Coupling Electromagnetism to Global Charge

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Abstract

It is shown that an alternative to the standard scalar QED is possible. In this new version there is only global gauge invariance as far as the charged scalar fields are concerned although local gauge invariance is kept for the electromagnetic field. The electromagnetic coupling has the form $j_\mu(A^\mu + \partial^\mu B)$ where B is an auxiliary field and the current j_μ is A_μ independent so that no "sea gull terms" are introduced. In a model of this kind spontaneous breaking of symmetry does not lead to photon mass generation, instead the Golstone boson becomes a massless source for the electromagnetic gield, Infrared questions concerning the theory when spontaneous symmetry breaking takes place and generalizations to global vector QED are discussed.

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I. INTRODUCTION: CONVENTIONAL SCALAR QED AND ITS SEA GULLS

In conventional scalar QED, we "minimally couple" a globally invariant action (under global phase transformations). To be concrete, for a complex scalar field ψ with mass, m whose Lagrangian density can be represented in relativistic invariant form in the absence of interactions to electromagnetism as

$$\mathcal{L} = \hbar^2 g^{\mu\nu} \frac{\partial \psi^*}{\partial x^\mu} \frac{\partial \psi}{\partial x^\nu} - m^2 c^2 \psi^* \psi \tag{1}$$

Then, in the standard scalar QED model we introduce the electromagnetic interaction with scalar charged particles by introducing the minimal coupling in the Lagrangian for charged particles (see Eq. 1). As we recall, minimal coupling requires that we let the momentum p_μ be replaced by $p_\mu \rightarrow p_\mu - eA_\mu$ where $p_\mu = -i\ \hbar\frac{\partial}{\partial x^\mu}$ and where A_μ is the electromagnetic 4-vector whose Lagrangian is given by

$$\mathcal{L}_{\mathcal{EM}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{2}$$

with $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. We can now write the total Lagrangian after using the minimal coupling substitution into Eq. 1

$$\mathcal{L}_{\mathcal{T}} = g^{\mu\nu} \left[(\hbar \frac{\partial}{\partial x^\mu} - ieA_\mu)\psi^*][(\hbar \frac{\partial}{\partial x^\nu} + ieA_\nu)\psi \right] - m^2 c^2 \psi^* \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{3}$$

This leads to the equation of motion for the scalar field ψ

$$(i\ \hbar \frac{\partial}{\partial t} - e\phi)^2 \psi = (\frac{c\ \hbar}{i} \nabla - e\mathbf{A})^2 \psi + m^2 c^4 \psi \tag{4}$$

This equation and the lagrangian density from which it is derived are invariant under local gauge transformations:

$$\mathbf{A} \rightarrow \mathbf{A}' = \mathbf{A} + \nabla\chi; \quad \phi \rightarrow \phi' = \phi - \frac{1}{c}\frac{\partial \chi}{\partial t} \quad \text{with} \quad \psi \rightarrow \exp{[\frac{ie\chi}{\hbar c}]}\psi \tag{5}$$

Furthermore the electromagnetic field satisfies the Maxwell's equations where the electric charge density ρ and the current density $\mathbf{j}(x)$ are given by (now set $c = \hbar = 1$).

$$\rho(x) = i(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}) - 2e\phi\psi^*\psi \text{ and } \mathbf{j}(x) = -i(\psi^* \nabla \psi - \psi \nabla \psi^*) - 2e\mathbf{A}\psi^*\psi \tag{6}$$

There is an example, the BCS theory of superconductivity [1], where the effective theory in terms of the composite Cooper pairs retains the local gauge invariance which involves the local phase transformations of the composite scalar, however we may ask if this is a general rule, may be not.

When thinking of the electromagnetic interactions of pions, the quadratic dependence of the interactions on the potentials characterises the sea gull behaviour of standard scalar QED. As pointed out by Feynman [2], it is somewhat puzzling that spinor electrodynamics does not lead to any of such sea gulls . Considering that the microscopic description of charged pions is really the spinor electrodynamics of quarks, shouldn't we search for an effective scalar electrodynamics devoid of sea gulls?, is this possible?. In the next section we will see that this can be achieved in global scalar QED.

II. GLOBAL SCALAR QED

Since the macroscopic hadron is a very non local construction in terms of the fundamental quark fields and gluon fields as has been revealed from both the theoretical point of view [3] and from the experimental point of view [4], we do not necessarily have to keep a local gauge invariance in terms of the composite scalar fields (the hadrons), although global phase invariance must be respected. Also local gauge transformations for the photon should be mantained.

We work therefore with the following lagrangian density

$$\mathcal{L} = g^{\mu\nu}\frac{\partial\psi^*}{\partial x^\mu}\frac{\partial\psi}{\partial x^\nu} - U(\psi^*\psi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + j_\mu(A^\mu + \partial^\mu B) \tag{7}$$

where

$$j_\mu = ie(\psi^*\frac{\partial\psi}{\partial x^\mu} - \psi\frac{\partial\psi^*}{\partial x^\mu}) \tag{8}$$

and where we have also allowed an arbitrary potential $U(\psi^*\psi)$ to allow for the possibility of spontaneous breaking of symmetry. The model is separately invariant under local gauge transformations

$$A^\mu \to A^\mu + \partial^\mu\Lambda; \quad B \to B - \Lambda \tag{9}$$

and the independent global phase transformations

$$\psi \to exp(i\chi)\psi \tag{10}$$

The use of a gauge invariant combination $(A^\mu + \partial^\mu B)$ can be utilized for the construction of mass terms[5] or both mass terms and couplings to a current defined from the gradient of a scalar in the form $(A^\mu + \partial^\mu B)\partial_\mu A$ [6]. Since the subject of this paper is electromagnetic couplings of photons and there is absolutely no evidence for a photon mass, we will disregard such type of mass terms and concentrate on the implications of the $(A^\mu + \partial^\mu B) j_\mu$ couplings.

III. A DOUBLE CHARGE THEORY

As we will see the scalar QED model has two charge conservation laws associated with it. We see that Maxwell's equations are satisfied with j_μ being the source, that is

$$\partial^\nu F_{\nu\mu} = j_\mu \tag{11}$$

of course this implies $\partial^\nu \partial^\mu F_{\nu\mu} = \partial^\mu j_\mu = 0$. The same conclusion can be obtained from the equation of motion obtained from the variation with respect to B.

The Noether current obtained from the independent global phase transformations $\psi \rightarrow exp(i\chi)\psi$, χ being a constant, is

$$J_\mu = ie(\psi^* \frac{\partial \psi}{\partial x^\mu} - \psi \frac{\partial \psi^*}{\partial x^\mu}) + 2e(A_\mu + \partial_\mu B)\psi^* \psi \tag{12}$$

Therefore

$$j^B_\mu = J_\mu - j_\mu = 2e(A_\mu + \partial_\mu B)\psi^* \psi \tag{13}$$

is also conserved, that is $\partial^\mu((A_\mu + \partial_\mu B)\psi^* \psi) = 0$

IV. NO KLEIN PARADOX

V. BEHAVIOUR UNDER SPONTANEOUS BREAKING OF SYMMETRY, NEW COUPLINGS OF GOLDSTONE BOSONS TO ELECTROMAGNETISM AND ASSOCIATED INFRARED PROBLEMS

The absence of quadratic terms in the vector potential implies that no mass generation for the photon takes place. Furthermore the Goldstone boson that results from this s.s.b. ,writing $\psi = \rho exp(i\theta)$, where ρ is real and positive, we obtain that the phase of the ψ field, is not eaten, it remains in the theory, in fact it couples derivatively to $(A_\mu + \partial_\mu B)$, like the

A field studied in [6] and it produces a gradient type charge. In fact under s.s.b. regarding ρ as a constant, $j^\mu = 2e\rho^2\partial^\mu\theta$ the coupling $(A_\mu + \partial_\mu B)j^\mu$ implies the coupling of $(A_\mu + \partial_\mu B)$ to a gradient current, as discussed in [6].

It should be pointed out that this type of gradient current $j^\mu = 2e\rho^2\partial^\mu\theta$ for $\rho = constant$ generates an infrared problem, since the θ field now represents a massless field, which instead of being eaten becomes a source of electromagnetism. The normal way of solving for the electromagnetic field, using the Green's function method does not work straightforwardly, since the source now in Fourier space has support only in the light-cone and the Green's function has a pole like behaviour at the light-cone as well, so we encounter an undefined product of distributions. This is very similar to the solution of a forced harmonic oscillator when the external force has exactly the same frequency to that of the oscillator, that is the resonant case.

To resolve this problem, we note first that considering $F_{\nu\mu}$ as an antisymmetric tensor field (without at first considering whether this field derives from a four vector potential), then a solution of the equation $\partial^\nu F_{\nu\mu} = j_\mu$ is

$$F_{\nu\mu} = \int_0^1 d\lambda\lambda^2(x_\nu j_\mu(\lambda x) - x_\mu j_\nu(\lambda x)) \tag{14}$$

For a generic current the above $F_{\nu\mu}$ does not derive from a potential, however if the current is the gradient of a scalar field, the above $F_{\nu\mu}$ derives from a potential and provides a solution of the problem, where the Green's function method fails. Notice that the similarity with the the resonant case of the forced harmonic oscillator is very close, there the solution is of the form of an oscillating function times time and in the above solution we see the similar x_ν dependence appearing.

The resulting gauge potentials displays also a linear dependence on x_ν, which is interesting, since the central issue in the confinement problem for example is how to obtain potentials with linear dependence on the coordinates, although it is not clear how the very specific solution studied here is relevant to the confinement problem.

Axions are an example of Goldstone bosons with non trivial electromagnetic interactions

VI. GLOBAL VECTOR QED

In this case we consider a complex vector field W_μ and consider the action

$$\mathcal{L} = -\frac{1}{4} g^{\mu\nu} g^{\alpha\beta} G_{\mu\alpha} G^*_{\nu\beta} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j_\mu (A^\mu + \partial^\mu B) + M^2 W_\mu W^{*\mu} \tag{15}$$

with $G^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu$ and where

$$j_\mu = ie(W^{*\alpha} G_{\alpha\mu} - W^\alpha G^*_{\alpha\mu}) \tag{16}$$

This model displays global phase invariance for the complex vector field W_μ and local gauge invariance for the photon and B fields (7), as was the case of global scalar QED. Once again, no sea gull terms are present here.

VII. Q BALLS AND OTHER GLOBAL $U(1)$ SOLITONS AS ELECTROMAGNETICALLY CHARGED PARTICLES

A more complicated situation could present itself when considering solitons that connect a true vacuum where spontaneous symmetry breaking takes place and a core region without such spontaneous symmetry breaking, or as in the case of Q-Balls, the opposite case.

VIII. DISCUSSION AND CONCLUSIONS

Discussing the new global QED makes sense from both the purely theoretical point of view, since it provides a new type of viewing interactions of charged scalar particles with electromagnetism, as well as from a phenomenological point of view, since standard scalar QED contains the sea gull contributions for which apparently do not represent any known physical process in the electrodynamics of charged pions for example, so it makes sense to build a theory without such sea gulls.

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