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What Comes Beyond the Standard Models

Bled, July 11–19, 2015

Edited by

Norma Susana Mankoč Borštnik
Holger Bech Nielsen
Dragan Lukman

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Contents

Preface

The series of workshops on "What Comes Beyond the Standard Models?" started in 1998 with the idea of Norma and Holger for organizing a real workshop, in which participants would spend most of the time in discussions, confronting different approaches and ideas. It is the eighteenth workshop which took place this year in the picturesque town of Bled by the lake of the same name, surrounded by beautiful mountains and offering pleasant walks and mountaineering.

In our very open minded, friendly, cooperative, long, tough and demanding discussions several physicists and even some mathematicians have contributed. Most of topics presented and discussed in our Bled workshops concern the proposals how to explain physics beyond the so far accepted and experimentally confirmed both standard models - in elementary particle physics and cosmology. Although most of participants are theoretical physicists, many of them with their own suggestions how to make the next step beyond the accepted models and theories, experts from experimental laboratories were very appreciated, helping a lot to understand what do measurements really tell and which kinds of predictions can best be tested.

The (long) presentations (with breaks and continuations over several days), followed by very detailed discussions, have been extremely useful, at least for the organizers. We hope and believe, however, that this is the case also for most of participants, including students. Many a time, namely, talks turned into very pedagogical presentations in order to clarify the assumptions and the detailed steps, analysing the ideas, statements, proofs of statements and possible predictions, confronting participants' proposals with the proposals in the literature or with proposals of the other participants, so that all possible weak points of the proposals showed up very clearly. The ideas therefore seem to develop in these years considerably faster than they would without our workshops.

In the eighteen years of our workshops the organizers, together with the participants, are trying to answer several open questions of the elementary particle physics and cosmology. Experiments have made large steps in this time. Among the most notable and might be also among the most important ones was two years ago the LHC confirmation that the scalar field, the higgs is like other fermionic and bosonic fields - just a field. And yet it is a very unusual field: A boson with the fractional weak and hyper charges, resembling fermion charges. Do we have the explanation for that? Can we explain the origin of families and Yukawa couplings? Can we understand and explain all the assumptions of the *standard model*? That is, can we explain the appearance of the charges of the family members, quarks

and leptons, the left handed members distinguishing in charges from the right handed ones? Can we understand the appearance of the vector and scalar gauge fields of these charges? How many are they? Can we understand the origin of the matter-antimatter asymmetry, of the dark matter?
The behaviour of the nature, that is the evolution of our universe and the dynamics of it can be understood on all levels — from the elementary fermionic and bosonic particles (fields) and their mutual interactions to matter of all kinds, forming galaxies, clusters of galaxies and our universe — only if we have the theory behind, which explains the observed phenomena and predicts new phenomena. It is hard to distinguish among theories, which all explain the same observed phenomena, unless one of them is more predictive, better mathematically supported, offering more detailed predictions for the future observations.
Should we design theories and models in steps, each one more or less adapted for explaining a new experimental observation, in particular if such models can help to explain a small next step? Or can we suggest the theory which answers many open questions at the same time?
Can it happen that at the LHC no new fields - scalars, vectors or fermions - will be observed, so that there will be no sign which will help to make a trustable step beyond the standard model?
This can hardly happen. The (so far) observed three families, (only) one scalar field and several Yukawa couplings call for explanation for the origin of families and of the higgs, suggesting that there are several scalar fields, which manifest as the higgs and the Yukawa couplings.
If trusting the *spin-charge-family* theory, presented in this workshop in details, there exists the fourth family, coupled to the observed three families, and several scalar fields, all with the weak and the hyper charges of the higgs, carrying additional quantum numbers — either the family ones or the family members ones — which explain the origin of Yukawa couplings. This theory, which offers the explanation for all the assumptions of the *standard model* — for the appearance of charges of the family members, for families, for all the properties of fermions and of the vector gauge fields, explaining why there are scalar fields with the weak and the hyper charge of the *standard model* higgs — offers also the explanation for the appearance of the matter/anti-matter asymmetry in the universe by predicting that there are scalars, which are colour triplets, causing transitions between anti-leptons into quarks and anti-quarks into quarks and back. In the presence of the scalar condensate of the two right handed neutrinos, which breaks the CP symmetry, might in the expanding universe in thermal inequilibrium take care of the matter/anti-matter asymmetry. Predicting two groups of four families of quarks and leptons the theory might explain also the appearance of the dark matter.
The fermionization/bosonization theory, showing that at least for free massless particles it is possible to construct a boson theory, which is equivalent (in terms of momenta and energy) to a fermion theory, might help to understand whether the *spin-charge-family* theory, doing so well in explaining the assumptions of the *standard model*, is the acceptable next step beyond the *standard model*, and what might be beyond the *spin-charge-family* theory.

The models, like a model of supersymmetry in the presence of a very small cosmological constant, which is able to manifest at the low energy regime the observed data, might help to understand how has the nature proceeded in the expanding universe. In particular since the model shows correspondence with string and other models.
The conformal field theory action and its correspondence to the Chern-Simons action, discussed in this workshop, is manifesting the use of mathematical approaches while making the correspondence among approaches, in order to better understand "thoughts" of the nature.
The mathematical proofs are essential for all the theories. The proof is discussed and presented, showing the equivalence between the vielbeins and the spin connection fields in the Kaluza-Klein theories when representing the vector (and the scalar) gauge fields in $d = 3 + 1$.
The studies of the origin of families, which might explain, why the nature manifests in the low energy regime the families of quarks and leptons with the observed properties, go in this workshop in several attempts: Besides with the *spin-charge-family* theory, which is able to explain the properties of families of quarks and leptons, also by using a triple tensor products of the Dirac spinors, the representations of which can be identified with the three observed families of quarks and leptons and one more family, offering the explanation for the existence of the dark matter. It is also the attempt, presented in this workshop, which demonstrates, how close to the democratic matrix can the mass matrices of quarks and leptons be parametrized while still manifesting the quarks mixing matrices. There is the attempt, presented in this workshop, to extend the *standard model* with new fermion and boson fields to explain mass matrices of quarks and leptons and correspondingly their masses and mixing matrices.
There is also the possibility, that the dark matter might consist of the -2 electromagnetically charged particles, bound by the ordinary Coulomb interaction with primordial helium in OHe. The author discussed the solved and not yet solved problems of this model.
The last progress in experiments, manifesting that the measured annual modulation, can hardly be something else but the interaction of the dark matter from our galaxy with the scintillators in DAMA/NaI and DAMA/LIBRA experiments is reported.
There were works on many body problems on hadron physics, interesting for high energy physics as well. It is happening many times in physics, that experiences from one field of physics can successfully be used also on other fields, provided that the symmetry of the systems is comparable.
It is suggested to use the experiences with the effective action, developed for studying hadron resonances, to calculate the scattering of two higgses or two heavy bosons in the energy region of 1-3 TeV.
It is claimed and represented that the light-front is providing a physical, frame independent formalism, offering a new inside into the hadronic mass scale, hadronic spectrum and the running coupling constants in nonperturbative domain.

It is demonstrated that the theory of Dirac particles in curved space-times caused by several central potential confirms the weak equivalence principle in deep gravitational potentials.
As every year also this year there has been not enough time to mature the very discerning and innovative discussions, for which we have spent a lot of time, into the written contributions.
Since the time to prepare the proceedings is indeed very short, three months if vacations included, authors did not have a time to polish their contributions carefully enough.
Bled Workshops owe their success to participants who have at Bled in the heart of Slovene Julian Alps enabled friendly and active sharing of information and ideas, yet their success was boosted by videoconferences. Questions and answers as well as lectures enabled by M.Yu. Khlopov via Virtual Institute of Astroparticle Physics (viavca.in2p3.fr/site.html) of APC have in ample discussions helped to resolve many dilemmas.
The reader can find the records of all the talks delivered by cosmovia since Bled 2009 on viavca.in2p3.fr/site.html in Previous - Conferences. The six talks delivered by: L. Bonora (Regularization of conformal correlators), R. Cerulli (Particle Dark Matter direct detection), N.S. Mankoč Borštnik (How many answers of the open questions of the Standard Model can the Spin-Charge-Family theory offer?), S. Brodsky (New perspectives for hadron physics and the cosmological constant problem), M. Yu. Khlopov (Composite dark matter) and H.B.F. Nielsen (Fermionization in an Arbitrary Number of Dimensions), can be accessed directly at
http://viavca.in2p3.fr/what_comes_beyond_the_standard_models_xviii.html
Most of the talks can be found on the workshop homepage
http://bsm.fmf.uni-lj.si/.
Let us conclude this preface by thanking cordially and warmly to all the participants, present personally or through the teleconferences at the Bled workshop, for their excellent presentations and in particular for really fruitful discussions and the good and friendly working atmosphere.
The workshops take place in the house gifted to the Society of Mathematicians, Physicists and Astronomers of Slovenia by the Slovenian mathematician Josip Plemelj, well known to the participants by his work in complex algebra.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(the Organizing comittee)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(the Editors)

Ljubljana, December 2015

1 Predgovor (Preface in Slovenian Language)

Serija delavnic "Kako preseči oba standardna modela, kozmološkega in elektrošibkega" ("What Comes Beyond the Standard Models?") se je začela leta 1998 z idejo Norme in Holgerja, da bi organizirali delavnice, v katerih bi udeleženci v izčrpnih diskusijah kritično soočili različne ideje in teorije. Letos smo imeli osemnajsto delavnico na Bledu ob slikovitem jezeru, kjer prijetni sprehodi in pohodi na čudovite gore, ki kipijo nad mestom, ponujajo priložnosti in vzpodbudo za diskusije.
K našim zelo odprtim, prijateljskim, dolgim in zahtevnim diskusijam, polnim iskrivega sodelovanja, je prispevalo veliko fizikov in celo nekaj matematikov. Večina predlogov teorij in modelov, predstavljenih in diskutiranih na naših Blejskih delavnicah, išče odgovore na vprašanja, ki jih v fizikalni skupnosti sprejeta in s številnimi poskusi potrjena standardni model osnovnih fermionskih in bozonskih polj ter kozmološki standardni model puščata odprta. Čeprav je večina udeležencev teoretičnih fizikov, mnogi z lastnimi idejami kako narediti naslednji korak onkraj sprejetih modelov in teorij, so še posebej dobrodošli predstavniki eksperimentalnih laboratorijev, ki nam pomagajo v odprtih diskusijah razjasniti resnično sporočilo meritev in kakšne napovedi so potrebne, da jih lahko s poskusi dovolj zanesljivo preverijo.
Organizatorji moramo priznati, da smo se na blejskih delavnicah v (dolgih) predstavitvah (z odmori in nadaljevanji čez več dni), ki so jim sledile zelo podrobne diskusije, naučili veliko, morda več kot večina udeležencev. Upamo in verjamemo, da so veliko odnesli tudi študentje in večina udeležencev. Velikokrat so se predavanja spremenila v zelo pedagoške predstavitve, ki so pojasnile predpostavke in podrobne korake, soočile predstavljene predloge s predlogi v literaturi ali s predlogi ostalih udeležencev ter jasno pokazale, kje utegnejo tičati šibke točke predlogov. Zdi se, da so se ideje v teh letih razvijale bistveno hitreje, zahvaljujoč prav tem delavnicam.
V teh osemnajstih letih delavnic smo organizatorji skupaj z udeleženci poskusili odgovoriti na marsikatero odprto vprašanje v fiziki osnovnih delcev in kozmologiji. Na vsakoletnem napovedniku naše delavnice objavimo zbirko odprtih vprašanj, na katera bi udeleženci utegnili predlagati rešitve. V osemnajstih letih so eksperimenti napravili velike korake. Med najpomembnejšimi dosežki je potrditev LHC, da je skalarno pole, Higgsov delec, prav tako polje kot ostala fermionska in bozonska polja. In vendar je to skalarno polje zelo nenavadno polje: Je bozon s polovičnim šibkim in hiper nabojem, kot pritičeta fermionom. Ali to razumemo? Ali lahko pojasnimo izvor družin in Yukawinih sklopitev? Znamo pojasniti nesimetrijo med snovjo in antisnovjo v vesolju? Znamo razložiti privzetke *standardnega modela*?

Dinamiko vesolja na vseh nivojih, od osnovnih delcev do snovi, lahko razumemo samo, če ponudimo teorijo, ki opažanja razloži in napove nova spoznanja. Je prava pot pri postavljanju teorij ta, da prilagodimo teorijo eksperimentalnim spoznanjem po korakih in s tem omogočimo napovedi za naslednje majhne korake? Ali pa ponudimo teorijo, ki odgovori na mnoga (morda vsa) doslej odprta vprašanja?
Naravne zakone, to je razvoj našega vesolja in njegovo dinamiko, lahko razumemo na vseh nivojih — od elementarnih fermionskih in bozonskih delcev (polj) in njihovih vzajemnih interakcij s snovjo vseh vrst, ki tvori galaksije, gruče galaksij in naše vesolje — le, če imamo teorijo, ki pojasni opažene pojave in napove nove. Med teorijami, ki pojasnjujejo iste pojave, je težko ločevati, razen, če je kakšna bolj elegantna, preprosteša, ima boljše matematične temelje in daje bolj natančne napovedi za prihodnja opazovanja.
Kaj pa, če na LHC ne bodo izmerili nobenega novega polja, ne skalarnega, ne vektorskega, ne fermionskega in ne bo ponudil eksperiment nobenega napotka, kako napraviti naslednji korak od *standardnega modela?*
Menimo, da je to malo verjetno. Dosedaj opažene (tri) družine, (samo) eno skalarno polje in več Yukawinih sklopitev kliče po razlagi izvora družin in higgsa in namiguje, da je skalarnih polj več in da se kažejo kot higgs in Yukawine sklopitve.
Če ima prav teorija *spina, naboja in družin*, ki je bila podrobno predstavljena na tej delavnici, potem obstaja četrta družina, ki je sklopljena z že opaženimi. Je tudi več skalarnih polj, vsa s šibkim in hiper nabojem kot ga ima higgs, in še z dodatnimi kvantnimi števili — s kvantnimi števili družin in njihovih članov — ki pojasnijo izvor Yukawinih sklopitev. Ta teorija ponudi razlago za vse privzetke *standardnega modela* — pojasni izvor nabojev članov družin, izvor družin in družinskih kvantnih števil, pojasni lastosti fermionov in vektorskih umeritvenih polj, pojasni, zakaj nosijo skalarna polja šibki in hiper naboj higgsa — in ponudi tudi razlago za pojav asimetrije med snovjo in antisnovjo opaženo v vesolju, saj napove obstoj skalarjev, barvnih tripletov, ki sprožijo prehod antileptonov v kvarke in antikvarkov v kvarke ter obratne procese. Skalarni kondenzat dveh desnoročnih nevtrinov, ki zlomi CP simetrijo, lahko povzroči asimetrijo snovi in antisnovi v vesolju, ki se razširja in je v toplotnem neravnovesju. Ker napove dve skupini po štiri družine kvarkov in leptonov, lahko morda pojasni pojav temne snovi.
Teorija fermionizacije/bozonizacije, ki pokaže, da ja vsaj za proste brezmasne delce možno konstruirati bozonsko teorijo, ki je ekvivalentna (vsaj kar se tiče gibalne količine in energije) fermionski teoriji, morda lahko pomaga razumeti, ali je teorija *spinov-nabojev-družin*, ki je tako uspešna pri razlagi predpostavk *standardnega modela*, sprejemljiv naslednji korak onkraj *standardnega modela* in kaj je morda onkraj te teorije.
Modeli, kot je model supersimetrije v prisotnosti zelo majhne kozmološke konstante, ki lahko v območju nizkih energij ponudi ujemanje napovedi modela z meritvami, lahko pomagajo razumeti, kako se narava razvija v razširjajočem se vesolju, zlasti če model pokaže povezavo s strunami in ostalimi modeli.
Akcija konformne teorije polja in njena povezava z akcijo Cherna in Simonsa, predstavljena na delavnici, je demonstracija uporabe različnih matematičnih pristopov ter povezav med njimi, ki mnogo prispevajo k boljšemu razumevanju narave.

Matematični dokaz, prikazan na delavnici, pokaže ekvivalentnost predstavitve umeritvenih vektorskih (in skalarnih) polj v $d = 3 + 1$ med vektorskimi svežnji in polji spinskih povezav v teorijah Kaluze-Kleinovega tipa v poljubno razsežnih prostorih.
Raziskave izvora družin, ki naj pojasnijo, zakaj se v naravi pojavijo pri nizkih energijah družine kvarkov in leptonov z opaženimi lastnostmi, se na tej delavnici lotimo na več načinov: poleg teorije *spinov-nabojev-družin*, ki lahko pojasni lastnosti kvarkov in leptonov, še s pristopom s trojnim tenzorskim produktom Diracovih spinorjev, katerega upodobitve lahko identificiramo s tremi poznanimi družinami kvarkov in leptonov ter s še eno družino, ki pojasni temno snov. Na tej delavnici je prikazan uspešen poskus takšne parametrizacije kvarkovskih masnih matrik, ki so zelo blizu demokratičnima, tako da produkt unitarnih transformacij teh matrik pojasni izmerjene lastnosti mešalne matrike kvarkov. V zborniku je predstavljen poskus razširitve *standardnega modela* z novimi fermionskimi in bozonskimi polji z družinskimi kvantnimi števili simetrije $SU(3)$, kar lahko pojasni masne matrike kvarkov in leptonov, njihove mase in mešalne matrike.
Obravnavamo tudi možnost, da temno snov sestavljajo delci z elektromagnetnim nabojem -2, ki jih običajna Coulombska interakcija veže s prvotnim helijem v OHe. Avtor obravnava rešene in odprte probleme v tem modelu.
Poročilo o vseh dosedaj narejenih meritvah na eksperimentu DAMA/NaI in DAMA/LIBRA, ki kažejo izrazito letno modulacijo, skoraj ne dopušča dvoma, da gre za interakcijo temne snovi naše galaksije s scintilatorji DAMA/NaI in DAMA/LIBRA.
Zanimivo je tudi delo, ki prenaša izkušnje iskanja rešitev sistemov mnogih teles v fiziki hadronov, na fiziko visokih energij. Kot se zgodi mnogokrat v fiziki, lahko tudi tu izkušnje iz enega področja fizike prenesemo na drugo področje, kadar so simetrije sistemov primerljive. Avtorji predlagajo uporabo efektivne akcije, razvito za študij hadronskih resonanc, za študij sipanja dveh higgsov ali dveh težkih bozonov v energijskem območju 1-3 TeV.
Avtor uporabe koordinatnega sistema na svetlobnem stožcu (light front) in z od opazovalnega sistema neodvisnim formalizmom ponudi nov vpogled v masno skalo hadronov, spekter hadronov in spremenljivo sklopitveno konstanto v neperturbacijskem območju kromodinamike.
Iskanje rešitev Diracovih delcev v ukrivljenih prostor-časih, ki jih povzročijo različni potencali, ovrže domnevo, da v v globokih gravitacijskih potencialih načelo šibke ekvivalencene velja.
Kot vsako leto nam tudi letos ni uspelo predstaviti v zborniku kar nekaj zelo obetavnih diskusij, ki so tekle na delavnici. Premalo je bilo ča do zaključka redakcije.
Četudi so k uspehu „Blejskih delavnic" največ prispevali udeleženci, ki so na Bledu omogočili prijateljsko in aktivno izmenjavo mnenj v osrčju slovenskih Julijcev, so k uspehu prispevale tudi videokonference, ki so povezale delavnice z laboratoriji po svetu. Vprašanja in odgovori ter tudi predavanja, ki jih je v zadnjih letih omogočil M.Yu. Khlopov preko Virtual Institute of Astroparticle Physics (viavca.in2p3.fr/site.html, APC, Pariz), so v izčrpnih diskusijah pomagali razčistiti marsikatero dilemo.

Bralec najde zapise vseh predavanj, objavljenih preko "cosmovia" od leta 2009, na viavca.in2p3.fr/site.html v povezavi Previous - Conferences. Šest letošnjih predavanj,

L. Bonora (Regularization of conformal correlators), R. Cerulli (Particle Dark Matter direct detection), N.S. Mankoč Borštnik (How many answers of the open questions of the Standard Model can the Spin-Charge-Family theory offer?), S. Brodsky (New perspectives for hadron physics and the cosmological constant problem), M. Yu. Khlopov (Composite dark matter) in H.B.F. Nielsen (Fermionization in an Arbitrary Number of Dimensions), je dostopnih na

http://viavca.in2p3.fr/what_comes_beyond_the_standard_models_xviii.html

Večino predavanj najde bralec na spletni strani delavnice na

http://bsm.fmf.uni-lj.si/.

Naj zaključimo ta predgovor s prisrčno in toplo zahvalo vsem udeležencem, prisotnim na Bledu osebno ali preko videokonferenc, za njihova predavanja in še posebno za zelo plodne diskusije in odlično vzdušje.

Delavnica poteka v hiši, ki jo je Društvu matematikov, fizikov in astronomov Slovenije zapustil v last slovenski matematik Josip Plemelj, udeležencem delavnic, ki prihajajo iz različnih koncev sveta, dobro poznan po svojem delu v kompleksni algebri.

Norma Mankoč Borštnik, Holger Bech Nielsen, Maxim Y. Khlopov,
(Organizacijski odbor)

Norma Mankoč Borštnik, Holger Bech Nielsen, Dragan Lukman,
(uredniki)

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1 Aspects of String Phenomenology in Particle Physics and Cosmology

I. Antoniadis ⋆

LPTHE, UMR CNRS 7589 Sorbonne Universités, UPMC Paris 6,
75005 Paris, France
and Albert Einstein Center, Institute for Theoretical Physics,
Bern University, Sidlerstrasse 5, 3012 Bern, Switzerland

Abstract. We describe the phenomenology of a model of supersymmetry breaking in the presence of a tiny (tunable) positive cosmological constant. It utilises a single chiral multiplet with a gauged shift symmetry, that can be identified with the string dilaton (or an appropriate compactification modulus). The model is coupled to the MSSM, leading to calculable soft supersymmetry breaking masses and a distinct low energy phenomenology that allows to differentiate it from other models of supersymmetry breaking and mediation mechanisms.

Povzetek. Avtor obravnava lastnosti modela za zlom supersimetrije, ko majhno pozitivno kozmološko konstanto prilagaja fenomenološkim lastnostim. Obravnava primer kiralnega multipleta, ko postane umeritvena simetrija dilatacijska simetrija strune (uporabiti pa je mogoče tudi kak drug model kompaktifikacije). Model poveže s standardnim modelom z minimalno supersimetrijo, kar omogoči izračun mas pri mehki zlomitvi supersimetrije. Model uspešno opiše fenomenološke lastnosti polj, kar ga loči od ostalih modelov za zlomitev supersimetrije.

1.1 Introduction

If String Theory is a fundamental theory of Nature and not just a tool for studying systems with strongly coupled dynamics, it should be able to describe at the same time particle physics and cosmology, which are phenomena that involve very different scales from the microscopic four-dimensional (4d) quantum gravity length of 10^{-33} cm to large macroscopic distances of the size of the observable Universe $\sim 10^{28}$ cm spanned a region of about 60 orders of magnitude. In particular, besides the 4d Planck mass, there are three very different scales with very different physics corresponding to the electroweak, dark energy and inflation. These scales might be related via the scale of the underlying fundamental theory, such as string theory, or they might be independent in the sense that their origin could be based on different and independent dynamics. An example of the former constrained and more predictive possibility is provided by TeV strings with a fundamental scale at low energies due for instance to large extra dimensions transverse to a

⋆ E-mail: ignatios.antoniadis@polytechnique.edu

four-dimensional braneworld forming our Universe [1]. In this case, the 4d Planck mass is emergent from the fundamental string scale and inflation should also happen around the same scale [2].

Here, we will adopt the second more conservative approach, assuming that all three scales have an independent dynamical origin. Moreover, we will assume the presence of low energy supersymmetry that allows for an elegant solution of the mass hierarchy problem, a unification of fundamental forces as indicated by low energy data and a natural dark matter candidate due to an unbroken R-parity. The assumption of independent scales implies that supersymmetry breaking should be realized in a metastable de Sitter vacuum with an infinitesimally small (tunable) cosmological constant independent of the supersymmetry breaking scale that should be in the TeV region. In a recent work [3], we studied a simple $N = 1$ supergravity model having this property and motivated by string theory. Besides the gravity multiplet, the minimal field content consists of a chiral multiplet with a shift symmetry promoted to a gauged R-symmetry using a vector multiplet. In the string theory context, the chiral multiplet can be identified with the string dilaton (or an appropriate compactification modulus) and the shift symmetry associated to the gauge invariance of a two-index antisymmetric tensor that can be dualized to a (pseudo)scalar. The shift symmetry fixes the form of the superpotential and the gauging allows for the presence of a Fayet-Iliopoulos (FI) term, leading to a supergravity action with two independent parameters that can be tuned so that the scalar potential possesses a metastable de Sitter minimum with a tiny vacuum energy (essentially the relative strength between the F- and D-term contributions). A third parameter fixes the Vacuum Expectation Value (VEV) of the string dilaton at the desired (phenomenologically) weak coupling regime. An important consistency constraint of our model is anomaly cancellation which has been studied in [5] and implies the existence of additional charged fields under the gauged R-symmetry.

In a more recent work [6], we analyzed a small variation of this model which is manifestly anomaly free without additional charged fields and allows to couple in a straight forward way a visible sector containing the minimal supersymmetric extension of the Standard Model (MSSM) and studied the mediation of supersymmetry breaking and its phenomenological consequences. It turns out that an additional 'hidden sector' field z is needed to be added for the matter soft scalar masses to be non-tachyonic; although this field participates in the supersymmetry breaking and is similar to the so-called Polonyi field, it does not modify the main properties of the metastable de Sitter (dS) vacuum. All soft scalar masses, as well as trilinear A-terms, are generated at the tree level and are universal under the assumption that matter kinetic terms are independent of the 'Polonyi' field, since matter fields are neutral under the shift symmetry and supersymmetry breaking is driven by a combination of the $U(1)$ D-term and the dilaton and z-field F-term. Alternatively, a way to avoid the tachyonic scalar masses without adding the extra field z is to modify the matter kinetic terms by a dilaton dependent factor.

A main difference of the second analysis from the first work is that we use a field representation in which the gauged shift symmetry corresponds to an ordinary $U(1)$ and not an R-symmetry. The two representations differ by a Kähler transformation that leaves the classical supergravity action invariant. However, at the quantum level, there is a Green-Schwarz term generated that amounts an extra dilaton dependent contribution to the gauge kinetic terms needed to cancel the anomalies of the R-symmetry. This creates an apparent puzzle with the gaugino masses that vanish in the first representation but not in the latter. The resolution to the puzzle is based to the so called anomaly mediation contributions [7,8] that explain precisely the above apparent discrepancy. It turns out that gaugino masses are generated at the quantum level and are thus suppressed compared to the scalar masses (and A-terms).

1.2 Conventions

Throughout this paper we use the conventions of [9]. A supergravity theory is specified (up to Chern-Simons terms) by a Kähler potential $\mathcal{K}$, a superpotential W, and the gauge kinetic functions $f_{AB}(z)$. The chiral multiplets z^α, χ^α are enumerated by the index α and the indices A, B indicate the different gauge groups. Classically, a supergravity theory is invariant under Kähler tranformations, viz.

$$\begin{aligned}\mathcal{K}(z,\bar{z}) &\longrightarrow \mathcal{K}(z,\bar{z}) + J(z) + \bar{J}(\bar{z}),\\ W(z) &\longrightarrow e^{-\kappa^2 J(z)} W(z),\end{aligned} \tag{1.1}$$

where κ is the inverse of the reduced Planck mass, $m_p = \kappa^{-1} = 2.4 \times 10^{15}$ TeV. The gauge transformations of chiral multiplet scalars are given by holomorphic Killing vectors, i.e. $\delta z^\alpha = \theta^A k_A^\alpha(z)$, where θ^A is the gauge parameter of the gauge group A. The Kähler potential and superpotential need not be invariant under this gauge transformation, but can change by a Kähler transformation

$$\delta\mathcal{K} = \theta^A \left[r_A(z) + \bar{r}_A(\bar{z})\right], \tag{1.2}$$

provided that the gauge transformation of the superpotential satisfies $\delta W = -\theta^A \kappa^2 r_A(z) W$. One then has from $\delta W = W_\alpha \delta z^\alpha$

$$W_\alpha k_A^\alpha = -\kappa^2 r_A W, \tag{1.3}$$

where $W_\alpha = \partial_\alpha W$ and α labels the chiral multiplets. The supergravity theory can then be described by a gauge invariant function

$$\mathcal{G} = \kappa^2 \mathcal{K} + \log(\kappa^6 W \bar{W}). \tag{1.4}$$

The scalar potential is given by

$$\begin{aligned} V &= V_F + V_D\\ V_F &= e^{\kappa^2 \mathcal{K}} \left(-3\kappa^2 W\bar{W} + \nabla_\alpha W g^{\alpha\bar{\beta}} \bar{\nabla}_{\bar{\beta}} \bar{W}\right)\\ V_D &= \frac{1}{2} (\mathrm{Re} f)^{-1\, AB} \mathcal{P}_A \mathcal{P}_B, \end{aligned} \tag{1.5}$$

where W appears with its Kähler covariant derivative

$$\nabla_\alpha W = \partial_\alpha W(z) + \kappa^2 (\partial_\alpha \mathcal{K}) W(z). \tag{1.6}$$

The moment maps $\mathcal{P}_A$ are given by

$$\mathcal{P}_A = i(k_A^\alpha \partial_\alpha \mathcal{K} - r_A). \tag{1.7}$$

In this paper we will be concerned with theories having a gauged R-symmetry, for which $r_A(z)$ is given by an imaginary constant $r_A(z) = i\kappa^{-2}\xi$. In this case, $\kappa^{-2}\xi$ is a Fayet-Iliopoulos [10] constant parameter.

1.3 The model

The starting point is a chiral multiplet S and a vector multiplet associated with a shift symmetry of the scalar component s of the chiral multiplet S

$$\delta s = -ic\theta\,, \tag{1.8}$$

and a string-inspired Kähler potential of the form $-p\log(s+\bar{s})$. The most general superpotential is either a constant $W = \kappa^{-3}a$ or an exponential superpotential $W = \kappa^{-3}ae^{bs}$ (where a and b are constants). A constant superpotential is (obviously) invariant under the shift symmetry, while an exponential superpotential transforms as $W \to We^{-ibc\theta}$, as in eq. (1.3). In this case the shift symmetry becomes a gauged R-symmetry and the scalar potential contains a Fayet-Iliopoulos term. Note however that by performing a Kähler transformation (1.1) with $J = \kappa^{-2}bs$, the model can be recast into a constant superpotential at the cost of introducing a linear term in the Kähler potential $\delta K = b(s+\bar{s})$. Even though in this representation, the shift symmetry is not an R-symmetry, we will still refer to it as $U(1)_R$. The most general gauge kinetic function has a constant term and a term linear in s, $f(s) = \delta + \beta s$.

To summarise,[1]

$$\begin{aligned}
\mathcal{K}(s,\bar{s}) &= -p\log(s+\bar{s}) + b(s+\bar{s}),\\
W(s) &= a,\\
f(s) &= \delta + \beta s\,,
\end{aligned} \tag{1.9}$$

where we have set the mass units $\kappa = 1$. The constants a and b together with the constant c in eq. (1.8) can be tuned to allow for an infinitesimally small cosmological constant and a TeV gravitino mass. For $b > 0$, there always exists a supersymmetric AdS (anti-de Sitter) vacuum at $\langle s+\bar{s}\rangle = b/p$, while for $b = 0$ (and $p < 3$) there is an AdS vacuum with broken supersymmetry. We therefore focus on $b < 0$. In the context of string theory, S can be identified with a compactification modulus or the universal dilaton and (for negative b) the exponential superpotential may be generated by non-perturbative effects.

[1] In superfields the shift symmetry (1.8) is given by $\delta S = -ic\Lambda$, where Λ is the superfield generalization of the gauge parameter. The gauge invariant Kähler potential is then given by $\mathcal{K}(S,\bar{S}) = -p\kappa^{-2}\log(S+\bar{S}+cV_R) + \kappa^{-2}b(S+\bar{S}+cV_R)$, where V_R is the gauge superfield of the shift symmetry.

The scalar potential is given by:

$$
\begin{aligned}
V &= V_F + V_D \\
V_F &= a^2 e^{\frac{b}{l}} l^{p-2} \left\{ \frac{1}{p}(pl-b)^2 - 3l^2 \right\} \qquad l = 1/(s+\bar{s}) \\
V_D &= c^2 \frac{l}{\beta + 2\delta l}(pl-b)^2
\end{aligned} \tag{1.10}
$$

In the case where S is the string dilaton, V_D can be identified as the contribution of a magnetized D-brane, while V_F for $b = 0$ and $p = 2$ coincides with the tree-level dilaton potential obtained by considering string theory away its critical dimension [11]. For $p \geq 3$ the scalar potential V is positive and monotonically decreasing, while for $p < 3$, its F-term part V_F is unbounded from below when $s + \bar{s} \to 0$. On the other hand, the D-term part of the scalar potential V_D is positive and diverges when $s + \bar{s} \to 0$ and for various values for the parameters an (infinitesimally small) positive (local) minimum of the potential can be found.

If we restrict ourselves to integer p, tunability of the vacuum energy restricts $p = 2$ or $p = 1$ when $f(s) = s$, or $p = 1$ when the gauge kinetic function is constant. For $p = 2$ and $f(s) = s$, the minimization of V yields:

$$
b/l = \alpha \approx -0.183268 \quad , \quad p = 2 \tag{1.11}
$$

$$
\frac{a^2}{bc^2} = A_2(\alpha) + B_2(\alpha)\frac{\Lambda}{b^3c^2} \approx -50.6602 + \mathcal{O}(\Lambda), \tag{1.12}
$$

where Λ is the value of V at the minimum (i.e. the cosmological constant), α is the negative root of the polynomial $-x^5 + 7x^4 - 10x^3 - 22x^2 + 40x + 8$ compatible with (1.12) for $\Lambda = 0$ and $A_2(\alpha)$, $B_2(\alpha)$ are given by

$$
A_2(\alpha) = 2e^{-\alpha}\frac{-4+4\alpha-\alpha^2}{\alpha^3-4\alpha^2-2\alpha} \quad ; \quad B_2(\alpha) = 2\frac{\alpha^2 e^{-\alpha}}{\alpha^2-4\alpha-2} \tag{1.13}
$$

It follows that by carefully tuning a and c, Λ can be made positive and arbitrarily small independently of the supersymmetry breaking scale. A plot of the scalar potential for certain values of the parameters is shown in figure 1.1.

At the minimum of the scalar potential, for nonzero a and $b < 0$, supersymmetry is broken by expectation values of both an F and D-term. Indeed the F-term and D-term contributions to the scalar potential are

$$
\begin{aligned}
V_F|_{s+\bar{s}=\frac{\alpha}{b}} &= \frac{1}{2}a^2b^2e^{\alpha}\left(1-\frac{2}{\alpha}\right)^2 > 0, \\
V_D|_{s+\bar{s}=\frac{\alpha}{b}} &= \frac{b^3c^2}{\alpha}\left(1-\frac{2}{\alpha}\right)^2 > 0\,.
\end{aligned} \tag{1.14}
$$

The gravitino mass term is given by

$$
(m_{3/2})^2 = e^{\mathcal{G}} = \frac{a^2b^2}{\alpha^2}e^{\alpha}\,. \tag{1.15}
$$

Due to the Stueckelberg coupling, the imaginary part of s (the axion) gets eaten by the gauge field, which acquires a mass. On the other hand, the Goldstino, which is

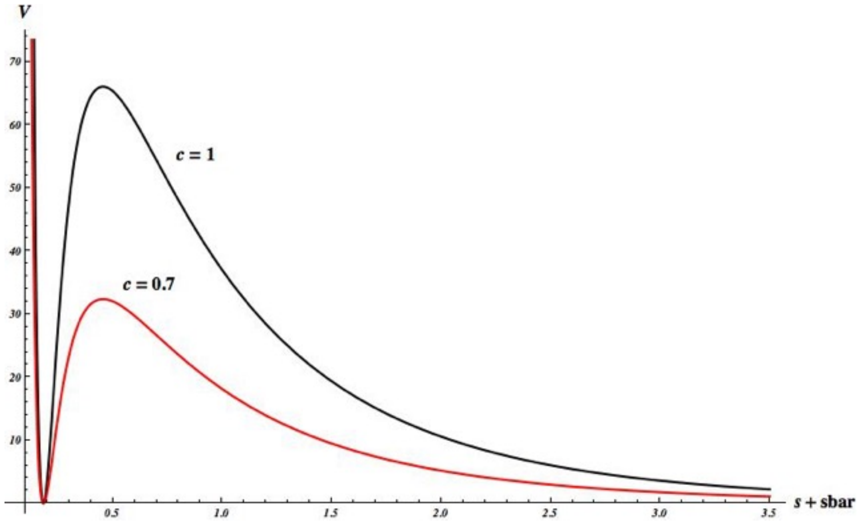

Fig. 1.1. A plot of the scalar potential for $p = 2$, $b = -1$, $\delta = 0$, $\beta = 1$ and a given by equation (1.12) for $c = 1$ (black curve) and $c = 0.7$ (red curve).

a linear combination of the fermion of the chiral multiplet χ and the gaugino λ gets eaten by the gravitino. As a result, the physical spectrum of the theory consists (besides the graviton) of a massive scalar, namely the dilaton, a Majorana fermion, a massive gauge field and a massive gravitino. All the masses are of the same order of magnitude as the gravitino mass, proportional to the same constant a (or c related by eq. (1.12) where b is fixed by eq. (1.11)), which is a free parameter of the model. Thus, they vanish in the same way in the supersymmetric limit $a \to 0$.

The local dS minimum is metastable since it can tunnel to the supersymmetric ground state at infinity in the s-field space (zero coupling). It turns out however that it is extremely long lived for realistic perturbative values of the gauge coupling $l \simeq 0.02$ and TeV gravitino mass and, thus, practically stable; its decay rate is [5]:

$$\Gamma \sim e^{-B} \quad \text{with} \quad B \approx 10^{300}\,. \tag{1.16}$$

1.4 Coupling a visible sector

The guideline to construct a realistic model keeping the properties of the toy model described above is to assume that matter fields are invariant under the shift symmetry (1.8) and do not participate in the supersymmetry breaking. In the simplest case of a canonical Kähler potential, MSSM-like fields ϕ can then be added as:

$$\begin{aligned} \mathcal{K} &= -\kappa^{-2}\log(s+\bar{s}) + \kappa^{-2}b(s+\bar{s}) + \sum \varphi\bar{\varphi}, \\ W &= \kappa^{-3}a + W_{MSSM}, \end{aligned} \tag{1.17}$$

where $W_{MSSM}(\phi)$ is the usual MSSM superpotential. The squared soft scalar masses of such a model can be shown to be positive and close to the square of the gravitino mass (TeV^2). On the other hand, for a gauge kinetic function with a linear term in s, $\beta \neq 0$ in eq. (1.9), the Lagrangian is not invariant under the shift symmetry

$$\delta\mathcal{L} = -\theta \frac{\beta c}{8}\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}. \tag{1.18}$$

and its variation should be canceled. As explained in Ref. [5], in the 'frame' with an exponential superpotential the R-charges of the fermions in the model can give an anomalous contribution to the Lagrangian. In this case the 'Green-Schwarz' term $\mathrm{Im}s F\tilde{F}$ can cancel quantum anomalies. However as shown in [5], with the minimal MSSM spectrum, the presence of this term requires the existence of additional fields in the theory charged under the shift symmetry.

Instead, to avoid the discussion of anomalies, we focus on models with a constant gauge kinetic function. In this case the only (integer) possibility[2] is $p = 1$. The scalar potential is given by (1.10) with $\beta = 0$, $\delta = p = 1$. The minimization yields to equations similar to (1.11), (1.12) and (1.13) with a different value of α and functions A_1 and B_1 given by:

$$\begin{aligned} b\langle s + \bar{s}\rangle &= \alpha \approx -0.233153 \\ \frac{bc^2}{a^2} &= A_1(\alpha) + B_1(\alpha)\frac{\Lambda}{a^2 b} \approx -0.359291 + \mathcal{O}(\Lambda) \\ A_1(\alpha) &= 2e^{\alpha}\alpha\frac{3-(\alpha-1)^2}{(\alpha-1)^2} \quad , \quad B_1(\alpha) = \frac{2\alpha^2}{(\alpha-1)^2}\,, \end{aligned} \tag{1.19}$$

where α is the negative root of $-3+(\alpha-1)^2(2-\alpha^2/2) = 0$ close to -0.23, compatible with the second constraint for $\Lambda = 0$. However, this model suffers from tachyonic soft masses when it is coupled to the MSSM, as in (1.17). To circumvent this problem, one can add an extra hidden sector field which contributes to (F-term) supersymmetry breaking. Alternatively, the problem of tachyonic soft masses can also be solved if one allows for a non-canonical Kähler potential in the visible sector, which gives an additional contribution to the masses through the D-term.

Let us discuss first the addition of an extra hidden sector field z (similar to the so-called Polonyi field [12]). The Kähler potential, superpotential and gauge kinetic function are given by

$$\begin{aligned} \mathcal{K} &= -\kappa^{-2}\log(s+\bar{s}) + \kappa^{-2} b(s+\bar{s}) + z\bar{z} + \sum \varphi\bar{\varphi}\,, \\ W &= \kappa^{-3} a(1+\gamma\kappa z) + W_{\mathrm{MSSM}}(\varphi)\,, \\ f(s) &= 1 \quad , \quad f_A = 1/g_A^2\,, \end{aligned} \tag{1.20}$$

where A labels the Standard Model gauge group factors and γ is an additional constant parameter. The existence of a tunable dS vacuum with supersymmetry

[2] If $f(s)$ is constant, the leading contribution to V_D when $s+\bar{s} \to 0$ is proportional to $1/(s+\bar{s})^2$, while the leading contribution to V_F is proportional to $1/(s+\bar{s})^p$. It follows that $p < 2$; if $p > 2$, the potential is unbounded from below, while if $p = 2$, the potential is either positive and monotonically decreasing or unbounded from below when $s+\bar{s} \to 0$ depending on the values of the parameters.

breaking and non-tachyonic scalar masses implies that γ must be in a narrow region:

$$0.5 \lesssim \gamma \lesssim 1.7\,. \tag{1.21}$$

In the above range of γ the main properties of the toy model described in the previous section remain, while Rez and its F-auxiliary component acquire non vanishing VEVs. All MSSM soft scalar masses are then equal to a universal value m_0 of the order of the gravitino mass, while the B_0 Higgs mixing parameter is also of the same order:

$$\begin{aligned} m_0^2 &= m_{3/2}^2\left[(\sigma_s+1)+\frac{(\gamma+t+\gamma t)^2}{(1+\gamma t)^2}\right],\\ A_0 &= m_{3/2}\left[(\sigma_s+3)+t\frac{(\gamma+t+\gamma t^2)}{1+\gamma t}\right],\\ B_0 &= m_{3/2}\left[(\sigma_s+2)+t\frac{(\gamma+t+\gamma t^2)}{(1+\gamma t)}\right], \end{aligned} \tag{1.22}$$

where $\sigma_s = -3+(\alpha-1)^2$ with α and $t \equiv \langle \mathrm{Re}\, z\rangle$ determined by the minimization conditions as functions of γ. Also, A_0 is the soft trilinear scalar coupling in the standard notation, satisfying the relation [13]

$$A_0 = B_0 + m_{3/2}\,. \tag{1.23}$$

On the other hand, the gaugino masses appear to vanish at tree-level since the gauge kinetic functions are constants (see (1.20)). However, as mentioned in Section 1.3, this model is classically equivalent to the theory[3]

$$\begin{aligned} \mathcal{K} &= -\kappa^{-2}\log(s+\bar{s}) + z\bar{z} + \sum_\alpha \varphi\bar{\varphi},\\ W &= \left(\kappa^{-3}a(1+z) + W_{\mathrm{MSSM}}(\varphi)\right)e^{bs}, \end{aligned} \tag{1.24}$$

obtained by applying a Kähler transformation (1.1) with $J = -\kappa^{-2}bs$. All classical results remain the same, such as the expressions for the scalar potential and the soft scalar masses (1.22), but now the shift symmetry (1.8) of s became a gauged R-symmetry since the superpotential transforms as $W \longrightarrow We^{-ibc\theta}$. Therefore, all fermions (including the gauginos and the gravitino) transform[4] as well under this $U(1)_R$, leading to cubic $U(1)_R^3$ and mixed $U(1) \times G_{\mathrm{MSSM}}$ anomalies. These anomalies are cancelled by a Green-Schwarz (GS) counter term that arises from a quantum correction to the gauge kinetic functions:

$$f_A(s) = 1/g_A^2 + \beta_A s \quad \text{with} \quad \beta_A = \frac{b}{8\pi^2}\left(T_{R_A} - T_{G_A}\right), \tag{1.25}$$

where T_G is the Dynkin index of the adjoint representation, normalized to N for SU(N), and T_R is the Dynkin index associated with the representation R of

[3] This statement is only true for supergravity theories with a non-vanishing superpotential where everything can be defined in terms of a gauge invariant function $G = \kappa^2\mathcal{K} + \log(\kappa^6 W\bar{W})$ [14].

[4] The chiral fermions, the gauginos and the gravitino carry a charge $bc/2$, $-bc/2$ and $-bc/2$ respectively.

dimension d_R, equal to 1/2 for the $SU(N)$ fundamental. An implicit sum over all matter representations is understood. It follows that gaugino masses are non-vanishing in this representation, creating a puzzle on the quantum equivalence of the two classically equivalent representations. The answer to this puzzle is based on the fact that gaugino masses are present in both representations and are generated at one-loop level by an effect called Anomaly Mediation [7,8]. Indeed, it has been argued that gaugino masses receive a one-loop contribution due to the super-Weyl-Kähler and sigma-model anomalies, given by [8]:

$$M_{1/2}=-\frac{g^2}{16\pi^2}\left[(3T_G-T_R)m_{3/2}+(T_G-T_R)\mathcal{K}_\alpha F^\alpha+2\frac{T_R}{d_R}(\log\det\mathcal{K}|_R{}'')_{,\alpha}F^\alpha\right] \tag{1.26}$$

The expectation value of the auxiliary field F^α, evaluated in the Einstein frame is given by

$$F^\alpha = -e^{\kappa^2\mathcal{K}/2}g^{\alpha\bar\beta}\bar\nabla_{\bar\beta}\bar W. \tag{1.27}$$

Clearly, for the Kähler potential (1.20) or (1.24) the last term in eq. (1.26) vanishes. However, the second term survives due to the presence of Planck scale VEVs for the hidden sector fields s and z. Since the Kähler potential between the two representations differs by a linear term $b(s+\bar s)$, the contribution of the second term in eq. (1.26) differs by a factor

$$\delta m_A = \frac{g_A^2}{16\pi^2}(T_G-T_R)be^{\kappa^2\mathcal{K}/2}g^{\alpha\bar\beta}\bar\nabla_{\bar\beta}\bar W, \tag{1.28}$$

which exactly coincides with the 'direct' contribution to the gaugino masses due to the field dependent gauge kinetic function (1.25) (taking into account a rescaling proportional to g_A^2 due to the non-canonical kinetic terms).

We conclude that even though the models (1.20) and (1.24) differ by a (classical) Kähler transformation, they generate the same gaugino masses at one-loop. While the one-loop gaugino masses for the model (1.20) are generated entirely by eq. (1.26), the gaugino masses for the model (1.24) after a Kähler transformation have a contribution from eq. (1.26) as well as from a field dependent gauge kinetic term whose presence is necessary to cancel the mixed $U(1)_R\times G$ anomalies due to the fact that the extra $U(1)$ has become an R-symmetry giving an R-charge to all fermions in the theory. Using (1.26), one finds:

$$M_{1/2} = -\frac{g^2}{16\pi^2}m_{3/2}\left[(3T_G-T_R)-(T_G-T_R)\left((\alpha-1)^2+t\frac{\gamma+t+\gamma t^2}{1+\gamma t}\right)\right]. \tag{1.29}$$

For $U(1)_Y$ we have $T_G=0$ and $T_R=11$, for $SU(2)$ we have $T_G=2$ and $T_R=7$, and for $SU(3)$ we have $T_G=3$ and $T_R=6$, such that for the different gaugino masses this gives (in a self-explanatory notation):

$$\begin{aligned}
M_1 &= 11\frac{g_Y^2}{16\pi^2}m_{3/2}\left[1-(\alpha-1)^2-\frac{t(\gamma+t+\gamma t)}{1+\gamma t}\right],\\
M_2 &= \frac{g_2^2}{16\pi^2}m_{3/2}\left[1-5(\alpha-1)^2-5\frac{t(\gamma+t+\gamma t^2)}{1+\gamma t}\right],\\
M_3 &= -3\frac{g_3^2}{16\pi^2}m_{3/2}\left[1+(\alpha-1)^2+\frac{t(\gamma+t+\gamma t^2)}{1+\gamma t}\right].
\end{aligned} \tag{1.30}$$

1.5 Phenomenology

The results for the soft terms calculated in the previous section, evaluated for different values of the parameter γ are summarised in Table 1.1. For every γ, the corresponding t and α are calculated by imposing a vanishing cosmological constant at the minimum of the potential. The scalar soft masses and trilinear terms are then evaluated by eqs. (1.22) and the gaugino masses by eqs. (1.30). Note that the relation (1.23) is valid for all γ. We therefore do not list the parameter B_0.

γ	t	α	m_0	A_0	M_1	M_2	M_3	$\tan\beta$ $\mu>0$	$\tan\beta$ $\mu<0$
0.6	0.446	-0.175	0.475	1.791	0.017	0.026	0.027		
1	0.409	-0.134	0.719	1.719	0.015	0.025	0.026		
1.1	0.386	-0.120	0.772	1.701	0.015	0.024	0.026	46	29
1.4	0.390	-0.068	0.905	1.646	0.014	0.023	0.026	40	23
1.7	0.414	-0.002	0.998	1.588	0.013	0.022	0.025	36	19

Table 1.1. The soft terms (in terms of $m_{3/2}$) for various values of γ. If a solution to the RGE exists, the value of $\tan\beta$ is shown in the last columns for $\mu > 0$ and $\mu < 0$ respectively.

In most phenomenological studies, B_0 is substituted for $\tan\beta$, the ratio between the two Higgs VEVs, as an input parameter for the renormalization group equations (RGE) that determine the low energy spectrum of the theory. Since B_0 is not a free parameter in our theory, but is fixed by eq. (1.23), this corresponds to a definite value of $\tan\beta$. For more details see [15] (and references therein). The corresponding $\tan\beta$ for a few particular choices for γ are listed in the last two columns of table 1.1 for $\mu > 0$ and $\mu < 0$ respectively. No solutions were found for $\gamma \lesssim 1.1$, for both signs of μ. The lighest supersymmetric particle (LSP) is given by the lightest neutralino and since $M_1 < M_2$ (see table 1.1) the lightest neutralino is mostly Bino-like, in contrast with a typical mAMSB (minimal anomaly mediation supersymmetry breaking) scenario, where the lightest neutralino is mostly Wino-like [16].

To get a lower bound on the stop mass, the sparticle spectrum is plotted in Figure 1.2 as a function of the gravitino mass for $\gamma = 1.1$ and $\mu > 0$ (for $\mu < 0$ the bound is higher). The experimental limit on the gluino mass forces $m_{3/2} \gtrsim 15$ TeV. In this limit the stop mass can be as low as 2 TeV. To conclude, the lower end mass spectrum consists of (very) light charginos (with a lightest chargino between 250 and 800 GeV) and neutralinos, with a mostly Bino-like neutralino as LSP ($80 - 230$ GeV), which would distinguish this model from the mAMSB where the LSP is mostly Wino-like. These upper limits on the LSP and the lightest chargino imply that this model could in principle be excluded in the next LHC run. In order for the gluino to escape experimental bounds, the lower limit on the gravitino mass is about 15 TeV. The gluino mass is then between 1-3 TeV. This however forces the squark masses to be very high ($10 - 35$ TeV), with the exception of the stop mass which can be relatively light ($2 - 15$ TeV).

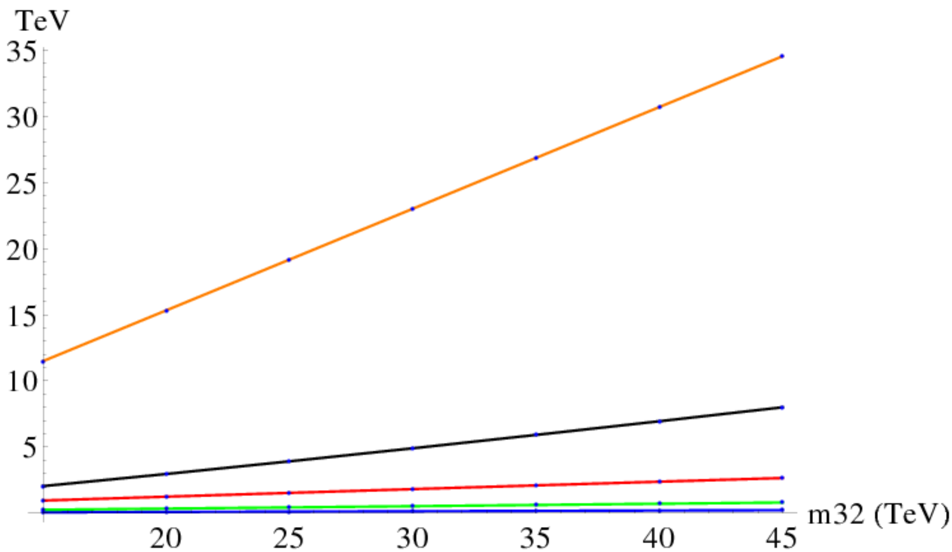

Fig. 1.2. The masses (in TeV) of the sbottom (yellow), stop (black), gluino (red), lightest chargino (green) and lightest neutralino (blue) as a function of $m_{3/2}$ for $\gamma = 1.1$ and for $\mu > 0$. No solutions to the RGE were found when $m_{3/2} \gtrsim 45$ TeV. The lower bound corresponds to a gluino mass of 1 TeV.

1.6 Non-canonical Kähler potential for the visible sector

As mentioned already in Section 4, an alternative way to avoid tachyonic soft scalar masses for the MSSM fields in the model (1.17), instead of adding the extra Palonyi-type field z in the hidden sector, is by introducing non-canonical kinetic terms for the MSSM fields, such as:

$$\begin{aligned} \mathcal{K} &= -\kappa^{-2}\log(s+\bar{s}) + \kappa^{-2}b(s+\bar{s}) + (s+\bar{s})^{-\nu}\sum \varphi\bar{\varphi}, \\ W &= \kappa^{-3}a + W_{MSSM}, \\ f(s) &= 1, \qquad f_A(s) = 1/g_A^2\,, \end{aligned} \tag{1.31}$$

where ν is an additional parameter of the theory, with $\nu = 1$ corresponding to the leading term in the Taylor expansion of $-\log(s+\bar{s}-\varphi\bar{\varphi})$. Since the visible sector fields appear only in the combination $\varphi\bar{\varphi}$, their VEVs vanish provided that the scalar soft masses squared are positive. Moreover, for vanishing visible sector VEVs, the scalar potential and is minimization remains the same as in eqs. (refbsalpha). Therefore, the non-canonical Kähler potential does not change the fact that the F-term contribution to the soft scalar masses squared is negative. On the other hand, the visible fields enter in the D-term scalar potential through the derivative of the Kähler potential with respect to s. Even though this has no effect on the ground state of the potential, the φ-dependence of the D-term scalar potential does result in an extra contribution to the scalar masses squared which become positive

$$\nu > -\frac{e^{\alpha}(\sigma_s+1)\alpha}{A(\alpha)(1-\alpha)} \approx 2.6\,. \tag{1.32}$$

The soft MSSM scalar masses and trilinear couplings in this model are:

$$\begin{aligned} m_0^2 &= \kappa^2 a^2 \left(\frac{b}{\alpha}\right)\left(e^{\alpha}(\sigma_s+1)+\nu\frac{A(\alpha)}{\alpha}(1-\alpha)\right) \\ A_0 &= m_{3/2}(s+\bar{s})^{\nu/2}\,(\sigma_s+3) \\ B_0 &= m_{3/2}(s+\bar{s})^{\nu/2}\,(\sigma_s+2) \end{aligned} \tag{1.33}$$

where σ_s is defined as in (1.22), eq. (1.20) has been used to relate the constants a and c, and corrections due to a small cosmological constant have been neglected. A field redefinition due to a non-canonical kinetic term $g_{\varphi\bar{\varphi}} = (s+\bar{s})^{-\nu}$ is also taken into account. The main phenomenological properties of this model are not expected to be different from the one we analyzed in section 1.5 with the parameter ν replacing γ. Gaugino masses are still generated at one-loop level while mSUGRA applies to the soft scalar sector. We therefore do not repeat the phenomenological analysis for this model.

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2 Results on DAMA/LIBRA-Phase1 and Perspectives of the Phase2 ⋆

R. Bernabei[1], P. Belli[1], A. Di Marco[1], F. Montecchia[1 †],
A. d'Angelo[2], A. Incicchitti[2],
F. Cappella[3], V. Caracciolo[3], R. Cerulli[3],
C.J. Dai[4], H.L. He [4], H.H. Kuang [4], X.H. Ma[4],
X.D. Sheng [4], R.G. Wang [4], Z.P. Ye [4 ‡]

[1]Dip. di Fisica, Università di Roma "Tor Vergata" and
INFN, sez. Roma "Tor Vergata", I-00133 Rome, Italy
[2]Dip. di Fisica, Università di Roma "La Sapienza" and
INFN, sez. Roma, I-00185 Rome, Italy
[3]Laboratori Nazionali del Gran Sasso
I.N.F.N., I-67100 Assergi, Italy
[4]Key Laboratory of Particle Astrophysics
IHEP, 100049 Beijing, China

Abstract. The DAMA/LIBRA experiment consists of ~ 250 kg of highly radio-pure NaI(Tl) and it is in data taking in the underground Laboratory of Gran Sasso (LNGS). The data collected in its first 7 annual cycles, corresponding to the so called DAMA/LIBRA–phase1, have been released. Considering also of the former DAMA/NaI experiment (their cumulative exposure is 1.33 ton × yr), the data of 14 independent annual cycles have been analysed to exploit the model-independent Dark Matter (DM) annual modulation signature. An annual modulation effect has been observed at 9.3 σ C.L., giving evidence for the presence of DM particles in the galactic halo. No systematic or side reaction able to mimic the exploited DM signature has been found or suggested by anyone. At present DAMA/LIBRA is running after an upgrade of the experiment in its phase2 with increased sensitivity. The model independent result of DAMA is compatible with a wide set of scenarios regarding the nature of the DM candidate and related astrophysical, nuclear and particle Physics. Here, after briefly reporting the DAMA model independent results, the recent analysis in terms of Mirror Dark Matter candidate will be mentioned.

Povzetek. Experiment DAMA/LIBRA, ki je postavljen v podzemeljskem laboratoriju Gran Sasso (LNGS), uporablja ~ 250 kg NaI(Tl) z visoko čistočo. Posebej predstavijo analizo meritev Faze I iz zadnjih sedmih let, tem meritvam pa dodajo tudi meritve sedmih let prednodnega experimenta DAMA/NaI (s kumulativno ekspozicijo 1.33 ton × let). Letno modulacijo sipanih delcev potrdijo z zanesljivostjo 9.3 σ, kar je, ob sistematičnem iskanju drugih vzrokov za izmerjeno modulacijo, mogoče pripisati samo prisotnosti delcev temne snovi v

† also Dip. di Ingegneria Civile e Ingegneria Informatica, Università di Roma "Tor Vergata", I-00133 Rome, Italy
‡ also University of Jing Gangshan, Jiangxi, China
⋆ Talk delivered by R. Cerulli

galaktičnem haloju. Zdaj teče posodobljen experiment, faza 2, s povečano občutljivostjo merjenja ozadja. Rezultate poskusa je mogoče razložiti z različimi modeli, ki poskušajo pojasniti temno snov v vesolju. Omenijo možnost, da pojasni prisotnost temne snovi delec, ki pripada zrcalni temni snovi.

2.1 Introduction

The DAMA project is based on the development and use of low background scintillators. In particular, the second generation DAMA/LIBRA apparatus [1–15], as the former DAMA/NaI (see for example Refs. [8,16,17] and references therein), is further investigating the presence of DM particles in the galactic halo by exploiting the model independent DM annual modulation signature [18]. At present DAMA/LIBRA is running in its phase2 with increased sensitivity.

The signature exploited by DAMA/LIBRA (the model independent DM annual modulation) is a consequence of the Earth's revolution around the Sun; in fact, the Earth should be crossed by a larger flux of DM particles around $\simeq 2$ June (when the projection of the Earth orbital velocity on the Sun velocity with respect to the Galaxy is maximum) and by a smaller one around $\simeq 2$ December (when the two velocities are opposite). This DM annual modulation signature is very effective since the effect induced by DM particles must simultaneously satisfy many requirements: the rate must contain a component modulated according to a cosine function (1) with one year period (2) and a phase peaked roughly $\simeq 2$ June (3); this modulation must only be found in a well-defined low energy range, where DM particle induced events can be present (4); it must apply only to those events in which just one detector of many actually "fires" (*single-hit* events), since the DM particle multi-interaction probability is negligible (5); the modulation amplitude in the region of maximal sensitivity must be $\simeq 7\%$ for usually adopted halo distributions (6), but it can be larger (even up to $\simeq 30\%$) in case of some possible scenarios. Thus this signature is model independent and it allows the test a large range of cross sections and halo densities. This DM signature might be mimicked only by systematic effects or side reactions able to account for the whole observed modulation amplitude and to simultaneously satisfy all the requirements given above. No one is available [1–4,7,8,12,19,16,17,13].

2.2 The annual modulation results

The total exposure of DAMA/LIBRA–phase1 is 1.04 ton $\times$ yr in 7 annual cycles; when including also the data collected by the first generation DAMA/NaI experiment, the exposure is 1.33 ton $\times$ yr, corresponding to 14 annual cycles [2–4,8].

To investigate the presence of an annual modulation in the data many analyses have been carried out. Here, as example, the time behaviour of the experimental residual rate of the *single-hit* scintillation events for DAMA/NaI and DAMA/LIBRA–phase1 in the (2–6) keV energy interval is plotted in Fig. 2.1. The χ^2 test excludes the hypothesis of absence of modulation in the data (P-value = 2.2×10^{-3}). When fitting the *single-hit* residual rate of DAMA/LIBRA–phase1 together with the DAMA/NaI ones, with the function: $A \cos \omega(t - t_0)$, considering a

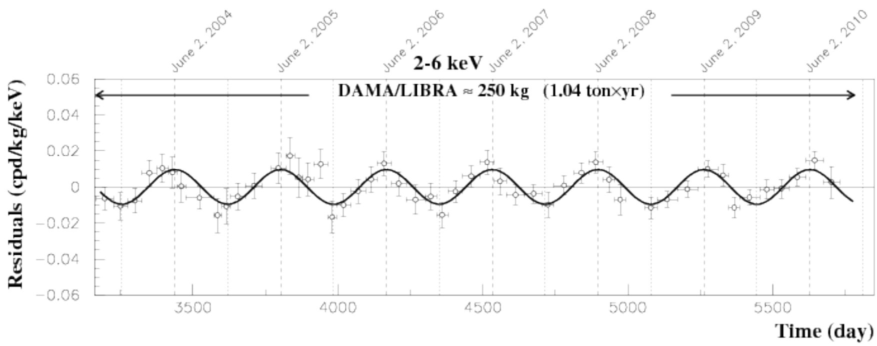

Fig. 2.1. Experimental residual rate of the *single-hit* scintillation events measured by DAMA/NaI and DAMA/LIBRA–phase1 in the (2–6) keV energy interval as a function of the time. The data points present the experimental errors as vertical bars and the associated time bin width as horizontal bars. The superimposed curves are the cosinusoidal functions expected for a Dark Matter signal (period 1 yr and phase June 2^{nd}) and modulation amplitudes, A, as obtained by the fit on the data. The dashed vertical lines correspond to the maximum expected for the DM signal (June 2^{nd}), while the dotted vertical lines correspond to the minimum.

period $T = \frac{2\pi}{\omega} = 1$ yr and a phase $t_0 = 152.5$ day (June 2^{nd}) as expected by the DM annual modulation signature, the following modulation amplitude is obtained: $A = (0.0110 \pm 0.0012)$ cpd/kg/keV, corresponding to 9.2 σ C.L.. When the period, and the phase are kept free in the fitting procedure, the modulation amplitude is (0.0112 ± 0.0012) cpd/kg/keV (9.3 σ C.L.), the period $T = (0.998 \pm 0.002)$ year and the phase $t_0 = (144 \pm 7)$ day, values well in agreement with expectations for a DM annual modulation signal. In particular, the phase is consistent with about June 2^{nd} and is fully consistent with the value independently determined by Maximum Likelihood analysis [4][1]. The run test and the χ^2 test on the data have shown that the modulation amplitudes singularly calculated for each annual cycle of DAMA/NaI and DAMA/LIBRA–phase1 are normally fluctuating around their best fit values [2–4].

We have also performed a power spectrum analysis of the *single-hit* residuals of DAMA/LIBRA–phase1 and DAMA/NaI [8], obtaining a clear principal mode in the (2–6) keV energy interval at a frequency of 2.737×10^{-3} d^{-1}, corresponding to a period of $\simeq 1$ year, while only aliasing peaks are present in other energy intervals.

Absence of any other significant background modulation in the energy spectrum has been verified in energy regions not of interest for DM [4]; it is worth noting that the obtained results account for whatever kind of background and, in addition, no background process able to mimic the DM annual modulation signature (that is able to simultaneously satisfy all the peculiarities of the signature and to account for the whole measured modulation amplitude) is available (see also discussions e.g. in Refs. [1–4,7,8,12,13]).

[1] For completeness, we recall that a slight energy dependence of the phase could be expected in case of possible contributions of non-thermalized DM components to the galactic halo, such as e.g. the SagDEG stream [20–22] and the caustics [23].

A further relevant investigation in the DAMA/LIBRA–phase1 data has been performed by applying the same hardware and software procedures, used to acquire and to analyse the *single-hit* residual rate, to the *multiple-hit* one. In fact, since the probability that a DM particle interacts in more than one detector is negligible, a DM signal can be present just in the *single-hit* residual rate. Thus, the comparison of the results of the *single-hit* events with those of the *multiple-hit* ones corresponds practically to compare between them the cases of DM particles beam-on and beam-off. This procedure also allows an additional test of the background behaviour in the same energy interval where the positive effect is observed. In particular, the residual rates of the *single-hit* events measured over the DAMA/LIBRA–phase1 annual cycles are reported in Ref. [4] together with the residual rates of the *multiple-hit* events, in the (2–6) keV energy interval. A clear modulation is present in the *single-hit* events, while the fitted modulation amplitude of the *multiple-hit* residual rate in the same energy region (2–6) keV is well compatible with zero: $-(0.0005 \pm 0.0004)$ cpd/kg/keV. Thus, again evidence of annual modulation with the features required by the DM annual modulation signature is present in the *single-hit* residuals (events class to which the DM particle induced events belong), while it is absent in the *multiple-hit* residual rate (event class to which only background events belong). Since the same identical hardware and the same identical software procedures have been used to analyse the two classes of events, the obtained result offers an additional strong support for the presence of a DM particle component in the galactic halo.

By performing a maximum-likelihood analysis of the *single-hit* scintillation events, it is possible to extract from the data the modulation amplitude, S_m, as a function of the energy considering T =1 yr and $t_0 = 152.5$ day. Again the results have shown that positive signal is present in the (2–6) keV energy interval, while S_m values compatible with zero are present just above; for details see Ref. [4]. Moreover, as described in Refs. [2–4,8], the observed annual modulation effect is well distributed in all the 25 detectors, the annual cycles and the energy bins at 95% C.L. Further performed analyses confirm that the evidence for the presence of an annual modulation in the data satisfy all the requirements of a DM signal.

Sometimes naive statements were put forward as the fact that in nature several phenomena may show some kind of periodicity. The point is whether they might mimic the annual modulation signature in DAMA/LIBRA (and former DAMA/NaI), i.e. whether they might be not only quantitatively able to account for the observed modulation amplitude but also able to satisfy at the same time all the requirements of the DM annual modulation signature. The same is also for side reactions. A deep investigation is reported in Refs. [1–4] and references therein; additional arguments can be found in Refs. [7,8,12,13]. No modulation has been found in any possible source of systematics or side reactions; thus, cautious upper limits on possible contributions to the DAMA/LIBRA measured modulation amplitude have been obtained (see Refs. [2–4]). It is worth noting that they do not quantitatively account for the measured modulation amplitudes, and also are not able to simultaneously satisfy all the many requirements of the signature. Similar analyses have also been performed for the DAMA/NaI data [16,17]. In particular, in Ref. [13] a simple and intuitive way why the neutrons, the muons and the solar neutrinos cannot give any significant contribution to the DAMA annual modulation results is outlined.

In conclusion, DAMA give model-independent evidence (at 9.3σ C.L. over 14 independent annual cycles) for the presence of DM particles in the galactic halo.

As regards comparisons, we recall that no direct model independent comparison is possible in the field when different target materials and/or approaches are used; the same is for the strongly model dependent indirect searches. In particular, the DAMA model independent evidence is compatible with a wide set of scenarios regarding the nature of the DM candidate and related astrophysical, nuclear and particle Physics; for examples some given scenarios and parameters are discussed e.g. in Refs. [2,8,16] and references therein. Further large literature is available on the topics. In conclusion, both negative results and possible positive hints reported in literature are compatible with the DAMA model-independent DM annual modulation results in various scenarios considering also the existing experimental and theoretical uncertainties.

Recently an investigation of possible diurnal effects in the *single-hit* low energy scintillation events collected by DAMA/LIBRA–phase1 has been carried out [12]. In particular, a model-independent diurnal effect with the sidereal time is expected for DM because of Earth rotation. At the present level of sensitivity the presence of any significant diurnal variation and of diurnal time structures in the data can be excluded for both the cases of solar and sidereal time; in particular, the DM diurnal modulation amplitude expected, because of the Earth diurnal motion, on the basis of the DAMA DM annual modulation results is below the present sensitivity [12]. It will be possible to investigate such a diurnal effect with adequate sensitivity only when a much larger exposure will be available; moreover better sensitivities can also be achieved by lowering the software energy threshold as in the presently running DAMA/LIBRA–phase2.

For completeness we recall that recently we have performed also an analysis considering the so called "Earth Shadow Effect" [14]. Other rare processes have also been searched for by DAMA/LIBRA-phase1; see for details Refs. [9–11].

2.3 The case of asymmetric mirror matter

The model independent annual modulation effect observed by the DAMA experiments can be related to a variety of interaction mechanisms of DM particles with the detector materials (see for example Ref. [8]). Among all the many possibilities recently the case where the signal is induced by mirror-type dark matter candidates in some scenarios has been considered in collaboration with A. Addazi and Z. Berezhiani (see Ref. [15] and references therein). Here we just recall some arguments.

In the framework of asymmetric mirror matter, the DM origiates from hidden (or shadow) gauge sectors which have particles and interaction content similar to that of ordinary particles. Such a dark sector would consist of elementary leptons (similar to our electron) and baryons (similar to our proton or neutron) composed of shadow quarks which are confined by strong gauge interactions like in our QCD. These two types of particles can be combined in atoms by electromagnetic forces mediated by dark photons. The stability of the dark proton is guaranteed by the conservation law of the related baryon number, as the stability of our proton is related to the conservation of the ordinary baryon number. On the other hand, the cosmological abundance of DM in the Universe can be induced by the violation of such baryon number in the early Universe which could produce dark baryon asymmetry by mechanisms similar to those considered for the primordial baryogenesis in the observable sector. In this respect, such type of DM is also known as asymmetric dark matter [15]. In the asymmetric mirror matter considered scheme, it is assumed that the mirror parity is spontaneously broken and the electroweak symmetry breaking scale v' in the mirror sector is much larger than that in our Standard Model, $v = 174$ GeV. In this case, the mirror world becomes a heavier and deformed copy of our world, with mirror particle masses scaled in different ways with respect to the masses of the ordinary particles. Taking the mirror weak scale e.g. of the order of 10 TeV, the mirror electron would become two orders of magnitude heavier than our electron while the mirror nucleons p' and n' only about 5 times heavier than the ordinary nucleons. Then dark matter would exist in the form of mirror hydrogen composed of mirror proton and electron, with mass of about 5 GeV which is a rather interesting mass range for dark matter particles. Owing to the large mass of mirror electron, mirror atoms should be more compact and tightly bound with respect to ordinary atoms. Asymmetric mirror model can be considered as a natural benchmark for more generic types of atomic dark matter with *ad hoc* chosen parameters.

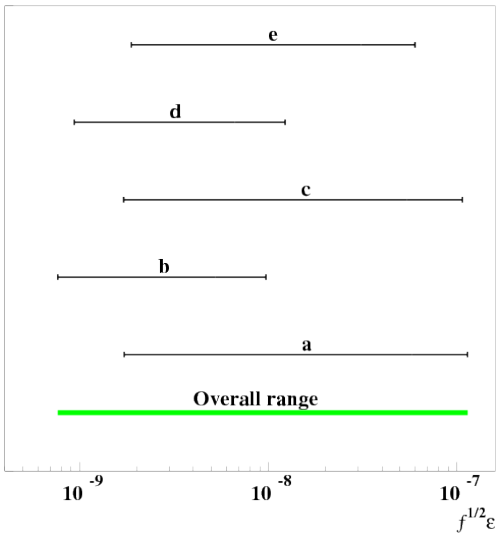

Fig. 2.2. DAMA allowed intervals for the $\sqrt{f}\epsilon$ parameter, obtained by marginalizing all the models for each considered scenario as given in Ref. [15]. The overall range is also reported [15].

The annual modulation observed by DAMA in the framework of asymmetric mirror matter has been analysed in the light of the very interesting interaction portal which is kinetic mixing $\frac{\epsilon}{2}F^{\mu\nu}F'_{\mu\nu}$ of two massless states, ordinary photon and mirror photon. This mixing mediates the mirror atom (that are very compact objects) scattering off the ordinary target nuclei in the NaI(Tl) detectors at DAMA/LIBRA set-up with the Rutherford-like cross sections.

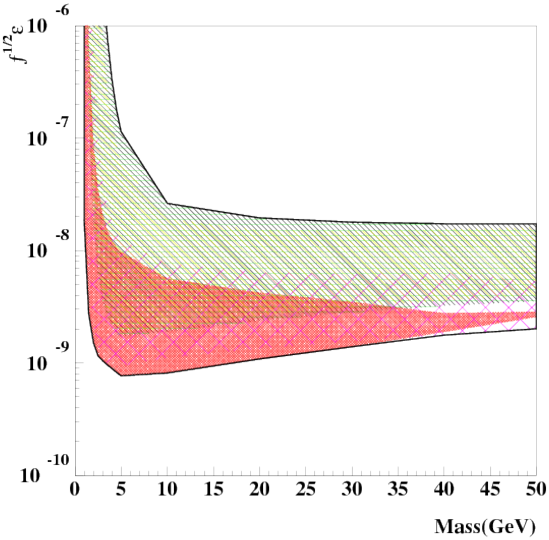

Fig. 2.3. Allowed regions for the $\sqrt{f}\epsilon$ parameter as function of $M_{A'}$, mirror hydrogen mass, obtained by marginalizing all the models for each considered scenario. The $M_{A'}$ interval from few GeV up to 50 GeV is explored. These allowed intervals identify the $\sqrt{f}\epsilon$ values corresponding to C.L. larger than 5σ from the *null hypothesis*, that is $\sqrt{f}\epsilon = 0$. The allowed regions corresponding to the five different scenarios are depicted in different hatching; the black line is the overall boundary [15].

The data analysis in the Mirror DM model framework allows the determination of the $\sqrt{f}\epsilon$ parameter (where f is the fraction of DM in the Galaxy in form of mirror atoms and $epsilon$). In the analysis several uncertainties on the astrophysical, particle physics and nuclear physics models have been taken into account in the calculation. For detailed discussion see [15]. In particular in the analysis five scenarios have been considered depending on: i) the adopted quenching factors; ii) either inclusion or not of the channeling effect; iii) either inclusion or not of the Migdal effect. For each scenario the 138 halo models and the relative uncertainties have been considered. To estimate the free parameter of the analysis (e.g. $\sqrt{f}\epsilon$ in the DM model) a comparison of the expectations of the mirror DM with the experimental results has been performed considering a χ^2 analysis [15].

In Fig. 2.2 the cumulative allowed intervals of the $\sqrt{f}\epsilon$ parameter selected by the DAMA data for the mentioned scenario are depicted; the overall allowed band is also shown. The obtained values of the $\sqrt{f}\epsilon$ parameter are well compatible with cosmological bounds.

Finally, releasing the assumption $M_{A'} \simeq 5m_p$, the allowed regions for the $\sqrt{f}\epsilon$ parameter as function of $M_{A'}$, mirror hydrogen mass, obtained by marginalizing all the models for each considered scenario, are shown in Fig. 2.3 where the $M_{A'}$ interval from few GeV up to 50 GeV is explored. The five scenarios are reported with different hatching of the allowed regions; the black line is the overall boundary.

In conclusion, the allowed values for $\sqrt{f}\epsilon$ in the case of mirror hydrogen atom, $Z' = 1$, ranges between 7.7×10^{-10} to 1.1×10^{-7}. The values within this overall range are well compatible with cosmological bounds.

2.4 DAMA/LIBRA–phase2 and perspectives

After a first upgrade of the DAMA/LIBRA set-up in 2008, a more important upgrade has been performed at the end of 2010 when all the PMTs have been replaced with new ones having higher Quantum Efficiency (Q.E.), realized with a special dedicated development by HAMAMATSU co.. Details on the developments and on the reached performances are reported in Ref. [6] where the feasibility to decrease the software energy threshold below 2 keV has also been demonstrated.

DAMA/LIBRA–phase2 is continuously running in order: (1) to increase the experimental sensitivity lowering the software energy threshold of the experiment; (2) to improve the corollary investigation on the nature of the DM particle and related astrophysical, nuclear and particle physics arguments; (3) to investigate other signal features. This requires long and heavy full time dedicated work for reliable collection and analysis of very large exposures. Another upgrade at the end of 2012 was concluded: new-concept pre-amplifiers were installed. Further improvements are planned.

Finally, other possibility to further increase the sensitivity of the set-up can be considered; in particular, the use of high Q.E. and ultra-low background PMTs directly coupled to the NaI(Tl) crystals is an interesting possibility. This possible configuration can allow a further large improvement in the light collection and a further lowering of the software energy threshold. Moreover, efforts towards a possible "general purpose" experiment with highly radiopure NaI(Tl) (DAMA/1ton) having full sensitive mass of 1 ton (we already proposed in 1996 as a general purpose set-up) have been continued in various aspects.

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3 Pure Contact Term Correlators in CFT ⋆

L. Bonora ⋆⋆ and B.L. de Souza ⋆⋆⋆

International School for Advanced Studies (SISSA),
Via Bonomea 265, 34136 Trieste and INFN, Sezione di Trieste, Italy

Abstract. We discuss the case of correlators in CFT made of pure contact terms, without a corresponding bare part. We show two examples. The first is provided by the conformal limits of a free massive fermion theory in 3d. We show that the (conserved) current correlators are in one-to-one correspondence with the terms of the 3d gauge CS action. The second is the Pontryagin trace anomaly. The corresponding 3-point correlator is nonvanishing even though the corresponding untraced correlator vanishes.

Povzetek. Avtorja obravnavata korelatorje v konformni teoriji polja, ki vsebujejo le kontaktne člene, brez ustreznih 'golih' členov. Obravnavata dva primera. Prvi je konformna limita teorije masivnih prostih fermionov v 3d. Pokažeta, da so korelatorji (ohranjenih) tokov v bijekciji s členi Chern-Simonsove akcije v 3d. Drugi primer je Pontrjaginova sledna anomalija. Ustrezni 3-točkovni korelator je neničelen, čeprav je pridruženi nesledni korelator enak nič.

3.1 Introduction

Correlators in conformal field theories can be formulated both in configuration space and, via Fourier transform, in momentum space. In the first form they may happen to be singular at coincident insertion points and in need to be regularized. In coordinate space they are therefore simply distributions. In the simplest cases such distrubutions have been studied and can be found in textbooks. But in general the correlators of CFT are very complicate expressions and their regularization has to be carried out from scratch. This can be done directly in configuration space, in which case a well known procedure is the differential regularization. An alternative, and often more accessible, technique consists in formulating the same problem in momentum space via Fourier transform and proceeding to regularize the Fourier transform of the relevant correlators. This procedure produces various types of terms, which we refer to as *non-local, partially local and local terms*. Local terms are represented by polynomials of the external momenta in momentum space, and by delta functions and derivatives of delta functions in configuration space. The unregularized correlators will be referred to as *bare* correlators; they are ordinary regular functions at non-coincident points and are classified as non-local in the previous classification. While regularizing the latter one usually produces

⋆ Talk delivered by L. Bonora
⋆⋆ bonora@sissa.it
⋆⋆⋆ blima@sissa.it

not only local terms, but also intermediate ones, which are product of bare functions and delta functions or derivatives thereof. These are referred to as partially local.

From the above introduction one might be led to think that local terms (i.e. polynomials of the external momenta, in momentum space representation) can come only from regularizing bare correlators. This is not the case, there are important cases of local correlators that do not have a bare counterpart. We can say that they consist only of the quantum part. This is the main subject of this article. We will discuss two examples. The first, in 3d, is the case of pure contact terms in the parity-odd sector of the 2-point function of currents. There exist no bare terms corresponding to them. An important implication of these contact terms is that they give rise to a Chern-Simons term in the effective action.

The second example is that of the 3-point function of the energy-momentum tensor in 4d, in which one of the entries is the trace of the em tensor. Classically, the trace of the em tensor is zero in a Weyl invariant theory. At the quantum level this fact becomes a set of Ward identities that relate n-point functions with one insertion of the trace of the em tensor with $(n-1)$-point functions. When the theory possesses trace anomalies these Ward identities are complemented by a set of contact terms which reproduces the anomalies. What we would like to stress here is that such correlators containing one trace insertion can be nonvanishing even if there is no bare correlator corresponding to it. This is what happens with the Pontryagin trace anomaly. The latter is puzzling at first, but, in fact, when properly understood, it would be surprising if it did not exist.

There are of course other examples, beside the two above ones. All these examples are characterized by the fact that they break parity.

The paper is organized as follows. In the next section we introduce some basic CFT formulas in momentum space. In section 3 we work out the 3d example of pure contact term correlators and its connection with gauge CS. In section 4 we review the 4d example, which corresponds to the Pontryagin trace anomaly. In section 5 we add some new remarks concerning this anomaly.

3.2 Conformal invariance in momentum space

In this section we will lay down some introductory material on conformal invariance and conformal field theories, which will be needed in the sequel. The conformal group in d dimension encompasses the Poincaré transformations, the dilatation and the special conformal transformations (SCTs). The latter is

$$x'^{\mu} = \frac{x^{\mu} + b^{\mu}x^2}{1 + 2b\cdot x + b^2x^2} = x^{\mu} + b^{\mu}x^2 - 2b\cdot x\, x^{\mu} + \mathcal{O}(b^2)$$

for infinitesimal b^{μ}. In this paper we will mostly consider the effects of conformal invariance in momentum space. If we Fourier transform the generators of the conformal algebra we get (a tilde represents the transformed generator and $\tilde{\partial} = \frac{\partial}{\partial k}$)

$$\begin{aligned}
\tilde{P}_\mu &= -k_\mu, \\
\tilde{D} &= i(d + k^\mu \tilde{\partial}_\mu), \\
\tilde{L}_{\mu\nu} &= i(k_\mu \tilde{\partial}_\nu - k_\nu \tilde{\partial}_\mu), \\
\tilde{K}_\mu &= 2d\, \tilde{\partial}_\mu + 2k_\nu \tilde{\partial}^\nu \tilde{\partial}_\mu - k_\mu \tilde{\Box}.
\end{aligned}$$

Notice that $\tilde{P}_\mu$ is a multiplication operator and $\tilde{K}_\mu$ is a quadratic differential operator. The Leibniz rule does not hold for $\tilde{K}_\mu$ and $\tilde{P}_\mu$ with respect to the ordinary product. However it does hold for the convolution product:

$$\tilde{K}_\mu(\tilde{f} \star \tilde{g}) = (\tilde{K}_\mu \tilde{f}) \star \tilde{g} + \tilde{f} \star (\tilde{K}_\mu\, \tilde{g})$$

where $(\tilde{f} \star \tilde{g})(k) = \int dp\, f(k-p) g(p)$. Nevertheless these generators form a closed algebra

$$\begin{aligned}
&[\tilde{D}, \tilde{P}_\mu] = i\tilde{P}_\mu, \\
&[\tilde{D}, \tilde{K}_\mu] = i\tilde{K}_\mu, \\
&[\tilde{K}_\mu, \tilde{K}_\nu] = 0, \\
&[\tilde{K}_\mu, \tilde{P}_\nu] = 2i(\eta_{\mu\nu}\tilde{D} - \tilde{L}_{\mu\nu}), \\
&[\tilde{K}_\lambda, \tilde{L}_{\mu\nu}] = i(\eta_{\lambda\mu}\tilde{K}_\nu - \eta_{\lambda\nu}\tilde{K}_\mu), \\
&[\tilde{P}_\lambda, \tilde{L}_{\mu\nu}] = i(\eta_{\lambda\mu}\tilde{P}_\nu - \eta_{\lambda\nu}\tilde{P}_\mu), \\
&[\tilde{L}_{\mu\nu}, \tilde{L}_{\lambda\rho}] = i(\eta_{\nu\lambda}\tilde{L}_{\mu\rho} + \eta_{\mu\rho}\tilde{L}_{\nu\lambda} - \eta_{\mu\lambda}\tilde{L}_{\nu\rho} - \eta_{\nu\rho}\tilde{L}_{\mu\lambda}.
\end{aligned}$$

One should be aware that they do not generate infinitesimal transformation in momentum space. This notwithstanding, in momentum space we can write down the conformal Ward identities that the correlators must satisfy, see [3]. As an example, let us consider the SCT for the 2-point function of a current J_μ and the energy-momentum tensor $T_{\mu\nu}$ in d dimensions. For the 2-point function of currents we have the special conformal Ward identity

$$\begin{aligned}
\mathcal{K}_\kappa \langle J_\mu(\mathbf{k}) J_\nu(-\mathbf{k})\rangle = (2(\Delta - d)\tilde{\partial}_\kappa - 2k\cdot\tilde{\partial}\,\tilde{\partial}_\kappa + k_\kappa \tilde{\Box})\langle J_\mu(\mathbf{k}) J_\nu(-\mathbf{k})\rangle \\
+ 2(\eta_{\kappa\mu}\tilde{\partial}^\alpha - \delta^\alpha_\kappa \tilde{\partial}_\mu)\langle J_\alpha(\mathbf{k}) J_\nu(-\mathbf{k})\rangle = 0, \quad (3.1)
\end{aligned}$$

while for the 2-point function of the energy-momentum tensor we have

$$\begin{aligned}
&\mathcal{K}_\kappa \langle T_{\mu\nu}(\mathbf{k}) J_{\rho\sigma}(-\mathbf{k})\rangle = (2(\Delta - d)\tilde{\partial}_\kappa - 2k\cdot\tilde{\partial}\,\tilde{\partial}_\kappa + k_\kappa \tilde{\Box})\langle T_{\mu\nu}(\mathbf{k}) T_{\rho\sigma}(-\mathbf{k})\rangle \\
&+ 2(\eta_{\kappa\mu}\tilde{\partial}^\alpha - \delta^\alpha_\kappa \tilde{\partial}_\mu)\langle T_{\alpha\nu}(\mathbf{k}) T_{\rho\sigma}(-\mathbf{k})\rangle + 2(\eta_{\kappa\nu}\tilde{\partial}^\alpha - \delta^\alpha_\kappa \tilde{\partial}_\nu)\langle T_{\mu\alpha}(\mathbf{k}) T_{\rho\sigma}(-\mathbf{k})\rangle = 0. \quad (3.2)
\end{aligned}$$

3.3 2- and 3-point functions and CS effective action

The first example announced in the introduction is mostly pedagogical. It arises from a very simple model, a free massive fermion model in 3d coupled to a gauge field, see [14–16]. The action is

$$S = \int d^3x \, \left(i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi\right), \quad D_\mu = \partial_\mu + A_\mu \quad (3.3)$$

where $A_\mu = A_\mu^a(x)T^a$ and T^a are the generators of a gauge algebra in a given representation determined by ψ. The generators are antihermitean, $[T^a, T^b] = f^{abc}T^c$, with normalization $\mathrm{tr}(T^aT^b) = n\,\delta^{ab}$.

The current

$$J_\mu^a(x) = \bar{\psi}\gamma_\mu T^a \psi \tag{3.4}$$

is (classically) covariantly conserved on shell

$$(DJ)^a = (\partial^\mu \delta^{ac} + f^{abc}A^{b\mu})J_\mu^c = 0 \tag{3.5}$$

The generating functional of the connected Green functions is given by

$$W[A] = \sum_{n=1}^{\infty} \frac{i^{n+1}}{n!} \int \prod_{i=1}^{n} d^3x_i A^{a_1\mu_1}(x_1)\ldots A^{a_n\mu_n}(x_n)\langle 0|\mathcal{T}J_{\mu_1}^{a_1}(x_1)\ldots J_{\mu_n}^{a_n}(x_n)|0\rangle \tag{3.6}$$

The full 1-point function of J_μ^a in the presence of the source $A^{a\mu}$ is

$$\langle\langle J_\mu^a(x)\rangle\rangle = \frac{\delta W[A]}{\delta A^{a\mu}(x)} = -\sum_{n=1}^{\infty} \frac{i^n}{n!} \int \prod_{i=1}^{n} d^3x_i A^{a_1\mu_1}(x_1)\ldots A^{a_n\mu_n}(x_n) \langle 0|\mathcal{T}J_\mu^a(x)J_{\mu_1}^{a_1}(x_1)\ldots J_{\mu_n}^{a_n}(x_n)|0\rangle \tag{3.7}$$

The 1-loop conservation is

$$(D_\mu \langle\langle J_\mu(x)\rangle\rangle)^a = \partial^\mu \langle\langle J_\mu^a(x)\rangle\rangle + f^{abc}A_\mu^b(x)\langle\langle J^{\mu c}(x)\rangle\rangle = 0 \tag{3.8}$$

if there are no anomalies. By deriving this relation with respect to A we find the implications of conservation for the 2-point and 3-point correlators

$$k^\mu \tilde{J}_{\mu\nu}^{ab}(k) = 0 \tag{3.9}$$

$$-iq^\mu \tilde{J}_{\mu\nu\lambda}^{abc}(k_1,k_2) + f^{abd}\tilde{J}_{\nu\lambda}^{dc}(k_2) + f^{acd}\tilde{J}_{\lambda\nu}^{db}(k_1) = 0 \tag{3.10}$$

where $q = k_1 + k_2$ and $\tilde{J}_{\mu\nu}^{ab}(k)$ and $\tilde{J}_{\mu\nu\lambda}^{abc}(k_1,k_2)$ are Fourier transform of the 2- and 3-point functions, respectively.

The Feynman rules are easily extracted from the action. The propagator is $\frac{i}{\not{p}-m}$ and the gauge-fermion-fermion vertex is simply $\gamma_\mu T^a$, where μ, a are the labels of A_μ^a. Our next task will be to calculate the odd-parity 2- and 3-point correlators in this model and study their behaviour in the IR and UV limit.

3.3.1 The 2-point current correlator

The relevant diagram is the bubble one, with external momentum k. Its Fourier transform is

$$\tilde{J}_{\mu\nu}^{ab}(k) = -\int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\left(\gamma_\mu T^a \frac{1}{\not{p}-m}\gamma_\nu T^b \frac{1}{\not{p}-\not{k}-m}\right) = -2n\,\delta^{ab} \tag{3.11}$$

$$\cdot \int \frac{d^3p}{(2\pi)^3} \frac{p_\nu(p-k)_\mu - p\cdot(p-k)\eta_{\mu\nu} + p_\mu(p-k)_\nu + im\epsilon_{\mu\nu\sigma}k^\sigma + m^2\eta_{\mu\nu}}{(p^2-m^2)((p-k)^2-m^2)}$$

Let us focus from now on on the odd-parity part. After a Wick rotation and integration we get

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) = \frac{n}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^{\sigma}\,\frac{m}{k}\arctan\frac{k}{2m} \tag{3.12}$$

where $k = \sqrt{k^2}$. The conservation law (3.9) is readily seen to be satisfied.

We are interested in the IR and UV limits of this expression. To this end we notice that k is the total energy E of the process. Therefore the IR and UV limit correspond to $\frac{m}{E} \to \infty$ and 0, respectively. Therefore near the IR (3.12) becomes

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) = \frac{n}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^{\sigma}\left(\frac{1}{2} - \frac{1}{24}\left(\frac{k}{m}\right)^2 + \frac{1}{160}\left(\frac{k}{m}\right)^4 + \dots\right) \tag{3.13}$$

and near the UV

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) = \frac{n}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^{\sigma}\left(\frac{\pi}{2}\frac{m}{k} - 2\left(\frac{m}{k}\right)^2 + \frac{8}{3}\left(\frac{m}{k}\right)^4 + \dots\right) \tag{3.14}$$

In particular in the two limits we have

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) = \frac{n}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^{\sigma}\begin{cases}\frac{1}{2}\frac{m}{|m|} & \text{IR}\\ \frac{\pi}{2}\frac{m}{k} & \text{UV}\end{cases} \tag{3.15}$$

So far we have worked with a Euclidean metric, but the same result holds also for Lorentzian metric. Moreover from now on, for simplicity, we will consider only the case of m positive. For a more complete treatment see [2].

We notice that the UV limit is actually vanishing. However we could consider a model made of N identical copies of free fermions coupled to the same gauge field. Then the result (3.15) would be

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) = \frac{nN}{4}\delta^{ab}\epsilon_{\mu\nu\sigma}k^{\sigma}\frac{m}{k} \tag{3.16}$$

In this case we can consider the scaling limit $\frac{m}{k} \to 0$ and $N \to \infty$ in such a way that $N\frac{m}{k}$ is fixed. Then the UV limit (3.16) becomes nonvanishing.

Before discussing the implications of the previous results let us consider also the 3-current correlator.

3.3.2 The 3-point current correlator

The 3-point correlator for currents is given by the triangle diagram. The three external momenta are q, k_1, k_2. q is incoming, while k_1, k_2 are outgoing and, of course, momentum conservation implies $q = k_1 + k_2$. The Fourier transform is

$$\tilde{J}^{1,abc}_{\mu\nu\lambda}(k_1,k_2) = i\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left(\gamma_\mu T^a\frac{1}{\not{p}-m}\gamma_\nu T^b\frac{1}{\not{p}-\not{k}_1-m}\gamma_\lambda T^c\frac{1}{\not{p}-\not{q}-m}\right) \tag{3.17}$$

to which we have to add the cross graph corresponding to the exchange $b \leftrightarrow c, \nu \leftrightarrow \lambda, 1 \leftrightarrow 2$.

We will not go through all the calculation, which is rather more complicated than in 2-point case. For instance, near the IR fixed point we obtain a series expansion of the type

$$\tilde{J}^{1,abc(odd)}_{\mu\nu\lambda}(k_1,k_2) \approx i\frac{n}{32\pi}\sum_{n=0}^{\infty}\left(\frac{E}{m}\right)^{2n} f^{abc}\tilde{I}^{(2n)}_{\mu\nu\lambda}(k_1,k_2) \tag{3.18}$$

and, in particular,

$$I^{(0)}_{\mu\nu\lambda}(k_1,k_2) = -12\epsilon_{\mu\nu\lambda} \tag{3.19}$$

Let us pause to comment on this result. We expect the current (3.4) to be conserved also at the quantum level, because no anomaly is expected in this case. This should be true also in the IR limit. It would seem that conservation, if any, should hold order by order in the expansions we have considered in (3.18). In order to check conservation we have to verify (3.10). Conservation has a contribution from the 2-point function, so the LHS of equation (3.10) reads

$$-\frac{3}{8\pi}nf^{abc}q^{\mu}\epsilon_{\mu\nu\lambda} + \frac{1}{4\pi}f^{abc}\epsilon_{\nu\lambda\sigma}k_2^{\sigma} + \frac{1}{4\pi}f^{abc}\epsilon_{\nu\lambda\sigma}k_1^{\sigma} \neq 0. \tag{3.20}$$

Conservation is violated unless we add to $I^{(0)}_{\mu\nu\lambda}(k_1,k_2)$ a term $4\epsilon_{\mu\nu\lambda}$. In order to understand what is at stake here let us turn to the Chern-Simons action for the gauge field A in 3d.

3.3.3 The CS action

The CS action for the gauge field A is

$$\begin{aligned} CS &= \frac{\kappa}{4\pi}\int d^3x \mathrm{Tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) \\ &= \frac{n\kappa}{4\pi}\int d^3x \epsilon^{\mu\nu\lambda}\left(A^a_\mu\partial_\nu A^a_\lambda + \frac{1}{3}f^{abc}A^a_\mu A^b_\nu A^c_\lambda\right) \end{aligned} \tag{3.21}$$

Now let us return to the 2- and 3-point functions obtained above. The Fourier anti-transform of the 2-point function $\sim \epsilon_{\mu\nu\sigma}k^{\sigma}$ is

$$\mathcal{F}^{-1}[\epsilon_{\mu\nu\sigma}k^{\sigma}](x) = i\epsilon_{\mu\nu\sigma}\partial^{\sigma}\delta(x) \tag{3.22}$$

The Fourier anti-transform of the 3-point function $\sim \epsilon_{\mu\nu\lambda}$ is

$$\begin{aligned} &\mathcal{F}^{-1}[\epsilon_{\mu\nu\sigma}](x,y,z) \\ &\quad = \int\frac{d^3q}{(2\pi)^3}e^{-iqx}\int\frac{d^3k_1}{(2\pi)^3}e^{-ik_1y}\int\frac{d^3k_2}{(2\pi)^3}e^{ik_2z}\delta(q-k_1-k_2)\epsilon_{\mu\nu\lambda} \\ &\quad = \int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}e^{ik_1(y-x)}e^{ik_2(y-z)}\epsilon_{\mu\nu\lambda} = \delta(y-x)\delta(z-x)\epsilon_{\mu\nu\lambda} \end{aligned} \tag{3.23}$$

Inserting this into the functional generator $W[A]$ and integrating with respect to space time we obtain the two terms of the action (3.21). Therefore if we add to

$I^{(0)}_{\mu\nu\lambda}(k_1,k_2)$ a term $4\epsilon_{\mu\nu\lambda}$ the effective action of our model in the IR gives back the CS action with coupling $\kappa = 1$.

This corresponds to correcting the effective action by adding a counterterm

$$4\int dx \epsilon^{\mu\nu\lambda} f^{abc} A^a_\mu A^b_\nu A^c_\lambda \tag{3.24}$$

This counterterm simultaneously guarantees conservation, see (3.20), and reconstructs the correct CS action. We remark that for the effective action in the IR limit the CS coupling $\kappa = 1$, see (3.15). This guarantees invariance of the action also under large gauge transformations, [1].

Something similar can be done also for the UV limit. However in the UV limit the resulting effective action has a vanishing coupling, unless we consider an $N \to \infty$ limit theory, as outlined above. In order to guarantee invariance under large gauge transformations we have also to fine tune the limit in such a way that the κ coupling be an integer.

Free fermions in 3d can be coupled also to a background metric. In this case the relevant correlators are those of the energy-momentum tensor and the resulting effective action in the UV and IR is the gravitational CS action, see [2].

A few remarks We would like to stress a few points of the above construction. The first is the problem of non-conservation for the 3-point function we have met. This is a consequence of the particular regularization procedure we have used, that is of the fact the we have first computed the 3-point function of three currents and then contracted the correlator with the external momentum q^μ. We could have proceeded in another way, that is we could have contracted the 3-point correlator with $q^\mu = k_1^\mu + k_2^\mu$ before doing the integration over p. The triangle diagram contracted with q^μ is:

$$q^\mu \tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2) = -i\int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\left(\not{q} T^a \frac{1}{\not{p}-m}\gamma_\nu T^b \frac{1}{\not{p}-\not{k}_1-m}\gamma_\lambda T^c \frac{1}{\not{p}-\not{q}-m}\right). \tag{3.25}$$

Replacing $\not{q} = (\not{p}-m) - (\not{p}-\not{q}-m)$ considerably simplifies the calculation. The final result for the odd parity part (after adding the cross diagram contribution, $1 \leftrightarrow 2, b \to c, \nu \leftrightarrow \lambda$) is

$$\begin{aligned} q^\mu \tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2) = &-\frac{i}{4\pi} f^{abc}\epsilon_{\lambda\nu\sigma} k_1^\sigma \frac{2m}{k_1}\mathrm{arcot}\left(\frac{2m}{k_1}\right) \\ &-\frac{i}{4\pi} f^{abc}\epsilon_{\lambda\nu\sigma} k_2^\sigma \frac{2m}{k_2}\mathrm{arcot}\left(\frac{2m}{k_2}\right). \end{aligned} \tag{3.26}$$

Therefore, as far as the odd part is concerned, the 3-point conservation (3.10) reads

$$\begin{aligned} &-iq^\mu \tilde{J}^{(odd)abc}_{\mu\nu\lambda}(k_1,k_2) + f^{abd}\tilde{J}^{(odd)dc}_{\nu\lambda}(k_2) + f^{acd}\tilde{J}^{(odd)db}_{\lambda\nu}(k_1) \\ &\quad = -\frac{1}{4\pi} f^{abc}\epsilon_{\lambda\nu\sigma}\left(k_1^\sigma \frac{2m}{k_1}\mathrm{arcot}\left(\frac{2m}{k_1}\right) + k_2^\sigma \frac{2m}{k_2}\mathrm{arcot}\left(\frac{2m}{k_2}\right)\right) \\ &\quad + \frac{1}{4\pi} f^{abc}\epsilon_{\lambda\nu\sigma}\left(k_1^\sigma \frac{2m}{k_1}\mathrm{arcot}\left(\frac{2m}{k_1}\right) + k_2^\sigma \frac{2m}{k_2}\mathrm{arcot}\left(\frac{2m}{k_2}\right)\right) = 0. \end{aligned} \tag{3.27}$$

Thus conservation is secured for any value of the parameter m. The fact that in the UV or IR limit we find a violation of the conservation is an artifact of the procedure we have used and we have to remedy by subtracting suitable counterterms from the effective action. These subtractions are to be understood as (part of) the definition of our regularization procedure.

The second remark concerns the odd-parity correlators we have obtained above in the IR limit, the 2-point function $\sim \delta^{ab}\epsilon_{\mu\nu\sigma}k^{\sigma}$ and the 3-point function $\sim f^{abc}\epsilon_{\mu\nu\lambda}$. As expected from the fact that they are correlators at a RG fixed point, both satisfy the Ward identities of CFT, in particular the SCT one. They are both purely local and at least the 2-point one does not come from the regularization of any bare correlator. Ref.[4] provides a classification of all bare correlators in 3d CFT, both odd- and even-parity ones. These satisfy the simplest conservation law, in which lower order correlators are not involved. It is clear that, a complete classification of CFT correlators requires that we add also those considered above, which satisfy the conservation law (3.10).

Another remark is that in many cases correlators can be constructed directly from free field theory via the Wick theorem. It is evident that there is no conformal free field theory in 3d that can give rise to the parity odd 2- and 3-point correlators found above.

Finally let us remark that similar results are expected in other odd dimensional spacetimes. Interesting cases will be 7d for free fermions coupled to gravity, and 5d and 7d for fermions coupled to a gauge field alone or to both gravity and gauge fields.

3.4 The Pontryagin trace anomaly

The second example of a correlator made only of contact terms is in even dimension, specifically in 4d. It is provided by the parity-odd 3-point function of the energy-momentum tensor in which one of the entries is the trace of the e.-m. tensor. This 3-point function is the basic (but not exclusive) ingredient of the trace anomaly. It is well-known that in 4d a theory coupled to external gravity is generically endowed with an energy-momentum tensor whose trace takes the form

$$T_{\mu}{}^{\mu} = aE + c\mathcal{W}^2 + eP, \tag{3.28}$$

where E is the Euler density, $\mathcal{W}^2$ the square Weyl density and P the density of the Pontryagin class

$$P = \frac{1}{2}\left(\frac{\epsilon^{nmlk}}{\sqrt{|g|}} R_{nmpq} R_{lk}{}^{pq}\right) \tag{3.29}$$

where ϵ^{nmlk} is the numerical Levi-Civita symbol. Our interest here focus on this term[1]. The obvious question is whether there are models where this term appears

[1] Of course also the other anomalies, E and $\mathcal{W}^2$, are local terms, but they come from the regularization of nonvanishing bare correlators.

in the trace of the e.m. tensor, that is if there are models in 4d where the coefficient e does not vanish. The natural candidates are models involving chiral fermions, where the ϵ tensor may appear in the trace of γ matrices. The coefficient e has been recently calculated [5,6], following an early work [10], (see also [9,7,8]) in a model of free chiral fermions coupled to a background metric.

The model is the simplest possible one: a right-handed spinor coupled to external gravity in 4d. The action is

$$S = \int d^4x \sqrt{|g|}\, i\bar{\psi}_R \gamma^m \left(\nabla_m + \frac{1}{2}\omega_m\right) \psi_R \tag{3.30}$$

where $\gamma^m = e_a^m \gamma^a$, ∇ ($m, n, \ldots$ are world indices, $a, b, \ldots$ are flat indices) is the covariant derivative with respect to the world indices and ω_m is the spin connection:

$$\omega_m = \omega_m^{ab} \Sigma_{ab}$$

where $\Sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ are the Lorentz generators. Finally $\psi_R = \frac{1+\gamma_5}{2}\psi$. Classically the energy-momentum tensor

$$T_{\mu\nu} = \frac{i}{2}\bar{\psi}_R \gamma_\mu \overset{\leftrightarrow}{\nabla}_\nu \psi_R \tag{3.31}$$

is both conserved and traceless on shell. At one loop, to make sense of the calculations one must introduce regulators. The latter generally break both diffeomorphism and conformal invariance. A careful choice of the regularization procedure may preserve diff invariance, but anyhow breaks conformal invariance, so that the trace of the e.m. tensor takes the form (3.28), with specific nonvanishing coefficients a, c and e. There are various techniques to calculate the latter: cutoff, point splitting, dimensional regularization, and a few others. Here, for simplicity we limit ourselves to a short summary of dimensional regularization. First one expands the metric around a flat background: $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ represent the gravity fluctuation. Then one extracts from the action propagator and vertices. The essential ones are the fermion propagator $\frac{i}{\not p + i\epsilon}$ and the two-fermion-one-graviton vertex (V_{ffg})

$$-\frac{i}{8}\left[(p-p')_\mu \gamma_\nu + (p-p')_\nu \gamma_\mu\right] \frac{1+\gamma_5}{2} \tag{3.32}$$

where p, p' are the fermion momenta. The only contributing diagrams are the triangle diagram together with the crossed one. The triangle diagram is constructed by joining three vertices V_{ffg} with three fermion lines. The external momenta are q (incoming) with labels σ and τ, and k_1, k_2 (outgoing), with labels μ, ν and μ', ν' respectively. Of course $q = k_1 + k_2$. The internal momenta are p, $p - k_1$ and $p - k_1 - k_2$, respectively. After contracting σ and τ the total contribution to the 3-point e.m. tensor correlator, in which one of the entries is the trace, is

$$-\frac{1}{256}\int \frac{d^4p}{(2\pi)^4} \operatorname{tr}\left[\left(\frac{1}{\not p}\left((2p-k_1)_\mu \gamma_\nu + (\mu \leftrightarrow \nu)\right)\frac{1}{\not p - \not k_1}\right.\right. \tag{3.33}$$
$$\left.\left.\cdot\left((2p-2k_1-k_2)_{\mu'}\gamma_{\nu'} + (\mu' \leftrightarrow \nu')\right)\frac{1}{\not p - \not k_1 - \not k_2}(2\not p - \not k_1 - \not k_2)\right)\frac{1+\gamma_5}{2}\right]$$

to which we have to add the cross diagram where k_1, μ, ν is exchanged with k_2, μ', ν'. This integral is divergent. To regularize it we use dimensional regularization, which consists in introducing additional components of the momentum running in the loop: $p \to p + l$, $l = (l_4, \dots, l_{n-4})$. This regulates the integral, and one can now proceed to the integration. Full details of the calculation can be found in [5,6]. The result is as follows. Calling $\tilde{T}^{(tot)}_{\mu\nu\mu'\nu'}(k_1, k_2)$ the overall contribution of the two diagrams, with $k_1^2 = k_2^2 = 0$, one has

$$\tilde{T}^{(tot)}_{\mu\nu\mu'\nu'}(k_1, k_2) = \frac{1}{3072\pi^2}\left(k_1 \cdot k_2\, t_{\mu\nu\mu'\nu'\lambda\rho} - t^{(21)}_{\mu\nu\mu'\nu'\lambda\rho}\right) k_1^\lambda k_2^\rho \tag{3.34}$$

where

$$t_{\mu\nu\mu'\nu'\kappa\lambda} = \eta_{\mu\mu'}\epsilon_{\nu\nu'\kappa\lambda} + \eta_{\nu\nu'}\epsilon_{\mu\mu'\kappa\lambda} + \eta_{\mu\nu'}\epsilon_{\nu\mu'\kappa\lambda} + \eta_{\nu\mu'}\epsilon_{\mu\nu'\kappa\lambda},$$
$$t^{(21)}_{\mu\nu\mu'\nu'\kappa\lambda} = k_{2\mu}k_{1\mu'}\epsilon_{\nu\nu'\kappa\lambda} + k_{2\nu}k_{1\nu'}\epsilon_{\mu\mu'\kappa\lambda} + k_{2\mu}k_{1\nu'}\epsilon_{\nu\mu'\kappa\lambda} + k_{2\nu}k_{1\mu'}\epsilon_{\mu\nu'\kappa\lambda}.$$

Fourier transforming (3.34) and plugging the result in the full 1-point correlator of the e.m. tensor trace

$$\langle\langle T^\mu_\mu(x)\rangle\rangle = 2\sum_{n=1}^{\infty}\frac{i^{n+1}}{(n-1)!}\int\prod_{i=2}^{n} dx_i\, h_{\mu_i\nu_i}(x_i)\,\langle 0|\mathcal{T}\, T^\mu_\mu(x)\dots T^{\mu_n\nu_n}(x_n)|0\rangle \tag{3.35}$$

one obtains

$$\langle\langle T^\mu_\mu(x)\rangle\rangle = \frac{i}{768\pi^2}\epsilon^{\mu\nu\lambda\rho}\left(\partial_\mu\partial_\sigma h^\tau_\nu\,\partial_\lambda\partial_\tau h^\sigma_\rho - \partial_\mu\partial_\sigma h^\tau_\nu\,\partial_\lambda\partial^\sigma h_{\tau\rho}\right) + \mathcal{O}(h^3), \tag{3.36}$$

which is the lowest order expansion in $h_{\mu\nu}$ of

$$\langle\langle T^\mu_\mu(x)\rangle\rangle = \frac{i}{768\pi^2}\,\frac{1}{2}\,\epsilon^{\mu\nu\lambda\rho}R_{\mu\nu}{}^{\sigma\tau}R_{\lambda\rho\sigma\tau}, \tag{3.37}$$

i.e. the Pontryagin trace anomaly. Changing chirality in (3.30) leads to a change of sign in the RHS of (3.37). Therefore, in left-right symmetric models this anomaly is absent. The surprising aspect of (3.37) is the i in the RHS. In other words the coefficient e in (3.28) is imaginary. Before entering the discussion of this point in the next section, let us recall that the odd-parity 3-point correlator, with three (untraced) e.m. tensor insertions, in the model (3.30), calculated by means of the Wick theorem, identically vanishes in configuration space, [6]. An unsurprising result, because on the basis of a general theorem we know that the odd-parity conformal covariant 3-point e.m. tensor bare correlator in 4d vanishes identically, [11,12].

Finally let us remark that the one described in this section is not an isolated case. Similar pure contact terms correlators (and similar anomalies) exist in 4k dimensions, and mixed gauge-gravity pure contact terms correlators may exist also in other even dimensions.

3.4.1 Comments on the Pontryagin anomaly

The Pontryagin anomaly is puzzling at first because it looks like a challenge for many commonplaces. Several points have been already discussed in section 4 of

[5] and in section 7 of [6]. We would like to add here a few additional remarks. One surprising aspect of this anomaly is the appearance of an imaginary coefficient in front of it, with the consequence that the energy-momentum tensor at one loop becomes complex and may endanger unitarity, see [5]. The surprise is due to the fact that the action of the model (3.30) is hermitean and one would not expect the e.m. tensor to become complex at one loop. However this is a simple consequence of the regularization. For regularizing an expression may require to trespass on the complex plane, much in the same way as when one looks for solutions of a real algebraic equation. The simplest example of this effect is the regularization of the real function $\frac{1}{x}$ in one dimension given by $\mathcal{P}\frac{1}{x} + \pi i\delta(x)$ (the first term is the principal value). Something similar happens in our regularization of (3.33) and leads to the imaginary coefficient of eq.(3.37). Therefore, finally, this result is not at all surprising.

An important aspect of the anomaly we are considering, which was only sketched in [6], is the following: if instead of regularizing (3.33) (let's call it procedure (a)), as we have done above, we first regularize the 3-point function of the untraced e.m. tensor and *then* take the trace of one of the insertions (procedure (b)), we get a vanishing result. It was pointed out in [6] that the latter is not the correct way to proceed. However, although this statement was supported by explicit examples in 2d, it may leave the impression that our result in [6] and in the previous section is scheme dependent. This is not the case and we would like now to explain why. The point is that procedure (b), as just outlined, is incomplete. As we have pointed out above regularizing may break not only Weyl symmetry but also diffeomorphism covariance. This is in fact what happens with both procedure (a) and (b). But while, as was shown in [6], this breaking in case (a) is innocuous (one subtracts counterterms which restore covariance without modifying the trace anomaly), in case (b) the breaking of covariance is more substantial. In order to restore it one has to modify the (previously vanishing) trace anomaly. The explicit calculation in scheme (b), which is very challenging, has not been done yet, but we conjecture that the result will restore the Pontryagin anomaly with the same coefficient as in (3.37). If this is true, as we believe, choosing scheme (a) instead of (b) is only a matter of opportunity.

We would like to add also a few words on a frequent source of misunderstanding, which stems from a reckless identification of Majorana and Weyl spinors in 4d. In 4d they transform according to two different irreducible representations of the Lorentz group. The first belong to a real representation and the second to a complex one. Moreover, Weyl fermions have definite chirality while for Majorana fermions chirality is not defined. Majorana fermion admit a massive term in the action, whereas Weyl fermions are rigorously massless. The corresponding Dirac operators are different, even in the massless case. So in no way can one confuse Majorana and Weyl spinors, even when massless. However misnaming is very frequent and not always innocuous, especially when anomalies are involved.

For instance, given a Weyl spinor χ, one can construct a Majorana spinor ψ as follows

$$\psi = \chi + \gamma_0 C \chi^* \tag{3.38}$$

where C is the charge conjugation matrix (for notation, see [5]). If χ is left-handed, the conjugate spinor $\gamma_0 C\chi^*$ is right-handed. Thus we can see the reason why for Majorana fermions there is no Pontryagin anomaly. But, apart from this, (3.38) is not much more than saying that the sum of a complex number and its conjugate is real. In any case it is not a good reason to confuse Weyl and Majorana fermions.

On the other hand many theories, in particular the supersymmetric ones, are conveniently formulated in terms of the two-component formalism, i.e. on the basis of two-component spinors ξ_α and $\xi_{\dot\alpha}$ ($\alpha, \dot\alpha = 1, 2$). These two-component fields are the building blocks of the theory and, a priori, they can be the components of either a Weyl, Majorana or Dirac fermion. When the two-component formalism is used one must know the full content of the theory in order to decide that[2]. However the two-component formalism has many advantages, it serves well for many purposes and there is no reason not to use it. However the problem of anomalies must be dealt with carefully, anomalies come from a (regularized) variation of the fermion determinant, i.e. the determinant of the relevant Dirac operator, which is different in the different cases. So when anomalies are involved it is of course irrelevant what formalism we use, provided we unambiguously distinguish the true chiral nature of the fermions in the theory. For instance, it is a well known and important fact that consistent gravitational (Einstein) and Lorentz anomalies in 4d vanish. But this is not due to Weyl fermions being exchangeable with Majorana ones, but rather because the third order symmetric invariant tensor of the Lorentz algebra vanishes identically. If one understand this it is not difficult to understand the origin of the Pontryagin anomaly. In particular what is decisive for the latter is the overall balance of chirality.

3.5 Conclusion

Our purpose in this article was to show that in field theories, and in particular in conformal field theories, there are correlators made of pure contact terms, without a corresponding bare part. We have exhibited two examples. The first obtained by considering the conformal limits of a free massive fermion theory in 3d and the current correlators thereof; we have shown that such correlators are in one-to-one correspondence with the terms of the 3d gauge CS action. The second corresponds to the case of the Pontryagin trace anomaly. Such an anomaly appears in e.m. tensor correlators containing one trace insertion. We have shown that the corresponding 3-point correlator is nonvanishing even though the corresponding untraced correlator vanishes (that is, there is no bare correlator underlying it). In other words pure contact term correlators may live of their own.

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[2] Sometimes ξ_α and $\xi_{\dot\alpha}$ are called themselves Weyl spinors, which does not add to clarity.

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4 Novel Perspectives from Light-Front QCD, Super-Conformal Algebra, and Light-Front Holography

S.J. Brodsky ⋆

SLAC National Accelerator Laboratory
Stanford University

SLAC-PUB-16432

Abstract. Light-Front Quantization — Dirac's "Front Form" — provides a physical, frame-independent formalism for hadron dynamics and structure. Observables such as structure functions, transverse momentum distributions, and distribution amplitudes are defined from the hadronic LFWFs. One obtains new insights into the hadronic mass scale, the hadronic spectrum, and the functional form of the QCD running coupling in the non-perturbative domain using light-front holography. In addition, superconformal algebra leads to remarkable supersymmetric relations between mesons and baryons. I also discuss evidence that the antishadowing of nuclear structure functions is non-universal i.e., flavor dependent, and why shadowing and antishadowing phenomena may be incompatible with the momentum and other sum rules for the nuclear parton distribution functions.

Povzetek. Kvantizacija na svetlobnem stožcu — Diracove "frontne forme" —ponudi formalizem za opis dinamike in strukture hadronov, ki je neodvisen od opazovalnega sistema. Opazljivke — kot so strukturne funkcije, porazdelitev prečne gibalne količine in porazdelitev amplitud — so definirane z valovnimi funkcijami na hadronov na svetlobnme stožcu. Uporaba holografije svetlobnega stožca ponudi nov vpogled v masno skalo hadronov, hadronski spekter in funkcijsko obliko tekočih sklopitev v neperturbativnem območju kromodinamike. Superkonformna algebra pokaě zanimive supersimetrične povezave med mezoni in barioni. Avtor razpravlja tudi o tem, da 'antisenčenje' strukturnih funkcij jeder ni univerzalno, ampak je odvisno od okusnega števila, ter o tem, zakaj utegnejo biti pojavi senčenja in antisenčenja neskladni z vsotnimi pravili, denimo za gibalno količino in za porazdelitvene funkcije partonov v jedru.

4.1 Light-Front Wavefunctions and QCD

Measurements of hadron structure – such as the structure functions determined by deep inelastic lepton-proton scattering (DIS) – are analogous to a flash photograph: one observes the hadron at fixed $\tau = t + z/c$ along a light-front, not at a given

⋆ sjbth@slac.stanford.edu

instant of time t. The underlying physics follows from the the light-front wavefunctions (LFWFs) $\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$ with $x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}, \sum_i^n x_1 = 1, \sum_i^n \vec{k}_{\perp_i} = \vec{0}_\perp$ and spin projections λ_i. The LFWFs are the Fock state projections of the eigenstates of the QCD LF Hamiltonian $H_{LF}|\Psi >= M^2|\Psi >$ [5], where the LF Hamiltonian is the light-front time evolution operator defined directly from the QCD Lagrangian. One can avoid ghosts and longitudinal gluonic degrees of freedom by choosing.to work in the light-cone gauge $A^+ = 0$. The LFWFs are boost invariant; i.e., they are independent of the hadron's momentum $P^+ = P^0 + P^z, \vec{P}_\perp$. This contrasts with the wavefunctions defined at a fixed time t – the Lorentz boost of an instant-form wavefunction is much more complicated than a Melosh transform [1] – even the number of Fock constituents changes under a boost. Current matrix element such as form factors are simple overlaps of the initial-state and final-state LFWFs, as given by the Drell-Yan West formula [2–4]. There is no analogous formula for the instant form, since one must take into account the coupling of the external current to connected vacuum-induced currents. Observables such as structure functions, transverse momentum distributions, and distribution amplitudes are defined from the hadronic LFWFs. Since they are frame-independent, the structure functions measured in DIS are the same whether they are measured in an electron-proton collider or in a fixed-target experiment where the proton is at rest. There is no concept of length contraction of the hadron or nucleus at a collider – no collisions of "pancakes" – since the observations of the collisions of the composite hadrons are made at fixed τ, not at fixed time. The dynamics of a hadron is not dependent on the observer's Lorentz frame.

The LF Heisenberg equation can in principle be solved numerically by matrix diagonalization using "Discretized Light-Cone Quantization" (DLCQ) [6] where anti-periodic boundary conditions in x^- render the k^+ momenta discrete as well as limiting the size of the Fock basis. In fact, one can easily solve 1+1 quantum field theories such as QCD$(1+1)$ [7] for any number if colors, flavors and quark masses. Unlike lattice gauge theory, the nonpertubative DLCQ analysis is in Minkowski space, is frame-independent and is free of fermion-doubling problems. A new method for solving nonperturbative QCD "Basis Light-Front Quantization" (BLFQ) [8], uses the eigensolutions of a color-confining approximation to QCD (such as LF holography) as the basis functions, rather than the plane-wave basis used in DLCQ. The LFWFs can also be determined from covariant Bethe-Salpeter wavefunction by integrating over k^- [9].

Factorization theorems and DGLAP and ERBL evolution equations can be derived using the light-front Hamiltonian formalism [10]. In the case of an electron-ion collider, one can represent the cross section for e-p colisions as a convolution of the hadron and virtual photon structure functions times the subprocess cross-section in analogy to hadron-hadron colisions. This nonstandard description of $\gamma^* p \to X$ reactions gives new insights into electroproduction physics – physics not apparent using the usual usual infinite momentum frame description, such as the dynamics of heavy quark-pair production. I intrinsic heavy quarks also play an important role [11]. In the case of $ep \to e'X$, one can consider the collisions of the confining QCD flux tube appearing between the $q and \bar{q}$ of the virtual photon with the flux tube between the quark and diquark of the proton. Since the

$q\bar{q}$ plane is aligned with the scattered electron's plane, the resulting "ridge" of hadronic multiplicity produced from the $\gamma^* p$ collision will also be aligned with the scattering plane of the scattered electron. The virtual photon's flux tube will also depend on the photon virtuality Q^2, as well as the flavor of the produced pair arising from $\gamma^* \to q\bar{q}$. The resulting dynamics [12] is a natural extension of the flux-tube collision description of the ridge produced in $p-p$ collisions [13].

4.2 Color Confinement and Supersymmetry in Hadron Physics from LF Holography

A key problem in hadron physics is to obtain a first approximation to QCD which predicts both the hadron spectrum and the hadronic LFWFs. If one neglects the Higgs couplings of quarks, then no mass parameter appears in the QCD Lagrangian, and the theory is conformal at the classical level. Nevertheless, hadrons have a finite mass. de Teramond, Dosch, and I [14] have shown that a mass gap and a fundamental color confinement scale can be derived from a conformally covariant action when one extends the formalism of de Alfaro, Fubini and Furlan [15] to light-front Hamiltonian theory. Remarkably, the resulting light-front potential has a unique form of a harmonic oscillator $\kappa^4 \zeta^2$ in the light-front invariant impact variable ζ where $\zeta^2 = b_\perp^2 x(1-x)$. The result is a single-variable frame-independent relativistic equation of motion for $q\bar{q}$ bound states, a "Light-Front Schrödinger Equation" [16], analogous to the nonrelativistic radial Schrödinger equation in quantum mechanics. The Light-Front Schrödinger Equation incorporates color confinement and other essential spectroscopic and dynamical features of hadron physics, including a massless pion for zero quark mass and linear Regge trajectories with the same slope in the radial quantum number n and internal orbital angular momentum L. The same light-front equation for mesons of arbitrary spin J can be derived [17] from the holographic mapping of the "soft-wall model" modification of AdS_5 space with the specific dilaton profile $e^{+\kappa^2 z^2}$, where one identifies the fifth dimension coordinate z with the light-front coordinate ζ. The five-dimensional AdS_5 space provides a geometrical representation of the conformal group. It is holographically dual to 3+1 spacetime using light-front time $\tau = t + z/c$. The derivation of the confining LF Schrodinger Equation is outlined in Fig. 4.1.

The combination of light-front dynamics, its holographic mapping to AdS_5 space, and the dAFF procedure provides new insight into the physics underlying color confinement, the nonperturbative QCD coupling, and the QCD mass scale. A comprehensive review is given in ref. [19]. The $q\bar{q}$ mesons and their valence LF wavefunctions are the eigensolutions of a frame-independent bound state equation, the "Light-Front Schrödinger Equation". The mesonic $q\bar{q}$ bound-state eigenvalues for massless quarks are $M^2(n, L, S) = 4\kappa^2(n + L + S/2)$. The equation predicts that the pion eigenstate $n = L = S = 0$ is massless at zero quark mass, The Regge spectra of the pseudoscalar $S = 0$ and vector $S = 1$ mesons are predicted correctly, with equal slope in the principal quantum number n and the internal orbital angular momentum. The predicted nonperturbative pion distribution

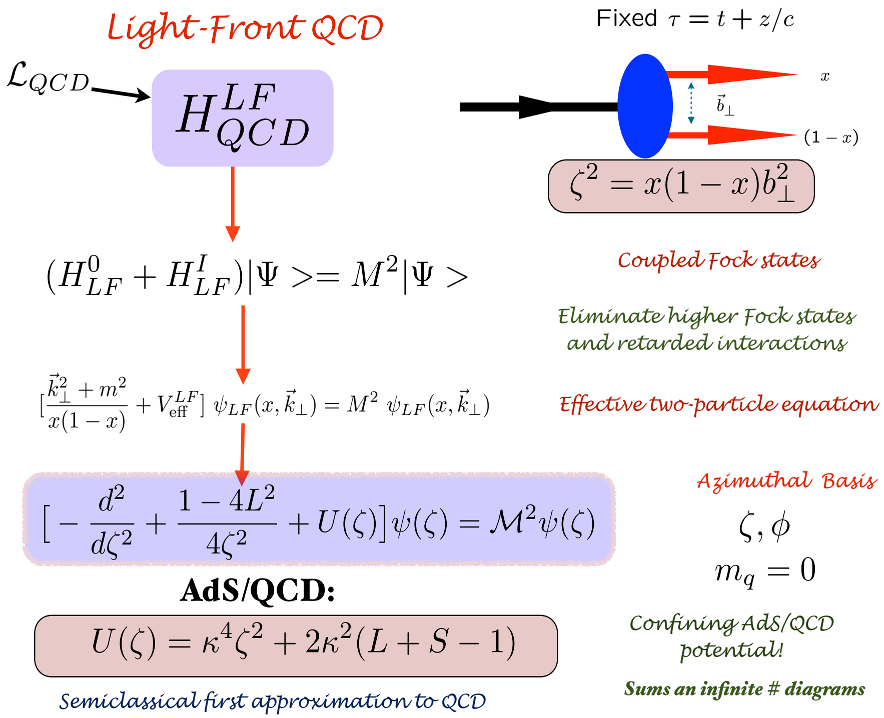


Fig. 4.1. Derivation of the Effective Light-Front Schrödinger Equation from QCD. As in QED, one reduces the LF Heisenberg equation $H_{LF}|\Psi>= M^2|\Psi>$ to an effective two-body eigenvalue equation for $q\bar{q}$ mesons by systematically eliminating higher Fock states. One utilizes the LF radial variable ζ, where $\zeta^2 = x(1-x)b^2_\perp$ is conjugate to the $q\bar{q}$ LF kinetic energy $\frac{k^2_\perp}{x(1-x)}$ for $m_q = 0$. This allows the reduction of the dynamics to a single-variable bound state equation acting on the valence $q\bar{q}$ Fock state. The confining potential $U(\zeta)$, including its spin-J dependence, is derived from the soft-wall AdS/QCD model with the dilaton $e^{+\kappa^2 z^2}$, where z is the fifth coordinate of AdS_5 holographically dual to ζ. See ref. [14]. The resulting light-front harmonic oscillator confinement potential $\kappa^4\zeta^2$ for light quarks is equivalent to a linear confining potential for heavy quarks in the instant form [18].

amplitude $\phi_\pi(x) \propto f_\pi\sqrt{x(1-x)}$ is consistent with the Belle data for the photon-to-pion transition form factor [20]. The prediction for the LFWF $\psi_\rho(x, k_\perp)$ of the ρ meson gives excellent predictions for the observed features of diffractive ρ electroproduction $\gamma^* p \to \rho p'$ [21].

These results can be extended [22–24] to effective QCD light-front equations for both mesons and baryons by using the generalized supercharges of superconformal algebra [25]. The supercharges connect the baryon and meson spectra and their Regge trajectories to each other in a remarkable manner: each meson has internal angular momentum one unit higher than its superpartner baryon

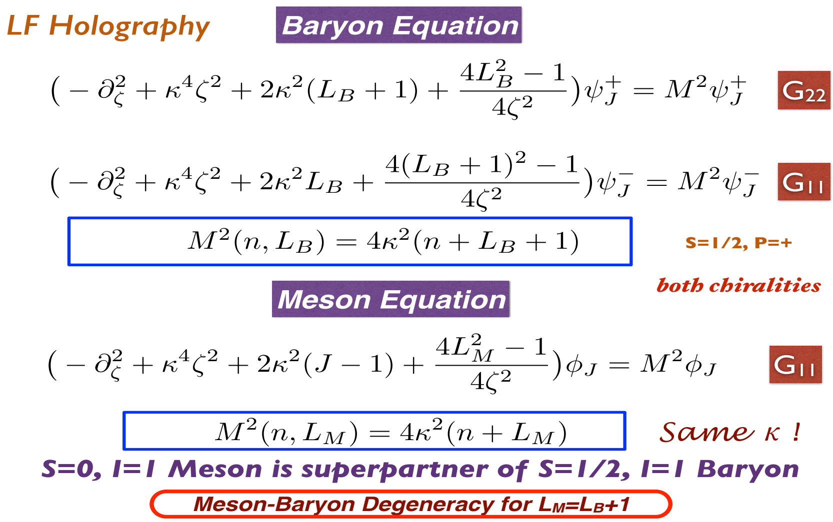
$L_M = L_B + 1$. See Fig. 4.2(A). Only one mass parameter κ appears; it sets the confinement and the hadron mass scale in the chiral limit, as well as the length scale which underlies hadron structure. "Light-Front Holography" not only predicts meson and baryon spectroscopy successfully, but also hadron dynamics: light-front wavefunctions, vector meson electroproduction, distribution amplitudes, form factors, and valence structure functions. The LF Schrödinger Equations for baryons and mesons derived from superconformal algebra are shown in Fig. 4.2. The comparison between the meson and baryon masses of the ρ/ω Regge trajectory with the spin-3/2 Δ trajectory is shown in Fig. 4.2(B). Superconformal algebra predicts the meson and baryon masses are identical if one identifies a meson with internal orbital angular momentum L_M with its superpartner baryon with $L_B = L_M - 1$. Notice that the twist $\tau = 2 + L_M = 3 + L_B$ of the interpolating operators for the meson and baryon superpartners are the same. Superconformal algebra also predicts that the LFWFs of the superpartners are identical, and thus they have identical dynamics, such their elastic and transition form factors. These features can be tested for spacelike form factors at JLab12.

4.3 The QCD Coupling at all Scales

The QCD running coupling can be defined [27] at all momentum scales from any perturbatively calculable observable, such as the coupling $\alpha^s_{g_1}(Q^2)$ which is defined from measurements of the Bjorken sum rule. At high momentum transfer, such "effective charges" satisfy asymptotic freedom, obey the usual pQCD renormalization group equations, and can be related to each other without scale ambiguity by commensurate scale relations [28]. The dilaton $e^{+\kappa^2 z^2}$ soft-wall modification of the AdS_5 metric, together with LF holography, predicts the functional behavior in the small Q^2 domain [29]: $\alpha^s_{g_1}(Q^2) = \pi e^{-Q^2/4\kappa^2}$. Measurements of $\alpha^s_{g_1}(Q^2)$ are remarkably consistent with this predicted Gaussian form. Deur, de Teramond, and I [30,29,26] have also shown how the parameter κ, which determines the mass scale of hadrons in the chiral limit, can be connected to the mass scale Λ_s controlling the evolution of the perturbative QCD coupling. The connection can be done for any choice of renormalization scheme, such as the $\overline{MS}$ scheme, as seen in Fig. 4.3. The relation between scales is obtained by matching at a scale Q_0^2 the nonperturbative behavior of the effective QCD coupling, as determined from light-front holography, to the perturbative QCD coupling with asymptotic freedom. The result of this perturbative/nonperturbative matching is an effective QCD coupling defined at all momenta.

4.4 Other Features of Light-Front QCD

There are a number of advantages if one uses LF Hamiltonian methods for perturbative QCD calculations. Unlike instant form, where one must sum $n!$ frame-dependent amplitudes, only the τ-ordered diagrams where every line has positive $k^+ = k^0 + k^z$ can contribute [31]. The number of nonzero amplitudes is also greatly reduced by noting that the total angular momentum projection



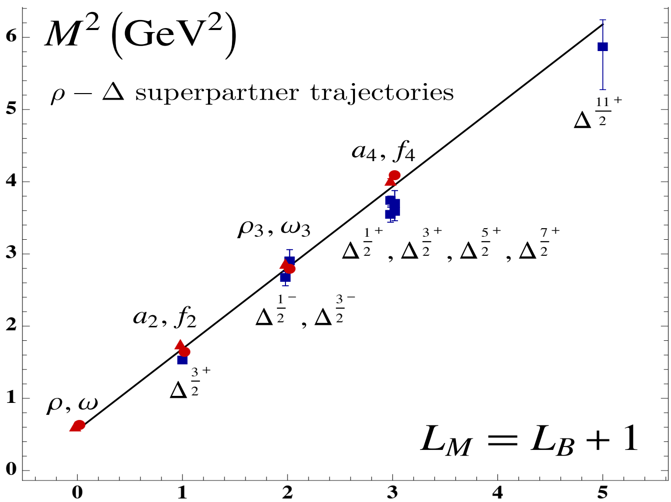
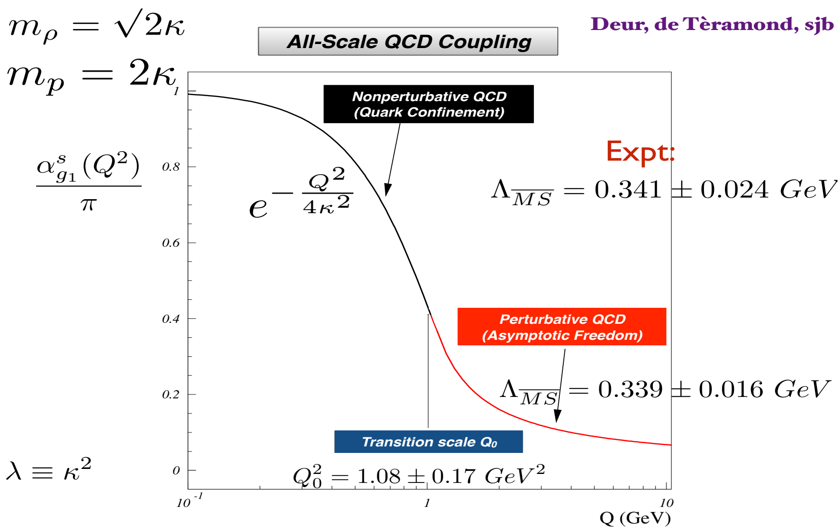


Fig. 4.2. (A). The LF Schrödinger equations for baryons and mesons for zero quark mass derived from the Pauli 2×2 matrix representation of superconformal algebra. The $\psi^\pm$ are the baryon quark-diquark LFWFs where the quark spin $S^z_q = \pm 1/2$ is parallel or antiparallel to the baryon spin $J^z = \pm 1/2$. The meson and baryon equations are identical if one identifies a meson with internal orbital angular momentum L_M with its superpartner baryon with $L_B = L_M - 1$. See ref. [22–24]. (B). Comparison of the ρ/ω meson Regge trajectory with the $J = 3/2$ Δ baryon trajectory. Superconformal algebra predicts the degeneracy of the meson and baryon trajectories if one identifies a meson with internal orbital angular momentum L_M with its superpartner baryon with $L_M = L_B + 1$. See refs. [22,23].



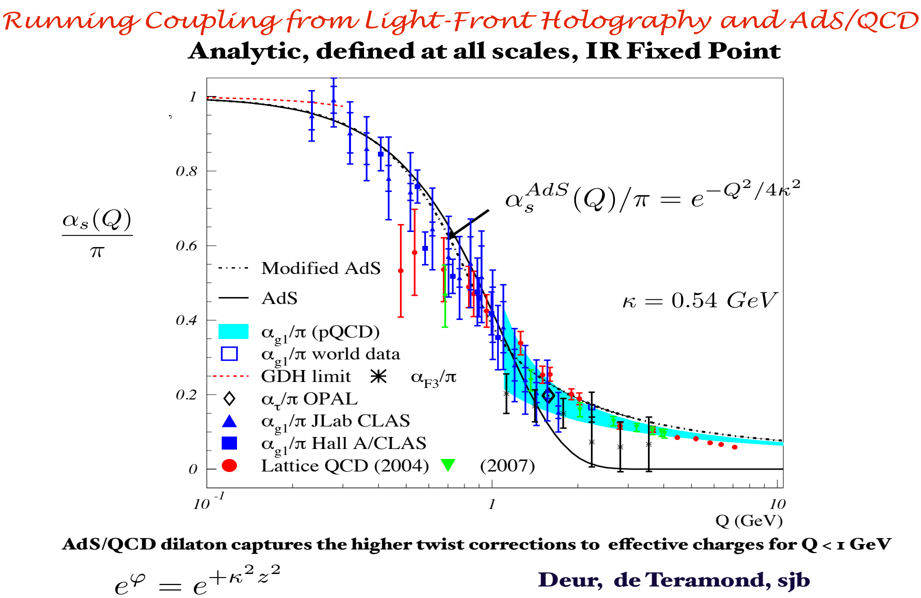


Fig. 4.3. (A) Prediction from LF Holography for the QCD Running Coupling $\alpha^s_{g_1}(Q^2)$. The magnitude and derivative of the perturbative and nonperturbative coupling are matched at the scale Q_0. This matching connects the perturbative scale $\Lambda_{\overline{MS}}$ to the nonpertubative scale κ which underlies the hadron mass scale. (B) Comparison of the predicted nonpertubative coupling with measurements of the effective charge $\alpha^s_{g_1}(Q^2)$ defined from the Bjorken sum rule. See ref. [26].

$J^z = \sum_i^{n-1} L_i^z + \sum_i^n S_i^z$ and the total P^+ are conserved at each vertex. In addition, in a renormalizable theory the change in orbital angular momentum is limited to $\Delta L^z = 0, \pm 1$ at each vertex. The calculation of a subgraph of any order in pQCD only needs to be done once; the result can be stored in a "history" file, since in LFPth the numerator algebra is independent of the process; the denominator changes, but only by a simple shift of the initial P^-. Loop integrations are three dimensional: $\int d^2\vec{k}_\perp \int_0^1 dx$. Renormalization can be done using the "alternate denominator" method which defines the required subtraction counterterms [32].

The LF vacuum in LF Hamlitonian theory is defined as the eigenstate of H_{LF} with lowest invariant mass. Since propagation with negative k^+ does not appear, there are no loop amplitudes in the LF vacuum – it is is thus trivial up to possible $k^+ = 0$ "zero" modes. The usual quark and gluon QCD vacuum condensates of the instant form =are replaced by physical effects, such as the running quark mass and the physics contained within the hadronic LFWFs in the hadronic domain. This is referred to as "in-hadron" condensates [33–35]. In the case of the Higgs theory, the traditional Higgs vacuum expectation value (VEV) is replaced by a zero mode analogous to a classical Stark or Zeeman field. [36] This again contrasts with the traditional view of the vacuum based on the instant form.

The instant-form vacuum, the lowest energy eigenstate of the instant-form Hamiltonian, is defined at one instant of time over all space; it is thus acausal and frame-dependent. It is usually argued that the QCD contribution to the cosmological constant – dark energy – is 10^{45} times larger that observed, and in the case of the Higgs theory, the Higgs VEV is argued to be 10^{54} larger than observed [37], estimates based on the loop diagrams of the acausal frame-dependent instant-form vacuum. However, the universe is observed within the causal horizon, not at a single instant of time. In contrast, the light-front vacuum provides a viable description of the visible universe [35]. Thus in agreement with Einstein, quantum effects do not contribute to the cosmological constant. In the case of the HIggs theory, the Higgs zero mode has no energy density, so again it gives no contribution to the cosmological constant. However, it is possible that if one solves the Higgs theory in a curved universe, the zero mode will be replaced with a field of nonzero curvature which could give a nonzero contribution.

4.5 Is the Momentum Sum Rule Valid for Nuclear Structure Functions?

Sum rules for DIS processes are analyzed using the operator product expansion of the forward virtual Compton amplitude, assuming it depends in the limit $Q^2 \to \infty$ on matrix elements of local operators such as the energy-momentum tensor. The moments of the structure function and other distributions can then be evaluated as overlaps of the target hadron's light-front wavefunction, as in the Drell-Yan-West formulae for hadronic form factors [4,38–40]. The real phase of the resulting DIS amplitude and its OPE matrix elements reflects the real phase of the stable target hadron's wavefunction.

The "handbag" approximation to deeply virtual Compton scattering also defines the "static" contribution [41,42] to the measured parton distribution functions

(PDF), transverse momentum distributions, etc. The resulting momentum, spin and other sum rules reflect the properties of the hadron's light-front wavefunction. However, final-state interactions which occur *after* the lepton scatters on the quark, can give non-trivial contributions to deep inelastic scattering processes at leading twist and thus survive at high Q^2 and high $W^2 = (q+p)^2$. For example, the pseudo-T-odd Sivers effect [43] is directly sensitive to the rescattering of the struck quark. Similarly, diffractive deep inelastic scattering involves the exchange of a gluon after the quark has been struck by the lepton [44]. In each case the corresponding DVCS amplitude is not given by the handbag diagram since interactions between the two currents are essential. These "lensing" corrections survive when both W^2 and Q^2 are large since the vector gluon couplings grow with energy. Part of the phase can be associated with a Wilson line as an augmented LFWF [45] which do not affect the moments.

The Glauber propagation of the vector system V produced by the diffractive DIS interaction on the nuclear front face and its subsequent inelastic interaction with the nucleons in the nuclear interior $V + N_b \to X$ occurs after the lepton interacts with the struck quark. Because of the rescattering dynamics, the DDIS amplitude acquires a complex phase from Pomeron and Regge exchange; thus final-state rescattering corrections lead to nontrivial "dynamical" contributions to the measured PDFs; i.e., they involve physics aspects of the scattering process itself [46]. The $I = 1$ Reggeon contribution to diffractive DIS on the front-face nucleon leads to flavor-dependent antishadowing [47,48]. This could explain why the NuTeV charged current measurement $\mu A \to \nu X$ scattering does not appear to show antishadowing in contrast to deep inelastic electron nucleus scattering as discussed in ref. [49]. Again the corresponding DVCS amplitude is not given by the handbag diagram since interactions between the two currents are essential.

Diffractive DIS is leading-twist and is the essential component of the two-step amplitude which causes shadowing and antishadowing of the nuclear PDF. It is important to analyze whether the momentum and other sum rules derived from the OPE expansion in terms of local operators remain valid when these dynamical rescattering corrections to the nuclear PDF are included. The OPE is derived assuming that the LF time separation between the virtual photons in the forward virtual Compton amplitude $\gamma^* A \to \gamma^* A$ scales as $1/Q^2$. However, the propagation of the vector system V produced by the diffractive DIS interaction on the front face and its inelastic interaction with the nucleons in the nuclear interior $V + N_b \to X$ are characterized by a longer LF time which scales as $1/W^2$. Thus the leading-twist multi-nucleon processes that produce shadowing and antishadowing in a nucleus are evidently not present in the $Q^2 \to \infty$ OPE analysis.

It should be emphasized that shadowing in deep inelastic lepton scattering on a nucleus involves nucleons at or near the front surface; i.e, the nucleons facing the incoming lepton beam. This geometrical orientation is not built into the frame-independent nuclear LFWFs used to evaluate the matrix elements of local currents. Thus the dynamical phenomena of leading-twist shadowing and antishadowing appear to invalidate the sum rules for nuclear PDFs. The same complications occur in the leading-twist analysis of deeply virtual Compton scattering $\gamma^* A \to \gamma^* A$ on a nuclear target.

4.6 Elimination of Renormalization Scale Ambiguities

The "Principle of Maximum Conformality", (PMC) [50] systematically eliminates the renormalization scale ambiguity in perturbative QCD calculations, order-by-order. The resulting scale-fixed predictions for physical observables using the PMC are *independent of the choice of renormalization scheme* – a key requirement of renormalization group invariance. The PMC predictions are also insensitive to the choice of the initial renormalization scale μ_0. The PMC sums all of the non-conformal terms associated with the QCD β function into the scales of the coupling at each order in pQCD. The resulting conformal series is free of renormalon resummation problems. The number of active flavors n_f in the QCD β function is also correctly determined at each order. The R_δ scheme – a generalization of t'Hooft's dimensional regularization. systematically identifies the nonconformal β contributions to any perturbative QCD series, thus allowing the automatic implementation of the PMC procedure [51]. The elimination of the renormalization scale ambiguity greatly increases the precision, convergence, and reliability of pQCD predictions. For example, PMC scale-setting has been applied to the pQCD prediction for $t\bar{t}$ pair production at the LHC, where subtle aspects of the renormalization scale of the three-gluon vertex and multi-gluon amplitudes, as well as large radiative corrections to heavy quarks at threshold play a crucial role. The large discrepancy of pQCD predictions with the $t\bar{t}$ forward-backward asymmetry measured at the Tevatron is significantly reduced from 3σ to approximately 1σ [52,53].

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5 Charged Fermion Masses and Mixing from a SU(3) Family Symmetry Model

A. Hernández-Galeana *

Departamento de Física, Escuela Superior de Física y Matemáticas, I.P.N.,
U. P. "Adolfo López Mateos". C. P. 07738, México, D.F., México

Abstract. Within the framework of a Beyond Standard Model augmented with a local $SU(3)$ family symmetry, we report an updated fit of parameters, which account for the known spectrum of quarks and charged lepton masses, and the quark mixing in a 4×4 non-unitary V_{CKM}. In this scenario, ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from Dirac See-saw mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlet vector-like fermions, U, D, E, N, with N a sterile neutrino. The $N_{L,R}$ sterile neutrinos allow the implementation of a 8×8 general See-saw Majorana neutrino mass matrix with four massless eigenvalues at tree level. Hence, light fermions, including light neutrinos obtain masses from loop radiative corrections mediated by the massive $SU(3)$ gauge bosons. $SU(3)$ family symmetry is broken spontaneously in two stages, whose hierarchy of scales yield an approximate $SU(2)$ global symmetry associated with the $Z_1, Y_1^{\pm}$ gauge boson masses of the order of 2 TeV. A global fit of parameters to include neutrino masses and lepton mixing is in progress.

Povzetek. Avtor poroča o prilagajanju vrednosti parametrov v razširjenem standardnem modelu z dodano družinsko simetrijo $SU(3)$, s katerim mu uspe pojasniti izmerjeni masni spekter kvarkov in leptonov ter neunitarni mešalni matriki za kvarke in leptone. V svojem scenariju doda običajnim fermionom še fermione (U, D, E, N), ki so šibki singleti $SU(2)_L$ z vektorskim značajem. Težki fermioni postanejo masivni že na drevesnem nivoju z Diracovim mehanizmom "see-saw". Sterilni nevtrini $N_{L,R}$ poskrbijo v nevtrinski masni matriki 8×8, na drevesnem nivoju, da so štiri lastne vrednosti enake 0. Maso lahkih kvarkov in leptonov, vključno z nevtrini, določajo bozonska polja z družinskimi kvantnimi števili v popravkih višjih redov. Avtor predvidi spontano zlomitev družinske simetrije $SU(3)$ v dveh korakih tako, da so mase $Z_1, Y_1^{\pm}$ umeritvenih bozonov $SU(2)$ reda 2 TeV.

5.1 Introduction

The origen of the hierarchy of fermion masses and mixing is one of the most important open problems in particle physics. Any attempt to account for this hierarchy introduce a mass generation mechanism which distinguish among the different Standard Model (SM) quarks and leptons.

After the discovery of the scalar Higgs boson on 2012, LHC has not found a conclusive evidence of new physics. However, there are theoretical motivations

* E-mail: albino@esfm.ipn.mx

to look for new particles in order to answer some open questions like; neutrino ossccillations, dark matter, stability of the Higgs mass against radiative corrections,,etc.

In this article, we address the problem of charged fermion masses and quark mixing within the framework of an extension of the SM introduced by the author in [1]. This Beyond Standard Model (BSM) proposal include a vector gauged $SU(3)$ family symmetry[1] commuting with the SM group and introduce a hierarchical massgeneration mechanism in which the light fermions obtain masses through loop radiative corrections, mediated by the massive bosons associated to the $SU(3)$ family symmetry that is spontaneously broken, while the masses of the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw"[3] mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlets U, D, E and N vector-like fermions. Due to the fact that these vector-like quarks do not couple to the W boson, the mixing of U and D vector-like quarks with the SM quarks gives rise to and extended 4×4 non-unitary CKM quark mixing matrix [4].

5.2 Model with $SU(3)$ flavor symmetry

5.2.1 Fermion content

Before "Electroweak Symmetry Breaking"(EWSB) all ordinary, "Standard Model"(SM) fermions remain massless, and the global symmetry in this limit of all quarks and leptons massless, including R-handed neutrinos, is:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R}$$

$$\supset SU(3)_{q_L+u_R+d_R+l_L+e_R+\nu_R} \equiv SU(3) \qquad (5.1)$$

We define the gauge symmetry group

$$G \equiv SU(3) \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \qquad (5.2)$$

where $SU(3)$ is the gaged family symmetry among families, eq.(5.1) , and G_{SM} is the "Standard Model" gauge group, with g_H, g_s, g and g' the corresponding coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under G as:

Ordinary Fermions: $q^o_{iL} = \begin{pmatrix} u^o_{iL} \\ d^o_{iL} \end{pmatrix}$, $l^o_{iL} = \begin{pmatrix} \nu^o_{iL} \\ e^o_{iL} \end{pmatrix}$, $Q = T_{3L} + \frac{1}{2}Y$

$$\Psi^o_q = (3, 3, 2, \frac{1}{3})_L = \begin{pmatrix} q^o_{1L} \\ q^o_{2L} \\ q^o_{3L} \end{pmatrix} \quad , \quad \Psi^o_l = (3, 1, 2, -1)_L = \begin{pmatrix} l^o_{1L} \\ l^o_{2L} \\ l^o_{3L} \end{pmatrix}$$

[1] See [1,2] and references therein for some other $SU(3)$ family symmetry model proposals.

$$\Psi_u^o = (3,3,1,\frac{4}{3})_R = \begin{pmatrix} u_R^o \\ c_R^o \\ t_R^o \end{pmatrix} \quad , \quad \Psi_d^o = (3,3,1,-\frac{2}{3})_R = \begin{pmatrix} d_R^o \\ s_R^o \\ b_R^o \end{pmatrix}$$

$$\Psi_e^o = (3,1,1,-2)_R = \begin{pmatrix} e_R^o \\ \mu_R^o \\ \tau_R^o \end{pmatrix}$$

where the last entry corresponds to the hypercharge Y, and the electric charge is defined by $Q = T_{3L} + \frac{1}{2}Y$. The model also includes two types of extra fermions:

Right Handed Neutrinos: $\Psi_{\nu_R}^o = (3,1,1,0)_R = \begin{pmatrix} \nu_{e_R} \\ \nu_{\mu_R} \\ \nu_{\tau_R} \end{pmatrix}$,

and the $SU(2)_L$ weak singlet vector-like fermions

Sterile Neutrinos: $N_L^o, N_R^o = (1,1,1,0)$,

The Vector Like quarks:

$$U_L^o, U_R^o = (1,3,1,\frac{4}{3}) \quad , \quad D_L^o, D_R^o = (1,3,1,-\frac{2}{3}) \tag{5.3}$$

and

The Vector Like electrons: $E_L^o, E_R^o = (1,1,1,-2)$

The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U\, \bar{U}_L^o\, U_R^o \,+\, M_D\, \bar{D}_L^o\, D_R^o \,+\, M_E\, \bar{E}_L^o\, E_R^o + h.c.\,, \tag{5.4}$$

and

$$m_D\, \bar{N}_L^o\, N_R^o \,+\, m_L\, \bar{N}_L^o\, (N_L^o)^c \,+\, m_R\, \bar{N}_R^o\, (N_R^o)^c \,+\, h.c \tag{5.5}$$

The above fermion content make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the usual form under the standard model group and in the fundamental 3-representation under the SU(3) family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies[5]. The $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. These vector like fermions play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses, together with the radiative corrections.

5.3 $SU(3)$ family symmetry breaking

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of SU(3), we introduce the flavon scalar fields: $\eta_i,\ i = 2,3,$

$$\eta_i = (3,1,1,0) = \begin{pmatrix} \eta^o_{i1} \\ \eta^o_{i2} \\ \eta^o_{i3} \end{pmatrix} \quad , \quad i = 2,3$$

and acquiring the "Vacuum ExpectationValues" (VEV's):

$$\langle \eta_2 \rangle^T = (0, \Lambda_2, 0) \quad , \quad \langle \eta_3 \rangle^T = (0, 0, \Lambda_3) \, . \tag{5.6}$$

The above scalar fields and VEV's break completely the $SU(3)$ flavor symmetry. The corresponding $SU(3)$ gauge bosons are defined in Eq.(5.20) through their couplings to fermions. Thus, the contribution to the horizontal gauge boson masses from Eq.(5.6) read

- η_2 : $\frac{g_{H_2}^2 \Lambda_2^2}{2}(Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2 Z_1 \frac{Z_2}{\sqrt{3}})$
- η_3 : $\frac{g_{H_3}^2 \Lambda_3^2}{2}(Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$

These two scalars in the fundamental representation is the minimal set of scalars to break down completely the $SU(3)$ *family symmetry*. Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms.

$$M_2^2 \, Y_1^+ Y_1^- + M_3^2 \, Y_2^+ Y_2^- + (M_2^2 + M_3^2) \, Y_3^+ Y_3^- + \frac{1}{2} M_2^2 \, Z_1^2 + \frac{1}{2} \frac{M_2^2 + 4M_3^2}{3} \, Z_2^2 - \frac{1}{2} (M_2^2) \frac{2}{\sqrt{3}} \, Z_1 \, Z_2 \tag{5.7}$$

$$M_2^2 = \frac{g_H^2 \Lambda_2^2}{2} \quad , \quad M_3^2 = \frac{g_H^2 \Lambda_3^2}{2} \quad , \quad y \equiv \frac{M_3}{M_2} = \frac{\Lambda_3}{\Lambda_2} \tag{5.8}$$

	Z_1	Z_2
Z_1	M_2^2	$-\frac{M_2^2}{\sqrt{3}}$
Z_2	$-\frac{M_2^2}{\sqrt{3}}$	$\frac{M_2^2+4M_3^2}{3}$

Table 5.1. $Z_1 - Z_2$ mixing mass matrix

Diagonalization of the $Z_1 - Z_2$ squared mass matrix yield the eigenvalues

$$M_-^2 = \frac{2}{3}\left(M_2^2 + M_3^2 - \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2} \right)_- \tag{5.9}$$

$$M_+^2 = \frac{2}{3}\left(M_2^2 + M_3^2 + \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2} \right)_+ \tag{5.10}$$

$$M_2^2\, Y_1^+ Y_1^- + M_3^2\, Y_2^+ Y_2^- + (M_2^2 + M_3^2)\, Y_3^+ Y_3^- + M_-^2\, \frac{Z_-^2}{2} + M_+^2\, \frac{Z_+^2}{2} \tag{5.11}$$

$$M_2^2\, Y_1^+ Y_1^- + M_2^2\, y^2\, Y_2^+ Y_2^- + M_2^2 (1 + y^2)\, Y_3^+ Y_3^- + M_2^2\, y_-\, \frac{Z_-^2}{2} + M_2^2\, y_+\, \frac{Z_+^2}{2} \tag{5.12}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} \tag{5.13}$$

$$\cos\phi\ \sin\phi = \frac{\sqrt{3}}{4}\, \frac{M_2^2}{\sqrt{M_2^4 + M_3^2 (M_3^2 - M_2^2)}}$$

Due to the $Z_1 - Z_2$ mixing, we diagonalize the propagators involving Z_1 and Z_2 gauge bosons according to Eq.(5.13)

$$Z_1 = \cos\phi\ Z_- - \sin\phi\ Z_+ \quad , \quad Z_2 = \sin\phi\ Z_- + \cos\phi\ Z_+$$

$$\langle Z_1 Z_1 \rangle = \cos^2\phi\ \langle Z_- Z_- \rangle + \sin^2\phi\ \langle Z_+ Z_+ \rangle$$

$$\langle Z_2 Z_2 \rangle = \sin^2\phi\ \langle Z_- Z_- \rangle + \cos^2\phi\ \langle Z_+ Z_+ \rangle$$

$$\langle Z_1 Z_2 \rangle = \sin\phi\ \cos\phi\ (\langle Z_- Z_- \rangle - \langle Z_+ Z_+ \rangle)$$

So, in the one loop diagrams contribution:

$$F_{Z_1} = \cos^2\phi\, F(M_-) + \sin^2\phi\, F(M_+)\ ,\ F_{Z_2} = \sin^2\phi\, F(M_-) + \cos^2\phi\, F(M_+)$$

Therefore, in the tree level single exchange diagrams

$$\frac{1}{M_{Z_1}^2} = \frac{\cos^2\phi}{M_-^2} + \frac{\sin^2\phi}{M_+^2} \quad , \quad \frac{1}{M_{Z_2}^2} = \frac{\sin^2\phi}{M_-^2} + \frac{\cos^2\phi}{M_+^2}$$

Notice that in the limit $y = \frac{M_3}{M_2} \gg 1$, $\sin\phi \to 0$, $\cos\phi \to 1$, *and there exist a* $SU(2)$ *global symmetry for the* $Z_1, Y_1^{\pm}$ *degenerated gauge boson masses.*

It is worth to emphasize that the hierarchy of scales in the SSB yields an approximate $SU(2)$ global symmetry in the spectrum of $SU(3)$ gauge boson masses. Actually this approximate $SU(2)$ symmetry plays the role of a custodial symmetry to suppress properly the tree level $\Delta F = 2$ processes mediated by the M_1 lower scale $Z_1,\ Y_1^1,\ Y_1^2$ horizontal gauge bosons.

5.4 Electroweak symmetry breaking

Recently ATLAS[6] and CMS[7] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. The electroweak symmetry breaking in the $SU(3)$ family symmetry model involves the introduction of two triplets of $SU(2)_L$ Higgs doublets, namely;

$$\Phi^u = (3, 1, 2, -1) = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_1^u \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_2^u \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_3^u \end{pmatrix} \quad , \quad \Phi^d = (3, 1, 2, +1) = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_1^d \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_2^d \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_3^d \end{pmatrix} ,$$

with the VEV?s

$$\Phi^u\rangle = \begin{pmatrix} \langle \Phi_1^u \rangle \\ \langle \Phi_2^u \rangle \\ \langle \Phi_3^u \rangle \end{pmatrix} \quad , \quad \langle \Phi^d \rangle = \begin{pmatrix} \langle \Phi_1^d \rangle \\ \langle \Phi_2^d \rangle \\ \langle \Phi_3^d \rangle \end{pmatrix} ,$$

where

$$\Phi_i^u\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{ui} \\ 0 \end{pmatrix} \quad , \quad \langle \Phi_i^d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{di} \end{pmatrix} .$$

The contributions from $\langle \Phi^u \rangle$ and $\langle \Phi^d \rangle$ yield the W and Z gauge boson masses and mixing with the $SU(3)$ gauge bosons

$$\frac{g^2}{4}\,(v_u^2+v_d^2)\,W^+W^- + \frac{(g^2+g'^2)}{8}\,(v_u^2+v_d^2)\,Z_o^2$$

$$+\frac{1}{4}\sqrt{g^2+g'^2}\,g_H\,Z_o\,\Big[\,(v_{1u}^2-v_{2u}^2-v_{1d}^2+v_{2d}^2)\,Z_1 + (v_{1u}^2+v_{2u}^2-2v_{3u}^2-v_{1d}^2-v_{2d}^2+2v_{3d}^2)\,\frac{Z_2}{\sqrt{3}}$$

$$+2\,(v_{1u}v_{2u}-v_{1d}v_{2d})\,\frac{Y_1^+ + Y_1^-}{\sqrt{2}} + 2\,(v_{1u}v_{3u}-v_{1d}v_{3d})\,\frac{Y_2^+ + Y_2^-}{\sqrt{2}} + 2\,(v_{2u}v_{3u}-v_{2d}v_{3d})\,\frac{Y_3^+ + Y_3^-}{\sqrt{2}}\Big]$$

$$+\frac{g_H^2}{4}\,\Big\{\frac{1}{2}\,(v_{1u}^2+v_{2u}^2+v_{1d}^2+v_{2d}^2)\,Z_1^2 + \frac{1}{2}\,(v_{1u}^2+v_{2u}^2+4v_{3u}^2+v_{1d}^2+v_{2d}^2+4v_{3d}^2)\,\frac{Z_2^2}{3}$$

$$+(v_{1u}^2+v_{2u}^2+v_{1d}^2+v_{2d}^2)\,Y_1^+Y_1^- + (v_{1u}^2+v_{3u}^2+v_{1d}^2+v_{3d}^2)\,Y_2^+Y_2^- + (v_{2u}^2+v_{3u}^2+v_{2d}^2+v_{3d}^2)\,Y_3^+Y_3^-$$

$$+(v_{1u}^2-v_{2u}^2+v_{1d}^2-v_{2d}^2)\,Z_1\,\frac{Z_2}{\sqrt{3}} + (v_{2u}v_{3u}+v_{2d}v_{3d})\,(Y_1^+Y_2^- + Y_1^-Y_2^+) + (v_{1u}v_{2u}+v_{1d}v_{2d})\,(Y_2^+Y_3^- + Y_2^-Y_3^+) + (v_{1u}v_{3u}+v_{1d}v_{3d})\,(Y_1^+Y_3^+ + Y_1^-Y_3^-)$$

$$+2\,(v_{1u}v_{2u}+v_{1d}v_{2d})\,\frac{Z_2}{\sqrt{3}}\,\frac{Y_1^+ + Y_1^-}{\sqrt{2}} + (v_{1u}v_{3u}+v_{1d}v_{3d})\,(Z_1-\frac{Z_2}{\sqrt{3}})\,\frac{Y_2^+ + Y_2^-}{\sqrt{2}} - (v_{2u}v_{3u}+v_{2d}v_{3d})\,(Z_1+\frac{Z_2}{\sqrt{3}})\,\frac{Y_3^+ + Y_3^-}{\sqrt{2}}\Big\} \quad (5.14)$$

$v_u^2 = v_{1u}^2+v_{2u}^2+v_{3u}^2\,,\, v_d^2 = v_{1d}^2+v_{2d}^2+v_{3d}^2$. Hence, if we define as usual $M_W = \frac{1}{2}gv$, we may write $v = \sqrt{v_u^2+v_d^2} \approx 246$ GeV.

$$Y_j^1 = \frac{Y_j^+ + Y_j^-}{\sqrt{2}} \quad , \quad Y_j^{\pm} = \frac{Y_j^1 \mp iY_j^2}{\sqrt{2}} \quad (5.15)$$

The mixing of Z_o neutral gauge boson with the $SU(3)$ gauge bosons modify the couplings of the standard model Z boson with the ordinary quarks and leptons

5.5 Fermion masses

5.5.1 Dirac See-saw mechanisms

Now we describe briefly the procedure to get the masses for fermions. The analysis is presented explicitly for the charged lepton sector, with a completely analogous procedure for the u and d quarks and Dirac neutrinos. With the fields of particles introduced in the model, we may write the gauge invariant Yukawa couplings, as

$$h\,\bar{\psi}_l^o\,\Phi^d\,E_R^o\,+\,h_2\,\bar{\psi}_e^o\,\eta_2\,E_L^o\,+\,h_3\,\bar{\psi}_e^o\,\eta_3\,E_L^o\,+\,M\,\bar{E}_L^o\,E_R^o\,+h.c \tag{5.16}$$

where M is a free mass parameter (because its mass term is gauge invariant) and h, h_2 and h_3 are Yukawa coupling constants. When the involved scalar fields acquire VEV's we get, in the gauge basis ${\psi_{L,R}^o}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms $\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + h.c$, where

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & h\,v_1 \\ 0 & 0 & 0 & h\,v_2 \\ 0 & 0 & 0 & h\,v_3 \\ 0 & h_2\Lambda_2 & h_3\Lambda_3 & M \end{pmatrix} \equiv \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & M \end{pmatrix} . \tag{5.17}$$

Notice that $\mathcal{M}^o$ has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call $\mathcal{M}^o$ a **"Dirac See-saw"** mass matrix. $\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi_{L,R}^o = V_{L,R}^o\,\chi_{L,R}$. The orthogonal matrices V_L^o and V_R^o are obtained explicitly in the Appendix 5.9 A. From V_L^o and V_R^o, and using the relationships defined in this Appendix, one computes

$${V_L^o}^T \mathcal{M}^o\; V_R^o = Diag(0, 0, -\lambda_3, \lambda_4) \tag{5.18}$$

$${V_L^o}^T \mathcal{M}^o {\mathcal{M}^o}^T\; V_L^o = {V_R^o}^T {\mathcal{M}^o}^T \mathcal{M}^o\; V_R^o = Diag(0, 0, \lambda_3^2, \lambda_4^2)\,. \tag{5.19}$$

where λ_3^2 and λ_4^2 are the nonzero eigenvalues defined in Eqs.(5.53-5.54), λ_4 being the fourth heavy fermion mass, and λ_3 of the order of the top, bottom and tau mass for u, d and e fermions, respectively. We see from Eqs.(5.18,5.19) that at tree level the See-saw mechanism yields two massless eigenvalues associated to the light fermions:

5.6 One loop contribution to fermion masses

Subsequently, the masses for the light fermions arise through one loop radiative corrections. After the breakdown of the electroweak symmetry we can construct the generic one loop mass diagram of Fig. 5.1. Internal fermion line in this diagram represent the Dirac see-saw mechanism implemented by the couplings in Eq.(5.16). The vertices read from the SU(3) flavor symmetry interaction Lagrangian

$$\begin{aligned} i\mathcal{L}_{int} = \frac{g_H}{2}\,(\bar{e^o}\gamma_\mu e^o - \bar{\mu^o}\gamma_\mu \mu^o)\,Z_1^\mu + \frac{g_H}{2\sqrt{3}}\,(\bar{e^o}\gamma_\mu e^o + \bar{\mu^o}\gamma_\mu \mu^o - 2\bar{\tau^o}\gamma_\mu \tau^o)\,Z_2^\mu \\ + \frac{g_H}{\sqrt{2}}\left(\bar{e^o}\gamma_\mu \mu^o Y_1^+ + \bar{e^o}\gamma_\mu \tau^o Y_2^+ + \bar{\mu^o}\gamma_\mu \tau^o Y_3^+ + h.c.\right)\,, \end{aligned} \tag{5.20}$$

where g_H is the SU(3) coupling constant, Z_1, Z_2 and Y_i^j , $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons. The crosses in the internal fermion line mean tree level mixing, and the mass M generated by the Yukawa couplings in Eq.(5.16) after the scalar

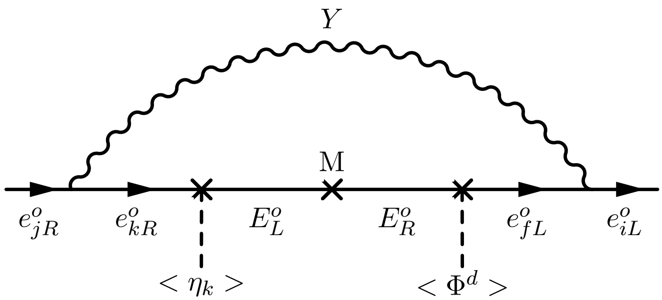

Fig. 5.1. Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

fields get VEV's. The one loop diagram of Fig. 1 gives the generic contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m^o_k\ (V^o_L)_{ik} (V^o_R)_{jk} f(M_Y, m^o_k) \qquad , \qquad \alpha_H \equiv \frac{g^2_H}{4\pi} \tag{5.21}$$

where M_Y is the gauge boson mass, c_Y is a factor coupling constant, Eq.(5.20), $m^o_3 = -\sqrt{\lambda^2_3}$ and $m^o_4 = \lambda_4$ are the See-saw mass eigenvalues, Eq.(5.18), and $f(x,y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$. Using the results of Appendix 5.9, we compute

$$\sum_{k=3,4} m^o_k\ (V^o_L)_{ik} (V^o_R)_{jk} f(M_Y, m^o_k) = \frac{a_i\ b_j\ M}{\lambda^2_4 - \lambda^2_3}\ F(M_Y)\,, \tag{5.22}$$

$i = 1, 2, 3\,,\ j = 2, 3$, and $F(M_Y) \equiv \frac{M^2_Y}{M^2_Y - \lambda^2_4} \ln \frac{M^2_Y}{\lambda^2_4} - \frac{M^2_Y}{M^2_Y - \lambda^2_3} \ln \frac{M^2_Y}{\lambda^2_3}$. Adding up all the one loop SU(3) gauge boson contributions, we get the mass terms $\bar{\psi^o_L} \mathcal{M}^o_1 \psi^o_R + h.c.$,

$$\mathcal{M}^o_1 = \begin{pmatrix} D_{11} & D_{12} & D_{13} & 0 \\ 0 & D_{22} & D_{23} & 0 \\ 0 & D_{32} & D_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{\alpha_H}{\pi}\,, \tag{5.23}$$

$$\begin{aligned}
D_{11} &= \mu_{11}\left(\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12} + F_m\right) + \frac{1}{2}(\mu_{22}F_1 + \mu_{33}F_2) \\
D_{12} &= \mu_{12}\left(-\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12}\right) \\
D_{13} &= -\mu_{13}\left(\frac{F_{Z_2}}{6} + F_m\right) \\
D_{22} &= \mu_{22}\left(\frac{F_{Z_1}}{4} + \frac{F_{Z_2}}{12} - F_m\right) + \frac{1}{2}(\mu_{11}F_1 + \mu_{33}F_3) \\
D_{23} &= -\mu_{23}\left(\frac{F_{Z_2}}{6} - F_m\right) \\
D_{32} &= -\mu_{32}\left(\frac{F_{Z_2}}{6} - F_m\right) \\
D_{33} &= \mu_{33}\frac{F_{Z_2}}{3} + \frac{1}{2}(\mu_{11}F_2 + \mu_{22}F_3)\,,
\end{aligned}$$

Here,

$$F_1 \equiv F(M_{Y_1}) \quad , \quad F_2 \equiv F(M_{Y_2}) \quad , \quad F_3 \equiv F(M_{Y_3}) \quad , \quad F_{Z_1} \equiv F(M_{Z_1}) \quad , \quad F_{Z_2} \equiv F(M_{Z_2})$$

$$M_{Y_1}^2 = M_2^2 \quad , \quad M_{Y_2}^2 = M_3^2 \quad , \quad M_{Y_3}^2 = M_2^2 + M_3^2$$

$$F_m = \frac{\cos\phi \sin\phi}{2\sqrt{3}} \left[F(M_-) - F(M_+) \right]$$

with M_2, M_3, M_{Z_1} and M_{Z_2} the horizontal boson masses, Eqs.(5.8-5.10),

$$\mu_{ij} = \frac{a_i\, b_j\, M}{\lambda_4^2 - \lambda_3^2} = \frac{a_i\, b_j}{a\, b} \lambda_3\, c_\alpha\, c_\beta\ , \tag{5.24}$$

and $c_\alpha \equiv \cos\alpha$, $c_\beta \equiv \cos\beta$, $s_\alpha \equiv \sin\alpha$, $s_\beta \equiv \sin\beta$, as defined in the Appendix 5.9, Eq.(5.55). Therefore, up to one loop corrections we obtain the fermion masses

$$\bar{\psi}_L^o \mathcal{M}^o\, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o\, \psi_R^o = \bar{\chi_L}\, \mathcal{M}\, \chi_R\ , \tag{5.25}$$

with $\mathcal{M} \equiv \left[Diag(0, 0, -\lambda_3, \lambda_4) + V_L^{o\,T} \mathcal{M}_1^o\, V_R^o \right]$.

Using V_L^o, V_R^o from Eqs.(5.51-5.52) we get the mass matrix:

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta\, m_{13} & s_\beta\, m_{13} \\ m_{21} & m_{22} & c_\beta\, m_{23} & s_\beta\, m_{23} \\ c_\alpha\, m_{31} & c_\alpha\, m_{32} & (-\lambda_3 + c_\alpha c_\beta\, m_{33}) & c_\alpha s_\beta\, m_{33} \\ s_\alpha\, m_{31} & s_\alpha\, m_{32} & s_\alpha c_\beta\, m_{33} & (\lambda_4 + s_\alpha s_\beta\, m_{33}) \end{pmatrix} , \tag{5.26}$$

where

$$m_{11} = \frac{1}{2}\frac{a_2}{a'}\Pi_1 \quad , \quad m_{12} = -\tfrac{1}{2}\tfrac{a_1 b_3}{a' b}(\Pi_2 - 6\mu_{22}F_m) \tag{5.27}$$

$$m_{21} = \frac{1}{2}\frac{a_1 a_3}{a' a}\Pi_1 \quad , \quad m_{31} = \tfrac{1}{2}\tfrac{a_1}{a}\Pi_1 \tag{5.28}$$

$$m_{13} = -\frac{1}{2}\frac{a_1 b_2}{a' b}[\Pi_2 + 2(2\frac{b_3^2}{b_2^2} - 1)\mu_{22}F_m] \tag{5.29}$$

$$m_{22} = \frac{1}{2}\frac{a_3 b_3}{a\, b}\left[\frac{a_2}{a'}(\Pi_2 - 6\mu_{22}F_m) + \frac{a' b_2}{a_3 b_3}(\Pi_3 + \Delta) \right] \tag{5.30}$$

$$m_{23} = \frac{1}{2}\frac{a_3 b_3}{a\, b}\left[\frac{a_2 b_2}{a' b_3}(\Pi_2 + 2(2\frac{b_3^2}{b_2^2} - 1)\mu_{22}F_m) - \frac{a'}{a_3}(\Pi_3 - \frac{b_2^2}{b_3^2}\Delta + 2\frac{b^2}{b_3^2}\mu_{33}F_m)\right] \quad (5.31)$$

$$m_{32} = \frac{1}{2}\frac{a_3 b_3}{a\, b}\left[\frac{a_2}{a_3}(\Pi_2 - 6\mu_{22}F_m) - \frac{b_2}{b_3}(\Pi_3 - \frac{a'^2}{a_3^2}\Delta - 2\frac{a^2}{a_3^2}\mu_{33}F_m)\right] \quad (5.32)$$

$$m_{33} = \frac{1}{2}\frac{a_3 b_3}{a\, b}\left[\frac{a_2 b_2}{a_3 b_3}(\Pi_2 - 2\mu_{22}F_m) + \Pi_3 + \frac{a'^2 b_2^2}{a_3^2 b_3^2}\Delta - \frac{1}{3}\frac{a^2 b^2}{a_3^2 b_3^2}\mu_{33}F_{Z_2} + 2(\frac{b_2^2}{b_3^2} + 2\frac{a_2^2}{a_3^2} - \frac{a'^2}{a_3^2})\mu_{33}F_m\right] \quad (5.33)$$

$$\Pi_1 = \mu_{22}F_1 + \mu_{33}F_2 \quad , \quad \Pi_2 = \mu_{22}F_{Z_1} + \mu_{33}F_3$$

$$\Pi_3 = \mu_{22}F_3 + \mu_{33}F_{Z_2} \quad , \quad \Delta = \frac{1}{2}\mu_{33}(F_{Z_2} - F_{Z_1}) \quad (5.34)$$

Notice that the m_{ij} mass terms depend just on the ratio $\frac{a_i}{a_j}$ and $\frac{b_i}{b_j}$ of the tree level parameters.

$$a' = \sqrt{a_1^2 + a_2^2}\ , \quad a = \sqrt{a'^2 + a_3^2}\ , \quad b = \sqrt{b_2^2 + b_3^2}\ , \quad (5.35)$$

The diagonalization of $\mathcal{M}$, Eq.(5.26) gives the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V_{L,R}^{(1)}\,\Psi_{L,R}$; $\bar{\chi}_L\,\mathcal{M}\,\chi_R = \bar{\Psi}_L\, {V_L^{(1)}}^T \mathcal{M}\, V_R^{(1)}\,\Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$${V_L^{(1)}}^T \mathcal{M}\,\mathcal{M}^T\, V_L^{(1)} = {V_R^{(1)}}^T \mathcal{M}^T\,\mathcal{M}\, V_R^{(1)} = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\ , \quad (5.36)$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$\psi_L^o = V_L^o\, V_L^{(1)}\,\Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\,\Psi_R \quad (5.37)$$

5.6.1 Quark $(V_{CKM})_{4\times4}$ and Lepton $(U_{PMNS})_{4\times8}$ mixing matrices

Within this SU(3) family symmetry model, the transformation from massless to physical mass fermion eigenfields for quarks and charged leptons is

$$\psi_L^o = V_L^o \, V_L^{(1)} \, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o \, V_R^{(1)} \, \Psi_R \,,$$

Recall now that vector like quarks, Eq.(5.3), are $SU(2)_L$ weak singlets, and hence, they do not couple to W boson in the interaction basis. In this way, the interaction of L-handed up and down quarks; ${f_{uL}^o}^T = (u^o, c^o, t^o)_L$ and ${f_{dL}^o}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}} \, \bar{f^o}_{uL} \gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}} \, \bar{\Psi}_{uL} \, [(V_{uL}^o \, V_{uL}^{(1)})_{3\times4}]^T \, (V_{dL}^o \, V_{dL}^{(1)})_{3\times4} \, \gamma_\mu \Psi_{dL} \, W^{+\mu} \,, \tag{5.38}$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V_{uL}^o \, V_{uL}^{(1)})_{3\times4}]^T \, (V_{dL}^o \, V_{dL}^{(1)})_{3\times4} \tag{5.39}$$

5.7 Numerical results

To illustrate the spectrum of masses and mixing, let us consider the following fit of space parameters at the M_Z scale [8]

Taking the input values

$$M_1 = 2\,\text{TeV} \quad , \quad M_2 = 2000\,\text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.2$$

for the M_1, M_2 horizontal boson masses, Eq.(5.8), and the SU(3) coupling constant, respectively, and the ratio of the electroweak VEV's: v_{iu} from Φ^u and v_{id} from Φ^d,

$$v_{1u} = 0 \quad , \quad \frac{v_{2u}}{v_{3u}} = 0.1$$

$$\frac{v_{1d}}{v_{2d}} = 0.23257 \quad , \quad \frac{v_{2d}}{v_{3d}} = 0.08373$$

5.7.1 Quark masses and mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 0. \\ 0 & 0 & 0 & 29834. \\ 0 & 0 & 0 & 298340. \\ 0 & 1.49495\times10^7 & -730572. & 1.58511\times10^7 \end{pmatrix} \text{MeV}\,, \tag{5.40}$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} 1.38 & 0. & 0. & 0. \\ 0. & -532.587 & -2587.14 & -2442.42 \\ 0. & 7064.64 & -172017. & 31927.1 \\ 0. & 70.6499 & 338.204 & 2.18023 \times 10^7 \end{pmatrix} \text{MeV} \tag{5.41}$$

and the u-quark masses

$$(\, m_u \,,\, m_c \,,\, m_t \,,\, M_U \,) = (\, 1.38 \,,\, 638.22 \,,\, 172181 \,,\, 2.18023 \times 10^7 \,)\, \text{MeV} \tag{5.42}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 13375.7 \\ 0 & 0 & 0 & 57510.3 \\ 0 & 0 & 0 & 686796. \\ 0 & 723708. & -37338.1 & 6.89219 \times 10^7 \end{pmatrix} \text{MeV} \tag{5.43}$$

$$\mathcal{M}_d = \begin{pmatrix} 2.82461 & 0.0338487 & -0.656039 & -0.00689715 \\ 0.65453 & -25.1814 & -217.369 & -2.28527 \\ 0.0562685 & 423.166 & -2820.62 & 46.5371 \\ 0.000562713 & 4.23187 & 44.2671 & 6.89291 \times 10^7 \end{pmatrix} \text{MeV} \tag{5.44}$$

$$(\, m_d \,,\, m_s \,,\, m_b \,,\, M_D \,) = (\, 2.82368 \,,\, 57.0005 \,,\, 2860 \,,\, 6.89291 \times 10^7 \,)\, \text{MeV} \tag{5.45}$$

and the quark mixing

$$V_{CKM} = \begin{pmatrix} 0.97362 & 0.225277 & -0.0362485 & 0.000194044 \\ -0.226684 & 0.973105 & -0.040988 & -0.000310055 \\ 0.0260403 & 0.0481125 & 0.998387 & -0.00999333 \\ -0.000234396 & -0.000826552 & -0.011432 & 0.000114632 \end{pmatrix} \tag{5.46}$$

5.7.2 Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 37956.9 \\ 0 & 0 & 0 & 189784. \\ 0 & 0 & 0 & 1.93543 \times 10^6 \\ 0 & 548257. & -30307.4 & 1.94497 \times 10^8 \end{pmatrix} \text{MeV} \tag{5.47}$$

$$\mathcal{M}_e = \begin{pmatrix} -0.486368 & -0.00536888 & 0.0971221 & 0.000274163 \\ -0.0967909 & -34.7536 & -250.305 & -0.706579 \\ -0.0096786 & 485.768 & -1661.27 & 10.8107 \\ -0.0000967909 & 4.85792 & 38.2989 & 1.94507 \times 10^8 \end{pmatrix} \text{MeV} \quad (5.48)$$

fit the charged lepton masses:

$$(m_e \,,\, m_\mu \,,\, m_\tau \,,\, M_E) = (0.486095 \,,\, 102.7 \,,\, 1746.17 \,,\, 3.15956 \times 10^8\,)\,\text{MeV}$$

and the charged lepton mixing

$$V^o_{e\,L}\, V^{(1)}_{e\,L} = \begin{pmatrix} 0.973942 & 0.221206 & 0.050052 & 0.000194 \\ -0.226798 & 0.949931 & 0.214927 & 0.0008342 \\ -2.90427 \times 10^{-6} & -0.220675 & 0.975296 & 0.009963 \\ 2.62189 \times 10^{-7} & 0.0013632 & -0.009906 & 0.99995 \end{pmatrix} \quad (5.49)$$

5.8 Conclusions

We reported recent numerical analysis on charged fermion masses and mixing within a BSM with a local $SU(3)$ family symmetry, which combines tree level "Dirac See-saw" mechanisms and radiative corrections to implement a successful hierarchical mass generation mechanism for quarks and charged leptons.

In section 5.7 we show a parameter space region where this scenario account for the hierarchical spectrum of ordinary quarks and charged lepton masses, and the quark mixing in a non-unitary $(V_{CKM})_{4\times4}$ within allowed values[2] reported in PDG 2014 [9].

Let me point out here that the solutions for fermion masses and mixing reported in section 5.7 suggest that the dominant contribution to Electroweak Symmetry Breaking comes from the weak doublets which couple to the third family.

It is worth to comment here that the symmetries and the transformation of the fermion and scalar fields, all together, forbid tree level Yukawa couplings between ordinary standard model fermions. Consequently, the flavon scalar fields introduced to break the symmetries: Φ^u, Φ^d, η_2 *and* η_3, *couple only ordinary fermions to their corresponding vector like fermion at tree level. Thus, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which as is shown in* $(V_{CKM})_{4\times4}$, *Eq.(5.46), may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.*

[2] except $(V_{CKM})_{13}$ and $(V_{CKM})_{31}$

5.9 Appendix: Diagonalization of the generic Dirac See-saw mass matrix

$$\mathcal{M}^o = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ 0 & b_2 & b_3 & c \end{pmatrix} \tag{5.50}$$

Using a biunitary transformation $\psi_L^o = V_L^o \, \chi_L$ and $\psi_R^o = V_R^o \, \chi_R$ to diagonalize $\mathcal{M}^o$, the orthogonal matrices V_L^o and V_R^o may be written explicitly as

$$V_L^o = \begin{pmatrix} \frac{a_2}{a'} & \frac{a_1 a_3}{a' a} & \frac{a_1}{a}\cos\alpha & \frac{a_1}{a}\sin\alpha \\ -\frac{a_1}{a'} & \frac{a_2 a_3}{a' a} & \frac{a_2}{a}\cos\alpha & \frac{a_2}{a}\sin\alpha \\ 0 & -\frac{a'}{a} & \frac{a_3}{a}\cos\alpha & \frac{a_3}{a}\sin\alpha \\ 0 & 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \tag{5.51}$$

$$V_R^o = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{b_3}{b} & \frac{b_2}{b}\cos\beta & \frac{b_2}{b}\sin\beta \\ 0 & -\frac{b_2}{b} & \frac{b_3}{b}\cos\beta & \frac{b_3}{b}\sin\beta \\ 0 & 0 & -\sin\beta & \cos\beta \end{pmatrix} \tag{5.52}$$

$$\lambda_3^2 = \frac{1}{2}\left(B - \sqrt{B^2 - 4D}\right) \quad , \quad \lambda_4^2 = \frac{1}{2}\left(B + \sqrt{B^2 - 4D}\right) \tag{5.53}$$

are the nonzero eigenvalues of $\mathcal{M}^o \mathcal{M}^{o\,T}$ ($\mathcal{M}^{o\,T}\mathcal{M}^o$), and

$$B = a^2 + b^2 + c^2 = \lambda_3^2 + \lambda_4^2 \quad , \quad D = a^2 b^2 = \lambda_3^2 \lambda_4^2 \ , \tag{5.54}$$

$$\begin{aligned} \cos\alpha &= \sqrt{\frac{\lambda_4^2 - a^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\alpha = \sqrt{\frac{a^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \ , \\ \cos\beta &= \sqrt{\frac{\lambda_4^2 - b^2}{\lambda_4^2 - \lambda_3^2}} \quad , \quad \sin\beta = \sqrt{\frac{b^2 - \lambda_3^2}{\lambda_4^2 - \lambda_3^2}} \ . \end{aligned} \tag{5.55}$$

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6 Gravitational Effects for Dirac Particles ⋆

U.D. Jentschura and J.H. Noble

Department of Physics, Missouri University of
Science and Technology, Rolla, Missouri 65409, USA

Abstract. We present an update on recent advances in the theory of Dirac particles in curved space-times. The basic formulation behind the covariant coupling of the Dirac bispinor to space-time geometry is briefly reviewed including the appropriate covariant action principle. A number of central-field problems have recently been analyzed; all of these depend on a concrete, explicit evaluation of the spin connection matrices for particular space-time geometries; the relevant results are discussed. The generalization of the formalism to tachyonic spin-1/2 particles is rather straightforward and allows the identification of the leading interaction terms for high-energy tachyons, which approach the light cone. The combination of quantum electrodynamics on curved space-time backgrounds may seem like a far-fetched field of research, but recent claims in the field have shaken the foundations of fundamental principles of general relativity. We show that a careful consideration of the vacuum polarization integral, with a gravitational effective mass, restores the validity of the weak equivalence principle in deep gravitational potentials.

Povzetek. Poročava o nedavnem napredku v teoriji Diracovih delcev v ukrivljenem prostoru-času. Na kratko predstaviva kovariantno sklopitev Diracovega bispinorja z geometrijo prostor-časa s in ustrezno kovariantno akcijo. Na kratko predstaviva nove dosžke pri iskanju rešitev za Diracov delec v več različnih centralnih potencialih, ki so se pojavili v zadnjem času. Vsi uporabijo matrike spinskih povezav. Ta pristop posplošiva na tahionske delce s spinom 1/2, kar nama omogoči prepoznati vodilne člene interakcije za skoraj brezmasne tahione na svetlobnemu stožcu. Povezava kvantne elektrodinamike v ukrivljenem prostoru-času se zdi zanimiva ob trditvah o morebitni neveljavnosti splošne teorije relativnosti. Vendar s skrbno obravnavo polarizacije vakuuma (z gravitacijsko efektivno maso) pokaževa veljavnost načela šibke ekvivalence v globokih gravitacijskih potencialih in s tem zmotnost teh trditev.

6.1 Introduction

The coupling of a spin-1/2 particle to the gauge fields via the covariant derivative, within the Dirac equation, has been central to the formulation of the Standard Model of Elementary Interactions, and to the understanding of the properties of antiparticles. It is much less common wisdom how to couple a Dirac particle to a curved space-time geometry. Some naive guesses fail. In flat space, the Clifford algebra of Dirac matrices was formulated [1,2] to fulfill the fundamental anticommutator relation

$$\{\gamma^{\mu}, \gamma^{\nu}\} = \eta^{\mu\nu} \tag{6.1}$$

⋆ Talk delivered by U.D. Jentschura

where $\eta^{\mu\nu}$ is the flat space-time metric $\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. In curved space, this relation has to be generalized to

$$\{\overline{\gamma}^{\mu}(x), \overline{\gamma}^{\nu}(x)\} = \overline{g}^{\mu\nu}(x)\,, \tag{6.2}$$

where $\overline{g}^{\mu\nu}(x)$ is the curved space-time metric, and the Dirac matrices become coordinate-dependent. (We choose to denote the curved-space metric by $\overline{g}$ in order to avoid confusion with the flat-space counterpart, which is usually denoted by g in elementary physics.)

However, one would be mistaken to simply replace $\gamma^{\mu} \to \overline{\gamma}^{\mu}$ in the Dirac equation in order to couple the Dirac particle to space-time curvature, or, to simple insert the gravitational potential $V = -G m_1 m_2/r$ into the Dirac equation by hand. Both approaches fail because they are not covariant with respect to Lorentz transformations of curved space-time. In particular, the simple insertion of the gravitational potential into the Dirac equation would lead to a different equation of motion for the Dirac particle under a change of the space-time coordinates, which is unacceptable. The requirement of covariance under local Lorentz transformations leads to the definition of the spin connection matrices, and to the covariant derivative for a spinor in curved space time.

Here, we briefly review some recent works on related topics, which are based on the concrete evaluation of the spin connection matrices for particular space-time geometries, and discuss an application to quantum electrodynamics in curved space-time, namely, the gravitational correction to vacuum polarization. We use units with $\hbar = c = \epsilon_0 = 1$ and employ the standard representation for the Dirac matrices [3,4] (the "standard" speed of light is sometimes denoted as c_0, for reasons apparent from the context of the discussion on conceivable tiny deviations induced by quantum phenomena).

6.2 Dirac Particles and Curved Space–Time

In order to fix ideas [5], let us recall that the vierbein e^{A}_{μ} (the "square root of the metric") describes the connection of the curved-space and flat-space metrics,

$$\overline{g}_{\mu\nu}(x) = e^{A}_{\mu}\, e^{B}_{\nu}\, \eta_{AB}\,, \qquad \eta_{AB} = \mathrm{diag}(1,-1,-1,-1)\,. \tag{6.3}$$

The completeness of the vierbein implies that both the "local" (nonholonomic) as well as the "global" (holonomic) indices can be raised and lowered using the metric(s) η and $\overline{g}$. In particular, one has

$$e^{\mu}_{A}\, e_{\mu B} = \eta_{AB}\,, \qquad e^{A}_{\mu}\, e_{\nu A} = \overline{g}_{\mu\nu}(x) \tag{6.4}$$

The connection of the flat-space ($\widetilde{\gamma}$) and curved-space ($\overline{\gamma}$) Dirac matrices is given as follows,

$$\overline{\gamma}_{\mu}(x) = e^{A}_{\mu}\, \widetilde{\gamma}_{A}\,, \qquad \{\widetilde{\gamma}^{A}, \widetilde{\gamma}^{B}\} = \eta^{AB}\,, \qquad \{\overline{\gamma}_{\mu}(x), \overline{\gamma}_{\nu}(x)\} = \overline{g}_{\mu\nu}(x)\,. \tag{6.5}$$

Local Lorentz transformations lead to a reparameterization of the "internal" space,

$$e'^{\mu A}(x) = \Lambda^{A}{}_{B}(x)\, e^{\nu B}(x)\,, \qquad e'^{\mu}_{A}(x) = \Lambda_{A}{}^{B}(x)\, e^{\nu}_{B}(x)\,, \tag{6.6}$$

The Ricci rotation coefficient ω_ν^{AB} is obtained from the covariant derivative of an anholonomic basis vector,

$$\vec{e}_A = e^\mu_A \, \vec{e}_\mu \,, \qquad \vec{e}^{\,B} = e^{\mu B} \, \vec{e}_\mu \,,$$

$$\partial_\nu \vec{e}^{\,B} = \left(\nabla_\nu \, e^{\mu B}\right) \vec{e}_\mu = e^A_\mu \, \left(\nabla_\nu \, e^{\mu B}\right) \vec{e}_A \equiv \omega_\nu^{AB} \; \vec{e}_A \,. \tag{6.7}$$

It is given, in terms of the vierbein and Christoffel symbols, as follows,

$$\omega_\nu^{AB} = e^A_\mu \, \nabla_\nu \, e^{\mu B} = e^A_\mu \, \partial_\nu \, e^{\mu B} + e^A_\mu \, \Gamma^\mu_{\nu\lambda} \, e^{\lambda B} \,. \tag{6.8}$$

A local spinor Lorentz transformation with generators $\Omega^{AB}(x)$ transforms the bispinor ψ according to

$$\psi'(x') = S(\Lambda(x)) \, \psi(x) = \left(1 - \frac{i}{4} \, \Omega^{AB}(x) \, \widetilde{\sigma}_{AB}\right) \psi(x) \tag{6.9}$$

The flat-space spin matrices $\widetilde{\sigma}_{AB}$ are given as

$$\widetilde{\sigma}_{AB} = \frac{i}{2} \, [\widetilde{\gamma}_A, \widetilde{\gamma}_B] \tag{6.10}$$

The covariant derivative ∇_μ of a spinor contains the spin connection matrix $\Gamma_\mu(x)$,

$$\Gamma_\mu(x) = \frac{i}{4} \, \omega_\mu{}^{AB}(x) \, \widetilde{\sigma}_{AB} \,,$$

$$\nabla_\mu \psi(x) = \left(\partial_\mu - \frac{i}{4} \, \omega_\mu{}^{AB}(x) \, \widetilde{\sigma}_{AB}\right) \psi(x) = (\partial_\mu - \Gamma_\mu) \, \psi(x) \tag{6.11}$$

A change of the vierbein according to Eq. (6.6) leads to a different form of the Ricci rotation coefficients, and of the spin connection matrices,

$$\nabla'_\mu \psi(x) = \left(\partial_\mu - \frac{i}{4} \, \omega'_\mu{}^{AB}(x) \, \widetilde{\sigma}_{AB}\right) \psi(x) = \left(\partial_\mu - \Gamma'_\mu\right) \psi(x) \tag{6.12}$$

but the covariance with respect to the local Lorentz transformation is ensured by the relationship [5],

$$\nabla'_\mu \, [S(\Lambda(x))\psi(x)] = S(\Lambda(x)) \, \nabla_\mu \, \psi(x) \,, \tag{6.13}$$

in accordance with the underlying idea of the covariant derivative. From the action

$$S = \int d^4x \, \sqrt{-\det \overline{g}(x)} \; \overline{\psi}(x) \, \left(\frac{i}{2}\gamma^\mu(x) \overleftrightarrow{\nabla}_\mu - m\right) \psi(x) \,,$$

one derives the gravitationally coupled Dirac equation

$$(i\gamma^\mu \, \nabla_\mu - m)\psi(x) = 0 \,. \tag{6.14}$$

It is straightforward to generalize this formalism to tachyons, which in flat space-time are described by the tachyonic Dirac equation,

$$(i\widetilde{\gamma}^\mu \, \nabla_\mu - \widetilde{\gamma}^5 \, m)\psi(x) = 0 \,. \tag{6.15}$$

In curved space-time [6,7], the generalization of the γ^5 matrix reads as

$$\overline{\gamma}^5(x) = \frac{i}{4!} \frac{\epsilon_{\mu\nu\rho\delta}}{\sqrt{-\det \overline{g}(x)}} \widetilde{\gamma}^\mu(x)\, \widetilde{\gamma}^\nu(x)\, \widetilde{\gamma}^\rho(x)\, \widetilde{\gamma}^\delta(x)\,, \tag{6.16}$$

the action becomes

$$S = \int d^4x\, \sqrt{-\det \overline{g}(x)}\; \overline{\psi}(x)\, \overline{\gamma}^5(x)\, \left(\frac{i}{2}\gamma^\rho(x) \overleftrightarrow{\nabla}_\rho - \overline{\gamma}^5(x)\, m\right) \psi(x)\,,$$

from which one derives the gravitationally coupled tachyonic Dirac equation as

$$\left[i\overline{\gamma}^\mu\, \nabla_\mu - \overline{\gamma}^5(x)\, m\right]\, \psi(x) = 0\,. \tag{6.17}$$

Based on this formalism, a number of very concrete and definite problems have recently been investigated [8–10,7], mainly for time-independent, central-field curved-spacetime configurations. The Dirac bispinor ψ describes both particle ("electron") as well as antiparticle ("positron") states. A symmetry of particle and anti-particle solutions has been uncovered in Ref. [8] for the Schwarzschild space-time geometry; it implies that, on the level of Newtonian and Einsteinian geometrodynamics, antiparticles are attracted in central gravitational fields in the same way as particles are (including all relativistic corrections of motion, and within a quantum dynamical formalism). A conceivable deviation of the gravitational interactions for particles and antiparticles therefore would be indicative of a fifth fundamental force [8]. We also found the nonrelativistic limit of the Dirac-Schwarzschild Hamiltonian and identified the gravitational spin-orbit coupling, and gravitational zitterbewegung term [9]. The leading relativistic corrections terms are obtained after a Foldy–Wouthuysen transformation [11], and read [9]

$$\begin{aligned} H_{FW} &= \beta\, \left(m + \frac{\vec{p}^{\,2}}{2m} - \frac{\vec{p}^{\,4}}{8m^3}\right) - \beta\, \frac{m\, r_s}{2\, r} \\ &+ \beta\, \left(-\frac{3r_s}{8m}\, \left\{\vec{p}^{\,2}, \frac{1}{r}\right\} + \frac{3\pi r_s}{4m} \delta^{(3)}(\vec{r}) + \frac{3r_s}{8m}\, \frac{\vec{\Sigma} \cdot \vec{L}}{r^3}\right)\,. \end{aligned} \tag{6.18}$$

Here, $r_s = 2GM$ is the Schwarzschild radius. The spectrum of a purely gravitationally coupled bound state in a central field was studied, including the relativistic corrections [12], and the analogue of the electromagnetic fine-structure constant for gravity was identified [12]. Furthermore, it has recently been clarified, based on tachyonic gravitationally coupled Dirac equation (6.17), that the leading term in gravitational central fields actually is *attractive* for tachyons, in full agreement with the fact that tachyons become "luxons" in the high-energy domain (they approach the light cone), and they thus are attracted to gravitational centers much like photons. However, several correction terms are repulsive for tachyons, in contrast to their attractive counterparts for tardyons [7]. Namely, according to Ref. [7], One finds for tardyons (in the high-energy limit, with $\mathcal{E} = -\vec{\Sigma} \cdot \vec{p}$)

$$\begin{aligned} \mathcal{H}_{ds} = \beta\, \Bigg(\mathcal{E} &+ \frac{m^2}{2\mathcal{E}} - \frac{1}{2}\left\{\mathcal{E}, \frac{r_s}{r}\right\} + \frac{9}{32}\left\{\mathcal{E}, \frac{r_s^2}{r^2}\right\} \\ &- \frac{7\, m^2}{64}\left\{\frac{1}{\mathcal{E}}, \frac{r_s^2}{r^2}\right\} + \frac{3\, m^2}{16} \frac{r_s}{r} \frac{1}{\mathcal{E}} \frac{r_s}{r}\Bigg)\,, \end{aligned} \tag{6.19}$$

while for tachyons

$$\mathcal{H}_{tg} = \beta \left(\mathcal{E} - \frac{m^2}{2\mathcal{E}} - \frac{1}{2} \left\{ \mathcal{E}, \frac{r_s}{r} \right\} + \frac{9}{32} \left\{ \mathcal{E}, \frac{r_s^2}{r^2} \right\} + \frac{7\,m^2}{64} \left\{ \frac{1}{\mathcal{E}}, \frac{r_s^2}{r^2} \right\} - \frac{3\,m^2}{16} \frac{r_s}{r} \frac{1}{\mathcal{E}} \frac{r_s}{r} \right) . \tag{6.20}$$

Here, "ds" and "tg" refer to the identifications of the Hamiltonians as "Dirac Schwarzschild" and "tachyonic gravitational", respectively. The final two terms in these Hamiltonians have opposite signs, indicating a difference in the gravitational interaction for tachyons and tardyons.

We should clarify that, in order to couple a Dirac particle to space-time curvature, it is not necessary to quantize space-time. The spin connection matrices mediate the coupling to the "classical" (not quantum) space-time geometry, and they ensure the covariance of the covariant derivative under local Lorentz transformations (in a nonholonomic basis).

6.3 Speed of Light in Deep Gravitational Potentials

First, it's necessary to remember that the speed of light is not as "constant" as one would a priori assume, when expressed in global coordinates. According to Eq. (5) of Ref. [13], the space-time metric for static, weak gravitational fields reads as

$$ds^2 = (1 + 2\,\Phi_G(\vec{r}))\; dt^2 - (1 - 2\,\Phi_G(\vec{r}))\; d\vec{r}^{\,2} , \tag{6.21}$$

where Φ_G is the gravitational potential. Light travels on a null geodesic, with $ds^2 = 0$, and so

$$\left(\frac{d\vec{r}}{dt} \right)^2 = \frac{1 + 2\,\Phi_G(\vec{r})}{1 - 2\,\Phi_G(\vec{r})} \approx 1 + 4\,\Phi_G(\vec{r}) . \tag{6.22}$$

The local speed of light, expressed in terms of the global coordinates, thus is

$$\left| \frac{d\vec{r}}{dt} \right| = 1 + 2\,\Phi_G(\vec{r}) , \qquad \Delta c = 2\,\Phi_G(\vec{r}) = (1 + \gamma)\,\Phi_G(\vec{r}) < 0 . \tag{6.23}$$

In a central field, we have $\Phi_G(\vec{r}) = -GM/r$. Deviations from $\gamma = 1$ parameterize departures from standard geometrodynamics [14–16]. For further discussion, we also refer to Chap. 4.4 on page 196 ff. of Ref. [17], Eq. (4.43) of Ref. [18] and Sec. 4.5.2 of Ref. [18], as well as Ref. [19]. The effect parameterized by Eq. (6.23) is known as the Shapiro time delay [20–24].

Some attention [25] was recently directed to a recent paper [26] where it was claimed that quantum electrodynamics, when considering the gravitational correction to the fermion propagators, yields an additional correction to the speed of light, parameterized as

$$\delta c_\gamma = \frac{9}{64}\, \alpha\, \frac{\Phi_G(\vec{r})}{c_0^2} < 0 , \tag{6.24}$$

slowing down photons as compared to other high-energy particles, which approach the "unperturbed light cone". The reason for the special role of photons is claimed to be due to the fact that vacuum polarization, on shell, receives a tiny correction due to the gravitational interactions of the fermions in the loop, which in turn displaces the photon ever so slightly from the flat space-time mass shell.

First doubts arise because the value

$$\gamma - 1 = \chi\,\alpha = \frac{9\,\alpha}{64} = 1.03 \times 10^{-3} \,. \tag{6.25}$$

is in disagreement with the bounds set by radar reflection from the the Cassini observations [27] in superior conjunction, which reads as follows,

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5} \,. \tag{6.26}$$

Further conceptual difficulties result because, one might otherwise perform a thought experiment and enter a region of deep gravitational potential with three freely falling, propagating wave packets, one describing a photon, the others describing a very highly energetic neutrino and a very highly energetic electron. The latter two propagate at a velocity (infinitesimally close to) the flat-space speed of light c_0. If a correction of the form δc_γ exists, then photons will have been decelerated to a velocity $c_0 - |\delta c_\gamma|$ within the gravitational potential, whereas *both* fermions retain a velocity (infinitesimally close to) c_0. If we regard the photons as particles, then we could argue that a "force" must have acted onto the photon, causing deceleration, even though the particles were in free fall, leading to violating of the weak equivalence principle. However, the claim (6.24) is of quantum origin and therefore beyond the realm of applicability of standard classical general relativity; it is thus hard to refute based on first principles.

It thus remains to calculate the leading correction to vacuum polarization in gravitational fields using the gravitationally coupled Dirac equation. According to Eq. (12) of Ref. [8], the leading term is

$$H = \vec{\alpha} \cdot \vec{p} + \beta\, m\, w(\vec{r}) \,, \qquad w(\vec{r}) \approx 1 + \Phi_G \,, \tag{6.27}$$

leading to an effective mass of the fermion,

$$m_{\rm eff} = m\, w(r) \approx m\,(1 + \Phi_G)\,, \tag{6.28}$$

which needs to be inserted into the covariant representation of the one-loop vacuum insertion into the photon propagator,

$$\frac{g_{\mu\nu}}{k^2} \to \frac{g_{\mu\nu}}{k^2\,[1 + \overline{\omega}^R(k^2)]} \,, \qquad k^2 = \omega^2 - \vec{k}^2 \,, \tag{6.29}$$

$$\overline{\omega}^R(k^2) = \frac{\alpha k^2}{3\pi} \int\limits_{4\,m_{\rm eff}^2}^{\infty} \frac{dk'^2}{k'^2}\, \frac{1 + 2m_{\rm eff}^2/k'^2}{k'^2 - k^2} \sqrt{1 - \frac{4m_{\rm eff}^2}{k'^2}} \,. \tag{6.30}$$

The asymptotic forms

$$\overline{\omega}^R(k^2) = \frac{\alpha}{15\pi} \frac{k^2}{m_{\rm eff}^2} + \mathcal{O}(k^4)\,, \qquad k^2 \to 0\,, \tag{6.31a}$$

$$\overline{\omega}^R(k^2) = -\frac{\alpha}{3\pi} \ln\left(-\frac{k^2}{m_{\rm eff}^2}\right) + \frac{5\alpha}{3\pi} + \mathcal{O}\left(\frac{\ln(-k^2)}{k^2}\right)\,, \quad k^2 \to \infty\,, \tag{6.31b}$$

imply that the correction on the mass shell, $\overline{\omega}^R(0) = 0$, invalidating the claim made in Ref. [26].

6.4 Conclusions

In Sec. 6.2, we have studied the gravitational coupling of Dirac particles to curved space-time backgrounds, and found that the covariant coupling to space-time implies the use of spin connection matrices; naive prescriptions based on the insertion of the gravitational potential into the Dirac equation can only be valid in an approximate sense. The central idea behind the covariant coupling is the covariance of the covariant derivative in spinor space, given in Eq. (6.13), from which by an explicit evaluation of the spin connection matrices, the results given in Eqs. (6.18), (6.19) and (6.20) can be derived.

The gravitational correction to vacuum polarization is discussed in Sec. 6.3, and a recent claim [26] regarding an additional modification of the speed of light in deep gravitational potentials [parameterized by the γ parameter, see Eq. (6.25)] is refuted. The vacuum polarization tensor in gravitational backgrounds is obtained, within the leading approximation, by a substitution of the gravitationally shifted effective electron mass into the fermion propagator of the one-loop vacuum polarization integral [see Eq. (6.29)]. In summary, we have shown that one can apply the gravitational coupling of Dirac particles in order to solve a number of problems of practical interest, including central-field problems and variations thereof, and potential gravitational corrections to quantum-field theoretical phenomena, like vacuum polarization.

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7 10 Years of Dark Atoms of Composite Dark Matter

M.Yu. Khlopov [1,2,3]⋆⋆

[1] Centre for Cosmoparticle Physics "Cosmion"
[2] National Research Nuclear University "Moscow Engineering Physics Institute", 115409 Moscow, Russia
[3] APC laboratory 10, rue Alice Domon et Léonie Duquet
75205 Paris Cedex 13, France

Abstract. In 2005 Sheldon Glashow has proposed his sinister model, opening the path to composite-dark-matter scenarios, in which heavy stable electrically charged particles bound in neutral atoms play the role of dark matter candidates. Though the general problem of new stable single charged particles, forming with ordinary electrons anomalous isotopes of hydrogen, turned out to be unresolvable in Glashow's scenario, this scenario stimulated development of composite dark matter models, which can avoid the trouble of anomalous isotope overproduction. In the simplest case composite dark matter may consist of -2 charged particles, bound by ordinary Coulomb interaction with primordial helium in OHe dark matter model. The advantage and open problems of this model are discussed.

Povzetek. Sheldon Glashow je leta 2005 predlagal model, ki je odprl pot modelom, v katerih so kandidati za temno snov nevtralni atomi sestavljeni iz (novih) težkih stabilnih delcev z električnim nabojem. Glashowih stabilnih delcev z električnim nabojem 1 sicer ni mogoče povezati z elektroni v anomalne izotope vodika, ki bi pojasnili lastnosti temne snovi. Je pa ta model spodbudil razvoj modelov, ki naj pojasnijo temno snov s gručami težkih delcev in se hkrati izognejo težavam s preobiljem anomalnih izotopov. Avtor obravnava model, v katerem temno snov sestavljajo delci z nabojem -2, ki jih Coulombska interakcija veže s helijem v OHe, ter predstavi prednosti in odprte probleme tega modela.

7.1 Introduction

The existence of dark matter, constituting dominant fraction of the matter content of the Universe, is one of the cornerstones of the modern cosmology, but its physical nature is still elusive. The results of direct searches for dark matter are reviewed in [1]. Though the apparent contradiction of these results comes from the uncritical comparison of the data, obtained with the use of different techniques, and even their interpretation in the terms of Weakly Interacting Massive Particles (WIMPs) is still not ruled out [1], a more general approach to a possible solution of the dark matter problem is appealing. Here we concentrate on a possibility that in the same way as the ordinary matter is composed by atoms, which consist of electrically charged electrons and nuclei, bound by Coulomb forces, new electrically charged

⋆⋆ khlopov@apc.univ-paris7.fr

stable particles may be bound by ordinary Coulomb field in the dark atoms of the dark matter. The electrically charged constituents of dark atoms may be not only elementary particles, but can be composite objects, as are ordinary nuclei and nucleons.

In 2005 the idea of such a multi-composite dark matter was put forward by Sheldon Glashow in his sinister model [2]. The model assumed a set of million times heavier partners of ordinary quarks and leptons related by a strict symmetry. The lightest of these partners (tera-electron and tera-U-quark) were stable and could form a stable tera-helium atom (UUU)EE, in which +2 charged quark cluster (UUU) was bound by ordinary Coulomb force with two tera-electrons. It was proposed that in the early Universe the excessive U-quarks first bind in (UUU) cluster, which recombines then with excessive tera-electrons to form tera-helium atom. The unrecoverable problem of this scenario, revealed in [3], was inevitable overproduction of +1 and +2 charged remnants of incomplete binding, like (Uud), (UUu) hadrons or (UUU)E ions, which bind with ordinary electrons in atomic states that look like anomalous isotopes of hydrogen and helium. Moreover all the free tera-electrons turned to bind with primordial helium, as soon as it was formed in Big Bang Nucleosynthesis, in +1 charged ion (EHe), increasing the list of undesirable +1 charged species and preventing any possible reduction of their abundance. It makes impossible to realize the dark atom scenario not only in Glashow's sinister model, but also in any other model predicting stable +1 and -1 charged species. However these studies stimulated further development of the idea of composite dark matter particles both in the form of stable heavy quark clusters and dark atoms, in which new stable charged particles are bound.

Starting from 2006 various realizations of possible solution for dark atom scenario were proposed [4–10], in which the important role of stable -2 charged species was revealed. These species are bound with primordial helium in neutral OHe atoms, which play important catalyzing role in reduction of all the undesirable positively charged heavy species that can give rise to anomalous isotopes, as well as can be the candidate for composite dark matter, dominating in the matter density of the Universe. Such candidates for dark matter should consist of negatively doubly-charged heavy (with the mass $\sim$ 1 TeV) particles, which are called O^{--}, coupled to primordial helium. Lepton-like technibaryons, technileptons, AC-leptons or clusters of three heavy anti-U-quarks of 4th generation with strongly suppressed hadronic interactions are examples of such O^{--} particles (see [4–6,8–10] for a review and for references). Another direction of composite dark matter scenario is to consider neutral stable heavy quark clusters as it is proposed in the approach of [11]. However, even in this approach heavy stable -2 charged clusters ($\bar{u}_5\bar{u}_5\bar{u}_5$) of stable antiquarks $\bar{u}_5$ of 5th generation can also find their physical basis [7].

Here we briefly outline the advantages of the OHe dark atom scenario in its ability to explain some puzzles of direct and indirect dark matter searches, specifying collider and non-collider probes for this scenario as well as its open problems.

7.2 OHe solutions for puzzles of dark matter searches

It is assumed that together with generation of baryon asymmetry the excess of O^{--} particles is generated. This assumption finds natural basis in Walking Technicolor models, which provide proper ratio of baryon and O^{--} excess due to sphaleron transitions in the early Universe. Similar relationship can take place in any other model with O^{--} coupling to electroweak sphalerons.

As soon as primordial helium is produced in Big Bang nucleosynthesis, it captures all the free O^{--} forming OHe atoms. These atoms catalyze binding and annihilation of all the undesirable positively charged species. Before OHe gas starts to dominate on matter dominated stage, it decouples from plasma and radiation, what is necessary for its role of proper dark matter candidate. It leads to scenario of Warmer than Cold Dark Matter (WtCDM) with a slight suppression of small scale fluctuations. This suppression is less pronounced than in the Warm Dark Matter scenario, but still it can be of interest for solution of small scale cusp problem of the standard CDM. In spite of its strong nuclear interaction OHe gas is collisionless at galactic scale, but all the dense matter objects like stars or planets are opaque for it. Due to this opacity the infalling flux of OHe is captured and thermalized in the terrestrial matter.

It is assumed that the effective potential between OHe and a normal nucleus would have a barrier, preventing He and/or O^{--} from falling into the nucleus. Under these conditions elastic collisions dominate in OHe interactions with matter, and lead to a successful OHe scenario. The cosmological and astrophysical effects of such composite dark matter (dark atoms of OHe) are dominantly related to the helium shell of OHe and involve only one parameter of new physics — the mass of O^{--}.

7.2.1 OHe solution for puzzles of direct dark matter search

Dark atom interpretation of the puzzles of direct dark matter search is based on the specifics of OHe nuclear interaction. If dark matter can bind to normal matter, the observations could come from radiative capture of thermalized OHe and could depend on the detector composition and temperature. In the matter of the underground detector local concentration of OHe is determined by the equilibrium between the infalling cosmic OHe flux and its diffusion towards the center of Earth. Since the infalling flux experiences annual changes due to Earth's rotation around Sun, this local OHe concentration possess annual modulations.

The positive results of the DAMA/NaI and DAMA/LIBRA experiments are then explained by the annual modulations of the rate of radiative capture of OHe by sodium nuclei. Such radiative capture to a low energy OHe-nucleus bound state is possible only for intermediate-mass nuclei: this explains the negative results of the XENON100 and LUX experiments. The rate of this capture can be calculated by the analogy with radiative capture of neutron by proton, taking into account the scalar and isoscalar nature of He nucleus, what makes possible only E1 transition with isospin violation in this process. In the result this rate is proportional to the temperature (to the square of relative velocity in the absence of local thermal equilibrium): this leads to a suppression of this effect in cryogenic detectors, such as CDMS.

7.2.2 OHe solution for positron line excess in the galactic bulge

The timescale of OHe collisions in the Galaxy exceeds the age of the Universe, what proves that the OHe gas is collisionless. However the rate of such collisions is nonzero and grows in the regions of higher OHe density, particularly in the central part of the Galaxy, where these collisions lead to OHe excitations. De-excitations of OHe with pair production in E0 transitions can explain the excess of the positron-annihilation line, observed by INTEGRAL in the galactic bulge [9,10,12–16]. The calculated rate of collisions and OHe excitation in them strongly depends on OHe density and relative velocity and the explanation of positron excess was found to be very sensitive to the dark matter density in the central part of Galaxy, where baryonic matter dominates and theoretical estimations are very uncertain. The latest analysis of dark matter distribution favors more modest values of dark matter central density, what fixes the explanation of the excess of the positron-annihilation line by OHe collisions and de-excitation in a very narrow range of the mass of O^{--} near 1.25 TeV.

7.2.3 OHe solution for high energy positron excess

In a two-component dark atom model, based on Walking Technicolor, a sparse WIMP-like component of atom-like state, made of positive and negative doubly charged techniparticles, is present together with the dominant OHe dark atom and the decays of doubly positive charged techniparticles to pairs of same-sign leptons can explain the excess of high-energy cosmic-ray positrons, found in PAMELA and AMS02 experiments [17]. This explanation is possible for the mass of decaying +2 charged particle below 1 TeV and depends on the branching ratios of leptonic channels. Since even pure lepton decay channels are inevitably accompanied by gamma radiation the important constraint on this model follows from the measurement of cosmic gamma ray background in FERMI/LAT experiment. The multi-parameter analysis of decaying dark atom constituent model is under way in order to determine the maximal model independent value of the mass of decaying +2 charge particle, at which this explanation is possible.

7.2.4 The LHC probes for OHe solutions for cosmic positron excess

These astroparticle data can be fitted, avoiding many astrophysical uncertainties of WIMP models, for a mass of $O^{--} \sim 1$ TeV, which stimulates searches for stable doubly charged lepton-like particles at the LHC as a test of the composite-dark-matter scenario. The search for stable multi-charge particles in ATLAS and CMS experiments gives the lower value for double charged particles around 700 GeV [18]. This search will continue in the current Run of the LHC, giving the hope on the complete experimental test of composite dark matter explanation for the observed low and high energy positron excess.

7.3 Open problems of OHe scenario

7.3.1 The problem of OHe nuclear barrier

The crucial problem of OHe scenario is the existence of a dipole barrier in OHe nuclear interaction. The scenario in which such a barrier does not appear was considered in [19]. This lead to a significant role of inelastic reactions for OHe, and strongly modified the main features of the OHe scenario. In the period of Big Bang Nucleosynthesis, when OHe was formed, it captured an additional He nucleus, so that the dominant form of dark matter became charged, recombining with electrons in anomalous isotopes of helium and heavier elements. The resulting over-abundance of anomalous isotopes in terrestrial matter seems to be unavoidable in this case.

This makes the full solution of OHe nuclear physics, started in [20], vital. The answer to the possibility of the creation of a dipole Coulomb barrier in OHe interaction with nuclei is crucial. Without that barrier one gets no suppression of inelastic reactions, in which O^{--} binds with nuclei. These charged species form atoms (or ions) with atomic cross sections, and that strongly suppresses their mobility in terrestrial matter, leading to their storage and over-abundance near the Earth's surface and oceans. Hence, the model cannot work if no repulsive interaction appears at some distance between OHe and the nucleus, and the solution to this question of OHe nuclear physics is vital for the composite-dark-matter OHe scenario.

7.3.2 The problem of the Earth's shadowing

The terrestrial matter is opaque for OHe, what should inevitably lead to an effect of Earth matter shadowing for the OHe flux and corresponding diurnal modulation. This effect needs special study in the confrontation with the constraints, recently obtained in DAMA/LIBRA experiment [21].

7.4 Conclusion

The existence of new stable electrically charged particles poses an immediate question on their presence in the surrounding matter in the form of anomalous isotopes, whose possible abundance is severely constrained by the experimental data. The original approach of the sinister model [2] could not overcome the trouble of overproduction of anomalous hydrogen and helium [3]. However, this approach revealed two important aspects of composite dark matter: possibility of clusters of heavy stable quarks with suppressed QCD interaction and a possibility of stable charged particles hidden in neutral dark atoms. The development of dark atom scenario during the past decade gave rise to the OHe composite-dark-matter scenario.

The advantages of this scenario is that it is minimally related to the parameters of new physics and is dominantly based on the effects of known atomic and nuclear physics. However, the full quantum treatment of this problem turns out to be rather complicated and remains an open (see [22] for the most recent review).

At the mass of long living stable double charged particles near 1 TeV dark atom scenario can explain the observed excess of low energy positrons in the galactic bulge and the excess of high energy positrons above 10 GeV, challenging experimental search for such particles at the LHC.

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8 Describing 2-TeV Scale $W_L W_L$ Resonances with Unitarized Effective Theory

F.J. Llanes-Estrada **, A. Dobado and R.L. Delgado

Dept. Fisica Teorica I, Univ. Complutense, 28040 Madrid, Spain

Abstract. The LHC is now exploring the 1-3 TeV scale where resonances of the Electroweak Symmetry Breaking Sector might exist. If so, Unitarized Effective Theory can be used to describe the data with all the constraints of unitarity, causality and global-symmetry breaking, and to find the resonance positions in the complex s-plane. From any resonances found, one can infer the parameters of the universal Effective Lagrangian, and those may be used to inform higher-energy theories (UV completions) that can be matched to it. We exemplify with two-body resonances in the coupled channels hh and $W_L W_L - Z_L Z_L$ employing the Equivalence Theorem and comment on the apparent excess in the ATLAS dijet data at 2 TeV.

Povzetek. Pričakuje se, da bodo meritve na pospeševalniku LHC potrdile obstoj resonanc pri energijah od 1–3 TeV. Avtorji uporabijo unitarni efektivni model za opis dvodelčnih resonanc v dvodelčnih kanalih (hh in $W_L W_L - Z_L Z_L$), ki uspešno opiše te vrste resonanc v energijskem območju nekaj sto GeV, v novem energijskem območju. Komentirajo rezultate meritev z dvema curkoma na merilniku Atlas pri energiji 2 TeV.

8.1 Non-linear EFT for $W_L W_L$ and hh

The LHC has found a scalar boson with $m_h = 125$ GeV and not much more. It is natural to describe the Electroweak Symmetry Breaking Sector of the Standard Model (SM) in terms of the low-energy spectrum alone. The resulting effective Lagrangian for the Higgs-like particle h and the longitudinal gauge bosons $W_L,\ Z_L \sim \omega^a$ in the non-linear representation appropriate for the global symmetry breaking scheme $SU(2) \times SU(2) \to SU(2)_c$ (leaving the approximate custodial subgroup as a good isospin symmetry) is as given by us [1], the Barcelona group [2] and others [3,4],

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\left[1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2\right]\partial_\mu\omega^i\partial^\mu\omega^j\left(\delta_{ij} + \frac{\omega^i\omega^j}{v^2}\right) + \frac{1}{2}\partial_\mu h\partial^\mu h \\ &+ \frac{4a_4}{v^4}\partial_\mu\omega^i\partial_\nu\omega^i\partial^\mu\omega^j\partial^\nu\omega^j + \frac{4a_5}{v^4}\partial_\mu\omega^i\partial^\mu\omega^i\partial_\nu\omega^j\partial^\nu\omega^j + \frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2 \\ &+ \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^i\partial^\nu\omega^i + \frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^i\partial_\nu\omega^i \end{aligned} \tag{8.1}$$

** Speaker, fllanes@fis.ucm.es

The parameters of this Lagrangian, neglecting the masses of all quasi-Goldstone bosons ω^a and of the Higgs h, adequate to explore the energy region 1-3 TeV $\gg$ 100 GeV, are seven. Their status is given in table 8.1.

a	b	a_4	a_5	g	d	e
$(0.88, 1.34)$	$\in (-1, 3)a^2$ (this work)	0?	0?	0?	0?	0?

Table 8.1. From the ATLAS and CMS reported [5] hWW, hZZ couplings we infer the approximate 2σ level constraint on a shown (a recent communication to the LHCP2015 conf. finds similar results [8]). In our recent work [6,7] on unitarized perturbation theory we could also put a coarse constraint on b due to the absence of a coupled-channel resonance in $hh - \omega\omega$ (the second channel is visible while the first is much harder). Basically no bounds have been reported on the NLO parameters: their SM value is zero.

We emphasize that with seven parameters, this is a reasonably manageable Lagrangian for LHC exploration of electroweak symmetry breaking, granted, under the approximation of $M_W \simeq M_Z \simeq m_h \simeq 0$ which is fair enough in the TeV region, and this is in contrast to the very large parameter space of the fully fledged effective theory [3].

The perturbative scattering amplitudes $A_I^J(s) = A_{IJ}^{(LO)}(s) + A_{IJ}^{(NLO)}(s)\ldots$ for $\omega\omega$ and hh, projected into partial waves, are given to NLO in [6]. For example, the LO amplitudes of $I = 0, 1$ and 2, proportional to $(1 - a^2)$, and the channel-coupling amplitude $\omega\omega \to hh$, to $(a^2 - b)$,

$$\begin{aligned} A_0^0(s) &= \frac{1}{16\pi v^2}(1 - a^2)s \\ A_1^1(s) &= \frac{1}{96\pi v^2}(1 - a^2)s \\ A_2^0(s) &= -\frac{1}{32\pi v^2}(1 - a^2)s \\ M^0(s) &= \frac{\sqrt{3}}{32\pi v^2}(a^2 - b)s \end{aligned}$$

show how a tiny separation of the parameters from the SM value leads to an energy-growing, eventually strongly interacting set of amplitudes.

Including the NLO, these amplitudes take a form characteristic of chiral perturbation theory

$$A_{IJ}^{(LO+NLO)}(s) = Ks + \left(B(\mu) + D\log\frac{s}{\mu^2} + E\log\frac{-s}{\mu^2}\right)s^2 \qquad (8.2)$$

with a left cut carried by the $Ds^2 \log s$ term, a right cut in the $Es^2 \log(-s)$ term, and the $Ks + Bs^2$ tree-level polynomial. B, D and E have been calculated, reported in [6] and allow for perturbative renormalizability, where the chiral counterterms contained in B absorb one-loop divergences from iterating the tree-level Lagrangian and run to make Eq. (8.2) scale invariant.

The energy reach of the Effective Theory with the Lagrangian density in Eq. (8.1) is nominally $4\pi v \sim 3$ TeV. If the LHC finds no clear new phenomenon through this scale, experimental data on $W_L W_L$ spectra can eventually be compared with the effective theory predictions. In this precision work, separations of a from 1 or of b from a^2 or any NLO parameter from 0 can then be used to predict the scale of new physics, or if measurements are null, at least to constrain it.

8.2 Resonances

On the other hand, if the LHC finds new resonances that couple to two longitudinal gauge bosons (and potentially also to two Higgs bosons), then a purely perturbative approach is inadequate. A deffect of the amplitudes in Eq. (8.2) is that they violate the unitarity relation $\text{Im}A_{IJ} = |A_{IJ}|^2$, which is satisfied only order by order in perturbation theory, namely $\text{Im}A_{IJ}^{(NLO)} = |A_{IJ}^{(LO)}|^2$. This introduces an error which is only acceptably small when s is much smaller than the mass of the first resonance in the IJ channel. But of course, since near resonances the imaginary part of the amplitude is large, the effective theory is of no use there. The solution is sometimes called Unitarized Effective Theory and is described in subsection (8.2.1).

8.2.1 Unitarization

Unitarization of effective theory amplitudes is a technique well-known [9] in hadron physics that we describe only briefly. It is possible because scattering amplitudes in field theory are very constrained functions due to Lorentz invariance, causality and unitarity. Dispersion relations, known from old in optics, are a way of incorporating all the constraints [10] leaving little freedom to determine the amplitudes, though they remain ambiguous without dynamical knowledge. To fully obtain them though, one needs a few key numbers which are provided by the effective theory at low-energy (see the lectures [11] for an introduction). This powerful method of combining dispersion relations with effective theory, which basically exhausts all underlying-model independent information in the experimental data for two-body channels, was deployed for the electroweak symmetry breaking sector early on [12]. Usually the resulting amplitudes for $W_L W_L \sim \omega\omega$ scattering are encoded in simple algebraic forms that avoid the complications of the dispersion relations, such as the K-matrix [4] that introduce a small amount of model dependence in the discussion.

To address this, we have compared [6] three unitarization methods that agree in predicting the same resonances at the same positions within 1 to 10% when all three can be used. These are the Inverse Amplitude Method, the N/D method, and an improved version of the K-matrix method that ensures complex-plane analyticity where appropriate. Table 8.2 shows the IJ channels where each one is currently applicable in the Electroweak sector.

As an example, consider the Inverse Amplitude Method. In its simplest form it requires two orders of the perturbative expansion, that are combined in the

Table 8.2. Channels where each unitarization method can currently be used.

IJ	00	02	11	20	22
Method	All	N/D, IK	IAM	All	N/D, IK

following simple formula,

$$A_{IJ} = \frac{\left(A_{IJ}^{(LO)}\right)^2}{A_{IJ}^{(LO)} - A_{IJ}^{(NLO)}} \,. \tag{8.3}$$

To obtain it, one realizes that a dispersion relation for $A(s)$ may be exact but of little use because of insufficient low-energy information. On the contrary, a dispersion relation for the perturbative $A^{(LO)} + A^{(NLO)}$ can be fully studied, but it is trivial because the perturbative amplitude is known everywhere. The trick is to write one for $\left(A^{(LO)}\right)^2 A^{-1}$ (hence the name "Inverse Amplitude Method") because the integral over the right, unitarity cut of $1/A$ is exactly calculable when only two-body channels are important. The result is the formula in Eq. (8.3). Its generalization to two (massless) channels is straightforward by turning the quantities therein into matrices, each element being an elastic $\omega\omega \to \omega\omega$, $hh \to hh$ or a cross-channel $\omega\omega \to hh$ amplitude. In Fig. 8.1 we show the IAM and also the other two methods with NLO parameters set to 0 at a scale of $\mu = 3$ TeV and with LO parameters $a = 0.88$ and $b = 3$. This set generates a characteristic coupled-channel resonance seen in all three amplitudes.

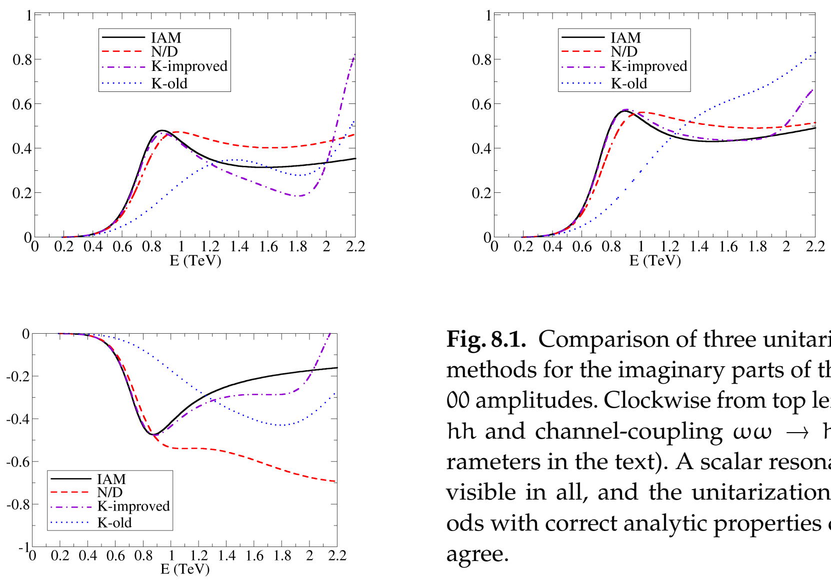


Fig. 8.1. Comparison of three unitarization methods for the imaginary parts of the $IJ = 00$ amplitudes. Clockwise from top left, $\omega\omega$, hh and channel-coupling $\omega\omega \to hh$ (parameters in the text). A scalar resonance is visible in all, and the unitarization methods with correct analytic properties closely agree.

The variable s in Eq. (8.3) may be extended to the complex plane, allowing to search for resonances in its second Riemann sheet. We locate the pole positions and report selected ones below in subsection 8.2.3.

8.2.2 ATLAS excess in two-jet events

The interest in TeV-scale resonances has recently rekindled because of an apparent excess in ATLAS data [13] plotted in Fig. 8.2 together with comparable, older CMS data [16] that does not show such an enhancement.

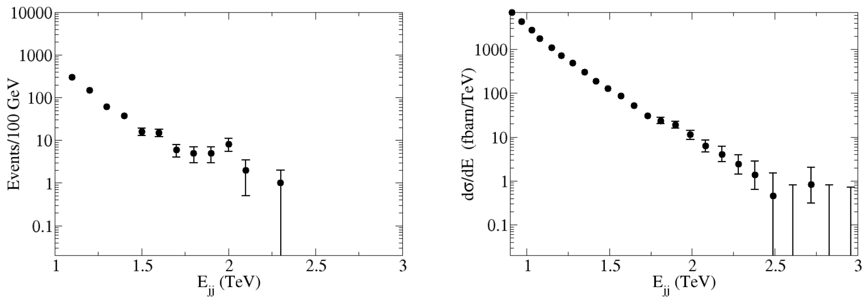


Fig. 8.2. Left: rerendering of the ATLAS data[13] for $WZ \to 2$ jet in pp collisions at the LHC, that shows a slight excess at 2 TeV (same in the other isospin combinations WW and ZZ, not shown). Criticism on the jet analysis has been presented in [15]. Right: CMS data [16] in the same 2-jet channel with jets tagged as vector bosons. Here the collaboration provides the absolute normalization of the cross-section. No excess is visible at 2 TeV (if at all, a tiny one at 1.8-1.9 TeV).

The excess is seen in two-jet events, each one containing the entire debris of a respective gauge boson. Their invariant mass reconstruction allows the assignment of a W or of a Z tag (82 and 91 GeV respectively) but the experimental error makes the identification loose, so that the three-channels cross-feed and we should not take seriously the excess to be seen in all three yet. Because WZ is a charged channel, an $I = 0$ resonance cannot decay there. Likewise ZZ cannot come from an $I = 1$ resonance because the corresponding Clebsch-Gordan coefficient $\langle 1010|10\rangle$ vanishes. A combination of both isoscalar and isovector could explain all three signals simultaneously, as would also an isotensor $I = 2$ resonance. In the isotensor case, the resonance should be visible in the doubly charged channel W^+W^+ whereas not in the other (to tag the charge requires to study leptonic decays instead of jets, so it is a whole other measurement, but worth carrying out).

Numerous models have been proposed to explain the presumed excess, but the model-independent information is still sparse [14].

One statement that we can make, based on the so-called KSFR relation that the IAM naturally incorporates (as do broad classes of theories such as Composite Higgs models [17] with vector resonances [18]), is that if a ρ-like isovector resonance is in the ATLAS data, it will be quite narrower than the bump seen (perhaps broadened due to experimental resolution). The relation, given here in the absence of further channels [7], links the mass and width of the isovector resonance with the low-energy constants v and a in a quite striking manner,

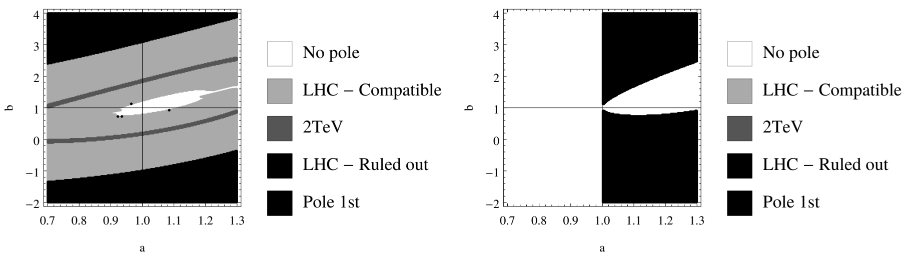
$$\Gamma^{\rm IAM} = \frac{M_{\rm IAM}^3}{96\pi v^2}(1 - a^2)\,. \tag{8.4}$$

For $M \sim 2$ TeV and $\Gamma \sim 0.2$ TeV as obtained by rule of thumb in Fig. 8.2, one gets $a \sim 0.73$ which is in tension with the ATLAS-deduced bound $a|_{2\sigma} > 0.88$ at 4-5σ level; Eq. (8.4) predicts that an isovector $W_L W_L$ resonance at 2 TeV, with present understanding of the low-energy constants, needs to have a width of order 50 GeV at most.

8.2.3 IAM parameter map

At last, we map out part of the seven-parameter space in search for resonances at 2 TeV that can be brought to bear on the new ATLAS data.

For $a < 1$ the scalar-isoscalar channel can be resonant from the LO Lagrangian alone (generating a σ-like resonance that was described in [1]). In fact, even for $a = 1$, there is a resonance generated for large enough b that oscillates between the $\omega\omega$ and hh, a "pinball" resonance, reported in [6]. This can be seen in the left plot of Fig. 8.3, where, for $a < 1$ so that $(1 - a^2) > 1$ there is a pole in the second Riemann sheet.



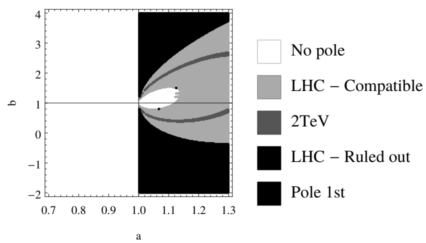


Fig. 8.3. We explore the ab parameter space in search for resonant poles of $\omega\omega$ scattering; clockwise from top left, $IJ = 00$, 11 and 20.

The isoscalar wave resonates for a broad swipe of ab parameter space, and near 2 TeV (the thin band), though the structure is generally broad, and feeds the *WW* and *ZZ* channels seen in the ATLAS data. In that case, the charged *WZ* experimental excess must be ascribed to misidentification of one of the two bosons, since an isoscalar resonance is of necessity neutral.

For $a > 1$ an isotensor resonance exists (see again Fig. 8.3, bottom plot). This is possible for $a > 1$ (light gray band marked "LHC compatible") as the

LO amplitude in Eq. (8.2) becomes attractive. Of course, for this negative sign of $(1-a^2)$, as seen in Eq. (8.2), the usual roles of the isoscalar and isotensor waves are reversed, with the first now being repulsive.

In a narrow curved strip (middle gray, immersed in that band) this resonance appears at about 2 TeV and can decay to all of *WW*, *WZ* and *ZZ* charge-channels. The darkest area corresponds here to "LHC ruled out" and means that the resonance is light and might already be excluded.

We need to make sure that the other waves don't present causality-violating poles in the first Riemann sheet that rule out a certain parameter region. Returning to Fig. 8.3 we see that the isovector wave indeed violates causality for much of the parameter space where the isotensor resonance exists, though there are perhaps small patches where the isotensor resonance is still allowed, for not too large values of b.

Since this allowed parameter space is so small and because, even if the isotensor resonance were there its production cross-section would be smaller (requiring two intermediate *W* bosons) than the production of an isovector one as reported in [19], we proceed to the NLO amplitude.

We likewise look for poles in the complex s plane as function of the a_4, a_5 parameters with fixed $a = 0.95$ and $b = 1$, as shown in Fig. 8.4. The bottom plot

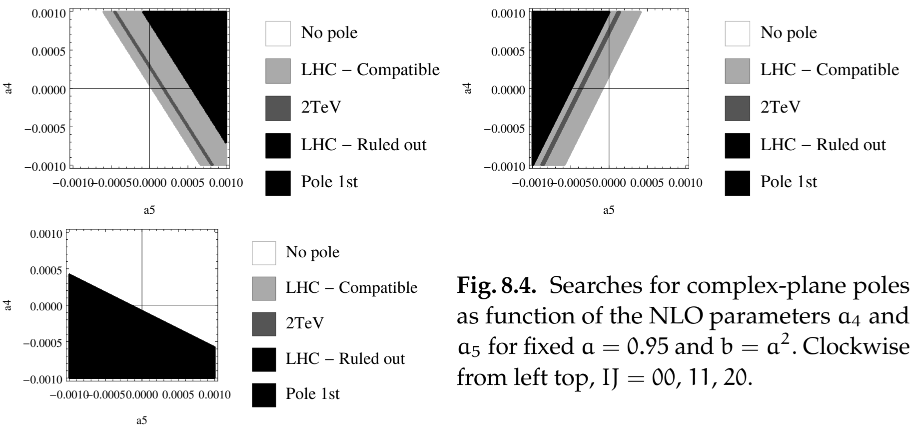


Fig. 8.4. Searches for complex-plane poles as function of the NLO parameters a_4 and a_5 for fixed $a = 0.95$ and $b = a^2$. Clockwise from left top, IJ = 00, 11, 20.

shows how a large swath of parameter space towards negative a_4 is excluded by displaying a pole in the first Riemann sheet of the 20 channel. Because here we chose $a < 1$, this channel does not resonate in the second sheet, whereas the scalar one (left, top plot) does, as well as the 11 channel (that is seen, by comparing with Fig. 8.3, to present "intrinsic" resonances driven by the NLO counterterms).

The two diagonal bands in the 00 and 11 channels that support poles at around 2 TeV intersect for slightly negative a_5 and a_4 of order 5×10^{-4}. There, we find both isoscalar and isovector poles, that jointly could explain all of the extant *WW*, *WZ* and *ZZ* excesses in two-jet data.

8.3 Conclusion

The LHC is now taking data at 13 TeV and production cross-sections sizeably increase. This is necessary as the typical σ for ωω resonances are currently at or below the LHC sensitivity limit as shown in Fig. 8.5. The large rate at which a resonance would have to be produced to explain the ATLAS excess is a bit puzzling.

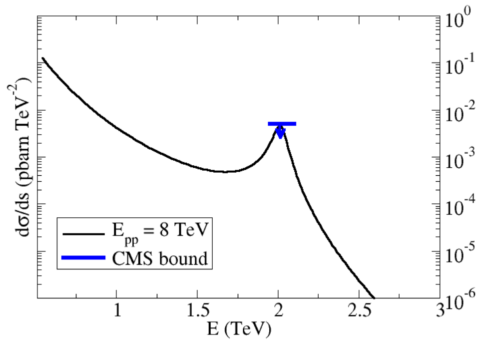


Fig. 8.5. Tree-level W production of ωω [19] with final-state resonance; non-zero parameters are $a = 0.9$, $b = a^2$, $a_4 = 7\times10^{-4}$ (at $\mu = 3$ TeV). Also shown is the CMS upper bound on the cross-section obtained from fig 8.2.

We hope that this ATLAS excess will soon be confirmed or refuted. In any case, the combination of effective theory and unitarity, as encoded for example in the IAM, is a powerful tool to describe data up to 3 TeV of energy in the electroweak sector if new, strongly interacting phenomena appear, with only few independent parameters. The content of new, Beyond the Standard Model theories, can then be matched onto those parameters for quick tests of their phenomenological viability.

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9 The Spin-Charge-Family Theory Offers the Explanation for all the Assumptions of the Standard Model, for the Dark Matter, for the Matter/Anti-matter Asymmetry, Making Several Predictions.

N.S. Mankoč Borštnik

Department of Physics, FMF, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

Abstract. The *spin-charge-family* theory, which is a kind of the Kaluza-Klein theories, but with two kinds of the spin connection fields — the gauge fields of the two kinds of spins [1–5] — is offering the explanation for the appearance and properties of family members (quarks and leptons), of families, of vector gauge fields (weak, hyper, colour), of scalar higgs and Yukawa couplings and gravity. It also explains the appearance of the dark matter and matter/anti-matter asymmetry. In this talk the achievements of this theory, its predictions and also its not yet solved problems are briefly presented and discussed.

Povzetek. Teorija *spinov-nabojev-družin* [1–5] ponuja odgovor na vsa odprta vprašanja standardnega modela fizike osnovnih delcev in polj, pa tudi na marsikatero odprto vprašanje v kozmologiji. Pojasni lastnosti ene družine kvarkov in leptonov, nastanek družin, nastanek barvnega, šibkega in hiper polja, nastanek skalarnih polj, ki pojasnijo pojav Higgsovega polja in Yukavinih sklopitev. Pojasni tudi pojav temne snovi in asimetrijo med snovjo in antisnovjo v vesolju. Teorija, ki ima marsikaj skupnega s Kaluza-kleinovimi teorijami, ponudi dve vrsti spinov. Ena vrsta določa vse naboje osnovnih delcev, druga družinska kvantna števila. V predavanju predstavim dosedanje dosežke te teorije, njene napovedi, pa tudi še nerešena odprta vprašanja.

9.1 Introduction

More than 40 years ago the *standard model* offered the elegant new step in understanding elementary fermion and boson fields. It postulated:

- The existence of the massless family members - coloured quarks and colourless leptons, both left and right handed, the left handed members distinguishing from the right handed ones in the weak and hyper charges and correspondingly mass protected.
- The existence of massless families to each of a family member.
- The existence of the massless gauge fields (colour octet, weak triplet, hyper singlet) to the observed (colour, weak and hyper) charges of the family members. They all are vectors in $d = (3 + 1)$, in the adjoint representations with respect to the weak, colour and hyper charges.

- The existence of a massive self interacting scalar field carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$, respectively, obviously doublets (in the fundamental representation with respect to the weak charge - like fermions), with the "nonzero vacuum expectation values", what breaks the weak and the hyper charge, breaking correspondingly the mass protection of fermions and weak and hyper bosons.
- The existence of the Yukawa couplings, which together with (the gluons and) the scalar higgs take care of the properties of the fermions and heavy bosons, after the break of the weak and the hyper charge.

The *standard model* offers no explanation for the assumptions, suggested by the phenomenology. Its assumptions have been confirmed without offering surprises. The last unobserved field, the higgs scalar, was detected in June 2012 and confirmed in March 2013.

There are several attempts in the literature, offering the extensions of the *standard model*, but do not really offer the explanation for the *standard model* assumptions. The SU(5) and SU(10) *grand unified theories* unify all the charges, but neither they explain why the spin (the handedness) is connected with the (weak and hyper) charges nor why and from where do families appear. *Supersymmetric* theories, assuming the existence of bosons with the charges of quarks and leptons and fermions with the charges of the gauge vector fields, although having several nice properties, do not explain the occurrence of families except by assuming larger groups. Also the theories of *strings* and *membranes*, again having desired features with respect to several requirements, like renormalizability, also do not offer the explanation for the appearance of families, although they do have families, if assuming a large enough group. The*Kaluza-Klein* theories do unify spin and charges, but do not offer the explanation for the appearance of families.

To see the next step beyond the standard model one should be able to answer the following questions:
i. Where do families originate and why there exist families at all? How many families are there?
ii. How are the origin of the scalar field - the higgs - and the Yukawa couplings connected with the origin of families?
iii. How many scalar fields determine properties of the so far (and others possibly be) observed fermions and masses of the heavy bosons?
iv. Why is the higgs, or are all the scalar fields, if there are several, doublets with respect to the weak and the hyper charge, while all the other bosons have charges in the adjoint representations of the group?
v. Why do the left and the right handed family members distinguish so much in charges and why do they - quarks and leptons - manifest so different properties if they all start as massless? **vi.** Are there also scalar bosons with the colour charge in the fundamental representation of the colour group and where, if they are, do they manifest?
vii. Where does the dark matter originate?
viii. Where does the matter/anti-matter asymmetry originate?
ix. Where do the charges and correspondingly the so far (and others possibly be) observed gauge fields originate?

x. Where does the dark energy originate and why is it so small?
xi. And several other questions, like: What is the dimension of space-time?

My statement is: An elegant trustworthy step beyond the *standard model* must offer answers to several of the above open questions, explaining: **o** the origin of the charges of the fermions, **o** the origin of the families of the fermions and their properties, **o** the origin of the vector gauge fields and their properties, **o** the origin of the scalar field, its properties and the Yukawa couplings, **o** the origin of the dark matter, **o** the origin of the "ordinary" matter/anti-matter asymmetry.

Inventing a next step, which covers only one of the open questions, can hardly be the right step.

The *spin-charge-family* theory [1–14] does offer the explanation for all the assumptions of the *standard model*, offering answers to many of the above cited open questions. The more I am working (together with the collaborators) on the *spin-charge-family* theory, the more answers to the open questions of the elementary fermion and boson fields and cosmology the theory is offering. Although still many theoretical proofs, more precise, and first of all the experimentally confirmed, predictions are needed, the theory is becoming more and more trustworthy.

I shall briefly present the achievements of the *spin-charge-family* theory, still open questions and answers to some of the most often posed questions and criticisms.

9.2 *Spin-charge-family* theory, action and assumptions

I present in this section, following a lot the similar one from Refs. [1,5], the *assumptions* of the *spin-charge-family* theory, on which the theory is built.

A i. In the action [1,4,2,5] fermions ψ carry in $d = (13+1)$ as the *internal degrees of freedom only two kinds of spins* (no charges), which are determined by the two kinds of the Clifford algebra objects (there exist no additional Clifford algebra objects) (9.7)) - γ^a and $\tilde{\gamma}^a$ - and *interact correspondingly with the two kinds of the spin connection fields* - $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, the *gauge fields* of $S^{ab} = \frac{i}{4}\,(\gamma^a\gamma^b - \gamma^b\gamma^a)$, the generators of $SO(13,1)$ and $\tilde{S}^{ab} = \frac{i}{4}\,(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$ (the generators of $\widetilde{SO}(13,1)$) - and the *vielbeins* $f^{\alpha}{}_a$.

$$
\begin{aligned}
\mathcal{A} &= \int d^dx\; E\; \mathcal{L}_f + \int d^dx\; E\;(\alpha\, R + \tilde{\alpha}\,\tilde{R})\,,\\
\mathcal{L}_f &= \frac{1}{2}\,(\bar{\psi}\,\gamma^a p_{0a}\psi) + \text{h.c.},\\
p_{0a} &= f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\,\{p_\alpha, E f^{\alpha}{}_a\}_-\,,\quad p_{0\alpha} = p_\alpha - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}\,,\\
R &= \frac{1}{2}\,\{f^{\alpha[a}f^{\beta b]}\;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^{c}{}_{b\beta})\} + \text{h.c.}\,,\\
\tilde{R} &= \frac{1}{2}\,\{f^{\alpha[a}f^{\beta b]}\;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\,\tilde{\omega}^{c}{}_{b\beta})\} + \text{h.c.}\,.
\end{aligned}
\tag{9.1}
$$

Here [1] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$. R and $\tilde{R}$ are the two scalars (R is a curvature).

A ii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ (manifesting as $SO(7,1) \times SU(3) \times U(1)$), affecting both internal degrees of freedom - the one represented by γ^a and the one represented by $\tilde{\gamma}^a$. Since the left handed (with respect to $M^{(7+1)}$) spinors couple differently to scalar (with respect to $M^{(7+1)}$) fields than the right handed ones, the break can leave massless and mass protected $2^{((7+1)/2-1)}$ massless families (which decouple into twice four families). The rest of families get heavy masses [2].

A iii. The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)} \times M^{(4)}$.

A iv. The scalar condensate (Table 9.1) of two right handed neutrinos with the family quantum numbers of one of the two groups of four families, brings masses of the scale of unification ($\propto 10^{16}$ GeV) to all the vector and scalar gauge fields, which interact with the condensate [1].

A v. There are nonzero vacuum expectation values of the scalar fields with the space index $s = (7, 8)$, conserving the electromagnetic and colour charge, which cause the electroweak break and bring masses to all the fermions and to the heavy bosons.

Comments on the assumptions:

C i.: This starting action enables to represent the *standard model* as an effective low energy manifestation of the *spin-charge-family* theory [1–13]. It offers the explanation for all the *standard model* assumptions: **a.** One representation of $SO(13,1)$ contains, if analyzed with respect to the *standard model* groups ($SO(3,1) \times SU(2) \times U(1) \times SU(3)$) all the members of one family (Table 9.4), left and right handed, with the quantum numbers required by the *standard model* [3]. **b.** The action explains the appearance of families due to the two kinds of generators

[1] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b,\ e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$, $E = \det(e^a{}_\alpha)$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[2] A toy model [20,21] was studied in $d = (5+1)$ with the same action as in Eq. (9.1). The break from $d = (5+1)$ to $d = (3+1)\times$ an almost S^2 was studied. For a particular choice of vielbeins and for a class of spin connection fields the manifold $M^{(5+1)}$ breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families remain massless and mass protected. Equivalent assumption, its proof is is in progress, is made in the $d = (13+1)$ case.

[3] It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons (neutrinos and electrons), and the right handed weak chargeless and $SU(2)_{II}$ charged coloured quarks and colourless leptons, as well as the right handed weak charged and $SU(2)_{II}$ chargeless colour anti-triplet anti-quarks and (anti)colourless anti-leptons, and the left handed weak chargeless and $SU(2)_{II}$ charged anti-quarks and anti-leptons. The anti-fermion states are reachable from the fermion states by the application of the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$, presented in Ref. [22].

of groups, the infinitesimal generators of one being S^{ab}, of the other $\tilde{S}^{ab}$ [4]. **c.** The action explains the appearance of the gauge fields of the *standard model* [1,5]. (In Ref [5] the proof is presented, that gauge fields can in the Kaluza-Klein theories be equivalently represented with either the vielbeins or spin connection fields.) [5]. **d.** It explains the appearance of the scalar higgs and Yukawa couplings [6]. **e.** The starting action contains also additional $SU(2)_{II}$ (from $SO(4)$) vector gauge fields (one of the components contributes to the hyper charge gauge fields as explained above), as well as the scalar fields with the space index $s \in (5, 6)$ and $t \in (9, 10, \ldots, 14)$. All these fields gain masses of the scale of the condensate (Table 9.1), which they interact with. They all are expressible with the superposition of $f^{\mu}{}_{m}\, \omega_{ab\mu}$ or of $f^{\mu}{}_{m}\, \tilde{\omega}_{ab\mu}$ [7].

C ii., C iii.: There are many ways of breaking symmetries from $d = (13 + 1)$ to $d = (3 + 1)$. The assumed breaks explain the connection between the weak and the hyper charge and the handedness of spinors, manifesting correspondingly the observed properties of the family members - the quarks and the leptons, left and right handed (Table 9.4) - and of the observed vector gauge fields. After the break from $SO(13, 1)$ to $SO(3, 1) \times SU(2) \times U(1) \times SU(3)$ the anti-particles are accessible from particles by the application of the operator $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$, as explained in Refs. [22,23] [8].

[4] There are before the electroweak break two massless decoupled groups of four families of quarks and leptons, in the fundamental representations of $\widetilde{SU}(2)_{R,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II,\widetilde{SO}(4)}$ and $\widetilde{SU}(2)_{L,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{I,\widetilde{SO}(4)}$ groups, respectively - the subgroups of $\widetilde{SO}(3, 1)$ and $\widetilde{SO}(4)$ (Table 9.5). These eight families remain massless up to the electroweak break due to the "mass protection mechanism", that is due to the fact that the right handed members have no left handed partners with the same charges.

[5] Before the electroweak break are all observable gauge fields massless: the gravity, the colour octet vector gauge fields (of the group $SU(3)$ from $SO(6)$), the weak triplet vector gauge field (of the group $SU(2)_{I}$ from $SO(4)$), and the hyper singlet vector gauge field (a superposition of $U(1)$ from $SO(6)$ and the third component of $SU(2)_{II}$ triplet).All are the superposition of the $f^{\alpha}{}_{c}\, \omega_{ab\alpha}$ spinor gauge fields

[6] There are scalar fields with the space index $(7, 8)$ and with respect to the space index with the weak and the hyper charge of the Higgs's scalar. They belong with respect to additional quantum numbers either to one of the two groups of two triplets, (either to one of the two triplets of the groups $\widetilde{SU}(2)_{R\,\widetilde{SO}(3,1)}$ and $\widetilde{SU}(2)_{II\,\widetilde{SO}(4)}$, or to one of the two triplets of the groups $\widetilde{SU}(2)_{L\,\widetilde{SO}(3,1)}$ and $\widetilde{SU}(2)_{I\,\widetilde{SO}(4)}$, respectively), which couple through the family quantum numbers to one (the first two triplets) or to another (the second two triplets) group of four families - all are the superposition of $f^{\sigma}{}_{s}\ \tilde{\omega}_{ab\sigma}$, or they belong to three singlets, the scalar gauge fields of (Q, Q', Y'), which couple to the family members of both groups of families - they are the superposition of $f^{\sigma}{}_{s}\ \omega_{ab\sigma}$. Both kinds of scalar fields determine the fermion masses (Eq. (9.6)), offering the explanation for the higgs, the Yukawa couplings and the heavy bosons masses.

[7] In the case of free fields (if no spinor source, carrying their quantum numbers, is present) both $f^{\mu}{}_{m}\, \omega_{ab\mu}$ and $f^{\mu}{}_{m}\, \tilde{\omega}_{ab\mu}$ are expressible with vielbeins, correspondingly only one kind of the three gauge fields are the propagating fields.

[8] The discrete symmetry operator $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$, Refs. [22,23], does not contain $\tilde{\gamma}^{a}$'s degrees of freedom. To each family member there corresponds the anti-member, with the same family quantum number.

C iv.: It is the condensate (Table 9.1) of two right handed neutrinos with the quantum numbers of one group of four families, which makes massive all the scalar gauge fields (with the index $(5, 6, 7, 8)$, as well as those with the index $(9, \ldots, 14)$) and the vector gauge fields, manifesting nonzero τ^4, τ^{23}, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{N}^3_R$ [1,5]. Only the vector gauge fields of Y, $SU(3)$ and $SU(2)$ remain massless, since they do not interact with the condensate.

C v.: At the electroweak break the scalar fields with the space index $s = (7, 8)$ - originating in $\tilde{\omega}_{abs}$, as well as some superposition of $\omega_{s's"s}$ with the quantum numbers (Q, Q', Y'), conserving the electromagnetic charge - change their mutual interaction, and gaining nonzero vacuum expectation values change correspondingly also their masses. They contribute to mass matrices of twice the four families, as well as to the masses of the heavy vector bosons.

All the rest scalar fields keep masses of the scale of the condensate and are correspondingly unobservable in the low energy regime.

The fourth family to the observed three ones is predicted to be observed at the LHC. Its properties are under consideration [13,14], the baryons of the stable family of the upper four families is offering the explanation for the dark matter [12].

Let us (formally) rewrite that part of the action of Eq.(9.1), which determines the spinor degrees of freedom, in the way that we can clearly see that the action does in the low energy regime manifest by the *standard model* required degrees of freedom of the fermions, vector and scalar gauge fields [2–13].

$$\begin{aligned}
\mathcal{L}_f = \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_m)\psi + \\
\{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\, \psi\} + \\
\{\sum_{t=5,6,9,\ldots,14} \bar{\psi}\gamma^t p_{0t}\, \psi\},
\end{aligned} \tag{9.2}$$

where $p_{0s} = p_s - \frac{1}{2}S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$, $p_{0t} = p_t - \frac{1}{2}S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}$, with $m \in (0, 1, 2, 3)$, $s \in (7, 8)$, $(s', s") \in (5, 6, 7, 8)$, (a, b) (appearing in $\tilde{S}^{ab}$) run within either $(0, 1, 2, 3)$ or $(5, 6, 7, 8)$, t runs $\in (5, \ldots, 14)$, $(t', t")$ run either $\in (5, 6, 7, 8)$ or $\in (9, 10, \ldots, 14)$. The spinor function ψ represents all family members of all the $2^{\frac{7+1}{2}-1} = 8$ families.

The first line of Eq. (9.2) determines (in $d = (3+1)$) the kinematics and dynamics of spinor (fermion) fields, coupled to the vector gauge fields. The generators τ^{Ai} of the charge groups are expressible in terms of S^{ab} through the complex coefficients ${c^{Ai}}_{ab}$ [9].

$$\tau^{Ai} = \sum_{a,b} {c^{Ai}}_{ab}\, S^{ab}\,, \tag{9.3}$$

[9] $\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78})$, $\vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78})$, $\vec{\tau}^3 := \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}$, $\tau^4 := -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$.
After the electroweak break the charges $Y := \tau^4 + \tau^{23}$, $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}$, $Q := \tau^{13} + Y$, $Q' := -Y\tan^2\vartheta_1 + \tau^{13}$ manifest. θ_1 is the electroweak angle, breaking $SU(2)_I$, θ_2 is the angle of the break of the $SU(2)_{II}$ from $SU(2)_I \times SU(2)_{II}$.

fulfilling the commutation relations

$$\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak} \,. \tag{9.4}$$

They represent the colour, the weak and the hyper charge. The corresponding vector gauge fields A^{Ai}_m are expressible with the spin connection fields ω_{stm}, with (s,t) either $\in (5,6,7,8)$ or $\in (9,\ldots,14)$, in agreement with the assumptions **A ii.** and **A iii.**. I demonstrate in Ref. [5] the equivalence between the usual Kaluza-Klein procedure leading to the vector gauge fields through the vielbeins and the procedure with the spin connections proposed by the *spin-charge-family* theory.

All vector gauge fields, appearing in the first line of Eq. (9.2), except $A^{2\pm}_m$ and $A^{Y'}_m$ $(= \cos\vartheta_2\, A^{23}_m - \sin\vartheta_2\, A^4_m$, Y' and τ^4 are defined in [10], are massless before the electroweak break. $\vec{A}^3_m$ carries the colour charge $SU(3)$ (originating in $SO(6)$), $\vec{A}^1_m$ carries the weak charge $SU(2)_I$ ($SU(2)_I$ and $SU(2)_{II}$ are the subgroups of $SO(4)$) and A^Y_m $(= \sin\vartheta_2\, A^{23}_m + \cos\vartheta_2\, A^4_m)$ carries the corresponding $U(1)$ charge, $Y = \tau^{23} + \tau^4$, τ^4 originates in $SO(6)$ and τ^{23} is the third component of the second $SU(2)_{II}$ group, A^4_m and $\vec{A}^2_m$ are the corresponding vector gauge fields). The fields $A^{2\pm}_m$ and $A^{Y'}_m$ get masses of the order of the condensate scale through the interaction with the condensate of the two right handed neutrinos with the quantum numbers of one of the group of four families (the assumption **iv.**, Table 9.1). (See Ref. [5].)

Since spinors (fermions) carry besides the family members quantum numbers also the family quantum numbers, determined by $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$, there are correspondingly $2^{(7+1)/2-1} = 8$ families [5], which split into two groups of $\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)}$ families.

If there are no fermions present then the vector gauge fields of the family members and family charges - ω_{abm} and $\tilde{\omega}_{abm}$ - are all expressible with the vielbeins [1,5], which are then the only propagating fields.

The scalar fields, the gauge fields with the space index ≥ 5, which are either the superposition of $\tilde{\omega}_{abs}$ or the superposition of $\omega_{s'ts}$, determine, when gaining nonzero vacuum expectation values (the assumption **v.**), masses of fermions (belonging to two groups of four families of family members of spinors) and weak bosons.

The condensate (the assumption **iv.**), Table 9.1, gives masses of the order of the scale of its appearance to all the scalar gauge fields, presented in the second and the third line of Eq. (9.2).

The vector gauge fields of the (before the electroweak break) conserved charges ($\vec{\tau}^3$, $\vec{\tau}^1$, Y) do not interact with the condensate and stay correspondingly massless. After the electroweak break - when the scalar fields (those with the family quantum numbers and those with the family members quantum numbers (Q, Q', Y')) with the space index $s = (7,8)$ start to self interact and gain nonzero vacuum expectation values - only the charges $\vec{\tau}^3$ and $Q = Y + \tau^{13}$ are the conserved charges. No family quantum numbers are conserved, since all scalar fields with the family quantum numbers and the space index $s = (7,8)$ gain nonzero vacuum expectation values.

[10] $Y' := -\tau^4 \tan^2\vartheta_2 + \tau^{23}, \tau^4 = -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^{4}	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^{4}$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$(\vert\nu^{\mathbf{VIII}}_{\mathbf{1R}} >_{\mathbf{1}}\ \vert\nu^{\mathbf{VIII}}_{\mathbf{2R}} >_{\mathbf{2}})$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\vert\nu^{VIII}_{1R} >_1 \vert e^{VIII}_{2R} >_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\vert e^{VIII}_{1R} >_1 \vert e^{VIII}_{2R} >_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 9.1. This table is taken from [1]. The condensate of the two right handed neutrinos ν_R, with the $VIII^{th}$ family quantum numbers, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The right handed neutrino has $Q = 0 = Y$. The triplet carries $\tau^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{\tau}^4 = -1$, $\tilde{N}^3_R = 1$, $\tilde{N}^3_L = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers are presented in Table 9.5.

Quarks and leptons have the "spinor" quantum number (τ^4, originating in $SO(6)$ presented in Table 9.4) equal to $\frac{1}{6}$ and $-\frac{1}{2}$, respectively. In the Pati-Salam model [24] twice this "spinor" quantum number is named $\frac{B-L}{2}$ quantum number, for quarks equal to $\frac{1}{3}$ and for leptons to -1.

Let me introduce a common notation A^{Ai}_s for all the scalar fields, independently of whether they originate in ω_{abs} or $\tilde{\omega}_{abs}$, $s \geq 5$. In the case that we are interested in the scalar fields which contribute to masses of fermions and weak bosons, then $s = (7, 8)$. If A^{Ai}_s represent ω_{abs}, $Ai = (Q,Q', Y')$, while if A^{Ai}_s represent $\tilde{\omega}_{\tilde{a}\tilde{b}s}$, all the family quantum numbers of all eight families contribute to Ai.

$$A^{Ai}_s \in (\, A^{Q}_s \,, A^{Q'}_s \,, A^{Y'}_s \,, \vec{\tilde{A}}^{\tilde{1}}_s \,, \vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_s \,, \vec{\tilde{A}}^{\tilde{2}}_s \,, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_s \,)\,,$$
$$\tau^{Ai} \supset (Q,\, Q',\, Y',\, \vec{\tilde{\tau}}^1,\, \vec{\tilde{N}}_L,\, \vec{\tilde{\tau}}^2,\, \vec{\tilde{N}}_R)\,. \tag{9.5}$$

Here τ^{Ai} represent all the operators, which apply on the spinor states. These scalars, the gauge scalar fields of the generators τ^{Ai} and $\tilde{\tau}^{Ai}$, are expressible in terms of the spin connection fields.

9.3 Achievements of the *spin-charge-family* theory and its predictions

The achievements of the *spin-charge-family* theory.

I. The *spin-charge-family* theory does offer the explanation for all the assumptions of the *standard model*:
I A. It explains all the properties of family members of one family - their spins and all the charges - clarifying the relationship between the spins and charges, Table 9.4 [11].
I B. It explains the properties of the vector fields, the gauge fields of the corresponding charges. They are in the *spin-charge-family* theory represented by the superposition of the spin connection fields ω_{stm}. It is proven in Sect. II of Ref. [5]

[11] The *spin-charge-family* theory explains, why the left handed and the right handed quarks and leptons differ in the weak and the hyper charge. It also explains, why quarks and leptons differ in the colour charge.

that the spin connection fields representation is equivalent to the usual Kaluza-Klein representation with the vielbeins $f^{\sigma}{}_{m} = \vec{\tau}^{A\sigma}\vec{A}^{A}_{m}$, where $\vec{\tau}^{A} = \vec{\tau}^{A\sigma}p_{\sigma}$, $\vec{\tau}^{A}$ determine symmetry properties of the space with $s \geq 5$ and $\vec{A}^{A}_{m}$ are the corresponding gauge fields.

I C. The scalar fields with the space index $s \in (7,8)$ belong to two doublets with respect to the space index s, while they belong with respect to additional quantum numbers either to three singlets with one of the family members charges (Y, Y', Q') or to twice two triplets of the family charges belonging to the groups $\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)}$. These scalar fields explain the appearance of the higgs and Yukawa couplings.

I D. The theory explains why these scalar fields, and consequently the higgs, which is the superposition of several scalar fields [4,5], have the weak and the hyper charge equal to $(\pm\frac{1}{2}, \mp\frac{1}{2})$, respectively, although they are bosons. They do transform as bosons with respect to $\mathcal{S}^{ab}$ [12], but due to the fact that they belong with respect to the space index $s = (5,6,7,8)$ to two $SU(2)$ groups with $\tau^{13} = \frac{1}{2}(S^{56} - S^{78})$ and $\tau^{23} = \frac{1}{2}(S^{56} + S^{78})$, respectively, their weak (τ^{13}) and hyper charge ($\tau^{23} + \tau^{4}$, where $\tau^{4} = -\frac{1}{3}\,(S^{9\,10} + S^{11\,12} + S^{13\,14})$ does not influence $s = (7,8)$) are the ones required by the *standard model*. Table 9.2 presents these two doublets and their quantum numbers.

I E. There are the nonzero vacuum expectation values of the scalar gauge fields

	state	τ^{13}	τ^{23}	spin	τ^{4}	Q
$A^{Ai}_{\underset{(-)}{78}}$	$A^{Ai}_{7} + iA^{Ai}_{8}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
$A^{Ai}_{\underset{(-)}{56}}$	$A^{Ai}_{5} + iA^{Ai}_{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1
$A^{Ai}_{\underset{(+)}{78}}$	$A^{Ai}_{7} - iA^{Ai}_{8}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0
$A^{Ai}_{\underset{(+)}{56}}$	$A^{Ai}_{5} - iA^{Ai}_{6}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	+1

Table 9.2. The two scalar weak doublets, one with $\tau^{23} = -\frac{1}{2}$ and the other with $\tau^{23} = +\frac{1}{2}$, both with the "spinor" quantum number $\tau^{4} = 0$, are presented. In this table all the scalar fields carry besides the quantum numbers determined by the space index also the quantum numbers $\mathcal{A}i$, which represent either the family members quantum numbers (Q, Q', Y') or the family quantum numbers (twice two triplets), $A^{Ai}_{\underset{(\pm)}{78}} = A^{Ai}_{7} \pm iA^{Ai}_{8}$, Eq. (9.5)

.

with the space index $s = (7,8)$, (with the weak charge equal to $\pm\frac{1}{2}$ and the hyper charge correspondingly equal to $\mp\frac{1}{2}$, both with respect to the space index), and with the family (twice two triplets) and family member quantum numbers (three singlets) in adjoint representations, which start to interact among themselves, gain nonzero vacuum expectation values, causing the break of the weak and the hyper

[12] $\mathcal{S}^{ab}$, which applies on the spin connections ω_{bde} $(= f^{\alpha}{}_{e}\ \omega_{bd\alpha})$ and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ $(= f^{\alpha}{}_{e}\ \tilde{\omega}_{\tilde{b}\tilde{d}\alpha})$, on either the space index e or the indices $(b, d, \tilde{b}, \tilde{d})$, is equal to $\mathcal{S}^{ab}\, A^{d\dots e\dots g} = i\,(\eta^{ae}\, A^{d\dots b\dots g} - \eta^{be}\, A^{d\dots a\dots g})$, or equivalently, in the matrix notation, $(\mathcal{S}^{ab})^{c}{}_{e}A^{d\dots e\dots g} = i(\eta^{ac}\delta^{b}_{e} - \eta^{bc}\delta^{a}_{e})\, A^{d\dots e\dots g}$.

charge symmetry.

II. The *spin-charge-family* theory does offer the explanation for the dark matter and for matter/anti-matter asymmetry:

II A. Neutral clusters of the members of the stable among the upper four families explain the appearance of the dark matter [12].
II B. The scalar fields with the space index $s \in (9, \ldots, 14)$ belong with respect to the space index s to a triplet or an anti-triplet, Table 9.3. They cause transitions of anti-leptons into quarks and anti-quarks into quarks and back, transforming matter into anti-matter and back. The condensate breaks CP symmetry. In the expanding universe, fulfilling the Sakharov request for appropriate non thermal equilibrium, these colour triplet and anti-triplet scalars have a chance to explain the matter/anti-matter asymmetry in the universe [1], as well as the proton decay.
II C. It is the *scalar condensate* of two right handed neutrinos (Table 9.1), which

	state	τ^{33}	τ^{38}	spin	τ^4	Q
$A^{Ai}_{\overset{9\,10}{(+)}}$	$A^{Ai}_9 - iA^{Ai}_{10}$	$+\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{\overset{11\,12}{(+)}}$	$A^{Ai}_{11} - iA^{Ai}_{12}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{\overset{13\,14}{(+)}}$	$A^{Ai}_{13} - iA^{Ai}_{14}$	0	$-\frac{1}{\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{\overset{9\,10}{(-)}}$	$A^{Ai}_9 + iA^{Ai}_{10}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A^{Ai}_{\overset{11\,12}{(-)}}$	$A^{Ai}_{11} + iA^{Ai}_{12}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A^{Ai}_{\overset{13\,14}{(-)}}$	$A^{Ai}_{13} + iA^{Ai}_{14}$	0	$\frac{1}{\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$

Table 9.3. The triplet and the anti-triplet scalar gauge fields, the triplet with the "spinor" quantum number equal to $\mathcal{S}^4 = -\frac{1}{3}$, $\mathcal{S}^4 = -\frac{1}{3}$ $(\mathcal{S}^{9\,10} + \mathcal{S}^{11\,12} + \mathcal{S}^{13\,14})$ and the anti-triplet with the "spinor" quantum number equal to $\mathcal{S}^4 = +\frac{1}{3}$. In this table all the scalar fields carry, besides the quantum numbers determined by the space index, (only) the family quantum numbers, not pointed out in this table. The table is taken from Ref. [1].

gives masses to all the vector and scalar gauge fields appearing in the *spin-charge-family* theory, except to the gravity, colour vector gauge fields, weak vector gauge fields and hyper $U(1)$ gauge field, since they do not interact with the condensate.
II D. The scalar fields, the members of the weak doublets (Table 9.2) with the space index $s = (5, 6)$, and the colour triplets and anti-triplets with the space index $t = (9, \ldots, 14)$ [1], which contribute to transitions of anti-particles into particles and to proton decay, keep masses of the condensate scale, as also do $A^{2\pm}_m$ and $A^{Y'}_M = \cos\theta_2 A^{2m}_m - \sin\theta A^4_m$.

III. The theory might have a chance to explain the hierarchy of the fermion and boson masses.
III A. By the theory predicted existence of the fourth family to the observed three families with the masses of the fourth family members at 1 TeV or even above [13,14] makes the mass matrices of the family members very close to the

democratic matrix, which suggests that the lower four families masses expand in the interval from less than eV to 10^{12} eV. Correspondingly would the interval of the higher four families be within the interval from $\approx$ 100 TeV [12] to $\approx 100\times 10^{12}$ TeV, which is above the unification scale 10^{16} GeV) (10^{13} TeV), explaining why are the masses of fermions spreading from few orders of magnitude below eV to TeV and above up to the unification scale.

Predictions of the *spin-charge-family* theory.

I. The *spin-charge-family* theory predicts in the low energy regime two decoupled groups of four families. The scalar fields with the space index $s = (7, 8)$, which are the gauge fields of the family charges, the superposition of $\tilde{S}^{ab}$ belonging to the subgroups $\widetilde{SU}(2)_{II\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II\widetilde{SO}(4)}$, determine the symmetry of each of the two groups of families.
I A. The symmetry of mass matrices

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha},$$

enables to tell what are the masses and matrix elements of the fourth family quarks and leptons within the interval of the accuracy of the experimental data. Any $(n-1) \times (n-1)$ submatrix of the $n \times n$ unitary matrix, $n \geq 4$ determines uniquely the $n \times n$ unitary matrix.

Present experimental data for the mixing matrices are not accurate enough even for quarks to tell, what are the fourth family masses. The estimation: most probably they are above 1 TeV. We can, however, for the chosen fourth family masses, predict the mixing matrix elements of quarks [13,14]. It comes out that the fourth family matrix elements are not very sensitive either to the lower three or to the fourth family quark masses. Our calculations [14] show that the new experimental data are in better agreement with the *spin-charge-family* theory predictions than the old ones.

For leptons the experimental data are less accurate and correspondingly the estimated mixing matrix elements for the fourth family leptons are less predictable.

The higher are the fourth family members masses, the closer are the mass matrices to the democratic matrices for either quarks or leptons - which is expected.
The fourth of the lower four families will be measured at the LHC.
I B. Scalar fields, which cause electroweak phase transition and are responsible for masses of the lower four families of quarks and leptons and weak bosons, determine the higss and the Yukawa couplings.
Besides the higgs, additional superposition of scalar fields are predicted to be measured at the LHC.
I C. The properties of the upper four family members, (almost) decoupled from the lower four families (their mass matrices still manifest the $\widetilde{SU}(2)_{II\widetilde{SO}(3,1)}$ $\times\widetilde{SU}(2)_{II\widetilde{SO}(4)}$ symmetry, provided that the condensate respect this symmetry, and are influenced by the family scalar fields of the upper four families, by the

family members scalar field with the quantum numbers (Q, Q', Y') and by the interaction with the condensate), can be evaluated within this theory by following the evolution of the universe [12].
The masses of the lowest of the upper four families are estimated [12] to be in the interval of several 10 *TeV to several* 10^4 *TeV.*
I D. Very heavy dark matter baryons are opening an interesting new "fifth family nuclear" dynamics.

III. There are besides the scalar fields, which are, like higgs, $SU(2)$ doublets, also the scalar fields, which are $SU(3)$ triplets, involved and responsible for the matter/anti-matter asymmetry of our universe.

9.4 Most common questions about the *spin-charge-family* theory

Let me present and offer a brief answers to the most common questions and complains about the validity and the ability that the*spin-charge-family* theory might be the right answer to the open questions of the *standard model* by attentive participants of the conferences, readers or referees. To most of such questions the answers can be found by carefully reading papers [5,4,1–3,6–14], some of them are discussed in special sections of these papers or in contributions to the Discussion section of the Bled 2015 workshop.

There are also the assumptions in this theory, represented in this talk, chosen in order that the theory manifests in the low energy regime the *standard model* properties, which also need, and want for better answer than the one, that obviously our universe has chosen among many other possibilities, those required by the assumptions.

The most needed are, of course, the experimental data confirmation of the predictions of this theory, making it trustworthy as the right next step beyond the *standard model*. But what does speak for this theory is that the simple starting action (Eq. (9.1)) and only a few assumptions explain all the assumptions of the *standard model*, offering the explanation also for the existence of the dark matter and the matter/anti-matter asymmetry, and might be for more open questions in the elementary particle physics and cosmology.

The order of questions presented below have no special meaning.

1. Can the fourth family (to the observed three ones) with the masses close to or larger than 1 TeV exist at all, since the masses of the higgs, top quark and heavy bosons are all below 200 GeV?
2. If there are so many scalar fields carrying the weak and the hyper charges of the higss (three singlets with the quantum numbers (Q, Q', Y') and two times two triplets carrying the family quantum numbers), how can the masses of the heavy bosons, to which all the scalars contribute, be so low, ≈ 100 GeV?
3. If there are two kinds of charges, the family and the family members ones, why after the electroweak break the colour and the electromagnetic charges are the only conserved charges ?
4. Can there be at all two kinds of the spin connection fields and only one kind of the vielbeins?

5. How can the vector gauge fields at all be represented by spin connection fields and not, like in the Kaluza-Klein ordinary procedure, by vielbeins [5,1]?
6. The two $SO(d-1,1)$ groups - $SO(13,1)$ and $\widetilde{SO}(13,1)$ - have so many representations that there is not difficult to make a choice of the needed ones, but there are many more left.
7. Can the higher loops contributions, making all the off diagonal matrix elements of the mass matrices depending on the scalar singlets with the quantum numbers (Q, Q', Y') keep the symmetry of the three level (Eq. 9.6)?
8. And several others.

Let me try to answer the above questions.

1. Due to not accurate enough experimental data the prediction for the fourth family masses is, that they might be at around one TeV or above. Since for the masses of the fourth families the theory predicts the mass matrices which are very close to the democratic ones, although still keeping the symmetry of Eq. (9.6), the matrix elements of the mixing matrices for the fourth family members are very small. Correspondingly the predictions can hardly be inconsistent with the so far made measurements. I expect that the new experiments on the LHC will confirm the existence of the fourth family of quarks and leptons.
2. The question, which remains to be answered, is, whether the scalar fields belonging to either the three singlets with the quantum numbers $(Q, Q', Y)'$ or to the two times two triplets with respect to the family charges, all carrying the weak and the hyper charge of the higgs, do all together contribute only ≈ 100 GeV to the masses of heavy bosons after the electroweak break (Ref. [5], Eq. (14)). Although it looks like that under certain conditions (the masses and nonzero vacuum expectation values of these scalars) this is possible, the study is not yet finished and the answer is not yet convincing.
3. The answer to the third questions is that all the scalar fields with the space index $s = (7, 8)$ - all having the weak and the hyper charges of the higgs - with the family quantum numbers gain nonzero vacuum expectation values, causing correspondingly the breaking of all the family charges, while their weak and hyper charges cause the breaking of the weak and hyper charge. Correspondingly the only conserved charges after the electroweak break are the electromagnetic and colour charges.
4. The answer to the question number 4. is explained in details in Ref. [5], Sect. IV., and in App. A., Sect. 2.. A short answer to this question is that either γ^a's or $\tilde{\gamma}^a$'s transform in the flat space under the Lorentz transformations as vectors. The curved coordinate space is only one, while both kinds of spin connection fields are expressible in terms of the vielbeins, if there are no spinors (fermion) sources present, while spin connections of both kinds differ among themselves and are not expressible by vielbeins, if there are spinor sources present (Ref. [5], App. C., Eq. (C9)).
5. The relation of the vector gauge fields when they are expressed with the spin connection fields (as it is done in the *spin-charge-family* theory) and the vector gauge fields when they are expressed with the vielbeins (as it is usually in the

Kaluza-Klein theories) is explained in Refs. [5,1]. The vector (as well as the scalar) gauge fields - $A^{Ai}_{m} = \sum_{st} c^{Aist}\omega_{stm}$ - are (Ref. [5], Eq. (C9)) expressible with vielbeins. In Sect. II. of this Ref. the proof is presented that the vielbein $f^{\sigma}{}_{m} = i x^{\tau}\, \vec{\tau}^{A\sigma}_{\tau}\, \vec{A}^{A}_{m}$, where $A^{Ai}_{m} = \sum_{st} c^{Aist}\omega_{stm}$ and $\vec{\tau}^{A} = \vec{\tau}^{A\sigma}\, p_{\sigma} = \vec{\tau}^{A\sigma}{}_{\tau}$ $x^{\tau}\, p_{\sigma}$ (Eqs. (5-13) of Ref [5]). This is true when the space with $d \geq 5$ has the rotational symmetry, $x'^{\mu} = x^{\mu}$, $x'^{\sigma} = x^{\sigma} - i\vec{\alpha}^{1}(x^{\mu})\, \vec{\tau}^{A}(x^{\tau})\, x^{\sigma}$. This symmetry manifests in $f^{\sigma}{}_{s} = \delta^{\sigma}_{s}\, f$, for **any** f, which is the *scalar function of the coordinates* x^{σ} in $d \geq 5$.
For $f = (1 + \frac{\rho^2}{2\rho_0^2})$ the space is an almost S^{d-4} sphere, with one point missing, and the curvature R is equal to

$$R = \frac{d(d-1)}{(\rho_0)^2}\,. \tag{9.6}$$

6. One representation of $SO(13,1)$ contains just all the members of one family of quarks and leptons, left and right handed with respect to $d = (3+1)$, with the quantum numbers required by the *standard model*. Although it contains also anti-quarks and anti-leptons, after the break of the symmetry of space from $SO(13,1)$ (and simultaneously of $\widetilde{SO}(13,1)$) to $SO(7,1)\times SU(3)\times U(1)$ the transformations of quarks into leptons as well as those, which transform spins to charges, are at low energies not possible. All the scalar fields, which would cause such transformations, become too massive.
All the scalar fields with the space index $s \geq 5$ have phenomenological meaning, either as scalars causing the electroweak break ($s = (7,8)$) or as scalars which contribute to the matter/anti-matter asymmetry of our universe. All the scalar, as well as the vector gauge fields, with the quantum numbers of the condensate, gain masses through the interaction with the condensate as discussed in Sect. II. of this talk and in Ref. [1,5].
7. In Ref. [30] the authors discussed this problem. Although in this paper the proof is not yet done, later studies show that the $U(1)\times SU(2)\times SU(2)$ symmetry remains in all orders of loop corrections.

9.5 Conclusions

I represent in this talk very briefly the so far obtained achievements of the *spin-charge-family* theory, which offers the explanation for all the assumptions of the *standard model*, with the families included, as well as some answers to the open questions in cosmology. Answering so far to so many open questions of the elementary particles and fields physics, this theory might be the right next step beyond the *standard model*.

The theory predicts that there are two triplet (with respect to the family quantum numbers) and three singlet (with respect to the family members quantum numbers) scalar fields, all with the weak and hyper charges of the higgs ($\mp\frac{1}{2}$, $\pm\frac{1}{2}$, respectively, with respect to the space index $s = (7,8)$), which explain the appearance of the scalar higgs and the Yukawa couplings. Some superposition of these scalar fields will be observed at the LH. The LHC will measure also the fourth family to the observed three ones.

I present in this talk also the most often asked questions about the validity of this theory, replying briefly to these questions and discuss the not yet solved problems of this theory.

9.6 Appendix: Short presentation of spinor technique [4,8,17,18]

This appendix is a short review (taken from [4]) of the technique [8,19,17,18], initiated and developed in Ref. [8], while proposing the *spin-charge-family* theory [2–4,6–13,1,29]. All the internal degrees of freedom of spinors, with family quantum numbers included, are describable in the space of d-anticommuting (Grassmann) coordinates [8], if the dimension of ordinary space is also d. There are two kinds of operators in the Grassmann space fulfilling the Clifford algebra and anticommuting with one another.The technique was further developed in the present shape together with H.B. Nielsen [19,17,18].

In this last stage we rewrite a spinor basis, written in Ref. [8] as products of polynomials of Grassmann coordinates of odd and even Grassmann character, chosen to be eigenstates of the Cartan subalgebra defined by the two kinds of the Clifford algebra objects, as products of nilpotents and projections, formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s.

The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

App. B of Ref. [5]briefly represents the starting point [8] of this technique in order to better understand the Lorentz transformation properties of both Clifford algebra objects, γ^a's and $\tilde{\gamma}^a$'s, as well as of spinor, vector, tensor and scalar fields, appearing in the *spin-charge-family* theory, that is of the vielbeins and spin connections of both kinds, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, and of spinor fields, family members and families.

The objects γ^a and $\tilde{\gamma}^a$ have properties

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,, \tag{9.7}$$

If B is a Clifford algebra object, let say a polynomial of γ^a, $B = a_0 + a_a\,\gamma^a + a_{ab}\,\gamma^a\gamma^b + \cdots + a_{a_1 a_2 \ldots a_d}\,\gamma^{a_1}\gamma^{a_2}\ldots\gamma^{a_d}$, one finds

$$\begin{aligned}&(\tilde{\gamma}^a B := i(-)^{n_B}\, B\gamma^a\,)\,|\psi_0\,>,\\ &\qquad B = a_0 + a_{a_0}\gamma^{a_0} + a_{a_1 a_2}\gamma^{a_1}\gamma^{a_2} + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\cdots\gamma^{a_d}\,,\end{aligned} \tag{9.8}$$

where $|\psi_0>$ is a vacuum state, defined in Eq. (9.22) and $(-)^{n_B}$ is equal to 1 for the term in the polynomial which has an even number of γ^b's, and to -1 for the term with an odd number of γ^b's, for any d, even or odd, and I is the unit element in the Clifford algebra.

It follows from Eq. (9.8) that the two kinds of the Clifford algebra objects are connected with the left and the right multiplication of any Clifford algebra objects B.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned} S^{ab} &: = (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,, \\ \tilde{S}^{ab} &: = (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,, \end{aligned} \tag{9.9}$$

$\{S^{ab}, \tilde{S}^{cd}\}_- = 0\,, \{S^{ab}, S^{cd}\}_- = i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac})\,, \{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- = i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})\,.$

We assume the "Hermiticity" property for γ^a's

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a\,, \tag{9.10}$$

in order that γ^a are compatible with (9.7) and formally unitary, i.e. $\gamma^{a\,\dagger}\,\gamma^a = I$.

One finds from Eq. (9.10) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(9.9) that the two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$\begin{aligned} S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4, \\ S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,, \\ \tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4\,, \\ \tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,. \end{aligned} \tag{9.11}$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$\begin{aligned} \Gamma^{(d)} &: = (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n, \\ \Gamma^{(d)} &: = (i)^{(d-1)/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n+1\,. \end{aligned} \tag{9.12}$$

One proceeds equivalently for $\tilde{\Gamma}^{(d)}$, subtituting γ^a's by $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from Eq.(9.10) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma$, $\Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) , while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows

$$\stackrel{ab}{(k)}: = \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \stackrel{ab}{[k]}:= \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,, \tag{9.13}$$

where $k^2 = \eta^{aa}\eta^{bb}$. It follows then

$$\begin{aligned} \gamma^a &= \stackrel{ab}{(k)} + \stackrel{ab}{(-k)}\,, \qquad \gamma^b = ik\eta^{aa}\,(\stackrel{ab}{(k)} - \stackrel{ab}{(-k)})\,, \\ S^{ab} &= \frac{k}{2}(\stackrel{ab}{[k]} - \stackrel{ab}{[-k]}) \end{aligned} \tag{9.14}$$

One can easily check by taking into account the Clifford algebra relation (Eq. (9.7)) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq. (9.9)) that the nilpotent $\stackrel{ab}{(k)}$ and the projector $\stackrel{ab}{[k]}$ are "eigenstates" of S^{ab} and $\tilde{S}^{ab}$

$$
\begin{aligned}
S^{ab} \stackrel{ab}{(k)} &= \frac{1}{2} k \stackrel{ab}{(k)} , \qquad & S^{ab} \stackrel{ab}{[k]} &= \frac{1}{2} k \stackrel{ab}{[k]} , \\
\tilde{S}^{ab} \stackrel{ab}{(k)} &= \frac{1}{2} k \stackrel{ab}{(k)} , \qquad & \tilde{S}^{ab} \stackrel{ab}{[k]} &= -\frac{1}{2} k \stackrel{ab}{[k]} ,
\end{aligned}
\tag{9.15}
$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognize that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$
\begin{aligned}
&\gamma^a \stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}, \; \gamma^b \stackrel{ab}{(k)} = -ik \stackrel{ab}{[-k]}, \; \gamma^a \stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, \; \gamma^b \stackrel{ab}{[k]} = -ik\eta^{aa} \stackrel{ab}{(-k)} , \\
&\tilde{\gamma}^a \stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]}, \; \tilde{\gamma}^b \stackrel{ab}{(k)} = -k \stackrel{ab}{[k]}, \; \tilde{\gamma}^a \stackrel{ab}{[k]} = \; i \stackrel{ab}{(k)}, \; \tilde{\gamma}^b \stackrel{ab}{[k]} = -k\eta^{aa} \stackrel{ab}{(k)} \; .
\end{aligned}
\tag{9.16}
$$

From Eq.(9.16) it follows

$$
\begin{aligned}
S^{ac} \stackrel{ab}{(k)}\stackrel{cd}{(k)} &= -\frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[-k]}\stackrel{cd}{[-k]} , \qquad & \tilde{S}^{ac} \stackrel{ab}{(k)}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{aa}\eta^{cc} \stackrel{ab}{[k]}\stackrel{cd}{[k]} , \\
S^{ac} \stackrel{ab}{[k]}\stackrel{cd}{[k]} &= \frac{i}{2} \stackrel{ab}{(-k)}\stackrel{cd}{(-k)} , \qquad & \tilde{S}^{ac} \stackrel{ab}{[k]}\stackrel{cd}{[k]} &= -\frac{i}{2} \stackrel{ab}{(k)}\stackrel{cd}{(k)} , \\
S^{ac} \stackrel{ab}{(k)}\stackrel{cd}{[k]} &= -\frac{i}{2}\eta^{aa} \stackrel{ab}{[-k]}\stackrel{cd}{(-k)} , \qquad & \tilde{S}^{ac} \stackrel{ab}{(k)}\stackrel{cd}{[k]} &= -\frac{i}{2}\eta^{aa} \stackrel{ab}{[k]}\stackrel{cd}{(k)} , \\
S^{ac} \stackrel{ab}{[k]}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{cc} \stackrel{ab}{(-k)}\stackrel{cd}{[-k]} , \qquad & \tilde{S}^{ac} \stackrel{ab}{[k]}\stackrel{cd}{(k)} &= \frac{i}{2}\eta^{cc} \stackrel{ab}{(k)}\stackrel{cd}{[k]} \; .
\end{aligned}
\tag{9.17}
$$

From Eq. (9.17) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$
\begin{aligned}
&\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0 , \quad & &\stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa} \stackrel{ab}{[k]} , \quad & &\stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]} , \quad & &\stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0 , \\
&\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]} , \quad & &\stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0 , \quad & &\stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0 , \quad & &\stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]} , \\
&\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0 , \quad & &\stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)} , \quad & &\stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)} , \quad & &\stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0 , \\
&\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)} , \quad & &\stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, \quad & &\stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0 , \quad & &\stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)} \; .
\end{aligned}
\tag{9.18}
$$

We recognize in Eq. (9.18) the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\overset{ab}{(k)}$ and $\overset{ab}{[k]}$, respectively. Defining

$$\overset{ab}{(\tilde{\pm i})}= \frac{1}{2}\,(\tilde{\gamma}^a \mp \tilde{\gamma}^b)\,, \quad \overset{ab}{(\tilde{\pm 1})}= \frac{1}{2}\,(\tilde{\gamma}^a \pm i\tilde{\gamma}^b)\,, \tag{9.19}$$

one recognizes that

$$\begin{aligned} &\overset{ab}{(\tilde{k})}\,\overset{ab}{(k)} = 0\,, \qquad \overset{ab}{(\tilde{-k})}\,\overset{ab}{(k)}= -i\eta^{aa}\;\overset{ab}{[k]}\,,\\ &\overset{ab}{(\tilde{k})}\,\overset{ab}{[k]} = i\;\overset{ab}{(k)}\,, \qquad \overset{ab}{(\tilde{k})}\,\overset{ab}{[-k]}= 0\,. \end{aligned} \tag{9.20}$$

Recognizing that

$$\overset{ab}{(k)}^{\dagger} = \eta^{aa}\;\overset{ab}{(-k)}\,, \quad \overset{ab}{[k]}^{\dagger} =\overset{ab}{[k]}\,, \tag{9.21}$$

we define a vacuum state $|\psi_0>$ so that one finds

$$\begin{aligned} &< \overset{ab}{(k)}^{\dagger}\;\overset{ab}{(k)} >= 1\,,\\ &< \overset{ab}{[k]}^{\dagger}\,\overset{ab}{[k]} >= 1\,. \end{aligned} \tag{9.22}$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of $d/2$, let us say, only nilpotents $\overset{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(9.11)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$\begin{aligned} &\overset{0d}{(k_{0d})}\overset{12}{(k_{12})}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0\;>\\ &\overset{0d}{[-k_{0d}]}\overset{12}{[-k_{12}]}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0\;>\\ &\overset{0d}{[-k_{0d}]}\overset{12}{(k_{12})}\overset{35}{[-k_{35}]}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0\;>\\ &\qquad\vdots\\ &\overset{0d}{[-k_{0d}]}\overset{12}{(k_{12})}\overset{35}{(k_{35})}\cdots\overset{d-1\;d-2}{[-k_{d-1\;d-2}]}\;|\psi_0\;>\\ &\overset{od}{(k_{0d})}\overset{12}{[-k_{12}]}\overset{35}{[-k_{35}]}\cdots\overset{d-1\;d-2}{(k_{d-1\;d-2})}\;|\psi_0\;>\\ &\qquad\vdots \end{aligned} \tag{9.23}$$

All the states have the same handedness Γ, since $\{\Gamma, S^{ab}\}_- = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrates that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $(\stackrel{ab}{k_{ab}})$, by transforming all possible pairs of $(\stackrel{ab}{k_{ab}})(\stackrel{mn}{k_{mn}})$ into $[\stackrel{ab}{-k_{ab}}][\stackrel{mn}{-k_{mn}}]$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (9.11)) generate all the states of one representation, $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra (Eq. (9.11)) generate the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set (Eq. (9.11)) of the algebra S^{ab} and $\tilde{S}^{ab}$

$$\begin{aligned} &S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}\,,\\ &\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\,10}, \tilde{S}^{11\,12}, \tilde{S}^{13\,14}\,, \end{aligned} \tag{9.24}$$

a left handed ($\Gamma^{(13,1)} = -1$) eigenstate of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,, 1/(2\sqrt{3}))$, for example, can be written as

$$\begin{aligned} &\stackrel{03}{(+i)}\stackrel{12}{(+)} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\; |\psi_0\rangle = \\ &\qquad \frac{1}{2^7}(\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2)|(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8)|| \\ &\qquad\qquad (\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi_0\rangle\,. \end{aligned} \tag{9.25}$$

This state is an eigenstate of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (9.11)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (9.11)), generate families from the starting u_R quark, transforming the u_R quark from Eq. (9.25) to the u_R of another family, keeping all of the properties with respect to S^{ab} unchanged. In particular, $\tilde{S}^{01}$ applied on a right handed u_R-quark from Eq. (9.25) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,, 1/(2\sqrt{3}))$

$$\begin{aligned} &\tilde{S}^{01}\;\stackrel{03}{(+i)}\stackrel{12}{(+)} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(-)}\,\stackrel{13\,14}{(-)} = \\ &\qquad -\frac{i}{2}\;\stackrel{03}{[+i]}\stackrel{12}{[+]} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\,\stackrel{11\,12}{(-)}\,\stackrel{13\,14}{(-)}\;. \end{aligned} \tag{9.26}$$

Below some useful relations [6] are presented

$$
\begin{aligned}
&N^{\pm}_{+} = N^{1}_{+} \pm i\, N^{2}_{+} = -\stackrel{03}{(\mp i)}\stackrel{12}{(\pm)}\,, \quad N^{\pm}_{-} = N^{1}_{-} \pm i\, N^{2}_{-} = \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\,,\\
&\tilde{N}^{\pm}_{+} = -\stackrel{03}{\tilde{(\mp i)}}\stackrel{12}{\tilde{(\pm)}}\,, \quad \tilde{N}^{\pm}_{-} = \stackrel{03}{\tilde{(\pm i)}}\stackrel{12}{\tilde{(\pm)}}\,,\\
&\tau^{1\pm} = (\mp)\,\stackrel{56}{(\pm)}\stackrel{78}{(\mp)}\,, \quad \tau^{2\mp} = (\mp)\,\stackrel{56}{(\mp)}\stackrel{78}{(\mp)}\,,\\
&\tilde{\tau}^{1\pm} = (\mp)\,\stackrel{56}{\tilde{(\pm)}}\stackrel{78}{\tilde{(\mp)}}\,, \quad \tilde{\tau}^{2\mp} = (\mp)\,\stackrel{56}{\tilde{(\mp)}}\stackrel{78}{\tilde{(\mp)}}\,.
\end{aligned}
\tag{9.27}
$$

i		$\vert{}^{a}\psi_i\rangle$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^{4}	Y	Q
		(Anti)octet, $\Gamma^{(1,7)} = (-1)\,1\,,\ \Gamma^{(6)} = (1)\,-1$ of (anti)quarks and (anti)leptons										
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
9	u_R^{c2}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
10	u_R^{c2}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...												
17	u_R^{c3}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
18	u_R^{c3}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...												
25	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
26	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
27	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
28	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
29	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
30	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
31	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
32	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
33	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
34	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
35	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
36	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
37	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
38	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
39	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
40	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{[-]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$

Continued on next page

i		$\vert^a\psi_i>$ (Anti)octet, $\Gamma^{(1,7)} = (-1)\,1\,,\ \Gamma^{(6)} = (1)\,-1$ of (anti)quarks and (anti)leptons	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
41	$\bar{d}^{\bar{c}2}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
. . .												
49	$\bar{d}^{\bar{c}3}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
. . .												
57	$\bar{e}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
58	$\bar{e}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
59	$\bar{\nu}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
60	$\bar{\nu}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
61	$\bar{\nu}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
62	$\bar{\nu}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
63	$\bar{e}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
64	$\bar{e}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 9.4. The left handed ($\Gamma^{(13,1)} = -1$) ($= \Gamma^{(7,1)} \times \Gamma^{(6)}$) multiplet of spinors - the members of the $SO(13,1)$ group, manifesting the subgroup $SO(7,1)$ - of the colour charged quarks and anti-quarks and the colourless leptons and anti-leptons, is presented in the massless basis using the technique presented in App. 9.6. It contains the left handed ($\Gamma^{(3,1)} = -1$) weak charged ($\tau^{13} = \pm\frac{1}{2}$) and $SU(2)_{II}$ chargeless ($\tau^{23} = 0$) quarks and the right handed weak chargeless and $SU(2)_{II}$ charged ($\tau^{23} = \pm\frac{1}{2}$) quarks of three colours ($c^i = (\tau^{33}, \tau^{38})$) with the "spinor" charge ($\tau^4 = \frac{1}{6}$) and the colourless left handed weak charged leptons and the right handed weak chargeless leptons with the "spinor" charge ($\tau^4 = -\frac{1}{2}$). S^{12} defines the ordinary spin $\pm\frac{1}{2}$. It contains also the states of opposite charges, reachable from particle states by the application of the discrete symmetry operator $\mathcal{C_N}\,\mathcal{P_N}$, presented in Refs. [22,23]. The vacuum state, on which the nilpotents and projectors operate, is not shown. The reader can find this Weyl representation also in Refs. [1,29,4].

I present at the end one Weyl representation of $SO(13+1)$ and the family quantum numbers of the two groups of four families.

One Weyl representation of $SO(13+1)$ contains left handed weak charged and the second $SU(2)$ chargeless coloured quarks and colourless leptons and right handed weak chargeless and the second $SU(2)$ charged quarks and leptons (electrons and neutrinos). It carries also the family quantum numbers, not mentioned in this table. The table is taken from Ref. [22].

The eight families of the first member of the eight-plet of quarks from Table 9.4, for example, that is of the right handed u_{1R} quark, are presented in the left column of Table 9.5 [4]. In the right column of the same table the equivalent eight-plet of the right handed neutrinos ν_{1R} are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
I	$u^{c1}_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	$\nu_{R\,2}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	$\nu_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	$\nu_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
I	$u^{c1}_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	$\nu_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[+]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
II	$u^{c1}_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	$\nu_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	$\nu_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[+]}\stackrel{78}{[+]}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	$\nu_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$
II	$u^{c1}_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	$\nu_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$

Table 9.5. Eight families of the right handed u^{c1}_R (9.4) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$), the other (II) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a doublet with with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (9.27). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (9.27)) transform u_{1R} to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino ν_{1R} to all the colourless members of the same family.

which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

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10 Fermionization in an Arbitrary Number of Dimensions

N.S. Mankoč Borštnik[1] and H.B.F. Nielsen[2]

[1]Department of Physics, FMF, University of Ljubljana,
Jadranska 19, SI-1000 Ljubljana, Slovenia
[2]The Niels Bohr Institute,
Blegdamsvej 17 - 21, Copenhagen Ø; Denmark

Abstract. One purpose of this proceedings-contribution is to show that at least for free massless particles it is possible to construct an explicit boson theory which is exactly equivalent in terms of momenta and energy to a fermion theory. The fermions come as $2^{d/2-1}$ families and the to this whole system of fermions corresponding bosons come as a whole series of the Kalb-Ramond fields, one set of components for each number of indexes on the tensor fields.

Since Kalb-Ramond fields naturally (only) couple to the extended objects or branes, we suspect that inclusion of interaction into such for a bosonization prepared system - except for the lowest dimensions - without including branes or something like that is not likely to be possible.

The need for the families is easily seen just by using the theorem long ago put forward by Aratyn and one of us (H.B.F.N.), which says that to have the statistical mechanics of the fermion system and the boson system to match one needs to have the number of the field components in the ratio $\frac{2^{d-1}-1}{2^{d-1}} = \frac{\#bosons}{\#fermions}$, enforcing that the number of fermion components must be a multiple of 2^{d-1}, where d is the space-time dimension. This "explanation" of the number of dimension is potentially useful for the explanation for the number of dimension put forward by one of us (S.N.M.B.) since long in the Spin-Charge-Family theory, and leads like the latter to typically (a multiple of) 4 families.

And this is the second purpose for our work on the fermionization in an arbitrary number of dimensions - namely to learn how "natural" is the inclusion of the families in the way the Spin-Charge-Family theory does.

Povzetek. Eden od namenov tega prispevka je pokazati, da je za brezmasne bozone mogoče postaviti teorijo, ki je glede na energijo in gibalno količino lahko tudi teorija brezmasnih fermionov v poljubno razsežnih prostorih. Bozoni so v tej teoriji opisani z $2^{d-1}-1$ realnimi polji Kalb-Ramond-ove vrste, za ekvivalentna fermionska polja, ki so kompleksi Weylovi spinorji, pa Aratyn-Nielsen-ov teorem zahteva, da se pojavijo v sodo razsežnih prostor-časih d v $2^{d/2-1}$ družinah, ker mora biti po tem teoremu razmerje bozonskih in fermionskih polj enako $\frac{2^{d-1}-1}{2^{d-1}} = \frac{\#bozonov}{\#fermionov}$.

Pojav sodega števila družin, ki ga zahteva ta teorija fermionizacije bozonov (ali ekvivalentno bozonizacije fermionov), pritrjuje teoriji spinov-nabojev-družin, ki jo je postavila soavtorica tega prispevka, in ki napoveduje, da je število družin cel mnogokratnik števila 4. Ta prispevek pritrdi teoriji spinov-nabojev-družin, da je pojav družin fermionov v naravi osnovnega pomena.

10.1 Introduction

This is the first draft to the paper, prepared so far only to appear in the Proceedings as the talk of one of the authors (H.B.F.N.). Although many things are not yet strictly proven, the fermionization/bosonization seems, hopefully, to work in any dimensional space-time and also, hopefully, in the presence of a weak background field. We hope, that the fermionization/bosonization procedure might help to better understand why nature has made of choice of spins, charges and families of fermions and of the corresponding gauge and scalar fields, observed in the low energy regime and why the Spin-Charge-Family theory [7,6] might be the right explanations for all the assumptions of the Standard Model.

This talk demonstrates that:

- Bosonization/fermionization is possible in an arbitrary number of dimensions (although the fermions theories are non-local due to the anticommuting nature of fermions, while bosons commute).
- The number of degrees of freedom for fermions versus bosons obeys in our procedure in any d the Aratyn-Nielsen theorem [1].
- The number of families in four dimensional space-time is (a multiple of) four families.

To prove for massless fermions and bosons that the bosonization/fermionization is possible in an arbitrary number of dimensions we use the Jacoby's triple product formula, presented by Leonhard Euler in 1748 [3] and is a special case of Glaisher's theorem [5]

$$\frac{1}{2}\prod_{n=0,1,2,\ldots}(1+x^n) = \prod_{m=1,3,5,\ldots}\frac{1}{1-x^m}\,. \tag{10.1}$$

Let the reader notices that the product on the left hand side runs over 0 and all positive integers, while on the right hand side it runs only over odd positive integers. One can recognize also that for all positive numbers the number of partitions with odd parts equals the number of partitions with distinct parts. Let us demonstrate this in a special case:

Among the 22 partitions of the number 8 there are 6 that contain only odd parts, namely

(7 + 1, 5 + 3, 5 + 1 + 1 + 1, 3 + 3 + 1 + 1, 3 + 1 + 1 + 1 + 1 + 1,
1 + 1 + 1 + 1 + 1 + 1 + 1 + 1).

If we count partitions of 8, in which no number occurs more than once, that is with distinct parts, we obtain again 6 such partitions, namely

(8, 7 + 1, 6 + 2, 5 + 3, 5 + 2 + 1, 4 + 3 + 1).

For every type of restricted partition there is a corresponding function for the number of partitions satisfying the given restriction. An important example is q(n), the number of partitions of n into distinct parts [4]. The generating function for q(n), partitions into distinct parts, is given by

$$\sum_{n=0}^{\infty} q(n)x^n = \prod_{k=1}^{\infty}(1+x^k) = \prod_{k=1}^{\infty}\frac{1}{1-x^{2k-1}}\,. \tag{10.2}$$

The first few values of q(n) are (starting with q(0)=1):

(1, 1, 1, 2, 2, 3, 4, 5, 6, 8, 10,

The pentagonal number theorem can be applied giving a recurrence for q [4]:

$$q(k) = ak + q(k-1) + q(k-2) - q(k-5) - q(k-7) + q(k-12) + q(k-15) - q(k-22) - \ldots \quad (10.3)$$

where ak is $(1)^m$, if $k = (3m^2 - m)$ for some integer m, and is 0 otherwise.

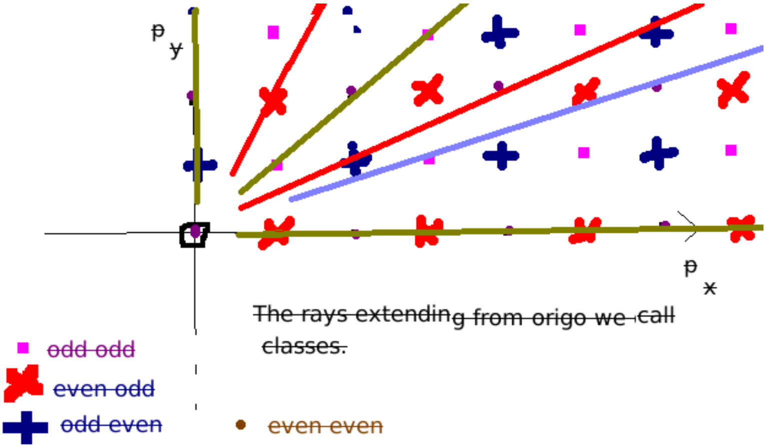

Fig. 10.1. Bosonization Illustrating Formula: the d_{space} space dimensional version (for only a "quadrant") is presented.

$$\frac{1}{2} \prod_{(m_1, m_2, \ldots, m_{d_{space}}) \in \mathbf{N}_0^{d_{space}}} (1 + x_1^{m_1} x_2^{m_2} \cdots x_{d_{space}}^{m_{d_{space}}}) = \\ = \prod_{\substack{(n_1, n_2, \ldots, n_{d_{space}}) \in \mathbf{N}_0^{d_{space}} \\ \text{but not all } n_i\text{'s even}}} \frac{1}{1 - x_1^{n_1} x_2^{n_2} \cdots x_{d_{space}}^{n_{d_{space}}}} \quad (10.4)$$

$$\frac{1}{2} \prod_{(m_1, m_2, \ldots, m_{d_{space}}) \in \mathbf{Z}^{d_{space}}} (1 + x_1^{m_1} x_2^{m_2} \cdots x_{d_{space}}^{m_{d_{space}}} z^{\sqrt{m_1^2 + m_2^2 + \ldots + m_{d_{space}}^2}}) = \\ = \prod_{\substack{(n_1, n_2, \ldots, n_{d_{space}}) \in \mathbf{Z}^{d_{space}} \\ \text{but not all } n_i\text{'s even}}} \frac{1}{1 - x_1^{n_1} x_2^{n_2} \cdots x_{d_{space}}^{n_{d_{space}}} z^{\sqrt{n_1^2 + n_2^2 + \ldots + n_{d_{space}}^2}}}. \quad (10.5)$$

The Idea for the Procedure for a Proof of the Multidimensional Bosonization Formula

- 1. Divide the whole system of all the discretized momentum vectors into "classes" of proportional vector (meaning in practice vectors deviating by a rational factor only), or rays (we might call them the rays of the module).
- 2. For each "class" the proof is given by the 1+1 dimensional case which means by just using the formula by Euler and extending it to both positive and negative integers.

Thinking of the Formulas of Bosonization as Products over Rays/Classes

$$\prod_{c\in rays}\;\prod_{m\in \mathbf{Z}, m\neq 0} (1 + x_1^{m_1(c)*m} x_2^{m_2(c)*m} \cdots x_{d_{space}}^{m_{d_{space}}(c)*m} z^{\sqrt{m_1^2(c)+m_2^2(c)+\ldots+m_{d_{space}}^2(c)}\;*|m|}) =$$

$$= \prod_{c\in rays}\;\prod_{n\ \mathrm{odd}} \frac{1}{1 - x_1^{n_1(c)*n} x_2^{n_2(c)*n} \cdots x_{d_{space}}^{n_{d_{space}}(c)*n} z^{\sqrt{n_1^2(c)+n_2^2(c)+\ldots+n_{d_{space}}^2(c)}\;*|n|}}\,.$$

where c runs over the set $rays$ of the d_{space}-tuples of non-negative integers, that cannot be written as such a tuple multiplied by an over all integer factor.

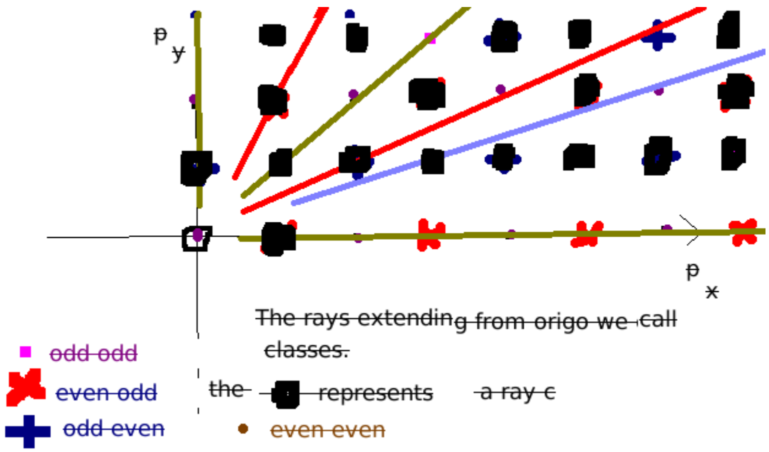

Fig. 10.2. Splitting the Fock space into Cartesian Product Factors from Each Ray c.

Denoting the Fock space for the theory - it be a boson or a fermion one - as $\mathcal{H}$ for the d_{space}-dimensional theory, and by $\mathcal{H}_c$ the Fock space for the - essentially 1 + 1 dimensional theory associated with the ray/or class c describing the particles with momenta being an integer (though not 0) times the representative for c, namely $(m_1(c), m_2(c), ..., m_{d_{space}}(c))$ - it is suggested that we write the full Fock space as the product

$$\mathcal{H} = \otimes_{c \in RAYS} \mathcal{H}_c. \tag{10.6}$$

Introduction of Creation and Annihilation Operators

We shall introduce for a boson interpretation of the Hilbert space the Fock space $\mathcal{H}$:

$$a(n_1, n_2, ..., n_{d_{space}}) \; \textit{annihilates} \text{ a boson with momentum } (n_1, n_2, ..., n_{d_{space}}),$$
$$a^\dagger(n_1, n_2, ..., n_{d_{space}}) \quad \textit{creates} \quad \text{a boson with momentum } (n_1, n_2, ..., n_{d_{space}}),$$

where the integers can be any, except that they must *not* all d_{space} ones be even.

Similarly for fermions:

$$f(n_1, n_2, ..., n_{d_{space}}) \; \textit{annihilates} \text{ a fermion with momentum } (n_1, n_2, ..., n_{d_{space}}),$$
$$f^\dagger(n_1, n_2, ..., n_{d_{space}}) \quad \textit{creates} \quad \text{a fermion with momentum } (n_1, n_2, ..., n_{d_{space}}),$$

where now the n_i numbers can be any integers.

Boson Operators Dividable into rays or classes c, also Fermions Except for one Type

We can write any "not all even" (discretized) momentum $(n_1, n_2, ..., n_{d_{space}})$ as an *odd* integer n times a representative for a class/ray c

$$a(n_1, n_2, ..., n_{d_{space}}) = a(n_1(c) * n, n_2(c) * n, ..., n_{d_{space}}(c) * n),$$
$$a^\dagger(n_1, n_2, ..., n_{d_{space}}) = a^\dagger(n_1(c) * n, n_2(c) * n, ..., n_{d_{space}}(c) * n)$$

The boson momentum with a given even/odd combination for its momentum components (say oe...o) goes to a ray/class c with the *same combination of even/oddness*.

Similarly one can proceed also for fermions with not all momentum components even; but the fermion momenta that have all components even *get divided into rays/classes with different even/odd combinations*. There are no rays with the even combination ee...e, of course, because a tuple of only even numbers could be divided by 2.

10.2 Thoughts on Construction of Fermion Operators

We have made an important step arriving at a model suggesting how it could be possible to match momenta and energies for a system with either fermions or bosons. To completely show the existence of fermionization (or looking the opposite way, bosonization) we should, however, write down the formula for how the fermion (boson) creation and annihilation operators are constructed in terms of the boson (fermion) operators, so that it can become clear (be proven) that the phase conventions and identification of the specific states with a given total momentum and energy for fermions can be identified with specific states for the boson system.

Such a construction is well known for 1+1 dimensions, where it looks like

$$\psi_e(x) + i\psi_o(x) = \exp(i\phi_R(x)) \tag{10.7}$$

in the "position" representation, meaning that

$$\psi_e(x) = \sum_{m \text{ even}} \exp(imx)\; b_e(m) \tag{10.8}$$

$$\psi_o(x) = \sum_{m \text{ odd}} \exp(imx)\; b_e(m) \tag{10.9}$$

$$\phi_o(x) = \sum_{m \text{ odd}} \exp(imx)\; a_e(m)\,. \tag{10.10}$$

Let us think of the case of making the field operators in position space

$$\phi_e(x),\; \phi_o(x),\; \psi_o(x)$$

Hermitean by assumming

$$a_o(m) = a_o^{\dagger}(-m);\;\; \text{for all } m \text{ odd}, \tag{10.11}$$

$$b_o(m) = b_o^{\dagger}(-m);\;\; \text{for all } m \text{ odd}, \tag{10.12}$$

$$b_e(m) = b_e^{\dagger}(-m);\;\; \text{for all } m \text{ even}\,. \tag{10.13}$$

10.2.1 Problem of Extending to Higher Dimensions Even if we Have Bosonization Ray for Ray

At first one might naively think that - since each of our rays (or classes) c functions as the 1+1 dimensional system and we can write the whole fermion, as well as the whole boson, space according to (10.6) - it would be trivial to obtain the bosonization for the whole system and thereby have achieved the bosonization in the arbitrary dimension, which is the major goal of this article.

However, one should notice that constructing in a simple way a system composed from several independent subsystems such it is the whole system $\mathcal{H}$, composed from the subsystems $\mathcal{H}_c$ (for $c \in rays$), one obtains *commutation* between operators acting solely inside one subsystem c, say, and operators acting solely inside another subsystem c', say. But we want for the fermions the *anticommutation* relations rather than the commutation ones, and thus some (little ?) trick is needed to achieve this anticommutation.

First we shall show how this anticommutation can be achieved by means of an ordering of all the rays $c \in rays$ by some ordering inequality being chosen between these rays: $>$. But this is a very ugly procedure and we shall develop a slightly more general attempt in which we construct a phase $\delta(c, c')$ for each pair of rays c and c'. Then we shall go on seeking to make the choice of this phase $\delta(c, c')$ in a continuous and more elegant way. Since that shall turn out to be non-trivial, we shall develop the ideas by first seeking for such a construction of the phase for the odd dimensional space of $d = 3$, meaning $d_{space} = 2$, to learn the idea, although we are most keen on even space-time dimensions, such it is the experimentally observed number of space-time dimensions, $d = 4$.

10.2.2 The > Ordered Rays Construction

Let us suppose that we have a formal way of constructing the fermion creation and annihilation operators in terms of the boson operators. We do indeed have such a construction, since we can Fourier transform back and forth the construction in the position representation (10.7) and the 1+1 dimensional bosonization is so well understood. Since for the present problem the details of this 1+1 dimensional bosonization relations are not so important, we shall just assume that we are able to deduce for each ray or class c a series of fermion creation - $b^{\dagger}_{naive\ o}(m, c)$ and $b^{\dagger}_{naive\ e}(m, c)$ - and annihilation - $b_{naive\ o}(m, c)$ and $b_{naive\ e}(m, c)$ - operators, that function well as fermion operators *inside the ray* c, so to speak. o and e denotes odd and even respectively. The only important thing is that these operators *can be expressed in terms of the bosons annihilation and creation operators belonging to the same ray* c:

$$b^{\dagger}_{naive\ o}(m, c) = b^{\dagger}_{naive\ o}(m, c; a_o(n, c)\,,\ \text{for } n \text{ odd}), \tag{10.14}$$
$$b_{naive\ o}(m, c) = b_{naive\ o}(m, c; a_o(n, c)\,,\ \text{for } n \text{ odd}), \tag{10.15}$$
$$b^{\dagger}_{naive\ e}(m, c) = b^{\dagger}_{naive\ e}(m, c; a_o(n, c)\,,\ \text{for } n \text{ odd}), \tag{10.16}$$
$$b_{naive\ e}(m, c) = b_{naive\ e}(m, c; a_o(n, c)\,,\ \text{for } n \text{ odd})\,. \tag{10.17}$$

For these operators we know form the 1+1 dimensional bosonization that we can take them to obey the usual anticommutation rules *provided we keep to only one ray* c:

$$\begin{aligned}
&\{b^{\dagger}_{naive\ o}(m, c; a_o(n, c)\,,\ \text{for } n \text{ odd}), b^{\dagger}_{naive\ o}(p, c; a_o(n, c)\,,\ \text{for } n \text{ odd})\}_{-}\\
&\qquad = \delta_{n,-p}\,,\ \text{for } m, p \text{ both odd}\,,\\
&\{b_{naive\ o}(m, c; a_o(n, c)\,,\ \text{for } n \text{ odd}), b^{\dagger}_{naive\ o}(p, c; a_o(n, c)\,,\ \text{for } n \text{ odd})\}_{-}\\
&\qquad = \delta_{n,p}\,,\ \text{for } m, p \text{ both odd}\,,\\
&\{b_{naive\ o}(m, c; a_o(n, c)\,,\ \text{for } n \text{ odd}), b_{naive\ o}(p, c; a_o(n, c)\,,\ \text{for } n \text{ odd})\}_{-}\\
&\qquad = \delta_{n,-p}\,,\text{for } m, p \text{ both odd}\,.
\end{aligned}$$

We have similar anticommutation rules for annihilation and creation operators if exchanging the index o (meaning odd) by the index e (meaning even), but now we should take into account that the fermion operators with zero momentum, i.e. $m, p = 0$, are not constructed from a single ray c. Rather there are - referring to our little problem with the explicit factor $\frac{1}{2}$ in the state counting formulae - not enough degrees of freedom in the 1+1 dimensional boson system to deliver a fermion operator with a zero momentum.

We should therefore imagine that we do not have these zero momentum fermion operators attached to our rays either. This is actually good for our hopes of bosonizing in higher dimensions because the zero momentum fermion operators would have had to be common for the infinitely many rays and we would have had too many candidates for the zero momentum fermion mode. Now instead we totally miss the zero momentum creation and annihilation fermion operators for the many dimensional system. That is, however, not at all so bad as it would have been to get an infinity of them, because we fundamentally can not expect to produce all fermion operators from boson ones because we cannot possibly build up a sector with an odd number of fermions from boson operators acting on say some vacuum with an even number. Therefore one fermion operator must be missing. This becomes the zero momentum one and that is o.k..

Our real problem remains that these naive fermion operators taken for two different rays c and c' will commute

$$\begin{aligned} &\{b^{\dagger}_{naive\ o}(m, c; a_o(n, c)\text{ , for } n \text{ odd}), b^{\dagger}_{naive\ o}(p, c'; a_o(n, c')\text{ , for } n \text{ odd})\}_{-} \\ &\qquad = 0 \qquad \text{for } m, p \text{ both odd} \\ &\quad \text{etc.}\,. \end{aligned} \tag{10.18}$$

We could define an $(-1)^F$-operator, where F is the fermion number operator. It sounds at first very easy just to write

$$F_c = \sum_m b^{\dagger}_{naive\ o}(m, c)\ b_{naive\ o}(m, c) + \sum_m b^{\dagger}_{naive\ e}(m, c)\ b_{naive\ e}(m, c), \tag{10.19}$$

where the sums run over respectively the odd and the even positive values for m for the o and the e components. But now this fermion number operator- as taken as a function of the *naive* operators - ends necessarily up being an expression in purely boson operators (from the ray c), and thus it looks at first as being valid except when the expression $(-1)^{F_c}$, which we are interested in, is equal to 1 on all states that can truly be constructed from boson operators. If it were indeed so, our idea of using $(-1)^{F_c}$ to construct the multidimensional fermion operators, would not be so good. However, there is a little detail that we did not have enough bosonic degrees of freedom to construct the zero momentum fermion operator in 1+1 dimensions. Therefore we can not really include in the definition of the "fermion number operator for the ray c", F_c, the term coming from $m = 0$. This term would formally have been $b^{\dagger}_{naive\ e}(m = 0, c)\ b_{naive\ e}(m = 0, c)$, but we decided to leave it out. This then means that the fermion number operator, for which we decide to use F_c as the number of fermions operator in the ray c is *not the full fermion number operator for the corresponding 1+1 dimensional theory, but rather only for those fermions, that avoid the zero momentum state*. To require this avoidance of the zero momentum is actually very attractive for defining a fermion number operator *for the ray* c as far as the momentum states included in such a ray really must exclude the zero momentum state in a similar way as a ray in a vector space is determined from the set of vectors in the ray not being zero.

But this precise definition avoiding the zero-momentum fermion operator contribution to the fermion number operator F_c leads to the avoidance of the just mentioned problem that this fermion number F_c looked as always having to be even when constructed in terms of boson operators.

Now there should namely be enough boson degrees of freedom that one should be able to construct by boson operators all the different possible combinations for fermion states being filled or unfilled (still not the zero momentum included). Thus one does by pure bosons construct both - the even F_c and the odd F_c - states and thus the F_c with the zero momentum fermion state not counted can indeed be a function of the boson operators and can *take on both even and odd values* for momentum, depending on the boson system state. So, we can have - using this leaving out the zero momentum fermion state in the rays - an operator

$$F_c = F_{naive\ c}(a_e(n)\,,\ \text{for } n \text{ odd}) \tag{10.20}$$

The operator $(-1)^{F_c}$ for each ray c counts if the number of fermions in the 1+1 dimensional system is even, then $(-1)^{F_c} = 1$, or odd, then $(-1)^{F_c} = -1$. We construct the following improved fermion operator (annihilation or creation),

$$b_e(m, c) = b_{naive\ e}(m, c) \prod_{c'<c} (-1)^{F_{c'}}. \tag{10.21}$$

The inclusion of this extra operator factor helps to convert the commutation relations between the fermion annihilation and creation operators for different rays into anticommutation relations, as it can easily be seen

$$\begin{aligned}
&b_e\,(m, c)\ b_e\,(p, c') = \\
&b_{naive\ e}\,(m, c) \cdot \prod_{c''<c} (-1)^{F_{c''}}\ \ b_{naive\ e}\,(p, c') \cdot \prod_{c'''<c'} (-1)^{F_{c'''}} = \\
&b_{naive\ e}\,(m, c) \cdot \prod_{c'\le c''<c} (-1)^{F_{c''}}\ \ b_{naive\ e}\,(p, c')\ = \\
&-b_{naive\ e}\,(m, c)\ \ b_{naive\ e}\,(p, c') \cdot \prod_{c'<c''<c} (-1)^{F_{c''}} = \\
&-b_{naive\ e}\,(p, c')\ \ b_{naive\ e}\,(m, c) \cdot \prod_{c'<c''<c} (-1)^{F_{c''}} = \\
&-b_{naive\ e}\,(p, c') \cdot \prod_{c'''<c'} (-1)^{F_{c'''}}\ \ b_{naive\ e}\,(m, c) \cdot \prod_{c''<c} (-1)^{F_{c''}} \\
&= -b_e(p, c')\ b_e(m, c)\,,\ \text{still for } c > c'\,.
\end{aligned} \tag{10.22}$$

Thus we deduced, for $c > c'$ in our in fact at first just chosen ordering of $<$, that the fermion operators do anticommute. It is not difficult to show similarly also in the case $c' > c$, that the fermion operators anticommute. The crux of the matter is that when e.g. $c' > c$ there is the factor $(-1)^{F_c}$ contained in the product $\prod_{c''<c'}(-1)^{F_{c''}}$, which is attached to $b_{naive\ o}\,(m, c')$ in order to correct it into $b_o\,(m, c')$, while there is no analogous factor $(-1)^{F_{c'}}$ contained in the factor $\prod_{c''<c}(-1)^{f_{c''}}$ attached at $b_{naive\ o}\,(m, c)$ in order to bring it into $b_o\,(m, c)$. In this way one gets just the one extra minus sign in the product of the fermion operators that makes them anticommute.

10.2.3 Slight Generalization to have a Phase Factor

It is not difficult to see that the idea of using such an ordering < could be slightly generalized to have instead of the factors only minus or plus phase factors of the form $\exp(\delta(c, c'))$

$$b_e^\dagger(m, c) = b_{naive\ e}^\dagger(m, c) \prod_{c' \neq c,\ \text{but}\ c' \in rays} e^{(i\delta(c,c')F_{c'})}. \quad (10.23)$$

It is also not difficult to see that, in order to obtain the anticommutation relations instead of the commutation ones (which we have for $b_{naive\ e}^\dagger(m, c)$), the phases must obey the rule

$$\delta(c, c') - \delta(c', c) = \pi(\text{mod } 2\pi). \quad (10.24)$$

We may in fact seek to evaluate the product of two fermion creation operators with the ansatz (10.23)

$$\begin{aligned}
& b_e^\dagger(m, c)\ \ b_e^\dagger(m', c') = \\
& = b_{naive\ e}^\dagger(m, c) \prod_{\substack{c'' \neq c, \\ \text{but} \\ c'' \in \text{rays}}} e^{i\delta(c,c'')\,F_{c''}}\ \ b_{naive\ e}^\dagger(m', c') \prod_{\substack{c''' \neq c', \\ \text{but} \\ c''' \in \text{rays}}} e^{i\delta(c',c''')F_{c'''}} \\
& + b_{naive\ e}^\dagger(m, c)\ e^{i\delta(c,c')} \prod_{\substack{c'' \neq c \\ \text{nor } c', \text{but } c'' \in \text{rays}}} e^{i(\delta(c,c'')+\delta(c',c''))F_{c''}}\ \ b_{naive\ e}^\dagger(m', c') \\
& = b_{naive\ e}^\dagger(m, c)\ e^{i\delta(c,c')F_{c'}}\ \ b_{naive\ e}^\dagger(m', c')\ e^{i\delta(c',c)F_c} \\
& \cdot \prod_{\substack{c'' \neq c \\ \text{nor } c', \text{ but } c'' \in \text{rays}}} e^{i(\delta(c,c'')+\delta(c',c''))F_{c''}} \\
& = e^{i(\delta(c,c')-\delta(c',c))}\ b_{naive\ e}^\dagger(m', c')\ e^{i\delta(c',c)F_c}\ \ b_{naive\ e}^\dagger(m, c)\ e^{i\delta(c,c')F_{c'}} \\
& \cdot \prod_{\substack{c'' \neq c \\ \text{nor } c', \\ \text{but } c'' \in \text{rays}}} e^{i(\delta(c,c'')+\delta(c',c'')} F_{c''}) = e^{i(\delta(c,c')-\delta(c',c))}\ b_e^\dagger(m', c')\ \ b_e^\dagger(m, c) = \\
& = -b_e^\dagger(m', c')\ \ b_e^\dagger(m, c),
\end{aligned} \quad (10.25)$$

where in the last step we used (10.24). Thus we see that in this way we can get - really in infinitely many ways - some algebraicly defined fermion operators that do indeed anticommute as they should. But it should be had in mind that both these procedure, by choosing $\delta(c, c')$ and the forgoing proposal with the ordering <, are a priori discontinuous and arbitrary.

We expect, however, that the latter method with $\delta(c, c')$ can be lead to a smooth and attractive scheme in the case of $d_{space} = 2$ or equivalently $d = 3$.

10.2.4 Exercise with Next to Simplest Case $d_{space} = 2$

In the case of $d_{space} = 2$ we can say that the set of our rays $rays$ form a kind of a set of "rational angles" in the sense that each ray specifies modulo π (rather than modulo 2π as for an oriented arrow it would specify) an angle, but that one only obtains those angles which rationalize tangenses. But the fact that they are after all implemented as angles - although only modulo π, means that they are at least locally ordered as numbers along a real or rather rational axis. So apart from troubles at the end and beginning we have an ordering and we could attempt to use it even for the implementation of the ordered set of rays method by proposing a "nice" $<$ ordering. However, we think we get a better chance by using the $\delta(c, c')$ method in this $d = 3$ and thus $d_{space} = 2$ case.

We have to think about what topological properties we shall and can achieve for the function $\delta(c, c')$ depending on a pair of rays c and c'.

Since the classes or rays are "a kind of rational" directions, though without orientation, the topological space of the rays is like the sphere $S^{d_{space}-1}$ with opposite points identified. This topological space obtained by the identification of the opposite point on the $S^{d_{space}-1}$ sphere is actually topologically identical to the projective space of $d_{space} - 1$ dimensions. For the case $d = 3$ or $d_{space} = 2$ the topological space rays thus becomes simply the projective line (using real numbers), but that is topologically just the S^1 circle. Had this topological space been naturally orderable we could have used the ordering as the $<$ above. However, it is a circle S^1 and not a simple line with plus and minus infinity; the infinities have so to speak been identified to only one point in the projective line. This means that using the method to define the fermion fields/operators by means of $<$-method would be very non-elegant, and would probably violate almost everything wanted.

Let us now think about a slight generalization by using the $\delta(c, c')$. We need to make a choice of a function $\delta(\cdot, \cdot)$ defined on the cross product of two projective spaces of dimension $d_{space} - 1$ each. Since it shall obey the condition (10.24), it cannot at all be a smooth or continuous function at the points where $c = c'$. Let us, for a while, take care that this method works well for $d = 3$ only.

In this $d = 3$ case the cross product of the two projective lines becomes topologically simply a two-dimensional torus. So we face topologically to define $\delta(c, c')$ on a two-dimensional torus. However, we are forced to give up having continuity along the "diagonal" - meaning the set of points on this torus with $c = c'$ - and it is thus rather a $\delta(c, c')$ defined as a continuous function on the torus minus its "diagonal", which we must choose/find.

This two dimensional torus minus its "diagonal" is rather like a belt. I.e., it is topologically like the outer surface of a finite piece of a tube. It has two separate edges, each being topologically an S^1 circle, namely two images of the "diagonal" seen from the two sides. In between there is then the two-dimensional bulk area of the topological shape of the surface of the finite piece of a tube. It is inside this bulk region that we shall attempt to construct $\delta(c, c')$ to be smooth and "nice". Choosing

$$\delta(c, c') = 2\text{"clock average angle"}(c, c') \tag{10.26}$$

might be a good choice. directions c and c' forms with some coordinate axis (in momentum space). The precise way of defining this "clock average angle"(c, c') is illustrated on the figure 10.3 and consists in the following (let us remind the reader that we are still in the $d = 3$; $d_{space} = 2$ case):

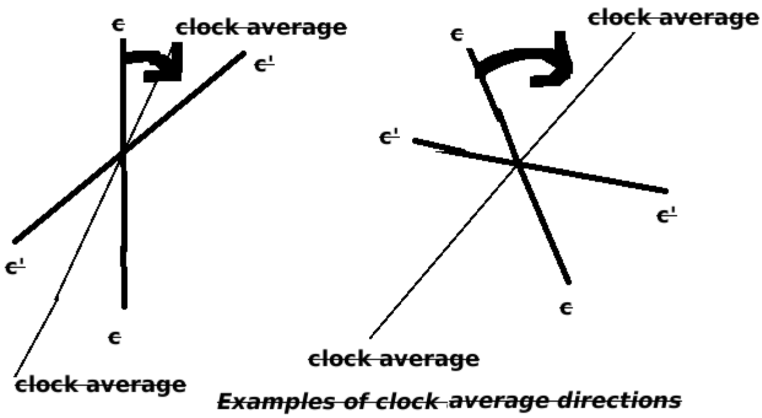

Fig. 10.3. Examples of clock average directions.

- a. We introduce a ″clockwise rotation orientation″ in the spatial momentum plane.
- b. We draw a circle arrow from one of the two ″ends″ (half lines) of which the line c (the ray c is basically just a line) in this clockwise direction, and note the angle between this end of c and the first ″end″ (=half line) of c' (met in the following the circular arrow), which measures less than 180^0.
- c. We draw a line, that divide this under b. noted angular region into halves. This line through the (momentum space) origo is denoted ″clock average″ (as marked on the figure).
- d. Such an unoriented line as the ″clock average″ defines relative to a coordinate system in spatial momentum space an angle-value modulo π. We call this angle-value "clock average angle"(c, c') and it is as just said defined modulo π (but only modulo π, because the line "clock average" is unoriented).
- e. Multiplying this angle - "clock average angle"(c, c') - by 2 its ambiguity to be only defined modulo π becomes instead an ambiguity modulo 2π. Thus our proposed expression (10.26) for $\delta(c, c')$ is defined modulo 2π, and that is what we need, since in our construction we exponentiate $\delta(c, c')$ after multiplication by i and an operator $F_{c'}$ that has only integer eigenvalues. Thus the expression, which we use, $\exp(i\delta(c, c')F_{c'})$ becomes well defined even though 2″clock average angle″ (c, c') makes sense only modulo 2π.

Let us see whether this proposal is indeed is a good one. To see that our proposal (10.26) is a good one we must first of all check that it obeys (10.24). That is we must see what happens to the expression when we permute the two independent variables c and c'. Since by definition the circular arrow constructed in step b. goes out from the c-line, the first of the two arguments in $\delta(c, c')$, we must draw this circle-arrow after the permutation from c' instead. Therefore the half-angle noted under point b. above will after the permutation differ from the one before the permutation. This means that the line (through the origo) ″clock average″ gets after the permutation perpendicular to its direction before the permutation of c and c'. Ttherefore "clock average angle"(c', c) = "clock average angle"$(c, c') + \pi/2 (\mathrm{mod}\ \pi)$, which means that this angle gets shifted modulo π with $\pi/2$. After the multiplication by 2 (point e.) it means that $\delta(c', c) = \delta(c, c') + \pi(\mathrm{mod}\ 2\pi)$, which is just (10.24). Thus we got indeed by proposal (10.26) the condition (10.24) fulfilled.

We can now remark that quite obviously our proposal (10.26) is continuous as function of the directions c and c' *except where* c *and* c' *just coincide - what means that it is zero (mod* π*) angle between them*.

Let us note that had we not chosen the clock-wise rule, but instead taken, say, the smallest angle between c and c' and just found the halfening line between those "ends", we would have got a discontinuity when c and c' were perpendicular to each other. But by our precise choice we avoided that singularity. (For a point close to the diagonal the two arguments, c and c', are approximately the same ray. Permuting them will for a continuous function $\delta(c, c')$ make almost no difference, and thus it cannot possibly change by π, while crossing the "diagonal" the function δ would ask to jump by π.)

10.3 A Guess for Arbitrary Dimension

We propose the generalization of Eq. (10.7) to an arbitrary dimension, due to our experience with the Clifford objects (apart from some modifications due to whether we choose Weyl or Majorana fermions for family or for geometrical components), by using the relation

$$(\psi + \psi_\mu \gamma^\mu + \psi_{\mu\nu}\gamma^\mu\gamma^\nu + ... + \psi_{1235\ldots d}\,\Gamma^{(d-1)} = e^{\phi_\mu\gamma^\mu + \phi_{\mu\nu}\gamma^\mu\gamma^\nu + \ldots + \phi_{1235\ldots d}\,\Gamma^{(d-1)}}\,. \quad (10.27)$$

10.4 Outlook on Supporting the Spin-Charge-Family theory [7,6]

We started with massless noninteracting bosons or fermions. But we like to work with the interacting fields. There are many Kalb-Ramond fields appearing in our type of fermionizable boson model in higher dimensions and correspondingly it is not easy to see how to make an interacting theory.

There are many ways to come from noninteracting bosonisable (fermionizable) fermion (boson) fields, which might lead to the fermion fields interacting with the boson fields as it is in the spin-charge-family theory.

But on the level of our fermionizable (bosonizable) boson (fermion) model with many Kalb-Ramond fields we must keep in mind that the conserved charges in the Kalb-Ramond theories are vectorial and thus one gets very many vectorial conserved quantities. This makes scattering processes (unless all the scattering particles are without these vectorial charges) very non-trivial.

One chance would be to let either fermion or boson fields to interact with gravity. Crudely speaking gravity couples to energy and momentum, and since in the free bosonization procedure we have at least sought to get the total d-momentum be the same in the corresponding states of fermions and bosons there might be a chance that we fermionize a theory with both - the bosons of the Kalb-Ramond type *and* gravity through the vierbein formulation - and correspondingly obtain a theory with both fermions *and* bosons, the later would be the gravity degrees of freedom. This might lead to exactly the theory [6,7] that one of us (N.S.M.B.) has postulated as the true model for Nature beyond the standard model (the spin-charge-family theory).

Since our scheme a priori looks to require the Majorana fermions to have real fields like the bosons - at least in the simplest version - we only expect to get chiral fermions in those dimensions wherein Majorana fermions can simultaneously be Weyl (=chiral)as in d=2,6,10,14,... It is therefore even a slight support for the spin-charge-family theory that its phenomenologically favoured dimension is just 13+1 =14.

One should for appreciating this idea of adding gravity without fermionizing it have in mind that one *does not have to bosonize all degrees of freedom,* but rather can - if one wishes - decide to fermionize some degrees of freedom but not all. Especially, if the motivation were to make all fermions from bosons because one claims that fermions are not properly local and should not be allowed to exist, then of course it is enough that we start with a purely boson theory as the fundamental one - and then we better only fermionize a part of bosons unless we could identify a purely fermionic theory with nature. But of course there seemingly are bosons in nature and we thus must end phenomenologically with a theory with *both* bosons and fermions.

Starting from fundamental bosons only that is only achievable by only a *partial fermionization*.

Hope for the Progress

The hope is, which is evidently from we haveproposed in this contribution, that we shall construct formulas for the higher dimensional cases by generalizing the formulas we already have for the one dimensional case, generalizing as well the ”classes” to higher dimensions. In the spirit of seeking to identify the fields characterized by their ”odd/even” indices with spin components, we hope to derive from the bosonization formula a scheme formally stating the relation between the boson and the fermion second quantized fields, $2^{d_{space}} - 1$ boson field components, while there will be $2^{d_{space}}$ fermion components.

10.5 Outlook on the Connection to the Spin-Charge-Family Theory

Let us try to clarify how the here discussed fermionization procedure is supposed to be, so to speak, the root for a theory beyond the Spin-Charge-Family theory of Norma Susana Mankoč Borštnik [7,6] (and her collaborators), The (one of) way we see as a very promising hope that one could justify this Spin-Charge-Family theory by the hoped fermionization is as follows:

We build up a model with only bosons as the fundamental theory in say - 13 +1 dimensions - in the sense that this 13 +1 dimensional purely bosonic theory with a series of the Kalb-Ramond fields and *with usual 13+1 dimensional gravity should be the fundamental choice of nature* (not necessarily starting in d=13+1). Then this theory should be *partly* fermionized in the sense that only the series of Kalb Ramond fields get fermionized, but not the gravity (bosonic) degrees of freedom. The latter remain gravitational degrees of freedom hopefully now functioning as gravity for the fermions that came out of the fermionization. The Spin-Charge-Family theory will show up out of the Kalb-Ramond components.

1. The first assumption of our new scheme, which might be the pre-scheme of the Spin-Charge-Family theory, is that *fermions a priori do not obey proper locality*. The accusation towards all the theories with fermions which are fundamental fermions rather than fermionized bosons is that the axiom of locality in a quantum field theory is for the fermions
$$\{\psi_\alpha(x), \psi_\beta(y)\}_+ = 0\,, \quad \text{for the space like separation of } y \text{ and } x, \tag{10.28}$$
while true physical locality should have been a *commutation rule* like the one obeyed by the boson fields
$$\{\phi(x), \phi(y)\}_- = 0\,, \quad \text{for the space like separation of } x \text{ and } y. \tag{10.29}$$
True locality means, one would think, that each little region in space is approximately a completely separate system that only interacts very indirectly with a far away different little region. If so, the physical operators describing the situation in one little region should commute with those describing the situation in a different little region, and not anticommute as the fermion fields do. One might like to assume that only products of an even number of fermion fields are considered as proper operators describing the little system region, what satisfies the requirement of getting commutation relations between the field variables describing the situation in different regions. But such an assumption must be justified as a physical assumption, discussing seriously also odd products of fermion fields.
The point of view we suggest here is that we admit that we cannot have fermions at all in a truly local way! This then means that the fundamental physics should be a model *without fermions* so that all fermions come from bosons that become fermionized.
2. Since it is not easy to find so terribly many systems of bosons that can be fermionized, and thus if one finds some way of fermionizing, then this way is presumably already likely to be almost the only one possible.
At least we expect that the fermionization of a boson system of fields can only be made provided the number of fermions and the number of bosons agree with the theorem which one of us and Aratyn [1] put forward many years ago. For massless free fermions on the one side and massless free bosons on the other side we obtained that the number of components for the bosons and the fermions counted in the same way with respect to the fields being real or

complex, should be in the ratio

$$\frac{\#\text{fermion components}}{\#\text{boson components}} = \frac{2^{d_{space}}}{2^{d_{space}} - 1}, \tag{10.30}$$

where the dimension is $d = d_{space} + 1$, or the spatial dimension is d_{space}. The number of components - at least the number of real counted components - must of course be positive integer or zero. Thus the minimal number of fermion components must be $2^{d_{space}}$, while the number of boson components must be $2^{d_{space}} - 1$ or the numbers must be an integer multiplum of these numbers.
Alone this theorem of ours [1] makes appreciable restriction for when bosonization/fermionization is at all possible.

3. We are suggesting here the starting point with the bosonic degrees of freedom only, consisting of "series of the Kalb-Ramond fields, all the chain through, except for one (pseudo)scalar". By this we mean that we have as the bosons a series of separate fields $A_{\mu\nu\cdots\rho}$ with all the values of the number of indexes, antisymmetric with respect to all their indexes.
There is a simple way in which one could get the number $2^{d_{space}} - 1$ of boson components, if we arrange to have - by some gauge choice - only spatial values of the indexes μ, ν,... on the A-fields, removing the A field with zero indexes. The number of components become equal to the number of subsets of d_{space} letters, which is $2^{d_{space}}$. Removing pure scalar, we get this number $2^{d_{space}} - 1$, as we want for the theorem of[1].
4. From the $2^{d_{space}} - 1$ bosons represented by the Kalb-Ramond fields with the scalar removed, then according to the Aratyn *et al.* theorem [1] theorem there must be the $2^{d_{space}}$ components of fermion fields. This means for the Weyl spinor representation of fermion fields in even $d = d_{space} + 1$, with $2^{d/2-1}$ members that there are $2\times\, 2^{d/2-1}$ real fermion fields. To get $2^{d_{space}}$ real Weyl spinor representation fermion fields there must be $\frac{2^{d_{space}}}{2^{(d_{space}+1)/2}} = 2^{d/2-1} = 2^{d_{space}/2-1/2}$ families.
5. From the bosonization requirement we obviously get out that there must exist an even number of families as it also comes out from the Spin-Charge-Family theory of one of us [7,6].
6. But now there is correction due to the components of the KalbRamond fields with time indices, the 0. This gives very interesting corrections as we may postpone till later.

10.5.1 A Hope for that the Gravity Interaction Can Be Added

There is an interesting hope for that actually our at first free bosons being fermionized to free fermions could be generalized to have an universal coupling to a gravitational field - the bosonic field, which we do not fermionize, keeping it as gravitation, interactiing with the fermions - so that we finally arrive at a theory with several families of fermions and gravity.

Above we wrote down a formula for counting the number of states for the fermion and the boson systems having the same number of Fock states with given momentum and energy for the free massless case of our bosonization/fermionization.

We used in reality an infrared cut off that meant that we in fact considered a torus world with for different components different periodicity conditions: Some components of fields had antiperiodicity while the others had periodicity property along various coordinate directions.

We shall note now that we could consider these momentum eigenstates for the single particles with given periodicity restriction as *topologically* specified in the following sense:

The wave functions for the momentum eigenstates are as is well known all along taking on only pure phase factor values, i.e. they obey $|\phi(x)| = 1$ all along. The number of turns around zero, which they perform when one goes around the torus along the different coordinates, is an integer (or a half integer depending on the boundary condition). We can consider this number of turns going around the torus in different ways (along different coordinates) a topological quantity in the sense that it as an integer cannot change under a small deformation.

Our main idea is at this point that we in this way can introduce at least a not too strong gravitational field and still have single particle solutions to the equations of motion characterized by the same system of (topological) quantum numbers.

That should suggest that we have the same set up for making the in this work studied bosonization in a not too strong gravitational field as in the free case. We namely should be able to classify the single particle states as functions of the space-time variables x on the by gravitational fields deformed torus (torus due to infra red cut off) according to a topological classification in terms of the number of times the wave function encircles the value zero in the complex plane when the one follows a closed curve, following, say, the coordinates of the deformed torus. For the massless theory we have scale-invariance for the matter fields - the series of the Kalb-Ramond fields or the fermions - so, as long as we consider the gravitational field as a background field, i.e. we ignore the dynamics of the gravitational field itsef - we can scale up the momenta of the single particles by just letting the phase of an eigen-solution be scaled up by a factor. Only the periodicity conditions will enforce such scalings to be by integer factors, just as they must be also in the free flat case.

So we argue that with a background gravitational field, that is with a not too strong field, we have a possible description in terms of a discretized enumeration quite in the correspondence with the one for the flat case.

Remembering that we obtained the bosonization w.r.t. state counting in fact class c for class, meaning that the momentum eigenstates in the classes corresponding to rays went separately from boson to fermion or oppositely, we may have given arguments at least suggesting that a corresponding bosonization correspondence as the one in the free flat case also applies to the case with some (may be not too large though) gravitational field as a background field.

This may require further study but we take it that there is at least a hope for that the bosonization/fermionization procedure *can also be performed in a background gravitational field*.

Since we now with our expansions in power series seek to guarantee that we shall make the bosonization or fermionizations just in such a way that the d-momentum will be the same for the fermion configuration and the boson configuration corresponding to each other, we might hope that we could formulate the exact correspondence and the interpretations in terms of fields with spin indexes so that indeed the momentum densities would be the same for the fermions and for the corresponding bosons. If we succeed in that then the action on the gravitational fields which only feel the matter via the energy momentum tensor $T_{\mu\nu}$ would be the same for the bosons and the fermions in the corresponding states. In that case the development of the gravitational fields would be the same for the corresponding fermion and boson configurations. Thus the bosonization/fermionization procedure would truly have been made also in the with gravity interacting models. Just the gravity field itself should *not be fermionized*.

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11 Unified Description of Quarks and Leptons in a Multi-spinor Field Formalism

I.S. Sogami ⋆

Yukawa Institute of Theoretical Physics, Kyoto University
Department of Physics, Kyoto Sangyo University

Abstract. Multi-spinor fields which behave as triple-tensor products of the Dirac spinors and form reducible representations of the Lorentz group describe three families of ordinary quarks and leptons in the visible sector and an additional family of exotic dark quarks and leptons in the dark sector of the Universe. Apart from the ordinary set of the gauge and Higgs fields in the visible sector, another set of gauge and Higgs fields belonging to the dark sector are assumed to exist. Two sectors possess channels of communication through gravity and a bi-quadratic interaction between the two types of Higgs fields. A candidate for the main component of the dark matter is a stable dark hadron with spin 3/2, and the upper limit of its mass is estimated to be 15.1 GeV/c^2.

Povzetek. Avtor obravnava spinorje, ki se obnašajo kot trojni tenzorski produkt Diracovih spinorjev in tvorijo nerazcepno upodobitev Lorentzove grupe. Z njimi uspe opisati tri izmerjene družine kvarkov in leptonov, upodobitve pa ponudijo obstoj še četrte družine (imenuje jo ”nevidno”). Umeriotvenim poljem izmerjenih clanov družin in Higgsovega skalarnega polja doda ustrezna umeritvena vektorska polja in skalarno polje, s katerimi se sklopi ”nevidna” družina. Gravitacija in bikvadratna interakcij poskrbita za interakcijo med med obema tipoma Higgsovih polj. Kot kandidata za poglavitno sestavino temne snovi predlaga hadron s spinom 3/2. Zgornjo mejo za njegovo maso oceni na 15.1 GeV/c^2.

11.1 Introduction

The Standard Model (SM) has been accepted as an almost unique effective scheme for phenomenology of particle physics in the energy region around and lower than the electroweak scale. Nevertheless it should be considered that the SM is still in an incomplete stage, since its fermionic and Higgs parts are full of unknowns. It is not yet possible to answer the question why quarks and leptons exist in the modes of three families with the color and electroweak gauge symmetry $G = SU_c(3) \times SU_L(2) \times U_Y(1)$, and we have not yet found definite rules to determine their interactions with the Higgs field. It is also a crucial issue to inquire whether the SM can be extended so as to accommodate the degrees of freedom of dark matter.

To extend the SM to a more comprehensive theory which can elucidate its unknown features, we introduce the algebra, called *triplet algebra*, consisting of

⋆ sogami@cc.kyoto-su.ac.jp

triple-tensor products of the Dirac algebra and construct a unified field theory with the multi-spinor field, called *triplet field*, which behaves as triple-tensor products of the four-component Dirac spinor [1,2]. The chiral triplet fields forming reducible representations of the Lorentz group include the three families of ordinary quarks and leptons and also an additional fourth family of exotic quarks and leptons which are assumed to belong to the dark sector of the Universe.

The bosonic part of the theory consists of the ordinary gauge and Higgs fields of the G symmetry and also the dark gauge and Higgs fields of the new $G_\star = SU_{c\star}(3) \times SU_R(2) \times U_{Y_\star}(1)$ symmetry. While the gauge fields of the G symmetry interact with the ordinary quarks and leptons of the three families, the gauge fields of the $G_\star$ symmetry are presumed to interact exclusively with the quarks and leptons of the fourth family in the dark sector. The gauge fields of the extra color symmetry $SU_{c\star}(3)$ work to confine the dark quarks into dark hadrons. Apart from the ordinary Higgs field φ which breaks the electroweak symmetry $G_{EW} = SU_L(2) \times U_Y(1)$ at the scale Λ, another Higgs field $\varphi_\star$ is assumed to exist to break the left-right twisted symmetry $G_{EW\star} = SU_R(2) \times U_{Y_\star}(1)$ at the scale $\Lambda_\star$ ($\Lambda_\star > \Lambda$).

Our theory predicts existence of a stable dark hadron with spin 3/2 as a candidate for the main component of the dark matter. From a heuristic argument, we estimate the upper limit of its mass to be 15.1 GeV/c^2.

11.2 Triplet field and triplet algebra

To describe all fermionic species of the SM, we introduce the triplet field $\Psi(x)$ which behaves as triple-tensor products of the Dirac spinors as

$$\Psi_{abc} \sim \psi_a \psi_b \psi_c \tag{11.1}$$

where ψ is the four-component Dirac spinor. Operators acting on the triplet field belong to the triplet algebra A_T composed of the triple-tensor products of the Dirac algebra $A_\gamma = \langle \gamma_\mu \rangle = \{1, \gamma_\mu, \sigma_{\mu\nu}, \gamma_5\gamma_\mu, \gamma_5\}$ as follows:

$$\begin{aligned} A_T &= \{ p \otimes q \otimes r : \ p, q, r \in A_\gamma \} \\ &= \langle \gamma_\mu \otimes 1 \otimes 1, \ 1 \otimes \gamma_\mu \otimes 1, \ 1 \otimes 1 \otimes \gamma_\mu \rangle. \end{aligned} \tag{11.2}$$

The triplet algebra A_T is too large for all its elements to acquire physical meanings. To extract its subalgebras being suitable for physical description in the SM energy region, we impose the criterion [1] that the subalgebra bearing physical interpretation is closed and irreducible under the action of the permutation group S_3 which works to exchange the order of A_γ elements in the tensor product. With this criterion, the triplet algebra can be decomposed into three mutually commutative subalgebras, i.e., an external algebra defining external properties of fermions and two internal subalgebras that have the respective roles of prescribing family and color degrees of freedom.

The four elements

$$\Gamma_\mu = \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \in A_T, \quad (\mu = 0, 1, 2, 3) \tag{11.3}$$

satisfy the anti-commutation relations $\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = 2\eta_{\mu\nu}I$ where $I = 1 \otimes 1 \otimes 1$. With them, let us construct an algebra A_Γ by

$$A_\Gamma = \langle \Gamma_\mu \rangle = \{ I,\ \Gamma_\mu,\ \Sigma_{\mu\nu},\ \Gamma_5\Gamma_\mu,\ \Gamma_5 \} \tag{11.4}$$

where $\Sigma_{\mu\nu} = -\frac{i}{2}(\Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu) = \sigma_{\mu\nu} \otimes \sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$ and $\Gamma_5 = -i\Gamma_0\Gamma_1\Gamma_2\Gamma_3 = \Gamma^5 = \gamma_5 \otimes \gamma_5 \otimes \gamma_5$. The algebra A_Γ being isomorphic to the original Dirac algebra A_γ fulfills the S_3 criterion and works to specify the external characteristics of the triplet field. Namely, we postulate that the operators $M_{\mu\nu} = \frac{1}{2}\Sigma_{\mu\nu}$ generate the Lorentz transformations for the triplet field $\Psi(x)$ in the four dimensional Minkowski spacetime $\{ x^\mu \}$ where we exist as observers. The subscripts of operators Γ_μ are related and contracted with the superscripts of the spacetime coordinates x^μ.

Under the proper Lorentz transformation $x'^\mu = \Omega^\mu{}_\nu x^\nu$, the triplet field and its adjoint field $\overline{\Psi}(x) = \Psi^\dagger(x)\Gamma_0$ are transformed as

$$\Psi'(x') = S(\Omega)\Psi(x), \quad \overline{\Psi}'(x') = \overline{\Psi}(x)S^{-1}(\Omega) \tag{11.5}$$

where the transformation matrix is given by

$$S(\Omega) = \exp\left(-\frac{i}{2}M_{\mu\nu}\omega^{\mu\nu}\right) \tag{11.6}$$

with the angles $\omega^{\mu\nu}$ in the μ–ν planes. The Lorentz invariant scalar product is formed as

$$\overline{\Psi}(x)\Psi(x) = \sum_{abc} \overline{\Psi}_{abc}(x)\Psi_{abc}(x). \tag{11.7}$$

For discrete transformations such as space inversion, time reversal and the charge conjugation, the present scheme retains exactly the same structure as the ordinary Dirac theory. The chirality operators are given by

$$L = \frac{1}{2}(I - \Gamma_5), \quad R = \frac{1}{2}(I + \Gamma_5) \ \in\ A_\Gamma \tag{11.8}$$

which are used to assemble algebras for electroweak symmetries.

Note that the Dirac algebra A_γ possesses two $\mathfrak{su}(2)$ subalgebras

$$A_\sigma = \{ \sigma_1 = \gamma_0,\ \sigma_2 = i\gamma_0\gamma_5,\ \sigma_3 = \gamma_5 \} \tag{11.9}$$

and

$$A_\rho = \{ \rho_1 = i\gamma_2\gamma_3,\ \rho_2 = i\gamma_3\gamma_1,\ \rho_3 = i\gamma_1\gamma_2 \} \tag{11.10}$$

which are commutative and isomorphic with each other. By taking the triple-tensor products of elements of the respective subalgebras A_σ and A_ρ in A_T, we are able to construct two sets of commutative and isomorphic subalgebras with compositions "$\mathfrak{su}(3)$ plus $\mathfrak{u}(1)$" which satisfy the criterion of S_3 irreducibility. Those algebras are postulated to have the roles to describe internal family and color degrees of freedom of the triplet fields.

11.3 Algebra for extended family degrees of freedom

From the elements of the algebra $A_\sigma = \{\sigma_1, \sigma_2, \sigma_3\}$ in Eq.(11.9), we can make up eight elements of A_T as follows:

$$\begin{cases} \pi_1 = \frac{1}{2}(\sigma_1 \otimes \sigma_1 \otimes 1 + \sigma_2 \otimes \sigma_2 \otimes 1), \\ \pi_2 = \frac{1}{2}(\sigma_1 \otimes \sigma_2 \otimes \sigma_3 - \sigma_2 \otimes \sigma_1 \otimes \sigma_3), \\ \pi_3 = \frac{1}{2}(1 \otimes \sigma_3 \otimes \sigma_3 - \sigma_3 \otimes 1 \otimes \sigma_3), \\ \pi_4 = \frac{1}{2}(\sigma_1 \otimes 1 \otimes \sigma_1 + \sigma_2 \otimes 1 \otimes \sigma_2), \\ \pi_5 = \frac{1}{2}(\sigma_1 \otimes \sigma_3 \otimes \sigma_2 - \sigma_2 \otimes \sigma_3 \otimes \sigma_1), \\ \pi_6 = \frac{1}{2}(1 \otimes \sigma_1 \otimes \sigma_1 + 1 \otimes \sigma_2 \otimes \sigma_2), \\ \pi_7 = \frac{1}{2}(\sigma_3 \otimes \sigma_1 \otimes \sigma_2 - \sigma_3 \otimes \sigma_2 \otimes \sigma_1), \\ \pi_8 = \frac{1}{2\sqrt{3}}(1 \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes 1 \otimes \sigma_3 - 2\sigma_3 \otimes \sigma_3 \otimes 1). \end{cases} \tag{11.11}$$

which are proved to obey the commutation and anti-commutation relations of the Lie algebra $\mathfrak{su}(3)$ as

$$[\pi_j, \pi_k] = 2f^{(3)}_{jkl}\pi_l, \quad \{\pi_j, \pi_k\} = \frac{4}{3}\delta_{jk}\Pi_{(\nu)} + 2d^{(3)}_{jkl}\pi_l \tag{11.12}$$

where

$$\Pi_{(\nu)} = \frac{1}{4}(3I - 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_3 - 1 \otimes \sigma_3 \otimes 1 \otimes \sigma_3 - 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1) \tag{11.13}$$

and

$$\Pi_{(d)} = \frac{1}{4}(I + 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_3 + 1 \otimes \sigma_3 \otimes 1 \otimes \sigma_3 + 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1) \tag{11.14}$$

are projection operators satisfying the relations

$$\Pi_{(a)}\Pi_{(b)} = \delta_{ab}\Pi_{(a)}, \quad \Pi_{(a)}\pi_j = \delta_{a\nu}\pi_j \tag{11.15}$$

for $(a, b = \nu, d)$ and $(j = 1, \cdots, 8)$.

Here we impose a crucial postulate that the operators $\Pi_{(\nu)}$ and $\Pi_{(d)}$ work to divide the triplet field into the orthogonal component fields as

$$\Psi_{(\nu)}(x) = \Pi_{(\nu)}\Psi(x), \quad \Psi_{(d)}(x) = \Pi_{(d)}\Psi(x) \tag{11.16}$$

which represent, respectively, fundamental fermionic species belonging to the visible and dark sectors of the Universe. The visible part $\Psi_{(\nu)}(x)$ can be further decomposed into the sum of the three component fields as follows:

$$\Psi_{(\nu)}(x) = \sum_{j=1,2,3} \Psi_j(x) = \sum_{j=1,2,3} \Pi_j \Psi(x) \tag{11.17}$$

where the projection operators Π_j are defined symmetrically by

$$\begin{cases} \Pi_1 = \frac{1}{4}\left(I + 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_3 - 1 \otimes \sigma_3 \otimes 1 \otimes \sigma_3 - 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1\right), \\ \Pi_2 = \frac{1}{4}\left(I - 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_3 + 1 \otimes \sigma_3 \otimes 1 \otimes \sigma_3 - 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1\right), \\ \Pi_3 = \frac{1}{4}\left(I - 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_3 - 1 \otimes \sigma_3 \otimes 1 \otimes \sigma_3 + 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1\right). \end{cases} \tag{11.18}$$

which obey the relations $\sum_{j=1}^{3} \Pi_j = \Pi_{(\nu)}$ and $\Pi_j \Pi_k = \delta_{jk} \Pi_j$. The component fields $\Psi_j(x)$ $(j = 1, 2, 3)$ are interpreted to be the fields for three ordinary families of quarks and leptons in interaction modes.

With the operators π_j and $\Pi_{(a)}$, let us construct the set of the $\mathfrak{su}(3)$ and $\mathfrak{u}(1)$ algebras by

$$A_{(\nu)} = \{\,\Pi_{(\nu)},\ \pi_1,\ \pi_2, \cdots, \pi_8\,\}, \quad A_{(d)} = \{\,\Pi_{(d)}\,\} \tag{11.19}$$

which are closed and irreducible under the action of S_3 permutation. The algebras $A_{(\nu)}$ and $A_{(d)}$ specify, respectively, the characteristics of the three ordinary families of the visible sector and the exotic family of the dark sector. Accordingly, the set $A_f = \{A_{(\nu)},\ A_{(d)}\}$ is the algebra of operators specifying the family structure in the triplet field theory.

Rich varieties observed in flavor physics are presumed in the SM to result from the Yukawa couplings of quarks and leptons with the Higgs field. In low energy regime of flavor physics, quarks and leptons manifest themselves in both of the dual modes of electroweak interaction and mass eigen-states. It is the elements of the algebra $A_{(\nu)}$ in Eq.(11.19) that determine the structures of the Yukawa coupling constants which brings about varieties in the mass spectra and the electroweak mixing matrices.

It is the algebra $A_{(d)}$ consisting of the single element $\Pi_4 \equiv \Pi_{(d)}$ and the fourth component field $\Psi_4 \equiv \Psi_{(d)}$ of the triplet field that determine characteristics of exotic quarks and leptons belonging to the dark sector of the Universe. The projection operators Π_j $(j = 1, 2, 3, 4)$ satisfy the relations $\sum_{j=1}^{4} \Pi_j = I$ and $\Pi_j \Pi_k = \delta_{jk} \Pi_j$.

11.4 Algebra for extended color degrees of freedom

In parallel with the arguments in the preceding section, it is possible to construct another set of "$\mathfrak{su}(3)$ plus $\mathfrak{u}(1)$" subalgebras from the algebra $A_\rho = \{\,\rho_1,\ \rho_2,\ \rho_3\,\}$ in Eq.(11.10). Replacing σ_a with ρ_a in Eq.(11.11), we obtain a new set of operators λ_j $(j = 1, \cdots 8)$ in place of π_j. Likewise, corresponding to $\Pi_{(a)}$ $(a = \nu, d)$ in Eqs.(11.13) and (11.14), we obtain operators $\Lambda^{(a)}$ $(a = q, \ell)$ which play respective roles to project out component fields representing the quark-like and lepton-like modes of the triplet field.

Then, from Eqs.(11.12) and (11.15), we find that the operators λ_j and $\Lambda^{(a)}$ satisfy the relations

$$[\,\lambda_j,\ \lambda_k\,] = 2f^{(3)}_{jkl}\lambda_l, \quad \{\,\lambda_j,\ \lambda_k\,\} = \frac{4}{3}\delta_{jk}\Lambda^{(q)} + 2d^{(3)}_{jkl}\lambda_l \tag{11.20}$$

and

$$\Lambda^{(a)}\Lambda^{(b)} = \delta^{ab}\Lambda^{(a)}, \quad \Lambda^{(a)}\lambda_j = \delta^{aq}\lambda_j \tag{11.21}$$

for $(a, b = q, \ell)$ and $(j = 1, \cdots, 8)$. The operators λ_j and $\Lambda^{(a)}$ enable us to build up the new set of $\mathfrak{su}(3)$ and $\mathfrak{u}(1)$ algebras as follows:

$$A^{(q)} = \{\, \Lambda^{(q)},\ \lambda_1, \lambda_2, \cdots, \lambda_8 \,\}, \quad A^{(\ell)} = \{\, \Lambda^{(\ell)} \,\}. \tag{11.22}$$

It is readily proved that these algebras $A^{(a)}$ satisfy the criterion of S_3 irreducibility and are commutative with the algebras A_Γ, $A_{(\nu)}$ and $A_{(d)}$.

The operator for the "baryon number minus lepton number" defined by

$$\begin{aligned} Q_{B-L} &= \tfrac{1}{3}\Lambda^{(q)} - \Lambda^{(\ell)} \\ &= -\tfrac{1}{3}\,(1 \otimes 1 \otimes \rho_3 \otimes \rho_3 + 1 \otimes \rho_3 \otimes 1 \otimes \rho_3 + 1 \otimes \rho_3 \otimes \rho_3 \otimes 1) \end{aligned} \tag{11.23}$$

obeys the minimal equation

$$(Q_{B-L} + I)\left(Q_{B-L} - \frac{1}{3}I\right) = 0 \tag{11.24}$$

and has the eigenvalues $\frac{1}{3}$ and -1. Therefore, $\Lambda^{(q)}\Psi$ and $\Lambda^{(\ell)}\Psi$ form, respectively, the quark-like and lepton-like modes of the triplet field.

At this stage, we construct the generators for extended color gauge symmetries $SU_c(3)$ and $SU_{c\star}(3)$ which act, respectively, to the visible and dark fields $\Psi_{(\nu)}$ and $\Psi_{(d)}$. Combining the elements of the core algebras $A^{(q)}$ with the projection operators $\Pi_{(a)}$, we can make up the operators as

$$\Lambda^{(q)}_{(a)} = \Pi_{(a)}\Lambda^{(q)},\ \lambda_{(a)j} = \Pi_{(a)}\lambda_j \tag{11.25}$$

which form the algebras

$$A^{(q)}_{(a)} = \{\, \Lambda^{(q)}_{(a)},\ \lambda_{(a)j} : j = 1, \cdots, 8 \,\} \tag{11.26}$$

where $a = \nu$, d. The elements of the algebras $A^{(q)}_{(a)}$ satisfy the commutation and anti-commutation relations

$$[\lambda_{(a)j},\ \lambda_{(a)k}] = 2f^{(3)}_{jkl}\lambda_{(a)l},\ \ \{\lambda_{(a)j},\ \lambda_{(a)k}\} = \frac{4}{3}\delta_{jk}\Lambda^{(q)}_{(a)} + 2d^{(3)}_{jkl}\lambda_{(a)l} \tag{11.27}$$

for $a = \nu,\ d$ and $j,\ k,\ l = 1, 2, \cdots, 8$.

In this formalism, the quark-like species in the visible and dark sectors are presumed to be confined separately by different color gauge interactions associated with the groups $SU_c(3)$ and $SU_{c\star}(3)$. Those gauge groups are defined by the exponential mappings of the algebras $A^{(q)}_{(a)}$ $(a = \nu,\ d)$ as

$$SU_c(3) \times SU_{c\star}(3) = \left\{ \exp\left(-\frac{i}{2} \sum_{a=\nu,\, d} \sum_j \lambda_{(a)j}\theta^j_{(a)}(x) \right) \Lambda^{(q)} \right\} \tag{11.28}$$

where $\theta^{j}_{(a)}(x)$ are arbitrary real functions of space-time. In addition to the ordinary gauge fields $A^{(3)j}_{\mu}(x)$ with coupling constant $g^{(3)}$ of the $SU_c(3)$ symmetry, our theory necessitates the new gauge fields $A^{(3)j}_{\star\mu}(x)$ with coupling constant $g^{(3)}_{\star}$ of the $SU_{c\star}(3)$ symmetry.

For the lepton-like species also, we have to introduce the algebras

$$A^{(\ell)}_{(a)} = \{\Lambda^{(\ell)}_{(a)} \equiv \Pi_{(a)}\Lambda^{\ell}\}. \tag{11.29}$$

with $a = \nu$, d, which act to the visible and dark sectors, respectively. The set $A^c = \{A^{(q)}_{(\nu)}, A^{(\ell)}_{(\nu)}; A^{(q)}_{(d)}, A^{(\ell)}_{(d)}\}$ is the algebra of operators characterizing the extended color degrees of freedom in the triplet field theory. The operators $Q^{(a)}_{B-L}$ $(a = \nu, d)$ of "baryon number minus lepton number" in the visible and dark sectors are defined, respectively, by

$$Q^{(\nu)}_{B-L} = \Pi_{(\nu)}Q_{B-L},\ Q^{(d)}_{B-L} = \Pi_{(d)}Q_{B-L}. \tag{11.30}$$

11.5 Fundamental representation of the group $G \times G_{\star}$

The triplet field Ψ with $4 \times 4 \times 4$ spinor-components is decomposed as

$$\Psi = \Psi_{(\nu)} + \Psi_{(d)} = \left[\Psi^{(q)}_{(\nu)} + \Psi^{(\ell)}_{(\nu)}\right] + \left[\Psi^{(q)}_{(d)} + \Psi^{(\ell)}_{(d)}\right] \tag{11.31}$$

where $\Psi^{(c)}_{(a)} = \Lambda^{(c)}\Pi_{(a)}\Psi$ $(a = \nu,\ d; c = q,\ \ell)$ are the four component fields of Dirac-type with degrees of freedom of $(3+1)$-families and $(3+1)$-colors. Hence the triplet field Ψ which is considered to be the basic unit of fermionic species has no more freedom.

In order to incorporate the Weinberg-Salam symmetry G_{EW} and its left-right twisted symmetry $G_{EW\star}$ in the visible and dark sectors, respectively, we have to postulate that there exists a two-storied compound field $\boldsymbol{\Psi}$ consisting of two triplet fields. The compound field $\boldsymbol{\Psi}$ forms the fundamental representations of the gauge group $G \times G_{\star}$ as

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}_L + \boldsymbol{\Psi}_R = \begin{pmatrix} \boldsymbol{\Psi}_{(\nu)} & \begin{matrix} U_{(d)} \\ D_{(d)} \end{matrix} \end{pmatrix}_L + \begin{pmatrix} \begin{matrix} U_{(\nu)} \\ D_{(\nu)} \end{matrix} & \boldsymbol{\Psi}_{(d)} \end{pmatrix}_R \tag{11.32}$$

where $\boldsymbol{\Psi}_{(\nu)L}$, $U_{(\nu)L}$ and $D_{(\nu)L}$ ($\boldsymbol{\Psi}_{(d)R}$, $U_{(d)R}$ and $D_{(d)R}$) are, respectively, the chiral multi-spinor fields of the doublet, the up singlet and the down singlet of the $SU_L(2)$ ($SU_R(2)$) symmetry. We interpret that all fermionic species in the visible and dark sectors of the Universe are described by the component fields of the single compound field $\boldsymbol{\Psi}$.

To name the fermionic species in the dark sector, let us assign new symbols $u_{\star}$ and $d_{\star}$ for up and down *dark quarks*, and $\nu_{\star}$ and $e_{\star}$ for up and down *dark leptons*. Then, the quark parts of the chiral compound fields $\boldsymbol{\Psi}_L$ and $\boldsymbol{\Psi}_R$ are schematically expressed, respectively, by

$$\boldsymbol{\Psi}^{(q)}_{(\nu)} = \begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}_L,\ U^{(q)}_{(d)} = (u_{\star})_L,\ D^{(q)}_{(d)} = (d_{\star})_L \tag{11.33}$$

and

$$U^{(q)}_{(\nu)} = \begin{pmatrix} u\ c\ t \end{pmatrix}_R,\ D^{(q)}_{(\nu)} = \begin{pmatrix} d\ s\ b \end{pmatrix}_R,\ \mathbf{\Psi}^{(q)}_{(d)} = \begin{pmatrix} u_\star \\ d_\star \end{pmatrix}_R. \tag{11.34}$$

Similarly, the lepton parts have the following expressions as

$$\mathbf{\Psi}^{(\ell)}_{(\nu)} = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{pmatrix}_L,\ U^{(\ell)}_{(d)} = (\nu_\star)_L,\ D^{(\ell)}_{(d)} = (e_\star)_L \tag{11.35}$$

and

$$U^{(\ell)}_{(\nu)} = \begin{pmatrix} \nu_e\ \nu_\mu\ \nu_\tau \end{pmatrix}_R,\ D^{(\ell)}_{(\nu)} = \begin{pmatrix} e\ \mu\ \tau \end{pmatrix}_R,\ \mathbf{\Psi}^{(\ell)}_{(d)} = \begin{pmatrix} \nu_\star \\ e_\star \end{pmatrix}_R. \tag{11.36}$$

The kinetic and gauge parts of the Lagrangian density for all fermions can now be written down in terms of the chiral compound fields $\mathbf{\Psi}_L$ and $\mathbf{\Psi}_R$ by

$$\mathcal{L}_{kg} = \bar{\mathbf{\Psi}}_L i\Gamma^\mu \mathcal{D}_\mu \mathbf{\Psi}_L + \bar{\mathbf{\Psi}}_R i\Gamma^\mu \mathcal{D}_\mu \mathbf{\Psi}_R \tag{11.37}$$

in which the covariant derivatives act as follows:

$$\begin{aligned} i\mathcal{D}_\mu \mathbf{\Psi}_L = \Big\{ i\partial_\mu - \Big[g^{(3)} A^{(3)j}_\mu \tfrac{1}{2}\lambda_{(\nu)j} + g^{(2)} A^{(2)j}_\mu \tfrac{1}{2}\tau_{Lj} + g^{(1)} A^{(1)}_\mu \tfrac{1}{2} Y \Big] \Pi_{(\nu)} \\ - \Big[g^{(3)}_\star A^{(3)j}_{\star\mu} \tfrac{1}{2}\lambda_{(d)j} + g^{(1)}_\star A^{(1)}_{\star\mu} \tfrac{1}{2} Y_\star \Big] \Pi_{(d)} \Big\} \mathbf{\Psi}_L \end{aligned} \tag{11.38}$$

and

$$\begin{aligned} i\mathcal{D}_\mu \mathbf{\Psi}_R = \Big\{ i\partial_\mu - \Big[g^{(3)} A^{(3)j}_\mu \tfrac{1}{2}\lambda_{(\nu)j} + g^{(1)} A^{(1)}_\mu \tfrac{1}{2} Y \Big] \Pi_{(\nu)} \\ - \Big[g^{(3)}_\star A^{(3)j}_{\star\mu} \tfrac{1}{2}\lambda_{(d)j} + g^{(2)}_\star A^{(2)j}_{\star\mu} \tfrac{1}{2}\tau_{Rj} + g^{(1)}_\star A^{(1)}_{\star\mu} \tfrac{1}{2} Y_\star \Big] \Pi_{(d)} \Big\} \mathbf{\Psi}_R \end{aligned} \tag{11.39}$$

where $A^{(2)j}_\mu$ and $A^{(1)}_\mu$ ($A^{(2)j}_{\star\mu}$ and $A^{(1)}_{\star\mu}$) are gauge fields of the G_{EW} ($G_{EW\star}$) symmetry with coupling constants $g^{(2)}$ and $g^{(1)}$ ($g^{(2)}_\star$ and $g^{(1)}_\star$). The operators $\frac{1}{2}\tau_{Lj}$ ($\frac{1}{2}\tau_{Rj}$) are the generators of the visible (dark) $SU_L(2)$ ($SU_R(2)$) symmetry, and the hypercharge Y ($Y_\star$) in the visible (dark) sector can be expressed by

$$Y = Q^{(\nu)}_{B-L} + y, \quad Y_\star = Q^{(d)}_{B-L} + y_\star \tag{11.40}$$

in which y ($y_\star$) takes 0, 1 and -1 for the doublet Ψ, the up singlet U and the down singlet D.

To break down the gauge symmetries G_{EW} and $G_{EW\star}$, we require two types of Higgs doublets φ and $\varphi_\star$ which, respectively, have the hypercharges ($Y = 1$, $Y_\star = 0$) and ($Y = 0$, $Y_\star = 1$). The Lagrangian density of the Yukawa interaction is given as follows:

$$\begin{aligned} \mathcal{L}_Y &= \bar{\mathbf{\Psi}}_L \{\text{Higgs fields}\} \mathbf{\Psi}_R + \text{h.c.} \\ &= \bar{\mathbf{\Psi}}_{(\nu)} \tilde{\varphi} \mathcal{Y}_U U_{(\nu)} + \bar{\mathbf{\Psi}}_{(\nu)} \varphi \mathcal{Y}_D D_{(\nu)} + y_{u\star} \bar{U}_{(d)} \tilde{\varphi}^\dagger_\star \mathbf{\Psi}_{(d)} + y_{d\star} \bar{D}_{(d)} \varphi^\dagger_\star \mathbf{\Psi}_{(d)} + \text{h.c.} \end{aligned} \tag{11.41}$$

where $\tilde{\varphi} = i\tau_{L2}\varphi^*$ and $\tilde{\varphi}_\star = i\tau_{R2}\varphi^*_\star$. The operators $\mathcal{Y}_U$ and $\mathcal{Y}_D$ consisting of elements of the algebra $A_{(\nu)}$ in Eq.(11.19) determine the patterns of Yukawa interactions among the fermions in the visible sector [2], and $y_{u\star}$ and $y_{d\star}$ are the Yukawa coupling constants of the fermions in the dark sector.

The Lagrangian density of the visible and dark Higgs fields takes the form

$$\mathcal{L}_H = (D^\mu \varphi)^\dagger (D_\mu \varphi) + (D^\mu \varphi_\star)^\dagger (D_\mu \varphi_\star) - V_H \tag{11.42}$$

in which the covariant derivatives act as follows:

$$iD_\mu \varphi = \left(i\partial_\mu - g^{(2)} A_\mu^{(2)a} \frac{1}{2} \tau_{La} - g^{(1)} A_\mu^{(1)} \frac{1}{2} \right) \varphi \tag{11.43}$$

and

$$iD_\mu \varphi_\star = \left(i\partial_\mu - g_\star^{(2)} A_{\star\mu}^{(2)a} \frac{1}{2} \tau_{Ra} - g_\star^{(1)} A_{\star\mu}^{(1)} \frac{1}{2} \right) \varphi_\star . \tag{11.44}$$

The Higgs potential is generally given by

$$V_H = V_0 - \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 - \mu_\star^2 \varphi_\star^\dagger \varphi_\star + \lambda_\star (\varphi_\star^\dagger \varphi_\star)^2 + 2\lambda_I (\varphi_\star^\dagger \varphi_\star)(\varphi^\dagger \varphi) \tag{11.45}$$

where λ, $\lambda_\star$ and λ_I are the constants of self-coupling and bi-quadratic mutual interaction.

11.6 A scenario for dark matter

If the quarks $u_\star$ and $d_\star$ acquire close masses like the u and d quarks of the first family when the $G_{EW\star}$ symmetry is spontaneously broken at the energy scale $\Lambda_\star$ ($\Lambda_\star > \Lambda$), many kinds of dark nuclei and a variety of dark elements come necessarily into existence. Thereby, the dark sector with such quarks $u_\star$ and $d_\star$ is destined to follow a rich and intricate path of thermal history of evolution like the visible sector of our Universe.

Here we consider a situation that, just like the case of the t and b quarks of the third family, the dark up quark $u_\star$ is much heavier than the dark down quark $d_\star$ as [1,3]

$$m_{u_\star} \gg m_{d_\star} + m_{e_\star} + m_{\nu_\star} . \tag{11.46}$$

In such a case, the $u_\star$ quark disappears quickly through the process $u_\star \to d_\star + \bar{e}_\star + \nu_\star$ leaving the $d_\star$ quark as the main massive components of the dark sector. Consequently, the gauge fields $A_{\star\mu}^{(3)}$ of $SU_{c\star}(3)$ symmetry act to confine the $d_\star$ quark into the dark color-singlet hadron

$$\Delta_\star^- = [d_\star\, d_\star\, d_\star] = \frac{1}{\sqrt{6}} \epsilon_{ijk} d_\star^i\, d_\star^j\, d_\star^k \tag{11.47}$$

which has the dark electric charge $Q_\star = -1$ and the spin angular momentum $\frac{3}{2}$ due to the Fermi statistics.

In this scenario, $\Delta_\star^-$ is the stable dark hadron which can exist as the only nucleus in the dark sector. Therefore, no rich nuclear reaction can occur and only a meager history of thermal evolution can take place. The stable atom which can exist in the matter-dominant stage of the dark sector is limited to be

$$\bar{H}_\star = (\Delta_\star^- + \bar{e}_\star). \tag{11.48}$$

It can be speculated that the dark molecule

$$(\bar{H}_\star)_2 = \bar{H}_\star \bar{H}_\star \tag{11.49}$$

in the spin 1 or 0 states can be stable entities prevailing over a broad spatial region of the late stage of the Universe. These features seem to be consistent with the characteristics of the dark matter which has the tendency to spread out rather monotonically over broad spatial regions as inferred by the observations of gravitational lensing.

11.7 Discussion

By generalizing the concept of Dirac spinor, we have developed a unified theory of multi-spinor fields that can describe the whole spectra of fermionic species in the visible and dark sectors of the Universe. The physical subalgebras of the triplet algebra satisfying the criterion of the S_3 irreducibility have the unique feature of the "$\mathfrak{su}(3)$ plus $\mathfrak{u}(1)$" structure for both of the color and family degrees of freedom. The triplet compound field in Eq.(11.32) possessing the component fields of the three visible and one dark family modes with the tri-color quarks and colorless leptons enables us to formulate a unified theory that can describe the flavor physics in the visible sector and cosmological phenomena related to both the visible and dark sectors.

To develop the present theory further, it is necessary to examine the thermal history of the dark sector and to confirm that the present scheme is consistent with the well-established standard theory of astrophysics and cosmology. In this note, we make heuristic analyses to estimate the mass of the stable dark hadron $\Delta_\star$ from the cosmological parameters for the densities of the cold dark matter and baryonic matter determined by WMAP [4] and Planck [5] observations.

The visible fermionic and bosonic fields can interact with the dark fermionic and bosonic fields through virtual loop corrections induced by the bi-quadratic interaction between the Higgs fields φ and $\varphi_\star$ in Eq.(11.45). Therefore, it is possible to assume that quanta of all fields of the visible and dark sectors constitute a common soup of inseparable phase in an early reheating period. Expansion of the Universe decreasing its temperature breaks both of the dark electroweak symmetry $G_{EW\star}$ and the visible electroweak symmetry G_{EW} symmetry in the quantum soup.

At present, there exists no reliable theory which can describe consistently the cosmic baryogenesis. So we set a simple working hypothesis [1] that the process of baryogenesis takes place cooperatively through the two-step breakdowns of $G_{EW\star}$ and G_{EW} symmetries in such a way that the excess of quark numbers is created and preserved equally for all families in the visible and dark sectors. The quarks getting heavy masses decay down to the lighter quarks. While the $u_\star$ quark disappears leaving only the $d_\star$ quark in the dark sector, four types of the heavy quarks decay into the u and d quarks which have almost degenerate masses in the visible sector. The $SU_{c\star}(3)$ gauge interaction works to confine the $d_\star$ quarks

[1] Technical details of some scenarios of 'Two-Step Electroweak Baryogenesis' can be found in the articles [6,7].

into the dark hadron $\Delta_\star$ with 4 spin degrees of freedom, and the $SU_c(3)$ gauge interaction confine the u and d quarks into the nucleon possessing 2 iso-spin (proton and neutron) states with 2 spin degrees of freedom.

Remark that the quark numbers are separately conserved in the dark and visible sectors after the decoupling of the two sectors. Therefore, the observed ratio of the densities of cold dark matter and baryonic matter can be identified with the ratio of energies (masses/c^2) stored by the stable particles in the dark and visible sectors. By assuming that the dark hadron $\Delta_\star$ is the dominant component of the cold dark matter, we obtain the following relation for the masses of $\Delta_\star$ and nucleon, $m_{\Delta_\star}$ and m_N, as

$$2m_{\Delta_\star} : 6m_N = \Omega_c h^2 : \Omega_b h^2 = 0.11889 : 0.022161 \tag{11.50}$$

where the values for the cosmological parameters $\Omega_c h^2$ and $\Omega_b h^2$ taken from the Table 10 of the reference [5] are the Planck best-fit including external data set. Consequently, the upper limit of the mass of the dark hadron $\Delta_\star$ is estimated to be $m_{\Delta_\star} = 16.1 m_N = 15.1 \mathrm{GeV}/c^2$.

Until now no affirmative result has been found by either of the direct and indirect dark matter searches. The recent observation by the LUX group [8] has proved that the background-only hypothesis is consistent with their data on spin-independent WIMP-nucleon elastic scattering with a minimum upper limit on the cross section of $7.6 \times 10^{-46}\mathrm{cm}^2$ at a WIMP mass of 33 GeV/c^2. More stringent experiments must be performed to confirm the possibility of scenarios for dark matter including stable particles with comparatively small masses such as the dark hadron $\Delta_\star$ of our theory.

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Discussion Section

The discussion section is reserved for those open problems discussed during the workshop. They might start new collaboration among participants or at least stimulate participants to start to think about possible solutions of particular open problems in a different way. Since the time between the workshop and the deadline for contributions for the proceedings is very short and includes for most of participants also their holidays, it is not so easy to prepare besides their presentation at the workshop also the common contributions to the discussion section. However, the discussions, even if not presented as a contribution to this section, influenced participants' contributions, published in the main section.

As it is happening every year also this year quite a lot of started discussions have not succeeded to appear in this proceedings. Organizers hope that they will be developed enough to appear among the next year talks, or will just stimulate the works of the participants.

Some of the discussion contributions are proofs of the statements, that is the mathematical support of some of the proposed theories offering a step beyond the *standard models*, or the contribution to better understanding the degrees of freedom in a generalized action, both concern the *spin-charge-family* theory.

One of them presents improvements on the previous works on the topic on almost democratic mass matrices of quarks.

This year we have started the discussion, which would connect the experiences from the hadron physics to possible new physics, which has already been measured at the LHC but not yet analysed. We also have started the discussions connecting many experiences in the many body systems in the hadron physics and the equivalent systems in the high energy physics. The organizers hope that this kind of discussions might continue up to the next workshop by emails. Felipe has summarized the starting point of these discussions in his contribution to this section.

All discussion contributions are arranged alphabetically with respect to the authors' names.

Ta razdelek je namenjen odprtim vprašanjem, o katerih smo med delavnico izčrpno razpravljali. Problemi, o katerih smo razpravljali, bodo morda privedli do novih sodelovanj med udeleženci, ali pa so pripravili udeležence, da razmislijo o možnih rešitvah odprtih vprašanj na drugačne načine. Ker je čas med delavnico in rokom za oddajo prispevkov zelo kratek, vmes so pa poletne počitnice, je zelo težavno poleg prispevka, v katerem vsak udeleženec predstavi lastno delo, pripraviti še prispevek k temu razdelku.

Tako se velik del diskusij ne bo pojavil v letošnjem zborniku. So pa gotovo vplivale na prispevek marsikaterega udeleženca. Organizatorji upamo, da bodo te diskusije do prihodnje delavnice dozorele do oblike, da jih bo mogoče na njej predstavit.

Eden od prispevkov je dokaz trditve, to je matematična podpora predlagani teoriji, ki ponuja (zanesljiv) korak onkraj *standardnih modelov*, drugi ponudi nadgradnjo tej teoriji. Oba se nanašata na teorijo *spina-nabojev-družin*.

Eden od prispevkov je nadgradnja dela o simetrijah skoraj demokratičnih masnih matrik.

To leto smo začeli diskusijo, ki naj bi povezala izkušnje iz hadronske fizike z morebitno novo fiziko, že izmerjeno na LHC, vendar analiza rezultatov meritev še ni dokončna. Začeli smo tudi z diskusijami, ki povezuje izkušnje v sistemih več teles v hadronski fiziki z ustreznimi sistemi v fiziki visokih energij. Organizatorji upamo, da se bodo te diskusije nadaljevale v času do naslednje delavnice po e-pošti.

Prispevki v tej sekciji so, tako kot prispevki v glavnem delu, urejeni po abecednem redu priimkov avtorjev.

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12 A Democratic Suggestion

A. Kleppe *

SACT, Oslo

Abstract. Within the framework of quark mass matrices with a democratic texture, the unitary rotation matrices that diagonalize the quark matrices are obtained by a specific parametrization of the Cabibbo-Kobayashi-Maskawa mixing matrix. Different forms of democratic quark mass matrices are derived from slightly different parametrizations.

Povzetek. Avtorica predstavi masne matrike kvarkov s skoraj demokratičnimi matrikami. Izbere različno parametrizacijo, ki preko unitarne transformacije vodijo do izmerjene mešalne matrike Cabibba-Kobayashija-Maskawe. Komentira sprejemljivost različnih parametrizacij.

12.1 Introduction

A main weakness of the Standard Model is the large number of free parameters. There is at present no explanation for their origin, and we don't know if there is some connection between them.

Most of the free parameters reside in "flavour space" - with six quark masses, six lepton masses, four quark mixing angles and ditto for the leptonic sector, as well as the strong CP-violating parameter $\bar{\Theta}$. The structure of flavour space is determined by the fermion mass matrices, i.e. by the form that the mass matrices take in the "weak interaction basis" where mixed fermion states interact weakly, in contrast to the "mass bases", where the mass matrices are diagonal.

One may wonder how one may ascribe such importance to the different bases in flavour space, considering that the information content of a matrix is contained in its matrix invariants, which in the case of a $N \times N$ matrix M are the N sums and products of the eigenvalues λ_j, such as $traceM$, $detM$,

$$\begin{aligned} I_1 &= \sum_j \lambda_j = \lambda_1 + \lambda_2 + \lambda_3 ... \\ I_2 &= \sum_{jk} \lambda_j \lambda_k = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + ... \\ I_3 &= \sum_{jkl} \lambda_j\lambda_k\lambda_l = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + ... \\ &\vdots \\ I_N &= \lambda_1\lambda_2 \cdots \lambda_N \end{aligned} \tag{12.1}$$

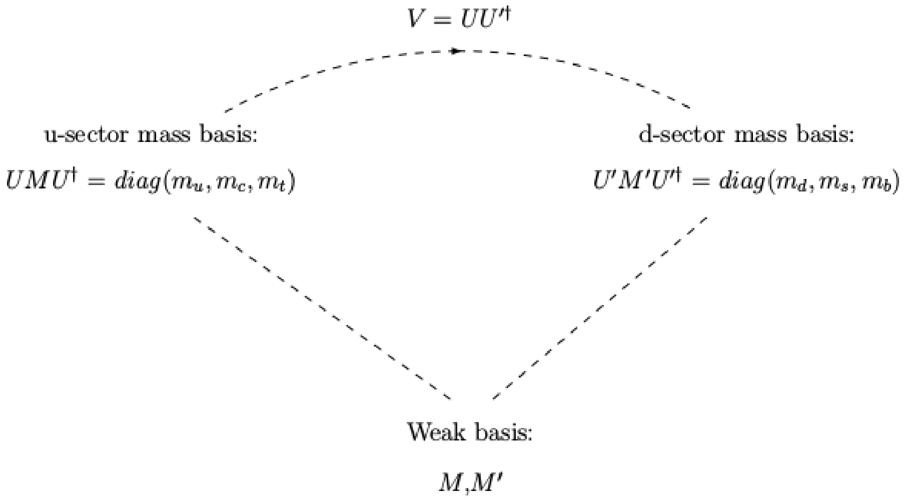
These expressions are invariant under permutations of the eigenvalues, which in the context of mass matrices means that they are flavour symmetric, and obviously independent of any choice of flavour space basis.

* astri@snofrix.org

Even if the information content of a matrix is contained in its invariants, the form of a matrix may also carry information, albeit of another type. The idea - the hope - is that the form that the mass matrices have in the weak interaction basis can give some hint about the origin of the unruly masses. There is a certain circularity to this reasoning; to make a mass matrix ansatz is in fact to define what we take as the weak interaction basis in flavour space. We denote the quark mass matrices of the up- and down-sectors in the weak interaction basis by M and M', respectively. We go from the weak interaction basis to the mass bases by rotating the matrices by the unitary matrices U and U',

$$M \to UMU^\dagger = D = diag(m_u, m_c, m_t) \tag{12.2}$$

$$M' \to U'M'U'^\dagger = D' = diag(m_d, m_s, m_b)$$



The lodestar in the hunt for the right mass matrices is the family hierarchy, with two lighter particles in the first and second family, and a much heavier particle in the third family. This hierarchy is present in all the charged sectors, with fermions in different families exhibiting very different mass values, ranging from the electron mass to the about 10^5 times larger top mass. It is still an open question whether the neutrino masses also follow this pattern [1].

12.2 "Democratic" mass matrices

In the "democratic" approach [2], [3], [4] the hierarchical pattern is taken very seriously. The basic assumption is that in the weak interaction basis the fermion mass matrices are next to “democratic”, in the sense that they have a structure close to the $S(3)_L \times S(3)_R$ symmetric “democratic” matrix

$$\mathbf{N} = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{12.3}$$

The underlying philosophy is that in the Standard Model, where the fermions get their masses from the Yukawa couplings by the Higgs mechanism, there is no reason why there should be a different Yukawa coupling for each fermion. The couplings to the gauge bosons of the strong, weak and electromagnetic interactions are identical for all the fermions in a given charge sector, it thus seems like a natural assumption that they should also have identical Yukawa couplings. The difference is that the weak interactions take place in a specific flavour space basis, while the other interactions are flavour independent.

The democratic assumption is thus that the fermion fields of the same charge initially have the same Yukawa couplings. With three families, the quark mass matrices in the weak interaction basis then have the (zeroth order) form

$$M^{(0)} = k_u \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad M'^{(0)} = k_d \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{12.4}$$

where k_u and k_d have dimension mass. The corresponding mass spectra $(m_1, m_2, m_3) \sim (0, 0, 3k_j)$ reflect the family hierarchy with two light families and a third much heavier family, a mass hierarchy that can be interpreted as the representation $\mathbf{1} \oplus \mathbf{2}$ of $S(3)$. In order to obtain realistic mass spectra with non-zero masses, the $S(3)_L \times S(3)_R$ symmetry must obviously be broken, and the different democratic matrix ansätze correspond to different schemes for breaking the democratic symmetry.

12.2.1 The lepton sector

We can apply the democratic approach to the lepton sector as well, postulating democratic (zeroth order) mass matrices for the charged leptons and the neutrinos, whether they are Fermi-Dirac or Majorana states,

$$M_l^{(0)} = k_l \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad M_\nu^{(0)} = k_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{12.5}$$

Relative to the quark ratio $k_u/k_d \sim m_t/m_b \sim 40 - 60$, the leptonic ratio $k_\nu/k_l < 10^{-8}$ is so extremely small that it seems unnatural. One way out is to simply assume that k_ν vanishes, meaning that the neutrinos get no mass contribution in the democratic limit [5]. According to the democratic philosophy, then there would be no reason for a hierarchical pattern à la the one observed in the charged sectors; the neutrino masses could even be of the same order of magnitude.

Data are indeed compatible with a much weaker hierarchical structure for the neutrino masses than the hierarchy displayed by the charged quark fermion masses.

Unlike the situation for the quark mixing angles, in lepton flavour mixing there are two quite large mixing angles and a third much smaller mixing angle, these large mixing angles can be interpreted as indicating weak hierachy of the neutrino mass spectrum. The neutrino mass spectrum hierarchy could even be inverted; if the solar neutrino doublet (ν_1, ν_2) has a mean mass larger than the remaining atmospheric neutrino ν_3, the hierarchy is called inverted, otherwise it is called normal.

Supposing that the neutrino masses do not emerge from a democratic scheme, a (relatively) flat neutrino mass spectrum could be taken as a support for the idea that the masses in the charged sectors emerge from a democratic scheme.

12.3 The democratic basis

At the level of the zeroth order mass matrices the quark mixing matrix is $V = UU'^{\dagger} = U_{dem}U_{dem}^{\dagger} = \mathbf{1}$, where

$$U_{dem} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \tag{12.6}$$

We use this to define the the democratic basis, meaning the flavour space basis where the mass matrices are diagonalized by (12.6) and the mass Lagrangian is symmetric under permutations of the fermion fields $(\varphi_1, \varphi_2, \varphi_3)$ of a given charge sector.

In the democratic basis the mass Lagrangian

$$\mathcal{L}_m = \bar{\varphi} M_{(democratic\ basis)} \varphi = k \sum_{jk=1}^{3} \bar{\varphi}_j \varphi_k$$

is symmetric under permutations of the fermion fields $(\varphi_1, \varphi_2, \varphi_3)$, while in the mass basis with

$$M_{(mass\ basis)} = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}$$

the mass Lagrangian has the form

$$\mathcal{L}_m = \lambda_1 \bar{\psi}_1 \psi_1 + \lambda_2 \bar{\psi}_2 \psi_2 + \lambda_3 \bar{\psi}_3 \psi_3 \tag{12.7}$$

which is clearly not invariant under permutations of the eigenvalues, nor under permutations of (ψ_1,ψ_3,ψ_3). We can perform a shift of the democratic matrix, by just adding a unit matrix $diag(a, a, a)$, so we get $M_0 \to M_1$,

$$M_1 = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} a & & \\ & a & \\ & & a \end{pmatrix} = \begin{pmatrix} k+a & k & k \\ k & k+a & k \\ k & k & k+a \end{pmatrix} \tag{12.8}$$

corresponding to the mass spectrum $(a, a, 3a + 3k)$. The matrix M_1 has a democratic texture, both because it is diagonalized by U_{dem}, and because the mass Lagrangian is invariant under permutations of the quark fields,

$$\mathcal{L}_{M_1} = (k + a) \sum \bar{\varphi}_j \varphi_j + k \sum_{j \neq k} \bar{\varphi}_j \varphi_j \tag{12.9}$$

.

If M_1 and M_1' both have a texture like (12.8), there is no CP-violation. This is independent of how many families there are, because of the degeneracy of the mass values. CP-violation only occurs once there are three or more non-degenerate families, because only then the phases can no longer be defined away.

We can repeat the democratic scheme with a number n of families, where the fermion mass matrices again are proportional to the $S(n)_L \times S(n)_R$ symmetric democratic matrix which is diagonalized by a unitary matrix analogous to U_{dem} in (12.6). To the $n \times n$-dimensional democratic matrix term, we can again add a $n \times n$-dimensional diagonal matrix $diag(a, a, ..., a)$, and get a $n \times n$-dimensional mass spectrum with n massive states, and $n - 1$ degenerate masses. The mass matrix still has a democratic texture, and there is still no CP-violation.

12.4 Breaking the democratic symmetry

In order to obtain non-degenerate, non-vanishing masses for the physical flavours (ψ_1, ψ_2, ψ_3), the permutation symmetry of the democratic fermion fields $(\varphi_1, \varphi_2, \varphi_3)$ must be broken. The proposal here is to derive the perturbed unitary rotation matrices U, U' for the up and down sectors from a specific parameterisation of the weak mixing matrix $V = UU'^{\dagger}$.

The idea is to embed the assumption of democratic symmetry into the Standard Model mixing matrix, by expressing the mixing matrix as a product

$$V = UU'^{\dagger} = (\tilde{U} U_{dem})(U_{dem}^{\dagger} \tilde{U}'^{\dagger}) \tag{12.10}$$

Since both the mixing matrix and its factors, according to the "standard" parameterisation [6], are so close to the unit matrix, the rotation matrices U, U' are effectively perturbations of the the democratic diagonalising matrix (12.6). In this way, the weak interaction basis remains close to the democratic basis.

12.4.1 Factorizing the mixing matrix

The Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix [7] can of course be parametrized - and factorized - in many different ways, and different factorizations correspond to different rotation matrices U and U'. The most obvious and "symmetric" factorization of the CKM mixing matrix is, following the "standard" parametrization [6] with three Euler angles α, β, 2θ,

$$V = \begin{pmatrix} c_\beta c_{2\theta} & s_\beta c_{2\theta} & s_{2\theta} e^{-i\delta} \\ -c_\beta s_\alpha s_{2\theta} e^{i\delta} - s_\beta c_\alpha & -s_\beta s_\alpha s_{2\theta} e^{i\delta} + c_\beta c_\alpha & s_\alpha c_{2\theta} \\ -c_\beta c_\alpha s_{2\theta} e^{i\delta} + s_\beta s_\alpha & -s_\beta c_\alpha s_{2\theta} e^{i\delta} - c_\beta s_\alpha & c_\alpha c_{2\theta} \end{pmatrix} = UU'^{\dagger} \tag{12.11}$$

with the diagonalizing rotation matrices for the up- and down-sectors

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{12.12}$$

and

$$U' = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix},$$

respectively, where α, β, θ and γ correspond to the parameters in the standard parametrization of the CKM mixing matrix, in such a way that $\gamma = \delta/2$, $\delta = 1.2 \pm 0.08$ rad, and $2\theta = 0.201 \pm 0.011°$, while $\alpha = 2.38 \pm 0.06°$ and $\beta = 13.04 \pm 0.05°$.

¿From the rotation matrices U and U' we then obtain the mass matrices $M = U^\dagger diag(m_u, m_c, m_t) U$ and $M' = U'^\dagger diag(m_d, m_s, m_b) U'$, such that

$$M = \frac{1}{6} \begin{pmatrix} X+H & \hat{M}_{12} & Z+W \\ \hat{M}^*_{12} & X-H & Z-W \\ Z^*+W^* & Z^*-W^* & 6T-2X \end{pmatrix} \tag{12.13}$$

where T is the trace $T = m_u + m_c + m_t$, and with $D = \sqrt{3}s_\theta - \sqrt{2}c_\theta$, $C = \sqrt{3}s_\theta + \sqrt{2}c_\theta$, $F = c_\alpha s_\alpha (m_t - m_c)$,

$$X = \tfrac{1}{2}(m_c s^2_\alpha + m_t c^2_\alpha - m_u)(D^2 + C^2 - 2) + F(D - C)\cos\gamma + T + 3m_u$$
$$H = \tfrac{1}{2}(m_c s^2_\alpha + m_t c^2_\alpha - m_u)(D^2 - C^2) + F\cos\gamma(D + C)$$

$$W = \tfrac{1}{4}(m_c s^2_\alpha + m_t c^2_\alpha - m_u)\ (D^2 - C^2) - F\ (D + C)\ e^{-i\gamma}$$
$$Z = (m_c s^2_\alpha + m_t c^2_\alpha - m_u)\left[2 + \tfrac{1}{4}(D - C)^2\right] + \tfrac{F}{2}\ (D - C)\ (e^{i\gamma} - 2\ e^{-i\gamma}) - 2T + 6\ m_u$$
$$\hat{M}_{12} = -(m_c s^2_\alpha + m_t c^2_\alpha - m_u)\ (D\ C + 1) - F\ (C\ e^{i\gamma} - D\ e^{-i\gamma}) + T - 3\ m_u$$

Similarly for the down-sector,

$$M' = \frac{1}{6} \begin{pmatrix} X'+H' & \hat{M}'_{12} & Z'+W' \\ \hat{M}'^*_{12} & X'-H' & Z'-W' \\ Z'^*+W'^* & Z'^*-W'^* & 6T'-2X' \end{pmatrix} \tag{12.14}$$

with the parameters $T' = m_d + m_s + m_b$, $G = \sqrt{2}s_\theta - \sqrt{3}c_\theta$, $J = \sqrt{2}s_\theta + \sqrt{3}c_\theta$ and $F' = c_\beta s_\beta (m_b - m_s)$, and

$$X' = \tfrac{1}{2}(m_s s^2_\beta + m_b c^2_\beta - m_d)(G^2 + J^2 - 2) - F'(J + G)\cos\gamma + T' + 3m_b$$
$$H' = \tfrac{1}{2}(m_s s^2_\beta + m_b c^2_\beta - m_d)(G^2 - J^2) + F'(J - G)\cos\gamma$$
$$W' = \tfrac{1}{4}(m_s s^2_\beta + m_b c^2_\beta - m_d)(G^2 - J^2) + F'(G - J)e^{i\gamma}$$
$$Z' = (m_s s^2_\beta + m_b c^2_\beta - m_d)\left[2 + \tfrac{1}{4}(J + G)^2\right] + \tfrac{F'}{2}(J + G)(2e^{i\gamma} - e^{-i\gamma}) - 2T' + 6m_b$$
$$\hat{M}'_{12} = (m_s s^2_\beta + m_b c^2_\beta - m_d)\ (G\ J - 1) - F'\ (J\ e^{i\gamma} - G\ e^{-i\gamma}) + T' - 3\ m_b$$

In order to evaluate to what degree these rather opaque matrices are "democratic", we evaluate the matrix elements by inserting numerical mass values. For the up-sector we get the (nearly democratic) matrix texture

$$M = C_u \left[\begin{pmatrix} 1 & & \\ & k\, e^{-i(\alpha+\beta)} & \\ & & kp\, e^{-i\alpha} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & k\, e^{i(\alpha+\beta)} & \\ & & kp\, e^{i\alpha} \end{pmatrix} + \Lambda \right] \tag{12.15}$$

where the "small" matrix

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon & \varepsilon' e^{-i\beta} \\ 0 & \varepsilon' e^{i\beta} & \eta \end{pmatrix},$$

with $\varepsilon \sim \varepsilon' \ll \eta < k, p$, is what breaks the democratic symmetry, supplying the two lighter families with non-zero masses. With mass values calculated at $\mu = M_Z$ (Jamin 2014) [8],

$$(m_u(M_Z), m_c(M_Z), m_t(M_Z)) = (1.24, 624, 171550)MeV,$$

we get $\alpha \sim 2.7895^o$, $\beta \sim 2.7852^o$, $C_u = 54240.36$ MeV $\approx m_t/3$, and

$k \approx 1.00438$, $p \approx 1.06646$, $\varepsilon' \approx 0.0000505$,
$\varepsilon \approx 0.00004596 \approx 2m_u/C_u$, $\eta = 0.018154 \approx \frac{1}{2}\frac{m_t}{C_u}\frac{m_c}{C_u}$.

For the down-sector, with

$$(m_d(M_Z), m_s(M_Z), m_b(M_Z)) = (2.69, 53.8, 2850)MeV$$

we get another democratic texture,

$$M' = C_d \begin{pmatrix} X + A & Ye^{-i\mu} & e^{-i\rho} \\ Ye^{i\mu} & X - A & (1 + 2A)e^{i\kappa} \\ e^{i\rho} & (1 + 2A)e^{-i\kappa} & X + Y - A - 1 \end{pmatrix} \tag{12.16}$$

where

$C_d = 966.5MeV$, $A = 0.0056$, $X = 1.0362$, $Y = 1.0305$ and
$\mu \leq \kappa \sim 0.22^o < \rho \sim 0.226^o$.

Just like in the up-sector mass matrix, the matrix elements in M' display a nearly democratic texture. In both the up-sector and the down-sector the mass matrices are thus approximately democratic.

12.5 Calculability

In the mass matrix literature there is an emphasis on "calculability". The ideal is to obtain mass matrices that have a manageable form, but there is nothing that forces nature to serve us such user-friendly formalism. It is however tempting to speculate that there are relations between the elements that could make the democratic matrices more calculable, and in the search for matrices that are reasonably transparent and calculable, we look at a more radical factorization of the mixing matrix, viz.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\omega & 0 & \sin\omega\, e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\omega\, e^{i\delta} & 0 & \cos\omega \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{12.17}$$

and

$$U' = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

where, as before, $\delta = 1.2 \pm 0.08$ rad, and $\omega = 2\theta = 0.201 \pm 0.011°$, while $\alpha = 2.38 \pm 0.06°$, and $\beta = 13.04 \pm 0.05°$. These rotation matrices are still "perturbations" of the democratic diagonalizing matrix (12.6), and the up-sector mass matrix has a texture similar to (12.13),

$$M = \frac{1}{6} \begin{pmatrix} R+Q+S\ \cos\delta & R-Q-iS\ \sin\delta & A-Be^{-i\delta} \\ R-Q+iS\ \sin\delta & R+Q-S\ \cos\delta & A+Be^{-i\delta} \\ A-Be^{i\delta} & A+Be^{i\delta} & T-2(R+Q) \end{pmatrix} \tag{12.18}$$

where T is the trace, $T = m_u + m_c + m_t$, and

$$\begin{aligned} &R = N\ (2\, c_\omega\, c_\omega - 1) + T - 2\,\sqrt{2}\, c_\omega\ F, \quad Q = 3\, s_\omega\, s_\omega\ N + 3\, m_u, \\ &S = -2\sqrt{6}\, c_\omega\, s_\omega\ N + 2\,\sqrt{3} s_\omega\ F \\ &A = N\ (2\, c_\omega\, c_\omega + 2) - 2\,T + \sqrt{2}\, c_\omega\ F + 6\, m_u, \quad B = \sqrt{6}\, c_\omega\, s_\omega\ N + 2\,\sqrt{3}\ F\ s_\omega \end{aligned}$$

with $N = m_c\, s_\alpha\, s_\alpha + m_t\, c_\alpha\, c_\alpha - m_u,\ F = c_\alpha\, s_\alpha\ (m_t - m_c)$. This matrix can be reformulated in a form similar to (12.15),

$$M_u = C_u \left[\begin{pmatrix} 1 & & \\ & k\, e^{-i\alpha} & \\ & & kp\ e^{-i(\alpha-\beta)} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & k\, e^{i\alpha} & \\ & & kp\ e^{i(\alpha-\beta)} \end{pmatrix} + \Lambda \right]$$

where $\alpha = \arctan(S\sin\delta/(R-Q))$, $\beta = \arctan(B\sin\delta/(A+B\cos\delta))$, and

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon & \varepsilon' e^{-i\beta} \\ 0 & \varepsilon' e^{i\beta} & \eta \end{pmatrix}$$

with

$$\begin{aligned} &k = |M_{12}|/M_{11} = \tfrac{|R-Q-iS\sin\delta|}{R+Q+S\cos\delta}, \quad p = |M_{13}|/|M_{12}| = \tfrac{|A-Be^{-i\delta}|}{|R-Q-iS\sin\delta|}, \\ &\varepsilon = (|M_{22}||M_{11}| - |M_{12}|^2)/|M_{11}|^2 = \tfrac{4RQ-S^2}{|R+Q+S\cos\delta|^2} \approx 2m_1/A, \\ &\varepsilon' = (|M_{23}||M_{11}| - |M_{13}||M_{12}|)/|M_{11}|^2, \\ &\eta = (|M_{33}||M_{11}| - |M_{13}|^2)/|M_{11}|^2 \approx \tfrac{1}{2}\tfrac{m_c}{A}\tfrac{m_t}{A} \end{aligned}$$

Inserting the masses $(m_u(M_Z), m_c(M_Z), m_t(M_Z)) = (1.24, 624, 171550)$ MeV, we get $C_u = 53723.5$MeV, $k = 1.00318$, $p = 1.0828$, and

$\varepsilon = 0.00004646 \approx 2(m_u/C_u),\ \varepsilon' = 0.0000444,\ \eta = 0.0185 \approx \frac{1}{2}(m_t/C_u)(m_c/C_u)$

For the down-sector, with

$$U' = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix},$$

the mass matrix $U'^{\dagger} diag(m_d, m_s, m_b) U'$ reads

$$M' = C_d \begin{pmatrix} X+A & Y & 1 \\ Y & X-A & 1+2A \\ 1 & 1+2A & X+Y-A-1 \end{pmatrix}$$

where

$C_d = 2(m_d c_\beta^2 + m_s s_\beta^2) - 2\sqrt{3} c_\beta s_\beta (m_s - m_d)) + 2(m_b - m_s - m_d)$
$X = (2m_b + m_s + m_d + 2(m_d c_\beta^2 + m_s s_\beta^2) + 2\sqrt{3} c_\beta s_\beta (m_s - m_d))/C_d$
$Y = (2m_b + m_s + m_d - 4(m_d c_\beta^2 + m_s s_\beta^2))/C_d,$
$A = 2\sqrt{3} c_\beta s_\beta (m_s - m_d))/C_d.$

Inserting the masses $(m_d(M_Z), m_s(M_Z), m_b(M_Z)) = (2.69, 53.8, 2850)$ MeV, we moreover get the numerical values

$C_d = 926.448 MeV \approx m_b/3$, $X = 1.0375$, $A = 0.0070$, $Y = 1.0318$.

12.6 Conclusion

By including the democratic rotation matrix in the parametrization of the weak mixing matrix, we obtain mass matrices with specific democratic textures. In this way we make contact between the democratic hypothesis and the experimentally derived parameters of the CKM mixing matrix, avoiding the introduction of additional concepts.

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13 Discussion Section on LHC Data

F.J. Llanes-Estrada

Fis. Teorica I, U. Complutense, 28040 Madrid, Spain

Abstract. We report on exchanges entertained and new developments reported and discussed at the workshop "What Comes Beyond the Standard Models" held in Bled, Slovenia, July 11th-19th 2015.

New LHC data, various unification schemes with and without gravity, the nature of fermions, flavor and the number of families, condensates, and other topics of current interest were all heatedly discussed.

Povzetek. Avtor poroča o diskusijah med predavanji in v diskusijskih sekcijah na izbrane teme, ter o napredku, ki ga je prinesla letošnja blejska delavnica "What Comes Beyond the Standard Models" . Posebej omenja zadnje analize meritev na LHC, o teorijah, ki prinašajo enotno sliko lastnosti fermionov, pomagajo razumeti pojav ustreznih bozonskih polj, vključno z gravitacijo, o napovedih o številu družin fermionov, o skalarnih poljih in lastnostih fermionov, o pojavu kondenzatov in ostalih temah na tem področju.

13.1 SM Electroweak Symmetry Breaking Sector

The LHC starts run II after having found a relatively light scalar particle that could be the predicted Standard Model Higgs at 125 GeV and not much more (to the disappointment of a part of the community that firmly expected TeV-scale supersymmetry). Still, this summer the ATLAS collaboration reported [1] a two-gauge boson spectrum in dijet searches (see talk by Llanes-Estrada in these proceedings) that shows an excess at 2 TeV not confirmed by CMS data.

We discussed whether this could be just a statistical fluctuation. Should increased data taking consolidate the excess, an interesting scenario to analyze was proposed, whether a top-ball [2] made of 6 top quarks and 6 top antiquarks all in an s-wave (with wavefunction antisymmetry allowed by the color and flavor degrees of freedom) might have been produced. The 2 TeV mass of the excess could be about right, since $12 \times m_t \simeq 2.1$ TeV which allows for some Higgs-exchange induced binding, though its production cross-section needed for the low-statistics LHC run-I would need to be very large. This cross-section needs to be estimated by theorists.

Independently of whether new resonances coupling to the Electroweak Symmetry Breaking Sector (EWSBS) are found, this can be studied by means of Effective Field Theory for the currently observed particles h (the new 125 GeV scalar) and $\omega^i \sim W_L,\ Z_L$.

In the non-linear realization of $SU(2)_L \times SU(2)_R \to SU(2)_V$, and neglecting masses of $O(100\text{GeV})$ as appropriate to study the 1-3 TeV region, the corresponding next-to-leading order (NLO) Lagrangian density is

$$\begin{aligned}
\mathcal{L} = &\frac{1}{2}\left[1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2\right]\partial_\mu\omega^i\partial^\mu\omega^j\left(\delta_{ij} + \frac{\omega^i\omega^j}{v^2}\right) + \frac{1}{2}\partial_\mu h\partial^\mu h \\
&+ \frac{4a_4}{v^4}\partial_\mu\omega^i\partial_\nu\omega^i\partial^\mu\omega^j\partial^\nu\omega^j + \frac{4a_5}{v^4}\partial_\mu\omega^i\partial^\mu\omega^i\partial_\nu\omega^j\partial^\nu\omega^j + \frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2 \\
&+ \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^i\partial^\nu\omega^i + \frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^i\partial_\nu\omega^i \qquad (13.1)
\end{aligned}$$

that was described in [3,4]. (Other researchers [5–7] are also pursuing this approach.) This Lagrangian has seven-parameters $(a, b, a_4, a_5, g, d, e)$, with the first two being LO and the other five NLO in the derivative expansion.

Two strategies can be followed. If the LHC run-II finds no new physics, it can conduct precision work to try to see a separation of the Standard Model $(1, 1, 0, 0, 0, 0, 0)$. Currently only $a \in (0.88, 1.3)$ is known at 2σ confidence level. Norma Mankoc triggered discussion on how Effective Theory is philosophically not too satisfactory as its predictivity is moderate and it cannot "solve" the various puzzles of the Standard Model. It remains a powerful descriptive tool to classify data. Full theories will manifest themselves as separations from the SM values of one of those parameters, and matching those UV completions to the Effective Theory allows to classify them and quickly discard or constrain families thereof. Example theories that can soon be tested include for example Left-Right models or Composite Higgs models that include spin-1 resonances within reach of the LHC [8,9].

If the LHC confirms resonant structures in $W_L W_L$ in the 2 TeV region, Effective Theory fails, since a derivative expansion cannot saturate unitarity. The second strategy then activates: the use of Unitarized Chiral Perturbation Theory based on the Lagrangian of Eq. (13.1) is appropriate to describe resonances [10–12].

13.2 The flavor problem and the SM parameters

The largest number of parameters in the Standard Model comes from the flavor sector. There is at present no compelling theory explaining them.

At the workshop, strong arguments were presented in favor of the existence of a fourth family. For example, the Ljubljana unified theory of spin and charge based on $SO(1, 13)$ predicts such a fourth fermion family. Also the concept of fermionization, by which SM fermions can be constructed from boson fields alone, was discussed by H. Nielsen and matching the number of degrees of freedom for the fundamental bosons and the generated fermions required that fourth family.

At present, strong phenomenological obstacles to this fourth family exist that require all its members to have high masses.

For a start, CKM unitarity closes very well with three families, so that the parameters of the unitarity triangle $\bar{\rho}$ and $\bar{\eta}$ are known to 20% and 3% respectively [13].

Second, direct searches at the LHC put excited quarks above the 3 TeV scale [14] so that they start being irrelevant for electroweak-scale physics.

Another hurdle for fourth-family extensions of the SM is that the number of relativistic degrees of freedom is tightly constrained from cosmology. For example, the Planck collaboration [15] reports an analysis of Baryon Acoustic Oscillations and the Cosmic Microwave Background that yields $N_{\rm eff} = 3.30 \pm 0.27$ for the effective number of relativistic degrees of freedom. This clearly excludes a fourth light neutrino, in agreement with LEP bounds at the Z-pole. The presumed fourth family therefore comes with an additional hierarchy problem in which $m_{\nu_4} \gg m_{\nu_{1,2,3}}$. (Planck finds that the sum of the three light neutrino masses is 0.23 eV.)

13.3 Other physics at very high scales: unification, condensates and gravity

13.3.1 Gauge symmetry groups

A topic widely discussed at the workshop is why nature chose the symmetry group $U(1) \times SU(2)_L \times SU(3)_c$ to charge the Standard Model fermions. Several possibilities were discussed. A widely accepted one is that the symmetry group at a very high energy scale is larger and we only perceive a remainder subgroup. Well-known are the $SU(5)$ and $SO(10)$ extensions [16] of the Standard Model, in strong tension with proton lifetime bounds. $SO(1, 13)$ has also been presented as an important alternative because of the entailed unification of spins and charges [17] under a common framework.

The first type of groups under discussion do not involve space-time and thus make no statement about gravity. The unification happens at the level of internal degrees of freedom only on a fixed space-time background. The scale must then be smaller than the Planck scale and is usually taken around 10^{15} to 10^{16} GeV where the running couplings of the $U(1)$, $SU(2)$ and $SU(3)$ SM subgroups are all approximately equal (see fig. 13.1).

The second possibility entails unification of internal and space-time symmetries and is a more general concept.

Many questions remain open. One is why given a large group G, the symmetry breaking pattern brings us to the SM group, i.e. $G \to U(1) \times SU(2)_L \times SU(3)$. Currently we know of no good argument why fermion condensates perform exactly this breaking and not something else. (Arranging symmetry breaking by fundamental scalar fields is equally ad-hoc as the potentials must be tuned to produced the wanted results.) One recent alley of investigation [19] addresses the smallness of the SM group dimension. Should there be larger unbroken groups under which certain fermions would be charged, and all couplings being equal at the GUT scale, the finding is that these fermions would be very massive and beyond reach of current collider experiments. This comes about because the large antiscreening for 1-flavor of fermions charged under a large-dimension group (left plot of figure 13.2) forces chiral symmetry breaking at a much larger scale than QCD's $SU(3)$. The corresponding fermion mass is proportional to that scale, $M(0) \propto \Lambda_{\chi SB}$ (right plot of figure 13.2) and out of reach. Fermions charged under

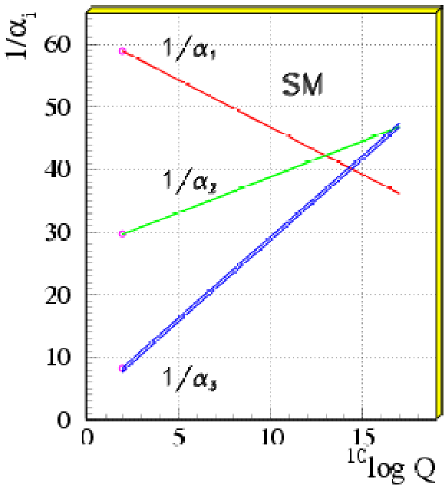

Fig. 13.1. Running coupling constants of $U(1)$, $SU(2)_L$ and $SU(3)_c$ in the absence of new physics through the GUT scale. Reprinted with permission of Particle Data Group [18]
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$SU(4)$ would have masses of $O(10)$ TeV and be not too far in the energy scale, but larger groups yet would yield hopelessly heavy masses.

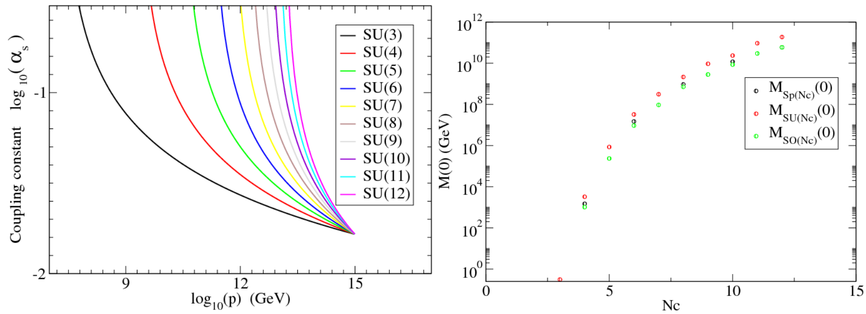

Fig. 13.2. Left: 1-loop running coupling constants for several groups (all equal at the GUT scale). Right: corresponding fermion masses due to chiral symmetry breaking. Were there fermions charged under $SU(4)$ or larger groups, their large mass would have impeded their production at colliders.

13.3.2 Condensates

Probably the biggest current embarrassment of physics is the smallness of the cosmological constant. The Planck collaboration [15] quotes $\Omega_\Lambda = 0.686(20)$ which is more than two thirds of the total energy density in the universe, but only slightly above 3 GeV per cubic meter in absolute value, or about 3×10^{-47} GeV^4. This is an absurdly small number by all SM measures. For example, the QCD condensate is typically found to be $(0.77(4)\Lambda_{\overline{\rm MS}})^3$ with the scale at 0.31(2) GeV [20], or about 0.023 GeV^3. The entailed energy density is 45 orders of magnitude off.

A solution for confining gauge theories such as QCD is to argue that this condensate is active only inside hadrons [21], that is, that the condensate itself is confined around quarks themselves. Dynamical studies of the corresponding domain walls between the condensed and uncondensed phases have to our knowledge not been carried out.

For the electroweak symmetry breaking sector the situation is worse since the corresponding vacuum energy density scale $v^4 = (246\text{GeV})^4$ is now off by 56 orders of magnitude. And it is not obvious that the fundamental scalar Higgs field reported so far will have anything to do with a confining gauge theory, so that the same mechanism can be invoked. In fact, technicolor theories were already discarded at the time of LEP. Likewise, condensates breaking higher symmetry groups will bring about energy densities disparate from the tiny number found by cosmologists.

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14 Vector and Scalar Gauge Fields with Respect to d = (3 + 1) in Kaluza-Klein Theories and in the *Spin-charge-family theory*

D. Lukman and N.S. Mankoč Borštnik

Department of Physics, Jadranska 19, University of Ljubljana, SI-1000 Ljubljana, Slovenia

Abstract. This contribution is to prove that in the Kaluza-Klein like theories the vielbeins and the spin connection fields — as used in the *spin-charge-family* theory — lead in $d = (3+1)$ space to equivalent vector (and scalar) gauge fields. The authors demonstrate this equivalence in spaces with the symmetry: $g_{\alpha\beta} = \eta_{\alpha\beta} e$, for any scalar function e of the coordinates x^{α}.

Povzetek. Prispevek dokazuje na posebnem primeru izometričnih prostorov, da vodijo v teorijah Kaluza-Kleinovega tipa vektorski svežnji in spinske povezave (uporabljene v teoriji *spinov-nabojev-družin*) v prostoru z $d = (3+1)$ do ekvivalentnih vektorskih (in skalarnih) umeritvenih polj. Avtorja demonstrirata enakovrednost obeh pristopov za prostore s simetrijo: $g_{\alpha\beta} = \eta_{\alpha\beta}\, e$, kjer je e poljubna skalarna funkcija koordinat x^{α}.

14.1 Introduction

This contribution is to demonstrate that in spaces with the symmetry of metric tensor $g_{\alpha\beta} = \eta_{\alpha\beta} e$, where $\eta_{\alpha\beta}$ is the diagonal matrix and e any scalar function of the coordinates, both procedures - the ordinary Kaluza-Klein procedure with vielbeins and the procedure with the spin connections used in the *spin-charge-family* theory - lead in $d = (3+1)$ to the same gauge vector and scalar fields.

In the starting action of the *spin-charge-family* theory[1–3] fermions interact with the vielbeins $f^{\alpha}{}_{a}$ and the two kinds of the spin-connection fields - $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ - the gauge fields of $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$ and $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$, respectively.

$$\mathcal{A} = \int d^d x\, E\, \frac{1}{2}\, (\bar{\psi}\, \gamma^a p_{0a} \psi) + h.c. + \int d^d x\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \tag{14.1}$$

here $p_{0a} = f^{\alpha}{}_{a} p_{0\alpha} + \frac{1}{2E}\{p_{\alpha}, E f^{\alpha}{}_{a}\}_{-}$, $p_{0\alpha} = p_{\alpha} - \frac{1}{2} S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}$, $R = \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\,(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^{c}{}_{b\beta})\} + h.c.$, $\tilde{R} = \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\,(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\,\tilde{\omega}^{c}{}_{b\beta})\} + h.c.$. The action introduces two kinds of the Clifford algebra objects, γ^a and $\tilde{\gamma}^a$,

$$\{\gamma^a, \gamma^b\}_{+} = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_{+}\,. \tag{14.2}$$

$f^{\alpha}{}_{a}$ are vielbeins inverted to $e^{a}{}_{\alpha}$, Latin letters $(a, b, ..)$ denote flat indices, Greek letters $(\alpha, \beta, ..)$ are Einstein indices, $(m, n, ..)$ and $(\mu, \nu, ..)$ denote the corresponding indices in $(0, 1, 2, 3)$, $(s, t, ..)$ and $(\sigma, \tau, ..)$ denote corresponding indices in $d \geq 5$:

$$e^{a}{}_{\alpha} f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}\,, \quad e^{a}{}_{\alpha} f^{\alpha}{}_{b} = \delta^{a}_{b}\,, \tag{14.3}$$

$E = \det(e^{a}{}_{\alpha})$. The action $\mathcal{A}$ offers the explanation for all the properties of the observed fermions and their families and of the observed vector gauge fields, the scalar higgs and the Yukawa couplings.

The spin connection fields and the vielbeins are related fields, and if there are no spinor (fermion) sources both kinds of the spin connection fields are expressible with the vielbeins. In Ref. [2] (Eq. (C9)) the expressions related the spin connection fields of both kinds with the vielbeins and the spinor sources are presented.

We prove in this contribution that in the spaces with the maximal number of the Killing vectors [4] (p. 333–340) and no spinor sources either the vielbeins or the spin connections can be used in Kaluza-Klein theories [5] to derive all the vector and scalar gauge fields. We present below the relation among the $\omega_{ab\alpha}$ fields and the vielbeins with no sources present, which is relevant for our discussions ([2], Eq. (C9)).

$$\begin{aligned}
\omega_{ab}{}^{e} = & \frac{1}{2E}\{e^{e}{}_{\alpha}\,\partial_{\beta}(E f^{\alpha}{}_{[a} f^{\beta}{}_{b]}) - e_{a\alpha}\,\partial_{\beta}(E f^{\alpha}{}_{[b} f^{\beta e]}) \\
& - e_{b\alpha}\partial_{\beta}(E f^{\alpha[e} f^{\beta}{}_{a]})\} \\
& - \frac{1}{d-2}\{\delta^{e}_{a}\frac{1}{E}\, e^{d}{}_{\alpha}\partial_{\beta}(E f^{\alpha}{}_{[d} f^{\beta}{}_{b]}) - \delta^{e}_{b}\frac{1}{E} e^{d}{}_{\alpha}\partial_{\beta}(E f^{\alpha}{}_{[d} f^{\beta}{}_{a]})\}\,,
\end{aligned} \tag{14.4}$$

(The expression for the spin connection fields carrying family quantum numbers is in the case that there are no spinor sources identical with the right hand side of Eq. 14.4.) One notices that if there are no spinor sources, carrying the spinor quantum numbers S^{ab}, then ω_{abc} is completely determined by the vielbeins (and so is $\tilde{\omega}_{abc}$).

14.2 Proof that spin connections and vielbeins lead to the same vector gauge fields in $d = (3 + 1)$

We discuss relations between spin connections and vielbeins when there are no spinor sources present in order to prove that both ways, either using the vielbeins or using the spin connection, lead to equivalent vector gauge fields.

Let the space manifest the rotational symmetry, determined by the infinitesimal coordinate transformations of the kind

$$x'^{\mu} = x^{\mu}\,, \quad x'^{\sigma} = x^{\sigma} + \varepsilon^{st}(x^{\mu})\, E^{\sigma}_{st}(x^{\tau}) = x^{\sigma} - i\varepsilon^{st}(x^{\mu})\, M_{st}\, x^{\sigma}\,, \tag{14.5}$$

where $M^{st} = S^{st} + L^{st}$, $L^{st} = x^{s}p^{t} - x^{t}p^{s}$, S^{st} concern internal degrees of freedom of boson and fermion fields, $\{M^{st}, M^{s't'}\}_{-} = i(\eta^{st'}M^{ts'} + \eta^{ts'}M^{st'} - \eta^{ss'}M^{tt'} - \eta^{tt'}M^{ss'})$. From Eq. (14.5) then follows that

$$-i\, M_{st}\, x^{\sigma} = E^{\sigma}_{st} = x_{s}\, f^{\sigma}{}_{t} - x_{t}\, f^{\sigma}{}_{s}\,, \tag{14.6}$$

and correspondingly $M_{st} = E^{\sigma}_{st} p_{\sigma}$. One derives, when taking into account the last relation and the commutation relations among generators of the infinitesimal rotations, the relation

$$E^{\sigma}_{st} p_{\sigma} E^{\tau}_{s't'} p_{\sigma} - E^{\sigma}_{s't'} p_{\sigma} E^{\tau}_{st} p_{\sigma} = -i(\eta_{st'} E^{\tau}_{ts'} + \eta_{ts'} E^{\tau}_{st'} - \eta_{ss'} E^{\tau}_{tt'} - \eta_{tt'} E^{\tau}_{ss'}) p_{\sigma} \,. \tag{14.7}$$

Let the corresponding background field ($g_{\alpha\beta} = e^{a}{}_{\alpha}\, e^{a}{}_{\beta}$) be

$$e^{a}{}_{\alpha} = \begin{pmatrix} \delta^{m}{}_{\mu} & e^{m}{}_{\sigma} = 0 \\ e^{s}{}_{\mu} & e^{s}{}_{\sigma} \end{pmatrix} , \quad f^{\alpha}{}_{a} = \begin{pmatrix} \delta^{\mu}{}_{m} & f^{\sigma}{}_{m} \\ 0 = f^{\mu}{}_{s} & f^{\sigma}{}_{s} \end{pmatrix} , \tag{14.8}$$

so that the background field in $d = (3+1)$ is flat. From $e^{a}{}_{\mu} f^{\sigma}{}_{a} = \delta^{\sigma}_{\mu} = 0$ it follows

$$e^{s}{}_{\mu} = -\delta^{m}_{\mu} e^{s}{}_{\sigma} f^{\sigma}{}_{m} \,. \tag{14.9}$$

This leads to

$$g_{\alpha\beta} = \begin{pmatrix} \eta_{mn} + f^{\sigma}{}_{m} f^{\tau}{}_{n} e^{s}{}_{\sigma} e_{s\tau} & -f^{\tau}{}_{m} e^{s}{}_{\tau} e_{s\sigma} \\ -f^{\tau}{}_{n} e^{s}{}_{\tau} e_{s\sigma} & e^{s}{}_{\sigma} e_{s\tau} \end{pmatrix} . \tag{14.10}$$

One can check properties of $f^{\sigma}{}_{m}\delta^{m}_{\mu}$ under general coordinate transformations $x'^{\mu} = x'^{\mu}(x^{\nu})$, $x'^{\sigma} = x'^{\sigma}(x^{\tau})$ $\left(g'_{\alpha\beta} = \frac{\partial x^{\rho}}{\partial x'^{\alpha}} \frac{\partial x^{\delta}}{\partial x'^{\beta}} g_{\rho\delta}\right)$

$$f'^{\sigma}{}_{m}\delta^{m}_{\mu} = \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\sigma}}{\partial x^{\tau}} f^{\tau}{}_{\nu} \,. \tag{14.11}$$

Let us introduce the field $\Omega^{st}{}_{m}(x^{\nu})$ as follows

$$f^{\sigma}{}_{m} := -\frac{1}{2}\, E^{\sigma}_{st}(x^{\tau})\, \Omega^{st}{}_{m}(x^{\nu}) \,. \tag{14.12}$$

From Eqs. (14.11,14.12) follow the transformation properties of $\Omega^{st}{}_{m}$ under the coordinate transformations of Eq. (14.5)

$$-E^{\sigma st}\, \delta_0\, \Omega_{stm} = -E^{\sigma st}\{-\varepsilon_{st,m} + i2(\varepsilon_{s}{}^{s'}\, \Omega_{s'tm} - \varepsilon_{t}{}^{s'}\, \Omega_{s'sm})\} \,. \tag{14.13}$$

If we look for the transformation properties of the superpositions of the fields Ω_{stm}, which are the gauge fields of let say τ^{Ai} with the commutation relations $\{\tau^{Ai}, \tau^{Bj}\}_{-} = i\delta^{A}_{B}\, f^{Aijk} \tau^{Ak}$, where $\tau^{Ai} = C^{Ai}{}_{st} M^{st}$, under the coordinate transformations of Eq. (14.5), one finds for the corresponding superposition of the fields Ω_{stm} the transformation properties

$$\delta_0\, A^{Ai}{}_{m} = \varepsilon^{Ai}{}_{,m} + i f^{Aijk} A^{Aj}_{m}\, \varepsilon^{Ak} \,. \tag{14.14}$$

Let us use the expression for $f^{\sigma}{}_{m}$ from Eq. (14.12) in Eq. (14.4) to see the relation among ω_{stm} and $f^{\sigma}{}_{m}$. One finds

$$\begin{aligned} \omega_{stm} = \frac{1}{2E}\{ & f^{\sigma}{}_{m}\, [e_{t\sigma}\partial_{\tau}(E f^{\tau}{}_{s}) - e_{s\sigma}\, \partial_{\tau}(E f^{\tau}{}_{t})] \\ & + e_{s\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m} f^{\tau}{}_{t} - f^{\tau}{}_{m} f^{\sigma}{}_{t})] - e_{t\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m} f^{\tau}{}_{s} - f^{\tau}{}_{m} f^{\sigma}{}_{s})]\} \end{aligned} \tag{14.15}$$

For $\Omega_{stm} = \Omega_{stm}(x^n)$ (as assumed above) and for $f^{\sigma}{}_s = f\delta^{\sigma}_s$ (which requires $e^s{}_{\sigma} = f^{-1}\delta^s_{\sigma}$) it follows for any f

$$\omega_{stm} = \Omega_{stm}. \tag{14.16}$$

Statement: Let the space with $s \geq 5$ *have the symmetry allowing the infinitesimal transformations of the kind*

$$x'^{\mu} = x^{\mu}, \quad x'^{\sigma} = x^{\sigma} - i \sum_{A,i,s,t} \varepsilon^{Ai}(x^{\mu})\, c_{Ai}{}^{st} M_{st}\, x^{\sigma}, \tag{14.17}$$

then the vielbein $f^{\sigma}{}_m$ in Eq. (14.8) manifest in $d = (3+1)$ the vector gauge fields $\mathcal{A}^{Ai}_m$

$$f^{\sigma}{}_m = i \sum_{A} \vec{\tau}^{A\sigma}{}_{\tau}\, \vec{\mathcal{A}}^{A}_m x^{\tau}, \tag{14.18}$$

where

$$\begin{aligned} \tau^{Ai} &= \sum_{Ai} c^{Ai}{}_{st}\, M^{st}, \\ \{\tau^{Ai}, \tau^{Bj}\}_- &= i f^{Aijk} \tau^{Ak}\, \delta^{AB}, \\ \vec{\tau}^{A} &= \vec{\tau}^{A\sigma}\, p_{\sigma} = x^{\tau} \vec{\tau}^{A\sigma}{}_{\tau}\, p_{\sigma} \\ A^{Ai}_m &= \sum_{st} c^{Ai}{}_{st} \omega^{st}{}_m\, x^{\tau}, \end{aligned} \tag{14.19}$$

while $\omega^{st}{}_m$ is determined in Eq. (14.15).

We shall prove this statement in the case, when the space $SO(7,1)$ breaks into $SO(3,1) \times SU(2) \times SU(2)$. One finds for the two $SU(2)$ generators

$$\begin{aligned} \vec{\tau}^1 &= \frac{1}{2}\,(M^{58} - M^{67}, M^{57} + M^{68}, M^{56} - M^{78}) \\ \vec{\tau}^2 &= \frac{1}{2}\,(M^{58} + M^{67}, M^{57} - M^{68}, M^{56} + M^{78}), \end{aligned} \tag{14.20}$$

and for the corresponding gauge fields

$$\begin{aligned} \vec{A}^1_a &= \frac{1}{2}\,(\omega_{58a} - \omega_{67a}, \omega_{57a} + \omega_{68a}, \omega_{56a} - \omega_{78a}) \\ \vec{A}^2_a &= \frac{1}{2}\,(\omega_{58a} + \omega_{67a}, \omega_{57a} - \omega_{68a}, \omega_{56a} + \omega_{78a}). \end{aligned} \tag{14.21}$$

One derives (Ref. [2], Eq. (11))

$$\begin{aligned} \vec{\tau}^1 &= \vec{\tau}^{1\sigma}\, p_{\sigma} = \vec{\tau}^{1\sigma}{}_{\tau} x^{\tau}\, p_{\sigma}, \\ \vec{\tau}^2 &= \vec{\tau}^{2\sigma}\, p_{\sigma} = \vec{\tau}^{2\sigma}{}_{\tau} x^{\tau}\, p_{\sigma}, \\ \vec{\tau}^{1\sigma}{}_{\tau} &= \frac{i}{2}\,(e^5{}_{\tau} f^{\sigma 8} - e^8{}_{\tau} f^{\sigma 5} - e^6{}_{\tau} f^{\sigma 7} + e^7{}_{\tau} f^{\sigma 6}, \\ &\quad e^5{}_{\tau} f^{\sigma 7} - e^7{}_{\tau} f^{\sigma 5} + e^6{}_{\tau} f^{\sigma 8} - e^8{}_{\tau} f^{\sigma 6}, \\ &\quad e^5{}_{\tau} f^{\sigma 6} - e^6{}_{\tau} f^{\sigma 5} - e^7{}_{\tau} f^{\sigma 8} + e^8{}_{\tau} f^{\sigma 7}), \\ \vec{\tau}^{2\sigma}{}_{\tau} &= \frac{i}{2}\,(e^5{}_{\tau} f^{\sigma 8} - e^8{}_{\tau} f^{\sigma 5} + e^6{}_{\tau} f^{\sigma 7} - e^7{}_{\tau} f^{\sigma 6}, \\ &\quad e^5{}_{\tau} f^{\sigma 7} - e^7{}_{\tau} f^{\sigma 5} - e^6{}_{\tau} f^{\sigma 8} + e^8{}_{\tau} f^{\sigma 6}, \\ &\quad e^5{}_{\tau} f^{\sigma 6} - e^6{}_{\tau} f^{\sigma 5} + e^7{}_{\tau} f^{\sigma 8} - e^8{}_{\tau} f^{\sigma 7}). \end{aligned} \tag{14.22}$$

The expressions for ${f^{\sigma}}_{m}$ are correspondingly as follows

$$ {f^{\sigma}}_{m} = i\,(\vec{\tau}^{1\sigma}{}_{\tau}\,\vec{\mathcal{A}}^{1}_{m} + \vec{\tau}^{2\sigma}{}_{\tau}\,\vec{\mathcal{A}}^{2}_{m})\,x^{\tau}\,. \tag{14.23}$$

Expressing the two $SU(2)$ gauge fields, $\vec{A}^{1}_{m}$ and $\vec{A}^{2}_{m}$, with ω_{stm} as required in Eqs. (14.21), and then using for each ω_{stm} the expression presented in Eq. (14.15), in which ${f^{\sigma}}_{m}$ is replaced by the relation in Eq. (14.23), while one takes for ${f^{\sigma}}_{s}$ $= f\delta^{\sigma}_{s}$, for any f, while then ${e^{s}}_{\mu} = -\delta^{m}_{\mu}{e^{s}}_{\sigma}{f^{\sigma}}_{m}$, Eq. (14.9), it follows after a longer but straightforward calculation that

$$\begin{aligned} \vec{A}^{1}_{m} &= \vec{\mathcal{A}}^{1}_{m}\,, \\ \vec{A}^{2}_{m} &= \vec{\mathcal{A}}^{2}_{m}\,. \end{aligned} \tag{14.24}$$

One obtains this result of any component of $A^{1i}_{m} and A^{2i}_{m}$, $i = 1,2,3$ separately.

It is not difficult to generalize this poof to any isometry of the space with $s \geq 5$ of any dimensional space, where then

$$ {f^{\sigma}}_{m} = -i\sum_{A} \vec{A}^{A}_{m}\vec{\tau}^{A\sigma}{}_{\tau}x^{\tau}\,, \tag{14.25}$$

where $\vec{A}^{A}_{m}$ are the superposition of ${\omega^{st}}_{m}$, $A^{Ai}_{m} = {c^{Ai}}_{st}{\omega^{st}}_{m}$, which demonstrate the symmetry of the space with $s \geq 5$.

This completes the proof of the above statement.

14.3 Conclusions

We presented the proof, that in spaces without fermion sources either the vielbeins or the spin connections lead in $d = (3+1)$ to the equivalent vector gauge fields. The proof offers indeed no surprise due to the fact that the spin connection fields ω_{abc} are expressible with the vielbeins as presented in (Eq. (14.4). This is true also for the scalar gauge fields, although not discussed in this contribution.

The proof is true for any f which is a scalar function of the coordinates x^{σ}, $\sigma \geq 5$. We have shown in Ref. [7,6] that for $f = (1 + \frac{\rho^2}{(2\rho_0)^2})$ the symmetry of the space with the coordinate x^{σ}, $\sigma = (5), (6)$, is a surface S^2, with one point missing.

14.4 Appendix: Derivation of the equality $\vec{A}^{1}_{m} = \vec{\mathcal{A}}^{1}_{m}$

We demonstrate for the particular case A^{11}_{m}, equal to $\omega_{58a} - \omega_{67a}$, Eq. (14.21), that this A^{11}_{m} is equal to $\mathcal{A}^{11}_{m}$, appearing in Eq. (14.23)

$$ {f^{\sigma}}_{m} = i\sum_{A} \mathcal{A}^{Ai}_{m}{\tau^{Ai\sigma}}_{\tau}x^{\tau}\,. \tag{14.26}$$

When using Eq. (14.15) for $A^{11}_{m} = \omega_{58a} - \omega_{67a}$ we end up with the expression

$$
\begin{aligned}
A^{11}_{m} = \frac{i^2}{2}\frac{1}{2E}\Big\{ & f^{\sigma}{}_{m}[e^{8}{}_{\sigma}\partial_{\tau}(Ef^{\tau 5}) - e^{5}{}_{\sigma}\partial_{\tau}(EF^{\tau 8})] \\
& - f^{\sigma}{}_{m}[e^{7}{}_{\sigma}\partial_{\tau}(Ef^{\tau 6}) - e^{6}{}_{\sigma}\partial_{\tau}(EF^{\tau 7})] \\
& + e^{5}{}_{\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m}f^{\tau 8}) - f^{\tau}{}_{m}f^{\sigma 8}] - e^{6}{}_{\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m}f^{\tau 7}) - f^{\tau}{}_{m}f^{\sigma 7}] \\
& - e^{8}{}_{\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m}f^{\tau 5}) - f^{\tau}{}_{m}f^{\sigma 5}] + e^{7}{}_{\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m}f^{\tau 6}) - f^{\tau}{}_{m}f^{\sigma 6}]\Big\}.
\end{aligned}
\tag{14.27}
$$

We must insert for $f^{\sigma}{}_{m}$ the expression from Eq. (14.23). We obtain

$$
\begin{aligned}
A^{11}_{m} = -\frac{1}{2}\frac{1}{2E}\sum_{i}\mathcal{A}^{1i}_{m}\Big\{ & \tau^{1i\sigma}{}_{\tau'}x^{\tau'}[e^{8}{}_{\sigma}\partial_{\tau}(Ef^{\tau 5}) - e^{5}{}_{\sigma}\partial_{\tau}(EF^{\tau 8}) \\
& - e^{7}{}_{\sigma}\partial_{\tau}(Ef^{\tau 6}) + e^{6}{}_{\sigma}\partial_{\tau}(EF^{\tau 7})] \\
& + e^{5}{}_{\sigma}\delta^{\tau'}_{\tau}E(f^{\tau 8}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 8}\tau^{1i\sigma}{}_{\tau'}) + e^{5}{}_{\sigma}x^{\tau'}\partial_{\tau}[E(f^{\tau 8}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 8}\tau^{1i\sigma}{}_{\tau'})] \\
& - e^{6}{}_{\sigma}E(f^{\tau 7}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 7}\tau^{1i\sigma}{}_{\tau'}) - e^{6}{}_{\sigma}x^{\tau'}\partial_{\tau}[E(f^{\tau 7}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 7}\tau^{1i\sigma}{}_{\tau'})] \\
& - e^{8}{}_{\sigma}E(f^{\tau 5}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 5}\tau^{1i\sigma}{}_{\tau'}) - e^{8}{}_{\sigma}x^{\tau'}\partial_{\tau}[E(f^{\tau 5}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 5}\tau^{1i\sigma}{}_{\tau'})] \\
& + e^{7}{}_{\sigma}E(f^{\tau 6}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 6}\tau^{1i\sigma}{}_{\tau'}) + e^{7}{}_{\sigma}x^{\tau'}\partial_{\tau}[E(f^{\tau 6}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 6}\tau^{1i\sigma}{}_{\tau'})]\Big\}.
\end{aligned}
\tag{14.28}
$$

We can write Eq. (14.28) in a compact way as follows

$$
A^{11}{}_{m} = -\frac{1}{2}\frac{1}{2E}\sum_{i}\mathcal{A}^{1i}{}_{m}\mathcal{C}^{1i}\,, \tag{14.29}
$$

where $\mathcal{C}^{1i}$ can be read off Eq. (14.28). Taking into account in Eqs. (14.22, 14.28) that $f^{\sigma}{}_{s} = f\delta^{\sigma}_{s}$ and $e^{s}{}_{\sigma} = f\delta^{s}_{\sigma}$ we find that most of terms in $\mathcal{C}^{11}$ cancel each other. The only term, which remains, originates in terms from coordinate derivatives, leading to

$$
\mathcal{C}^{11} = 0 + Ef(\tau^{11\,58} - \tau^{11\,67} - \tau^{11\,85} + \tau^{11\,76})\,, \tag{14.30}
$$

while we found that $\mathcal{C}^{12} = 0 = \mathcal{C}^{13}$.

Recognizing that $\mathcal{C}^{2i}$ contribute to A^{11}_{m} nothing, we can conclude that $A^{11}{}_{m} = \mathcal{A}^{11}{}_{m}$.

One easily see that to the expressions for $A^{Ai}{}_{m}$ only $\mathcal{C}^{Ai}$ contribute, while all $\mathcal{C}^{Bj}$, $B \neq A$ and $j \neq i$ contribute nothing. This completes the proof that $\vec{A}^{A}{}_{m} = \vec{\mathcal{A}}^{A}{}_{m}$, for all the gauge fields $\vec{A}^{A}{}_{m}$ of the charges $\vec{\tau}^{A}$, Eq. (14.22).

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15 Degrees of Freedom of Massless Boson and Fermion Fields in Any Even Dimension

N.S. Mankoč Borštnik[1] and H.B.F. Nielsen[2]

[1]Department of Physics, FMF, University of Ljubljana,
Jadranska 19, SI-1000 Ljubljana, Slovenia
[2]The Niels Bohr Institute,
Blegdamsvej 17 - 21, Copenhagen Ø; Denmark

Abstract. This is a discussion on degrees of freedom of massless fermion and boson fields, if they are free or weakly interacting. We generalize the gauge fields of the *spin-charge-family* to the gauge fields of all possible products of γ^a's and of all possible products of $\tilde{\gamma}^a$'s, the first taking care in the *spin-charge-family* theory of the spins and charges ($S^{ab}\ \omega_{abc}$) of fermions, the second ($\tilde{S}^{ab}\ \tilde{\omega}_{abc}$) taking care of families.

Povzetek. Avtorja diskutirata v prispevku prostostne stopnje brezmasnih prostih ali šibko sklopljenih fermionskih in ustreznih bozonskih polj, v primeru, da dovolita, da so bozonska polja umeritvena polja vseh produktov Cliffordovih operatorjev γ^a in umeritvena polja vseh operatorjev $\tilde{\gamma}^a$. Produkti dveh Cliffordovih operatorjev γ^a določajo v teoriji *spina-nabojev-družin* naboje ene družine kvarkov in leptonov, produkti dveh Cliffordovih operatorjev $\tilde{\gamma}^a$ pa družine kvarkov in leptonov.

15.1 Introduction

The purpose of this contribution to the Discussion section of this Proceedings to the Bled 2015 workshop is to hopefully better understand: : **a.** Why is the simple starting action of the *spin-charge-family* theory doing so well in manifesting the observed properties of the fermion and boson fields? **b.** Under which condition would more general action lead to the starting action of Eq. (15.1)? **c.** What would more general action, if leading to the same low energy physics, mean for the history of our Universe? **d.** Could the fermionization procedure of boson fields or the bosonization procedure of fermion fields, discussed in this Proceedings for any dimension d (by the authors of this contribution, while one of them, H.B.F.N. [5], has succeeded with another author to do the fermionization for $d = (1+1)$), help to find the answers to the questions under **a. b. c.**?

In the *spin-charge-family* theory of one of us (N.S.M.B.) [1–4], which offers the possibility to explain all the assumptions of the *standard model*, with the appearance of families, the scalar higgs and the Yukawa couplings included, as well as the matter-antimatter asymmetry in our universe and the appearance of the dark matter, a very simple starting action for massless fermions and bosons in $d =$

$(1+13)$ is assumed. In this action

$$\mathcal{A} = \int d^d x \, E \, \frac{1}{2} \, (\bar{\psi} \, \gamma^a p_{0a} \psi) + h.c. + \int d^d x \, E \, (\alpha \, R + \tilde{\alpha} \, \tilde{R}) \,, \tag{15.1}$$

where $p_{0a} = f^\alpha{}_a p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^\alpha{}_a\}_-$, $p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}$, $R = \frac{1}{2} \{f^{\alpha[a} f^{\beta b]} \; (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \, \omega^c{}_{b\beta})\} + h.c.$, $\tilde{R} = \frac{1}{2} \{f^{\alpha[a} f^{\beta b]} \; (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \, \tilde{\omega}^c{}_{b\beta})\} + h.c.$, the two kinds of the Clifford algebra objects, γ^a and $\tilde{\gamma}^a$,

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ \,, \tag{15.2}$$

which anticommute, $\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$ and determine one of them spins and charges of spinors, another determines families. Here [1] $f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$. There are correspondingly for spinors two kinds of the infinitesimal generators of the groups - S^{ab} for $SO(13,1)$ and $\tilde{S}^{ab}$ for $\widetilde{SO}(13,1)$. The generators $S^{ab} = \frac{i}{4} (\gamma^a \, \gamma^b - \gamma^b \, \gamma^a)$, $\tilde{S}^{ab} = \frac{i}{4} (\tilde{\gamma}^a \, \tilde{\gamma}^b - \tilde{\gamma}^b \, \tilde{\gamma}^a)$, determine in the theory the spin and charges of fermions, S^{ab}, and the family quantum numbers, $\tilde{S}^{ab}$.

The curvature R and $\tilde{R}$ determine dynamics of gauge fields.

The infinitesimal generators of the Lorentz transformations for bosons operate as follows $\mathcal{S}^{ab} \, A^{d \dots e \dots g} = i \, (\eta^{ae} \, A^{d \dots b \dots g} - \eta^{be} \, A^{d \dots a \dots g})$.

We discuss in what follows properties of free massless fermion fields, Sect. 15.1.1, of free massless boson fields and suggest the interaction among fermions and bosons, which fulfill the Aratyn-Nielsen theorem [5], but is in general not gauge invariance.

15.1.1 Properties of general fermion fields

Let us make a choice of one kind of the Clifford algebra objects, let say γ^a's, and express correspondingly the linear vector space of fermions as follows

$$\bar{\Psi}(\gamma) = \psi + \sum_{k=1}^{d} \psi_{a_1 a_2 \dots a_k} \, \gamma^{a_1} \gamma^{a_2} \dots \gamma^{a_k} \,, \quad a_i \le a_{i+1} \,. \tag{15.3}$$

We could as well make a choice of $\tilde{\gamma}^a$'s instead of γ^a's. We define that operation of γ^a and $\tilde{\gamma}^a$ on such a vector space is understood as the *left* and the *right* multiplication, respectively, of any Clifford algebra object. Let $f(\gamma)$ be one of the (orthogonal) fermion states in the Hilbert space. The *left* and the *right* multiplication

[1] $f^\alpha{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b$, $e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$, $E = \det(e^a{}_\alpha)$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indexes from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

can be understood as follows

$$
\begin{aligned}
\gamma^a f(\gamma)\,|\psi_0\ >: &= (\,a_0\,\gamma^a\ + a_{a_1}\,\gamma^a\,\gamma^{a_1} + \\
& \quad a_{a_1 a_2}\,\gamma^a\,\gamma^{a_1}\gamma^{a_2} + a_{a_1\cdots a_d}\,\gamma^a\,\gamma^{a_1}\cdots\gamma^{a_d}\,)\,|\psi_0\ >,\\
\tilde{\gamma}^a f(\gamma)\,|\psi_0\ >: &= (\,i\,a_0\gamma^a - i\,a_{a_1}\gamma^{a_1}\,\gamma^a + i\,a_{a_1 a_2}\gamma^{a_1}\gamma^{a_2}\,\gamma^a + \cdots + \\
& \quad i\,(-1)^d\,a_{a_1\cdots a_d}\gamma^{a_1}\cdots\gamma^{a_d}\,\gamma^a\,)\,|\psi_0\ >, \qquad (15.4)
\end{aligned}
$$

where $|\psi_0\ >$ is a vacuum state.

Eq. (15.3) represents 2^d internal degrees of freedom, that is 2^d basic states. Let us arrange the basis to be orthogonal in a way that operators S^{ab} transform $2^{\frac{d}{2}-1}$ members of these basic states among themselves. They represent one family. The operators $\tilde{S}^{ab}$ transform each family member into the same family member of one of $2^{\frac{d}{2}-1}$ families.

There are obviously four such groups of $2^{\frac{d}{2}-1}$ families with $2^{\frac{d}{2}-1}$ family members ($2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1} \times 2^2 = 2^d$). These four groups differ in the eigenvalues of the two operator of handedness, $\Gamma^{(1+(d-1))}$ and $\widetilde{\Gamma}^{(1+(d-1))}$,

$$
\begin{aligned}
\Gamma^{(1+(d-1))} &= (-i)^{\frac{d-2}{2}}\,\gamma^{a_1}\gamma^{a_2}\ldots\gamma^{a}_{d}\,,\\
\widetilde{\Gamma}^{(1+(d-1))} &= (-i)^{\frac{d-2}{2}}\,\tilde{\gamma}^{a_1}\tilde{\gamma}^{a_2}\ldots\tilde{\gamma}^{a}_{d}\,,\\
& \qquad a_k < a_{k+1}\,. \qquad (15.5)
\end{aligned}
$$

The eigenvalues of $[(\Gamma^{(1+(d-1))}, \widetilde{\Gamma}^{(1+(d-1))}]$ are $= [(+,+),(-,+),(+,-),(-,-)]$. Each of the groups can be extracted from the basis due to requirement

$$
\begin{aligned}
&A.\ \ (1-\widetilde{\Gamma}^{(1+(d-1))})\,(1-\Gamma^{(1+(d-1))})\,\bar{\Psi} = 0\,,\\
&B.\ \ (1-\widetilde{\Gamma}^{(1+(d-1))})\,(1+\Gamma^{(1+(d-1))})\,\bar{\Psi} = 0\,,\\
&C.\ \ (1+\widetilde{\Gamma}^{(1+(d-1))})\,(1-\Gamma^{(1+(d-1))})\,\bar{\Psi} = 0\,,\\
&D.\ \ (1+\widetilde{\Gamma}^{(1+(d-1))})\,(1+\Gamma^{(1+(d-1))})\,\bar{\Psi} = 0\,. \qquad (15.6)
\end{aligned}
$$

In ($d = 4n$)-dimensional spaces the first and the last condition share the space of spinors determined by an even number of γ^a's in each product, Eq. (15.3), while the second and the third share the rest half of the spinor space determined by an odd number of γ^a's in each product. In ($d = 2(2n + 1)$)-dimensional spaces is opposite: The first and the last condition determine spinor space of and odd number of γ^a's in each product, while the second and the third require an even number of γ^a's in each product.

Let us denote these four groups of states, defined in Eqs. (15.3,15.6) with the values of $[(\Gamma^{(1+(d-1))}, \widetilde{\Gamma}^{(1+(d-1))}] = [(+,+),(-,+),(+,-),(-,-)]$, by ($\bar{\Psi}_{++}$, $\bar{\Psi}_{-+}$, $\bar{\Psi}_{+-}$, $\bar{\Psi}_{--}$), respectively.

States of each group can be chosen to fulfill the Weyl dynamical equation for free massless spinors

$$
\begin{aligned}
&\gamma^0\,\gamma^a p_a \bar{\Psi}_{ij} = 0\,,\\
&\qquad (i,j) \in \{(+,+),(-,+),(+,-),(-,-)\}\,. \qquad (15.7)
\end{aligned}
$$

In the *spin-charge-family* theory one family contains, if analyzed with respect to the spin and charges of the *standard model*: the left handed weak charged quarks and the leptons - electrons and neutrinos - and the right handed weak chargeless quarks and leptons, with by the *standard model* assumed hyper charges, as well as the right handed weak charged quarks and leptons and left handed weak chargeless quarks and leptons. The break of the starting symmetry than leads to two groups of four families, which gain masses at the electroweak break. All the rest families ($2^{\frac{14}{2}-1}$ -8) gain masses interacting with the scalar fields.

These 2^d orthogonal basic states can be reached from any one of them by applying on such a state the products of operators: a constant, $\gamma^{a_1}, \tilde{\gamma}^{a_1}$, and products of γ^{a_i} and products of $\tilde{\gamma}^{b_1}$.

Let us see on the case of $d = 2$, how do these four groups of families and family members distinguish among themselves.

We shall check also conditions under which these fermion states fulfill the Weyl equation, (Eq. (15.7)), for free (massless) fermions.

Properties of four groups of fermion states defined in Eq. (15.6) To better understand the meaning of the four groups (Eq. (15.6)) of families and family members let start with the simplest case: $d = (1+1)$ - dimensional spaces.

- **d=(1+1) case.**

The requirement A. of Eq. (15.6) ($(1-\tilde{\Gamma}^{(1+1)})\,(1-\Gamma^{(1+1)})\,\bar{\Psi}_{\bar{\Psi}} = 0$, $\bar{\Psi}_{++} = \psi + \gamma^0\psi_0 + \gamma^1\psi_1 + \gamma^0\gamma^1\,\psi_{01}$) leads to $\psi_0 + \psi_1 = 0$, or consequently $\bar{\Psi}_{++} = \psi_{++}\,(\gamma^0 - \gamma^1)$. This state fulfills the Weyl equation provided that $(p_0 - p_1)\,\psi_{++} = 0$.

The requirement B. of Eq. (15.6) ($(1-\tilde{\Gamma}^{(1+1)})\,(1+\Gamma^{(1+1)})\,\bar{\Psi} = 0$) leads to $\psi + \psi_{01} = 0$, or consequently $\bar{\Psi}_{+-} = \psi_{+-}\,(1 - \gamma^0\gamma^1)$. This state fulfills the Weyl equation provided that $(p_0 + p_1)\,\psi_{+-} = 0$.

The requirement C. of Eq. (15.6) ($(1+\tilde{\Gamma}^{(1+1)})\,(1-\Gamma^{(1+1)})\,\bar{\Psi} = 0$) leads to $\psi - \psi_{01} = 0$, or consequently $\bar{\Psi}_{-+} = \psi_{-+}\,(1 + \gamma^0\gamma^1)$. This state fulfills the Weyl equation provided that $(p_0 - p_1)\,\psi_{-+} = 0$.

The requirement D. of Eq. (15.6) ($(1-\tilde{\Gamma}^{(1+1)})\,(1-\Gamma^{(1+1)})\,\bar{\Psi} = 0$) leads to $\psi_0 - \psi_1 = 0$, or consequently $\bar{\Psi}_{--} = \psi_{--}\,(\gamma^0 + \gamma^1)$. This state fulfills the Weyl equation provided that $(p_0 + p_1)\,\psi_{--} = 0$.

Making a choice of p_1 showing in the positive direction, the first and the third choice correspond to the positive energy solution, while the second and the fourth choice correspond to the negative energy solution of the Weyl equation (15.7).

Each of the four groups of states contains $2^{\frac{d}{2}-1} = 1$ state and $2^{\frac{d}{2}-1} = 1$ familiy. The operators $(1, \gamma^0\gamma^1, \tilde{\gamma}^0\tilde{\gamma}^1)$ are diagonal, the operators $(\gamma^0, \gamma^1, \tilde{\gamma}^0, \tilde{\gamma}^1)$ are off diagonal. Let us present the matrices for, let say, γ^0, $\tilde{\gamma}^0$ and $\gamma^0\tilde{\gamma}^0$ for the basic states, arranged as follows $\stackrel{01}{(+i)} = \frac{1}{2}(\gamma^0 - \gamma^1)$ (the case A.), $\stackrel{01}{(-i)} = \frac{1}{2}(\gamma^0 + \gamma^1)$ (the case D.), $\stackrel{01}{[+i]} = \frac{1}{2}(1 + \gamma^0\gamma^1)$ (the case C.), $\stackrel{01}{[-i]} = \frac{1}{2}(1 - \gamma^0\gamma^1)$ (the case B.). Let us notice that $\Gamma^{(1+1)}\,(\stackrel{01}{(+i)}, \stackrel{01}{(-i)}, \stackrel{01}{[+i]}, \stackrel{01}{[-i]}) = (\stackrel{01}{(+i)}, -\stackrel{01}{(-i)}, \stackrel{01}{[+i]}, -\stackrel{01}{[-i]})$, while $\widetilde{\Gamma}^{(1+1)}\,(\stackrel{01}{(+i)}, \stackrel{01}{(-i)}, \stackrel{01}{[+i]}, \stackrel{01}{[-i]}) = (\stackrel{01}{(+i)}, -\stackrel{01}{(-i)}, -\stackrel{01}{[+i]}, \stackrel{01}{[-i]})$. One finds the matrix representation for γ^0 and $\tilde{\gamma}^0$ and $\gamma^0\tilde{\gamma}^0$

$$\gamma^0 = \begin{pmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&1&0&0\\ 1&0&0&0 \end{pmatrix}, \tilde{\gamma}^0 = \begin{pmatrix} 0&0&i&0\\ 0&0&0&i\\ -i&0&0&0\\ 0&-i&0&0 \end{pmatrix}, \gamma^0\tilde{\gamma}^0 = \begin{pmatrix} 0&-i&0&0\\ -i&0&0&0\\ 0&0&0&i\\ 0&0&i&0 \end{pmatrix}. \quad (15.8)$$

While γ^0 causes the transformations among states, which have the opposite handedness $\Gamma^{(1+1)}$, while they have the same handedness $\tilde{\Gamma}^{(1+1)}$, transforms $\tilde{\gamma}^0$ among states of opposite handedness $\tilde{\Gamma}^{(1+1)}$, leaving handedness $\Gamma^{(1+1)}$ unchanged. The operator $\gamma^0\tilde{\gamma}^0$ causes transformations among the states, which differ in both handedness. Interaction of the type $S^{ab}\omega_{abc}$ and $\tilde{S}^{ab}\tilde{\omega}_{abc}$, appearing in the action Eq.(15.1) do not cause in this $d = (1+1)$ case transformations among the basic states $(\overset{01}{(+i)}, \overset{01}{(-i)}, \overset{01}{[+i]}, \overset{01}{[-i]})$.

o d=(13+1) case.

In the case of $d = (13+1)$-dimensional space the operators S^{ab} transform all the members of one family among themselves. Table IV of Ref. [4] represents one family representation analyzed with respect to the *standard model* gauge and spinor groups. The $2^{d/2-1} = 64$ members represent quarks and leptons, left and right handed, with spin up and down and with the hyper charges as required by the *standard model*. There are also the anti-members, reachable from members not only by S^{ab} but also by $\mathcal{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}}$ [7].

The operators $\tilde{S}^{ab}$ transform each family member of a particular family into another family, keeping the family member quantum numbers unchanged.

There are four groups of such families, having

$$(\Gamma^{(13+1)}, \tilde{\Gamma}^{(13+1)}) = ((+,+), (-,-), (+,-), (-,+)),$$

respectively. As seen in the simple case of $d = (1+1)$ all four groups could be reachable from the starting one only by the operators γ^a, $\tilde{\gamma}^a$ and $\gamma^a\tilde{\gamma}^b$.

We have some experience with the toy model in $d = (5+1)$, Refs. [8–10], that when breaking symmetries not only that only spinors of one handedness remain masless, but also most of families can get heavy masses.

After the break of $SO(13,1)$ to $SO(7,1) \times SO(6)$ (and correspondingly also of $\widetilde{SO}(13,1)$) S^{st}, $s \in (0,\ldots,8), t \in (9,\ldots,14)$ (and correspondingly also of $\tilde{S}^{st}$, $s \in (0,\ldots,8), t \in (9,\ldots,14)$) are no longer applicable. Anti spinors (spinors with quantum numbers of the second part, numerated by 33 up to 64, of Table IV in Ref. [4]) are after the break reachable only by $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ [7].

The break of $SO(6)$ to $SU(3) \times U(1)$ disables transformations from quarks to leptons.

When breaking symmetries, like from $SO(13,1)$ to $SO(7,1) \times SO(6)$, the break must be done in a way that only spinors of one handedness remain massless in order that the break leads to observed (almost massless) fermions and that most of families get masses of the energy of the break [8–10]. Our studies so far support the assumption that only the families with $\tilde{\Gamma}^{(7+1)} = 1$ and $\tilde{\Gamma}^{(6)} = -1$ remain massless.

Correspondingly only eight families ($2^{(7+1)/2-1}$) remain massless.

At the further break of $SO(7,1) \times SU(3) \times U(1)$ to $SO(3,1) \times SU(2) \times SU(3) \times U(1)$ all the eight families of quarks and leptons remain massless due to the fact that left handed and right handed quarks and leptons have different charges and are correspondingly mass protected.

15.1.2 Properties of general boson fields

We have discussed so far only fermion fields. The *spin-charge-family* theory action, Eq (15.1), introduces the vielbeins and the two kinds of the spin-connection fields, with which the fermions interact. These are the gauge fields of the two kinds of charges, which take care of the family members quantum numbers (S^{ab}) and of the family quantum numbers ($\tilde{S}^{ab}$).

The Lagrange density (15.1) of each kind of the spin connection fields is linear in the curvature. This action seems to be the simplest action of the Kaluza-Klein kinds of theories, in which fermions carry the family and the family members quantum numbers, while the gravitational field - the vielbeins and the two kinds of the spin connection fields take care of the interaction among fermions. Vielbeins and spin connections are the only boson fields in the theory. They manifest at the low energy regime all the phenomenologically needed vector and scalar bosons.

Let us define boson fields, which in the case of $d = (1+1)$, $d = (13+1)$, or any d, transform the 2^d fermion states among themselves? The fields $S^{ab}\omega_{abc}$ and $\tilde{S}^{ab}\tilde{\omega}_{abc}$ can, namely, cause transitions only among fermions with the same Clifford character: The Clifford even (odd) fermion states are transformed into the Clifford even (odd) fermion states, as we have seen in subsection 15.1.1.

Let us assume for this purpose that there exist to each of products

$$\gamma^{a_1}\gamma^{a_2}\ldots\gamma^{a_k},$$

the number of products of γ^a's running from zero to d, the corresponding gauge fields: $\omega_{a_1 a_2 \ldots a_k}$. There are obviously 2^d such gauge fields. These gauge fields, carrying k vector indexes $a_1 \ldots a_k$, transform a fermion state

$$\underline{\Psi}_{ij}, (i,j) = [(+,+),(-,+),(+,-),(-,-)]$$

belonging to one of the four groups (with the eigenvalues of ($\Gamma^{(d)}$,$\tilde{\Gamma}^{(d)}$= (i,j), respectively), discussed in subsection 15.1.1, into another state, belonging to the same or to one of the rest free groups: If starting with the state of either the A. or B. groups, these bosons transform such a state to one of the states belonging to either the group A. (if the number of a_j is even) or to the group B. (if the number of a_j is odd). If we start from the group C. or D., then the transformed state remains within these two groups.

Correspondingly we define to each of products $\tilde{\gamma}^{a_1}\ \tilde{\gamma}^{a_2}\ldots\tilde{\gamma}^{a_k}$, again the number of products of $\tilde{\gamma}^a$'s running from zero to d, the corresponding gauge fields $\tilde{\omega}_{a_1 a_2 \ldots a_k}$, which again transform the state $\underline{\Psi}_{ij}$, belonging to one of the four groups, discussed in subsection 15.1.1, into another state, belonging to the same (if the number of a_k is even), or to one of the rest free groups (if the number of a_k is odd). In this case the transformations go from A. to C., or from B. to D..

All the states of one group of fermions are reachable from the starting state under the application of ω_{abc} and $\tilde{\omega}_{abc}$. The operators S^{ab} and $\tilde{S}^{ab}$ keep the handedness $\Gamma^{(d)}$ and $\tilde{\Gamma}^{(d)}$, respectively, unchanged. (Let us remind the reader that all the $2^{(13+1)/2-1}$ states of one family (Table IV of Ref. [4]) are reachable by $S^{ab}\omega_{abc}$ and all the $2^{(7+1)/2-1}$ families (Table V of Ref. [4]) are reachable by $\tilde{S}^{ab}\tilde{\omega}_{abc}$).

The by the products of $\tilde{\gamma}^a$'s transformed state $\tilde{\Psi}$ differs in general from the one transformed by the product of γ^a's according to the definition in Eq. (15.2).

Let us assume that all the boson fields obey the equations of motion

$$\begin{aligned}\partial^a\partial_a\omega_{a_1 a_2\dots a_k} &= 0\,,\\ \partial^a\partial_a\tilde{\omega}_{a_1 a_2\dots a_k} &= 0\,.\end{aligned} \tag{15.9}$$

For the boson fields, which are the gauge fields of the products of $\tilde{\gamma}^{a_1}\,\tilde{\gamma}^{a_2}\dots\tilde{\gamma}^{a_k}$ or of $\gamma^{a_1}\gamma^{a_2}\dots\gamma^{a_k}$ Eq. (15.9), this can only be true in the weak fields limit.

Let us see the action of this boson fields on fermion basic states in the case of $d = (1+1)$. The boson fields bring to fermions the quantum numbers, which they carry. We can calculate these quantum numbers by taking into account Eq. (16) in Ref. [4]

$$\mathcal{S}^{ab}\,A^{d\dots e\dots g} = i\,(\eta^{ae}\,A^{d\dots b\dots g} - \eta^{be}\,A^{d\dots a\dots g})\,, \tag{15.10}$$

or we can simply calculate the action of the operators, the gauge fields of which are boson fields.

$$\begin{aligned}
1\ \ (1,\,\gamma^0,\gamma^1,\,\gamma^0\gamma^1) &= (1,\gamma^0,\gamma^1,\gamma^0\gamma^1)\,,\\
\tilde{1}\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (1,\gamma^0,\gamma^1,\gamma^0\gamma^1)\,,\\
\gamma^0\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (\gamma^0,1,\gamma^0\gamma^1,\gamma^1)\,,\\
\tilde{\gamma}^0\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= i\,(\gamma^0,-1,\gamma^0\gamma^1,-\gamma^1)\,,\\
\gamma^1\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (\gamma^1,-\gamma^0\gamma^1,-1,\gamma^0)\,,\\
\tilde{\gamma}^1\ \ (1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= i\,(\gamma^1,-\gamma^0\gamma^1,1,-\gamma^0)\,,\\
\gamma^0\gamma^1\,(1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (\gamma^0\gamma^1,-\gamma^1,-\gamma^0,1)\\
\tilde{\gamma}^0\tilde{\gamma^1}\,(1,\gamma^0,\gamma^1,\gamma^0\gamma^1) &= (i)^2\,(\gamma^0\gamma^1,\gamma^1,\gamma^0,1)\,,
\end{aligned} \tag{15.11}$$

It is obvious that the two kinds of fields influence states in a different way, except the two constants, which leave states untouched.

One can conclude that there are correspondingly $2\times 2^d - 1$ independent real boson fields (only one of the two constants has the meaning), and there are also, as we have learned in Subsec. 15.1.1 2^d complex fermion fields, which means 2×2^d real fermion fields in any dimension. This supports the Aratyn-Nielsen theorem [5].

o Comments on **d=(1+1) case**.

Let us make a choice of $2^{\frac{d}{2}-1}$ fermion states, which is for $d = 2$ only one state, say $\overset{01}{(+i)}$. It is the complex field and accordingly with two degrees of freedom.

One can make then (any) one choice of the boson field, let say ω_{01}, which is the gauge field of the "charge" $\Gamma^{(1+1)}$. This is in agreement with the Aratyn-Nielsen theorem.

All the (complex) Clifford odd fermion states, $(\stackrel{01}{(+i)}, \stackrel{01}{(-i)}$, need three of the independent boson fields, let say $(\gamma^1 \omega_1, \gamma^0\gamma^1 \omega_{01}, \tilde{\gamma}^1 \tilde{\omega}_1)$, to be in agreement with the Aratyn-Nielsen theorem.

Bosons in interaction with fermions If we expect gauge boson fileds to appear in the covariant derivative of fermions, as we are used to require, then all the gauge fields must curry the space index, like it is the case of the covariant derivative for fermions, presented in Eq. (15.1): $p_{0a} = p_a - \frac{1}{2} S^{bc} \omega_{bca} - \frac{1}{2} \tilde{S}^{bc} \tilde{\omega}_{bca}$.

Let us generalize this covariant momentum by replacing $\frac{1}{2} S^{a_1 a_2} \omega_{a_1 a_2 a} + \frac{1}{2} \tilde{S}^{a_1 a_2} \tilde{\omega}_{a_1 a_2 a}$ by

$$
\begin{aligned}
p_{0a} = p_a - & \\
& \{\omega_a + \gamma^{a_1} \omega_{a_1 a} + \gamma^{a_1}\gamma^{a_2} \omega_{a_1 a_2 a} + \cdots + \gamma^{a_1}\gamma^{a_2} \ldots \gamma^{a_d} \omega_{a_1 a_2 \ldots a_d\, a} \\
& + \tilde{\gamma}^{a_1} \tilde{\omega}_{a_1 a} + \tilde{\gamma}^{a_1}\tilde{\gamma}^{a_2} \tilde{\omega}_{a_1 a_2 a} + \cdots + \tilde{\gamma}^{a_1}\tilde{\gamma}^{a_2} \ldots \tilde{\gamma}^{a_d} \tilde{\omega}_{a_1 a_2 \ldots a_d\, a}\} \,.
\end{aligned}
\tag{15.12}
$$

We assumed that all the γ^a's in products appear in the ascending order. Correspondingly is $\frac{1}{2} S^{a_1 a_2} \omega_{a_1 a_2 a}$ replaced by $\frac{i}{2}\, \gamma^{a_1}\gamma^{a_2} \omega_{a_1 a_2 a}$, the factor $\frac{i}{2}$ appears due to $S^{a_1 a_2} = \frac{i}{2} \gamma^{a_1}\gamma^{a_2}$, $a_2 > a_1$.

This theory would neither be gauge invariant nor do the corresponding gauge fields fulfill the equations of motion, Eq. (15.9), except in the weak limit if the gauge fields appear as the background fields. The degrees of freedom of bosons and fermions no longer fulfill the Aratyn-Nielsen theorem, unless we again allow either only Clifford even or Clifford odd fermion states and only one of the two fields with the space index zero, let say ω_0 among the boson fields is allowed. And yet we have in addition nonphysical degrees of freedom due to gauge invariance for almost free massless fields in the weak limit, which should be possibly removed.

If nature has ever started with the boson fields as presented above, most of these fields do not manifest in $d = (3+1)$.

15.2 Conclusions

We have started the fermionization of boson fields (or bosonization of fermion fields) in any d (the reader can find the corresponding contribution in this proceedings)to understand better why, if at all, the nature has started in higher dimensions with the simple action as assumed in the *spin-charge-family* theory, offering in the low energy regime explanation for all observed degrees of freedom of fermion and boson fields, with the families of fermions included. This theory is a kind of the Kaluza-Klein theories with two kinds of the spin connection fields. We also hope that the fermionizasion can help to see which role can the same number of degrees of freedom of fermions and bosons play in the explanation, why the cosmological constant is so small.

This contribution is a small step towards understanding better the open problems of the elementary particle physics and cosmology. We discussed for any d-dimensional space the degrees of freedom for free massless fermions and the degrees of freedom for free massless bosons, which are the gauge fields of all possible products of both kinds of the Clifford algebra objects, either of γ^a or of $\tilde{\gamma}^a$.

Although we have not yet learned enough to be able to answer any of the four questions, presented in the introduction (**a.** Why is the simple starting action of the *spin-charge-family* theory doing so well in manifesting the observed properties of the fermion and boson fields? **b.** Under which condition can more general action lead to the starting action of Eq. (15.1)? **c.** What would more general action, if leading to the same low energy physics, mean for the history of our Universe? **d.** Could the fermionization procedure of boson fields or the bosonization procedure of fermion fields, discussed in this Proceedings for any dimension d (by the authors of this contribution, while one of them, H.B.F.N. [5], has succeeded with another author to do the fermionization for $d = (1 + 1)$), help to find the answers to the questions under **a. b. c.**?), yet we have started to understand better the topic.

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Virtual Institute of Astroparticle Physics Presentation

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16 Virtual Institute of Astroparticle Physics and Discussions at XVIII Bled Workshop

M.Yu. Khlopov [1,2,3]⋆⋆

[1] Centre for Cosmoparticle Physics "Cosmion"
[2] National Research Nuclear University "Moscow Engineering Physics Institute", 115409 Moscow, Russia
[3] APC laboratory 10, rue Alice Domon et Léonie Duquet
75205 Paris Cedex 13, France

Abstract. Being a unique multi-functional complex of science and education online, Virtual Institute of Astroparticle Physics (VIA) operates on website http://viavca.in2p3.fr/site.html. It supports presentation online for the most interesting theoretical and experimental results, participation in conferences and meetings, various forms of collaborative scientific work as well as programs of education at distance, combining online videoconferences with extensive library of records of previous meetings and Discussions on Forum. Since 2014 VIA online lectures combined with individual work on Forum is being elaborated in a specific tool for MOOC activity. The VIA facility is regularly effectively used in the programs of Bled Workshops. At XVIII Bled Workshop it provided a world-wide discussion of the open questions of physics beyond the standard model, supporting presentations at distance and world-wide propagation of discussions at this meeting.

Povzetek. Virtual Institute of Astroparticle Physics (VIA), ki deluje na spletni strani http://viavca.in2p3.fr/site.html, je vsestranski sistem za podporo znanosti in izobraževanja na spletu. Sistem podpira neposredne spletne predstavitve najbolj zanimivih teoretičnih in eksperimentalnih rezultatov, udeležbo na konferencah in srečanjih, različne oblike skupnega znanstvenega dela, pa tudi programe izobraževanja na daljavo, ki povezujejo neposredne videokonference in zapise prejšnjih srečanj in diskusij na spletnem forumu VIA. Od leta 2014 se kombinacija neposrednih spletnih predavanj in individualnega dela na spletnih forumih VIA razvija kot orodje za množično spletno izobraževanje na daljavo (MOOC). Sredstva VIA se redno učinkovito uporablja na blejskih delavnicah. Na letošnji, osemnajsti, delavnici je omogočila diskusijo udeležencev iz vseh koncev sveta o odprtih vprašanjih fizike onkraj standardnih modelov fizike osnovnih delcev in kozmologije ter podporo predstavitev na daljavo in svetovno dostopnost diskusij na delavnici.

16.1 Introduction

Studies in astroparticle physics link astrophysics, cosmology, particle and nuclear physics and involve hundreds of scientific groups linked by regional networks (like ASPERA/ApPEC [1,2]) and national centers. The exciting progress in these studies

⋆⋆ khlopov@apc.univ-paris7.fr

will have impact on the knowledge on the structure of microworld and Universe in their fundamental relationship and on the basic, still unknown, physical laws of Nature (see e.g. [3,4] for review).

Virtual Institute of Astroparticle Physics (VIA) [5] was organized with the aim to play the role of an unifying and coordinating structure for astroparticle physics. Starting from the January of 2008 the activity of the Institute takes place on its website [6] in a form of regular weekly videoconferences with VIA lectures, covering all the theoretical and experimental activities in astroparticle physics and related topics. The library of records of these lectures, talks and their presentations was accomplished by multi-lingual Forum. In 2008 VIA complex was effectively used for the first time for participation at distance in XI Bled Workshop [7]. Since then VIA videoconferences became a natural part of Bled Workshops' programs, opening the virtual room of discussions to the world-wide audience. Its progress was presented in [8–13]. Here the current state-of-art of VIA complex, integrated since 2009 in the structure of APC Laboratory, is presented in order to clarify the way in which VIA discussion of open questions beyond the standard model took place in the framework of XVIII Bled Workshop.

16.2 The structure of VIA complex and forms of its activity

16.2.1 The forms of VIA activity

The structure of VIA complex is illustrated on Fig. 16.1. The home page, presented on this figure, contains the information on VIA activity and menu, linking to directories (along the upper line from left to right): with general information on VIA (About VIA), entrance to VIA virtual rooms (Rooms), the library of records and presentations (Previous) of VIA Lectures (Previous → Lectures), records of online transmissions of Conferences(Previous → Conferences), APC Colloquiums (Previous → APC Colloquiums), APC Seminars (Previous → APC Seminars) and Events (Previous → Events), Calender of the past and future VIA events (All events) and VIA Forum (Forum). In the upper right angle there are links to Google search engine (Search in site) and to contact information (Contacts). The announcement of the next VIA lecture and VIA online transmission of APC Colloquium occupy the main part of the homepage with the record of the most recent VIA events below. In the announced time of the event (VIA lecture or transmitted APC Colloquium) it is sufficient to click on "to participate" on the announcement and to Enter as Guest (printing your name) in the corresponding Virtual room. The Calender links to the program of future VIA lectures and events. The right column on the VIA homepage lists the announcements of the regularly up-dated hot news of Astroparticle physics and related areas.

In 2010 special COSMOVIA tours were undertaken in Switzerland (Geneva), Belgium (Brussels, Liege) and Italy (Turin, Pisa, Bari, Lecce) in order to test stability of VIA online transmissions from different parts of Europe. Positive results of these tests have proved the stability of VIA system and stimulated this practice at XIII Bled Workshop. The records of the videoconferences at the XIII Bled Workshop are available on VIA site [14].

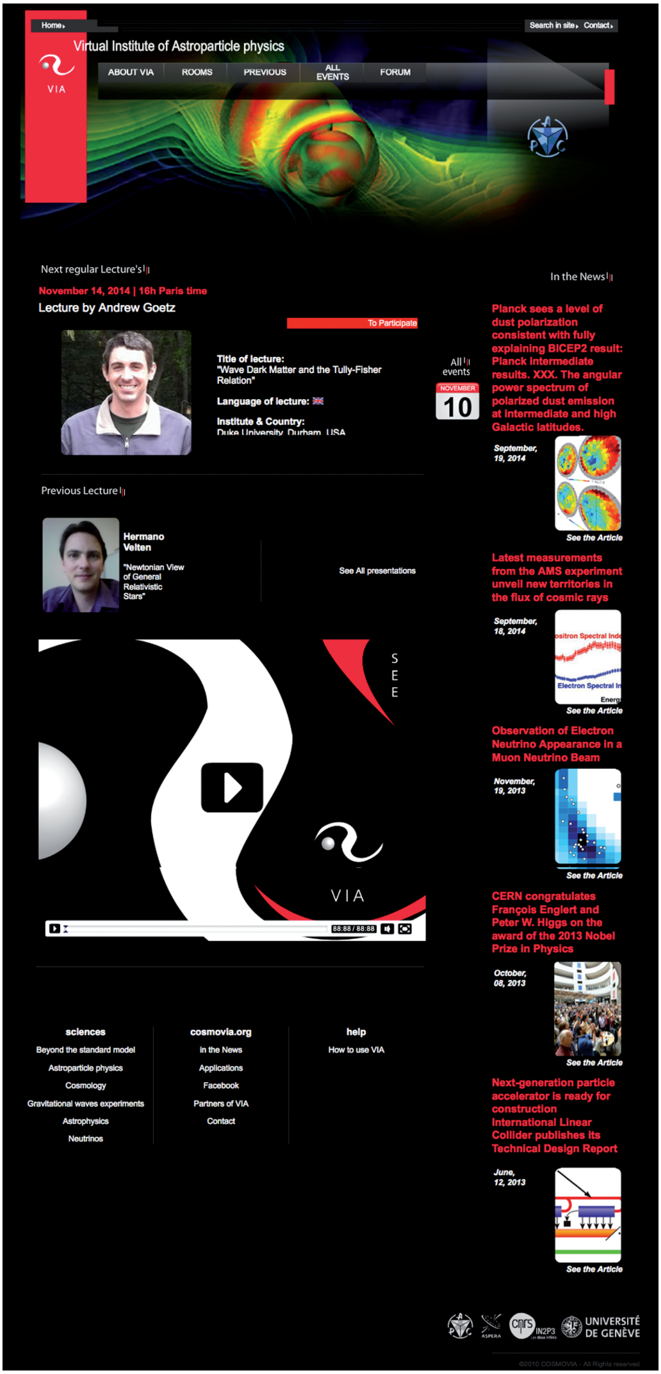

Fig. 16.1. The home page of VIA site

Since 2011 VIA facility was used for the tasks of the Paris Center of Cosmological Physics (PCCP), chaired by G. Smoot, for the public programme "The two infinities" conveyed by J.L.Robert and for effective support a participation at distance at meetings of the Double Chooz collaboration. In the latter case, the experimentalists, being at shift, took part in the collaboration meeting in such a virtual way.

The simplicity of VIA facility for ordinary users was demonstrated at XIV Bled Workshop in 2011. Videoconferences at this Workshop had no special technical support except for WiFi Internet connection and ordinary laptops with their internal video and audio equipments. This test has proved the ability to use VIA facility at any place with at least decent Internet connection. Of course the quality of records is not as good in this case as with the use of special equipment, but still it is sufficient to support fruitful scientific discussion as can be illustrated by the record of VIA presentation "New physics and its experimental probes" given by John Ellis from his office in CERN (see the records in [15]).

In 2012 VIA facility, regularly used for programs of VIA lectures and transmission of APC Colloquiums, has extended its applications to support M.Khlopov's talk at distance at Astrophysics seminar in Moscow, videoconference in PCCP, participation at distance in APC-Hamburg-Oxford network meeting as well as to provide online transmissions from the lectures at Science Festival 2012 in University Paris7. VIA communication has effectively resolved the problem of referee's attendance at the defence of PhD thesis by Mariana Vargas in APC. The referees made their reports and participated in discussion in the regime of VIA videoconference. In 2012 VIA facility was first used for online transmissions from the Science Festival in the University Paris 7. This tradition was continued in 2013, when the transmissions of meetings at Journes nationales du Dveloppement Logiciel (JDEV2013) at Ecole Politechnique (Paris) were organized [17].

In 2013 VIA lecture by Prof. Martin Pohl was one of the first places at which the first hand information on the first results of AMS02 experiment was presented [16].

In 2014 the 100th anniversary of one of the foundators of Cosmoparticle physics, Ya. B. Zeldovich, was celebrated. With the use of VIA M.Khlopov could contribute the programme of the "Subatomic particles, Nucleons, Atoms, Universe: Processes and Structure International conference in honor of Ya. B. Zeldovich 100th Anniversary" (Minsk, Belarus) by his talk "Cosmoparticle physics: the Universe as a laboratory of elementary particles" [18] and the programme of "Conference YaB-100, dedicated to 100 Anniversary of Yakov Borisovich Zeldovich" (Moscow, Russia) by his talk "Cosmology and particle physics" [19].

In 2015 VIA facility supported the talk at distance at All Moscow Astrophysical seminar "Cosmoparticle physics of dark matter and structures in the Universe" by Maxim Yu. Khlopov and the work of the Section "Dark matter" of the International Conference on Particle Physics and Astrophysics (Moscow, 5-10 October 2015). Though the conference room was situated in Milan Hotel in Moscow all the presentations at this Section were given at distance (by Rita Bernabei from Rome, Italy; by Juan Jose Gomez-Cadenas, Paterna, University of Valencia, Spain and by

Dmitri Semikoz, Martin Bucher and Maxim Khlopov from Paris) and its work was chaired by M.Khlopov from Paris [22].

The discussion of questions that were put forward in the interactive VIA events can be continued and extended on VIA Forum. The Forum is intended to cover the topics: beyond the standard model, astroparticle physics, cosmology, gravitational wave experiments, astrophysics, neutrinos. Presently activated in English,French and Russian with trivial extension to other languages, the Forum represents a first step on the way to multi-lingual character of VIA complex and its activity.

16.2.2 VIA e-learning and MOOC

One of the interesting forms of VIA activity is the educational work at distance. For the last six years M.Khlopov's course "Introduction to cosmoparticle physics" is given in the form of VIA videoconferences and the records of these lectures and their ppt presentations are put in the corresponding directory of the Forum [20]. Having attended the VIA course of lectures in order to be admitted to exam students should put on Forum a post with their small thesis. Professor's comments and proposed corrections are put in a Post reply so that students should continuously present on Forum improved versions of work until it is accepted as satisfactory. Then they are admitted to pass their exam. The record of videoconference with their oral exam is also put in the corresponding directory of Forum. Such procedure provides completely transparent way of evaluation of students' knowledge.

Since 2014 the second part of this course is given in English in order to develop VIA system as a possible tool for Massive Online Open Courses (MOOC) activity [21]. The students must write their small thesis, present it and being admitted to exam pass it in English. The restricted number of online connections to videoconferences with VIA lectures is compensated by the wide-world access to their records on VIA Forum and in the context of MOOC VIA Forum and videoconferencing system can be used for individual online work with advanced participants.

16.2.3 Organisation of VIA events and meetings

First tests of VIA system, described in [5,7–9], involved various systems of videoconferencing. They included skype, VRVS, EVO, WEBEX, marratech and adobe Connect. In the result of these tests the adobe Connect system was chosen and properly acquired. Its advantages are: relatively easy use for participants, a possibility to make presentation in a video contact between presenter and audience, a possibility to make high quality records, to use a whiteboard tools for discussions, the option to open desktop and to work online with texts in any format.

Initially the amount of connections to the virtual room at VIA lectures and discussions usually didn't exceed 20. However, the sensational character of the exciting news on superluminal propagation of neutrinos acquired the number of participants, exceeding this allowed upper limit at the talk "OPERA versus

Maxwell and Einstein" given by John Ellis from CERN. The complete record of this talk and is available on VIA website [23]. For the first time the problem of necessity in extension of this limit was put forward and it was resolved by creation of a virtual "infinity room", which can host any reasonable amount of participants. Starting from 2013 this room became the only main virtual VIA room, but for specific events, like Collaboration meetings or transmissions from science festivals, special virtual rooms can be created. This solution strongly reduces the price of the licence for the use of the adobeConnect videoconferencing, retaining a possibility for creation of new rooms with the only limit to one administrating Host for all of them.

The ppt or pdf file of presentation is uploaded in the system in advance and then demonstrated in the central window. Video images of presenter and participants appear in the right window, while in the lower left window the list of all the attendees is given. To protect the quality of sound and record, the participants are required to switch out their microphones during presentation and to use the upper left Chat window for immediate comments and urgent questions. The Chat window can be also used by participants, having no microphone, for questions and comments during Discussion. The interactive form of VIA lectures provides oral discussion, comments and questions during the lecture. Participant should use in this case a "raise hand" option, so that presenter gets signal to switch out his microphone and let the participant to speak. In the end of presentation the central window can be used for a whiteboard utility as well as the whole structure of windows can be changed, e.g. by making full screen the window with the images of participants of discussion.

Regular activity of VIA as a part of APC includes online transmissions of all the APC Colloquiums and of some topical APC Seminars, which may be of interest for a wide audience. Online transmissions are arranged in the manner, most convenient for presenters, prepared to give their talk in the conference room in a normal way, projecting slides from their laptop on the screen. Having uploaded in advance these slides in the VIA system, VIA operator, sitting in the conference room, changes them following presenter, directing simultaneously webcam on the presenter and the audience.

16.3 VIA Sessions at XVIII Bled Workshop

VIA sessions of XVIII Bled Workshop have developed from the first experience at XI Bled Workshop [7] and their more regular practice at XII, XIII, XIV, XV, XVI and XVII Bled Workshops [8–13]. They became a regular part of the Bled Workshop's programme.

In the course of XVIII Bled Workshop meeting the list of open questions was stipulated, which was proposed for wide discussion with the use of VIA facility. The list of these questions was put on VIA Forum (see [24]) and all the participants of VIA sessions were invited to address them during VIA discussions. During the XVIII Bled Workshop the test of not only minimal necessary equipment, but either of the use of VIA facility by ordinary non-experienced users was undertaken. VIA Sessions were supported by personal laptop with WiFi Internet connection only, as well as in 2015 the members of VIA team were physically absent in Bled and all the videoconferences were directed by M.Khlopov at distance. It principally confirmed a possibility to provide effective interactive online VIA videoconferences even in the absence of any special equipment and qualified personnel at place. Only laptop with microphone and webcam together with WiFi Internet connection was proved to support not only attendance, but also VIA presentations and discussions. In the absence of WiFi connection, the 3G connection of iPhone was sufficient for VIA management and presentations. However technical problems didn't provide time for all the talks scheduled for VIA Session.

In the framework of the program of XVIII Bled Workshop, M. Khlopov, gave his talk ”Composite dark matter” (Fig. 16.2). It provided an additional demonstration of the ability of VIA to support the creative non-formal atmosphere of Bled Workshops (see records in [25]). VIA facility has provided presentation at distance

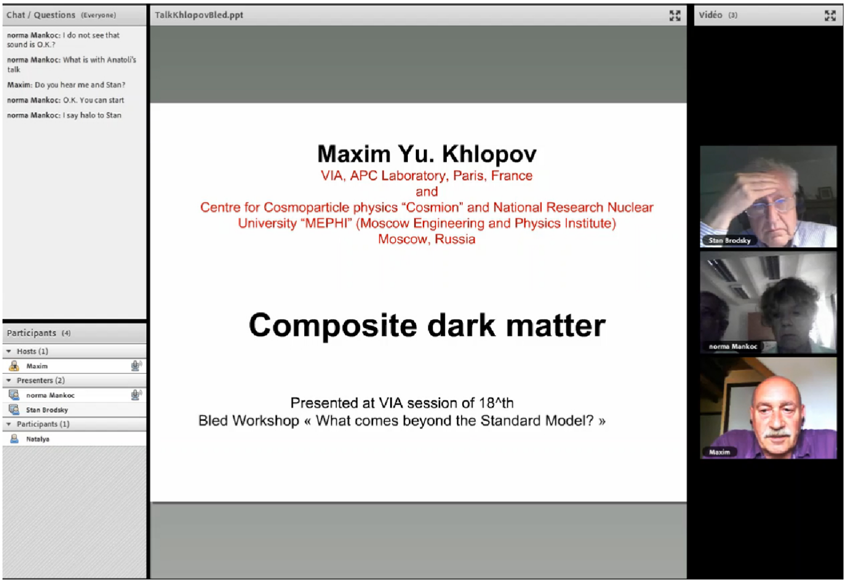

Fig. 16.2. VIA talk by M.Khlopov given from Paris at XVIII Bled Workshop

for talks ”Particle Dark Matter direct detection” by Rita Bernabei and Riccardo Cerulli (Rome University TorVergata, Italy) (Fig. 16.3) and ”New Perspectives for Hadron Physics and the Cosmological Constant Problem” by Stan Brodsky (SLAC, USA) (Fig. 16.4)

VIA sessions also included talks of Bled participants of the Workshop: ”Regularization of conformal correlators” by Loriano Bonora (Fig. 16.5) and ”Fermionization in an Arbitrary Number of Dimensions” by Holger Bech Nielsen.

Due to technical problems and the lack of time during the Workshop it was not possible to support by VIA the talk ”The Spin-Charge-Family theory offers the explanation for all the assumptions of the Standard model, for the Dark matter, for the Matter-antimatter asymmetry, for... ,making several predictions” by Norma

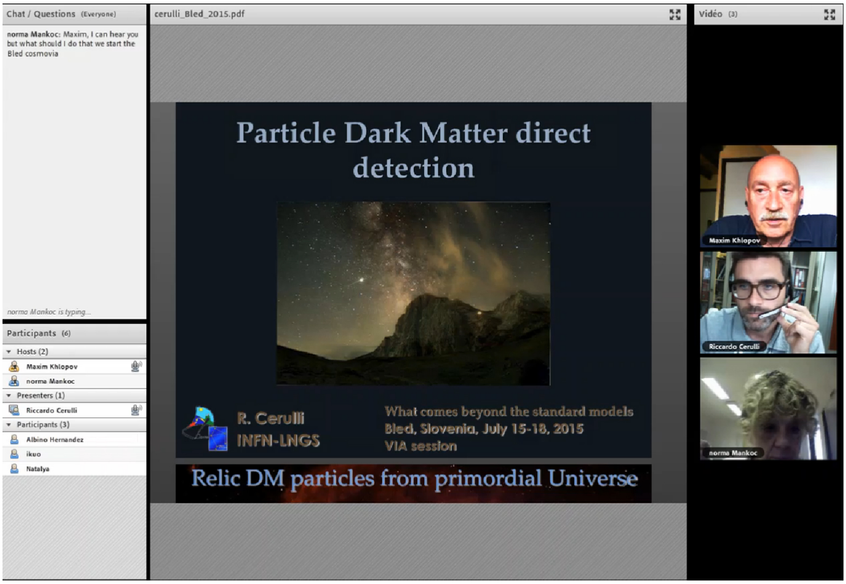

Fig. 16.3. VIA talk of R.Bernabei and R.Cerulli presented by R.Cerulli from Rome, Italy at XVIII Bled Workshop

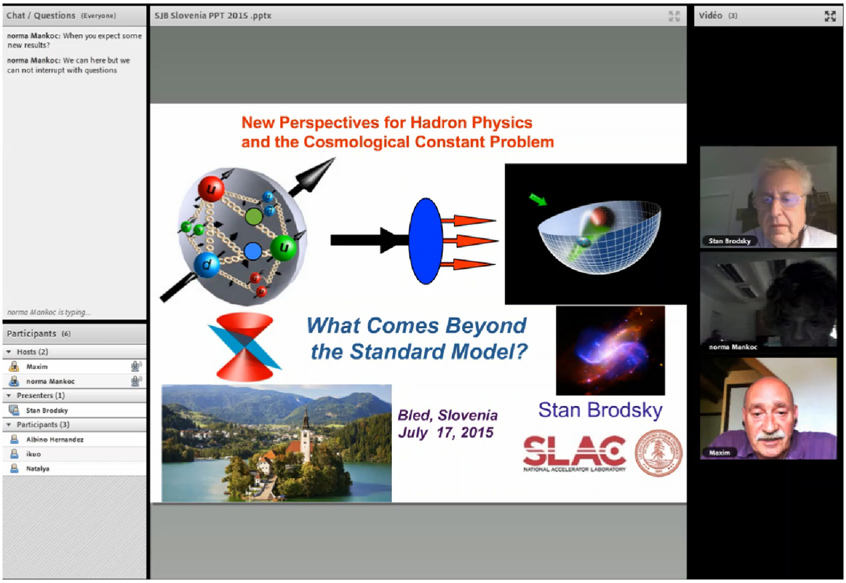

Fig. 16.4. VIA talk by Stan Brodsky from SLAC, USA at XVIII Bled Workshop

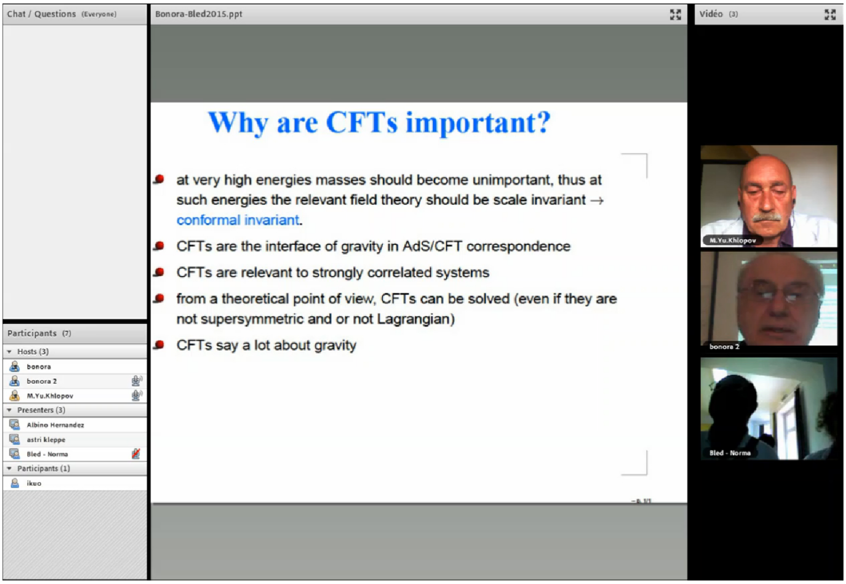

Fig. 16.5. VIA talk by Loriano Bonora at XVIII Bled Workshop

Mankoc-Borstnik. This talk was given in VIA specially later, so that its record has appeared in the list of VIA presentations at the XVIII Bled Workshop (Fig. 16.6). The records of all these lectures and discussions can be found in VIA library [25].

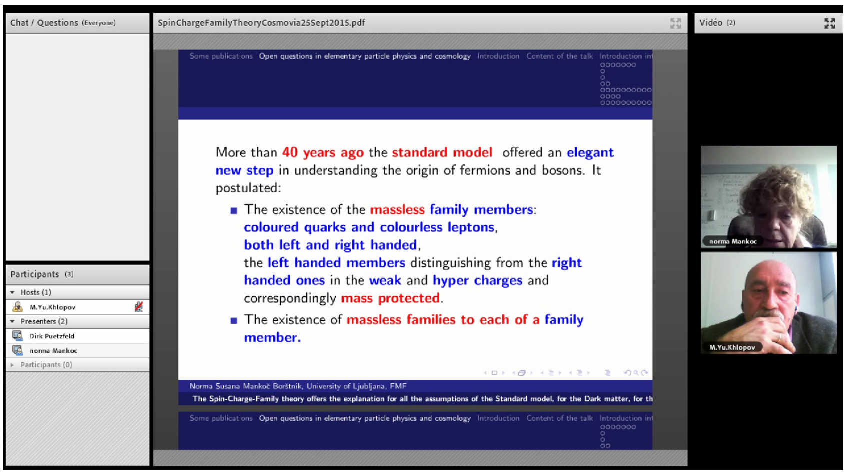

Fig. 16.6. VIA talk by N. Mankoc-Borstnik at XVIII Bled Workshop

16.4 Conclusions

The Scientific-Educational complex of Virtual Institute of Astroparticle physics provides regular communication between different groups and scientists, working in different scientific fields and parts of the world, the first-hand information on the newest scientific results, as well as support for various educational programs at distance. This activity would easily allow finding mutual interest and organizing task forces for different scientific topics of astroparticle physics and related topics. It can help in the elaboration of strategy of experimental particle, nuclear, astrophysical and cosmological studies as well as in proper analysis of experimental data. It can provide young talented people from all over the world to get the highest level education, come in direct interactive contact with the world known scientists and to find their place in the fundamental research. These educational aspects of VIA activity is now being evolved in a specific tool for MOOC. VIA applications can go far beyond the particular tasks of astroparticle physics and give rise to an interactive system of mass media communications.

VIA sessions became a natural part of a program of Bled Workshops, maintaining the platform of discussions of physics beyond the Standard Model for distant participants from all the world. The experience of VIA applications at Bled Workshops plays important role in the development of VIA facility as an effective tool of e-science and e-learning.

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