New Physics Hypotheses on Muonic Hydrogen and the Proton Radius Puzzle (Part II)

Ulrich D. Jentschura
Missouri University of Science and Technology
Rolla, Missouri, USA

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Muonic Hydrogen and the Proton Radius Puzzle (Part II)

Unifying Element: The Central-Field Problem

Now: QED of Bound States

Theory of Bound Systems: Three Developments

Schrödinger Theory:

$$E_n = -\frac{(Z\alpha)^2\, m}{2\,n^2} = -\frac{Z^2\,\hbar\,(2\pi R_\infty c)}{n^2} \qquad (\hbar = c = \epsilon_0 = 1)\,.$$

Dirac Theory:

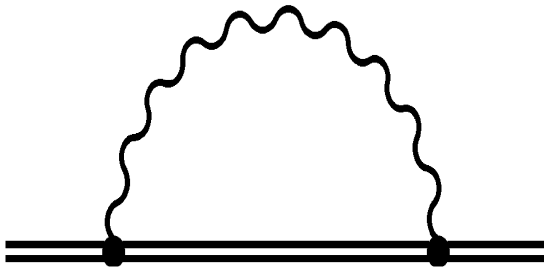
Relativistic Correction Terms: Dirac and Foldy-Wouthuysen

$$E_{nj} = m - \frac{(Z\alpha)^2\, m}{2n^2} - \frac{(Z\alpha)^4\, m}{n^3}\left[\frac{1}{2j+1} + \frac{3}{8\,n}\right] + \mathcal{O}[(Z\alpha)^6].$$

QED:

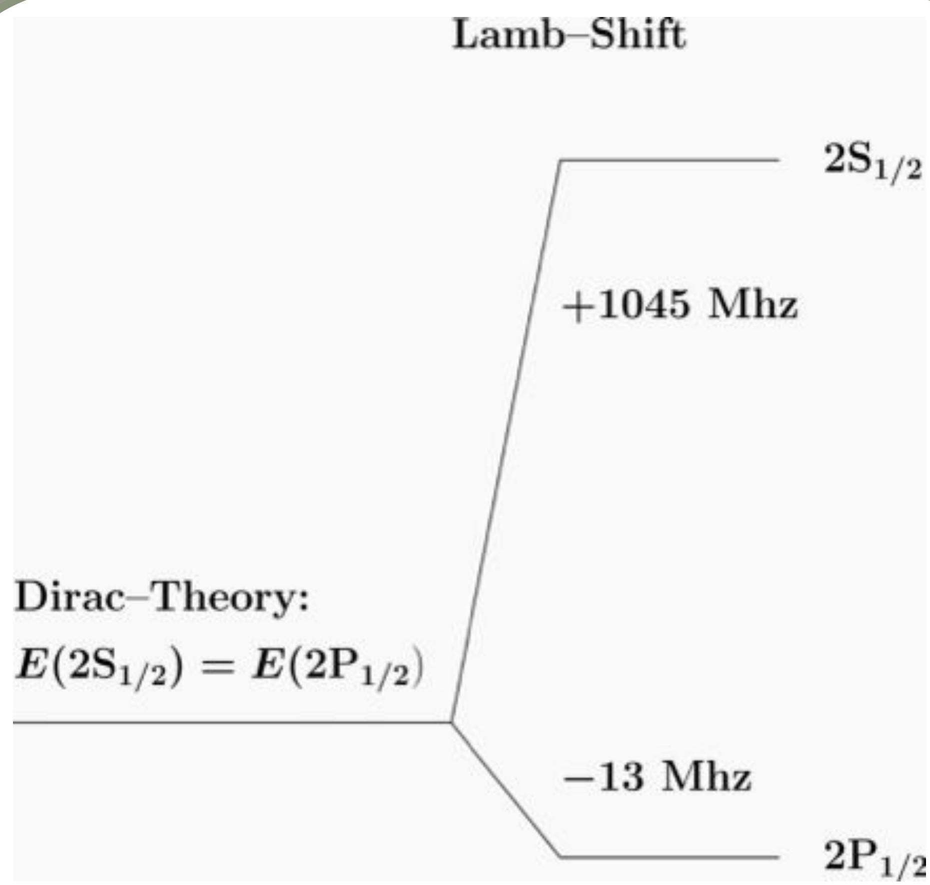
Beyond the Dirac formalism.

Self-energy effects,
corrections to the Coulomb force law,
So-called recoil corrections,
Feynman diagrams…



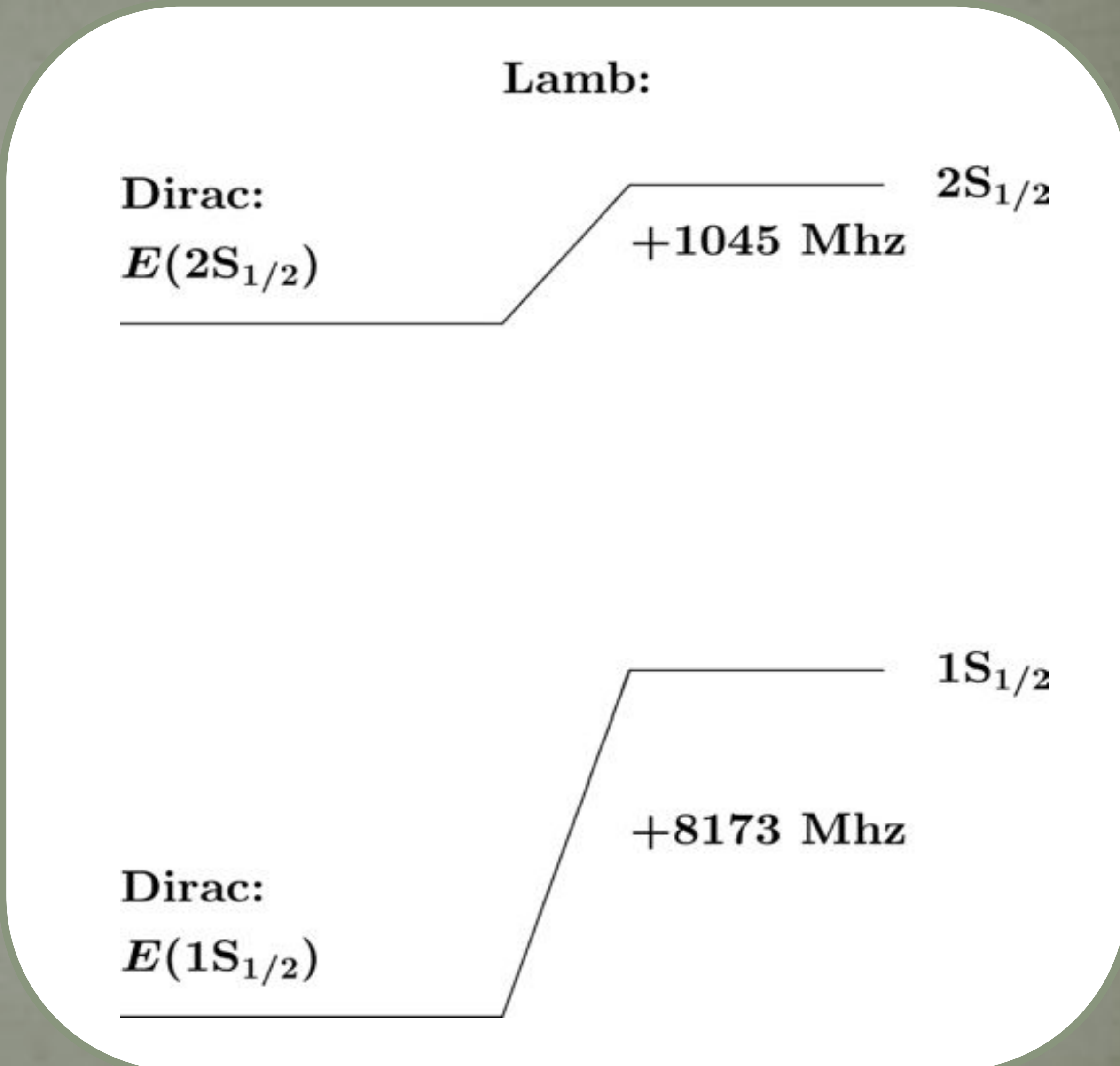
Lamb-Shift Phenomenology

Lifts 2S-2P degeneracy:



Lamb-Shift Phenomenology

Shifts nS-n'S transition frequencies:



The Predictive Power of QED…

TABLE III: Calculated transition frequencies in hydrogen and deuterium from the 1S state to the 3S and 3D excited states.

Excited state	Hydrogen ν_H/kHz	Deuterium ν_D/kHz
$3S_{1/2}$	2 922 743 278 671.6(1.4)	2 923 538 534 391.8(1.4)
$3D_{3/2}$	2 922 746 208 551.21(70)	2 923 541 464 741.56(72)
$3D_{5/2}$	2 922 747 291 888.42(70)	2 923 542 548 374.47(72)

TABLE IV: Examples of calculated transition frequencies in hydrogen and deuterium from the 2S state to various S and D excited states.

Excited state	Hydrogen ν_H/kHz	Deuterium ν_D/kHz
$3S_{1/2}$	456 681 865 484.5(1.4)	456 806 126 870.1(1.4)
$3D_{3/2}$	456 684 795 364.11(69)	456 809 057 219.82(69)
$3D_{5/2}$	456 685 878 701.32(69)	456 810 140 852.73(69)
$4S_{1/2}$	616 520 150 628.5(2.0)	616 687 903 590.7(2.0)
$4D_{3/2}$	616 521 386 393.2(1.7)	616 689 139 553.7(1.7)
$4D_{5/2}$	616 521 843 426.6(1.7)	616 689 596 711.8(1.7)

For this level of accuracy, one needs bound-state QED,
i.e., the formalism of relativistic bound-state quantum field theory.

[U.D.J., S. Kotochigova. E. Le Bigot, P. J. Mohr and B. N. Taylor, Phys. Rev. Lett. 95 (2005) 163003]

Muonic Hydrogen and Lamb Shift

What is the proton charge radius puzzle?

Which options have been tried for a resolution?

What are the remaining possible explanations?

Up to 2010:
QED and experiment were
essentially in agreement, but then…

Muonic Hydrogen Puzzle

CODATA: r_p = 0.8768(69) fm

electronic H: r_p = 0.8802(80) fm

Scattering (Mainz, 2010): r_p = 0.879(8) fm

Scattering (Jefferson Lab, 2011): r_p = 0.875(10) fm

(essentially 0.88 fm) BUT

muonic H: r_p = 0.84184(67) fm

(essentially 0.84 fm)

[R. Pohl et al., Nature 466 (2010) 213] [A. Antognini et al., Science 339 (2013) 417]

Why Can You Determine Nuclear Radii from Spectroscopy?

You calculate the spectrum.
[Nonrelativistic Theory.]

You calculate the spectrum more accurately.
[Relativistic Effects.]

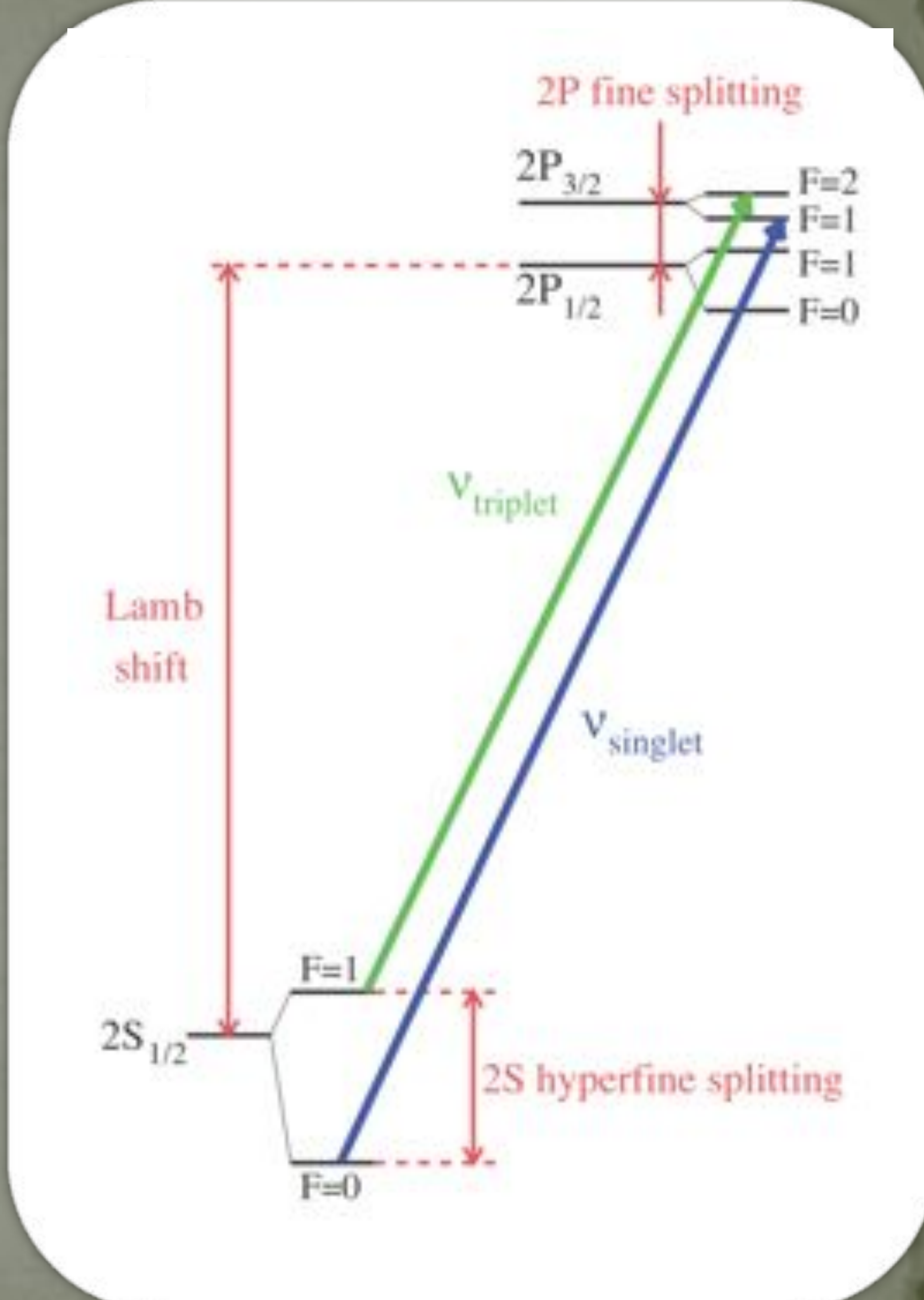
You calculate the spectrum even more accurately.
[QED effects.]

At some point the nuclear size becomes important.
[Distortion of Coulomb Potential.]

Someone else measures the spectrum.
[And then you can tell what the nuclear size is.]

$2S_{1/2}^{F=0}$-$2P_{3/2}^{F=1}$

$2S_{1/2}^{F=1}$-$2P_{3/2}^{F=2}$



Finite-Size Hamiltonian
[Affects S States with a Nonvanishing Probability Density at the Origin]

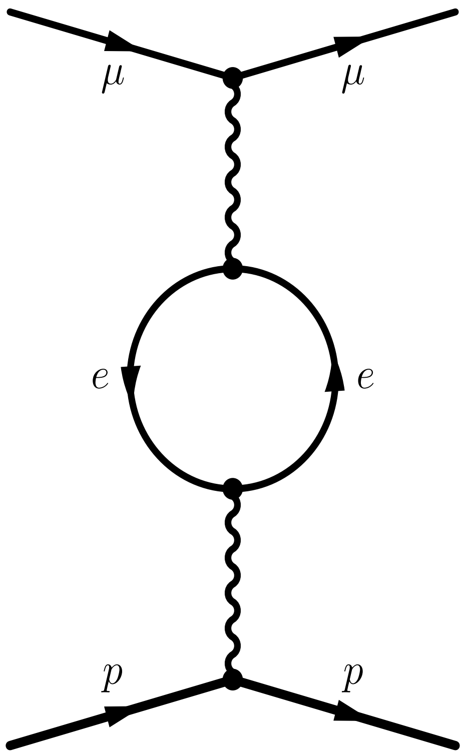
(proportional to the Dirac-δ function, measures probability density of the electronic wave function at the origin)

$$\Delta H_{\text{fs}} = \sum_i \frac{2}{3} \langle r^2 \rangle \; [\pi Z \alpha \delta^3(\vec{r_i})]$$

[R. Pohl et al., Nature 466 (2010) 213]

[A. Antognini et al., Science 339 (2013) 417]

Now the Theory: Vacuum Polarization Diagram



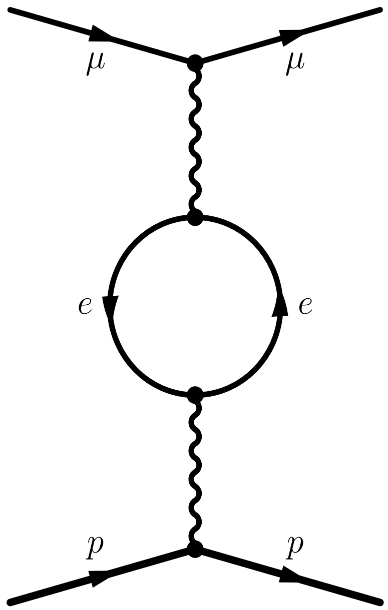
Vacuum Polarization Effects.

The Coulomb law is incorrect at small distances.

Muonic hydrogen is smaller than atomic hydrogen by a factor of 207 (mass ratio of muon to electron).

The vacuum polarization energy shift is 40,000 times larger in muonic hydrogen.

Reason:
Generation of virtual electron-positron pairs in the vicinity of the proton.
The quantum vacuum has structure!



Generation of so-called virtual electron-positron pairs leads to *modifications of the Coulomb force law at distance scales comparable to the electron Compton wavelength.* For long distances, the modification in exponentially suppressed.

(The 2P state is energetically higher, for muonic hydrogen)

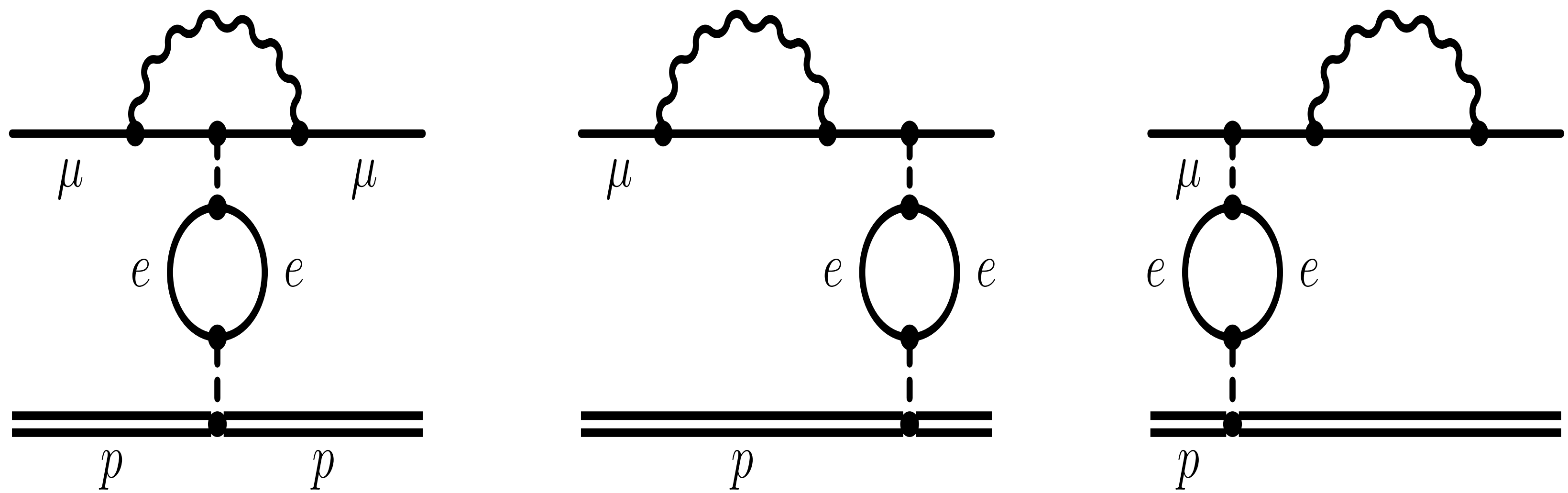
$$V(r) = -\frac{\alpha}{r} = -\frac{\alpha^2 m_r}{\rho}. \qquad \text{(Coulomb Law)}$$

(Mass Ratio) $$x = \frac{m_e}{\alpha m_r} = 0.73738368\ldots$$

(Quantum Correction) $$V_{\rm vp}(r) \sim \frac{\alpha^3 m_r}{\pi\rho}\left[\frac{2}{3}(\ln(\rho x) + \gamma_E) - \frac{\pi}{2}\rho x + \frac{5}{9}\right] + \mathcal{O}(\rho).$$

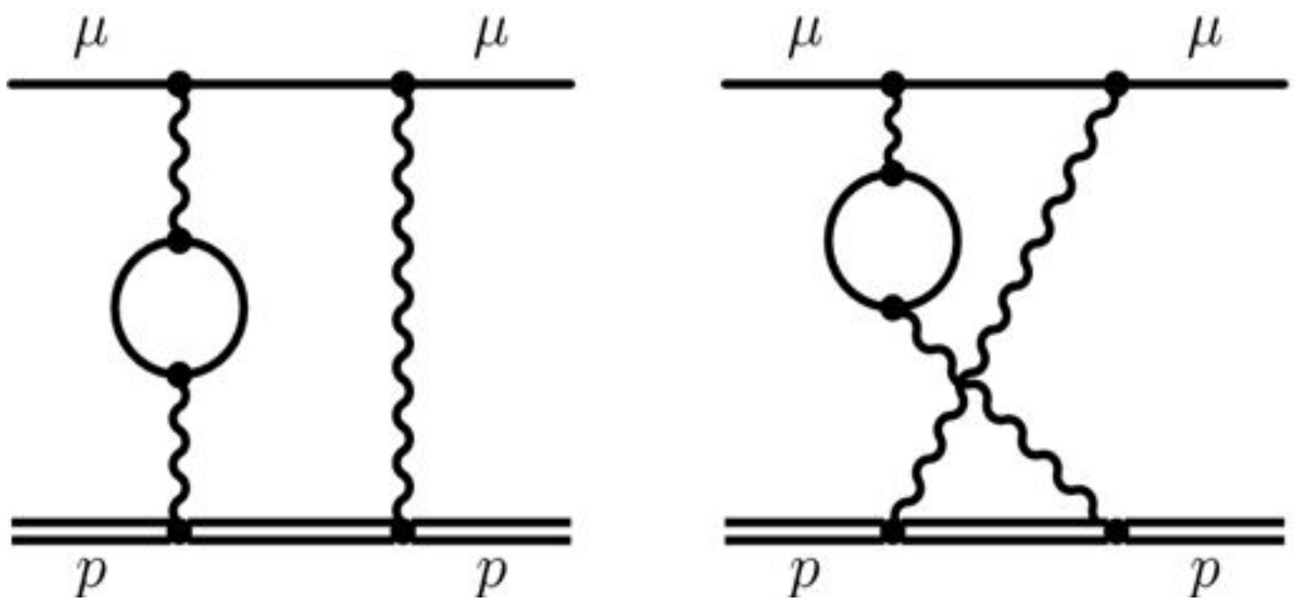
$$V_{\rm vp}(r) \sim -\frac{\alpha^3 m_r}{\sqrt{\pi}} e^{-2\rho x}\left[\frac{1}{4\rho^{5/2}x^{3/2}} - \frac{29}{64\rho^{7/2}x^{5/2}} + \frac{2225}{2048\rho^{9/2}x^{7/2}} + \mathcal{O}\left(\frac{1}{\rho^{11/2}}\right)\right],$$

Conspiracy of Self-Energy and Vacuum Polarization



-0.0025 meV for 2P-2S μH

Dominant Theoretical Uncertainty: Recoil Correction to Vacuum Polarization [Vacuum-Polarization Insertion in Two-Photon Exchange]



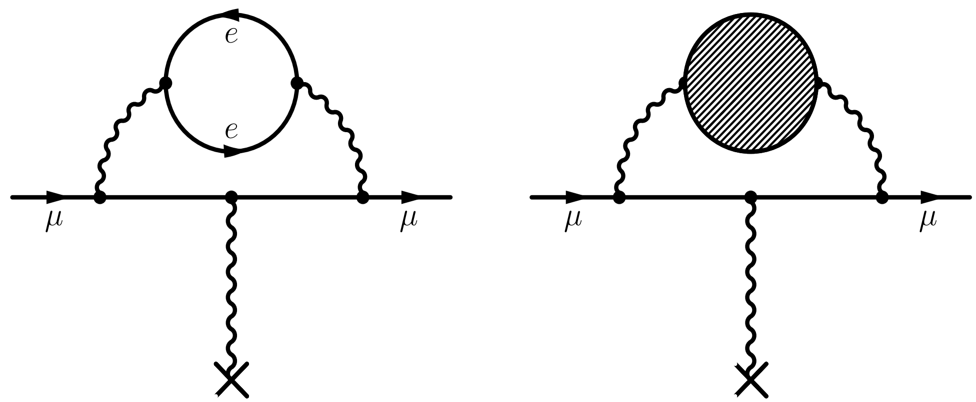
Now, that calculation is difficult.

Logarithmic Terms Calculated: < 0.0005 meV for 2P-2S μH

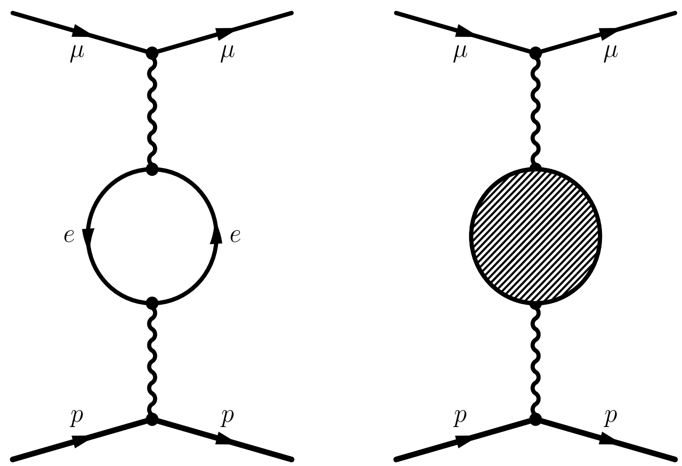
[Eur. Phys. J. D 65, 357–366 (2011)]

Subversive Particles: Influence on Muon g-2 and Muonic H

Muon g-2



Muonic H



[U.D.J., Ann. Phys. 326, 516 (2011)]

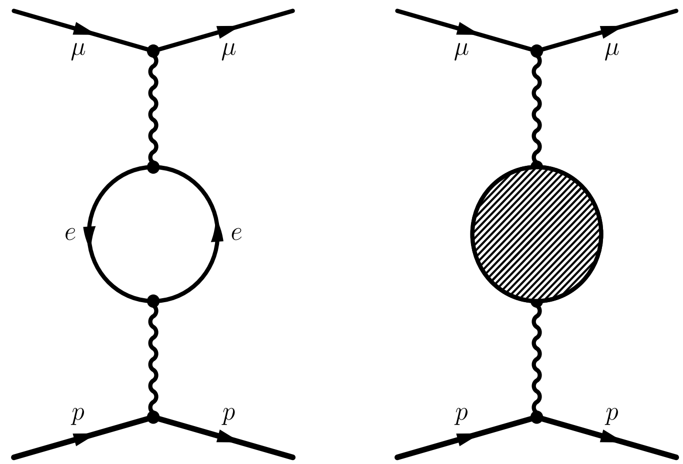
Subversive Millicharged Particles: Influence on Muonic H

$$\frac{1}{q^2 + i\epsilon} \to \frac{\alpha}{3\pi} \int_{4m_e^2}^{\infty} \frac{dt}{t} \rho_e(t) \frac{1}{q^2 - t + i\epsilon}$$

$$\rho_e(t) = \sqrt{1 - \frac{4m_e^2}{t}} \left(1 + \frac{2m_e^2}{t}\right)$$

$$\delta\rho(t) \approx \epsilon^2 \Theta(t - 4m_M^2)$$

$$G(m_M) = \frac{\chi_a}{\chi_\mu} = \frac{\int_0^\infty \frac{dt}{t} \eta_a(t) \delta\rho(t)}{\int_0^\infty \frac{dt}{t} \eta_\mu(t) \delta\rho(t)}$$



Function G measures the ratio of the two-loop effect on the g factor to the one-loop effect on the muonic hydrogen Lamb shift [U.D.J., Ann. Phys. 326, 516 (2011)]

Subversive Millicharged Particles: Influence on Muonic H expressed in Terms of the Remainder G Function

$$G(m_M) = \frac{\chi_a}{\chi_\mu} = \frac{\int_0^\infty \frac{\mathrm{d}t}{t}\eta_a(t)\delta\rho(t)}{\int_0^\infty \frac{\mathrm{d}t}{t}\eta_\mu(t)\delta\rho(t)}$$

Suppose that the energy discrepancy δE in muonic hydrogen were due to the millicharged particle, then how much would the muon anomalous magnetic moment be changed by that same millicharged particle, in terms of the observed discrepancy in the muonic *g*-2 experiment?

[U.D.J., Ann. Phys. 326, 516 (2011)]

Exclusion of a "Heavy" Millicharged Particle

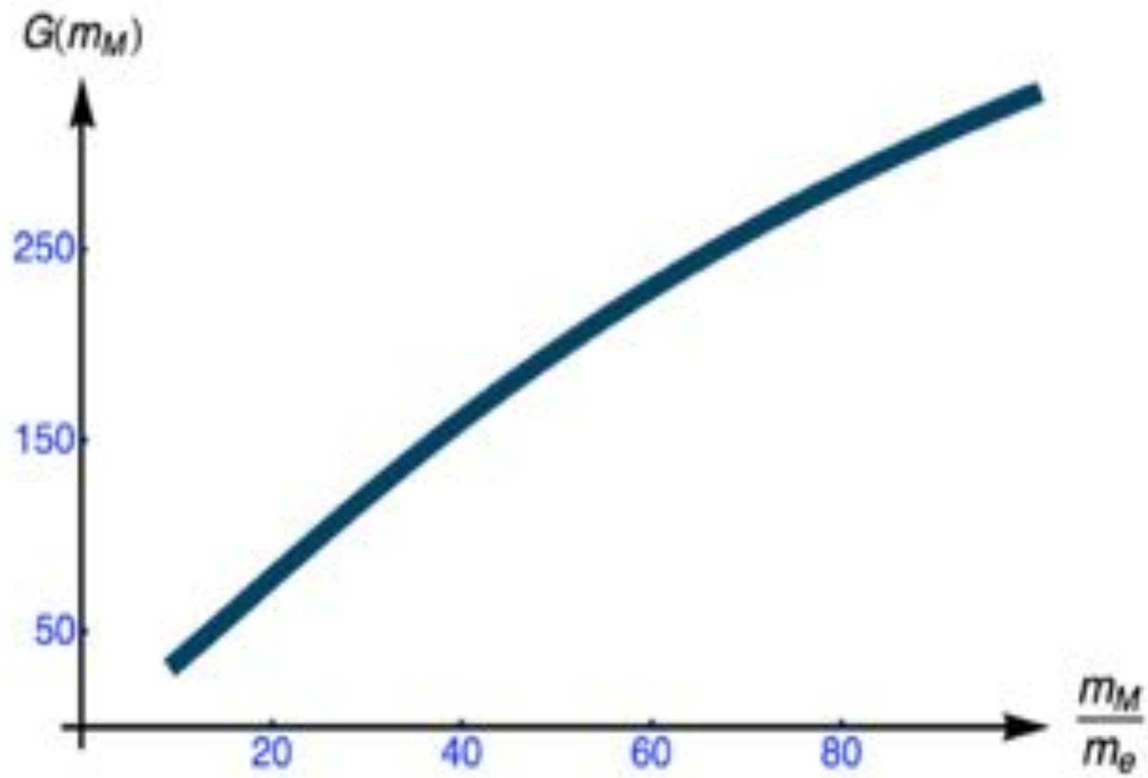


Fig. 3. In the range $10\,m_e < m_M < 100\,m_e$, the function $G(m_M)$ is a lot larger than unity, as shown in the plot. If a hypothetical millicharged particle in the given mass range were responsible for the discrepancy observed in muonic hydrogen, then the same particle would lead to complete disagreement for the anomalous magnetic moment of the muon.

[U.D.J., Ann. Phys. 326, 516 (2011)]

Exclusion of a "Light" Millicharged Particle

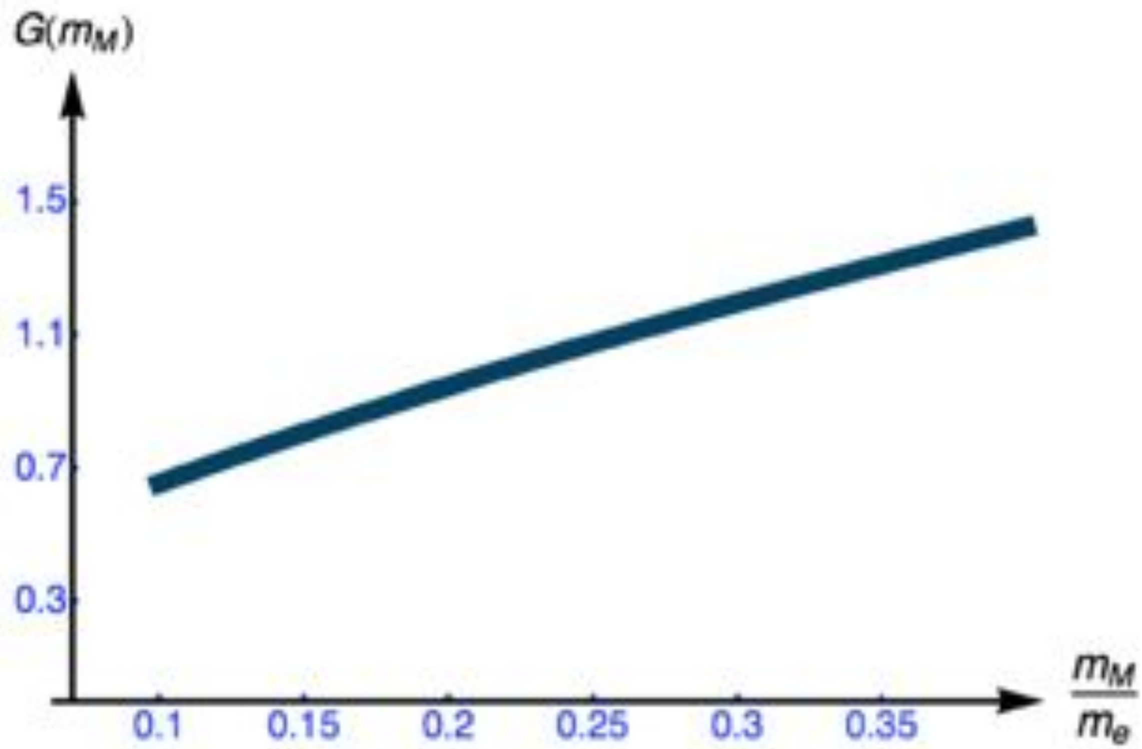


Fig. 4. Both the muon anomalous magnetic moment discrepancy as well as the muonic hydrogen Lamb shift discrepancy can be explained by a millicharged particle of mass $m_M = 0.221\, m_e$ and charge $q = \pm 0.0179e$, as shown in the graph. Indeed, one finds $G(0.221\, m_e) = 1$. However, in the indicated mass range, the correction to the electronic hydrogen Lamb shift induced by the millicharged particle becomes so large that it leads to an inconsistent, sizeable shift of the the proton radius inferred from the hydrogen Lamb shift. See text for further explanations.

[U.D.J., Ann. Phys. 326, 516 (2011)]

Subversive Particles: Influence on Muon g-2 and Muonic H

Conclusion Reached in Various Preprints on Web:

It is difficult if not impossible to even

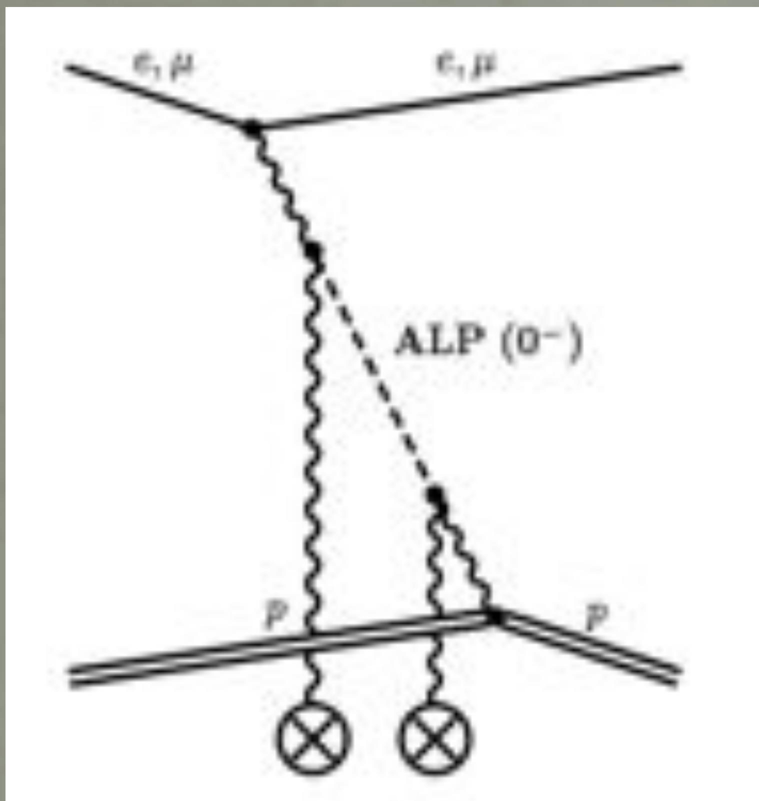
invent

a virtual particle that could explain the muonic hydrogen discrepancy and bring the muonic hydrogen proton radius into agreement with the proton radius derived from electronic hydrogen spectroscopic without messing something else up (e.g. the muon g factor).

[Same is true for direct exchange of "hidden photons", see J. Jaeckel and S. Roy, Phys. Rev. D 82 (2010) 125020]
[A hidden photon is simply too constrained by other spectroscopic and g factor experiments]

Maybe an Axion-Like Particle?

We have a 5T field in the hydrogen trap...



$$V_{\mathrm{ALP}\,0^-}(\vec{r}) = Z\alpha g^2 \left(\frac{\vec{B}^2}{3m_\phi} - \frac{\vec{B}^2\,\vec{r}^{\,2} + (\vec{B}\cdot\vec{r})^2}{8\,r} \right) \sim -\frac{Z\alpha g^2}{8\,r}\left(\vec{B}^2\,\vec{r}^{\,2} + (\vec{B}\cdot\vec{r})^2\right)$$

$$g < 4.9\times 10^{-7}\,\mathrm{GeV}\,, \qquad m_\phi \lesssim 0.5\,\mathrm{meV}\,.$$

$$\delta E = \left\langle 1S \left| -\frac{Z\alpha g^2}{8\,r}\left(\vec{B}^2\,\vec{r}^{\,2} + (\vec{B}\cdot\vec{r})^2\right) \right| 1S \right\rangle = -\frac{g^2\,\vec{B}^2}{4m_r} = -\epsilon_0\,(\hbar c)^3\,\frac{g^2\,\vec{B}^2}{4m_r}\,,$$

$$\delta E_H = -1.67\times 10^{-31}\,\mathrm{eV}\,, \qquad \delta E_{\mu H} = -6.28\times 10^{-34}\,\mathrm{eV}\,.$$

No!

Status Regarding 2S-2P Lamb Shift in mH

Muonic Hydrogen Discrepancy: 0.420 meV.

Largest Conceivable Uncertainty within Standard Model: +/- 0.010 meV.

Theory agrees.

Conundrum remains unsolved!

Observations:

All simple explanations of the proton radius conundrum have been explored (in particular, an error in the theory) and the consensus of the community implies agreement of theory on the level of the proton radius discrepancy.

Furthermore, the relatively good agreement of the muon g-2 experiment with theory sets important constraints for any conceivable "subversive" particles (or, "new physics") as potential explanations of the discrepancy. Theory has a problem even inventing a particle that might explain the effect.

Any remaining theoretical explanation must necessarily be a little bit contrived.

Remaining possibilities:

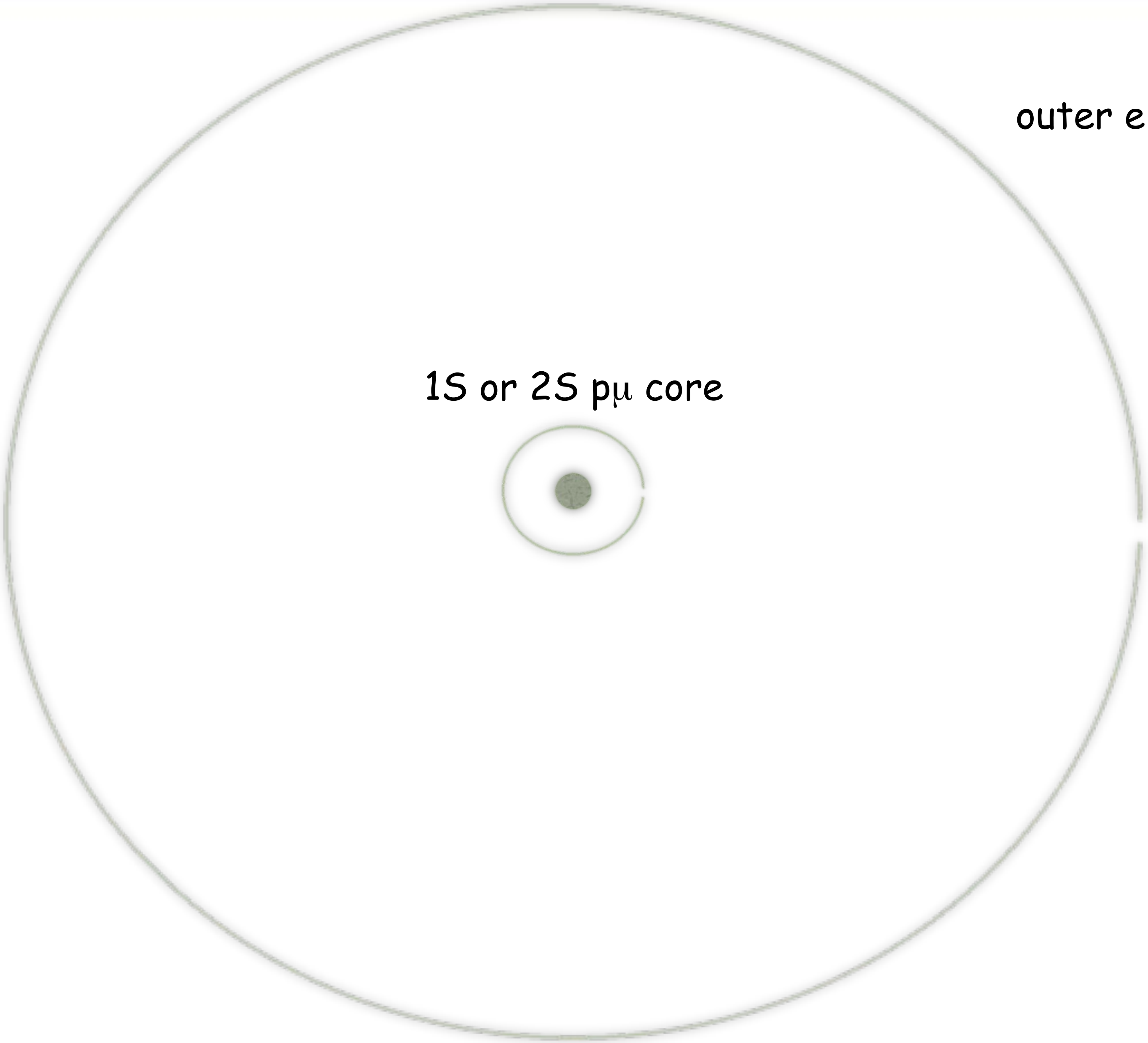
(a) Is there a possible role for three-body physics in muonic hydrogen spectroscopy?

(b) Is there room for novel effects in the extreme electric fields in muonic bound systems?

(c) What about nonresonant corrections to transitions in hydrogen?

[U.D.J. Phys. Rev. A, in press (2015)]

(a) Resonances in the pμe system bound to a 1S or 2S core [probably do not exit] …
… binding potential would be an atom-ion term proportional to $-\alpha_{core}/(2\ r^4)$…



…Karr and Hilico argue that the "ion-atom" interaction of core and the outer electron is too weak to form a bound state [Phys.Rev.Lett. 109, 103401 (2012)]…

Why Three-Body Physics Does Not Solve the Proton-Radius Puzzle

Jean-Philippe Karr[1,2,*] and Laurent Hilico[1,2]

[1]*Laboratoire Kastler Brossel, UPMC-Paris 6, ENS, CNRS; Case 74, 4 place Jussieu, 75005 Paris, France*

[2]*Université d'Evry-Val d'Essonne, Boulevard François Mitterrand, 91025 Evry Cedex, France*

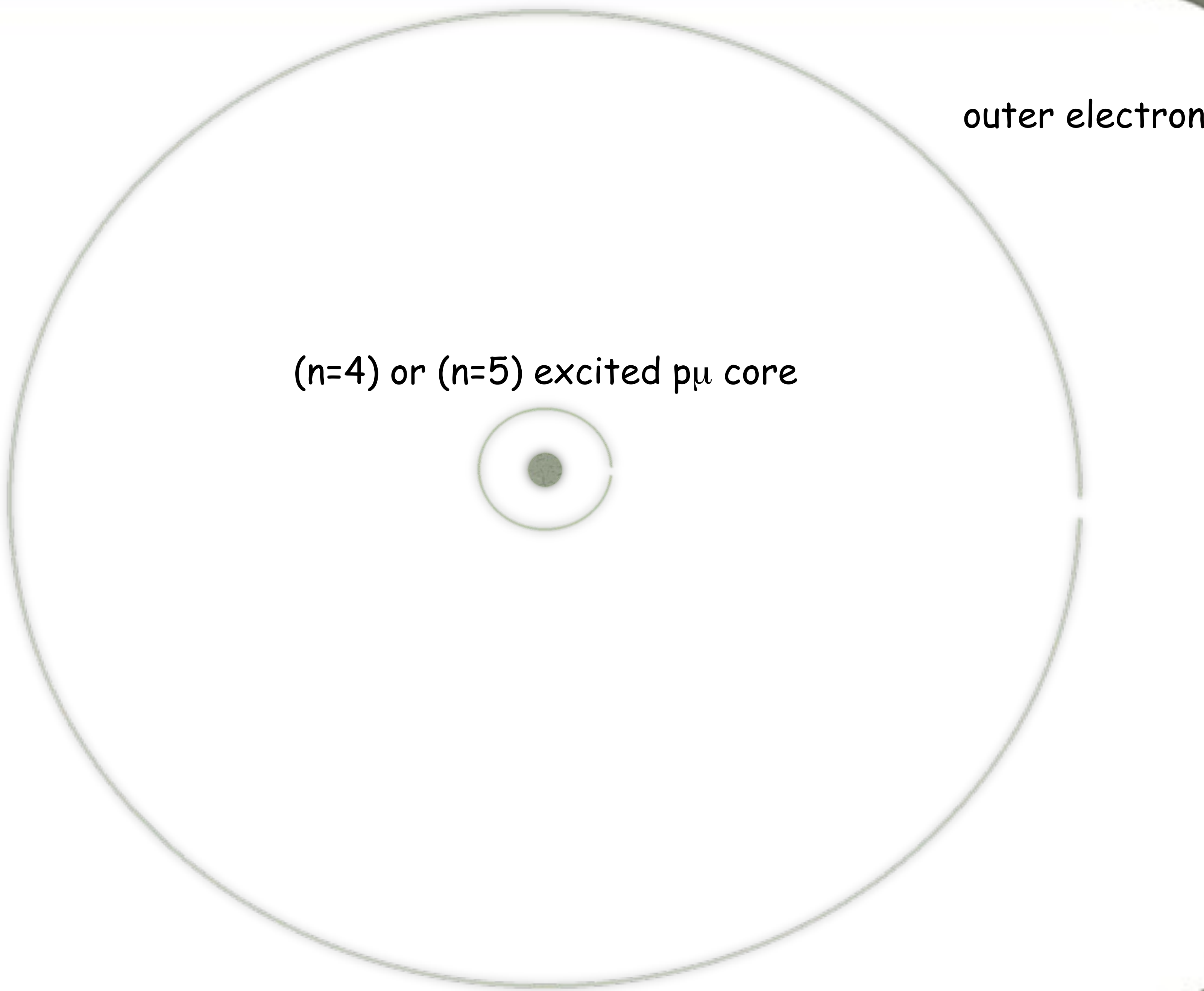
(Received 24 May 2012; published 7 September 2012)

The possible involvement of weakly bound three-body systems in the muonic hydrogen spectroscopy experiment, which could resolve the current discrepancy between determinations of the proton radius, is investigated. Using variational calculations with complex coordinate rotation, we show that in the $p\mu e$ ion, which was recently proposed as a possible candidate, the $p\mu$ core fails to bind the outer electron tightly enough to explain the discrepancy. It is also shown that the $pp\mu$ molecular ion cannot play any role in the observed line.

DOI: 10.1103/PhysRevLett.109.103401 PACS numbers: 36.10.Ee, 31.15.ac, 31.15.xt

…but does this conclusion hold if we consider excited core states, and/or if we include resonances of an excited core with an outer electron!?…work in progress…

…Bound pμe system involving an excited core state…



Question: *Are there really no resonances in the pμe bound system?* The core polarizability goes up as the principal quantum number of the core state increases! [This question is interesting in its own right.]

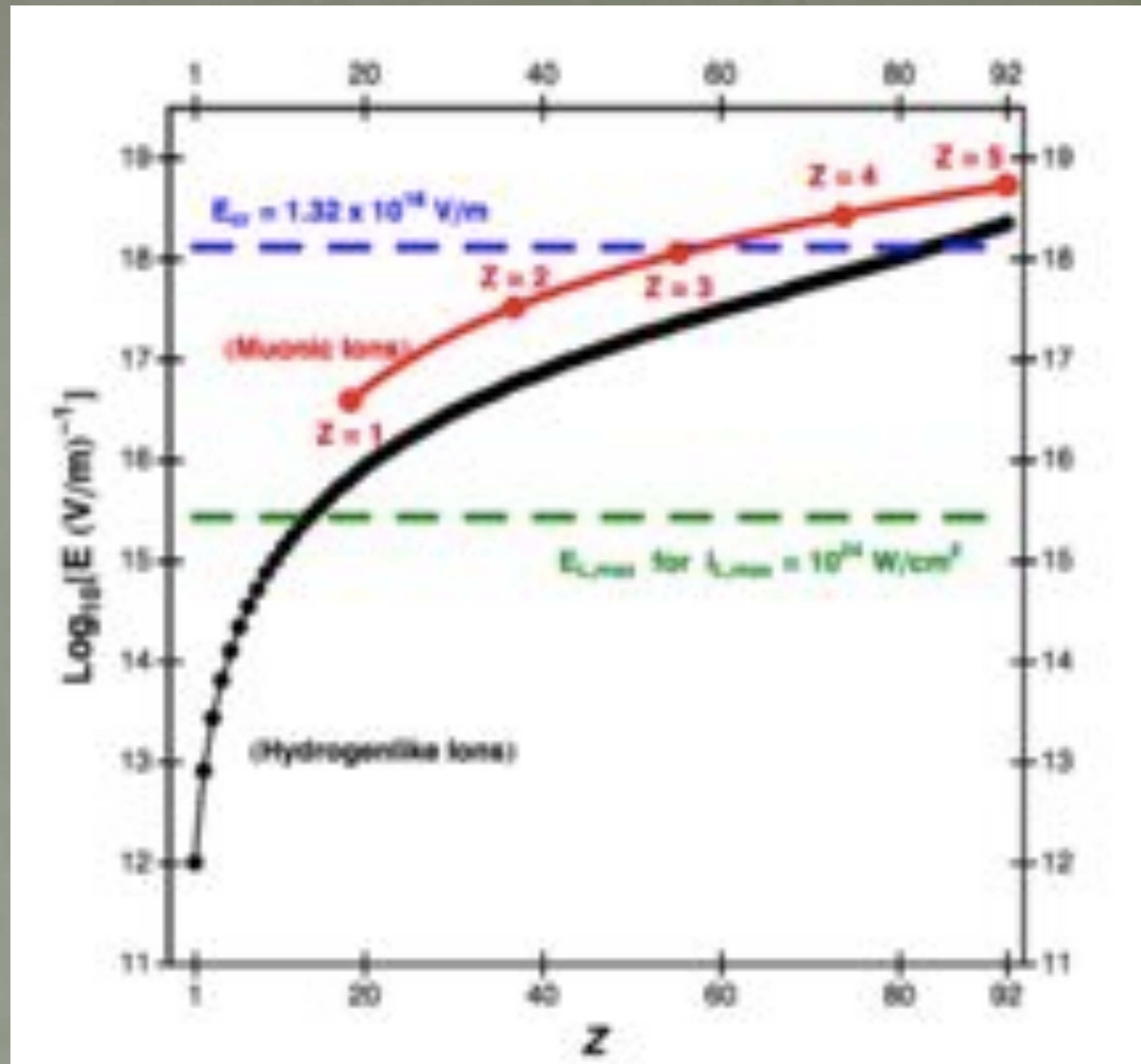
Answer: …there probably are, the 2s state is just unable to able to bind an electron…

Question: If these resonances exist, then, could they be relevant for a partial explanation of the proton radius puzzle?

Answer: …one could counter argue that the resonances could only be relevant if the 1S or 2S core can bind an electron. But perhaps, one should think harder…

Preliminary calculations indicate that higher excited core states of the muon-proton system can bind an outer electron via the atom-ion interaction. Details to follow soon. Role in any conceivable explanation of the discrepancy is unclear.

(b) Extreme Fields and Novel Phenomena?



$$p_{\rm cr}(Z=1) = 0.17\,\%\,,$$
$$p_{\rm cr}(Z=2) = 1.18\,\%\,,$$
$$p_{\rm cr}(Z=3) = 3.36\,\%\,,$$
$$p_{\rm cr}(Z=4) = 6.73\,\%\,,$$
$$p_{\rm cr}(Z=5) = 11.2\,\%\,.$$

$$\langle E \rangle = \left\langle 1S \left| \left(-\frac{\partial}{\partial r} \frac{Z|e|}{4\pi\epsilon_0\, r} \right) \right| 1S \right\rangle = 2\, Z^3\, \frac{m_r^2}{m_e^2}\, \mathcal{E}_0\,,$$

$$\mathcal{E}_0 = \frac{e\, \alpha_{\rm QED}^2\, m_e^2\, c^2}{4\pi\, \epsilon_0\, \hbar^2} = 5.14 \times 10^{11}\, \frac{\rm V}{\rm m}\,.$$

[U.D.J. Phys. Rev. A, in press (2015)]

(c) Nonresonant or Quantum Interference Effects in Normal (Electronic, Atomic) Hydrogen

$$\delta E = \frac{(\hbar\Gamma)^2}{\Delta E_{\mathrm{fs}}} \sim \alpha_{\mathrm{QED}}^6 \, m_e \, c^2 \,,$$

$$\xi = \frac{r_p}{\lambda_C} = 2.27 \times 10^{-3} \,.$$

$$R_P = \frac{E_{\mathrm{PP}}}{\hbar\Gamma_{nP}} = \frac{n^3 \, \chi_{\mathrm{PP}} \, \xi^2}{12 \, \alpha \, \chi_{\Gamma P}} = 1.68 \times 10^{-5} \, n^3 \,. \qquad R_P(n = 4) = 0.0011,$$

$$R_D = \frac{E_{\mathrm{PP}}}{\hbar\Gamma_{nD}} = \frac{n^3 \, \chi_{\mathrm{PP}} \, \xi^2}{12 \, \alpha \, \chi_{\Gamma D}} = 4.86 \times 10^{-5} \, n^3 \,. \qquad R_D(n = 12) = 0.084$$

2S-4P is problematic but 2S-12D is not so hard in terms of "splitting the line".

[U.D.J. Phys. Rev. A, in press (2015)]

Something strange is going on….

Historical Perspective: [PRL 23 (1969) 153]

VOLUME 23, NUMBER 3 PHYSICAL REVIEW LETTERS 21 JULY 1969

HIGH-ENERGY MUON-PROTON SCATTERING: MUON-ELECTRON UNIVERSALITY[*]

L. Camilleri,[†] J. H. Christenson, M. Kramer,[‡] and L. M. Lederman
Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

and

Y. Nagashima and T. Yamanouchi
University of Rochester, Rochester, New York 14627,
and Brookhaven National Laboratory, Upton, New York 11973
(Received 10 April 1969)

Measurements of the μ-p elastic cross section in the range $0.15 < q^2 < 0.85$ $(\mathrm{GeV}/c)^2$ are compared with similar e-p data. We find an apparent disagreement between the muon and electron experiments which can possibly be accounted for by a combination of systematic normalization errors.

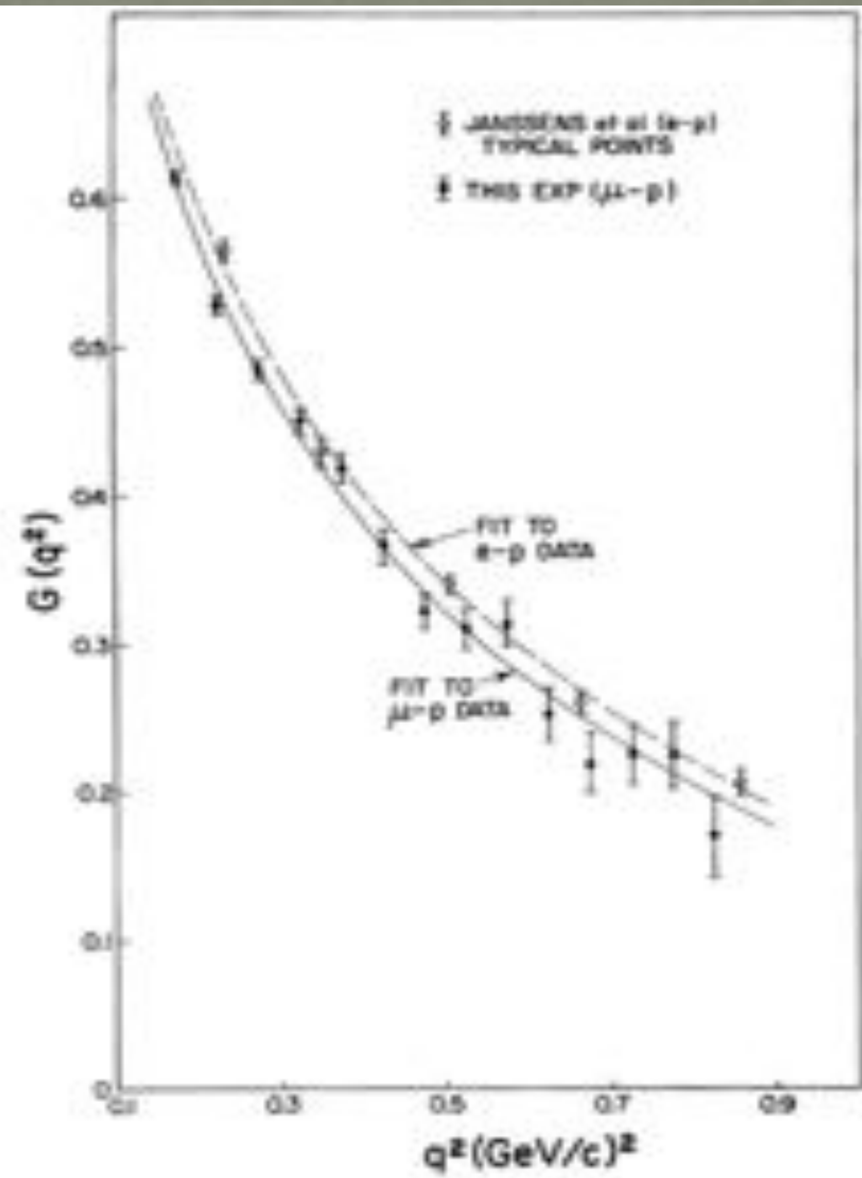


FIG. 1. Measurements of the form factor $G(q^2)$ vs q^2 for this experiment and for the $e-p$ data of Janssens et al. Not all of the electron data are shown. The solid and dashed curves represent fits to the muon and electron data, respectively.

[PRL 23 (1969) 153]

[PRD 6 (1972) 106]

PHYSICAL REVIEW D VOLUME 6, NUMBER 1 1 JULY 1972

Comparison of Muon-Proton and Electron-Proton Inelastic Scattering*

T. J. Braunstein,† W. L. Lakin, F. Martin, M. L. Perl, W. T. Toner,‡ and T. F. Zipf
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 17 January 1972)

Measurements of the differential cross section for the inelastic scattering of 12-GeV/c muons on protons are reported. These measurements cover a kinematic range of $|q^2|$ (the square of the four-momentum transferred from the lepton) up to 4.0 $(\mathrm{GeV}/c)^2$ and of muon energy losses (ν) up to 9.0 GeV. Only the scattered muon is observed in an optical spark-chamber apparatus. The data are compared with electron-proton inelastic scattering, and analyzed in terms of possible lepton form factors and anomalous interactions. μ-p inelastic scattering is found to exhibit the same mild $|q^2|$ behavior as does e-p inelastic scattering. No experimentally significant deviation from the predictions of muon-electron universality has been found. If the ratio of muon to electron inelastic cross sections is parametrized by the form $(1.0 + |q^2|/\Lambda_D^2)^{-2}$, we find with 97.7% confidence that $\Lambda_D > 4.1$ GeV/c. The muon-proton cross sections on the average are slightly smaller than the electron-proton cross sections. This observation is not experimentally significant because such a difference might be caused by systematic errors, but this observation is used to speculate as to the most fruitful direction for future experiments.

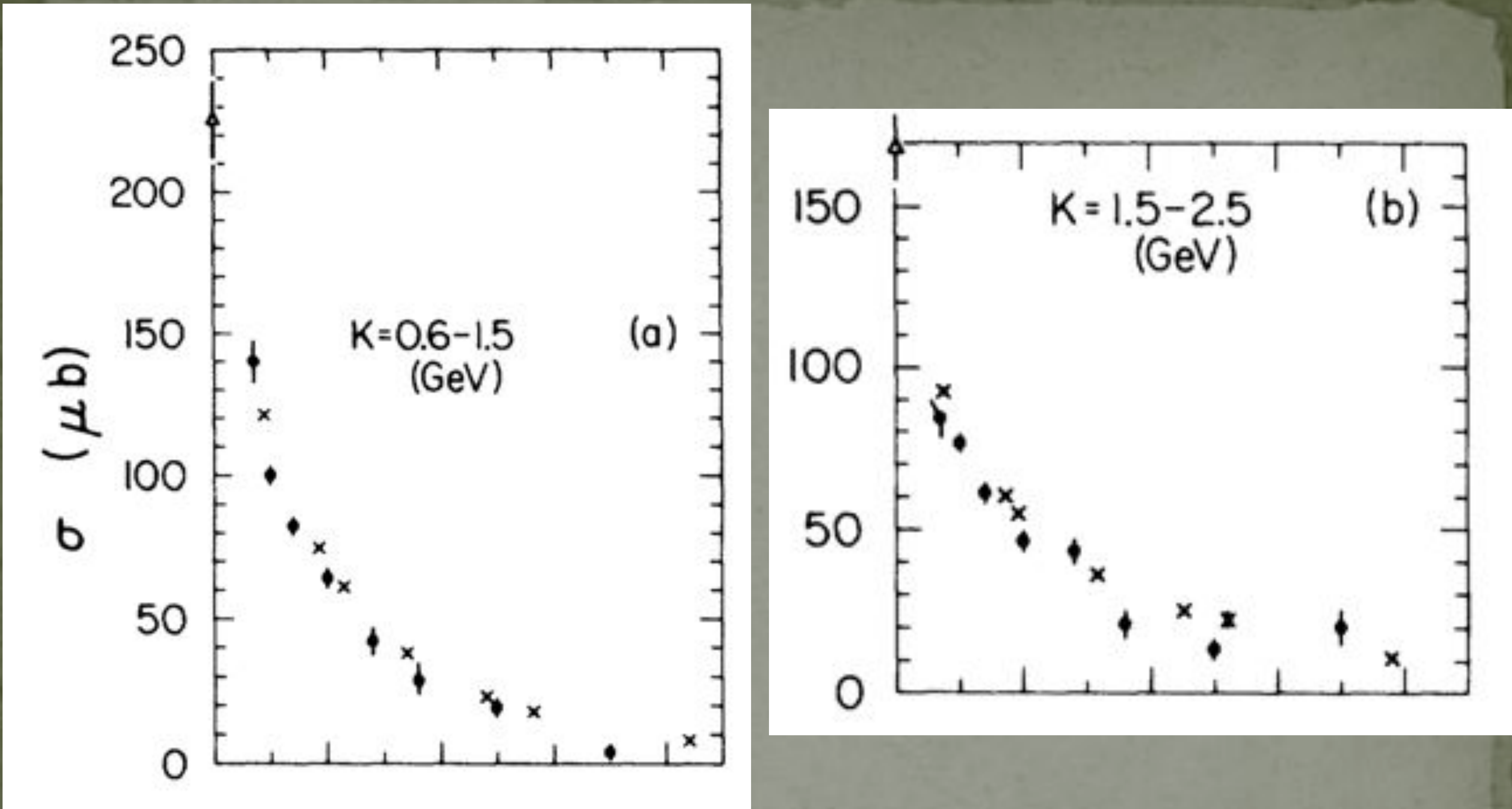


FIG. 15. For each K interval the upper plot gives the experimental values of $\sigma_{\mathbf{exp},\mu}(q^2,K,p_\mu)$ denoted by a solid circle, $\sigma_{\mathbf{exp},e}(q^2,K,p_\mu)$ denoted by an x, and $\sigma_{\gamma p}(K)$

$$N^2 = 0.922 \pm 0.013 \quad (48)$$

VIII. SPECULATIONS

In our inelastic experiment and in the two elastic experiments, there are no statistically significant indications of any q^2-dependent differences between the muon and the electron. But in all of these experiments the muon cross sections turn out to be lower than the electron cross sections. We emphasize that in our experiment this difference is not significant because the over-all normalization uncertainty is about 8%. In the elastic experiments the authors give a smaller normalization uncertainty for the muon data, but the combined over-all normalization uncertainty of the muon and electron data might be as large as our 8%. Thus the low muon cross section in any one experiment is not significant. However, we should not totally ignore these "normalization differences.'

Part I: One can apply the gravitational coupling of Dirac particles in order to solve a number of problems of practical interest, including central-field problems and variations thereof, and potential gravitational corrections to quantum field theoretical phenomena, like vacuum polarization.

Part II: The muonic hydrogen experiment has led to a very hard-to-resolve discrepancy regarding the proton radius. Scattering experiments and the "reverse engineering" of ordinary hydrogen transitions lead to a value of rp=0.88fm, while muonic hydrogen leads to rp=0.84fm. Many conceivable explanations have been tried. Electron versus muon scattering off of protons has led to discrepancies in the past.

Conclusions

The muonic hydrogen experiment has led to a very hard-to-resolve discrepancy regarding the proton radius. Scattering experiments and the "reverse engineering" of ordinary hydrogen transitions lead to a value of r_p=0.88fm, while muonic hydrogen leads to r_p=0.84fm. Many conceivable explanations have been tried. Electron versus muon scattering off of protons has led to discrepancies in the past.

Thank you for your Attention!