whereby the volume of the hypercube is $l^N = N^{-N/2}$, which shrinks with the dimension. There is however nothing magical with the higher N, this result is valid in any dimensions. Just compare the diagonals of a square and a 3D-cube:

In 2D we would get that $l_2 = 1/\sqrt{2}$, and in 3D that $l_3 = 1/\sqrt{3}$, the crux is that the normalizing "**1**" is not the same in the two expressions. If we were to carry the normalization in 2D over to 3D, we would actually get: $l_3^{normalized} = \sqrt{3}l_2 = \sqrt{\frac{3}{2}}$.

In conclusion, it is difficult to visualize, or "derive" a picture of one dimension by analogy of another. To go from three to four dimensions is not like going from two to three dimensions, both because we lack phenomenological experience of four dimensions, and also because the degree of complexity is so much higher in four than in three dimensions.

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15 Massless and Massive Representations in the *Spinor Technique*

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Abstract. This contribution uses the technique [1] for representing spinors and the definition of the discrete symmetries [3] to illustrate on a toy model [2] properties of massless and massive solutions of spinors. It might help to solve the problem about representations of Dirac, Weyl and Majorana, presented in the ref. [4] in this proceedings.

Povzetek. Prispevek uporablja tehniko [1] predstavitve spinorjev in definicijo diskretnih simetrij [3] za ilustracijo lastnosti brezmasnih in masivnih rešitev spinorjev v preprostem modelu. Prispeva lahko k rešitvi problema upodobitev Diracovih, Weylovih in Majoraninih spinorjev predstavljenega v prispevku [4] v tem zborniku.

15.1 Introduction

We study in a toy model defined in $d = (5 + 1)$, presented in the refs. [2], massless and massive positive and negative energy solutions of the equations of motion, and look for, by taking into account the definition of the discrete symmetry operators in the second quantized picture ($\mathbb{C}_{\mathcal{N}}$, $\mathcal{P}_{\mathcal{N}}$ and $\mathcal{T}_{\mathcal{N}}$, presented in the paper [3]) the antiparticle states to the particle ones. We present the representations in the *spinor technique* [1].

In this toy model the $\mathcal{M}^{5+1}$ manifold is assumed to break into $\mathcal{M}^{3+1}$ × an almost S^2 sphere due to the zweibein in $d = (5, 6)$.

We first study massless solutions in $d = (3 + 1)$ assuming that the extra dimensions bring no contribution to the masses in $d = (3+1)$. We correspondingly solve the Weyl equation in $d = (5 + 1)$ and present the representations and comment on particle and antiparticle states.

Requiring that there is only one massless of a particular handedness and mass protected solution in $d = (3 + 1)$, what is achieved by a particular choice of the spin connection fields on this (almost) S^2 sphere, what consequently forces the rest of solutions to be massive, we comment on the corresponding particle and antiparticle states. In these two cases the spin in $d = (5, 6)$ is a conserved quantity.

Assuming nonzero vacuum expectation values of the spin connection fields [1–3], the gauge fields of S^{56} with indices $d = (5, 6)$, which manifest as scalars in $d = (3+1)$ and carry the $U(1)$ charge S^{56}, all the spinors become massive and no charge is the conserved quantity any longer. The Weyl equation in $d = (5 + 1)$ manifests

$d = (3+1)$ as the Dirac equation for massive states. We present representations and comment on particle and antiparticle states also in this case.

15.2 Massless solutions

Let us look for the solutions of the Weyl equations $\gamma^a p_a \psi = 0$ in $d = (5+1)$ for a particular choice of the coordinate system: $p^a = (p^0, 0, 0, |p^3|, 0, 0)$. Then the Weyl equations read

$$(-2iS^{03}p^0 = p^3)\psi\,. \tag{15.1}$$

In Table I, taken from the paper [3], the solutions of Eq. 15.1 are presented, using the technique of the refs. [1]. We found for the basic states

$$\begin{aligned}
\Psi_1 &= \overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)}\ |vac>_{fam},\\
\Psi_2 &= \overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]}\ |vac>_{fam},\\
\Psi_3 &= \overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)}\ |vac>_{fam},\\
\Psi_4 &= \overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]}\ |vac>_{fam}\,,
\end{aligned} \tag{15.2}$$

where $|vac>_{fam}$ is defined so that there are $2^{\frac{d}{2}-1}$ family members (this is, however, not a second quantized vacuum). All the basic states are eigenstates of the Cartan subalgebra (of the Lorentz transformation Lie algebra), for which we take: S^{03}, S^{12}, S^{56}, with the eigenvalues, which can be read from Eq. (15.2) if taking $\frac{1}{2}$ times the numbers $\pm i$ or ± 1 in the parentheses of nilpotents $\overset{ab}{(k)}$ and projectors $\overset{ab}{[k]}$: $S^{ab}\overset{56}{(k)}= \frac{k}{2}\overset{ab}{(k)}$, $S^{ab}\overset{56}{[k]}= \frac{k}{2}\overset{ab}{[k]}$.

The first two positive energy solutions ($\psi_i^{\mathbf{pos}}, i = (1,2)$) and the last two negative energy solutions ($\psi_i^{\mathbf{neg}}, i = (3,4)$) correspond to $p^3 = |p^3|$. These all are the solutions of the Weyl equations

$$(\Gamma^{(3+1)}\frac{p^0}{|p^0|} = \frac{2\vec{p}\cdot\vec{S}}{|p^0|})\,\psi\,. \tag{15.3}$$

for the choice $\vec{p} = (p^1, p^2, p^3)$ (in our case is $(0, 0, p^3)$), presented in all text books. Here $\vec{S} = (S^{23}, S^{31}, S^{12})$, $S^{ab} = \frac{i}{2}(\gamma^a\gamma^b - \gamma^b\gamma^a)$, and $\Gamma^{((d-1)+1)}$ (in usual notation is for $d = (3+1)$ named γ^5) determines handedness for fermions in any d. For $d = (5+1)$ $\Gamma^{(5+1)} = \Pi_a\gamma^a$ in ascending order, equal also to $\Gamma^{(3+1)}(-2S^{56})$. For

ψ^{pos}_i	positive energy state	$\frac{p^0}{\lvert p^0\rvert}$	$\frac{p^3}{\lvert p^3\rvert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	S^{56}	$\frac{2p^3 S^{12}}{\lvert p^0\rvert}$
ψ^{pos}_1	$\overset{03}{(+i)}\,\overset{12}{(+)}\mid\overset{56}{(+)}\; e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	$+1$	$+1$	$+1$	$+1$	$\frac{1}{2}$	1
ψ^{pos}_2	$\overset{03}{(+i)}\,\overset{12}{[-]}\mid\overset{56}{[-]}\; e^{-i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	$+1$	$+1$	$+1$	-1	$-\frac{1}{2}$	-1
ψ^{pos}_3	$\overset{03}{[-i]}\,\overset{12}{[-]}\mid\overset{56}{(+)}\; e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	$+1$	-1	-1	$+1$	$\frac{1}{2}$	1
ψ^{pos}_4	$\overset{03}{[-i]}\,\overset{12}{(+)}\mid\overset{56}{[-]}\; e^{-i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	$+1$	-1	-1	-1	$-\frac{1}{2}$	-1
ψ^{neg}_i	negative energy state	$\frac{p^0}{\lvert p^0\rvert}$	$\frac{p^3}{\lvert p^3\rvert}$	$(-2iS^{03})$	$\Gamma^{(3+1)}$	S^{56}	$\frac{2p^3 S^{12}}{\lvert p^0\rvert}$
ψ^{neg}_1	$\overset{03}{(+i)}\,\overset{12}{(+)}\mid\overset{56}{(+)}\; e^{i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	-1	-1	$+1$	$+1$	$\frac{1}{2}$	-1
ψ^{neg}_2	$\overset{03}{(+i)}\,\overset{12}{[-]}\mid\overset{56}{[-]}\; e^{i\lvert p^0\rvert x^0-i\lvert p^3\rvert x^3}$	-1	-1	$+1$	-1	$-\frac{1}{2}$	1
ψ^{neg}_3	$\overset{03}{[-i]}\,\overset{12}{[-]}\mid\overset{56}{(+)}\; e^{i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	-1	$+1$	-1	$+1$	$\frac{1}{2}$	-1
ψ^{neg}_4	$\overset{03}{[-i]}\,\overset{12}{(+)}\mid\overset{56}{[-]}\; e^{i\lvert p^0\rvert x^0+i\lvert p^3\rvert x^3}$	-1	$+1$	-1	-1	$-\frac{1}{2}$	1

Table 15.1. Four positive energy states and four negative energy states, the solutions of Eq. (15.1), half have $\frac{p^3}{|p^3|}$ positive and half negative. $p^a = (p^0, 0, 0, p^3, 0, 0)$, $\Gamma^{(5+1)} = -1$, $\Gamma^{((d-1)+1)}$ defines the handedness in d-dimensional space-time, S^{56} defines the charge in $d = (3+1)$, $\frac{2p^3 S^{12}}{|p^0|}$ defines the helicity. Nilpotents $\overset{ab}{(k)}$ and projectors $\overset{ab}{[k]}$ operate on the vacuum state $|vac>_{fam}$ not written in the table. Table is taken from [3].

the choice $p^a = (p^1, p^2, p^3, 0, 0)$ the solutions read

$$
\begin{aligned}
p^0 &= |p^0|\,,\\
\psi^{pos}_1(\vec{p}) &= \mathcal{N}_1\left(\overset{03}{(+i)}\,\overset{12}{(+)}\mid\overset{56}{(+)} + \frac{p^1+ip^2}{|p^0|+p^3}\,\overset{03}{[-i]}\,\overset{12}{[-]}\mid\overset{56}{(+)}\right) e^{-i(|p^0|x^0-\vec{p}\cdot\vec{x})}\,,\\
\psi^{pos}_2(\vec{p}) &= \mathcal{N}_2\left(\overset{03}{[-i]}\,\overset{12}{(+)}\mid\overset{56}{[-]} - \frac{p^1+ip^2}{|p^0|-p^3}\,\overset{03}{(+i)}\,\overset{12}{[-]}\mid\overset{56}{[-]}\right) e^{-i(|p^0|x^0-\vec{p}\cdot\vec{x})}\,,\\
p^0 &= -|p^0|\,,\\
\psi^{neg}_1(\vec{p}) &= \mathcal{N}_2\left(\overset{03}{(+i)}\,\overset{12}{(+)}\mid\overset{56}{(+)} - \frac{p^1+ip^2}{|p^0|-p^3}\,\overset{03}{[-i]}\,\overset{12}{[-]}\mid\overset{56}{(+)}\right) e^{i(|p^0|x^0+\vec{p}\cdot\vec{x})}\,,\\
\psi^{neg}_2(\vec{p}) &= \mathcal{N}_1\left(\overset{03}{[-i]}\,\overset{12}{(+)}\mid\overset{56}{[-]} + \frac{p^1+ip^2}{|p^0|+p^3}\,\overset{03}{(+i)}\,\overset{12}{[-]}\mid\overset{56}{[-]}\right) e^{i(|p^0|x^0+\vec{p}\cdot\vec{x})}\,,
\end{aligned}
\tag{15.4}
$$

These solutions are obviously possible only if both kinds of "charges" in $d = (3+1)$ are allowed for particles and antiparticles and are not mass protected [2]. We shall see in sect. 15.4 that the first two states in Table 15.1 (ψ^{pos}_1 and ψ^{pos}_2), put on the top of the Dirac sea, describe a particle state, while the second two (ψ^{pos}_4 and ψ^{pos}_3, respectively) are the corresponding antiparticle states put on the top of the Dirac sea. The antiparticle states are obtained by emptying the negative energy states (ψ^{neg}_3 and ψ^{neg}_4, respectively).

For a particular choice of the spin connection one gets (normalizable) massless solutions of only one handedness and one charge and mass protected [2], say: the right handed ones of the "charge" $S^{56} = \frac{1}{2}$. In Table 15.1 is this state presented by a particle state ψ_1^{pos} put on the top of the Dirac sea. Its antiparticle state is ψ_4^{pos} put on the top of the Dirac sea. It is obtained by emptying the negative energy solution ψ_3^{neg}.

15.3 Massive solutions

To find the massive states we must solve the Weyl equation in which we allow the scalar fields, the gauge fields of S^{56}, that is $f^{\sigma}_{s}\omega_{56\sigma}$, with $s = (5,6)$ and $\sigma = ((5),(6))$, to have non zero vacuum expectation values. These scalar fields are then, analogously as there is the Higgs scalar in the *standard model* but carrying in our case only the "hyper" charge S^{56}, responsible for the spinors masses. The charge, which is the spin in $d = (5,6)$, is now not a conserved quantity any longer.

The Weyl equation in $(5+1)$, leading to the massive Dirac equation in $d = (3+1)$, reads [2]

$$(\gamma^m p_m + \overset{56}{(+)}\, p_{0+} + \overset{56}{(-)}\, p_{0-})\psi = 0\,,$$
$$p_{0\pm} = p^5_0 \mp p^6_0\,, \quad p_{0s} = f^{\sigma}_{s}(p_\sigma - \frac{1}{2} S^{ab}\omega_{ab\sigma}) = p_s - S^{56} i\omega_{56s}\,. \tag{15.5}$$

Looking for the solution of Eq. (15.5) by making superposition of states of a particular spin one observes that the term $< p_{0\pm} >= -S^{56} i < (f^{\sigma}_{5} \mp i f^{\sigma}_{6})\omega_{56\sigma} >$ causes on the tree level the mass m of spinors: $< p_{0\pm} >= -iS^{56} 2m$. Solutions of the Weyl equation in $d = (5+1)$ manifest in $d = (3+1)$ as massive states with the mass $2m =< \omega_{56+} >=< \omega_{56-} >$. Let us make a choice of the coordinate system so that $p^a = (p^0, 0, 0, 0, 0, 0)$. One obtains two positive and two negative energy solutions

$$\begin{aligned}
\psi^{pos}_{1\,m} &= \mathcal{N}(\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)} - i\,\overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]})\, e^{-imx^0}\,,\\
\psi^{pos}_{2\,m} &= \mathcal{N}(\overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)} - i\,\overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]})\, e^{-imx^0}\,,\\
\psi^{neg}_{1\,m} &= \mathcal{N}(\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)} + i\,\overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]})\, e^{imx^0}\,,\\
\psi^{neg}_{2\,m} &= \mathcal{N}(\overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)} + i\,\overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]})\, e^{imx^0}\,,
\end{aligned} \tag{15.6}$$

with $m^2 = (p^0)^2$, $m = 2 < (f^{\sigma}_{5} \mp i f^{\sigma}_{6})\omega_{56\sigma} >$ [1]. (To obtain true masses of spinors one must take into account loop corrections in all orders, to which also the dynamical scalar and vector gauge fields contribute.) In this discussion only one family is assumed.

Let us present massive positive and negative energy solutions, the ones which coincide with vectors $\begin{pmatrix} \varphi \\ \frac{\vec{\sigma}\cdot\vec{p}}{|p^0|+m}\varphi \end{pmatrix}$, $\varphi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, and $\begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{|p^0|+m}\chi \\ \chi \end{pmatrix}$, $\chi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, with

---

[1] One sees that the other two superposition, $\psi^{pos}_3 = \mathcal{N}(\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)} + i\,\overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]})\; e^{-imx^0}$ and $\psi^{pos}_4 = \mathcal{N}(\overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)} + i\,\overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]})\; e^{-imx^0}$ are not the solutions of the Weyl equation and so are not also $\psi^{neg}_3 = \mathcal{N}(\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)} - i\,\overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]})\; e^{imx^0}$ and $\psi^{neg}_4 = \mathcal{N}(\overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)} - i\,\overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]})\; e^{imx^0}$.

$\vec{\sigma} = (S^{23}, S^{31}, S^{12})$, in the usual notation for any $p^m = (p^0, p^1, p^2, p^3)$

$$
\begin{aligned}
\psi^{pos}_{1\,m}(\vec{p}) &= \mathcal{N}(\vec{p}, m)\{\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)} - i\frac{|p^0| - p^3 + m}{|p^0| + p^3 + m}\;\overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]} \\
&+ \frac{p^1 + ip^2}{|p^0| + p^3 + m}\,(\overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)} + i\;\overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]})\} \cdot e^{-i(|p^0| - i\vec{p}\cdot\vec{x})} \\
\psi^{pos}_{2\,m}(\vec{p}) &= \mathcal{N}(\vec{p}, m)\{\frac{p^1 - ip^2}{|p^0| + p^3 + m}\,(\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)} + i\;\overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]}) \\
&+ \frac{p^0 - p^3 + m}{|p^0| + p^3 + m}\cdot\;\overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)} - i\;\overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]}\} \cdot e^{-i(|p^0| - i\vec{p}\cdot\vec{x})}\,, \\
\psi^{neg}_{1\,m}(\vec{p}) &= \mathcal{N}(\vec{p}, m)\{\frac{p^1 + ip^2}{|p^0| + p^3 + m}\,(\overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)} - i\;\overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]}) \\
&+ \frac{-p^0 + p^3 - m}{|p^0| + p^3 + m}\;\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)} - i\;\overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]}\} \cdot e^{i(|p^0| + \vec{p}\cdot\vec{x})}\,, \\
\psi^{neg}_{2\,m}(\vec{p}) &= \mathcal{N}(\vec{p}, m)\{\overset{03}{[-i]}\overset{12}{[-]}\overset{56}{(+)} + i\frac{|p^0| - p^3 + m}{|p^0| + p^3 + m}\;\overset{03}{(+i)}\overset{12}{[-]}\overset{56}{[-]} \\
&+ \frac{p^1 - ip^2}{|p^0| + p^3 + m}\,(\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)} - i\;\overset{03}{[-i]}\overset{12}{(+)}\overset{56}{[-]})\} \cdot e^{i(|p^0| + \vec{p}\cdot\vec{x})}\,. \qquad (15.7)
\end{aligned}
$$

The antiparticle states to the particle state $\psi^{pos}_{i\,m}(\vec{p})$, $i = (1, 2)$, put on the top of the Dirac sea are the two states $\psi^{pos}_{i\,m}(-\vec{p})$, $i = (1, 2)$, respectively, put on the top of the Dirac sea.

15.4 Second quantized solutions

Let us now pay attention on the second quantized picture, discussions of which we already started in the above two sections. Following the proposal from the ref. [3] the discrete symmetries are in cases of the Kaluza-Klein kind for d even, as it is in our toy model, defined as follows

$$
\begin{aligned}
\mathcal{C}_{\mathcal{N}}\,\psi(x^0, \vec{x}) &= \Gamma^{(3+1)}\,\gamma^2\,K\,\psi(x^0, x^1, x^2, x^3, x^5, -x^6, x^7, -x^8, \dots, x^{d-1}, -x^d) \\
&= \Gamma^{(3+1)}\,\gamma^2\,K\,I_{6,8,\cdots,d}\,\psi(x^0, \vec{x})\,, \\
\mathcal{T}_{\mathcal{N}}\,\psi(x^0, \vec{x}) &= \Gamma^{(3+1)}\,\gamma^1\,\gamma^3\,K\,\psi(-x^0, x^1, x^2, x^3, -x^5, x^6, -x^7, \dots, -x^{d-1}, x^d) \\
&= \Gamma^{(3+1)}\,\gamma^1\,\gamma^3\,K\,I_{x^0}\,I_{5,7,\cdots,d-1}\,\psi(x^0, \vec{x})\,, \\
\mathcal{P}^{d-1}_{\mathcal{N}}\,\psi(x^0, \vec{x}) &= \gamma^0\,\Gamma^{(3+1)}\,\Gamma^{(d)}\,\psi(x^0, -x^1, -x^2, -x^3, x^5, x^6, \dots, x^{d-1}, x^d) \\
&= \gamma^0\,\Gamma^{(3+1)}\,\Gamma^{(d)}\,I_{\vec{x}_3}\,\psi(x^0, \vec{x})\,. \qquad (15.8)
\end{aligned}
$$

$I_{\vec{x}_3}$ reflects (x^1, x^2, x^3), $I_{6,8,\cdots,d}$ reflects $(x^6, x^8, \cdots, x^d)$, I_{x^0} reflects the time component x^0 and $I_{5,7,\cdots,d-1}$ reflects $(x^5, x^7, \cdots, x^{d-1})$. It is $\mathcal{C}_{\mathcal{N}} \cdot \mathcal{P}^{d-1}_{\mathcal{N}}$, which transforms in our case positive energy states into the corresponding negative energy states, staying within the same Weyl.

One finds

$$
\begin{aligned}
\{\mathcal{C}_{\mathcal{N}}, \gamma^a p_a\}_+ &= 0\,,\\
\{\mathcal{P}_{\mathcal{N}}, \gamma^a p_a\}_- &= 0\,,\\
\{\mathcal{T}_{\mathcal{N}}, \gamma^a p_a\}_+ &= 0\,,\\
\{\mathcal{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}, \gamma^a p_a\}_+ &= 0\,,\\
\{\mathcal{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}\,\mathcal{T}_{\mathcal{N}}, \gamma^a p_a\}_- &= 0\,.
\end{aligned} \tag{15.9}
$$

In even dimensional spaces namely neither $\mathcal{C}_{\mathcal{N}}$ nor $\mathcal{P}^{d-1}_{\mathcal{N}}$ transforms states within the same Weyl representation, it is only $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}^{d-1}_{\mathcal{N}}$, which does this and it is correspondingly a good symmetry, keeping the states within one Weyl representation.

To obtain an antiparticle state to a chosen particle state above the Dirac sea we must accordingly apply on a particle state the operator $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}^{d-1}_{\mathcal{N}}$ and then empty the obtained negative energy state. Emptying the corresponding negative energy state and putting it on the top of the Dirac sea determines the antiparticle state to the starting particle state.

In the ref. [3] the second quantized charge conjugation operator $\mathbb{C}_{\mathcal{N}}$ is defined as follows: First one applies on the particle state with positive energy put on the top of the Dirac sea, the operator $\mathcal{C}_{\mathcal{N}}$, which makes a choice of the corresponding negative energy state. Then by *emptying this negative energy state in the Dirac sea one creates an antiparticle with the positive energy and all the properties of the starting single particle state above the Dirac sea, that is with the same* d*-momentum and all the spin degrees of freedom the same, except the* S^{03} *value*, as the starting single particle state [2].

We make now a statement. It is the operator

$$
\mathtt{emptying} := \prod_{\gamma^a \in \Im} \gamma^a\ K\,\Gamma^{(3+1)}\,, \tag{15.10}
$$

operating on the negative energy state, which *empties the negative energy state* creating the antiparticle state above the Dirac sea.

Let us check on the massless states of Eq. (15.4) what we claimed:
First $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ applied on $\psi^{pos}_1(\vec{p})$ (this state is put on the top of the Dirac sea) transforms this state into the negative energy state $\psi^{neg}_1(\vec{p})$, then $\prod_{\gamma^a\in\Im}\gamma^a\ K\,\Gamma^{(3+1)}$ applied on $\psi^{\mathbf{neg}}{}_1(\vec{p})$ transforms this state into the positive energy antiparticle state $\psi^{pos}_2(-\vec{p}) = \mathcal{N}_2\left(\overset{03}{[-i]}\,\overset{12}{(+)}\mid\overset{56}{[-]} + \frac{p^1+ip^2}{|p^0|+p^3}\,\overset{03}{(+i)}\,\overset{12}{[-]}\mid\overset{56}{[-]}\right)\, e^{-i(|p^0|x^0+\vec{p}\cdot\vec{x})}$, (put on the top of the Dirac sea), up to phase factors, what can easily be checked. All these states, particle and antiparticle ones, are the solutions of the massless Weyl equation in $d=(3+1)$ (with $p^a=(p^0,p^1,p^2,p^3,0,0)$.

The two massless particle/antiparticle pairs are therefore $(\psi^{pos}_1(\vec{p}), \psi^{pos}_2(-\vec{p}))$ and $(\psi^{pos}_2(\vec{p}), \psi^{pos}_1(-\vec{p}))$.

For $p^a=(p^0,0,0,p^3,0,0)$ we find that the two particle/antiparticle pairs are correspondingly ($\psi^{\mathbf{pos}}_{\mathbf{1}}$ and ψ^{pos}_4) and ($\psi^{\mathbf{pos}}_{\mathbf{2}}$ and ψ^{pos}_3), presented in (Table 15.1).

[2] S^{03} is involved in the boost (contributing in $d=(3+1)$, together with the spin, to handedness) and does not determine the (ordinary) spin.

One checks this by applying the operator $\mathcal{C}_{\mathcal{N}}\,\mathcal{P}^{d-1}_{\mathcal{N}}$ on the state $\psi^{\mathbf{pos}}_{\mathbf{1}}$, transforming this state into the state $\psi^{\mathbf{neg}}_{\mathbf{3}}$, while $emptying$ this negative state generates $\psi^{\mathbf{pos}}_{\mathbf{4}}$. Similarly $\mathcal{C}_{\mathcal{N}}\,\mathcal{P}^{d-1}_{\mathcal{N}}\;\psi^{\mathbf{pos}}_{\mathbf{2}}$ into $\psi^{\mathbf{neg}}_{\mathbf{4}}$, while $emptying$ this negative state generates $\psi^{\mathbf{pos}}_{\mathbf{3}}$.

If only one massless solution, let say the right handed one with respect to $d = (3+1)$, is allowed, as in the case presented in the ref. [2], then the only allowed particle/antiparticle pair is $(\psi^{\mathbf{pos}}_{\mathbf{1}}, \psi^{pos}_{4})$.

Before discussing the discrete symmetries of the massive states presented in Eq. (15.5) let us pay attention that $K \stackrel{(56)}{(+)} = -\stackrel{(56)}{(+)}$, since $\stackrel{(56)}{(+)} = \frac{1}{2}(\gamma^5 + i\gamma^6)$ and we make a choice of γ^0, γ^1 real, γ^2 imaginary, γ^3 real, γ^5 imaginary, γ^6 real, and alternating real and imaginary ones we end up in even dimensional spaces with real γ^d. K makes complex conjugation, transforming i into $-i$.

Let us look at discrete symmetries of the massive solutions $\psi^{pos}_{i\,m}(\vec{p}=0)\,, i = 1, 2$. We see that $\mathcal{C}_{\mathcal{N}}\;\mathcal{P}_{\mathcal{N}}$ applied on $\psi^{pos}_{1\,m}$ (Eq. (15.6)), put on the top of the Dirac sea, transforms this state into the negative energy state $\psi^{neg}_{4\,m}$. Emptying this negative energy state, that is applying $\prod_{\gamma^a \in \Im} \gamma^a\;\; K\,\Gamma^{(3+1)}$ on $\psi^{neg}_{4\,m}$, makes the antiparticle state $\psi^{pos}_{1\,m}$ on the top of the Dirac sea. This state does not distinguish from the starting particle state. Massive states have no conserved charge and correspondingly are the particle and antiparticle solutions of the massive Weyl equation for $p^m = (m, 0, 0, 0)$ indistinguishable.

For the general momentum $p^m = (p^0, p^1, p^2, p^3)$ the state $\psi^{neg}_{2\,m}(\vec{p})$ in Eq. (15.7) follows if we apply the discrete operator $\mathcal{C}_{\mathcal{N}}\;\mathcal{P}_{\mathcal{N}}$ on the state $\psi^{pos}_{1\,m}(\vec{p})$. Emptying the state $\psi^{neg}_{2\,m}(\vec{p})$ leads to the antiparticle state above the Dirac sea, which is $\psi^{pos}_{1\,m}(-\vec{p})$. Let us remind the reader again that in this model no charge is the conserved one.

Let us say that the product of the operations $emptying \times \mathcal{C}_{\mathcal{N}}\;\mathcal{P}_{\mathcal{N}}$ leads to $\gamma^0\gamma^5\Gamma^{(3+1)}\, I_{\vec{x}_3}\, I_6 \Gamma^{(6)}$ in our $d = (5+1)$ model. In general even d case we have

$$emptying \times \mathcal{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}} = \gamma^0 \prod_{\gamma^a \in \Im, a \neq 2} \gamma^a\,\Gamma^{(3+1)}\, I_{\vec{x}_3}\, I_{6,8,\ldots,d}\Gamma^{(d)}\,. \qquad (15.11)$$

15.5 Conclusions

We discussed in this contribution representations of the Weyl equation in $d = (5+1)$, manifesting as: **i.** massless, **ii.** massive Dirac equation in $d = (3+1)$ in dependence on whether the fields, which manifest in $d = (3+1)$ as the scalar fields, not gain or gain, respectively, nonzero vacuum expectation values which causes the appearance of fermion masses. Using the technique from the ref. [1] for representing spinors, we present the basis and solutions of the equations of motion for the massless and massive states.

We also present the particle/antiparticle pairs in the second quantized picture, using the definition of the discrete symmetries from the ref. [3]. For general $p^a = (p^0, \vec{p}, 0, 0)$ the particle and antiparticle solutions are distinguished by their conserved charges. If no charge is conserved, then particle and the corresponding antiparticle state differ only by having opposite three momentum in $d + (3+1)$ space.

We conclude that *there are in all the cases two particle and two antiparticle states with the positive energy and the corresponding four states in the Dirac sea*. All these states solve the equations of motion, the Weyl or the Dirac ones, in $d = (3+1)$. Not necessarily are all of them either massless or massive. It can happen that under special conditions the number of massless solutions reduces, while the rest of representations belong to massive sector. The number of states enlarges, if spinors carry additional quantum numbers, which are or are not the conserved quantities.

Presenting the spinor basis in all these cases helps to understand what is happening with the degrees of freedom in massless case and after the scalar fields bring masses to the spinors (like the Higgs of the *standard model*.

We shall study in another paper what one of us calls the realistic case, that is the representations of massless and massive states of the *spin-charge-family* theory of N.S.M.B., before and after particular scalar fields cause that spinors manifest masses in $d = (3+1)$.

This might help to clarify the open problems which V. Dvoeglazov put in his contribution in this proceedings.

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Virtual Institute of Astroparticle Physics Presentation

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16 Virtual Institute of Astroparticle Physics in Online Discussion of Physics Beyond the Standard Model

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Abstract. Virtual Institute of Astroparticle Physics (VIA), integrated in the structure of Laboratory of AstroParticle physics and Cosmology (APC) is evolved in a unique multi-functional complex of science and education online. It supports participation in conferences and meetings, various forms of collaborative scientific work as well as programs of education at distance. The activity of VIA takes place on its website `http://viavca.in2p3.fr/site.html`. The format of VIA videoconferences was effectively used in the program of XVI Bled Workshop to provide a world-wide participation at distance in discussion of the open questions of physics beyond the standard model. The VIA system has demonstrated its high quality and stability for participation in discussions from different parts of the world without any technical assistance at place.

Povzetek. Virtual Institute of Astroparticle Physics (VIA), vključen v sestavo Laboratory of AstroParticle physics and Cosmology (APC) v Parizu, se je razvil v edinstven vsestranski kompleks za znanost in izobraževanje na spletu. Podpira sodelovanje na konferencah in sestankih, različne oblike skupnega znanstvenega dela in programe za izobraževanje na daljavo. Aktivnosti instituta VIA se odvijajo na njegovi spletni strani `http://viavca.in2p3.fr/site.html`. Oblika video konferenc, kot jih organizira VIA, je bila učinkovito uporabljena v sklopu programa 16. blejske delavnice za omogočanje sodelovanja na daljavo iz vsega sveta v diskusijah odprtih vprašanj fizike onkraj standardnega modela. Sistem instituta VIA je dokazal, da lahko omogoča visoko kvaliteto in stabilnost prisotnosti udeležencev iz različnih delov sveta v diskusijah, brez zahteve po tehnični pomoči na kraju samem.

16.1 Introduction

Studies in astroparticle physics link astrophysics, cosmology, particle and nuclear physics and involve hundreds of scientific groups linked by regional networks (like ASPERA/ApPEC [1,2]) and national centers. The exciting progress in these studies will have impact on the knowledge on the structure of microworld and Universe in their fundamental relationship and on the basic, still unknown, physical laws of Nature (see e.g. [3,4] for review).

Virtual Institute of Astroparticle Physics (VIA) [5] was organized with the aim to play the role of an unifying and coordinating structure for astroparticle physics. Starting from the January of 2008 the activity of the Institute takes place on its website [6] in a form of regular weekly videoconferences with VIA lectures, covering all the theoretical and experimental activities in astroparticle physics and related topics. The library of records of these lectures, talks and their presentations was accomplished by multi-lingual Forum. In 2008 VIA complex was effectively used for the first time for participation at distance in XI Bled Workshop [7]. Since then VIA videoconferences became a natural part of Bled Workshops' programs, opening the virtual room of discussions to the world-wide audience. Its progress was presented in [8–11]. Here the current state-of-art of VIA complex, integrated since the end of 2009 in the structure of APC Laboratory, is presented in order to clarify the way in which VIA discussion of open questions beyond the standard model took place in the framework of XVI Bled Workshop.

16.2 The structure of VIA complex and forms of its activity

16.2.1 The forms of VIA activity

The structure of VIA complex is illustrated on Fig. 16.1. The home page, presented

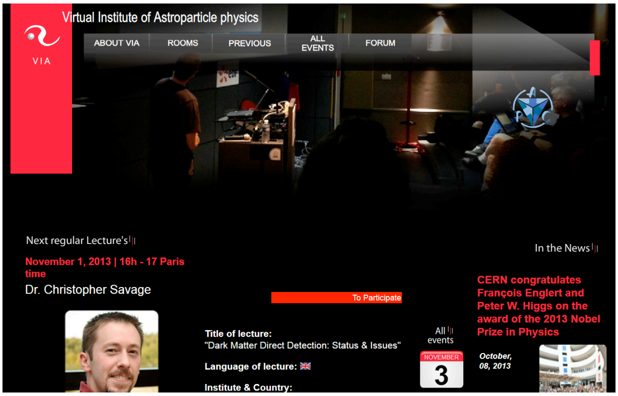

Fig. 16.1. The home page of VIA site.

on this figure, contains the information on VIA activity and menu, linking to directories (along the upper line from left to right): with general information on VIA (About VIA), entrance to VIA virtual rooms (Rooms), the library of records and presentations (Previous) of VIA Lectures (Previous → Lectures), records of online transmissions of Conferences(Previous → Conferences), APC Colloquiums (Previous → APC Colloquiums), APC Seminars (Previous → APC Seminars) and Events (Previous → Events), Calender of the past and future VIA events (All events) and VIA Forum (Forum). In the upper right angle there are links to Google search engine (Search in site) and to contact information (Contacts).

The announcement of the next VIA lecture and VIA online transmission of APC Colloquium occupy the main part of the homepage with the record of the most recent VIA events below. In the announced time of the event (VIA lecture or transmitted APC Colloquium) it is sufficient to click on ”to participate” on the announcement and to Enter as Guest (printing your name) in the corresponding Virtual room. The Calender links to the program of future VIA lectures and events. The right column on the VIA homepage lists the announcements of the regularly up-dated hot news of Astroparticle physics and related areas.

In 2010 special COSMOVIA tours were undertaken in Switzerland (Geneva), Belgium (Brussels, Liege) and Italy (Turin, Pisa, Bari, Lecce) in order to test stability of VIA online transmissions from different parts of Europe. Positive results of these tests have proved the stability of VIA system and stimulated this practice at XIII Bled Workshop. These tours involved special equipment, including, in particular, the use of the sensitive audio system KONFTEL 300W [12]. The records of the videoconferences at the XIII Bled Workshop are available on VIA site [13].

Sice 2011 VIA facility is used for the tasks of the Paris Center of Cosmological Physics (PCCP), chaired by G. Smoot and for the public programme ”The two infinities” conveyed by J.L.Robert. It regularly effectively supports participation at distance at meetings of the Double Chooz collaboration: the experimentalists, being at shift, took part in the collaboration meeting in such a virtual way.

The simplicity of VIA facility for ordinary users was demonstrated at XIV Bled Workshop. Videoconferences at this Workshop had no special technical support except for WiFi Internet connection and ordinary laptops with their internal video and audio equipments. This test has proved the ability to use VIA facility at any place with at least decent Internet connection. Of course the quality of records is not as good in this case as with the use of special equipment, but still it is sufficient to support fruitful scientific discussion as can be illustrated by the record of VIA presentation ”New physics and its experimental probes” given by John Ellis from his office in CERN (see the records in [14]).

In 2012 VIA facility, regularly used for programs of VIA lectures and transmission of APC Colloquiums, has extended its applications to support M.Khlopov’s talk at distance at Astrophysics seminar in Moscow, videoconference in PCCP, participation at distance in APC-Hamburg-Oxford network meeting as well as to provide online transmissions from the lectures at Science Festival 2012 in University Paris7. VIA communication has effectively resolved the problem of referee’s attendance at the defence of PhD thesis by Mariana Vargas in APC. The referees made their reports and participated in discussion in the regime of VIA videoconference.

In 2013 VIA lecture by Prof. Martin Pohl was one of the first places at which the first hand information on the first results of AMS02 experiment was presented [15].

In 2012 VIA facility was first used for online transmissions from the Science Festival in the University Paris 7. This tradition was continued in 2013, when the transmissions of meetings at Journes nationales du Dveloppement Logiciel (JDEV2013) at Ecole Politechnique (Paris) were organized [16].

The discussion of questions that were put forward in the interactive VIA events can be continued and extended on VIA Forum. The Forum is intended to cover the topics: beyond the standard model, astroparticle physics, cosmology, gravitational wave experiments, astrophysics, neutrinos. Presently activated in English,French and Russian with trivial extension to other languages, the Forum represents a first step on the way to multi-lingual character of VIA complex and its activity.

One of the interesting forms of Forum activity is the educational work at distance. For the last four years M.Khlopov's course "Introduction to cosmoparticle physics" is given in the form of VIA videoconferences and the records of these lectures and their ppt presentations are put in the corresponding directory of the Forum [17]. Having attended the VIA course of lectures in order to be admitted to exam students should put on Forum a post with their small thesis. Professor's comments and proposed corrections are put in a Post reply so that students should continuously present on Forum improved versions of work until it is accepted as satisfactory. Then they are admitted to pass their exam. The record of videoconference with their oral exam is also put in the corresponding directory of Forum. Such procedure provides completely transparent way of evaluation of students' knowledge.

16.2.2 Organisation of VIA events and meetings

First tests of VIA system, described in [5,7–9], involved various systems of videoconferencing. They included skype, VRVS, EVO, WEBEX, marratech and adobe Connect. In the result of these tests the adobe Connect system was chosen and properly acquired. Its advantages are: relatively easy use for participants, a possibility to make presentation in a video contact between presenter and audience, a possibility to make high quality records, to use a whiteboard facility for discussions, the option to open desktop and to work online with texts in any format. The regular form of VIA meetings assumes that their time and Virtual room are announced in advance. Since the access to the Virtual room is strictly controlled by administration, the invited participants should enter the Room as Guests, typing their names, and their entrance and successive ability to use video and audio system is authorized by the Host of the meeting. The normal amount of connections to the virtual room at VIA lectures and discussions usually didn't exceed 20. However, the sensational character of the exciting news on superluminal propagation of neutrinos acquired the number of participants, exceeding this allowed upper limit at the talk "OPERA versus Maxwell and Einstein" given by John Ellis from CERN. The complete record of this talk and is available on VIA website [18]. For the first time the problem of necessity in extension of this limit was put forward and it was resolved by creation of a virtual "infinity room", which can host any reasonable amount of participants. Starting from 2013 this room became the only main virtual VIA room, but for specific events, like Collaboration meetings or transmissions from science festivals, special virtual rooms can be created.

The ppt or pdf file of presentation is uploaded in the system in advance and then demonstrated in the central window. Video images of presenter and participants appear in the right window, while in the upper left window the list of all the attendees is given. To protect the quality of sound and record, the participants are required to switch out their microphones during presentation and to use lower left Chat window for immediate comments and urgent questions. The Chat window can be also used by participants, having no microphone, for questions and comments during Discussion. The interactive form of VIA lectures provides oral discussion, comments and questions during the lecture. Participant should use in this case a "raise hand" option, so that presenter gets signal to switch our his microphone and let the participant to speak. In the end of presentation the central window can be used for a whiteboard utility as well as the whole structure of windows can be changed, e.g. by making full screen the window with the images of participants of discussion.

Regular activity of VIA as a part of APC includes online transmissions of all the APC Colloquiums and of some topical APC Seminars, which may be of interest for a wide audience. Online transmissions are arranged in the manner, most convenient for presenters, prepared to give their talk in the conference room in a normal way, projecting slides from their laptop on the screen. Having uploaded in advance these slides in the VIA system, VIA operator, sitting in the conference room, changes them following presenter, directing simultaneously webcam on the presenter and the audience.

16.3 VIA Sessions at XVI Bled Workshop

VIA sessions of XVI Bled Workshop have developed from the first experience at XI Bled Workshop [7] and their more regular practice at XII, XIII, XIV and XV Bled Workshops [8–11]. They became a regular part of the Bled Workshop's programme.

In the course of XVI Bled Workshop meeting the list of open questions was stipulated, which was proposed for wide discussion with the use of VIA facility. The list of these questions was put on VIA Forum (see [19]) and all the participants of VIA sessions were invited to address them during VIA discussions. During the XVI Bled Workshop the test of not only minimal necessary equipment, but either of the use of VIA facility by ordinary users was undertaken. VIA Sessions were supported by personal laptop with WiFi Internet connection only, as well as in 2012 the members of VIA team were physically absent in Bled and all the videoconferences were directed by M.Khlopov and assisted by D.Rouable at distance from Paris. It proved the possibility to provide effective interactive online VIA videoconferences even in the absence of any special equipment and qualified personnel at place. Only laptop with microphone and webcam together with WiFi Internet connection was proved to be sufficient not only for attendance, but also for VIA presentations and discussions.

In the framework of the program of XVI Bled Workshop, R. Cerulli, staying in his office in LNGS, Gran Sasso gave his talk "DAMA/LIBRA results and perspectives" (Fig. 16.2) and took part in the discussion of puzzles of dark matter searches, which provided a brilliant demonstration of the interactivity of VIA in the way most natural for the non-formal atmosphere of Bled Workshops (see records in [20]). In the period of Workshop I.Antoniadis took part in Conference in Japan but

Fig. 16.2. VIA talk by R.Cerulli from LNGS Gran Sasso at XVI Bled Workshop and Discussion Bled-Paris- Moscow-CERN-Gran Sasso-Marburg-Liege.

owing to VIA facility he has managed to contribute his talk "Mass hierarchy and physics beyond the Standard Model" to the programme of XVI Bled Workshop (Fig. 16.3).

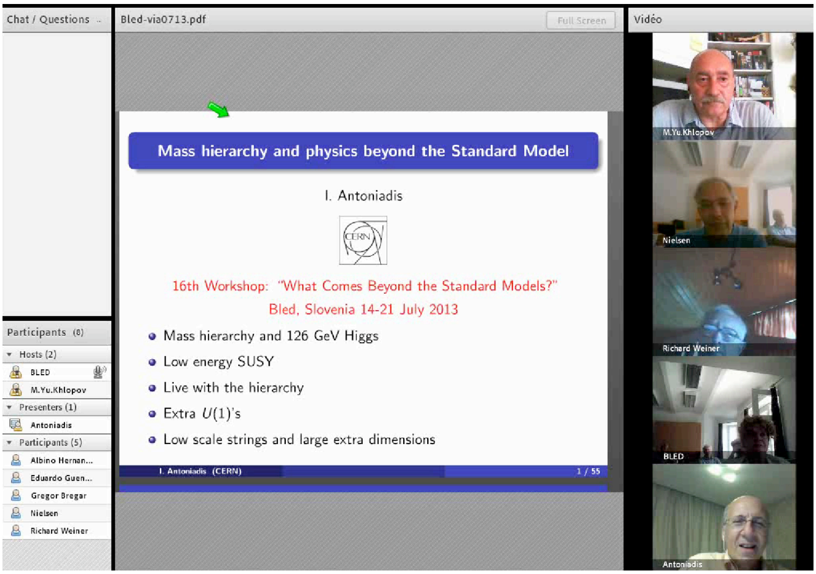

Fig. 16.3. VIA talk by I.Antoniadis from Japan at XVI Bled Workshop and Discussion Bled-Paris- Moscow-CERN-Tokyo-Marburg-Liege.

The videoconference with the talk The Spin-Charge-Family-Theory explains the origin of families, predicts their number, explains the origin of charges, of gauge vector and scalar fields by Norma Mankoc-Borstnik in Bled was followed by discussion with distant participants. In particular, in the course of this discussion M.Khlopov could present from Paris some aspects of cosmology of mirror world (Fig. 16.4).

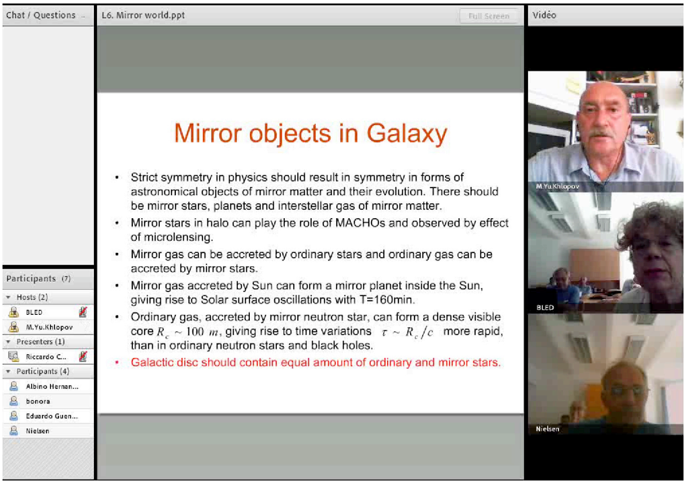

Fig. 16.4. VIA talk by N. Mankoc-Borstnik at XVI Bled Workshop and Discussion Bled-Paris-Moscow-CERN-Marburg-Liege.

VIA sessions also included the talk ”Status of the ATLAS experiment” by Anatoly Romaniouk (Fig. 16.5) followed by VIA discussion of problems of experimental search for new physics at accelerators. VIA sessions provided participation at distance in Bled discussions for M.Khlopov (APC, Paris, France), R. Cerulli (Gran Sasso, Italy), I.Antoniadis (CERN, participated from Japan), K.Belotsky (MEPhI, Moscow), J.-R. Cudell and Q.Wallemacq (Liege, Belgium), R.Weiner (Marburg, Germany) and many others.

16.4 Conclusions

The Scientific-Educational complex of Virtual Institute of Astroparticle physics provides regular communication between different groups and scientists, working in different scientific fields and parts of the world, the first-hand information on the newest scientific results, as well as support for various educational programs at distance. This activity would easily allow finding mutual interest and organizing task forces for different scientific topics of astroparticle physics and related topics. It can help in the elaboration of strategy of experimental particle, nuclear, astrophysical and cosmological studies as well as in proper analysis of experimental data. It can provide young talented people from all over the world to get the

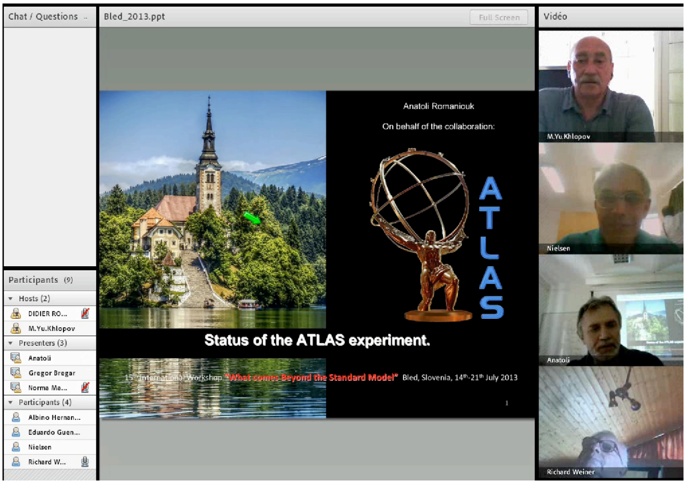

Fig. 16.5. VIA talk by A. Romaniouk at XVI Bled Workshop and Discussion Bled-Paris-Moscow-CERN-Marburg-Liege.

highest level education, come in direct interactive contact with the world known scientists and to find their place in the fundamental research. VIA applications can go far beyond the particular tasks of astroparticle physics and give rise to an interactive system of mass media communications.

VIA sessions became a natural part of a program of Bled Workshops, maintaining the platform of discussions of physics beyond the Standard Model for distant participants from all the world. The experience of VIA applications at Bled Workshops plays important role in the development of VIA facility as an effective tool of science and education online.

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