Multi-spinor field theory
and
extension of the Standard Model

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What Comes beyond
the Standard Model?

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Unorthodoxy

Motivation

Room for Dark Matter?

Rich spectra of quarks and leptons

Spin $\frac{1}{2}$ only	$u \; u \; u$	$c \; c \; c$	$t \; t \; t$
	$d \; d \; d$	$s \; s \; s$	$b \; b \; b$
	ν_e	ν_μ	ν_τ
	e	μ	τ

Three families (?) of quarks and leptons

Description: $\mathrm{SU}_c(3)$ and $\mathrm{SU}_L(2)\times\mathrm{U}(1)$

Varieties in Yukawa couplings

Keys to open the door to a next stage

Outline

1928

Dirac's spinor field for electron

$$\psi_a \qquad\qquad 4$$

$$\Downarrow$$

Diversity of Quarks and Leptons

Multi-spinor fields

Triplet fields for bundles of quarks and leptons

$$\Psi_{abc} \propto \psi_a \psi_b \psi_c \qquad\qquad 4\mathrm{x}4\mathrm{x}4 = 64$$

1968

Weinberg-Salam (WS) theory

$$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R \qquad \nu_R \qquad\qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Chiral doublet & singlet **Higgs doublet**

$$\Downarrow$$

Standard Model (SM)

$$\Downarrow$$

Extended SM in multi-spinor formalism

WS mechanism to triplet fields

Chiral spinor fields		Chiral triplet fields
EW doublet $$\psi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$ EW singlets $$\psi_{\nu R} = (\nu_e)_R$$ $$\psi_{eR} = (e)_R$$	$\Rightarrow$	EW doublet $$\Psi_L = \begin{pmatrix} \Psi_u \\ \Psi_d \end{pmatrix}_L$$ EW singlets $$U = (\Psi_u)_R$$ $$D = (\Psi_d)_R$$

Naïve extension

Sequential scheme with four families

$$\Psi_L = \begin{pmatrix}\Psi_u\\ \Psi_d\end{pmatrix}_L = \begin{pmatrix}\cdots & \boxed{\cdot}\\ \cdots & \boxed{\cdot}\end{pmatrix}_L \qquad \begin{matrix} U = (\cdots \ \boxed{\cdot})_R \\ D = (\cdots \ \boxed{\cdot})_R \end{matrix}$$

Excluded by LHC experiments (?, !)

L-R twisted scheme with (3+1) families

Fourth family for dark matter

$$\Psi_L = {}^t\left(\boxed{\Psi_{(v)}} : \boxed{\begin{matrix}U_{(d)}\\ D_{(d)}\end{matrix}}\right)_L \qquad \Psi_R = {}^t\left(\boxed{\begin{matrix}U_{(v)}\\ D_{(v)}\end{matrix}} : \boxed{\Psi_{(d)}}\right)_R$$

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

$$G_\star = SU_{c^*}(3) \times SU_R(2) \times U_{Y^*}(1)$$

$$2 \times 4 \times (3+1) \times (3+1) = 128$$

Triplet field and triplet algebra

$$\boldsymbol{\Psi}_{abc} \propto \boldsymbol{\psi}_a \boldsymbol{\psi}_b \boldsymbol{\psi}_c$$

$$\boldsymbol{A}_T = \boldsymbol{A}_\gamma \otimes \boldsymbol{A}_\gamma \otimes \boldsymbol{A}_\gamma$$

Triplet algebra

Dirac algebra

$$A_\gamma = \{1,\ \gamma_\mu,\ \sigma_{\mu\nu},\ \gamma_5\gamma_\mu,\ \gamma_5\} = \langle \gamma_\mu \rangle$$

Triplet algebra

$$\begin{aligned} A_T &= \{\, p \otimes q \otimes r : p,\ q,\ r \in A_\gamma \,\} \\ &= \langle\, \gamma_\mu \otimes 1 \otimes 1,\ 1 \otimes \gamma_\mu \otimes 1,\ 1 \otimes 1 \otimes \gamma_\mu \,\rangle \end{aligned}$$

Criterion for physical subalgebra

Closed and irreducible under permutation group S_3

$$p \otimes q \otimes r \ \rightarrow \ q \otimes r \otimes p \quad \text{etc}$$

A_Γ algebra

$$A_\Gamma = \langle\, \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \,\rangle$$

spacetime

$$\Gamma_\mu = \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \longleftrightarrow x^\mu$$

$$\Gamma_\mu \Gamma_\nu + \Gamma_\nu \Gamma_\mu = 2\eta_{\mu\nu} I$$

$$I = 1 \otimes 1 \otimes 1$$

$$\Sigma_{\mu\nu} = -\frac{i}{2}(\Gamma_\mu \Gamma_\nu - \Gamma_\nu \Gamma_\mu) = \sigma_{\mu\nu} \otimes \sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$$

$$\Gamma_5 = -i\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = \gamma_5 \otimes \gamma_5 \otimes \gamma_5$$

$$\Gamma^\mu = \eta^{\mu\nu}\Gamma_\nu$$

$$A_\Gamma = \{1,\ \Gamma_\mu,\ \Sigma_{\mu\nu},\ \Gamma_5\Gamma_\mu,\ \Gamma_5\} = \langle \Gamma_\mu \rangle \leftrightarrow A_\gamma$$

Lorentz transformation for triplet field $\Psi(x) \equiv (\Psi_{abc})(x)$

$$x'^{\mu} = \Omega^{\mu}{}_{\nu} x^{\nu} : \; \Omega_{\lambda\mu} \Omega^{\lambda}{}_{\nu} = \eta_{\mu\nu}$$

Dirac spinor field

$$\psi'(x') = s(\Omega)\psi(x) \qquad s(\Omega) = \exp\left(-\frac{i}{4}\sigma_{\mu\nu}\omega^{\mu\nu}\right)$$

Triplet fields

$$\Psi'(x') = S(\Omega)\Psi(x) \qquad S(\Omega) = \exp\left(-\frac{i}{4}\Sigma_{\mu\nu}\omega^{\mu\nu}\right)$$

$$\Sigma_{\mu\nu} = -\frac{i}{2}(\Gamma_{\mu}\Gamma_{\nu} - \Gamma_{\nu}\Gamma_{\mu}) = \sigma_{\mu\nu} \otimes \sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$$

External subalgebra $A_{ex} = \{ \Sigma_{\mu\nu} \} \subset A_{\Gamma}$

Chirality for triplet fields

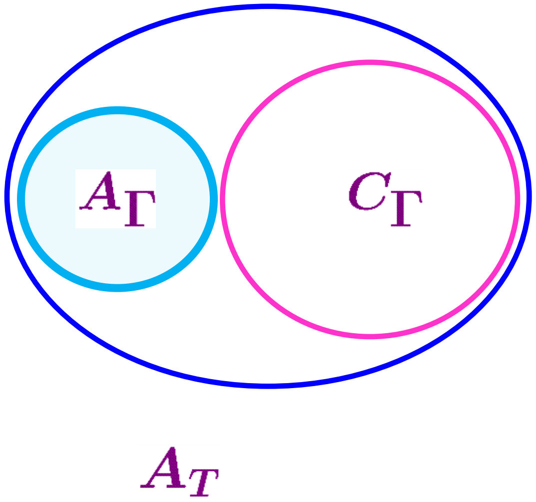
$$L = \frac{1}{2}(I - \Gamma_5), \quad R = \frac{1}{2}(I + \Gamma_5)$$

Centralizer of A_Γ algebra

$$C_\Gamma = \{ X \in A_T : [\, X,\ \Gamma_\mu \,] = 0 \} \qquad \Gamma_\mu = \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu$$

$$C_\Gamma = \langle\, 1 \otimes \gamma_\mu \otimes \gamma_\mu,\ \gamma_\mu \otimes 1 \otimes \gamma_\mu \,\rangle$$

$$A_\Gamma = \langle\, \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \,\rangle$$



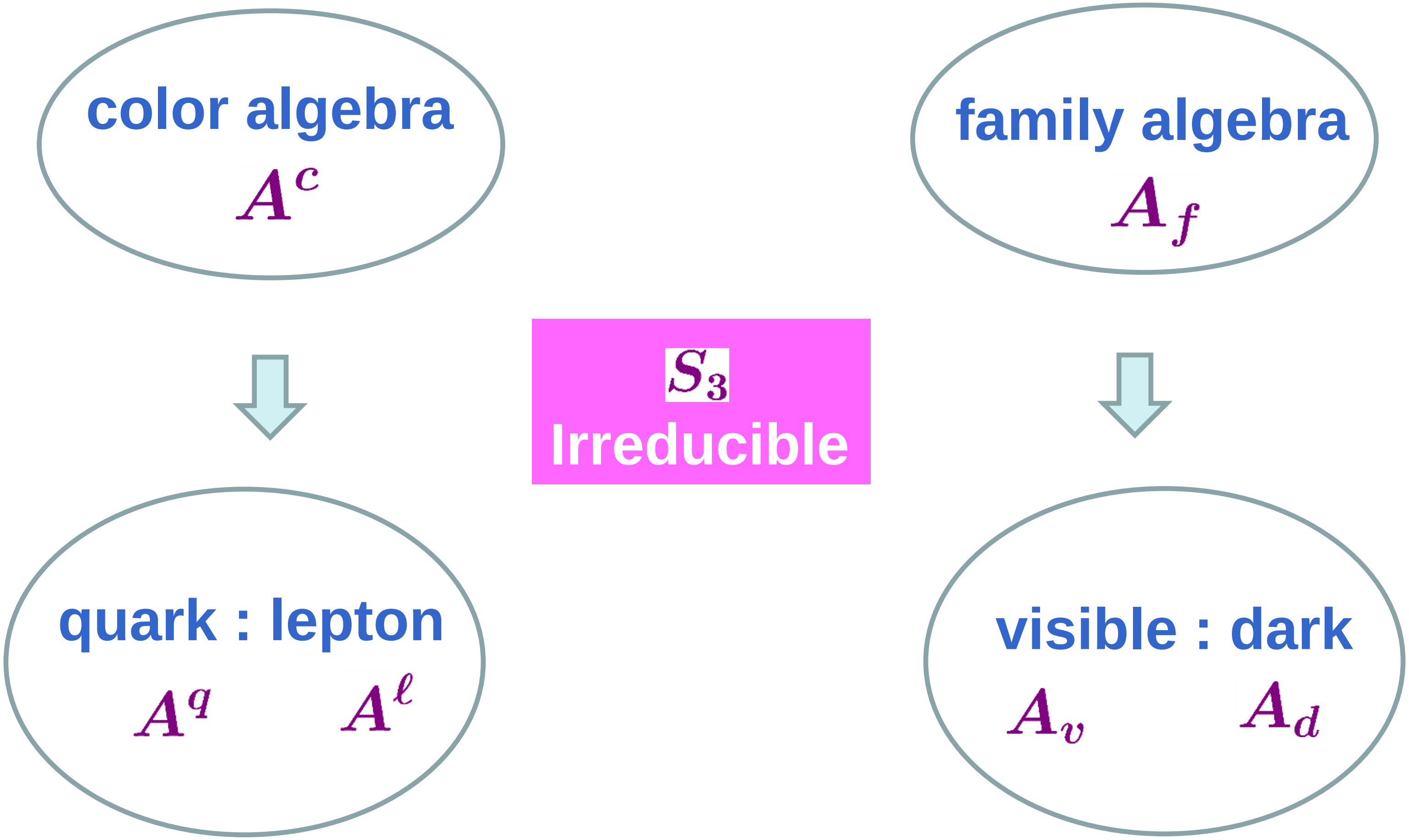
$$A_T = A_\Gamma C_\Gamma, \quad A_\Gamma \cap C_\Gamma = \emptyset$$

Coleman-Mandula theorem

External subalgebra $A_{ex} = \{ \Sigma_{\mu\nu} \} \subset A_\Gamma$

Internal subalgebra $A_{in} \subset C_\Gamma$

Internal subalgebra $A_{in} \subset C_{\Gamma}$



Isomorphic at algebraic level

commutative

Pauli algebra

$$\sigma_1 = \gamma_0,\ \sigma_2 = i\gamma_0\gamma_5,\ \sigma_3 = \gamma_5 \in A_\gamma$$

Triple-tensor-products

family algebra

$$A_f \subset \mathfrak{su}(4) \subset C_\Gamma$$

Pauli algebra

$$\rho_1 = i\gamma_2\gamma_3,\ \rho_2 = i\gamma_3\gamma_1,\ \rho_3 = i\gamma_1\gamma_2 \in A_\gamma$$

Triple-tensor-products

color algebra

$$A^c \subset \mathfrak{su}(4) \subset C_\Gamma$$

isomorphic

$$\pi_1 = \frac{1}{2}\left(\sigma_1 \otimes \sigma_1 \otimes 1 + \sigma_2 \otimes \sigma_2 \otimes 1\right),$$
$$\pi_2 = \frac{1}{2}\left(\sigma_1 \otimes \sigma_2 \otimes \sigma_3 - \sigma_2 \otimes \sigma_1 \otimes \sigma_3\right),$$
$$\pi_3 = \frac{1}{2}\left(1 \otimes \sigma_3 \otimes \sigma_3 - \sigma_3 \otimes 1 \otimes \sigma_3\right),$$

$\mathfrak{su}(2)$

$$\pi_4 = \frac{1}{2}\left(\sigma_1 \otimes 1 \otimes \sigma_1 + \sigma_2 \otimes 1 \otimes \sigma_2\right),$$
$$\pi_5 = \frac{1}{2}\left(\sigma_1 \otimes \sigma_3 \otimes \sigma_2 - \sigma_2 \otimes \sigma_3 \otimes \sigma_1\right),$$
$$\pi_6 = \frac{1}{2}\left(1 \otimes \sigma_1 \otimes \sigma_1 + 1 \otimes \sigma_2 \otimes \sigma_2\right),$$
$$\pi_7 = \frac{1}{2}\left(\sigma_3 \otimes \sigma_1 \otimes \sigma_2 - \sigma_3 \otimes \sigma_2 \otimes \sigma_1\right),$$
$$\pi_8 = \frac{1}{2\sqrt{3}}\left(1 \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes 1 \otimes \sigma_3 - 2\sigma_3 \otimes \sigma_3 \otimes 1\right)$$

$\mathfrak{su}(3)$

$\mathfrak{su}(4)$

$$\pi_9 = \frac{1}{2}\left(1 \otimes \sigma_1 \otimes \sigma_1 - 1 \otimes \sigma_2 \otimes \sigma_2\right),$$
$$\pi_{10} = -\frac{1}{2}\left(\sigma_3 \otimes \sigma_1 \otimes \sigma_2 + \sigma_3 \otimes \sigma_2 \otimes \sigma_1\right),$$
$$\pi_{11} = \frac{1}{2}\left(\sigma_1 \otimes 1 \otimes \sigma_1 - \sigma_2 \otimes 1 \otimes \sigma_2\right),$$
$$\pi_{12} = -\frac{1}{2}\left(\sigma_1 \otimes \sigma_3 \otimes \sigma_2 + \sigma_2 \otimes \sigma_3 \otimes \sigma_1\right),$$
$$\pi_{13} = \frac{1}{2}\left(\sigma_1 \otimes \sigma_1 \otimes 1 - \sigma_2 \otimes \sigma_2 \otimes 1\right),$$
$$\pi_{14} = \frac{1}{2}\left(\sigma_1 \otimes \sigma_2 \otimes \sigma_3 + \sigma_2 \otimes \sigma_1 \otimes \sigma_3\right),$$

S_3

Closed

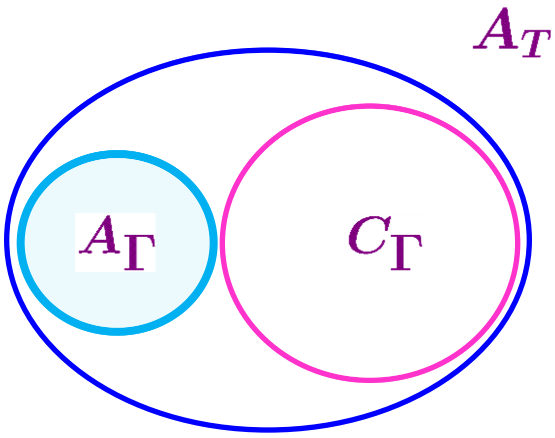
Irreducible

$$\pi_{15} = -\frac{1}{\sqrt{6}}\left(1 \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes 1 \otimes \sigma_3 + \sigma_3 \otimes \sigma_3 \otimes 1\right)$$

$\mathfrak{u}(1)$

$$A_T = \langle \gamma_\mu \otimes 1 \otimes 1,\ 1 \otimes \gamma_\mu \otimes 1,\ 1 \otimes 1 \otimes \gamma_\mu \rangle$$

$$16 \times 16 \times 16$$



$$A_\Gamma = \langle \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \rangle$$

$$16$$

$$C_\Gamma = \langle 1 \otimes \gamma_\mu \otimes \gamma_\mu,\ \gamma_\mu \otimes 1 \otimes \gamma_\mu \rangle$$

$$16 \times 16$$

$$\sigma_1 \otimes \sigma_1 \otimes \sigma_1 = \Gamma_0$$

$$\sigma_2 \otimes \sigma_2 \otimes \sigma_2 = -i\Gamma_0\Gamma_5$$

$$\sigma_3 \otimes \sigma_3 \otimes \sigma_3 = \Gamma_5$$

$$\sigma_1 \otimes 1 \otimes 1 = (\sigma_1 \times \sigma_1 \times \sigma_1)(1 \times \sigma_1 \times \sigma_1) = \Gamma_0(1 \times \sigma_1 \times \sigma_1)$$

$$15 + 3 + 9 = 27$$

Action of S_3

$$\pi_1 = \tfrac{1}{2}\left(\sigma_1 \otimes \sigma_1 \otimes 1 + \sigma_2 \otimes \sigma_2 \otimes 1\right),$$

$$\pi_2 = \tfrac{1}{2}\left(\sigma_1 \otimes \sigma_2 \otimes \sigma_3 - \sigma_2 \otimes \sigma_1 \otimes \sigma_3\right),$$

$$\pi_3 = \tfrac{1}{2}\left(1 \otimes \sigma_3 \otimes \sigma_3 - \sigma_3 \otimes 1 \otimes \sigma_3\right),$$

$\mathfrak{su}(2)$

$$\pi_4 = \tfrac{1}{2}\left(\sigma_1 \otimes 1 \otimes \sigma_1 + \sigma_2 \otimes 1 \otimes \sigma_2\right),$$

$$\pi_5 = \tfrac{1}{2}\left(\sigma_1 \otimes \sigma_3 \otimes \sigma_2 - \sigma_2 \otimes \sigma_3 \otimes \sigma_1\right),$$

$$\pi_6 = \tfrac{1}{2}\left(1 \otimes \sigma_1 \otimes \sigma_1 + 1 \otimes \sigma_2 \otimes \sigma_2\right),$$

$$\pi_7 = \tfrac{1}{2}\left(\sigma_3 \otimes \sigma_1 \otimes \sigma_2 - \sigma_3 \otimes \sigma_2 \otimes \sigma_1\right),$$

$$\pi_8 = \frac{1}{2\sqrt{3}}\left(1 \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes 1 \otimes \sigma_3 - 2\sigma_3 \otimes \sigma_3 \otimes 1\right)$$

$\mathfrak{su}(3)$

$$\pi_{15} = \frac{-1}{\sqrt{6}}\left(1 \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes 1 \otimes \sigma_3 + \sigma_3 \otimes \sigma_3 \otimes 1\right) \quad \mathfrak{u}(1)$$

S_3 **irreducibility** ⇒

$\mathfrak{su}(3)$ **Visible sector**

$\mathfrak{u}(1)$ **Dark sector**

Projection operators for family modes

$$\Pi_1 = \frac{1}{4}(I + 1 \otimes \sigma_3 \otimes \sigma_3 - \sigma_3 \otimes 1 \otimes \sigma_3 - \sigma_3 \otimes \sigma_3 \otimes 1)$$

$$\Pi_2 = \frac{1}{4}(I - 1 \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes 1 \otimes \sigma_3 - \sigma_3 \otimes \sigma_3 \otimes 1)$$

$$\Pi_3 = \frac{1}{4}(I - 1 \otimes \sigma_3 \otimes \sigma_3 - \sigma_3 \otimes 1 \otimes \sigma_3 + \sigma_3 \otimes \sigma_3 \otimes 1)$$

$$\Pi_4 = \frac{1}{4}(I + 1 \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes 1 \otimes \sigma_3 + \sigma_3 \otimes \sigma_3 \otimes 1)$$

$$\Pi_i \Pi_j = \delta_{ij} \Pi_i \qquad \sum_i \Pi_i = I$$

Projection operators to visible and dark modes

$$\begin{aligned} \Pi_{(v)} &= \Pi_1 + \Pi_2 + \Pi_3 \\ &= \frac{1}{4}(3I - 1 \otimes \sigma_3 \otimes \sigma_3 - \sigma_3 \otimes 1 \otimes \sigma_3 - \sigma_3 \otimes \sigma_3 \otimes 1) \end{aligned}$$

$$\begin{aligned} \Pi_{(d)} &= \Pi_4 \\ &= \frac{1}{4}(I + 1 \otimes \sigma_3 \otimes \sigma_3 + \sigma_3 \otimes 1 \otimes \sigma_3 + \sigma_3 \otimes \sigma_3 \otimes 1) \end{aligned}$$

Visible and dark subalgebras

$$A_f = A_v,\ A_d$$

$$A_v = \{\, \Pi_v,\ \pi_1,\ \pi_2,\ \cdots,\ \pi_8 \,\} \qquad \mathfrak{su}(3)$$

$$A_d = \{\, \Pi_d \,\} \qquad \mathfrak{u}(1)$$

$$\Pi_v = \frac{1}{4}(3I - 1\otimes\sigma_3\otimes\sigma_3 - \sigma_3\otimes 1\otimes\sigma_3 - \sigma_3\otimes\sigma_3\otimes 1)$$

$$\Pi_d = \frac{1}{4}(I + 1\otimes\sigma_3\otimes\sigma_3 + \sigma_3\otimes 1\otimes\sigma_3 + \sigma_3\otimes\sigma_3\otimes 1)$$

$$[\,\pi_j\,,\ \pi_k\,] = 2f^{(3)}_{jkl}\pi_l$$

$$\{\,\pi_j\,,\ \pi_k\,\} = \frac{4}{3}\delta_{jk}\Pi_v + 2d^{(3)}_{jkl}\pi_l$$

Projected family component fields

$$\Psi_i(x) = \Pi_i \Psi(x) \quad (i = 1,\ 2,\ 3;\ \ 4)$$

$$\{\Psi_1,\quad \Psi_2,\quad \Psi_3\} \qquad \{\Psi_4\}$$

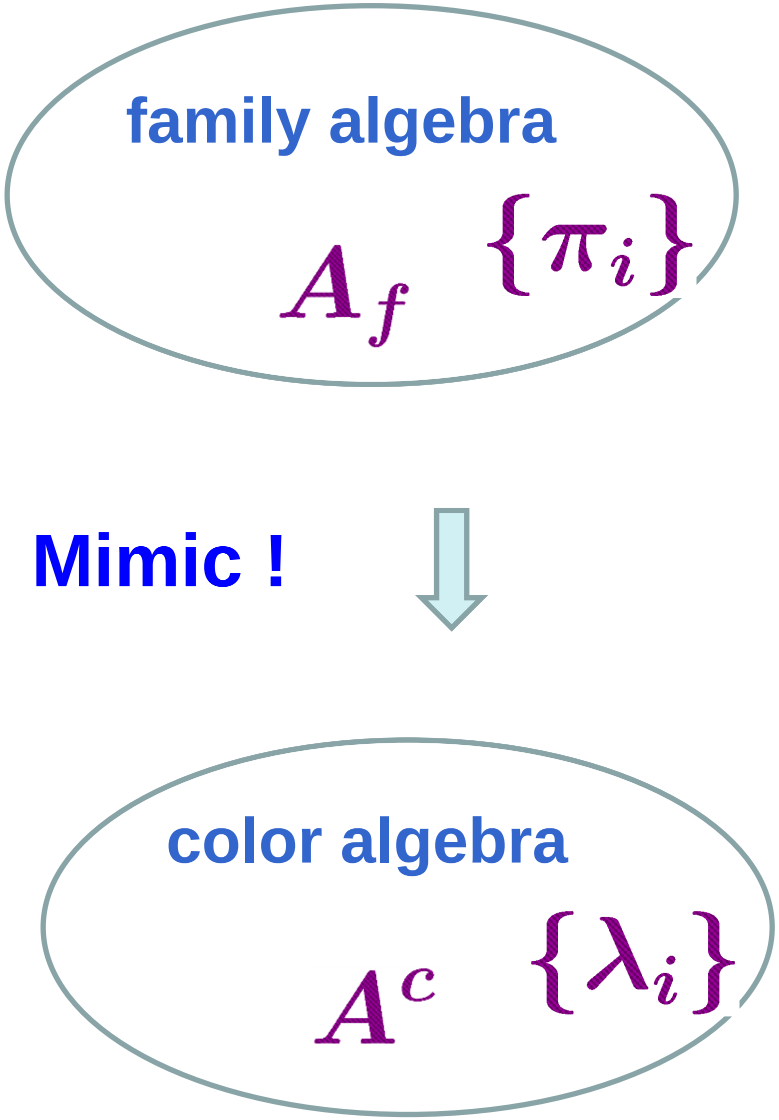
↑ Three family visible mode

↑ Additional 4-th dark mode

Chiral triplet fields

$$\Psi_{iL}(x) = L\Pi_i \Psi(x)$$

$$\Psi_{iR}(x) = R\Pi_i \Psi(x)$$


family algebra
A_f $\{\pi_i\}$
Mimic !
color algebra
A^c $\{\lambda_i\}$

Operators for extended color charge

Quarks

$$\Lambda_r = \tfrac{1}{4}\left(I + 1\otimes\rho_3\otimes\rho_3 - \rho_3\otimes 1\otimes\rho_3 - \rho_3\otimes\rho_3\otimes 1\right),$$

$$\Lambda_g = \tfrac{1}{4}\left(I - 1\otimes\rho_3\otimes\rho_3 + \rho_3\otimes 1\otimes\rho_3 - \rho_3\otimes\rho_3\otimes 1\right),$$

$$\Lambda_b = \tfrac{1}{4}\left(I - 1\otimes\rho_3\otimes\rho_3 - \rho_3\otimes 1\otimes\rho_3 + \rho_3\otimes\rho_3\otimes 1\right)$$

Leptons

$$\Lambda_\ell = \tfrac{1}{4}\left(I + 1\otimes\rho_3\otimes\rho_3 + \rho_3\otimes 1\otimes\rho_3 + \rho_3\otimes\rho_3\otimes 1\right)$$

$$\begin{aligned}\Lambda^{(q)} &= \Lambda_r + \Lambda_g + \Lambda_b \\ &= \tfrac{1}{4}\left(3I - 1\otimes\rho_3\otimes\rho_3 - \rho_3\otimes 1\otimes\rho_3 - \rho_3\otimes\rho_3\otimes 1\right)\end{aligned}$$

$$\Lambda^{(\ell)} = \Lambda_\ell = \frac{1}{4}\left(I + 1\otimes\rho_3\otimes\rho_3 + \rho_3\otimes 1\otimes\rho_3 + \rho_3\otimes\rho_3\otimes 1\right)$$

Operators for (baryon – lepton) charge

$$\begin{aligned}\hat{B} - \hat{L} &= \tfrac{1}{3}\Lambda^{(q)} - \Lambda^{(\ell)} \\ &= -\tfrac{1}{3}\left(1\otimes\rho_3\otimes\rho_3 + \rho_3\otimes 1\otimes\rho_3 + \rho_3\otimes\rho_3\otimes 1\right)\end{aligned}$$

Color algebra $\{\lambda_i\} = \mathfrak{su}(3)$

Lie group for Color SU(3)

$$
\begin{cases}
\lambda_1 = \frac{1}{2}(\rho_1 \otimes \rho_1 \otimes 1 + \rho_2 \otimes \rho_2 \otimes 1)\,, \\
\lambda_2 = \frac{1}{2}(\rho_1 \otimes \rho_2 \otimes \rho_3 - \rho_2 \otimes \rho_1 \otimes \rho_3)\,, \\
\lambda_3 = \frac{1}{2}(1 \otimes \rho_3 \otimes \rho_3 - \rho_3 \otimes 1 \otimes \rho_3)\,, \\
\lambda_4 = \frac{1}{2}(\rho_1 \otimes 1 \otimes \rho_1 + \rho_2 \otimes 1 \otimes \rho_2)\,, \\
\lambda_5 = \frac{1}{2}(\rho_1 \otimes \rho_3 \otimes \rho_2 - \rho_2 \otimes \rho_3 \otimes \rho_1)\,, \\
\lambda_6 = \frac{1}{2}(1 \otimes \rho_1 \otimes \rho_1 + 1 \otimes \rho_2 \otimes \rho_2)\,, \\
\lambda_7 = \frac{1}{2}(\rho_3 \otimes \rho_1 \otimes \rho_2 - \rho_3 \otimes \rho_2 \otimes \rho_1)\,, \\
\lambda_8 = \frac{1}{2\sqrt{3}}(1 \otimes \rho_3 \otimes \rho_3 + \rho_3 \otimes 1 \otimes \rho_3 - 2\rho_3 \otimes \rho_3 \otimes 1)
\end{cases}
$$

family algebra $A_f \ \{\pi_i\}$

Isomorphic at algebraic level

color algebra $A^c \ \{\lambda_i\}$

Projection operators to color subalgebras

$$\Lambda^q = \frac{1}{4}(3I - 1 \otimes \rho_3 \otimes \rho_3 - \rho_3 \otimes 1 \otimes \rho_3 - \rho_3 \otimes \rho_3 \otimes 1)$$

$$\Lambda^\ell = \frac{1}{4}(I + 1 \otimes \rho_3 \otimes \rho_3 + \rho_3 \otimes 1 \otimes \rho_3 + \rho_3 \otimes \rho_3 \otimes 1)$$

Color subalgebras

$$A^c = \boxed{A^q \quad A^\ell}$$

$$A^q = \{\, \Lambda^q, \lambda_1, \lambda_2, \cdots, \lambda_8 \,\} \qquad \mathfrak{su}(3)$$

$$A^\ell = \{\, \Lambda^\ell \,\} \qquad \mathfrak{u}(1)$$

$$[\,\lambda_j\,,\,\lambda_k\,] = 2f^{(3)}_{jkl}\lambda_l$$

$$\{\,\lambda_j\,,\,\lambda_k\,\} = \frac{4}{3}\delta_{jk}\Lambda^q + 2d^{(3)}_{jkl}\lambda_l$$

$$\Psi = \boxed{\Psi_v} + \boxed{\Psi_d}$$

Visible sector

$$\Psi_v = \Pi_v \Psi$$

$$SU_c(3)$$

Dark sector

$$\Psi_d = \Pi_d \Psi$$

$$SU_{c^*}(3)$$

Extended color subalgebras for visible and dark sectors

$$A^c = \boxed{A^q_{(u)} \quad A^q_{(d)} \quad A^\ell_{(u)} \quad A^\ell_{(d)}}$$

$$A^q_{(a)} = \left\{ \Lambda^q_{(a)}, \lambda_{(a)1}, \lambda_{(a)2}, \cdots, \lambda_{(a)8} \right\}$$

$$\Lambda^q_{(a)} = \Pi_{(a)} \Lambda^q \qquad \lambda_{(a)j} = \Pi_{(a)} \lambda_j$$

$$A^\ell_{(a)} = \left\{ \Lambda^\ell_{(a)} \right\}$$

$$\Lambda^\ell_{(a)} = \Pi_{(a)} \Lambda^\ell$$

Family modes

Three visible families

$$\Psi_i(x) = \Pi_i \Psi(x) \quad (i = 1,\ 2,\ 3)$$

Fourth dark family

$$\Psi_d(x) = \Pi_d \Psi(x)$$

Color states in visible and dark sectors

Tricolor fermions : Quark states

$$\Psi^i_{(a)}(x) = \Pi_{(a)} \Lambda^i \Psi(x) \quad (i = r,\ g,\ b)$$

Colorless fermions : Lepton states

$$\Psi^\ell_{(a)}(x) = \Pi_{(a)} \Lambda^\ell \Psi(x)$$

Model building
with
a pair of triplet fields

Sequential model with four families

Fundamental representations

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

L field

$$\Psi_L = {}^t\left(\ \Psi_{(1)},\ \Psi_{(2)},\ \Psi_{(3)}\ \vdots\ \Psi_{(4)}\ \right)_L$$

EW doublets

R field

$$\Psi_R = {}^t\begin{pmatrix} U_{(1)} & U_{(2)} & U_{(3)} & \vdots & U_{(4)} \\ D_{(1)}, & D_{(2)}, & D_{(3)} & & D_{(4)} \end{pmatrix}_R$$

EW singlets

Excluded by LHC experiments ?, !

⇩

We discard this possibility

L-R twisted model

$G = SU_c(3) \times SU_L(2) \times U_Y(1)$ for visible sector

$G_\star = SU_{c^*}(3) \times SU_R(2) \times U_{Y^*}(1)$ for dark sector

Fundamental representation

L field

$$\Psi_L = {}^t\begin{pmatrix} \Psi_{(v)} & : & \begin{matrix} U_{(d)} \\ D_{(d)} \end{matrix} \end{pmatrix}_L = \begin{pmatrix} \Psi_{(v)} \\ \cdot\cdot \\ U_{(d)} \\ D_{(d)} \end{pmatrix}_L$$

EW doublet

EW* singlets

R field

$$\Psi_R = {}^t\begin{pmatrix} \begin{matrix} U_{(v)} \\ D_{(v)} \end{matrix} & : & \Psi_{(d)} \end{pmatrix}_R = \begin{pmatrix} U_{(v)} \\ D_{(v)} \\ \cdot\cdot \\ \Psi_{(d)} \end{pmatrix}_R$$

EW singlets

EW* doublet

G $\quad$ $G_\star$

L field

$$\Psi_L = {}^t\left(\boxed{\Psi_{(v)}} \;:\; \boxed{\begin{matrix} U_{(d)} \\ D_{(d)} \end{matrix}} \right)_L$$

Star symbol

Quark states

$$\Psi^{(q)}_{(v)} = \begin{pmatrix} uuu & ccc & ttt \\ ddd & sss & bbb \end{pmatrix}_L$$

Lepton states

$$\Psi^{(\ell)}_{(v)} = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{pmatrix}_L$$

G

$$U^{(q)}_{(d)} = \left(u_\star u_\star u_\star \right)_L$$

$$D^{(q)}_{(d)} = \left(d_\star d_\star d_\star \right)_L$$

$$U^{(\ell)}_{(d)} = \left(\nu_\star \right)_L$$

$$D^{(\ell)}_{(d)} = \left(e_\star \right)_L$$

$G_\star$

R field

$$\Psi_R = {}^t\begin{pmatrix} U_{(v)} \\ D_{(v)} \end{pmatrix} : \Psi_{(d)} \Big)_R$$

Quark states

$$U^{(q)}_{(v)} = (\ uuu \quad ccc \quad ttt\ )_R$$

$$D^{(q)}_{(v)} = (\ ddd \quad sss \quad bbb\ )_R$$

Lepton states

$$U^{(\ell)}_{(v)} = (\ \nu_e \quad \nu_\mu \quad \nu_\tau\ )_R$$

$$D^{(\ell)}_{(v)} = (\ e \quad \mu \quad \tau\ )_R$$

G

$$\Psi^{(q)}_{(d)} = \begin{pmatrix} u_\star u_\star u_\star \\ d_\star d_\star d_\star \end{pmatrix}_R$$

$$\Psi^{(\ell)}_{(d)} = \begin{pmatrix} \nu_\star \\ e_\star \end{pmatrix}_R$$

$G_\star$

Lagrangian density of L-R twisted model

Kinetic and gauge parts

$$\mathcal{L}_{kg} = \bar{\Psi}_L i\Gamma^\mu \mathcal{D}_\mu \Psi_L + \bar{\Psi}_R i\Gamma^\mu \mathcal{D}_\mu \Psi_R$$

Yukawa parts

$$\mathcal{L}_Y = \bar{\Psi}_L \mathcal{Y}(\,\text{Higgs}\,) \Psi_R + \text{h.c.}$$

$\mathcal{Y}$: Kernel for Yukawa couplings in terms of π_i, $\Pi_{(t)}$

Two Higgs fields

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad \varphi_\star = \begin{pmatrix} \varphi_\star^+ \\ \varphi_\star^0 \end{pmatrix}$$

Visible Higgs

G

Dark Higgs

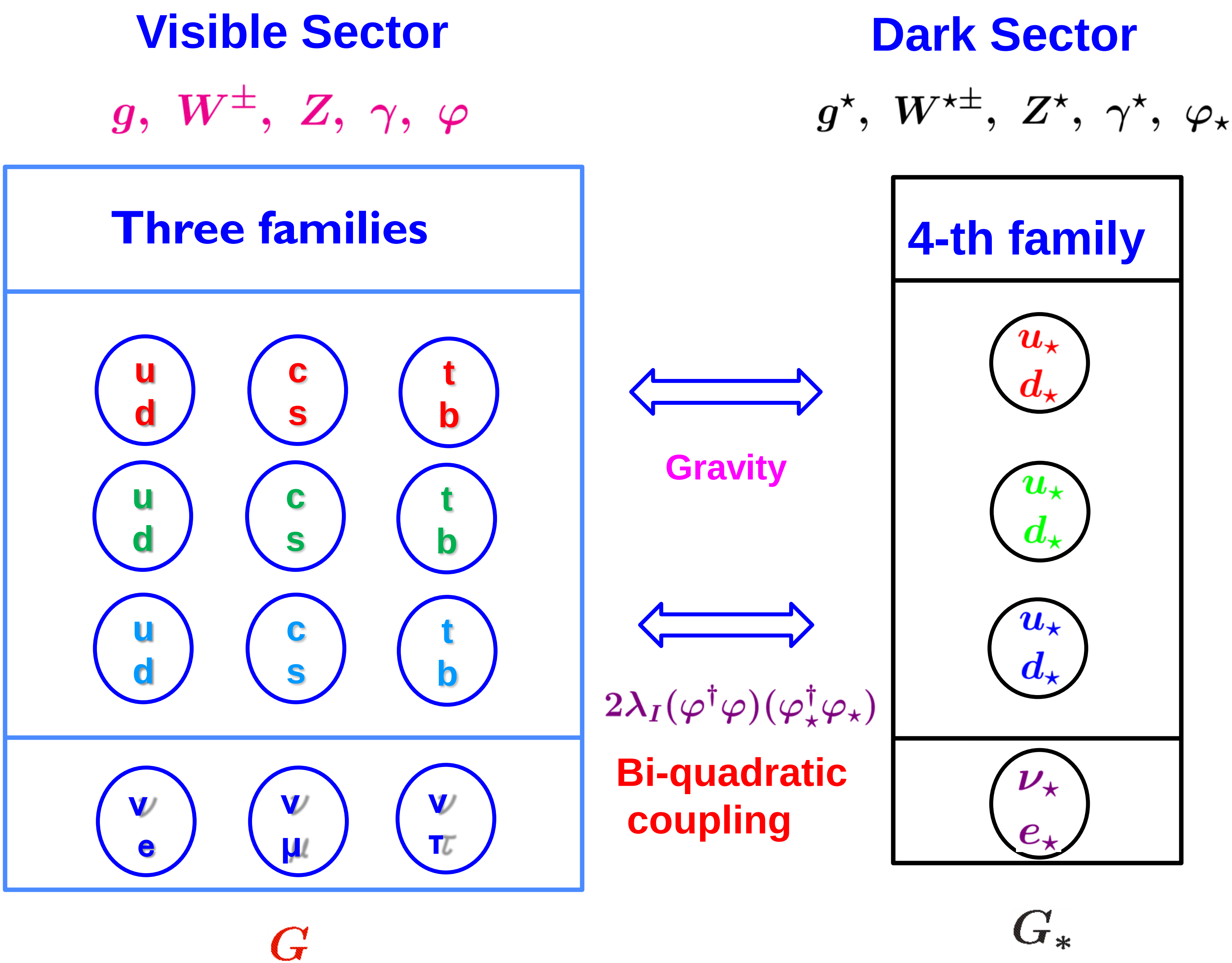
$G_\star$

Higgs potential

$$V_H = V_0 - \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 - \mu_\star^2 \varphi_\star^\dagger \varphi_\star + \lambda_\star (\varphi_\star^\dagger \varphi_\star)^2$$

$$+ 2\lambda_I (\varphi^\dagger \varphi)(\varphi_\star^\dagger \varphi_\star)$$

Bi-quadratic coupling



Visible Sector
$g, W^{\pm}, Z, \gamma, \varphi$
Three families
u d c s t b
u d c s t b
u d c s t b
ν e ν μ ν τ
G
Dark Sector
$g^{\star}, W^{\star\pm}, Z^{\star}, \gamma^{\star}, \varphi_{\star}$
4-th family
$u_{\star}$ $d_{\star}$
$u_{\star}$ $d_{\star}$
$u_{\star}$ $d_{\star}$
$\nu_{\star}$ $e_{\star}$
G_{*}
Gravity
$2\lambda_I(\varphi^{\dagger}\varphi)(\varphi_{\star}^{\dagger}\varphi_{\star})$
Bi-quadratic coupling

Simplest scheme

Breakdown of $G_{WS\star} = SU_R(2) \times U_{Y*}(1)$ symmetry

$$m_{u_\star} \gg m_{d_\star} + m_{e_\star}$$

$$u_\star \to d_\star + \bar{e}_\star + \nu_\star$$

Only one stable hadron

Spin $\frac{3}{2}$

Dark hadron $\Delta_\star^- = [d_\star\, d_\star\, d_\star] = \frac{1}{\sqrt{6}}\epsilon_{ijk} d_\star^i\, d_\star^j\, d_\star^k$

Dark atom $H_\star = (\Delta_\star^- \; + \; \bar{e}_\star)$

Dark molecule $(H_\star)_2 = H_\star H_\star$

Candidates of DM $(H_\star)_2,\; H_\star,\; \Delta_\star^-,\; e_\star,\; \nu_\star : \nu_{iR}$

No nuclear reaction : Simple thermal history

Breakdown of two symmetries

$$G_{WS} = SU_L(2) \times U_Y(1)$$

$$G_{WS\star} = SU_R(2) \times U_{Y*}(1)$$

$$V_H = V_0 - \mu^2\varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2 - \mu_\star^2\varphi_\star^\dagger\varphi_\star + \lambda_\star(\varphi_\star^\dagger\varphi_\star)^2$$

$$+ 2\lambda_I(\varphi^\dagger\varphi)(\varphi_\star^\dagger\varphi_\star)$$

Bi-quadratic coupling

Unitary decomposition

$$\varphi(x) = \frac{1}{\sqrt{2}}U(\vartheta(x))\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Visible Higgs

$$\varphi_\star(x) = \frac{1}{\sqrt{2}}U_\star(\vartheta_\star(x))\begin{pmatrix} 0 \\ v_\star + h_\star(x) \end{pmatrix}$$

Dark Higgs

$$V_H(h,\, h_\star) \;=\; \boxed{\lambda v^2 h^2} + \boxed{\lambda_\star v_\star^2 h_\star^2} + \boxed{2\lambda_I v v_\star h h_\star}$$

Interaction mode

$$+\;\; \lambda v h^3 + \lambda_\star v_\star h_\star^3 + \lambda_I v h h_\star^2 + \lambda_I v_\star h^2 h_\star$$

$$+\;\; \cdots$$

$$\boxed{m_h^2 = 2\lambda v^2 (\simeq \Lambda^2)} \qquad \boxed{m_{h\star}^2 = 2\lambda_\star v_\star^2 (\simeq \Lambda_\star^2)}$$

Mass eigen-modes

LHC Higgs → H

Visible Higgs → h

Dark Higgs → $h_\star$

$$\begin{cases} H = \cos\theta\, h - \sin\theta\, h_\star \\ H' = \sin\theta\, h + \cos\theta\, h_\star \end{cases}$$

Mass eigen-modes

LHC Higgs → H ; Visible Higgs → h ; Dark Higgs → $h_\star$

$$\begin{cases} H = \cos\theta\, h - \sin\theta\, h_\star \\ H' = \sin\theta\, h + \cos\theta\, h_\star \end{cases}$$

$$\tan 2\theta = \frac{2\lambda_I v v_\star}{\lambda_\star v_\star^2 - \lambda v^2}$$

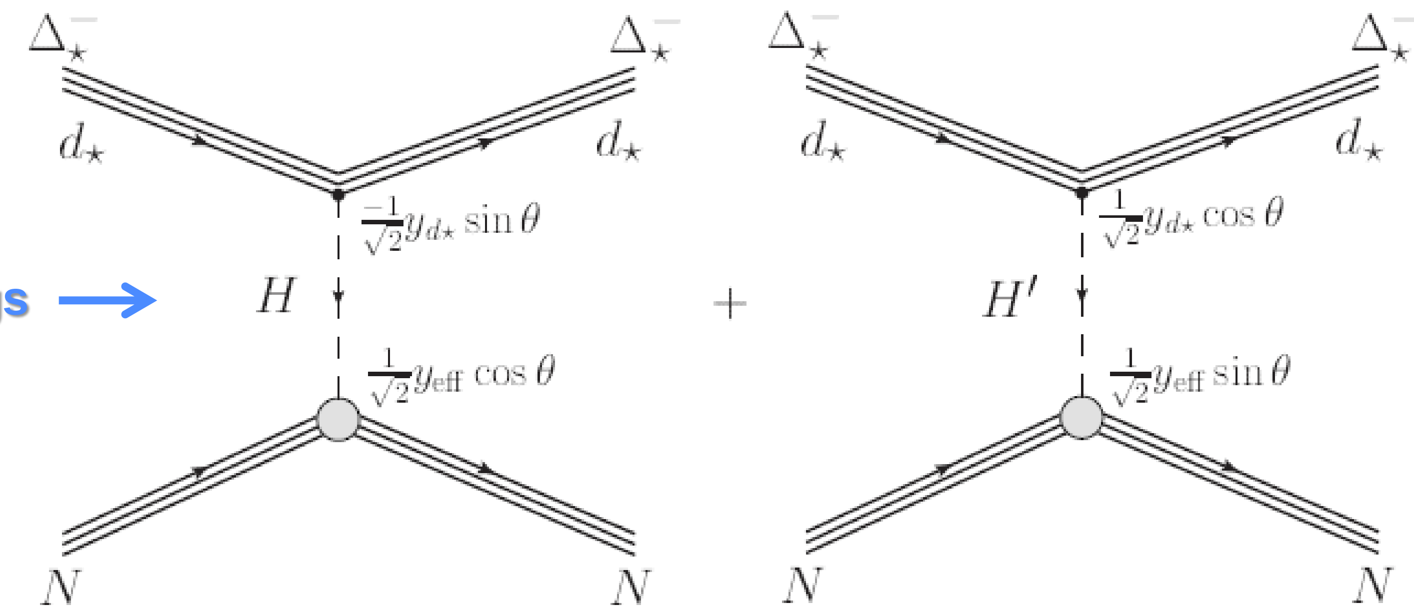
$$V_H(H, H') = \frac{1}{2} m_H^2 H^2 + \frac{1}{2} m_{H'}^2 H'^2 + \cdots$$

Λ ← m_H^2 ; $\Lambda_\star$ ← $m_{H'}^2$

$$\begin{pmatrix} m_H^2 \\ m_{H'}^2 \end{pmatrix} = (\lambda v^2 + \lambda_\star v_\star^2) \mp \sqrt{(\lambda v^2 - \lambda_\star v_\star^2)^2 + (2\lambda_I v v_\star)^2}$$

Direct detection of dark matter

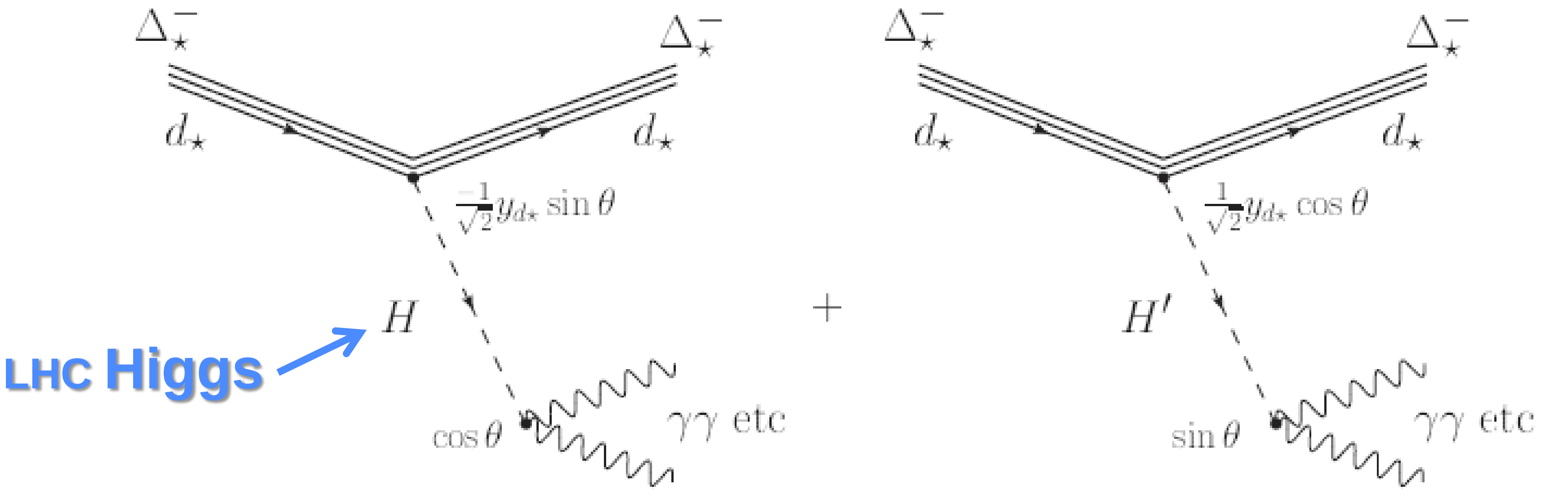
LHC Higgs →



$$\frac{1}{2} y_{d\star} y_{\rm eff} \sin\theta \cos\theta \left[\frac{1}{m_H^2} - \frac{1}{m_{H_\star}^2} \right] = \frac{1}{4} \frac{\lambda_I}{\lambda_\star \lambda - \lambda_I^2} \frac{y_{d\star} y_{\rm eff}}{v_{h\star} v_h}$$

LUX & Xenon 1t

Indirect detection of dark matter



Difficult to identify the process from decay products

Fermi & AMS-2

Early reheating stage

G and $G_\star$ symmetric stage

Quantum correction

Yukawa interaction

$$\mathcal{L}_Y = \bar{\Psi}_{(v)}\tilde{\varphi}\mathcal{Y}_U U_{(v)} + \bar{\Psi}_{(v)}\varphi\mathcal{Y}_D D_{(v)}$$
$$+ y_{*u}\bar{U}_{(d)}\tilde{\varphi}_*^\dagger\Psi_{(d)} + y_{*d}\bar{D}_{(d)}\varphi_*^\dagger\Psi_{(d)}$$

Higgs-Gauge interactions

$$\mathcal{L}_H = (\mathcal{D}^\mu\varphi)^\dagger(\mathcal{D}_\mu\varphi) + (\mathcal{D}^\mu\varphi_*)^\dagger(\mathcal{D}_\mu\varphi_*) - V_H$$

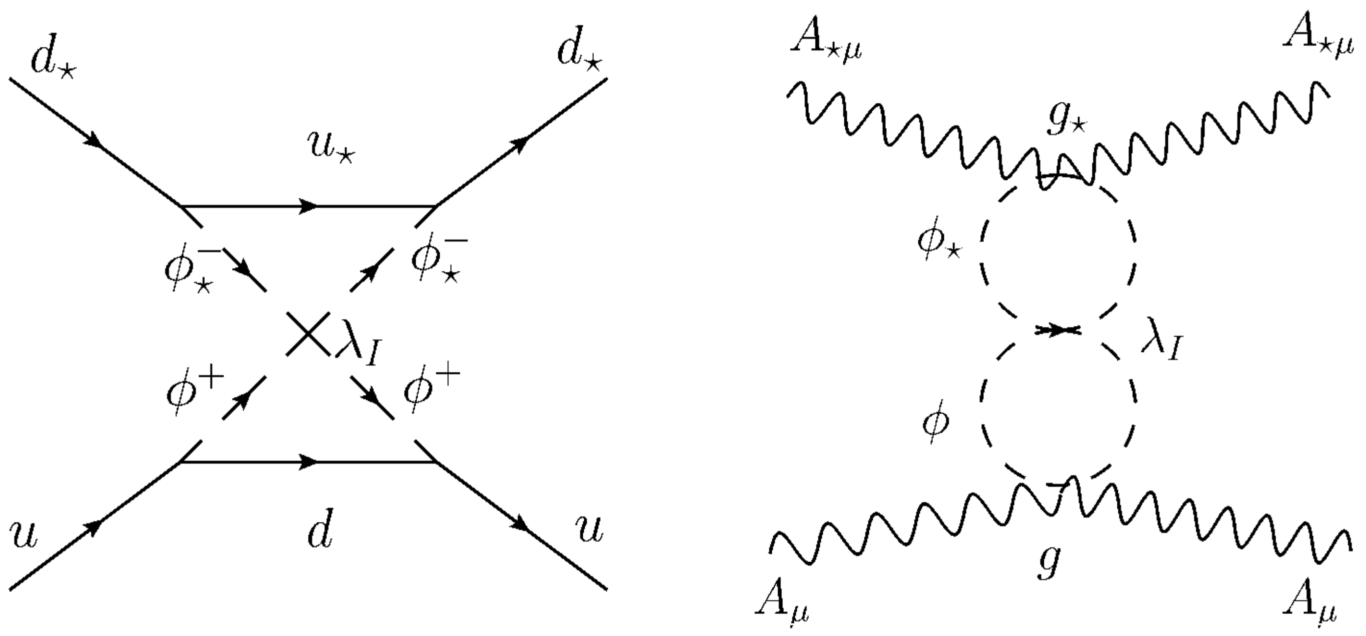
$$i\mathcal{D}_\mu\varphi = \left(i\partial_\mu - g^{(2)}A_\mu^{(2)a}\frac{1}{2}\tau_{La} - g^{(1)}A_\mu^{(1)}\frac{1}{2}\right)\varphi$$

$$i\mathcal{D}_\mu\phi_* = \left(i\partial_\mu - g_*^{(2)}A_{*\mu}^{(2)a}\frac{1}{2}\tau_{Ra} - g_*^{(1)}A_{*\mu}^{(1)}\frac{1}{2}\right)\phi_*$$

Higgs potential

$$V_H = \cdots + 2\lambda_I(\varphi^\dagger\varphi)(\varphi_*^\dagger\varphi_*) + \cdots$$

Higgs-induced interactions between visible and dark fermions $> \Lambda_*$



Era of quantum soup of visible & dark fields

Two sectors are in an inseparable phase of thermal equilibrium in an early reheating period

Early reheating period : Inseparable phase

G and $G_\star$ symmetric stage

Friedmann equation ⇦ All visible & dark fields

Effective number of relativistic d. o. f.

$$g_* = \left(28 + \frac{7}{8} \times 90\right) + \left(28 + \frac{7}{8} \times 30\right) = 106.75 + 54.25 = 161$$

⇩ $\Lambda_\star$

Decoupling of dark sector out of thermal equilibrium

⇩

Thermal histories of visible sector and dark sector

Mass of stable dark hadron

$$\Delta_\star^- = [d_\star\, d_\star\, d_\star] = \frac{1}{\sqrt{6}}\epsilon_{ijk} d_\star^i\, d_\star^j\, d_\star^k$$

(1) Era of common soup of visible & dark quanta

1 dark family : 3 visible families

(2) Same mechanism for baryogenesis in two sectors

(3) Era with stable nucleons N and dark hadrons Δ_*

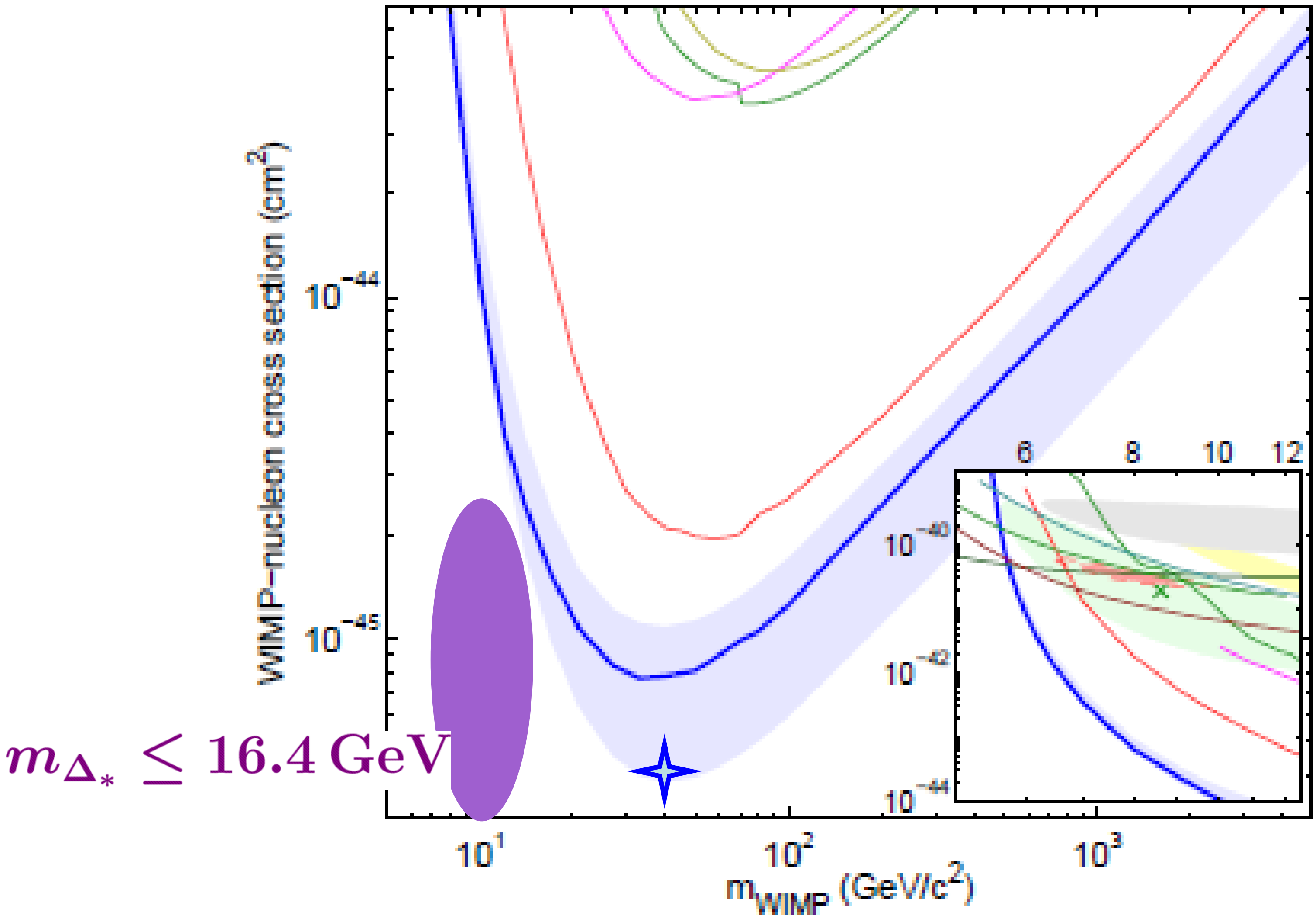
4 **4**

$$m_{\Delta_*} : 3m_N = 26.8 : 4.9 \;\Rightarrow\; m_{\Delta_*} = \frac{3\times 26.8}{4.9} m_N$$

$$m_{\Delta_*} \leq 16.4\,\mathrm{GeV}$$

Data from Planck

LUX detection



7.6×10^{-46} cm^2 at a WIMP mass of 33 GeV/c^2

Results, problems and speculation

Results

Fundamental representation of chiral triplet fields

$$\Psi_L(x) = {}^t\begin{pmatrix} \Psi_{(v)} & : & \begin{matrix} U_{(d)} \\ D_{(d)} \end{matrix} \end{pmatrix}_L \quad \Psi_R(x) = {}^t\begin{pmatrix} \begin{matrix} U_{(v)} \\ D_{(v)} \end{matrix} & : & \Psi_{(d)} \end{pmatrix}_R$$

Three families of visible Q's and L's

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

Additional single family of dark Q's and L's

$$G_\star = SU_{c^*}(3) \times SU_R(2) \times U_{Y^*}(1)$$

Monotone world of dark matter

⇩

Careful study of thermal history : Required

Problems

Different treatments for color and electroweak symmetries

$$\mathrm{SU(3)}$$

$$\mathrm{SU(2)\times U(1)}$$

A new type of multi-spinor field

Different treatment of color and family at the SM level

Isomorphic at algebraic level

$\{\, A_v + A_d \,\}$ **continuous symmetry?**

$\{\, A^q + A^\ell \,\}$ **gauge symmetry**

Question

Implication of triple-legs the triplet fields

$$\bigcirc \otimes \bigcirc \otimes \bigcirc$$

$$\begin{pmatrix} \gamma_\mu \\ \gamma_\mu \\ \gamma_\mu \end{pmatrix} \quad \begin{pmatrix} 1 & \gamma_\mu & \gamma_\mu \\ \gamma_\mu & 1 & \gamma_\mu \\ \gamma_\mu & \gamma_\mu & 1 \end{pmatrix} \begin{matrix} a \\ b \\ c \end{matrix} \qquad \Psi_{abc}$$

Composite particle

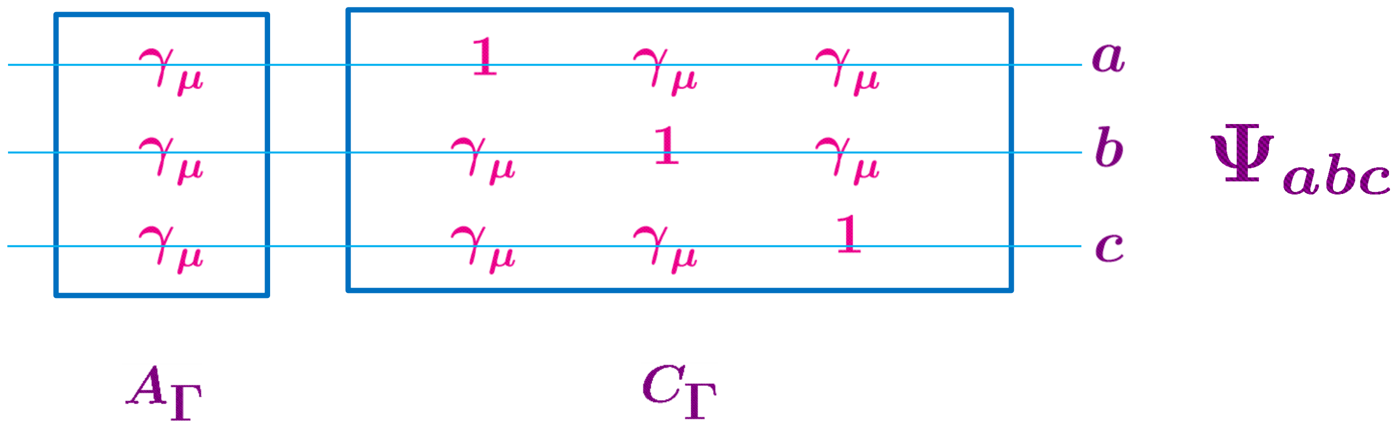
$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{3}{2}$$

Spin $\frac{1}{2}$ only

Composite spinor field?

Question

Implication of the triple-product of algebras



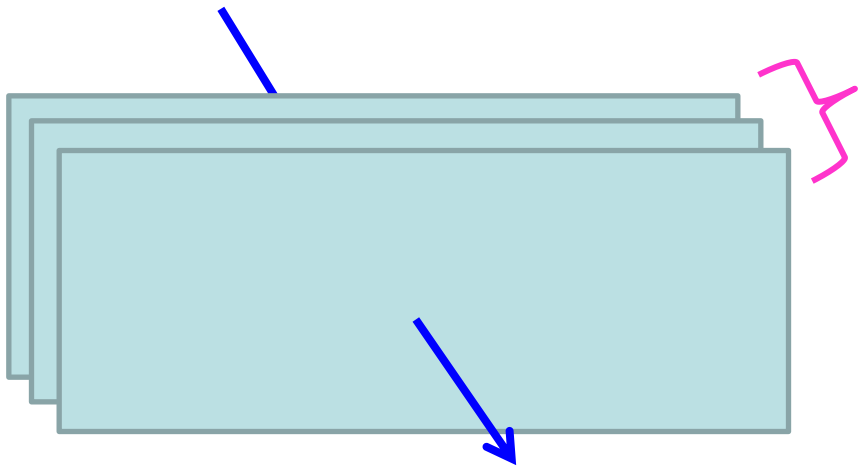
$$\Gamma_\mu = \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \qquad \{\, A_v + A_d \,\}$$

$$x^\mu \qquad \{\, A^q + A^\ell \,\}$$

$$\Gamma_\mu = \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu \quad \Longrightarrow \quad x^\mu$$

Speculation

Geometrical interpretation?



A_{ex}

$$\Gamma_\mu = \gamma_\mu \otimes \gamma_\mu \otimes \gamma_\mu$$

x^μ

spacetime

A_{in}

$$\{ A^q + A^\ell \}$$

$$\{ A_v + A_d \}$$

$$\{ A_v^{ew}, A_d^{ew} \}$$

World with three sheets a la Connes?