Neutrino Masses from a Low Energy $SU(3)$ Flavor Symmetry Model

Albino Hernandez-Galeana

ESFM, IPN

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MODEL WITH $SU(3)$ FLAVOUR SYMMETRY

$G \equiv SU(3) \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ "GAUGE GROUP"

$SU(3)$: Flavor symmetry

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: Standard Model

FERMION CONTENT

Ordinary Fermions: $Q = T_{3L} + \frac{1}{2} Y$

$$\Psi_q^o = (3, 3, 2, \frac{1}{3})_L \quad , \quad \Psi_l^o = (3, 1, 2, -1)_L$$

$$\Psi_u^o = (3, 3, 1, \frac{4}{3})_R \quad , \quad \Psi_d^o = (3, 3, 1, -\frac{2}{3})_R$$

$$\Psi_e^o = (3, 1, 1, -2)_R$$

EXTRA FERMIONS: $SU(2)_L$ SINGLETS

Anomaly conditions:

$$[SU(3)]^3\ ,\ [SU(2)]^2\ SU(3)\ ,\ [U(1)]^2\ SU(3)$$

$$\Psi^o_\nu = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} = (3,1,1,0)_R$$ **Right Handed Neutrinos**

Vector Like quarks

$$U^o_L, U^o_R = (1,3,1,\frac{4}{3}) \quad , \quad D^o_L, D^o_R = (1,3,1,-\frac{2}{3})$$

$$E^o_L, E^o_R = (1,1,1,-2)$$ **Vector Like electron**

$$N^o_L, N^o_R = (1,1,1,0)$$ **Sterile Neutrinos**

ELECTROWEAK SYMMETRY BREAKING

In this scenario we introduce two triplets of $SU(2)_L$ Higgs doublets: $\Phi^u = (3, 1, 2, -1)$ and $\Phi^d = (3, 1, 2, +1)$, with the VEVs:

$$\langle \Phi^u \rangle = \begin{pmatrix} \langle \Phi^u_1 \rangle \\ \langle \Phi^u_2 \rangle \\ \langle \Phi^u_3 \rangle \end{pmatrix} \quad , \quad \langle \Phi^d \rangle = \begin{pmatrix} \langle \Phi^d_1 \rangle \\ \langle \Phi^d_2 \rangle \\ \langle \Phi^d_3 \rangle \end{pmatrix},$$

where

$$\langle \Phi^u_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_i \\ 0 \end{pmatrix} \qquad , \qquad \langle \Phi^d_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ V_i \end{pmatrix} \ .$$

Contribute to the W and Z boson masses:

$$\frac{g^2}{4} \left(v_u^2 + v_d^2 \right) W^+ W^- + \frac{(g^2 + {g'}^2)}{8} \left(v_u^2 + v_d^2 \right) Z_o^2$$

$v_u^2 = v_1^2 + v_2^2 + v_3^2$, $v_d^2 = V_1^2 + V_2^2 + V_3^2$. Hence, if we define as usual $M_W = \frac{1}{2} g v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

$SU(3)$ **FLAVOUR SYMMETRY BREAKING**

$$\langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3)$$

The above scalar fields and VEV's break completely the $SU(3)$ flavor symmetry.

- $\eta_2 = (3, 1, 1, 0)$, $< \eta_2 >^T = (0, \Lambda_2, 0)$

$$\frac{g_{H_2}^2 \Lambda_2^2}{2}(Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2Z_1 \frac{Z_2}{\sqrt{3}})$$

- $\eta_3 = (3, 1, 1, 0)$, $< \eta_3 >^T = (0, 0, \Lambda_3)$

$$\frac{g_{H_3}^2 \Lambda_3^2}{2}(Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$$

$SU(3)$ GAUGE BOSON MASSES:

$$M_1^2\, Y_1^+ Y_1^- + \frac{M_1^2}{2}\, Z_1^2 + \left(\frac{4}{3}\, M_2^2 + \frac{1}{3}\, M_1^2\right)\frac{Z_2^2}{2} - \frac{M_1^2}{\sqrt{3}}\, Z_1\, Z_2 + M_2^2\, Y_2^+ Y_2^- + (M_1^2 + M_2^2)\, Y_3^+ Y_3^-$$

$$M_1^2 = \frac{g_{H_2}^2 \Lambda_2^2}{2} \quad , \quad M_2^2 = \frac{g_{H_3}^2 \Lambda_3^2}{2} \quad , \quad M_3^2 = M_1^2 + M_2^2$$

From the diagonalization of the $Z_1 - Z_2$ squared mass matrix, we obtain the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_1^2 + M_2^2 - \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2}\right)$$

$$M_+^2 = \frac{2}{3}\left(M_1^2 + M_2^2 + \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2}\right)$$

$$M_1^2\, Y_1^+ Y_1^- + M_-^2\, \frac{Z_-^2}{2} + M_+^2\, \frac{Z_+^2}{2} + M_2^2\, Y_2^+ Y_2^- + (M_1^2 + M_2^2)\, Y_3^+ Y_3^-$$

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix}$$

$$\cos\phi \, \sin\phi = \frac{\sqrt{3}}{4} \, \frac{M_1^2}{\sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2}}$$

$$\cos\phi = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{\frac{M_2^2}{M_1^2} - \frac{1}{2}}{\sqrt{(\frac{M_2^2}{M_1^2} - 1)^2 + \frac{M_2^2}{M_1^2}}}} \,, \; \sin\phi = \frac{1}{\sqrt{2}} \sqrt{1 - \frac{\frac{M_2^2}{M_1^2} - \frac{1}{2}}{\sqrt{(\frac{M_2^2}{M_1^2} - 1)^2 + \frac{M_2^2}{M_1^2}}}}$$

	Z_1	Z_2
Z_1	M_1^2	$-\frac{M_1^2}{\sqrt{3}}$
Z_2	$-\frac{M_1^2}{\sqrt{3}}$	$(\frac{4}{3} M_2^2 + \frac{1}{3} M_1^2)$

with the hierarchy $M_1, M_2 \gg M_W$.

Charged fermion masses

Dirac See-saw mechanisms

$$h\,\bar{\psi}_l^o\,\Phi^d\,E_R^o\;+\;h_2\,\bar{\psi}_e^o\,\eta_2\,E_L^o\;+\;h_3\,\bar{\psi}_e^o\,\eta_3\,E_L^o\;+\;M\,\bar{E}_L^o\,E_R^o\;+h.c$$

where M is a free mass parameter (because its mass term is gauge invariant) and h, h_1, h_2 and h_3 are Yukawa coupling constants.
u-quarks and neutrinos coupled to Φ^u, while
d-quarks and charged leptons couple to Φ^d

In the gauge basis ${\psi^o_{L,R}}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms $\bar{\psi}^o_L \mathcal{M}^o \psi^o_R + h.c$, where

	e^o_R	μ^o_R	τ^o_R	E^o_R
$\overline{e^o_L}$	0	0	0	$h\ v_1$
$\overline{\mu^o_L}$	0	0	0	$h\ v_2$
$\overline{\tau^o_L}$	0	0	0	$h\ v_3$
$\overline{E^o_L}$	0	$h_2\Lambda_2$	$h_3\Lambda_3$	M_D

Table : Tree level Dirac mass matrix $\mathcal{M}^o$

Notice that $\mathcal{M}^o$ has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call $\mathcal{M}^o$ a **"Dirac See-saw"** mass matrix. $\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi^o_{L,R} = V^o_{L,R}\, \chi_{L,R}$.

$$V^{o\,T}_L \mathcal{M}^o \; V^o_R = Diag(0, 0, -\sqrt{\lambda_-}, \sqrt{\lambda_+})$$

$$V^{o\,T}_L \mathcal{M}^o \mathcal{M}^{o\,T} \; V^o_L = V^{o\,T}_R \mathcal{M}^{o\,T} \mathcal{M}^o \; V^o_R = Diag(0, 0, \lambda_-, \lambda_+) \, .$$

where λ_- and λ_+ are the nonzero eigenvalues, $\sqrt{\lambda_+}$ being the fourth heavy fermion mass, and $\sqrt{\lambda_-}$ of the order of the top, bottom and tau mass for u, d and e fermions, respectively.

One loop contribution to charged fermion masses

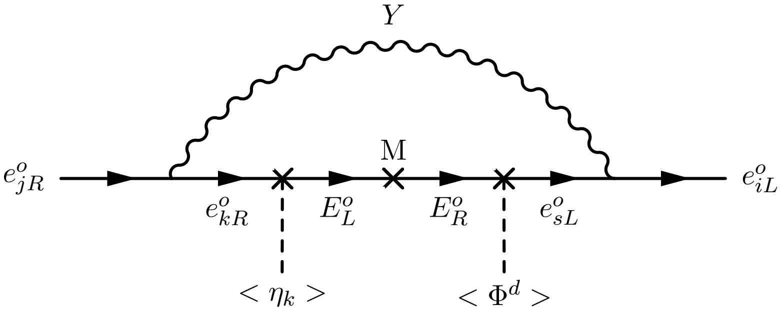

Figure : Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

$$i\mathcal{L}_{int} = \frac{g_H}{2}\left(\bar{e^o}\gamma_\mu e^o - \bar{\mu^o}\gamma_\mu \mu^o\right) Z_1^\mu + \frac{g_H}{2\sqrt{3}}\left(\bar{e^o}\gamma_\mu e^o + \bar{\mu^o}\gamma_\mu \mu^o - 2\bar{\tau^o}\gamma_\mu \tau^o\right) Z_2^\mu$$
$$+ \frac{g_H}{\sqrt{2}}\left(\bar{e^o}\gamma_\mu \mu^o Y_1^+ + \bar{e^o}\gamma_\mu \tau^o Y_2^+ + \bar{\mu^o}\gamma_\mu \tau^o Y_3^+ + h.c.\right)$$

where g_H is the $SU(3)$ coupling constant, Z_1, Z_2 and Y_i^j , $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons.

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o\, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

where M_Y is the gauge boson mass, c_Y is a factor coupling constant, $m_3^o = -\sqrt{\lambda_-}$ and $m_4^o = \sqrt{\lambda_+}$ are the See-saw mass eigenvalues, and $f(x, y) = \frac{x^2}{x^2 - y^2} \ln \frac{x^2}{y^2}$.

$$\sum_{k=3,4} m_k^o\, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i\, b_j\, M}{\lambda_+ - \lambda_-}\, F(M_Y)\,,$$

$i, j = 1, 2, 3$ and $F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_+} \ln \frac{M_Y^2}{\lambda_+} - \frac{M_Y^2}{M_Y^2 - \lambda_-} \ln \frac{M_Y^2}{\lambda_-}$. Adding up all the one loop $SU(3)$ gauge boson contributions, we get the mass terms

	e_R^o	μ_R^o	τ_R^o	E_R^o
$\overline{e_L^o}$	R_{11}	R_{12}	R_{13}	0
$\overline{\mu_L^o}$	0	R_{22}	R_{23}	0
$\overline{\tau_L^o}$	0	R_{32}	R_{33}	0
$\overline{E_L^o}$	0	0	0	0

Table : Generic one loop diagram contribution to the mass term $m_{ij}\, \bar{e}_{iL}^o\, e_{jR}^o$

$$\mathcal{M}_1^o = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ 0 & R_{22} & R_{23} & 0 \\ 0 & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} ,$$

$$\bar{\psi}_L^o \mathcal{M}^o \, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \, \psi_R^o = \bar{\chi}_L \, \mathcal{M} \, \chi_R \, ,$$

with $\mathcal{M} \equiv \Big[Diag(0, 0, -\sqrt{\lambda_-}, \sqrt{\lambda_+}) + {V_L^o}^T \mathcal{M}_1^o \, V_R^o \Big]$.

Acting V_L^o, V_R^o we get the mass matrix in the tree level mass eigenfields and eigenvalues basis

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta\ m_{13} & s_\beta\ m_{13} \\ m_{21} & m_{22} & c_\beta\ m_{23} & s_\beta\ m_{23} \\ c_\alpha\ m_{31} & c_\alpha\ m_{32} & (-\sqrt{\lambda_-} + c_\alpha c_\beta\ m_{33}) & c_\alpha s_\beta\ m_{33} \\ s_\alpha\ m_{31} & s_\alpha\ m_{32} & s_\alpha c_\beta\ m_{33} & (\sqrt{\lambda_+} + s_\alpha s_\beta\ m_{33}) \end{pmatrix}$$

The diagonalization of $\mathcal{M}$ yields the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\ \Psi_{L,R}$; $\bar{\chi}_L\ \mathcal{M}\ \chi_R = \bar{\Psi}_L\ {V_L^{(1)}}^T \mathcal{M}\ V_R^{(1)}\ \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$ {V_L^{(1)}}^T \mathcal{M}\,\mathcal{M}^T\ V_L^{(1)} = {V_R^{(1)}}^T \mathcal{M}^T\ \mathcal{M}\ V_R^{(1)} = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\,, $$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$ \psi_L^o = V_L^o\ V_L^{(1)}\ \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\ V_R^{(1)}\ \Psi_R $$

Quark Mixing and non-unitary $(V_{CKM})_{4\times 4}$

Recall that vector like quarks are $SU(2)_L$ weak singlets, and then they do not couple to W boson in the interaction basis. So, the interaction of ordinary quarks ${f_{uL}^o}^T = (u^o, c^o, t^o)_L$ and ${f_{dL}^o}^T = (d^o, s^o, b^o)_L$ to the W charged gauge boson is

$$\frac{g}{\sqrt{2}}\,\bar{f^o}_{uL}\gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}}\,\bar{\Psi}_{uL}\ {V_{uL}^{(1)}}^T\ [(V_{uL}^o)_{3\times 4}]^T\ (V_{dL}^o)_{3\times 4}\ V_{dL}^{(1)}\ \gamma_\mu \Psi_{dL}\ W^{+\mu},$$

with g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times 4} = {V_{uL}^{(1)}}^T\ [(V_{uL}^o)_{3\times 4}]^T\ (V_{dL}^o)_{3\times 4}\ V_{dL}^{(1)} \equiv {V_{uL}^{(1)}}^T\ Vo\ V_{dL}^{(1)}\ .$$

$$Vo = \begin{pmatrix} Co & -\frac{V_3}{V}So & -\frac{V'}{V}C^d_\alpha So & -\frac{V'}{V}S^d_\alpha So \\ \frac{v_3}{v}So & \frac{v_3V_3}{vV}Co + \frac{v'V'}{vV} & C^d_\alpha(\frac{v_3V'}{vV}Co - \frac{v'V_3}{vV}) & S^d_\alpha(\frac{v_3V'}{vV}Co - \frac{v'V_3}{vV}) \\ \frac{v'}{v}C^u_\alpha So & C^u_\alpha(\frac{v'V_3}{vV}Co - \frac{v_3V'}{vV}) & C^d_\alpha C^u_\alpha(\frac{v'V'}{vV}Co + \frac{v_3V_3}{vV}) & C^u_\alpha S^d_\alpha(\frac{v'V'}{vV}Co + \frac{v_3V_3}{vV}) \\ \frac{v'}{v}S^u_\alpha So & S^u_\alpha(\frac{v'V_3}{vV}Co - \frac{v_3V'}{vV}) & C^d_\alpha S^u_\alpha(\frac{v'V'}{vV}Co + \frac{v_3V_3}{vV}) & S^u_\alpha S^d_\alpha(\frac{v'V'}{vV}Co + \frac{v_3V_3}{vV}) \end{pmatrix}$$

$$s_o = \frac{v_1}{v'}\ \frac{V_2}{V'} - \frac{v_2}{v'}\ \frac{V_1}{V'} \quad , \quad c_o = \frac{v_1}{v'}\ \frac{V_1}{V'} + \frac{v_2}{v'}\ \frac{V_2}{V'}$$

$$c_o^2 + s_o^2 = 1$$

$V_i,\ v_i\ , i = 1, 2$ are related to (e,d) and (u,ν) fermions respectively.

It is important to comment here that the scalar fields introduced to break the symmetries in the model: Φ^u, Φ^d, η_2 and η_3, do not couple ordinary fermions directly. So, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which as is shown in $(V_{CKM})_{4\times 4}$, may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.

NEUTRINO MASSES

Tree level Dirac Neutrino masses

$$h_D \, \overline{\Psi_l^o} \, \Phi^u \, N_R^o \;\; + \;\; h_2 \, \overline{\Psi_\nu^o} \, \eta_2 \, N_L^o \;\; + \;\; h_3 \, \overline{\Psi_\nu^o} \, \eta_3 \, N_L^o \;\; + \;\; M_D \, \overline{N_L^o} \, N_R^o \;\; + h.c$$

h, h_2 and h_3 are Yukawa couplings, and M_D a Dirac type invariant neutrino mass for the sterile neutrino $N_{L,R}^o$. After electroweak symmetry breaking, we obtain in the interaction basis ${\Psi_\nu^o}^T_{L,R} = (\nu_e^o, \nu_\mu^o, \nu_\tau^o, N^o)_{L,R}$, the mass terms

$$h_D \, \left[v_1 \, \overline{\nu_{eL}^o} + v_2 \, \overline{\nu_{\mu L}^o} + v_3 \, \overline{\nu_{\tau L}^o} \right] \, N_R^o + \left[h_2 \, \Lambda_2 \, \overline{\nu_{\mu R}^o} + h_3 \, \Lambda_3 \, \overline{\nu_{\tau R}^o} \right] \, N_L^o + M_D \, \overline{N_L^o} \, N_R^o \; +$$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_D\ v_1$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_D\ v_2$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_D\ v_3$
$\overline{N^o_L}$	0	$h_2\Lambda_2$	$h_3\Lambda_3$	M_D

Table : Tree level Dirac mass terms $m_{ij}\ \bar{\nu}^o_{iL}\ \nu^o_{jR}$

Tree level Majorana masses:

Since $N^o_{L,R}$ are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L \, \overline{\Psi^o_l} \, \Phi^u (N^o_L)^c \quad + \quad m_L \, \overline{N^o_L} \, (N^o_L)^c$$

and

$$h_{2R} \, \overline{\Psi^o_\nu} \, \eta_2 \, (N^o_R)^c \;\; + \;\; h_{3R} \, \overline{\Psi^o_\nu} \, \eta_3 \, (N^o_R)^c \;\; + \;\; m_R \, \overline{N^o_R} \, (N^o_R)^c + h.c \; ,$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L \, \left[v_1 \, \overline{\nu^o_{eL}} + v_2 \, \overline{\nu^o_{\mu L}} + v_3 \, \overline{\nu^o_{\tau L}} \right] \, (N^o_L)^c \quad + \quad m_L \, \overline{N^o_L} \, (N^o_L)^c$$

$$+ \left[h_{2R} \, \Lambda_2 \, \overline{\nu^o_{\mu R}} + h_{3R} \, \Lambda_3 \, \overline{\nu^o_{\tau R}} \right] \, (N^o_R)^c \;\; + \;\; m_R \, \overline{N^o_R} \, (N^o_R)^c + h.c. \, ,$$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	0	0	0	$h_L v_1$
$\nu^o_{\mu L}$	0	0	0	$h_L v_2$
$\nu^o_{\tau L}$	0	0	0	$h_L v_3$
N^o_L	$h_L v_1$	$h_L v_2$	$h_L v_3$	m_L

Table : Tree level L-handed Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, (\nu^o_{jL})^T$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	0	0	$h_{2R}\Lambda_2$
$\nu^o_{\tau R}$	0	0	0	$h_{3R}\Lambda_3$
N^o_R	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	m_R

Table : Tree level R-handed Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iR}\, (\nu^o_{jR})^T$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	$(N^o_L)^c$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	0	0	0	$h_L v_1$	$h_D v_1$
$\overline{\nu^o_{\mu L}}$	0	0	0	0	0	0	$h_L v_2$	$h_D v_2$
$\overline{\nu^o_{\tau L}}$	0	0	0	0	0	0	$h_L v_3$	$h_D v_3$
$\overline{(\nu^o_{eR})^c}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	0	0	0	$h_2 \Lambda_2$	$h_{2R} \Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	0	0	0	$h_3 \Lambda_3$	$h_{3R} \Lambda_3$
$\overline{N^o_L}$	$h_L v_1$	$h_L v_2$	$h_L v_3$	0	$h_2 \Lambda_2$	$h_3 \Lambda_3$	m_L	M_D
$\overline{(N^o_R)^c}$	$h_D v_1$	$h_D v_2$	$h_D v_3$	0	$h_{2R} \Lambda_2$	$h_{3R} \Lambda_3$	M_D	m_R

Table : Tree Level Majorana mass matrix $\mathcal{M}^o_\nu$ in the Ψ^o_ν basis

Diagonalization of $\mathcal{M}_\nu^o$ yields 4 zero mass eigenvalues automatic, 5 zero mass eigenvalues also possible

$$(U_\nu^o)^T \, \mathcal{M}_\nu^o \, U_\nu^o = Diag(0, 0, 0, 0, m_5^o, m_6^o, m_7^o, m_8^o)$$

where

$$m_4 = \begin{pmatrix} 0 & 0 & \alpha & a \\ 0 & 0 & b & \beta \\ \alpha & b & m_L & m_D \\ a & \beta & m_D & m_R \end{pmatrix} \quad , \quad U_4 = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix}$$

$$U_4^T \, m_4 \, U_4 = Diag(m_5^o, m_6^o, m_7^o, m_8^o)$$

One loop neutrino masses

One loop Dirac Neutrino masses

Light neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 2, as well as L-handed and R-handed Majorana masses from Fig. 3 and Fig. 4, respectively. The contribution from these diagrams read

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=5,6,7,8} m_k^o \, U_{ik}^o U_{jk}^o f(M_Y, m_k^o) = c_Y \, m_\nu(M_Y)_{ij} \quad ,$$

with

$$m_\nu(M_Y)_{ij} \equiv \frac{\alpha_H}{\pi} \sum_{k=5,6,7,8} m_k^o \, U_{ik}^o U_{jk}^o \, f(M_Y, m_k^o), \quad , \quad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

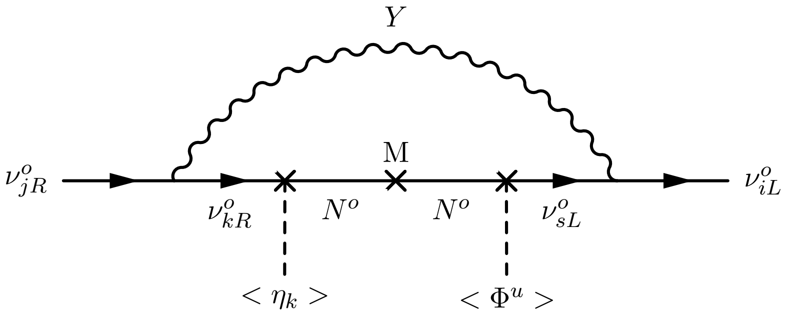

Figure : Generic one loop diagram contribution to the Dirac mass term $m_{ij}\ \bar{\nu}^o_{iL}\nu^o_{jR}$. $\mathrm{M} = M_D, m_L, m_R$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	$D_{\nu\, 14}$	$D_{\nu\, 15}$	$D_{\nu\, 16}$	0
$\overline{\nu^o_{\mu L}}$	0	$D_{\nu\, 25}$	$D_{\nu\, 26}$	0
$\overline{\nu^o_{\tau L}}$	0	$D_{\nu\, 35}$	$D_{\nu\, 36}$	0
$\overline{N^o_L}$	0	0	0	0

Table : One loop Dirac neutrino mass terms $m^{\nu}_{ij}\, \bar{\nu}^o_{iL}\, \nu^o_{jR}$

One loop L-handed Majorana masses

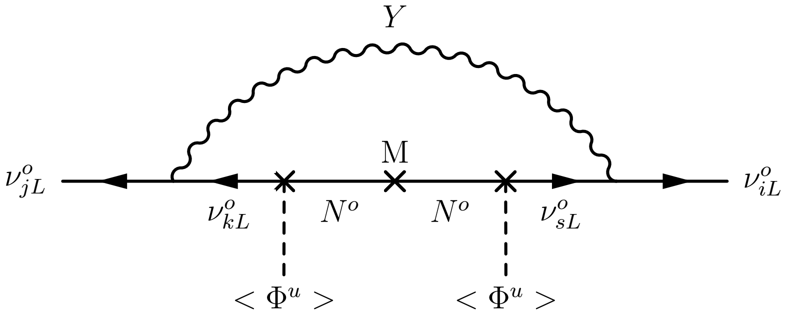

Figure : Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij}\, \bar{\nu}^o_{iL}(\nu^o_{jL})^T$. M $= M_D, m_L, m_R$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	0
$\nu^o_{\mu L}$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0
$\nu^o_{\tau L}$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0
N^o_L	0	0	0	0

Table : One loop L-handed neutrino Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, (\nu^o_{jL})^T$

One loop R-handed Majorana masses

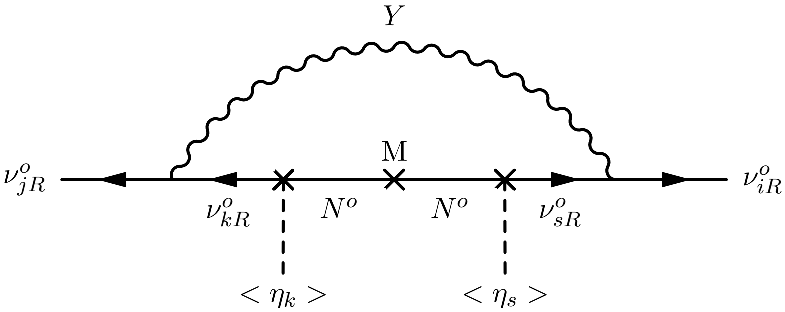

Figure : Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij}\, \bar{\nu}^o_{iR} (\nu^o_{jR})^T$. M $= M_D, m_L, m_R$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	$R_{\nu\,55}$	$R_{\nu\,56}$	0
$\nu^o_{\tau R}$	0	$R_{\nu\,56}$	$R_{\nu\,66}$	0
N^o_R	0	0	0	0

Table : One loop R-handed neutrino Majorana mass terms $m_{ij}\,\bar{\nu}^o_{iR}\,(\nu^o_{jR})^T$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	$(N^o_L)^c$
$\overline{\nu^o_{eL}}$	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	$D_{\nu\,14}$	$D_{\nu\,15}$	$D_{\nu\,16}$	0
$\overline{\nu^o_{\mu L}}$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0	$D_{\nu\,25}$	$D_{\nu\,26}$	0
$\overline{\nu^o_{\tau L}}$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0	$D_{\nu\,35}$	$D_{\nu\,36}$	0
$\overline{(\nu^o_{eR})^c}$	$D_{\nu\,14}$	0	0	0	0	0	0
$\overline{(\nu^o_{\mu R})^c}$	$D_{\nu\,15}$	$D_{\nu\,25}$	$D_{\nu\,35}$	0	$R_{\nu\,55}$	$R_{\nu\,56}$	0
$\overline{(\nu^o_{\tau R})^c}$	$D_{\nu\,16}$	$D_{\nu\,26}$	$D_{\nu\,36}$	0	$R_{\nu\,56}$	$R_{\nu\,66}$	0
$\overline{N^o_L}$	0	0	0	0	0	0	0
$\overline{(N^o_R)^c}$	0	0	0	0	0	0	0

Table : One Loop Majorana mass matrix $\mathcal{M}^o_{1\,\nu}$ in the Ψ^o_ν basis

NEUTRINO MASS MATRIX UP TO ONE LOOP

$$\mathcal{M}_\nu = (U^o_\nu)^T \, \mathcal{M}^o_{1\,\nu} \, U^o_\nu + Diag(0,0,0,0,m^o_5,m^o_6,m^o_7,m^o_8)$$

$$\mathcal{M}_\nu = \begin{pmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\ N_{12} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\ N_{13} & N_{23} & 0 & 0 & N_{35} & N_{36} & N_{37} & N_{38} \\ N_{14} & N_{24} & 0 & N_{44} & N_{45} & N_{46} & N_{47} & N_{48} \\ N_{15} & N_{25} & N_{35} & N_{45} & N_{55}+m^o_5 & N_{56} & N_{57} & N_{58} \\ N_{16} & N_{26} & N_{36} & N_{46} & N_{56} & N_{66}+m^o_6 & N_{67} & N_{68} \\ N_{17} & N_{27} & N_{37} & N_{47} & N_{57} & N_{67} & N_{77}+m^o_7 & N_{78} \\ N_{18} & N_{28} & N_{38} & N_{48} & N_{58} & N_{68} & N_{78} & N_{88}+m^o_8 \end{pmatrix}$$

Preliminary numerical results

SU(3) PARAMETERS: $\frac{\alpha_H}{\pi} = 0.12$, $M_1 = 1$ TeV , $M_2 = 10$ TeV

Charged leptons:

$$\frac{V_1}{V_2} \simeq 0 \quad , \quad \frac{V_2}{V_3} \simeq 0.22 \quad , \quad \frac{h_3}{h_2} \simeq -0.269$$

$$M_e = \begin{pmatrix} 0.486 & 0. & 0. & 0. \\ 0. & -53.3413 & 383.348 & 2.463 \\ 0. & -232.662 & -1689.89 & 5.92926 \\ 0. & -23.3834 & 92.7496 & 4.04616 \times 10^6 \end{pmatrix}$$

$$\sin\alpha = 0.1 \ , \ \sin\beta = 0.0064 \ , \ \sqrt{\lambda_-} = 2612.74\,\text{MeV} \ , \ \sqrt{\lambda_+} = 4.04\,\text{TeV}$$

$$(m_e \, , \, m_\mu \, , \, m_\tau \,) = (0.486 \, , \, 102.7 \, , \, 1746.17 \,) \ \text{MeV}$$

d-quarks:

$$\frac{V_1}{V_2} \simeq 0 \quad , \quad \frac{V_2}{V_3} \simeq 0.22 \quad , \quad \frac{h_3}{h_2} \simeq -0.238$$

$$M_d = \begin{pmatrix} -2.82 & 0 & 0 & 0 \\ 0 & -31.4906 & 333.018 & 14.2511 \\ 0 & -221.699 & -2832.28 & 48.0863 \\ 0 & -22.2816 & 112.933 & 919791. \end{pmatrix}$$

$$\sin\alpha = 0.1 \ , \ \sin\beta = \ , \ \sqrt{\lambda_-} = 3955.95\,\text{MeV} \ , \ \sqrt{\lambda_+} = 919.786\,\text{GeV}$$

$$(m_d \ , \ m_s \ , \ m_b\,) = (2.82\,,\ 57\,,\ 2860\,)\ \text{MeV}$$

u-quarks:

$$\frac{v_1}{v_2} \simeq 0.131 \quad , \quad \frac{\sqrt{v_1^2 + v_2^2}}{v_3} \simeq 0.165 \quad , \quad \frac{h_{3u}}{h_{2u}} \simeq -0.228$$

$$M_u = \begin{pmatrix} 1.40804 & 216.366 & -565.124 & -759.069 \\ 0.181992 & -564.987 & 2451.74 & 3293.16 \\ 0.0298782 & -2292.59 & -172100. & 7592.97 \\ 0.00300287 & -230.414 & 568.141 & 1.31749 \times 10^6 \end{pmatrix}$$

$$\sin\alpha = 0.1 \, , \; \sin\beta = \, , \; \sqrt{\lambda_-} = 177.753\,\text{GeV} \, , \; \sqrt{\lambda_+} = 1.31\,\text{TeV}$$

$$(m_u \, , \, m_c \, , \, m_t) = (1.382 \, , \, 637.459 \, , \, 172134) \text{ MeV}$$

QUARK MIXING

$$(V_{CKM})_{4\times4} = {V_{uL}^{(1)}}^T \; [(V_{uL}^{o})_{3\times4}]^T \; (V_{dL}^{o})_{3\times4} \; V_{dL}^{(1)}$$

$$(V_{CKM})_{4\times4} = \begin{pmatrix} 0.973479 & 0.226444 & 0.0325766 & -0.000625627 \\ 0.227665 & -0.972842 & -0.0409653 & -0.00719965 \\ -0.0224446 & -0.045686 & 0.988122 & -0.0992279 \\ 0.00198582 & 0.00894833 & -0.104875 & 0.0105793 \end{pmatrix}$$

NEUTRINO MASS MATRIX (eV)

$M_\nu =$

$$\begin{pmatrix}
0.0018 & -0.0070 & -0.0107 & 0.0210 & 0.0402 & 0.1242 & 0.0044 & -0.0047 \\
-0.0070 & 0.0034 & -0.0014 & -0.0127 & -0.0209 & -0.3602 & -0.0127 & 0.0137 \\
-0.0107 & -0.0014 & 0. & 0. & -0.0002 & 0.00007 & 2.5 \times 10^{-6} & -2.7 \times 10^{-6} \\
0.0210 & -0.0127 & 0. & 0.0453 & 0.0972 & 1.1315 & 0.0400 & -0.0431 \\
0.0402 & -0.0209 & -0.0002 & 0.0972 & -2.6872 & -1.6324 & -0.0577 & 0.0622 \\
0.1242 & -0.3602 & 0.00007 & 1.1315 & -1.6324 & -21.1625 & 0.0081 & -0.0088 \\
0.0043 & -0.0127 & 2.56 \times 10^{-6} & 0.0400 & -0.0577 & 0.0081 & 5445.6 & -0.0003 \\
-0.0047 & 0.0137 & -2.76 \times 10^{-6} & -0.0431 & 0.0622 & -0.0088 & -0.0003 & 10576.6
\end{pmatrix}$$

$\frac{\alpha}{\pi} = .12$, $M_1 = 1 \ TeV$, $M_2 = 10 \ TeV$,

NEUTRINO MASS EIGENVALUES (eV)

$(\, m_1 = 0 \, , \; m_2 = 0.0087 \, , \; m_3 = 0.01298 \, , \; m_4 = 0.122359 \,)$ eV

$(\, m_5 = 2.54454 \, , \; m_6 = 21.3726 \, , \; m_7 = 5445.6 \, , \; m_8 = 10576.6 \,)$ eV

$m_2^2 - m_1^2 = 7.55 \times 10^{-5}$, $m_3^2 - m_1^2 = 1.6 \times 10^{-4}$

NEUTRINO MIXING

$U_\nu =$

$$\begin{pmatrix}
0.121 & 0.608 & 0.746 & -0.239 & -0.011 & -0.005 & 8\times10^{-7} & -4\times10^{-7} \\
-0.940 & 0.118 & 0.146 & 0.280 & -0.004 & 0.016 & -2\times10^{-6} & \times10^{-6} \\
0.006 & -0.772 & 0.634 & 0.017 & 3\times10^{-5} & -4\times10^{-6} & 4\times10^{-10} & -2\times10^{-10} \\
-0.316 & -0.135 & -0.135 & -0.927 & \times10^{-4} & -0.052 & 7\times10^{-6} & -4\times10^{-6} \\
-0.002 & 0.007 & 0.008 & -0.005 & 0.996 & 0.087 & -\ \times10^{-5} & 5\times10^{-6} \\
0 & -0.006 & -0.006 & -0.055 & -0.087 & 0.994 & \times10^{-6} & -8\times10^{-6} \\
0 & 8\times10^{-7} & 8\times10^{-7} & 7\times10^{-6} & \times10^{-5} & -1\times10^{-7} & 1. & -6\times10^{-8} \\
0. & -4\times10^{-7} & -4\times10^{-7} & -4\times10^{-6} & -5\times10^{-6} & 7\times10^{-8} & 6\times10^{-8} & 1.
\end{pmatrix}$$

PREDICTIONS:

1. NEW MASSIVE GAUGE BOSONS AND SCALARS, (1-10) TeV
2. VECTOR LIKE QUARKS AND CHARGED LEPTONS,
   (400 GeV - few TeV)
3. STERILE NEUTRINOS:
   Light (eV), May be two
   Heavy (KeV-GeV),