

Novel String Field Theory Solving String Theory Liberating Left and Right Movers

Work by :

Holger Bech Nielsen

Masao Ninomiya

Our SFT model Equivalent to String Theory

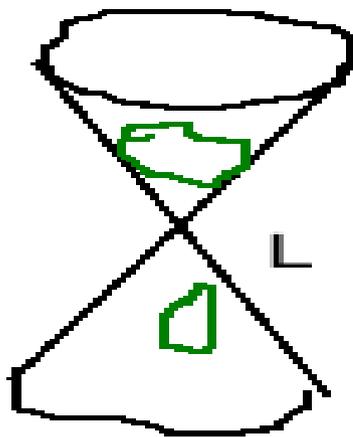
The final SFT-model of mine and Ninomiyas is described by the Fock space for massless non-interacting scalar bosons in 25 +1 dimensions. That is to say described by a Hilbert space which is generated by a series of creation $a^+(\vec{p})$ and destruction $a(\vec{p})$ operators for scalar particles with 25-momenta \vec{p} , which can act successively on a zero-particle state $|0\rangle$. That is to say that the typical states in the Fock space - or the Hilbert space describing the world state - are

$$a^+(\vec{p}_1)a^+(\vec{p}_2)\cdots a^+(\vec{p}_n)|0\rangle \quad (1)$$

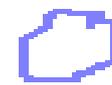
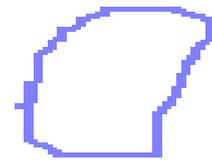
In the language, which we use, we call the scalar particles "even objects" and denote their momenta by J^μ instead by p^μ and their conjugate momenta by Π^μ (well really we only consider the conjugate momenta for the transverse components $i = 1, 2, \dots, 24$, i.e. J^i)

Detail of Open Versus Closed Strings in Our Model

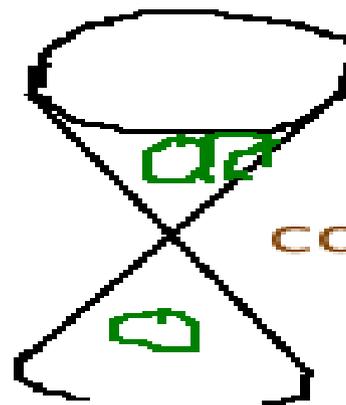
For Theory with Only Closed Strings:



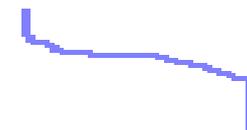
Strings



For Theories with also open strings:



common



Chains of Objects on the Lightcones

We Just Describe String Theory with Several Strings in Novel Way

- (Apart from null-sets) our string field theory should be just a rewriting of usual say string field theory.
- The Hilbert space describing all the possible world states in a string world is the Fock space of -either one or two – theories of massless noninteracting scalars (for the bosonic $25+1$ model).
- Two massless free scalar theories for purely closed string theory, while only one when there are open strings.
- But allowed states are restricted to obey – approximately? - some “chiral” invariant continuity condition: this means that the stringyness only comes in via initial state conditions.

Some Motivational Thoughts

- We – Ninomiya and I – think we have a new(novel) way of representing string theory, which because of being in some respects simpler could be helpful in understanding some aspects of string theory better.
- Even if string theory should not turn out to be the final truth – as can still be the case – its abilities for providing a cut off are so good that alone in looking for cut off it mat have inspiration.
- It happens generally thinking of seeking a cut off you easily get in the direction of the string theory, especially the aspect of not having any true interaction as is a trademark for Ninomias and my model.

String Field Theory Deviating from Earlier Ones by having Thrown Out Information

- Our – Ninomiyas and mine - novel field theory deviates from usual ones – Kaku Kikkawas or Wittens by including (a subset of) information less in its description of state of the world, i.e. of a set of strings present.
- We have rewritten the information – the kept part – on a state of several strings into a state of something (more like particles), which we call “objects”, to such a degree that one only sees the connection to genuine strings by quite a bit of complicated rewriting.

Our Novel SFT has character of a Solution of String Theory

- Our formulation in terms of the “objects” has the character of being a system of parameters describing the development of a system of strings, since:
- These “object”-parameters essentially do not change at all.
- The reconstruction of the strings involves integrating the object parameters up and even contain ambiguities (corresponding to that we left out some information).

Recent Victory: Veneziano Model comes out of our Novel String Field Theory

- Veneziano Model is obtained by putting up translated to our “objects” the state for the incoming set of strings and analogously for the outgoing set. Then the S-matrix of the Veneziano model appear as the overlap of these two states in the Hilbert space (“Fock space”) for the “objects”.
- The technical calculation quickly comes to remind strongly about a string scattering amplitude, but we should keep in mind that what I shall calculate in the seminar is only an overlap of two state-vectors in the “object” formalism.

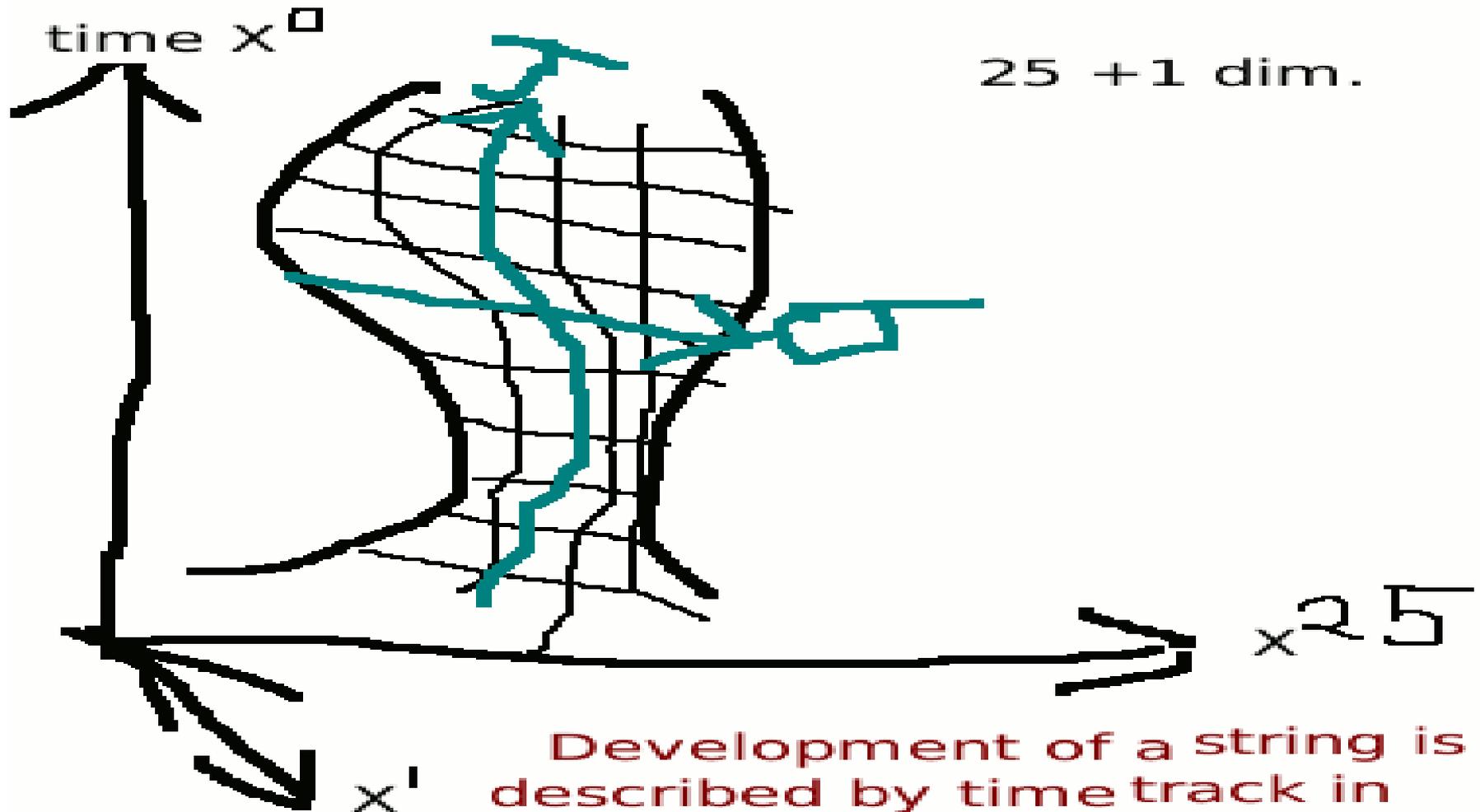
Our Throw *Away* of Information

Our Throw *Away* of Information

Our Throw Away of Information

- If two (open) strings cross, four pieces of string meet in one point.
- We throw away – before describing the situation in our model – the information about which pieces among these 4 are connected to form the strings with which of them.
- So we only keep the information as to where you find some string or the other (but do not keep which string it may be)
- The individuality of the separate strings is gone.

Single String Description by Parametrized Surface in Minkowski Space



Development of a string is
described by time track in
Minkowski space 'time.

Running point on time track $X(\sigma, \tau)$.

History , About Me and String

- The String Theory started with the Veneziano Model, but Veneziano knew nothing about that it were a string theory, he were about to make.
- It were then independently Nambu Ssuskind and myself (H.B. Nielsen) that found out that it were truly a theory of strings, that could deliver just the Veneziano model,
- Hereunder also the generalized Veneziano models for scattering of more than just four external particles – as several physisist had made among which also Koba and myself.

Important to Test New Formulation to Test for Veneziano Model

Veneziano Model, Euler's Beta function

$$B(-\alpha(s), -\alpha(t))$$

taken for Regge trajectory arguments.

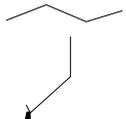
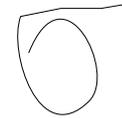
Important for Veneziano model were linear trajectories:

$$\alpha(s) = \alpha' s + \alpha(0)$$

$$\text{String tension} = \frac{1}{2\pi\alpha'}$$

Reminding Single String Theory

- First fix the main part of the “gauge choice” in the sense of fixing parametrization in τ and σ



to obey the conformal gauge choice.

- Next one can solve the equations of motion for the point of string position variable X by writing it as a sum of right and left mover parts.
- This solves it because these left and right movers only depend on one component of the two coordinates each.

Left and Right Mover Parts

In conformal gauge the equations of motion are solved by writing ~~X^μ~~ in terms of right and left movers

$$X^\mu(\sigma, \tau) = X_R^\mu(\tau - \sigma) + X_L^\mu(\tau + \sigma)$$

where these right ~~X_R^μ~~ and left mover ~~X_L^μ~~ only depend on 1 variable

For our model the tau derivatives of
left and right movers important

Our main variables considered are the
time, i.e. τ derivatives of the right
and left movers:

$$\dot{X}_R^\mu(\tau_R) \text{ and } \dot{X}_L^\mu(\tau_L)$$

(where

$$\tau_R = \tau - \sigma$$

$$\tau_L = \tau + \sigma$$

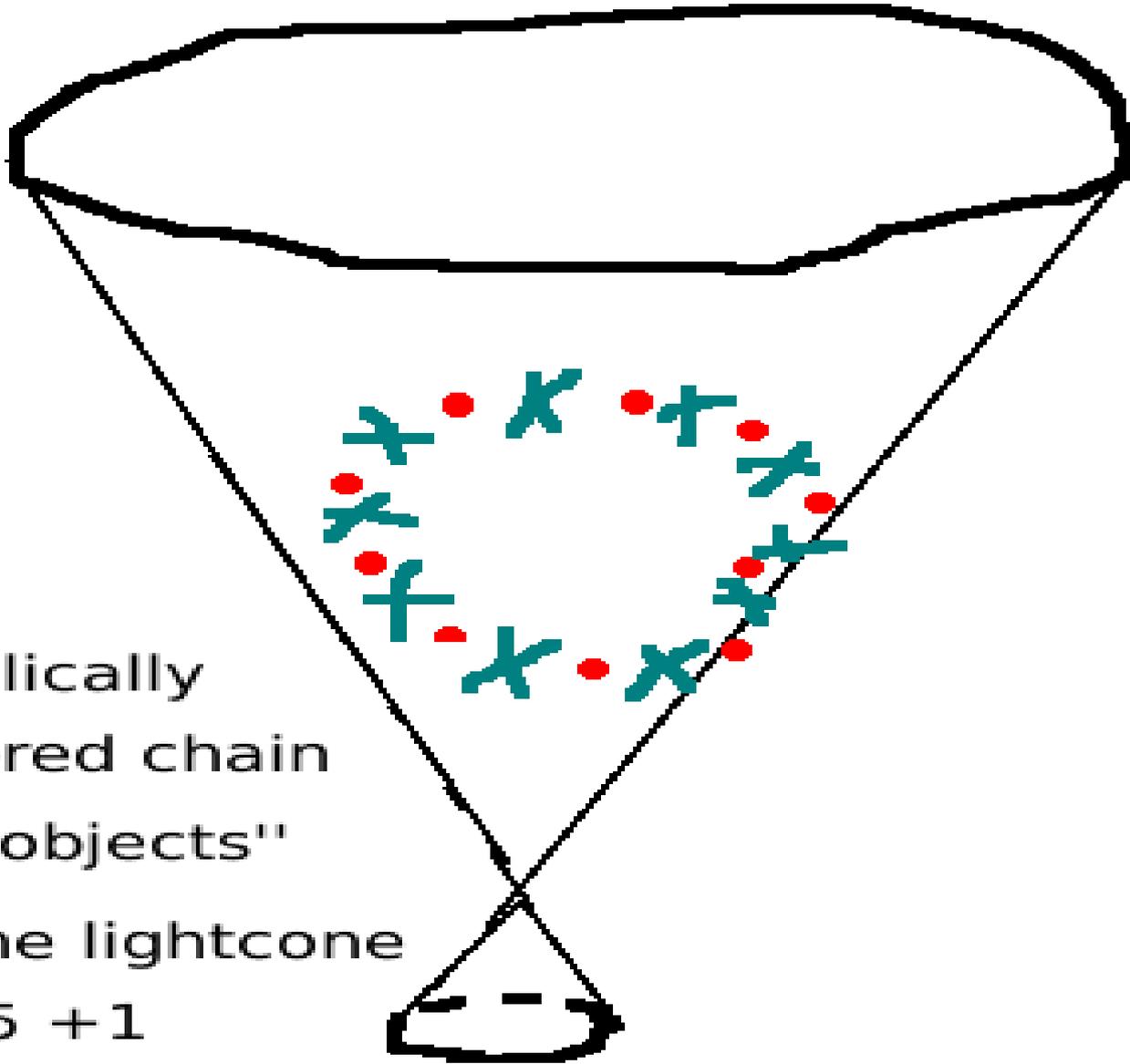
Important properties of the time/tau derivatives of left and right movers:

- The constraints just take the form of them being lightlike.
- We have a theorem about their images being conserved, even under scattering! (except for a nulset).
- They represent so many conserved degrees of freedom that we can say they “solved string theory”
- Except for perhaps some integration constants and nulsets one should be able to integrate them up and obtain almost the state of the string (in fact we shall get it for several strings in our picture)

Important Technical Details

- The main point is that we LIBERATE the right and left movers so that the tau-derivatives of the right and left movers – depending only on one variable – are replaced by CHAINS of “objects” one for each value of a discretization of this one variable.
- But only the “objects” with an EVEN number in the discretization are considered fundamental “objects” in our formulation. The ODD ones are instead replaced by differences of the conjugate variables for the neighboring even “objects”.
- This is done in order that the “objects” taken as “fundamental”, i.e. the even ones, shall have their variables J commute with each other (otherwise we cannot make a Fock space with them).

Picture of Chain of ``objects''



- A cyclically ordered chain of ``objects'' on the lightcone in $25 + 1$

Figure of Chain of ``Objects'' Illustrates

- That our ``objects'', when we count them as $25 + 1$ dimensional 26-vectors J , are light-like, i.e. They lie on the light-cone.
- That we discretize the a priori continuous series into discrete points (regularization).
- That we distinguish even numbered and odd numbered: Actually we take the philosophy that only the even ones truly exist in our formalism. The odd ones are described as proportional to a difference between the conjugate variables for the neighboring even numbered J 's.
- There is a continuity in the sense that the ``objects'' in the chain lie crudely on a one-dimensional curve.
- Because of the construction formula for the odd numbered ``objects'' and the fact that even the odd ones are also on the curve the continuity is actually orientation dependent: The curve may be continuous with one orientation but not with the opposite orientation!

Odd "objects" constructed from Even Ones:

Define an odd numbered "object" with say number l (l odd):

$$J^l(I) = -\pi \alpha \left(\pi^l(I+1) - \pi^l(I-1) \right)$$

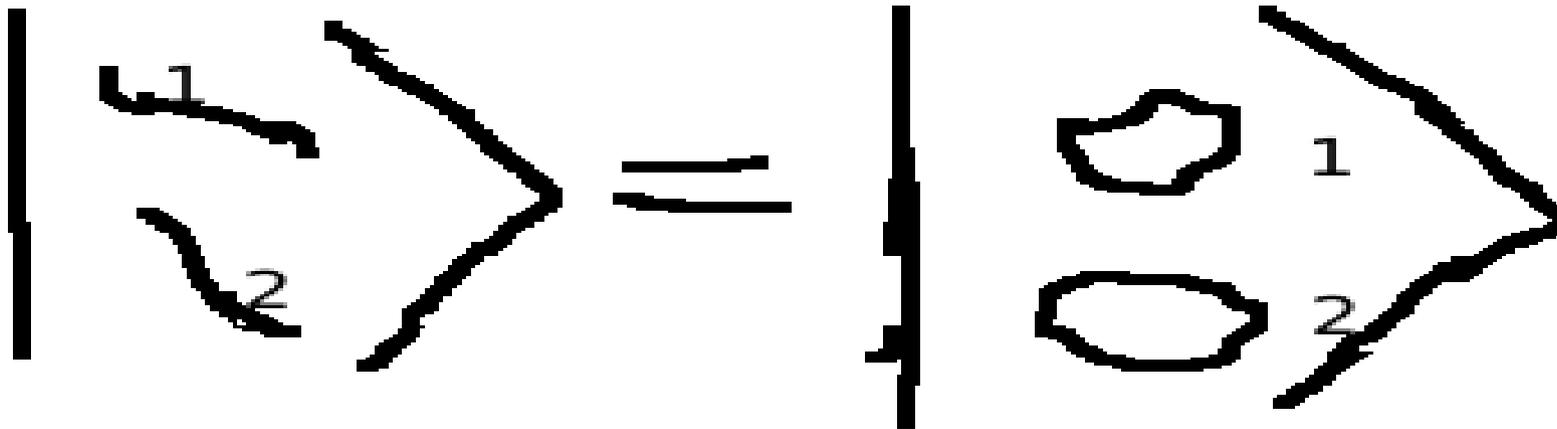
where $\pi^k(K)$ is the conjugate momentum of $J^k(K)$.
(K even)

Orientation dependent Continuity Condition:

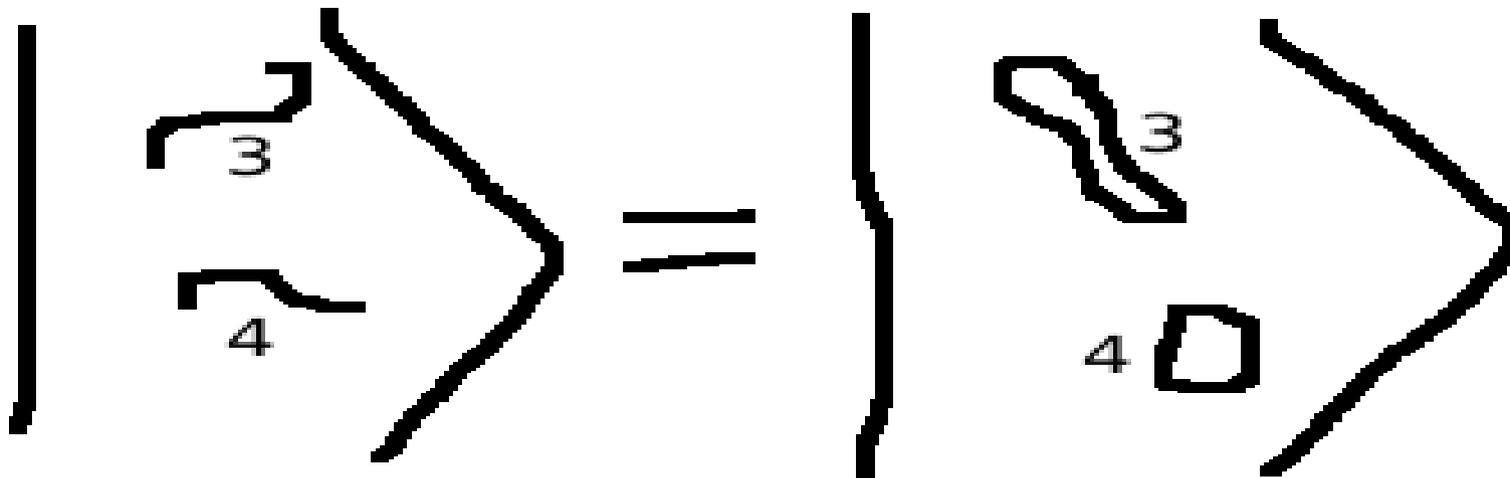
Continuity condition:

$$\begin{aligned}
 & J^i(I + \nu) \approx \\
 & \frac{2-\pi}{2+\pi} \alpha' (\pi^i(I + \nu) - \\
 & \quad - \pi^i(I - \nu)) \approx \\
 & \approx J^i(I - \nu) \\
 & \pi^i(K) \text{ conjugate of } J^i(K).
 \end{aligned}$$

Representing states of several strings as cyclic chains of objects

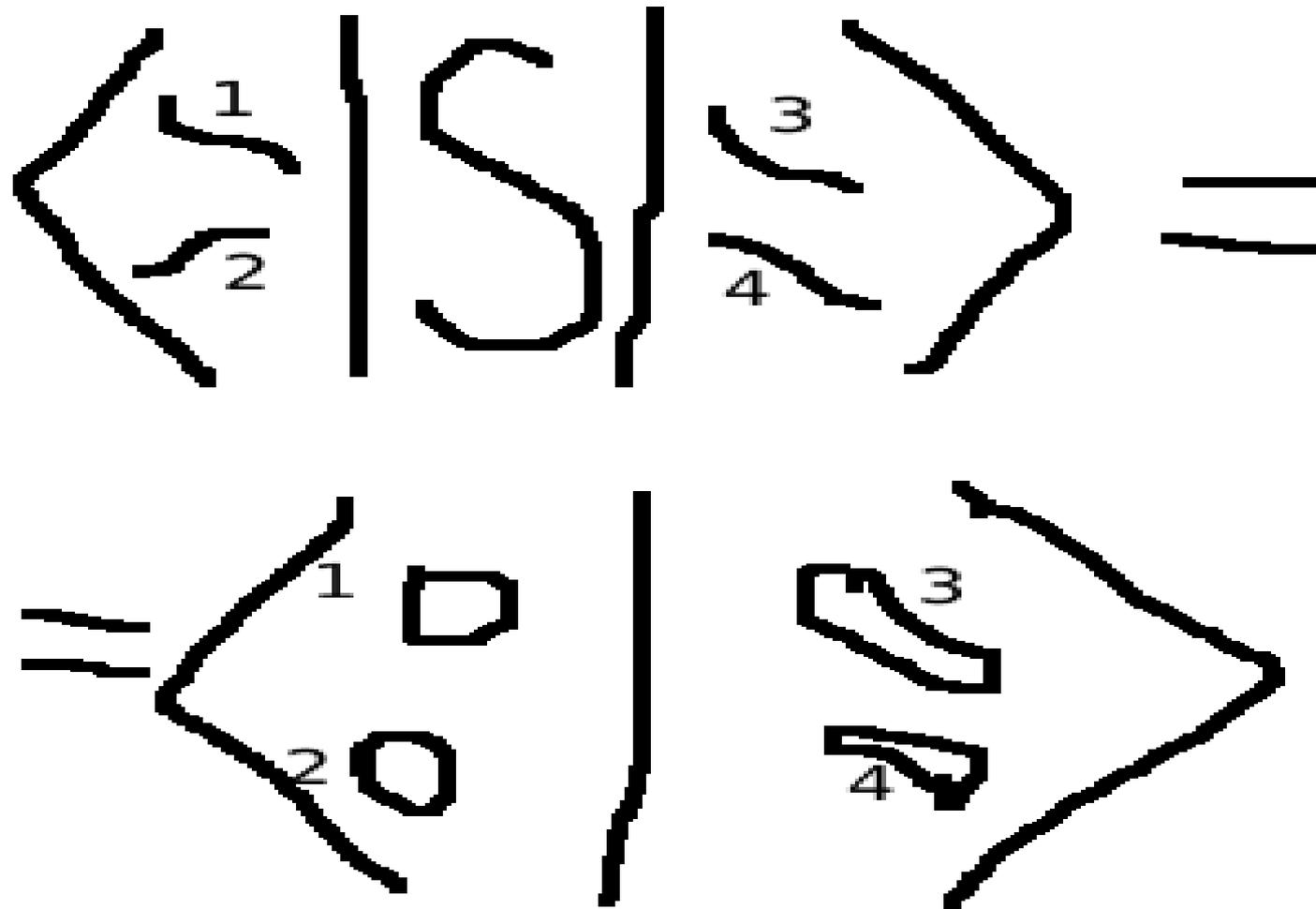


$$1 + 2 \dashrightarrow 3 + 4$$



Our S-matrix is just overlap of states
in our ``object'' describing (Fock)
space

Calculate as if S-matrix were 1:

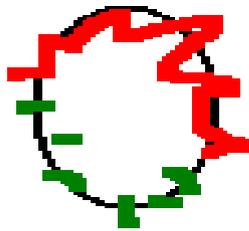


``Objects'' from an Initial String has Possibility of Going to Whatever String in Final State

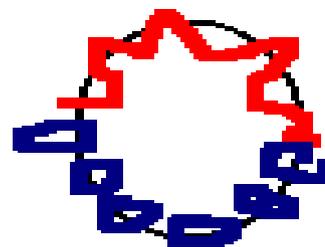
- In our model all the EVEN ``objects'' are just like bosons identical particles.
- So any even ``object'' in the initial state can go to become identified with any one in the final state.
- Now we assume the approximation that such an identification scheme of initial with final even ``objects'' dominates more the more the neighboring ``objects'' follow each other. So dominantly long chain-pieces should go collectively over from one string to another one.

Identifying “Objects” in Initial and Final Possibility

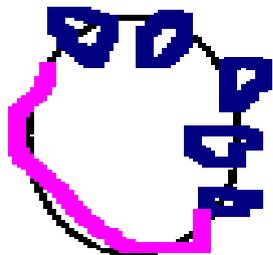
1



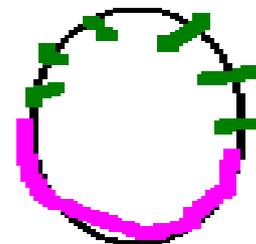
4



2

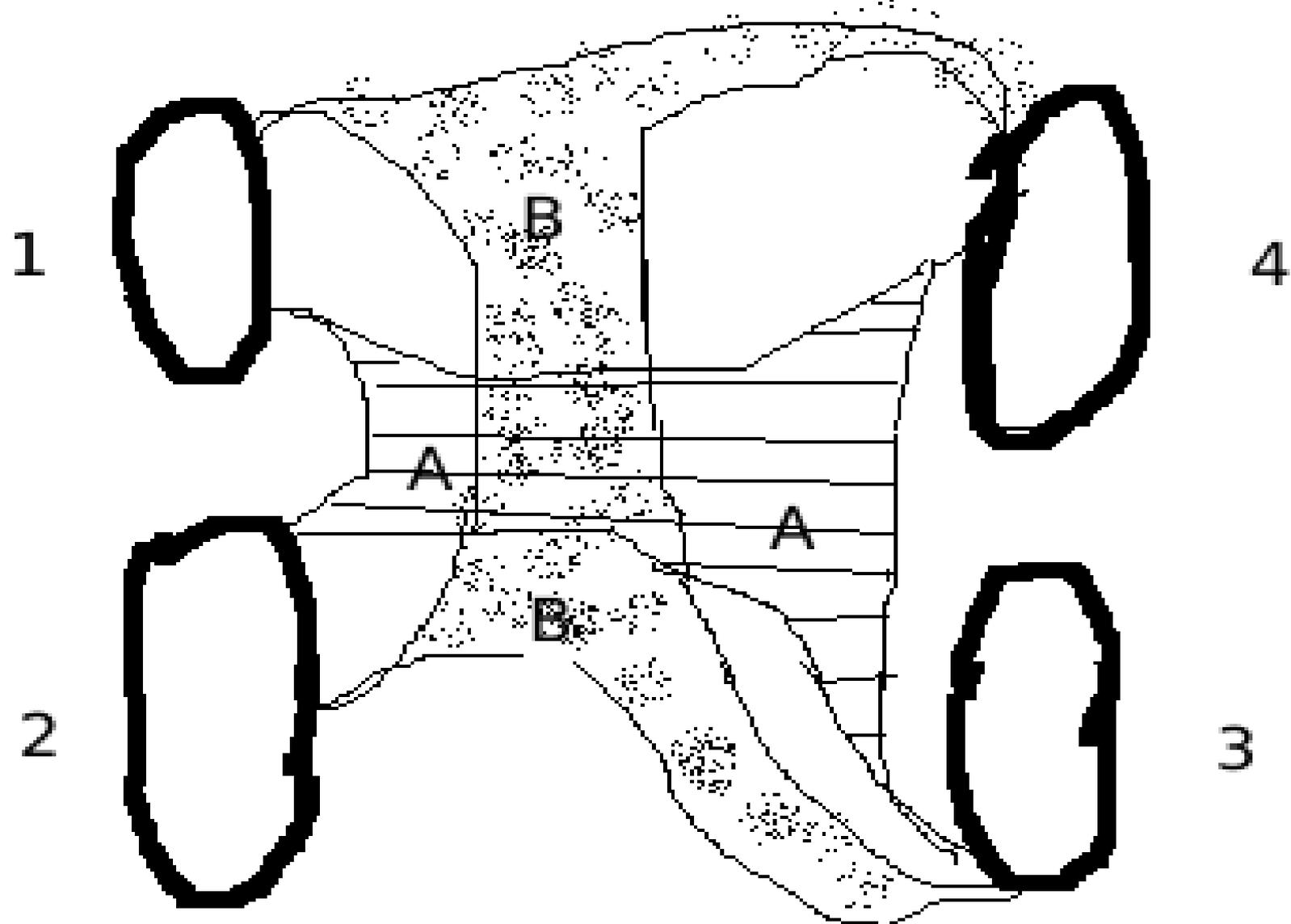


3



How different pieces of cyclically ordered chains of objects may get exchanged.

Diagrams to Describe Different Ways of Identification of ``objects'' in in and out



A bit nontrivial: The Wave Function(α) for the cyclically ordered object chains(describing strings)

- Depending on which say mass eigenstate of the (open) string we shall use as external particle, we need to put the cyclically ordered chain into that state, i.e. We need the appropriate wave function(α).
- We shall here take the ground state – the tachyon of the bosonic string model - .
- Its wave functional can be represented by a functional integral (trick) (or by analogue model).
- Physically we could say: we propagate a string in our doubled formulation for an infinitely long purely imaginary time; then only the ground state would survive at the end.

Functional Integral Represents the Wave Function

Our functional integrals to present wave functions are just like usual string theory functional integrals:

$$\int e^{-\int g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi} \mathcal{D}\varphi$$

where μ and ν runs over 1 and 2.

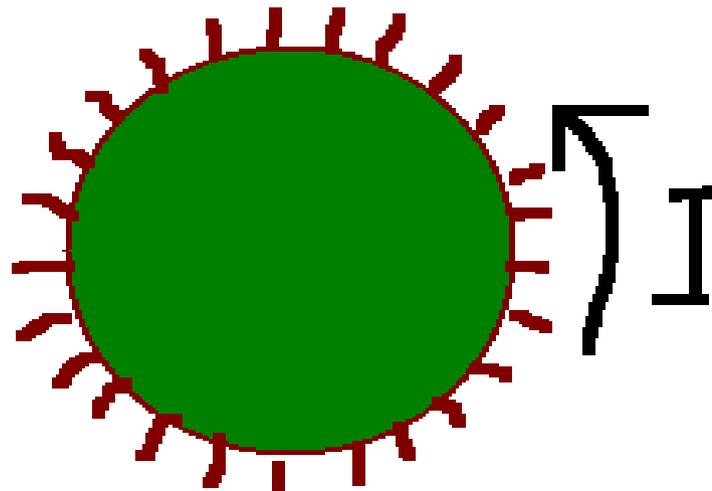
The functional integration is over $(\varphi, \partial_\mu \varphi)$ while $g^{\mu\nu}$ is a fixed metric in the space.

Functional Integral Represents wave function

- It shall be understood that we extract the wave function by FIXING THE DERIVATIVE OF THE INTEGRATION VARIABLE ALONG A BOUNDARY.
- To make precise sense a cutoff depending on the metric tensor is needed,
- although formally the functional integral looks invariant under a scaling of the metric tensor by a factor depending on the coordinates, this is not true due to the anomaly.
- But apart from this anomaly this type of functional integral is invariant under conformal transformations (of the region over which the coordinates σ run)

Wave Function for ``Objects'' from Functional Integral

We imagine the ``objects'' sitting along the edge of a disk, over which is defined a $\varphi(\sigma^1, \sigma^2)$ to be functionally integrated over:



The J 's of the ``objects'' are related to the derivatives $\partial_\mu \varphi$ at the edge.

Our Oriented Continuity

- Because our orientation dependent “continuity condition” that the “objects” both even and odd only vary slowly along the chain we should strictly speaking have found a functional integral prescription ensuring that,
- But for easiness we take an unoriented functional integral prescription for the wave functional and argue the result should be the same if we keep to orientable two dimensional manifolds.
- Then we must only include identifying pieces of cyclically ordered chains of “objects” , when they have the SAME ORIENTATION.
- But that is anyway physically needed, since overlap of opposite orientations vanish.

The Continuity Condition Changes if I-Orientation is Shifted

Continuity condition:

$$\begin{aligned}
 & J^i(I+1) \approx \\
 & \frac{z-\pi}{z} \alpha' (\pi^i(I+1) - \\
 & - \pi^i(I-1)) \approx \\
 & \approx J^i(I-1) \\
 & \pi^i(K) \text{ conjugate of } J^i(K).
 \end{aligned}$$

Conformal Transformations of the Two Dimensional Functional Integrals

- Have in mind that in our String Field Theory we have replaced the usual interval of sigma-coordinate describing the open string by one topologically circle shaped chain of “objects”, and thus already an open string looks by us more as a closed one in usual notation, while a closed string looks by us like TWO closed ones from usual notation.
- So to propagating an open string in imaginary time to make only its ground state survive becomes by us a half infinite cylinder (while in usual a half infinite belt).
- By conformal invariance of the functional integrals (apart from the anomaly) an half infinite cylinder can be replaced by a unit disk say.
- The wavefunction we look for is extracted from the fluctuation at the edge of the disk. The momentum of the ground state particle is inserted in the center of the disk.

Calculation Outline

- Notice already how the trick of introducing functional integrals for fields defined on two dimensional manifolds just because they can give the wave function for the string in our “object” description of the initial and final states is like getting the string in by a calculational trick (There is in a way no true string in our formalism but we get it in because it is one smart trick to calculate presumably among many) !
- Our calculational stringlike formulation is though still doubled compared to say Mandelstams usual string time track functional integrals.

Computational Outline Continued

- The crux of the matter of our calculation of the Veneziano amplitude is to make the overlap of the in terms of “objects” described states of the several string states – the initial and the final states.
- Our “perturbation-like” approximation consists in not taking – as we should – all possible combinations of one even object in the initial state with one even object in the final state, but only include the identification schemes for objects having the longest pieces of chains following each other in the sense of going from e.g. string 1 to string 3
- For each system of correspondance between initial and final “objects” (keeping only the ones with long pieces going same way) we then calculate the overlap contribution from that correspondance by combining the wave function describing functional integrals into a larger composed functional integral obtained by gluing together the two dimensional regions of the single functional integrals (the disks)

Yet Computational Outline

- When we for $1+2 \rightarrow 3+4$ identify pieces of the cyclically ordered chains of objects from initial and final states in the simplest (in the sense of having biggest unbroken pieces going between the various string-assigned cycles) and glue the corresponding disks for the functional integrals (giving the wave functions) together we obtain a functional integral for a two dimensional manifold, that turns out to be topologically a Riemann sphere.
- The inlets of the momenta for the four external strings(particles) sit of course in four points in this Riemann sphere.
- It turns out that they sit so that there is reflection symmetry – we talk about as a flattening symmetry – identifying pairs of points on the Riemann sphere, so that we for our calculational purpose can use a flattened two dimensional manifold of topology as a disk and with the four inlets from the four external strings sitting on the edge of this disk (as Koba Nielsen variables)

Yet Computational Outline

- Each way of distributing the initial state “objects” into the final state to be identified with the final state ones has to be summed up as separate contributions to the final “scattering” amplitude/ S-matrix / overlap.
- This becomes in the lowest order of our “perturbation like” approximation an integral, that ends up as the integration well known to represent Veneziano models. (A priori it would be the sum over permutations of the objects in initial state before being identified with final state ones)
- We strongly must use conformal transformations of the functional integrals used.
- But then there is an anomaly which actually is needed to get the right Veneziano model expression.

How to perform the Conformal Transformations of the stringlike functional integrals?

- Remember that the ground state wave function representing functional integrals were on either a half infinite cylinder surface or conformally equivalently a disk or even we could use the complement of a disk, with infinity as its center.
- Part of the cyclically ordered chain of ``objects'' for string 1 (one of the initial strings) go into 3 while another part goes to 4(the strings in the final state were called 3 and 4)
- Similarly for the other string 2 in the initial state part becomes a piece in 3 part in 4.

Concretely Constructing the Conformal Composed Functional Integral Region

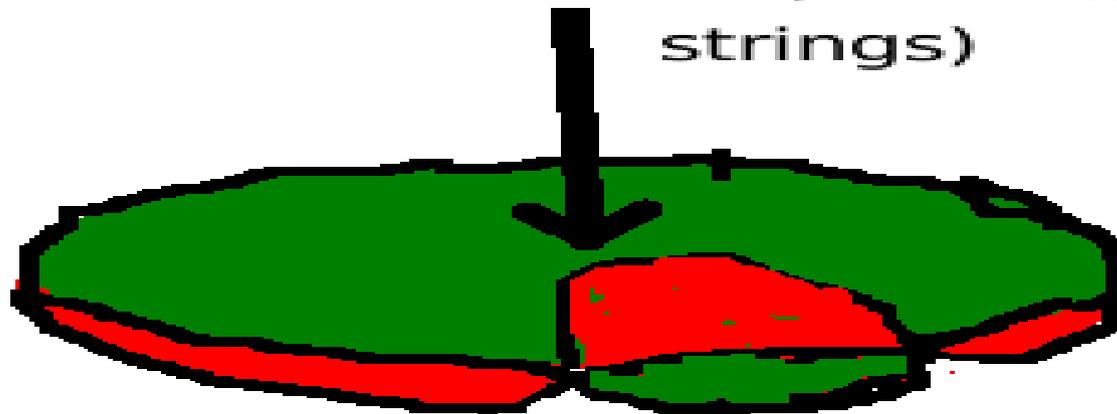
- We choose to use unit disks for the two initial state strings 1 and 2.
- We take complements of unit disks with center at infinity for the final state strings 3 and 4.
- We perform our calculation in a specially chosen frame or rather “gauge” so that all four external strings have equally many “objects” in the regularization and our each object a fixed “longitudinal momentum” P_+ .

Concretely to make Manifold for Functional Integral Composed from the Wave function producing ones

- Take the disks first of the initial state strings 1 and 2 and put them in two different layers in the complex plane.
- Next put the two complements of disks for the two final state strings in the remaining part of the complex plane including infinity (as the center) again into two different sheets/layers.
- Now the initial and the final state functional integral regions meet at the unit circle and lie on two layers. So we can very freely identify initial and final state "objects" by identifying these layers correspondingly.
- In the simplest case supposed to dominate we just have one piece of the unit circle where 1 goes to 4 while 2 to 3 and another piece where it is opposite: 1 goes to 3 and 2 to 4.

Two Layers from Initial State Strings

Inlet of current/momentum
for 1 and 2 (incoming
strings)



Two unit disks lying two layers in the complex plane, seen in perspective, and prepared for being glued to complements of disks for outgoing 3 + 4.

Green for 2; red for 1.

Building up the Manifold for the Functional integral Composed

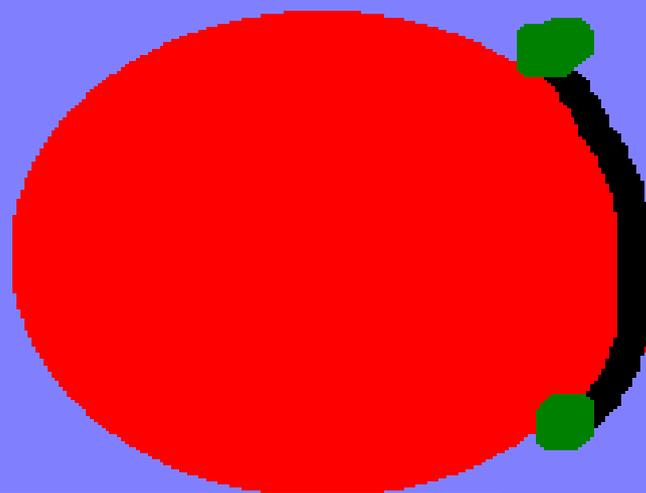
- Glue together the two layered disk with the edges permuted along part of the unit circle with the two layered complement of the unit disks representing the functional integral regions for the final state particles(strings), 3 and 4.
- Then we get the two dimensional region for the functional integral the result of which is the contribution from the identification correspondance related to the angle on the unit circle along which say 1 and 4 were identified.
- The functional integral is evaluated by using the essential conformal invariance and using analogy to Koba Nielsen disk and computing anharmonic ratios.
- But an anomaly correction has to be included. The conformal invariance is broken by the anomaly which occurs at the end of the cut. (Here one has a point with angle 720 degree around it)

Putting disks and complement disks into two layer complex plane

The complex plane with two sheets

Under the unit disk the disks of 1 & 2.

Under the exterior the 3 and 4 parts.



The cut
where 1 & 4
meet.

The cut where 1 &
3 meet.

Next use square root like transformation to get the two singular points straightened out

- The two singular points – green on the figure - are taken as singular points for a square root like function (product of two square roots) and we get the two sheets mapped into just one Riemann sphere.
- Now we managed to get the unit circle mapped to the real axis.
- And the two singular points to the 0 and infinity.
- And we got the inlet points where the four external momenta are let in to lie on the (new) unit circle.
- Seeing the inversion on the circle as a symmetry we do not need the exterior and just got Koba Nielsen disk.

The Surprise of our Calculation:

we found only one term in Veneziano model:

$$B(-\alpha(s), -\alpha(t))$$

But not those terms which have poles in the s-channel.

What went wrong?

Infinite momentum frame is bad!

One cannot have negative longitudinal P

In First Calculation we Gauge fixed

We use the parametrization choice, which combined with our discretization, gives that each object carry a fixed amount of "longitudinal momentum"

$$P^+ = \frac{1}{2} (P^0 + P^2 5)$$

so that

$$J^+ = \frac{\alpha \cdot \alpha'}{2}$$

for all "objects".

Is Usual IMF Formalism Wrong?

- Such gauge fixing that assigns the string or the cyclically ordered chain parameter to be proportional to the amount of longitudinal momentum is usual in Infinite Momentum Frame (IMF) formalism.
- But is it enough? We can never get the negative energy or longitudinal momentum frame states – such as the Dirac sea -.
- One has in IMF thrown vacuum away.
- That were presumably the goal but is it good enough also e.g. For our string field theory?
- We had chosen now to allow for “objects” with negative longitudinal momentum.

We Change Our gauge Choice to Allow both Signs!

Change the parametrization choice, which combined with our discretization, gives that each object carry a fixed amount of "longitudinal momentum"

$$P^+ = \frac{1}{2} (P^0 + P^2 5)$$

so that we allow also negative long. m.

$$J^+ = \pm \frac{\alpha \cdot \alpha'}{2}$$

for "two different sorts of "objects".

Extension of chains with Negative and Positive pieces

- Our introduction of negative P^+ momenta, negative longitudinal momenta, allows us to add to any cyclically ordered chain of “objects” an equal amount of the negative P^+ or J^+ ones and of the old positive P^+ or J^+ ones.
- Such an extension opens the possibility for “annihilating” a negative chain piece in one incoming string, say 1 with a positive piece in another 2.
- It turns out that we must then also have an annihilation in the final state between positive and negative pieces.
- And remarkably by inclusion of such possibilities we get the two remaining terms in the Veneziano model!

Conclusion

- We have constructed a new – since throws away information compared to older ones – string field theory, in the sense of a theory with potentially an arbitrary number of strings present.
- Our formalism is based on a “Fock space” of states for the universe of the strings, but formulated in terms of “objects”, in terms of which we have truly a Fock space: There can be any non-negative integer number of “objects” in any state for objects.
- The state of a single object is given its 24 transverse J^i components (and then their 24 conjugate variables).

Conclusion Continued

- The two last J-components for an “object” were reconstructed just by mathematical definition from the transverse – the 24 - components : J_+ is just fixed as a “gauge choice” and the J_- is determined to make the total J 26-vector lightlike.
- It is only the EVEN “objects” that truly exist in our model-formulation; the ODD “objects” are only derived by a formula from the conjugate variables to the even ones.

Conclusion Main Result

- We calculated the scattering amplitude – under a special assumption of all four strings having same longitudinal momentum – as just THE OVERLAP OF THE INITIAL AND THE FINAL STATE FORMULATED IN OUR MODEL “FOCK SPACE”!
- First we got only ONE of the three terms expected, but then introduction of “objects” with NEGATIVE longitudinal momentum P_+ or say negative J_+ led to also obtaining the two missing terms; and thereby a more crossing symmetric and boson symmetric scattering amplitude.

Conclusion, Forward Looking

- It is not totally trivial that our model gives string theory, but the derivation of the Veneziano model amplitude is a strong indication, that our model IS indeed string theory rewritten.
- Our model has the character of being a SOLUTION to string theory, even for many strings, string field theory, in the sense that it is in a Heisenberg-like formulation and nothing happens at all to our “objects” as time goes on. They are more like just the numbers in a certain solution to the string dynamical theory.

Conclusion, Hopes

- Since our string field theory is like a solution, you could expect it to solve also say the AdS world string theory in the Maldacena conjecture and thus probably lead to revealing that the Maldacena conjecture could be understood by such solution.
- I consider it an almost trivial thing to extend our model to include SUSY by giving our ``objects`` probably some spin.
- At the end we hopefully could also see through to an interpretation of branes in our formulation.
- But for the moment we concentrated too much on convincing ourselves and others that our model works even just for the bosonic strings

Conclusion, Phenomenological Question

- Could one believe that the true model for Nature were a SOLVABLE model? Well a priori random initial conditions could make the world random enough anyway.
- Could we possibly use solvability to argue that the effective dimension never would go down to four instead of say 10 in the susy version (yet to be constructed)? Could complicated initial conditions arrange for the wanted effective dimension?

Conclusion Question: Where is the String?

- Since our Fock space with fewer or more “objects” is essentially a free quantum field theory of massless scalar bosons, one may with good reason worry: where are the strings?
- Since the strings only appear as continuous one-dimensional structures because of the continuity in the cyclically ordered chains of “objects”, it is this continuity of chains of “objects”, that is the basis of the strings.
- Thus the strings are only there because of feature of the STATE of the “object” system.
- This means it is the INITIAL STATE assumed to have such continuity that makes up that there are strings. I.e. The strings are put in via an INITIAL STATE property!