Fermionization...

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Fermionization/Bosonization in Arbitrary Dimensions for Free Particles, Locality Predicts Four Families

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- Show that Bosonization/fermionization is at all pssible in an Arbtirary Number of Dimensions,
- Remind about Theorem about the Numbers of Degrees of Freedom for Fermions versus Bosons. (Aratyn and Nielsen)
- Complaint about Fermions theories being Non-local (if true fundametal fermions)
- Deduction of there being Four families (or a mutiplum thereof) in four dimensions.

Nice Version of Jacobys Triple Product Formula
(still to be checked. Euler did 1748)

$$\frac{1}{2}\prod_{n=0,1,2,\ldots}(1+x^n)=\prod_{m=1,3,5,\ldots}\frac{1}{1-x^m}. \tag{1}$$

Odd parts and distinct parts

Among the 22 partitions of the number 8, there are 6 that contain only odd parts:

$7 + 1$

$5 + 3$

$5 + 1 + 1 + 1$

$3 + 3 + 1 + 1$

$3 + 1 + 1 + 1 + 1 + 1$

$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$

Alternatively, we could count partitions in which no number occurs more than once. If we count the partitions of 8 with distinct parts, we also obtain 6:

8

$7 + 1$

$6 + 2$

$5 + 3$

$5 + 2 + 1$

$4 + 3 + 1$

For all positive numbers the number of partitions with odd parts equals the number of partitions with distinct parts.[19]

This result was proved by Leonhard Euler in 1748[20] and is a special case of Glaisher's theorem.

For every type of restricted partition there is a corresponding function for the number of partitions satisfying the given restriction. An important example is q(n), the number of partitions of n into distinct parts.[21] The first few values of q(n) are (starting with q(0)=1):

1, 1, 1, 2, 2, 3, 4, 5, 6, 8, 10, (sequence A000009 in OEIS).

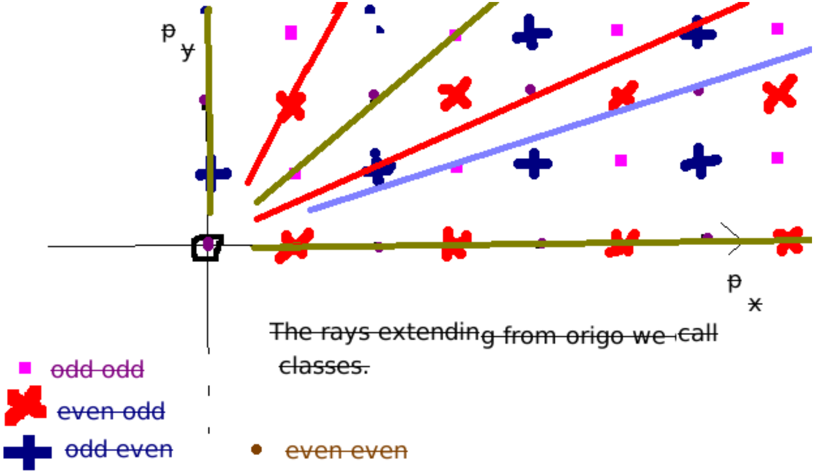
The generating function for q(n) (partitions into distinct parts) is given by[22]

$$\sum_{n=0}^{\infty} q(n)x^n = \prod_{k=1}^{\infty}(1+x^k) = \prod_{k=1}^{\infty} \frac{1}{1-x^{2k-1}}. \tag{2}$$

The second product can be written $\phi(x2)/\phi(x)$ whereϕ is Euler's function; the pentagonal number theorem can be applied to this as well giving a recurrence for q:[23]

$$q(k) = ak + q(k1) + q(k2)q(k5)q(k7) + q(k12) + q(k15)q(k22)... \quad (3)$$

where ak is (1)m if k = 3m2 m for some integer m and is 0 otherwise.


p y
p x
The rays extending from origo we call classes.
odd-odd
even-odd
odd-even
even-even

Bosonization Illustrating Formula

(d_{space} space dimensional version for only a "quadrant")

$$\frac{1}{2} \prod_{(m_1, m_2, \ldots, m_{d_{space}}) \in \mathbf{N}_0^{d_{space}}} (1 + x_1^{m_1} x_2^{m_2} \cdots x_{d_{space}}^{m_{d_{space}}}) = \tag{4}$$

$$= \prod_{(n_1, n_2, \ldots, n_{d_{space}}) \in \mathbf{N}_0^{d_{space}} \text{but not all } n_i\text{'s even}} \frac{1}{1 - x_1^{n_1} x_2^{n_2} \cdots x_{d_{space}}^{n_{d_{space}}}} \tag{5}$$

$$\frac{1}{2} \prod_{(m_1,m_2,\ldots,m_{d_{space}})\in \mathbf{Z}^{d_{space}}} \tag{6}$$

$$(1 + x_1^{m_1} x_2^{m_2} \cdots x_{d_{space}}^{m_{d_{space}}} z^{\sqrt{m_1^2+m_2^2+\ldots+m_{d_{space}}^2}}) \quad = \tag{7}$$

$$= \prod_{(n_1,n_2,\ldots,n_{d_{space}})\in \mathbf{Z}^{d_{space}} \text{but not all } n_i\text{'s even}} \tag{8}$$

$$\frac{1}{1 - x_1^{n_1} x_2^{n_2} \cdots x_{d_{space}}^{n_{d_{space}}} z^{\sqrt{n_1^2+n_2^2+\ldots+n_{d_{space}}^2}}}. \tag{9}$$