Small Representation Principle

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Continuation of work with Don Bennett on "What is special about the Standard Model Group" presented by Holger Bech Nielsen

Principle of SMALL representations being used for SEVERAL points:

- **The Standard Model Gauge Group**
- **Dimension of Space-time**
- **That we have Gauge symmetry** (a bit more doubtfull)
- **That we have "Small" representations of say Fermions**

What is then new ?:

- Group-volume discussion to justify some ad hoc rules
- Various attempts to include the Lorentz or Poincare group as being selected in the same game, thus obtaining the dimension of space-time! (for practical purposes; i.e. we hope to get 4 or 3+1 even if there should exist extra dimensions. Then one should just find a way of saying that dimensions would collaps or compactify somehow down to th dimension favoured by our principle.)
- Group of Gauge transformations co ntinue on next slide

What is new, continued

- Considering the group of all gauge transformations rather than what is usually called the "gauge group": The full group of gauge transformations for a "gauge group" - in usual sense - being G is rather $\times_x G(x) = G \times G \times \cdots \times G \times G$, where we have taken one with G isomorphic group $G(x)$ for every point x in space-time, $G(x) \approx G$. It is suggested that we then use the same representation for the "full" group of gauge transformations $\times_x G(x)$ and *even extend this group by the* **reparametrizations** *of the x-space-time*. But if we could extend the **group**, $\times_x G(x) \rightarrow (Diff_4$ extended with $\times_x G(x))$, then we would have made a **bigger group** than the one we started with have the **same representation space**. That should be more preferable from our principle!?
  Would we have "derived" the gauge symmetry of general relativity from our principle!?

Plan of talk:

- 1. Introduction: Sizes of representations.
- 2. Review and modern version of the winning Gauge group, essentially C_A/C_F
- 3. Main new point: Also dimension of space time gets predicted almost in the *same* way.
- 4. Can we say a gauge transformation has an exceptionally "small" representation by "cheeting" with Taylor expansion?
- Conclusion: Remarkably how much of Standad Model from just "small representations" !

Review, Gauge Group

We shall review my earlier Bled-talk og work with Don Bennett on seeking to adjust the rules of a game between Lie groups, so that the winner of that game - the Lie group, that obtains the highest value fir the "goal quantity" - becomes the Standard Model Group. Now I shall put it so that we in the contruction and justification for this "goal quantity" think mainly on the *volume* of the space of representation matrices for a representation F making this volume minimal and compared it to the volume of a certain standard representation - which we have problem to construct in a nice way -, which is for semi-simple groups the adjoint representation.
For a simple Lie algebra our "goal quantity", that is the quantiy to be maximized, is the ratio of the quadratic Casimir C_A for the adjoint representation and that of a representation F, i.e. C_F, to be chosen so as to make C_F minimal under the constraint that F be still faithfull. The last means that you select the representation F of the group G considered so as to maximize C_A/C_F the faithfulness constraint. For the simple Lie algebra we use as the "goal quantity" this ratio C_A/C_F. For other cases I have to explain

Don Bennett and I earlier presented a quantity “close to C_A/C_F” which maximized leads to the Standard Model Group.

I should like to somewhat give a physical interpretation and starting point for this quanity by first telling about a natural metric on unitary representation spaces:

Let $U(g)$ be a unitary representation matrix of an element g of a group G, i.e. $g \in G$. Then we can define a metric ds on the representation space V_F to which $U(g) \in V_F$ belongs by

$$ds^2 = Tr(dU^+ dU), \tag{1}$$

where dU is an infinitesimal matrix, $dU \in V_F$

Especially we can consider the length $ds(g, g+dg)$ of the image under the representation $U : G \to V_F$ of an infintesimal step g to $g + dg$ in the group G,

$$\begin{aligned} ds(g, g+dg)^2 &= Tr((U(g+dg) - U(g))(U(g+dg) - U(g))) \\ &= Tr((\sum_i \frac{\partial U}{\partial g_i})(\sum_j \frac{\partial U}{\partial g_j})dg_j)) \end{aligned} \tag{2}$$

Volume of the N-representation of $U(N)$

As an example to tell, that we can just calculate the volume of a given representation image of the group in the specified metric

$$ds^2 = Tr(dU^+ dU), \tag{3}$$

I give you the volume of the "defining representation" for $U(N)$:

$$Vol(U(N), \text{"defining repr."}) = \prod_{i=1,3,\ldots,2N-3,2N-1;} Area(S^i - unitsphere$$

$$= \prod_{i=1,3,\ldots 2N-3,2N-1} \frac{2\pi^{(i+1)/2}}{\Gamma((i+1)/2)} = \frac{2^N \pi^{(N^2+N)/2}}{\prod_{i=1,3,\ldots,2n-1} \Gamma((i+1)/2)}$$

Here $Area(S^i - unitsphere)$ stands for the surface area of the unit sphere imbedded in $i+1$ dimensinal space as $\{\vec{x} \in \mathbf{R}^{i+1} | vecx^2 = 1\}$ and thus having itself i dimensions. Remember that the complexwise N-diemsional representation space for the N-plet is realwise $2N$-dimensional.

Reminder: **What is a Quadratic Casirmir ?**

If a basis of the Lie algebra for a Lie group is

$$A_1, A_2, ..., A_h; \tag{6}$$

where h is the dimension of the Lie algebra, its representations under a representation ρ could be denoted

$$\rho(A_1), \rho(A_2), ..., \rho(A_h); \tag{7}$$

and the quadratic Casimir operator would be

$$C_\rho = \sum_{i=1,2,...,h} \rho(A_i)^2 \tag{8}$$

and on irreducible representations ρ the quadratic Casimir C_ρ would have the same eigenvalue whatever the vector (state) in the representation ρ it may act on. So for irreducible representations the quadratic Casimir is effectively just a number C_ρ.
Of course the normalization of C_ρ depends on the normalization of the Lie algebra basis vectors $A_1, A_2, ..., A_h$, but if we divide C_ρ for one irreducible representation ρ by that for another one ρ' say, i.e. $C_{\rho'}$, we get a number $C_\rho/C_{\rho'}$, not depending on this normalization.

Relation between quadratic Casimirs and Distances in Representations

For a given infinitesimal shift $g - - > g + dg$ we have in each of its representations, such as ρ and ρ', the corresponding infinitesimal shifts $U_\rho \rightarrow U_\rho + dU_\rho$ and $U_{\rho'} \rightarrow U_{\rho'} + dU_{\rho'}$.
With the definition of distance ds by

$$ds^2 = Tr(dU^\dagger dU) \tag{9}$$

we would get

$$ds^2_\rho / ds^2_{\rho'} = C_\rho / C_{\rho'}, \tag{10}$$

where we thought of irreducible representations and denoted the quadratic Casimirs C_ρ and $C_{\rho'}$ respectively.
For two faithful representations - i.e. which assigns only one element to each element in the group - the ratio of the volumes of the images of the of the group into the representations would be given as

$$\frac{Vol(\text{Image in}\rho)}{Vol(\text{Image in}\rho')} = (\frac{C_\rho}{C_{\rho'}})^{h/2}, \tag{11}$$

where h is the dimension of the Lie group.

Our Ratio of Adjoint to "Simplest" (or smallest) Quadratic Casimirs C_A/C_F:

For the simple Lie gruops in infinite series:

$$\frac{C_A}{C_F}|_{A_n} = \frac{2(n+1)^2}{n(n+2)} = \frac{2(n+1)^2}{(n+1)^2-1} = \frac{2}{1-\frac{1}{(n+1)^2}} \quad (12)$$

$$\frac{C_A}{C_{F\ vector}}|_{B_n} = \frac{2n-1}{n} = 2-\frac{1}{n} \quad (13)$$

$$\frac{C_A}{C_{F\ spinor}}|_{B_n} = \frac{2n-1}{\frac{2n^2+n}{8}} = \frac{16n-8}{n(2n+1)} \quad (14)$$

$$\frac{C_A}{C_F}|_{C_n} = \frac{n+1}{n/2+1/4} = \frac{4(n+1)}{2n+1} \quad (15)$$

$$\frac{C_A}{C_{F\ vector}}|_{D_n} = \frac{2(n-1)}{n-1/2} = \frac{4(n-1)}{2n-1} \quad (16)$$

$$\frac{C_A}{C_{F\ spinor}}|_{D_n} = \frac{2(n-1)}{\frac{2n^2-n}{8}} = \frac{16(n-1)}{n(2n-1)} \quad (17)$$

$$(18)$$

For the simple special Lie groups:

$$\frac{C_A}{C_F}|_{G_2} = \frac{4}{2} = 2 \quad (19)$$

$$\frac{C_A}{C_F}|_{F_4} = \frac{9}{6} = \frac{3}{2} \quad (20)$$

$$\frac{C_A}{C_F}|_{E_6} = \frac{12}{\frac{26}{3}} = \frac{18}{13} \quad (21)$$

$$\frac{C_A}{C_F}|_{E_7} = \frac{18}{\frac{57}{4}} = \frac{72}{57} = \frac{24}{19} \quad (22)$$

$$\frac{C_A}{C_F}|_{E_8} = \frac{30}{30} = 1 \quad (23)$$

Notation for the Lie group names

- n is the rank of the Lie algebra; it means the dimension of biggest Abelian subalgebra.
- A_n is the $SU(n+1)$, i.e. unitary Lie groups.
- B_n The odd dimensional orthogonal group $SO(2n+1)$ or its covering group $Spin(2n+1)$.
- C_n are symplectic Lie groups.
- D_n the even dimensional orthogonal groups $SO(2n)$ or the corresponding $Spin(2n)$.
- G_2, F_4, E_6, E_7, E_8 are special Lie groups.
- *vector* alludes to that we have attempted with the vector representation as to be the representation F that should be chosen to be the "smallest".
- spinor correspondingly mean we attempt with the spinro representation as F. The cross over as to which is smallest occur for $SO(8)$.

Extension of C_A/C_F to Semisimple Groups

If we have a semi-simple group $G_1 \times G_2 \times \cdots \times G_q$ represented by the reducible representation $F_1 \oplus F_2 \oplus ... \oplus F_q$ the metric in this full reducible representation ds is given as

$$ds^2 = ds_1^2 + ds_2^2 + ... + ds_q^2 \tag{24}$$

where the $(ds_1, ds_2, ..., ds_q)$ are the infinitesimal distances for the shift $dg = (dg_1, dg_2, ..., dg_q)$ in the semisimple group projected into the simple components of this group (locally).
If we would like to construct a quantity to be a "replacement" for C_A/C_F for the simple group to be applicable for the semisimple one $G_1 \times G_2 \times \cdots \times G_q$ and to keep the same relation to the volume ratio

$$\frac{Vol(\text{Image in}\rho)}{Vol(\text{Image in}\rho')} = (\frac{C_\rho}{C_{\rho'}})^{h/2}, \tag{25}$$

we must choose

$$\text{“}C_A/C_F\text{repl.''}(G_1 \times G_2 \times \cdots \times G_q) = C_A/C_F|_{G_1}^{h_1/h} * C_A/C_F|_{G_2}^{h_2/h} * \cdots C_A/C_F|_{G_q}^{h_q/h} \tag{26}$$

We shall choose the replacement for C_A/C_F to be in the semisimple case:

$$\text{“}C_A/C_F\text{repl.”}(G_1 \times G_2 \times \cdots \times G_q) = C_A/C_F|_{G_1}^{h_1/h} * C_A/C_F|_{G_2}^{h_2/h} * \cdots C_A/C_F|_{G_q}^{h_q/h} \quad (27)$$

where $C_A/C_F|_{G_1}^{h_1/h}$ etc. are the ratios C_A/C_F for the simple invariant subgroups G_1 etc. taken to the powers h_1/h etc. of their weight h_i/h counted in group dimension h_i relative to the full semi-simple group. Here h_i is the dimension of the simple subgroup G_i and the dimension of the semi-simple group $G_1 \times G_2 \times \cdots \times G_q$ is called $h = h_1 + h_2 + ... + h_q$.

That is to say we take in the semi-simple case the logarithmic average of the quantities C_A/C_F for the individual simple invariant subgroups weighted with their dimensions, i.e. each having the weight h_i/h.

Interpretation as Logarithmic Averaging

The quantity we suggest to use instead of C_A/C_F for semisimple Lie groups "C_A/C_Frepl."$(G_1 \times G_2 \times \cdots \times G_q)$ can be argued for as giving the $h/2$'th root of the volume ratio $Vol(A)/Vol(F)$ of the the representation image in the adjoint representation to that in the F-representation (chosen) to make this ratio maximal.(h is the total dimension of the semi-simple group). But it can also be concidered a "logarithmic averaging of the C_A/C_F's for the simple invariant subgroups G_i weighted with the Lie algebra dimensions",

$$\text{“}C_A/C_F\text{repl.”}(G_1 \times G_2 \times \cdots \times G_q) = \exp(\sum_{i=1,2,\ldots,q} \frac{dim(G_i)}{h} * \ln(G_A/C_F|_{G_i})). \tag{28}$$

Our Most ad hoc Rule: Abelian Subgroup

Our problem is -if we use the volume of the (faithful) image of the group - that we do not truly know what means the "adjoint" representation of a group with abelian invariant subgroups.
What to do, when asked for the adjoint representation of a $U(1)$ invariant subgroup?
We choose to choose a $U(1)$ counted with such a size, that it means the same volume for the "adjoint" representation as if we have a specified size say 2π together with the simple groups being taken as the images in the adjoint representations (i.e. with for say $SU(N)$ the N center-elements identified to only one).
In specified representations we can use our metric to even calculate the size of the abelian invariant subgroups it is only the concept of an "adjoint" representation, that makes no sense for these $U(1)$ invariant subgroups.

The Question of the $U(1)$ Charges, the ratio e_A^2/e_F^2 replacing C_A/C_F.

An Abelian group representation is charaterized by its "charge" e, which would be called e_F if the representation is called F and so on.
The in practise most important effect of the *group* rather than Lie algebra is the (quantiztion rule) for the representations, so it would be easiest for us to extract the e_A^2/e_F^2 effectively from the quantization rule for the $U(1)$-group, when we have some complicated group like say the Standard Model one $S(U(2) \times U(3))$ with Lie algebra as $U(1) \times SU(2) \times SU(3)$.
The Adjoint reprsentations of the simple groups such as say $SU(3)$ leads by exponentiation to a group isomorphic to $SU(3)/Z_3$, and can not allow any special fractional charge representation for the abelian group(s).
But e.g. the triplet representation of $U(3)$ is accompagnied by the abelian charge being $1/3$ of what it is for the adjoint representation.

It might be nice to have in mind what the significance of e.g. the factor 3 in the "charge ratio" e_A/e_F due to the $SU(3)$ contributes, namely a factor 9 before the 12th root is taken. Indeed $9^{\frac{1}{12}} = 1.200936955$. This means that obtaining the charge ratio due to the $SU(3)$ rather than there being no factor with $SO(5)$ ($\approx Spin(5)$) - not even a new Z_2 to divide out when we already have done so using $SU(2)$- we gain 20% in the goal quantity. The only 4.343315332% advantage of the semi-simple SO(5) over the SU(3) when combined to $SU(2) \times Spin(5)$ and $SU(2) \times SU(3)$ respectively is thus rather easily overshadowed by the effect of the e_A/e_F from the SU(3), which is of the order of 20 % in the goal quantity.

$U(2) = U(1) \times SU(2)/Z_2$, obtains the goal quantiy

$$\text{``}C_A/C_F\text{replacement for}U(2)'' = \left(2^2 * (\frac{8}{3})^3\right)^{\frac{1}{4}} = 2.951151786. \quad (29)$$

Indeed the advange of the Standard Model group over the so closely competing $U(2)$ (which would physically be that there were no strong interactions, but only the Weinberg Salam Glashow model say) is by $(\frac{2.957824511}{2.951151786} - 1) * 100\% = .2261058\%$

$$\text{“}C_A/C_F\text{replacement for} U(1)'' = (1)^1 = 1 \quad (30)$$
$$\text{“}C_A/C_F\text{replacement for}(U(1)\times SU(2))/Z_2'' \quad (31)$$
$$\left(2^2 * (8/3)^3\right) \quad (32)$$
$$\frac{1}{4} \quad (33)$$
$$= 2.951151786 \quad (34)$$
$$\text{“}C_A/C_F\text{replacement for}(U(1)\times SU(2)\times SU(3))/Z_6'' \quad (35)$$
$$= \left(6^2 * (8/3)^3 * (9/4)^8\right)^{\frac{1}{12}} \quad (36)$$
$$= 2.957824511 \quad (37)$$
$$\text{“}C_A/C_F\text{replacement for}(U(1)\times SU(2)\times SU(3)\times SU(5))/Z_{30}'' \quad (38)$$
$$\left(6^2 * (8/3)^3 * (9/4)^8 * (25/12)^{24}\right)^{\frac{1}{36}} \quad (39)$$
$$= 2.341513375 \quad (40)$$
$$(41)$$

We see that in this series of the most promising candidates with given centers of the covering groups for the simple Lie algebras the Standard Model lies at the (flat) maximum.

Dimension of Space-time,Also

The main point of my progress since last year is to say:
The choice of dimesionality of space time, that nature have made, - at least 3+1 for practical purpose - **can be considered also a choice of a group,** - and even a gauge group if we invoke general relativity -**namely say the Lorentz group or the Poincare group.** So if we have "game" or a "goal quanity" selecting by letting it be maximal the gauge group of the Standard Model, it is in principle possible to ask:
Which among the as Lorentz or Poincare group applicable groups get the highest "goal quanity" score? Which dimension wins the competition among Lorentz or Poincare groups?.

Continuation of start of section on "Space-time dimension, Also":

We would of course by extrapolation from the gauge group story (= previous work(with Don)) expect that Nature should againhave chosen the "winner".
It is my point now that - with only very little cheat - I can claim that indeed **Nature has chosen that dimension $d = 4$** (presumably meant to be the practical one, we see, and not necessarily the fundamental dimension, since our quantity could represent some stability against collapsing the dimension)**that gives the biggest score for the Poincare group!** (for the Lorentz groups $d = 4$ and $d = 3$ share the winner place !)

Development of Goal Quatities for dimension fitting.

In the present talk we shall ignore antropic principle arguments for what space time dimension should be and seek to get a statement that the experimental number of dimensions just maximizes some quantity, that is a relativly simple function of the group structure of say the Lorentz group, and which we then call a "goal quantity".

Making a “goal quantity” for Dimension a Two step Proceedure:

- 1) We first use the proposals in my work with Don Bennett to give a number - a goal quantity - for any Lie group.
- 2) We have to specify on which group we shall take and use the procedure of the previous work;shall it be the Lorentz group?, its covering group ? or somehow an attempt with the Poincare group ? :

Developping a "Goal quantity" for "predicting"(fitting) the Space Time dimension

A series of four proposals:

- a. Just take the Lorentz group and calculate for that the Dynkin index or rather the quantity which we already used as goal quantity in the previous work C_A/C_F. (Simple except for dimension $d = 2$ or smaller semi-simple Lorentz groups).

Second Development Proposal, b.

- b. We supplement in a somewhat ad hoc way the above *a.*, i.e. C_A/C_F by taking its $\frac{d+1}{d-1}$th power. The idea behind this proposal is that we think of the Poincare group instead of as under *a.* only on the Lorentz group part, though still in a crude way. This means we think of a group, which is the Poincare group, except that we for simplicity ignore that the translation generators do not commute with the Lorentz group part. Then we assign in accordance with the ad hoc rule used for the gauge group the Abelian sub-Lie-algebra a formal replacement 1 for the ratio of the quadratic Casimirs C_A/C_f: I.e. we put "$C_A/C_F|''_{Abelean\ formal} = 1$. Next we construct an "average" averaged *in a logarithmic way* (meaning that we average the logathms and then exponentiate again) weighted with the dimension of the Lie groups over all the dimensions of the Poincare Lie group. Since the dimension of the Lorentz group for d dimensional space-time is $\frac{d(d-1)}{2}$ while the Poincare group has dimension $\frac{d(d-1)}{2} + d = \frac{d(d+1)}{2}$ the logarithmic averageing means that we get

Continuing the b.-proposal.

That is to say we shall make a certain ad hoc partial inclusion of the Abelian dimensions in the Poincare groups.
To be concrete we here propose to say crudely: Let the poincare group have of course d "abelian" genrators or dimensions. Let the dimension of the Lorentz group be $d_{Lor} = d(d-1)/2$; then the total dimension of the Poincare group is
$d_{Poi} = d + d_{Lor} = d(d+1)/2$. If we crudely followed the idea of weighting proposed in the previous article [**?**] as if the d "abelian" generators were just simple cross product factors - and not as they really are: not quite usual by not commuting with the Lorentz generators - then since we formally are from this previous article suggested to use the *as if number 1 for the abelian groups*, we should use the quantity

$$(C_A/C_F)|_{Lor}^{\frac{d_{Lor}}{d_{Poi}}} = (C_A/C_F)|_{Lor}^{\frac{d-1}{d+1}} \tag{43}$$

as goal quantity.

- c. We could improve the above proposals for goal quantities *a.* or *b.* by including into the quadratic Casimir C_A for the adjoint representation also contributions from the translation generating generators, so as to define a quadratic Casimir for the whole Poincare group. This would mean that we for calculating our goal quantity would do as above but

$$\textbf{Replace}: C_A \to C_A + C_V, \tag{44}$$

where C_V is the vector representation quadratic Casimir, meaning the representation under which the translation generators transform under the Lorentz group. Since in the below table we in the lines denoted "no fermions" have taken the "small representation" F to be this vector representation V, this replacement means, that we replace the goal quantity ratio C_A/C_F like this:

$$\begin{array}{ccc} \textbf{(S)O(d),} & \textbf{"no spinors":} & \\ C_A/C_F = C_A/C_V & \to & (C_A + C_V)/C_F = C_A/C_F + 1 \\ \textbf{Spin(d),} & \textbf{"with spinors":} & \\ C_A/C_F & \to & (C_A + C_V)/C_F \end{array}$$

- d. To make the proposal *c.* a bit more “fair” we should at least say: Since we in *c.* considered a representation which were only faithfull w.r.t. the Lorentz subgroup of the Poincare group we should at least correct the quadratic Casimir - expected crudely to be “proportinal” to the number of dimensions of the (Lie)group - by a factor $\frac{d+1}{d-1}$ being the ratio of the dimension of the Poincare (Lie)group, $d + d(d-1)/2$ to that of actually faithfully represented Lorentz group $d(d-1)/2$. That is to say we should before forming the ratio of the improved C_A meaning $C_A + C_V$ (as calculated under *c.*) to C_F replace this C_F by $\frac{d+1}{d-1} * C_F$, i.e. we perform the replacement:

$$C_F \rightarrow C_F * \frac{d(d-1)/2 + d}{d(d-2)/2} = C_F * \frac{d+1}{d-1}. \qquad (51)$$

Inserted into $(C_A + C_V)/C_F$ from *c.* we obtain for the in this way made more “fair” approximate “goal quantity”

$$\text{“goal quantity”}|_{\text{no spinor}} = (C_A/C_F + 1) * \frac{d-1}{d+1}$$

The reader should consider these different proposals for a quantity to maximize (= use as goal quantiy) as rather closely related versions of a quantity suggested by a perhaps a bit vague idea being improved successively by treating the from our point of view a bit more difficult to treat Abelian part (the translation part of the Poincare group) at least in an approximate way. One should have in mind, that this somewhat vague basic idea behind is: The group selected by nature is the one that counted in a "normalization determined from the Lie algebra of the group" can be said to have a faithfull representation (F) the matrices of which move as little as possible, when the group element being represented move around in the group.
Let me at least clarify a bit, what is meant by this statement:
We think by representations as usual on linear representations, and thus it really means representation of the group by means of a homomorphism of the group into a group of matrices. The requirement of the representation being faithful then means, that this group of matrices shall actually be an isomorphic image of the original group. Now on a system of matrices we have a natural metric, namely the metric in which the distance between two

But let us stress that you can also look at the present work and the previous one in the following phenomenological philosophy: We wonder, why Nature has chosen just 4 (=3+1) dimensions and why Nature - at the present experimentally accessible scale at least - has chosen just the Standard Model group $S(U(2) \times U(3))$? Then we speculate that there might be some quantity characterizing groups, which measures how well they "are suited " to be the groups for Nature. And then we begin to *seek* that quantity as being some function defined on the class of abstract groups - i.e. giving a number for each abstract (Lie?) group - of course by proposing for ourselves at least various versions or ideas for what such a *relatively simple* function defined on the abstract Lie groups could be. Then the present works - this paper and the previous one[**?**] - represents the present status of the search: We found that with small variations the types of such functions representing the spirit of the *little motion of the "best" faithful representation*,i.e. essentially the largest C_A/C_F, turned out truly to bring Natures choices to be (essentially) the winners.
In this sense we may then claim that we have found by phenomenology that at least the "direction" of a quantity like

Dimension	Lorentz group, covering	Ratio C_A/C_F for spinor	Ratio C_A/C_F as no spinor	c.-quantity max c)	$\frac{d-1}{d+1}$	d.-quantity max d)
2^1	U(1)	-(formally 2)	-(formally 1)	4	1/3	4/3 =1.33
3	spin(3)	$\frac{8}{3} = 2.67$	1	$\frac{16}{3} = 5.3$	$\frac{2}{4}$	$\frac{8}{3} = 2.67$
4	$Spin(4)$ $SU(2)\times SU(2)$	$\frac{8}{3}$ =2.67	$\frac{4}{3}$	$\frac{14}{3}$ =4.67	$\frac{3}{5}$	$\frac{14}{5}$ =2.8
5	Spin(5)	$\frac{12}{5} = 2.4$	$\frac{3}{2} = 1.5$	4	$\frac{4}{6}$	$\frac{8}{3} = 2.67$
6	$Spin(6)$	$\frac{32}{15}$	$\frac{8}{5} = 1.6$	$\frac{52}{15} = 3.5$	$\frac{5}{7}$	$\frac{52}{21} = 2.5$
d odd	Spin(d)	$\frac{8(2n-1)}{n(2n+1)} = \frac{16(d-2)}{d(d-1)}$	$2 - 1/n = 2 - \frac{2}{d-1}$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{d-1}{d+1}$	$\frac{8(3d-5)}{d(d+1)}$
d even	$Spin(d)$	$\frac{16(d-2)}{d(d-1)}$	$\frac{4(n-1)}{2n-1} = \frac{2d-4}{d-1}$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{d-1}{d+1}$	$\frac{8(3d-5)}{d(d+1)}$
d ∞	Spin(d)	$\approx 16/d$	$\rightarrow 2$	$\approx 24/d$	$\rightarrow 1$	$\approx 24/d$ $\rightarrow 0$
d	Spin(d)	$\approx 16/d$	$\rightarrow 2$	$\approx 24/d$	$\rightarrow 1$	$\approx 24/d$

Caption:

We have put the goal-numbers for the third proposal *c* in which I (a bit more in detail) seek to make an analogon to the number used in the reference [**?**] in which we studied the gauge group of the Standard Model. The purpose of *c.* is to approximate using the *Poincare group* a bit more detailed, but still not by making a true representation of the Poincare group. I.e. it is still not truly the Poincare group we represent faithfully, but only the Lorentz group, or here in the table only the covering group $Spin(d)$ of the Lorentz group. However, I include in the column marked "*c.*, max c)" in the quadratic Casimir C_A of the Lorentz group an extra term coming from the structure constants describing the non-commutativity of the Lorentz group generators with the translation generators C_V so as to replace C_A in the starting expression of ours C_A/C_F by $C_A + C_V$. In the column marked "*d.*, max d) " we correct the ratio to be more "fair" by counting at least that because of truly faithfully represented part of the Poincare group in the representations, I use, has only dimension $d(d-1)/2$ (it is namely only the Lorentz group) while the full

Dimension	Lorentz group (covering)	Ratio C_F/C_A for spinor	Ratio C_A/C_F "no spinor"	*c.*-quantity max c)	*d.*-quantity max d)
2^2	U(1)	-(f.: 2)	-(f.: 1)	4	4/3=1.3
3	spin(3)	$\frac{8}{3} = 2.67$	1	$\frac{16}{3} = 5.33$	$\frac{8}{3} = 2.6$
4	$Spin(4) = SU(2) \times SU(2)$	$\frac{8}{3} = 2.67$	$\frac{4}{3}$	$\frac{14}{3} = 4.67$	$\frac{14}{5} = 2.8$
5	Spin(5)	$\frac{12}{5} = 2.4$	$\frac{3}{2} = 1.5$	4	$\frac{8}{3} = 2.6$
6	$Spin(6)$	$\frac{32}{15}$	$\frac{8}{5} = 1.6$	$\frac{52}{15} = 3.47$	$\frac{52}{21} = 2.4$
d odd	Spin(d)	$\frac{8(2n-1)}{n(2n+1)} = \frac{16(d-2)}{d(d-1)}$	$2 - 1/n = 2 - 2/(d-1)$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{8(3d-5)}{d(d+1)}$
d even	$Spin(d)$	$\frac{16(d-2)}{d(d-1)}$	$\frac{4(n-1)}{2n-1} = \frac{2d-4}{d-1}$	$\frac{8(3d-5)}{d(d-1)}$	$\frac{8(3d-5)}{d(d+1)}$
d odd $\to \infty$	Spin(d)	$\approx 16/d$	$\to 2$	$\approx 24/d$	$\approx 24/d$
d even $\to \infty$	Spin(d)	$\approx 16/d$	$\to 2$	$\approx 24/d$	$\approx 24/d$