Fermion Masses and Mixing from a Low Energy $SU(3)$ Family Symmetry Model

Albino Hernandez-Galeana

ESFM-IPN, MEXICO

BLED, July, 2014

$G \equiv SU(3) \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ "GAUGE GROUP"

$SU(3)$: Gauged Flavour Symmetry

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: Standard Model

The global symmetry in limit of all quarks and leptons massless, including R-handed neutrinos:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R}$$

$$\supset SU(3)_{q_L+u_R+d_R+l_L+e_R+\nu_R} \equiv SU(3)$$

Standard model fermions

Ordinary Fermions: $q_{iL}^o = \begin{pmatrix} u_{iL}^o \\ d_{iL}^o \end{pmatrix}$, $l_{iL}^o = \begin{pmatrix} \nu_{iL}^o \\ e_{iL}^o \end{pmatrix}$, $Q = T_{3L} + \frac{1}{2} Y$

$$\Psi_q^o = (3, 3, 2, \frac{1}{3})_L = \begin{pmatrix} q_{1L}^o \\ q_{2L}^o \\ q_{3L}^o \end{pmatrix} \quad , \quad \Psi_l^o = (3, 1, 2, -1)_L = \begin{pmatrix} l_{1L}^o \\ l_{2L}^o \\ l_{3L}^o \end{pmatrix}$$

$$\Psi_u^o = (3, 3, 1, \frac{4}{3})_R = \begin{pmatrix} u_R^o \\ c_R^o \\ t_R^o \end{pmatrix} \quad , \quad \Psi_d^o = (3, 3, 1, -\frac{2}{3})_R = \begin{pmatrix} d_R^o \\ s_R^o \\ b_R^o \end{pmatrix}$$

$$\Psi_e^o = (3, 1, 1, -2)_R = \begin{pmatrix} e_R^o \\ \mu_R^o \\ \tau_R^o \end{pmatrix}$$

EXTRA FERMIONS: $SU(2)_L$ SINGLETS

Sterile Neutrinos:
Anomaly conditions:
$[SU(3)]^3\ ,\ [SU(2)]^2\ SU(3)\ ,\ [U(1)]^2\ SU(3)$

Right Handed Neutrinos: $\Psi_\nu^o = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} = (3, 1, 1, 0)_R$

$N_L^o, N_R^o = (1, 1, 1, 0)$

Vector Like quarks:
$U_L^o, U_R^o = (1, 3, 1, \frac{4}{3}) \quad , \quad D_L^o, D_R^o = (1, 3, 1, -\frac{2}{3})$

Vector Like electron: $E_L^o, E_R^o = (1, 1, 1, -2)$

Sterile Neutrinos are new particles beyond the Standard Model which can mix with the standard active flavor neutrinos ν_e, ν_μ, ν_τ. In the standard three-neutrino mixing the three active flavor neutrinos are linear combinations of three massive neutrinos ν_1, ν_2, ν_3 with masses m_1, m_2, m_3. The squared mass differences $\Delta m^2_{21} \simeq 8 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} \simeq \Delta m^2_{32} \simeq 2 \times 10^{-3}$ eV^2 generate the neutrino oscillations observed in many solar, atmospheric and long-baseline experiments.

Short-baseline neutrino oscillation experiments, on the other hand, seem to provide some indications in favor of the possible existence of at least one light sterile neutrino in the eV scale.

The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U\,\bar{U}_L^o\,U_R^o + M_D\,\bar{D}_L^o\,D_R^o + M_E\,\bar{E}_L^o\,E_R^o + h.c.$$

and

$$m_D\,\bar{N}_L^o\,N_R^o + m_L\,\bar{N}_L^o\,(N_L^o)^c + m_R\,\bar{N}_R^o\,(N_R^o)^c + h.c$$

$$
\begin{aligned}
i\mathcal{L}_{int} = & \frac{g_H}{2}\left(\bar{f}_1^o \gamma_\mu f_1^o - \bar{f}_2^o \gamma_\mu f_2^o\right) Z_1^\mu \\
& + \frac{g_H}{2\sqrt{3}}\left(\bar{f}_1^o \gamma_\mu f_1^o + \bar{f}_2^o \gamma_\mu f_2^o - 2\bar{f}_3^o \gamma_\mu f_3^o\right) Z_2^\mu \\
& + \frac{g_H}{\sqrt{2}}\left(\bar{f}_1^o \gamma_\mu f_2^o Y_1^+ + \bar{f}_1^o \gamma_\mu f_3^o Y_2^+ + \bar{f}_2^o \gamma_\mu f_3^o Y_3^+ + h.c.\right)
\end{aligned}
$$

where g_H is the $SU(3)$ coupling constant, Z_1, Z_2 and Y_i^j , $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons.

Scalars introduced to break symmetries:

I. $SU(3)$: $\eta_2, \eta_3 = (3, 1, 1, 0)$

$$\eta_2 = \begin{pmatrix} \eta_{21}^o \\ \eta_{22}^o \\ \eta_{23}^o \end{pmatrix} \quad , \quad \eta_3 = \begin{pmatrix} \eta_{31}^o \\ \eta_{32}^o \\ \eta_{33}^o \end{pmatrix}$$

II. Electroweak symmetry:
$\Phi^u = (3, 1, 2, -1)$, $\Phi^d = (3, 1, 2, +1)$

$$\Phi^u = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_{u1} \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_{u2} \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_{u3} \end{pmatrix} \quad , \quad \Phi^d = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_{d1} \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_{d2} \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_{d3} \end{pmatrix}$$

$SU(3)$ FLAVOUR SYMMETRY BREAKING

$$\langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3)$$

The above scalar fields and VEV's break completely the $SU(3)$ flavor symmetry.

- $\eta_2 = (3, 1, 1, 0)\ ,\ < \eta_2 >^T = (0, \Lambda_2, 0)$

$$\frac{g_{H_2}^2 \Lambda_2^2}{2}(Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2Z_1 \frac{Z_2}{\sqrt{3}})$$

- $\eta_3 = (3, 1, 1, 0)\ ,\ < \eta_3 >^T = (0, 0, \Lambda_3)$

$$\frac{g_{H_3}^2 \Lambda_3^2}{2}(Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$$

$SU(3)$ GAUGE BOSON MASSES:

$$M_1^2\, Y_1^+ Y_1^- + \frac{M_1^2}{2}\, Z_1^2 + (\frac{4}{3}\, M_2^2 + \frac{1}{3}\, M_1^2)\frac{Z_2^2}{2} - \frac{M_1^2}{\sqrt{3}}\, Z_1\, Z_2 + M_2^2\, Y_2^+ Y_2^- + (M_1^2 + M_2^2)\, Y_3^+ Y_3^-$$

$$M_1^2 = \frac{g_{H_2}^2 \Lambda_2^2}{2} \quad , \quad M_2^2 = \frac{g_{H_3}^2 \Lambda_3^2}{2} \quad , \quad M_3^2 = M_1^2 + M_2^2$$

	Z_1	Z_2
Z_1	M_1^2	$-\frac{M_1^2}{\sqrt{3}}$
Z_2	$-\frac{M_1^2}{\sqrt{3}}$	$(\frac{4}{3}\, M_2^2 + \frac{1}{3}\, M_1^2)$

From the diagonalization of the $Z_1 - Z_2$ squared mass matrix:

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix}$$

$$\cos\phi \, \sin\phi = \frac{\sqrt{3}}{4} \, \frac{M_1^2}{\sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2}}$$

$$M_\mp^2 = \tfrac{2}{3} \left(M_1^2 + M_2^2 \mp \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2} \right)$$

$$M_1^2 \, Y_1^+ Y_1^- + M_-^2 \, \tfrac{Z_-^2}{2} + M_+^2 \, \tfrac{Z_+^2}{2} + M_2^2 \, Y_2^+ Y_2^- + (M_1^2 + M_2^2) \, Y_3^+ Y_3^-$$

with the hierarchy $M_1, M_2 \gg M_W$.

ELECTROWEAK SYMMETRY BREAKING

In this scenario we introduce two triplets of $SU(2)_L$ Higgs doublets: $\Phi^u = (3, 1, 2, -1)$ and $\Phi^d = (3, 1, 2, +1)$, with the

$$\text{VEV´s:} \quad \langle \Phi^u \rangle = \begin{pmatrix} \langle \Phi^u_1 \rangle \\ \langle \Phi^u_2 \rangle \\ \langle \Phi^u_3 \rangle \end{pmatrix} \quad , \quad \langle \Phi^d \rangle = \begin{pmatrix} \langle \Phi^d_1 \rangle \\ \langle \Phi^d_2 \rangle \\ \langle \Phi^d_3 \rangle \end{pmatrix} \ ,$$

$$\text{where} \quad \langle \Phi^u_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle \Phi^d_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix} \ .$$

Contribute to the W and Z boson masses:

$$\frac{g^2}{4} (v_u^2 + v_d^2) W^+ W^- + \frac{(g^2 + g'^2)}{8} (v_u^2 + v_d^2) Z_o^2$$

$v_u^2 = v_{u1}^2 + v_{u2}^2 + v_{u3}^2$, $v_d^2 = v_{d1}^2 + v_{d2}^2 + v_{d3}^2$. Hence, if we define $M_W = \frac{1}{2} g\, v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

Now we describe briefly the procedure to get the masses for quarks and leptons up to one loop corrections.
Before "Electroweak Symmetry Breaking"(EWSB) all ordinary, "Standard Model"(SM) fermions remain massless, and the quarks and leptons global symmetry is:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R}$$

Charged fermion masses:

$$h_e \, \bar{\psi}_l^o \, \Phi^d \, E_R^o \, + \, h_{e2} \, \bar{\psi}_e^o \, \eta_2 \, E_L^o \, + \, h_{e3} \, \bar{\psi}_e^o \, \eta_3 \, E_L^o \, + \, M_E \, \bar{E}_L^o \, E_R^o \, + h.c$$

where M is a free mass parameter ($M \, \bar{E}_L^o \, E_R^o$ is gauge invariant), h_e, h_{e2} and h_{e3} are Yukawa coupling constants.

u-quarks and neutrinos coupled only to Φ^u

d-quarks and charged leptons couple only to Φ^d

In the gauge basis ${\psi^o_{L,R}}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms read as $\bar{\psi}^o_L \mathcal{M}^o \psi^o_R + h.c$, where

	e^o_R	μ^o_R	τ^o_R	E^o_R
$\overline{e^o_L}$	0	0	0	$h\, v_{d1}$
$\overline{\mu^o_L}$	0	0	0	$h\, v_{d2}$
$\overline{\tau^o_L}$	0	0	0	$h\, v_{d3}$
$\overline{E^o_L}$	0	$h_{e2}\Lambda_2$	$h_{e3}\Lambda_3$	M_E

Table: Tree level Dirac mass matrix $\mathcal{M}^o$

Notice that $\mathcal{M}^o$ has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call $\mathcal{M}^o$ a **"Dirac See-saw"** mass matrix. $\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi^o_{L,R} = V^o_{L,R}\, \chi_{L,R}$.

$$V_L^{o\,T} \mathcal{M}^o \mathcal{M}^{o\,T}\ V_L^o = V_R^{o\,T} \mathcal{M}^{o\,T} \mathcal{M}^o\ V_R^o = Diag(0, 0, \lambda_-, \lambda_+)$$

$$V_L^{o\,T} \mathcal{M}^o\ V_R^o = Diag(0, 0, -\sqrt{\lambda_-}, \sqrt{\lambda_+})$$

where λ_- and λ_+ are the nonzero eigenvalues, $\sqrt{\lambda_+}$ being the fourth heavy fermion mass, and $\sqrt{\lambda_-}$ of the order of the top, bottom and tau mass for u, d and e fermions, respectively.

Tree level Dirac Neutrino masses

$$h_\nu \, \overline{\Psi^o_l} \, \Phi^u \, N^o_R \; + \; h_{\nu 2} \, \overline{\Psi^o_\nu} \, \eta_2 \, N^o_L \; + \; h_{\nu 3} \, \overline{\Psi^o_\nu} \, \eta_3 \, N^o_L \; + \; M_N \, \overline{N^o_L} \, N^o_R \; + h.c$$

h_ν, $h_{\nu 2}$ and $h_{\nu 3}$ are Yukawa couplings, and M_N a Dirac type invariant neutrino mass for the sterile neutrino $N^o_{L,R}$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi^{o\,T}_{\nu L,R} = (\nu^o_e, \nu^o_\mu, \nu^o_\tau, N^o)_{L,R}$, the mass terms

$$h_\nu \, \left[v_{u1} \, \overline{\nu^o_{eL}} + v_{u2} \, \overline{\nu^o_{\mu L}} + v_{u3} \, \overline{\nu^o_{\tau L}} \right] \, N^o_R + \left[h_{\nu 2} \, \Lambda_2 \, \overline{\nu^o_{\mu R}} + h_{\nu 3} \, \Lambda_3 \, \overline{\nu^o_{\tau R}} \right] \, N^o_L$$
$$+ M_N \, \overline{N^o_L} \, N^o_R + h.c.$$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_D\, v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_D\, v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_D\, v_{u3}$
$\overline{N^o_L}$	0	$h_{\nu 2}\Lambda_2$	$h_{\nu 3}\Lambda_3$	M_N

Table: Tree level Dirac mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, \nu^o_{jR}$

Tree level Majorana masses:

Since $N^o_{L,R}$ are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L \, \overline{\Psi^o_l} \, \Phi^u (N^o_L)^c \quad + \quad m_L \, \overline{N^o_L} \, (N^o_L)^c$$
$$+ h_{2R} \, \overline{\Psi^o_\nu} \, \eta_2 \, (N^o_R)^c \; + \; h_{3R} \, \overline{\Psi^o_\nu} \, \eta_3 \, (N^o_R)^c \; + \; m_R \, \overline{N^o_R} \, (N^o_R)^c + h.c \, ,$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L \, \left[v_{u1} \, \overline{\nu^o_{eL}} + v_{u2} \, \overline{\nu^o_{\mu L}} + v_{u3} \, \overline{\nu^o_{\tau L}} \right] \, (N^o_L)^c \quad + \quad m_L \, \overline{N^o_L} \, (N^o_L)^c$$
$$+ \left[h_{2R} \, \Lambda_2 \, \overline{\nu^o_{\mu R}} + h_{3R} \, \Lambda_3 \, \overline{\nu^o_{\tau R}} \right] \, (N^o_R)^c \; + \; m_R \, \overline{N^o_R} \, (N^o_R)^c + h.c.$$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	0	0	0	$h_L v_{u1}$
$\nu^o_{\mu L}$	0	0	0	$h_L v_{u2}$
$\nu^o_{\tau L}$	0	0	0	$h_L v_{u3}$
N^o_L	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L

Table: Tree level L-handed Majorana mass terms $m_{ij}\ \bar{\nu}^o_{iL}\ (\nu^o_{jL})^c$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	0	0	$h_{2R}\Lambda_2$
$\nu^o_{\tau R}$	0	0	0	$h_{3R}\Lambda_3$
N^o_R	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	m_R

Table: Tree level R-handed Majorana mass terms $m_{ij}\ \bar{\nu}^o_{iR}\ (\nu^o_{jR})^T$

Tree level Dirac and Majorana Neutrino masses

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	$(N^o_L)^c$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	0	0	0	$h_L v_{u1}$	$h_D v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	0	0	0	$h_L v_{u2}$	$h_D v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	0	0	0	$h_L v_{u3}$	$h_D v_{u3}$
$\overline{(\nu^o_{eR})^c}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	0	0	0	$h_{\nu 2}\Lambda_2$	$h_{2R}\Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	0	0	0	$h_{\nu 3}\Lambda_3$	$h_{3R}\Lambda_3$
$\overline{N^o_L}$	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	0	$h_{\nu 2}\Lambda_2$	$h_{\nu 3}\Lambda_3$	m_L	M_N
$\overline{(N^o_R)^c}$	$h_D v_{u1}$	$h_D v_{u2}$	$h_D v_{u3}$	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	M_N	m_R

Table: Tree Level Majorana mass matrix $\mathcal{M}^o_\nu$ in the Ψ^o_ν basis

Diagonalization of $\mathcal{M}_\nu^{(o)}$ yields four zero eigenvalues (five zero mass eigenvalues also possible if $a\,b = \alpha\,\beta$), associated to the neutrino fields: $ap = \sqrt{a_1^2 + a_2^2}$

$$\frac{a_2}{ap}\,\nu^o_{eL} - \frac{a_1}{ap}\,\nu^o_{\mu L} \quad , \quad \frac{a_1\,a_3}{ap\,a}\,\nu^o_{eL} + \frac{a_2\,a_3}{ap\,a}\,\nu^o_{\mu L} - \frac{a_p}{a}\,\nu^o_{\tau L},$$

$$\nu^o_{eR} \quad , \quad \frac{b_3}{b}\,\nu^o_{\mu R} - \frac{b_2}{b}\,\nu^o_{\tau R}$$

Assuming for simplicity, $\frac{h_2}{h_{2R}} = \frac{h_3}{h_{3R}}$, the Characteristic Polynomial for the nonzero eigenvalues of $\mathcal{M}_\nu^o$ reduce to the one of the matrix m_4, where

$$m_4 = \begin{pmatrix} 0 & 0 & \alpha & a \\ 0 & 0 & b & \beta \\ \alpha & b & m_L & m_N \\ a & \beta & m_N & m_R \end{pmatrix} \quad , \quad U_4 = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix}$$

$$U_4^T \, m_4 \, U_4 = Diag(m_5^o, m_6^o, m_7^o, m_8^o)$$

$$(U_\nu^o)^T \, \mathcal{M}_\nu^o \, U_\nu^o = Diag(0, 0, 0, 0, m_5^o, m_6^o, m_7^o, m_8^o)$$

Tree level Dirac and Majorana Neutrino masses

$$U_\nu^o = \begin{pmatrix} \frac{a_2}{ap} & \frac{a_1\, a_3}{a\, ap} & 0 & 0 & \frac{a_1}{a}u_{11} & \frac{a_1}{a}u_{12} & \frac{a_1}{a}u_{13} & \frac{a_1}{a}u_{14} \\ -\frac{a_1}{ap} & \frac{a_2\, a_3}{a\, ap} & 0 & 0 & \frac{a_2}{a}u_{11} & \frac{a_2}{a}u_{12} & \frac{a_2}{a}u_{13} & \frac{a_2}{a}u_{14} \\ 0 & -\frac{ap}{a} & 0 & 0 & \frac{a_3}{a}u_{11} & \frac{a_3}{a}u_{12} & \frac{a_3}{a}u_{13} & \frac{a_3}{a}u_{14} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{b_3}{b} & \frac{b_2}{b}u_{21} & \frac{b_2}{b}u_{22} & \frac{b_2}{b}u_{23} & \frac{b_2}{b}u_{24} \\ 0 & 0 & 0 & -\frac{b_2}{b} & \frac{b_3}{b}u_{21} & \frac{b_3}{b}u_{22} & \frac{b_3}{b}u_{23} & \frac{b_3}{b}u_{24} \\ 0 & 0 & 0 & 0 & u_{31} & u_{32} & u_{33} & u_{34} \\ 0 & 0 & 0 & 0 & u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix}$$

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R}$$

Therefore, in this scenario light fermion masses, including neutrinos, may get extremely small masses from radiative corrections mediated by the $SU(3)$ heavy gauge bosons.

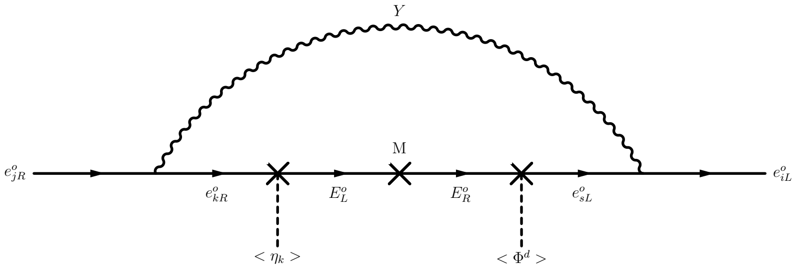
$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

M_Y is the gauge boson mass, c_Y is coupling constant, $m_3^o = -\sqrt{\lambda_-}$ and $m_4^o = \sqrt{\lambda_+}$, and $f(x, y) = \frac{x^2}{x^2 - y^2} \ln \frac{x^2}{y^2}$.

$$\sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i \, b_j \, M}{\lambda_+ - \lambda_-} \, F(M_Y) \, ,$$

$i, j = 1, 2, 3$, and $\quad F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_+} \ln \frac{M_Y^2}{\lambda_+} - \frac{M_Y^2}{M_Y^2 - \lambda_-} \ln \frac{M_Y^2}{\lambda_-}$.

Fig.1: Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$



	e^o_R	μ^o_R	τ^o_R	E^o_R
$\overline{e^o_L}$	R_{11}	R_{12}	R_{13}	0
$\overline{\mu^o_L}$	0	R_{22}	R_{23}	0
$\overline{\tau^o_L}$	0	R_{32}	R_{33}	0
$\overline{E^o_L}$	0	0	0	0

Table: Generic one loop diagram contribution to the mass term $m_{ij}\, \bar{e}^o_{iL} e^o_{jR}$

$$\mathcal{M}_1^o = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 \\ 0 & R_{22} & R_{23} & 0 \\ 0 & R_{32} & R_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} ,$$

$$\bar{\psi}_L^o \mathcal{M}^o \, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \, \psi_R^o = \bar{\chi}_L \, \mathcal{M} \, \chi_R \, ,$$

$$\mathcal{M} \equiv \left[Diag(0, 0, -\sqrt{\lambda_-}, \sqrt{\lambda_+}) + V_L^{o\,T} \mathcal{M}_1^o \, V_R^o \right] ,$$

Generic mass matrix up to one loop for quarks and charged leptons.

Explicitly

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta \, m_{13} & s_\beta \, m_{13} \\ m_{21} & m_{22} & c_\beta \, m_{23} & s_\beta \, m_{23} \\ c_\alpha \, m_{31} & c_\alpha \, m_{32} & -\sqrt{\lambda_-} + c_\alpha c_\beta \, m_{33} & c_\alpha s_\beta \, m_{33} \\ s_\alpha \, m_{31} & s_\alpha \, m_{32} & s_\alpha c_\beta \, m_{33} & \sqrt{\lambda_+} + s_\alpha s_\beta \, m_{33} \end{pmatrix}$$

The diagonalization of $\mathcal{M}$ yields the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\,\Psi_{L,R}$; $\bar{\chi}_L\,\mathcal{M}\,\chi_R = \bar{\Psi}_L\,{V^{(1)}_L}^T \mathcal{M}\,V^{(1)}_R\,\Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$ {V^{(1)}_L}^T \mathcal{M}\,\mathcal{M}^T\;V^{(1)}_L = {V^{(1)}_R}^T \mathcal{M}^T\,\mathcal{M}\;V^{(1)}_R = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\;, $$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$ \psi_L^o = V_L^o\,V_L^{(1)}\,\Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\,V_R^{(1)}\,\Psi_R $$

Quark Mixing and non-unitary $(V_{CKM})_{4\times 4}$

Recall that vector like quarks are $SU(2)_L$ weak singlets, and then they do not couple to W boson in the interaction basis. So, the interaction of ordinary quarks ${f_{uL}^o}^T = (u^o, c^o, t^o)_L$ and ${f_{dL}^o}^T = (d^o, s^o, b^o)_L$ to the W charged gauge boson is

$$\frac{g}{\sqrt{2}}\, \bar{f^o}_{uL} \gamma_\mu f_{dL}^o W^{+\mu}$$

$$= \frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\; {V_{uL}^{(1)}}^T\; [(V_{uL}^o)_{3\times 4}]^T\; (V_{dL}^o)_{3\times 4}\; V_{dL}^{(1)}\; \gamma_\mu \Psi_{dL}\; W^{+\mu}\;,$$

with g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times 4} = {V_{uL}^{(1)}}^T\; [(V_{uL}^o)_{3\times 4}]^T\; (V_{dL}^o)_{3\times 4}\; V_{dL}^{(1)}\;.$$

It is important to comment here that the scalar fields introduced to break the symmetries in the model: Φ^u, Φ^d, η_2 and η_3, do not couple ordinary fermions directly. So, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which as is shown in $(V_{CKM})_{4\times 4}$, may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.

One loop neutrino masses

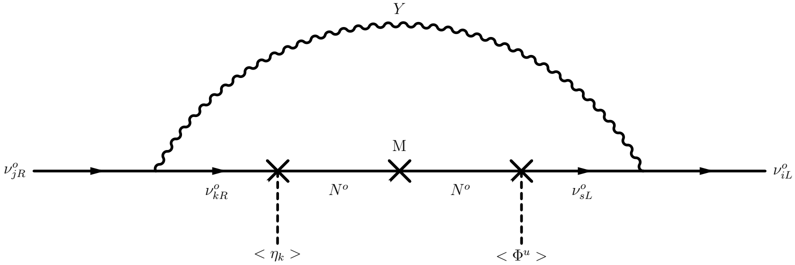
One loop Dirac Neutrino masses
Light neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 2, as well as L-handed and R-handed Majorana masses from Fig. 3 and Fig. 4, respectively. The contribution from these diagrams read

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=5,6,7,8} m_k^o \, U_{ik}^o U_{jk}^o f(M_Y, m_k^o) = c_Y \, m_\nu(M_Y)_{ij} \quad ,$$

with

$$m_\nu(M_Y)_{ij} = \frac{\alpha_H}{\pi} \sum_{k=5,6,7,8} m_k^o \, U_{ik}^o U_{jk}^o \, f(M_Y, m_k^o), \quad , \quad \alpha_H = \frac{g_H^2}{4\pi}$$

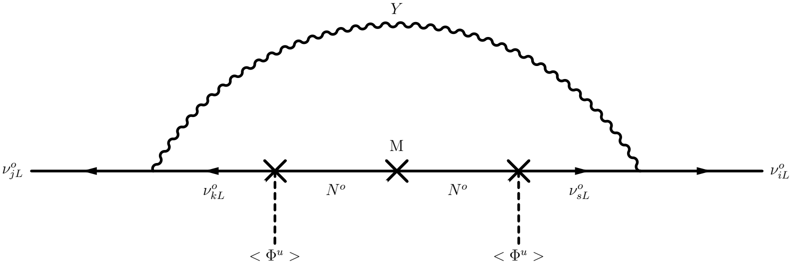
Fig.2: Generic one loop diagram contribution to the Dirac mass term $m_{ij}\ \bar{\nu}^o_{iL}\nu^o_{jR}$. M $= M_N, m_L, m_R$



	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	$D_{\nu\,14}$	$D_{\nu\,15}$	$D_{\nu\,16}$	0
$\overline{\nu^o_{\mu L}}$	0	$D_{\nu\,25}$	$D_{\nu\,26}$	0
$\overline{\nu^o_{\tau L}}$	0	$D_{\nu\,35}$	$D_{\nu\,36}$	0
$\overline{N^o_L}$	0	0	0	0

Table: One loop Dirac neutrino mass terms $m^{\nu}_{ij}\ \bar{\nu}^o_{iL}\ \nu^o_{jR}$

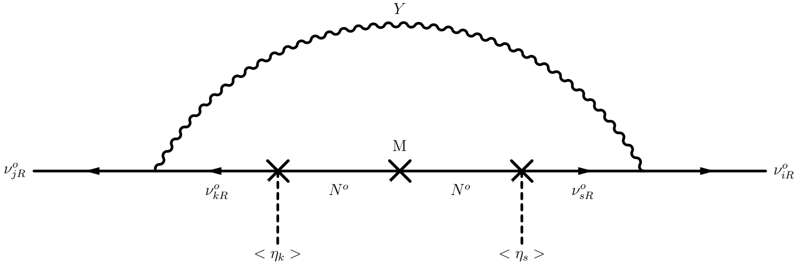
Fig.3: Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij}\,\bar{\nu}^o_{iL}(\nu^o_{jL})^T$. M $= M_N, m_L, m_R$



	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	0
$\nu^o_{\mu L}$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0
$\nu^o_{\tau L}$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0
N^o_L	0	0	0	0

Table: One loop L-handed neutrino Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, (\nu^o_{jL})^T$

Fig.4: Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij}\,\bar{\nu}^o_{iR}(\nu^o_{jR})^T$. M $= M_N, m_L, m_R$



	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	$R_{\nu\,55}$	$R_{\nu\,56}$	0
$\nu^o_{\tau R}$	0	$R_{\nu\,56}$	$R_{\nu\,66}$	0
N^o_R	0	0	0	0

Table: One loop R-handed neutrino Majorana mass terms
$m_{ij}\, \bar{\nu}^o_{iR}\, (\nu^o_{jR})^T$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	$(N^o_L)^c$	N^o_R
$\overline{\nu^o_{eL}}$	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	$D_{\nu\,14}$	$D_{\nu\,15}$	$D_{\nu\,16}$	0	0
$\overline{\nu^o_{\mu L}}$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0	$D_{\nu\,25}$	$D_{\nu\,26}$	0	0
$\overline{\nu^o_{\tau L}}$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0	$D_{\nu\,35}$	$D_{\nu\,36}$	0	0
$\overline{(\nu^o_{eR})^c}$	$D_{\nu\,14}$	0	0	0	0	0	0	0
$\overline{(\nu^o_{\mu R})^c}$	$D_{\nu\,15}$	$D_{\nu\,25}$	$D_{\nu\,35}$	0	$R_{\nu\,55}$	$R_{\nu\,56}$	0	0
$\overline{(\nu^o_{\tau R})^c}$	$D_{\nu\,16}$	$D_{\nu\,26}$	$D_{\nu\,36}$	0	$R_{\nu\,56}$	$R_{\nu\,66}$	0	0
$\overline{N^o_L}$	0	0	0	0	0	0	0	0
$\overline{(N^o_R)^c}$	0	0	0	0	0	0	0	0

Table: One Loop Majorana mass matrix $\mathcal{M}^o_{1\,\nu}$ in the Ψ^o_ν basis

NEUTRINO MASS MATRIX UP TO ONE LOOP

$$\mathcal{M}_\nu = (U_\nu^o)^T \, \mathcal{M}^o_{1\,\nu} \, U_\nu^o + Diag(0,0,0,0,m_5^o,m_6^o,m_7^o,m_8^o)$$

$$\mathcal{M}_\nu = \begin{pmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\ N_{12} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\ N_{13} & N_{23} & 0 & 0 & N_{35} & N_{36} & N_{37} & N_{38} \\ N_{14} & N_{24} & 0 & N_{44} & N_{45} & N_{46} & N_{47} & N_{48} \\ N_{15} & N_{25} & N_{35} & N_{45} & N_{55}+m_5^o & N_{56} & N_{57} & N_{58} \\ N_{16} & N_{26} & N_{36} & N_{46} & N_{56} & N_{66}+m_6^o & N_{67} & N_{68} \\ N_{17} & N_{27} & N_{37} & N_{47} & N_{57} & N_{67} & N_{77}+m_7^o & N_{78} \\ N_{18} & N_{28} & N_{38} & N_{48} & N_{58} & N_{68} & N_{78} & N_{88}+m_8^o \end{pmatrix}$$

How much consistent is this scenario?

Preliminary numerical results

Performing a numerical analysis of the free space parameter at the M_Z scale: Zhi-zhong Xing,He Zhang and Shun Zhou, Phys. Rev. D 86, 013013 (2012).

SU(3) PARAMETERS: $\frac{\alpha_H}{\pi} = 0.05$, $M_1 = 10$ TeV , $M_2 = 1$ TeV

$$\frac{v_{1d}}{v_{2d}} \simeq 0.099 \quad , \quad \frac{\sqrt{v_{1d}^2 + v_{2d}^2}}{v_{3d}} \simeq 0.543$$

$$\frac{v_{1u}}{v_{2u}} \simeq 0.1 \quad , \quad \frac{\sqrt{v_{1u}^2 + v_{2u}^2}}{v_{3u}} \simeq 0.5$$

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 7933.76 \\ 0 & 0 & 0 & 79337.6 \\ 0 & 0 & 0 & 159467. \\ 0 & 1.18613 \times 10^6 & -841128. & 374542. \end{pmatrix} \text{MeV} ,$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} -1.40509 & 187.442 & -66.8139 & -255.74 \\ -0.125675 & -609.844 & 408.793 & 1564.71 \\ -0.062809 & -1197.67 & -172100. & 1825.79 \\ -0.001885 & -35.9461 & 14.3165 & 1.502 \times 10^6 \end{pmatrix} \text{MeV}$$

and the u-quark masses

$$(m_u \, , \; m_c \, , \; m_t \, , \; M_U) = (1.3802 \, , \; 640.801 \, , \; 172105 \, , \; 1.502 \times 10^6) \, \text{MeV}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 1740.94 \\ 0 & 0 & 0 & 17442.3 \\ 0 & 0 & 0 & 32265.8 \\ 0 & 70019.9 & -41383.4 & 910004 \end{pmatrix} \text{ MeV}$$

$$\mathcal{M}_d = \begin{pmatrix} 3.09609 & 28.1593 & -47.4565 & -4.23475 \\ 0.271539 & -40.5966 & 215.617 & 19.2404 \\ 0.147401 & -176.235 & -2846.26 & 37.484 \\ 0.005900 & -7.05504 & 16.8159 & 914365. \end{pmatrix} \text{ MeV}$$

$$(\, m_d \,,\; m_s \,,\; m_b \,,\; M_D \,) = (\, 2.82 \,,\; 61.9998 \,,\; 2860 \,,\; 914365 \,) \text{ MeV}$$

non-unitary $(V_{CKM})_{4\times 4}$

and the quark mixing

$$(V_{CKM})_{4\times 4} = \begin{pmatrix} 0.974352 & 0.225001 & 0.003647 & 0.000410 \\ -0.224958 & 0.973502 & 0.041031 & -0.001417 \\ -0.005632 & 0.040662 & -0.997868 & -0.039994 \\ 0.000576 & -0.002325 & 0.031130 & 0.001251 \end{pmatrix}$$

Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 28340.3 \\ 0 & 0 & 0 & 283940. \\ 0 & 0 & 0 & 525249. \\ 0 & 17105.4 & -11570.9 & 5.94752 \times 10^6 \end{pmatrix} \text{MeV}$$

$$\mathcal{M}_e = \begin{pmatrix} -0.499137 & 29.7086 & -43.9181 & -0.15097 \\ -0.043776 & -72.8148 & 238.953 & 0.821414 \\ -0.023663 & -183.913 & -1720.65 & 1.18425 \\ -0.002378 & -18.4839 & 34.6241 & 5.977 \times 10^6 \end{pmatrix} \text{MeV}$$

fit the charged lepton masses:

$$(m_e\,,\, m_\mu\,,\, m_\tau\,,\, M_E) = (0.486\,,\, 102.7\,,\, 1746.17\,,\, 5.977 \times 10^6\,)\,\text{MeV}$$

Neutrino masses and mixing:

$$\mathcal{M}_\nu^o = \text{eV}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 53594.6 & 44137.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 535946. & 441372. \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.07\times 10^6 & 887147. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.80\times 10^6 & 1.49\times 10^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & -886604. & -730152. \\ 53594.6 & 535946. & 1.07\times 10^6 & 0 & 1.8097\times 10^6 & -886604. & 1.97\times 10^8 & 4.88\times 10^8 \\ 44137.2 & 441372. & 887147. & 0 & 1.49\times 10^6 & -730152. & 4.88\times 10^8 & 7.02\times 10^8 \end{pmatrix}$$

$\mathcal{M}_\nu =$ eV

$$\begin{pmatrix}
-0.011 & 0.052 & 0.0227 & -0.0878 & -0.069 & 0.1674 & -0.0016 & 0.0004 \\
0.0527 & -0.036 & 0.002 & 0.068 & 0.043 & -0.748 & 0.007 & -0.002 \\
0.0227 & 0.002 & 0. & 0. & 0.0008 & 0.0005 & -5.2\times 10^{-6} & 1.5\times 10^{-6} \\
-0.087 & 0.068 & 0. & -0.125 & -0.121 & 1.282 & -0.012 & 0.003 \\
-0.069 & 0.043 & 0.0008 & -0.121 & 3.206 & -0.743 & 0.0074 & -0.0021 \\
0.1674 & -0.748 & 0.0005 & 1.282 & -0.743 & 1749.96 & 0.0003 & -0.0001 \\
-0.001 & 0.007 & -5.2\times 10^{-6} & -0.012 & 0.0074 & 0.0003 & -1.\times 10^{8} & 1.1\times 10^{-6} \\
0.0004 & -0.002 & 1.5\times 10^{-6} & 0.003 & -0.002 & -0.0001 & 1.1\times 10^{-6} & 1.\times 10^{9}
\end{pmatrix}$$

neutrino mass eigenvalues

$$(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = \text{eV}$$
$$(\,0,\ 0.0085\,,\ 0.049\,,\ 0.22\,,\ 3.21\,,\ 1749.96\,,\ 1\times 10^8\,,\ 1\times 10^9\,)\,,$$

the squared mass differences

$$m_2^2 - m_1^2 \approx 0.0000723\,\text{eV}^2 \quad , \quad m_3^2 - m_1^2 \approx 0.0024\,\text{eV}^2$$

$$m_4^2 - m_1^2 \approx 0.0492\,\text{eV}^2 \quad , \quad m_5^2 - m_1^2 \approx 10.3182\,\text{eV}^2$$

Lepton mixing matrix U_{PMNS}

$$U_{PMNS} = \begin{pmatrix} 0.2104 & 0.3520 & 0.8658 & -0.2861 & 0.0060 & 0.0053 & 0.00005 & 0.00001 \\ -0.8282 & 0.0030 & 0.0186 & -0.5478 & 0.1038 & -0.0507 & -0.0005 & -0.0001 \\ 0.0807 & 0.0041 & 0.0052 & 0.0881 & 0.8475 & -0.5074 & -0.0050 & -0.0014 \\ 0.0034 & 0.0003 & 0.0011 & -0.0021 & -0.0857 & 0.0512 & 0.0005 & 0.0001 \end{pmatrix}$$

Flavour Violating Processes:

strongly suppressed from experimental data

	K	B	B_s	D
m_M [MeV]	497.614	5279.58	5366.77	1864.86
F_M [MeV]	156	220	205	191
B_M	0.571	0.87	0.86	0.87
$(\Delta m_M)_{EXP}$ [MeV]	3.484×10^{-12}	3.337×10^{-10}	1.164×10^{-8}	1.4×10^{-11}

Table: Masses, decay coupling constants, B parameters and Δm_M experimental bounds for neutral mesons.

Miriam A. Fuentes-González:

Meson K: $\Delta m_K = 8.50832 \times 10^{-8}$ MeV

Meson B: $\Delta m_B = 3.4375 \times 10^{-6}$ MeV

Meson B_s: $\Delta m_{B_s} = 8.25668 \times 10^{-5}$ MeV

Meson D: $\Delta m_D = 2.15158 \times 10^{-6}$ MeV

PREDICTIONS:

1. NEW MASSIVE GAUGE BOSONS AND SCALARS, (few-25) TeV
2. VECTOR LIKE QUARKS AND CHARGED LEPTONS,
   (1 TeV - 20 TeV)
3. STERILE NEUTRINOS:
   Light (eV), May be two
   Heavy (KeV-GeV),