$SU(3)$ Gauged Family Symmetry Model

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Main goal of this BSM is to account for the hierarchy of fermion masses

$$m_t \gg m_c \gg m_u \quad , \quad m_b \gg m_s \gg m_d \quad , \quad m_\tau \gg m_\mu \gg m_e$$

The global symmetry in limit of all quarks and leptons massless, including R-handed neutrinos:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R}$$

$$\supset SU(3)_{q_L+u_R+d_R+l_L+e_R+\nu_R} \equiv SU(3)$$

$SU(3)$: Gauged Family Symmetry

Completely vector-like and universal. That is, couple equally to Left and Right Handed quarks and leptons

$G \equiv SU(3) \otimes G_{SM}$ "GAUGE GROUP"

$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: Standard Model Group

Standard model fermions

Ordinary Fermions: $q^o_{iL} = \begin{pmatrix} u^o_{iL} \\ d^o_{iL} \end{pmatrix}$, $l^o_{iL} = \begin{pmatrix} \nu^o_{iL} \\ e^o_{iL} \end{pmatrix}$, $Q = T_{3L} + \frac{1}{2} Y$

$$\Psi^o_q = (3, 3, 2, \frac{1}{3})_L = \begin{pmatrix} q^o_{1L} \\ q^o_{2L} \\ q^o_{3L} \end{pmatrix} \quad , \quad \Psi^o_l = (3, 1, 2, -1)_L = \begin{pmatrix} l^o_{1L} \\ l^o_{2L} \\ l^o_{3L} \end{pmatrix}$$

$$\Psi^o_u = (3, 3, 1, \frac{4}{3})_R = \begin{pmatrix} u^o_R \\ c^o_R \\ t^o_R \end{pmatrix} \quad , \quad \Psi^o_d = (3, 3, 1, -\frac{2}{3})_R = \begin{pmatrix} d^o_R \\ s^o_R \\ b^o_R \end{pmatrix}$$

$$\Psi^o_e = (3, 1, 1, -2)_R = \begin{pmatrix} e^o_R \\ \mu^o_R \\ \tau^o_R \end{pmatrix}$$

All SM fermions transform as the fundamental representation under the $SU(3)$ family symmetry.

EXTRA FERMIONS: $SU(2)_L$ SINGLETS

Demanding the model to be theoretical consistent: Gauge Invariance & Anomaly Free

Anomaly conditions:
$[SU(3)]^3 \, , \; [SU(2)]^2 \; SU(3) \, , \; [U(1)]^2 \; SU(3)$

Right Handed Neutrinos: $\Psi^o_{\nu_R} = (3, 1, 1, 0)_R = \begin{pmatrix} \nu_{e_R} \\ \nu_{\mu_R} \\ \nu_{\tau_R} \end{pmatrix}$

So, in this scenario Right Handed Neutrinos are needed to cancel Anomalies

$$f_1^o = \begin{pmatrix} u^o \\ d^o \end{pmatrix}_L , \begin{pmatrix} \nu_e^o \\ e^o \end{pmatrix}_L , u_R^o , d_R^o , e_R^o , \nu_{e_R}^o$$

$$f_2^o = \begin{pmatrix} c^o \\ s^o \end{pmatrix}_L , \begin{pmatrix} \nu_\mu^o \\ \mu^o \end{pmatrix}_L , c_R^o , s_R^o , \mu_R^o , \nu_{\mu_R}^o$$

$$f_3^o = \begin{pmatrix} t^o \\ b^o \end{pmatrix}_L , \begin{pmatrix} \nu_\tau^o \\ \tau^o \end{pmatrix}_L , t_R^o , b_R^o , \tau_R^o , \nu_{\tau_R}^o$$

$$\begin{aligned} i\mathcal{L}_{int,SU(3)} = {} & \frac{g_H}{2} \left(\bar{f}_1^o \gamma_\mu f_1^o - \bar{f}_2^o \gamma_\mu f_2^o \right) Z_1^\mu \\ & + \frac{g_H}{2\sqrt{3}} \left(\bar{f}_1^o \gamma_\mu f_1^o + \bar{f}_2^o \gamma_\mu f_2^o - 2\bar{f}_3^o \gamma_\mu f_3^o \right) Z_2^\mu \\ & + \frac{g_H}{\sqrt{2}} \left(\bar{f}_1^o \gamma_\mu f_2^o Y_1^+ + \bar{f}_1^o \gamma_\mu f_3^o Y_2^+ + \bar{f}_2^o \gamma_\mu f_3^o Y_3^+ + h.c. \right) \end{aligned}$$

where g_H is the $SU(3)$ coupling constant, Z_1, Z_2 and $Y_i^\pm$, $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons.

We would like to be consistent with low energy Standard Model(SM) and simultaneously generate and account for the hierarchy of quark and lepton masses and mixing

$$SU(3) \times G_{SM} \longrightarrow G_{SM} \longrightarrow SU(3)_C \otimes U(1)_Q$$

SM fermions remain massless

SM fermions become massive (PDG known values)

Previous features of this BSM, all together, define the required scalars, V.E.V's, and additional vector-like fermions

NEW SET OF $SU(2)_L$ WEAK SINGLET VECTOR-LIKE FERMIONS:

A completely Sterile Neutrino: $N_L^o, N_R^o = (1, 1, 1, 0)$,

Vector Like quarks:
$U_L^o, U_R^o = (1, 3, 1, \frac{4}{3})$, $D_L^o, D_R^o = (1, 3, 1, -\frac{2}{3})$

and a **Vector Like electron:** $E_L^o, E_R^o = (1, 1, 1, -2)$

- **These new vector-like fermions do not spoil previous Anomaly Cancelation**
- **They are introduced to generate the top, bottom, and tau masses at tree level from Dirac See-saw mechanism**

The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U \, \bar{U}_L^o \, U_R^o + M_D \, \bar{D}_L^o \, D_R^o + M_E \, \bar{E}_L^o \, E_R^o + h.c.$$

and

$$m_D \, \bar{N}_L^o \, N_R^o + m_L \, \bar{N}_L^o \, (N_L^o)^c + m_R \, \bar{N}_R^o \, (N_R^o)^c + h.c$$

Scalars introduced to break symmetries:

I. $SU(3)$:

$$\eta_2 = (3,1,1,0) = \begin{pmatrix} \eta^o_{21} \\ \eta^o_{22} \\ \eta^o_{23} \end{pmatrix} \; , \; \eta_3 = (3,1,1,0) = \begin{pmatrix} \eta^o_{31} \\ \eta^o_{32} \\ \eta^o_{33} \end{pmatrix}$$

with VEV's:

$$\langle \eta_2 \rangle = \begin{pmatrix} 0 \\ \Lambda_2 \\ 0 \end{pmatrix} \; , \; \langle \eta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ \Lambda_3 \end{pmatrix}$$

$$SU(3) \times G_{SM} \quad \xrightarrow{\Lambda_3} \quad SU(2) \times G_{SM} \quad \xrightarrow{\Lambda_2} \quad G_{SM}$$

Λ_3: 5 very heavy boson masses ($\geq 100\ TeV's$)
Λ_2: 3 heavy boson masses (a few $TeV's$)

The above scalar fields and VEV's break completely the $SU(3)$ family symmetry, generating the mass terms

- $\langle\eta_2\rangle$: $\frac{g_{H_2}^2\Lambda_2^2}{2}(Y_1^+Y_1^- + Y_3^+Y_3^-) + \frac{g_{H_2}^2\Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2Z_1\frac{Z_2}{\sqrt{3}})$
- $\langle\eta_3\rangle$: $\frac{g_{H_3}^2\Lambda_3^2}{2}(Y_2^+Y_2^- + Y_3^+Y_3^-) + g_{H_3}^2\Lambda_3^2\frac{Z_2^2}{3}$

To suppress properly FCNC like, for instance: $\mu \rightarrow e\gamma$, $\mu \rightarrow e\,e\,e$, $K^o - \bar{K}^o$, it is relevant which gauge bosons are heavy, and which ones are very heavy

$SU(3)$ GAUGE BOSON MASSES:

Neglecting tiny contributions from electroweak symmetry breaking

$$M_2^2\, Y_1^+ Y_1^- + M_3^2\, Y_2^+ Y_2^- + (M_2^2 + M_3^2)\, Y_3^+ Y_3^- + \frac{1}{2} M_2^2\, Z_1^2 + \frac{1}{2}\frac{M_2^2 + 4M_3^2}{3}\, Z_2^2 - \frac{1}{2} M_2^2 \frac{2}{\sqrt{3}}\, Z_1\, Z_2$$

$$M_2^2 = \frac{g_{H_2}^2 \Lambda_2^2}{2} \quad , \quad M_3^2 = \frac{g_{H_3}^2 \Lambda_3^2}{2}$$

	Z_1	Z_2
Z_1	M_2^2	$-\frac{M_2^2}{\sqrt{3}}$
Z_2	$-\frac{M_2^2}{\sqrt{3}}$	$\frac{M_2^2+4M_3^2}{3}$

Table: $Z_1 - Z_2$ mixing mass matrix

From the diagonalization of the $Z_1 - Z_2$ squared mass matrix:

$$M_-^2 = \frac{2}{3}\left(M_2^2 + M_3^2 - \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2}\right)_-$$

$$M_+^2 = \frac{2}{3}\left(M_2^2 + M_3^2 + \sqrt{(M_3^2 - M_2^2)^2 + M_2^2 M_3^2}\right)_+$$

$$M_2^2\, Y_1^+ Y_1^- + M_3^2\, Y_2^+ Y_2^- + (M_2^2 + M_3^2)\, Y_3^+ Y_3^- + M_-^2\, \frac{Z_-^2}{2} + M_+^2\, \frac{Z_+^2}{2}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} Z_- \\ Z_+ \end{pmatrix}$$

$$\cos\phi\, \sin\phi = \frac{\sqrt{3}}{4}\, \frac{M_2^2}{\sqrt{M_2^4 + M_3^2(M_3^2 - M_2^2)}}$$

with the hierarchy $M_1, M_2 \gg M_W$.

II. Electroweak symmetry breaking: In this scenario we introduce two triplets of $SU(2)_L$ Higgs doublets: $\Phi^u = (3, 1, 2, -1), \Phi^d = (3, 1, 2, +1)$, explicitly:

$$\Phi^u = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_1^u \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_2^u \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_3^u \end{pmatrix} \quad , \quad \Phi^d = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_1^d \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_2^d \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_3^d \end{pmatrix}$$

ELECTROWEAK SYMMETRY BREAKING

with the VEV´s:

$$\langle \Phi^u \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u1} \\ 0 \end{pmatrix} \\ \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u2} \\ 0 \end{pmatrix} \\ \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u3} \\ 0 \end{pmatrix} \end{pmatrix} \quad , \quad \langle \Phi^d \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d1} \end{pmatrix} \\ \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d2} \end{pmatrix} \\ \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d3} \end{pmatrix} \end{pmatrix} \quad ,$$

Contribute to the W and Z boson masses:

$$\frac{g^2}{4}\,(v_u^2+v_d^2)\,W^+W^- + \frac{(g^2+{g'}^2)}{8}\,(v_u^2+v_d^2)\,Z_o^2$$
$$+\frac{1}{4}\sqrt{g^2+{g'}^2}\,g_H\,Z_o\,\Big[\,(v_{1u}^2-v_{2u}^2-v_{1d}^2+v_{2d}^2)\,Z_1$$
$$+(v_{1u}^2+v_{2u}^2-2v_{3u}^2-v_{1d}^2-v_{2d}^2+2v_{3d}^2)\,\frac{Z_2}{\sqrt{3}}$$
$$+2\,(v_{1u}v_{2u}-v_{1d}v_{2d})\,\frac{Y_1^+ + Y_1^-}{\sqrt{2}} + 2\,(v_{1u}v_{3u}-v_{1d}v_{3d})\,\frac{Y_2^+ + Y_2^-}{\sqrt{2}}$$
$$+2\,(v_{2u}v_{3u}-v_{2d}v_{3d})\,\frac{Y_3^+ + Y_3^-}{\sqrt{2}}\Big]$$

$+$ **tiny contributions to the** $SU(3)$ **gauge boson masses**

$v_u^2 = v_{u1}^2 + v_{u2}^2 + v_{u3}^2$, $v_d^2 = v_{d1}^2 + v_{d2}^2 + v_{d3}^2$. Hence, if we define $M_W = \frac{1}{2} g\, v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

Tree Level: Charged fermion masses:

Now we describe briefly the procedure to get the masses for quarks and leptons up to one loop corrections.
Before "Electroweak Symmetry Breaking"(EWSB) all ordinary, "Standard Model"(SM) fermions remain massless, and the quarks and leptons global symmetry is:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R}$$

$$h\,\bar{\psi}_l^o\,\Phi^d\,E_R^o \;+\; h_2\,\bar{\psi}_e^o\,\eta_2\,E_L^o \;+\; h_3\,\bar{\psi}_e^o\,\eta_3\,E_L^o \;+\; M\,\bar{E}_L^o\,E_R^o \;+ h.c$$

where M is a free mass parameter ($M\,\bar{E}_L^o\,E_R^o$ is gauge invariant), h, h_1, h_2 and h_3 are Yukawa coupling constants.

u-quarks and neutrinos coupled only to Φ^u

d-quarks and charged leptons couple only to Φ^d

Dirac See-saw mechanisms

In the gauge basis ${\psi^o_{L,R}}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms read as $\bar{\psi}^o_L \mathcal{M}^o \psi^o_R + h.c$, where

	e^o_R	μ^o_R	τ^o_R	E^o_R
$\overline{e^o_L}$	0	0	0	$h\, v_{d1}$
$\overline{\mu^o_L}$	0	0	0	$h\, v_{d2}$
$\overline{\tau^o_L}$	0	0	0	$h\, v_{d3}$
$\overline{E^o_L}$	0	$h_2\Lambda_2$	$h_3\Lambda_3$	M_E

Table: Tree level Dirac mass matrix $\mathcal{M}^o$

Notice that $\mathcal{M}^o$ has the same structure of a See-saw mass matrix, here for Dirac fermion masses. So, we call $\mathcal{M}^o$ a **"Dirac See-saw"** mass matrix. $\mathcal{M}^o$ is diagonalized by applying a biunitary transformation $\psi^o_{L,R} = V^o_{L,R}\,\chi_{L,R}$.

$$V_L^{o\,T}\mathcal{M}^o\mathcal{M}^{o\,T}\;V_L^o = V_R^{o\,T}\mathcal{M}^{o\,T}\mathcal{M}^o\;V_R^o = Diag(0,0,\lambda_3^2,\lambda_4^2)$$

$$V_L^{o\,T}\mathcal{M}^o\;V_R^o = Diag(0,0,-\lambda_3,\lambda_4)$$

where λ_3^2 and λ_4^2 are the nonzero eigenvalues, λ_4 being the fourth heavy fermion mass, and λ_3 of the order of the top, bottom and tau mass for u, d and e fermions, respectively.

Tree level Dirac Neutrino masses

$$h_D\,\overline{\Psi^o_l}\,\Phi^u\,N^o_R \;+\; h_{\nu 2}\,\overline{\Psi^o_{\nu_R}}\,\eta_2\,N^o_L \;+\; h_{\nu 3}\,\overline{\Psi^o_{\nu_R}}\,\eta_3\,N^o_L \;+\; m_D\,\overline{N^o_L}\,N^o_R \;+ h.c$$

h_D, h_1, h_2, and h_3 are Yukawa couplings, and m_D a Dirac type invariant neutrino mass for the sterile neutrino $N^o_{L,R}$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi^{o\,T}_{\nu\,L,R} = (\nu^o_e, \nu^o_\mu, \nu^o_\tau, N^o)_{L,R}$, the mass terms

$$h_D\,\left[v_{u1}\,\bar{\nu}^o_{eL} + v_{u2}\,\bar{\nu}^o_{\mu L} + v_{u3}\,\bar{\nu}^o_{\tau L}\right]\,N^o_R$$

$$+\left[h_{\nu 2}\,\Lambda_2\,\bar{\nu}^o_{\mu R} + h_{\nu 3}\,\Lambda_3\,\bar{\nu}^o_{\tau R}\right]\,N^o_L + m_D\,\bar{N}^o_L\,N^o_R + h.c.$$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_D\, v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_D\, v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_D\, v_{u3}$
$\overline{N^o_L}$	0	$h_{\nu 2}\, \Lambda_2$	$h_{\nu 3}\, \Lambda_3$	m_D

Table: Tree level Dirac mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, \nu^o_{jR}$

Tree level Majorana masses:

Since $N^o_{L,R}$ are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L \, \bar{\Psi}^o_l \, \Phi^u (N^o_L)^c \quad + \quad m_L \, \bar{N}^o_L \, (N^o_L)^c + h.c$$

and

$$h_{2R} \, \bar{\Psi}^o_\nu \, \eta_2 \, (N^o_R)^c \; + \; h_{3R} \, \bar{\Psi}^o_\nu \, \eta_3 \, (N^o_R)^c \; + \; m_R \, \bar{N}^o_R \, (N^o_R)^c + h.c.$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L \, \left[v_{u1} \, \bar{\nu}^o_{eL} + v_{u2} \, \bar{\nu}^o_{\mu L} + v_{u3} \, \bar{\nu}^o_{\tau L} \right] \, (N^o_L)^c \quad + \quad m_L \, \bar{N}^o_L \, (N^o_L)^c$$

$$+ \left[h_{2R} \, \Lambda_2 \, \bar{\nu}^o_{\mu R} + h_{3R} \, \Lambda_3 \, \bar{\nu}^o_{\tau R} \right] \, (N^o_R)^c \; + \; m_R \, \bar{N}^o_R \, (N^o_R)^c + h.c.$$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	0	0	0	$h_L v_{u1}$
$\nu^o_{\mu L}$	0	0	0	$h_L v_{u2}$
$\nu^o_{\tau L}$	0	0	0	$h_L v_{u3}$
N^o_L	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L

Table: Tree level L-handed Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, (\nu^o_{jL})^c$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	0	0	$h_{2R}\Lambda_2$
$\nu^o_{\tau R}$	0	0	0	$h_{3R}\Lambda_3$
N^o_R	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	m_R

Table: Tree level R-handed Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iR}\, (\nu^o_{jR})^T$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_L v_{u1}$	0	0	0	$h_D v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_L v_{u2}$	0	0	0	$h_D v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_L v_{u3}$	0	0	0	$h_D v_{u3}$
$\overline{N^o_L}$	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L	0	$h_2 \Lambda_2$	$h_3 \Lambda_3$	M_D
$\overline{(\nu^o_{eR})^c}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	$h_2 \Lambda_2$	0	0	0	$h_{2R} \Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	$h_3 \Lambda_3$	0	0	0	$h_{3R} \Lambda_3$
$\overline{(N^o_R)^c}$	$h_D v_{u1}$	$h_D v_{u2}$	$h_D v_{u3}$	M_D	0	$h_{2R} \Lambda_2$	$h_{3R} \Lambda_3$	m_R

Table: Tree Level Majorana masses

Thus, in the basis
$\Psi_{\nu}^{o\,T} = \left(\nu_{eL}^{o} \,, \nu_{\mu L}^{o} \,, \nu_{\tau L}^{o} \,, N_{L}^{o} \,, (\nu_{eR}^{o})^{c} \,, (\nu_{\mu R}^{o})^{c} \,, (\nu_{\tau R}^{o})^{c} \,, (N_{R}^{o})^{c} \right)$,
the Generic 8×8 tree level Majorana mass matrix for neutrinos $\mathcal{M}_{\nu}^{o}$, from Table 24, $\bar{\Psi}_{\nu}^{o}\, \mathcal{M}_{\nu}^{o}\, (\Psi_{\nu}^{o})^{c} \; + h.c.$, read

$$\mathcal{M}_{\nu}^{o} = \begin{pmatrix} 0 & 0 & 0 & \alpha_1 & 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & \alpha_3 & 0 & 0 & 0 & a_3 \\ \alpha_1 & \alpha_2 & \alpha_3 & m_L & 0 & b_2 & b_3 & m_D \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_2 & 0 & 0 & 0 & \beta_2 \\ 0 & 0 & 0 & b_3 & 0 & 0 & 0 & \beta_3 \\ a_1 & a_2 & a_3 & m_D & 0 & \beta_2 & \beta_3 & m_R \end{pmatrix}$$

Diagonalization of $\mathcal{M}_\nu^{(o)}$ yields four zero eigenvalues (five zero mass eigenvalues also possible if $a\,b = \alpha\,\beta$), associated to the neutrino fields: $ap = \sqrt{a_1^2 + a_2^2}$

$$\frac{a_2}{ap}\,\nu_{eL}^o - \frac{a_1}{ap}\,\nu_{\mu L}^o \quad , \quad \frac{a_1\,a_3}{ap\,a}\,\nu_{eL}^o + \frac{a_2\,a_3}{ap\,a}\,\nu_{\mu L}^o - \frac{a_p}{a}\,\nu_{\tau L}^o,$$

$$\nu_{eR}^o \quad , \quad \frac{b_3}{b}\,\nu_{\mu R}^o - \frac{b_2}{b}\,\nu_{\tau R}^o\,,$$

$b = \sqrt{b_2^2 + b_3^2}$.

Assuming for simplicity $\frac{h_{1R}}{h_1} = \frac{h_{2R}}{h_2} = \frac{h_{3R}}{h_3} \equiv c_R$,

$$\frac{\alpha_i}{a_i} = \frac{h_L}{h_D} = c_L \quad , \quad \frac{\beta_i}{b_i} = \frac{h_{iR}}{h_i} = c_R \, ,$$

the Characteristic Polynomial for the nonzero eigenvalues of $\mathcal{M}_\nu^o$ reduce to the one of the matrix m_4, Eq.(1), where

$$m_4 = \begin{pmatrix} 0 & \alpha & 0 & a \\ \alpha & m_L & b & m_D \\ 0 & b & 0 & \beta \\ a & m_D & \beta & m_R \end{pmatrix} \quad , \quad U_4 = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \qquad (1)$$

$$U_4^T \, m_4 \, U_4 = Diag(m_5^o, m_6^o, m_7^o, m_8^o)$$

$$(U_\nu^o)^T \, \mathcal{M}_\nu^o \, U_\nu^o = Diag(0,0,0,0,m_5^o, m_6^o, m_7^o, m_8^o)$$

Tree level Dirac and Majorana Neutrino masses

$$U_\nu^o = \begin{pmatrix} \frac{a_2}{ap} & \frac{a_1\, a_3}{a\, ap} & 0 & 0 & \frac{a_1}{a} u_{11} & \frac{a_1}{a} u_{12} & \frac{a_1}{a} u_{13} & \frac{a_1}{a} u_{14} \\ -\frac{a_1}{ap} & \frac{a_2\, a_3}{a\, ap} & 0 & 0 & \frac{a_2}{a} u_{11} & \frac{a_2}{a} u_{12} & \frac{a_2}{a} u_{13} & \frac{a_2}{a} u_{14} \\ 0 & -\frac{ap}{a} & 0 & 0 & \frac{a_3}{a} u_{11} & \frac{a_3}{a} u_{12} & \frac{a_3}{a} u_{13} & \frac{a_3}{a} u_{14} \\ 0 & 0 & 0 & 0 & u_{21} & u_{22} & u_{23} & u_{24} \\ 0 & 0 & \frac{b_2}{bp} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{b_2\, b_3}{b\, bp} & \frac{b_2}{b} u_{31} & \frac{b_2}{b} u_{32} & \frac{b_2}{b} u_{33} & \frac{b_2}{b} u_{34} \\ 0 & 0 & 0 & -\frac{b_p}{b} & \frac{b_3}{b} u_{31} & \frac{b_3}{b} u_{32} & \frac{b_3}{b} u_{33} & \frac{b_3}{b} u_{34} \\ 0 & 0 & 0 & 0 & u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \quad (2)$$

One loop contributions to fermion masses

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R}$$

Therefore, in this scenario light fermion masses, including neutrinos, may get extremely small masses from radiative corrections mediated by the $SU(3)$ heavy gauge bosons.

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

M_Y is the gauge boson mass, c_Y is coupling constant, $m_3^o = -\lambda_3$ and $m_4^o = \lambda_4$, and $f(x,y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$.

$$\sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i \, b_j \, M}{\lambda_4^2 - \lambda_3^2} \, F(M_Y) \, ,$$

$i,j = 1,2,3$, and $\quad F(M_Y) \equiv \frac{M_Y^2}{M_Y^2-\lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2-\lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$.

ONE LOOP CONTRIBUTION TO FERMION MASSES

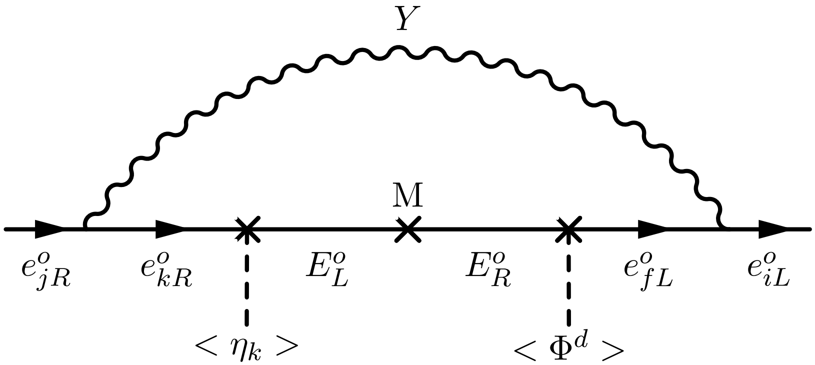

Figure: Generic one loop diagram contribution to the mass term $m_{ij}\, \bar{e}^o_{iL} e^o_{jR}$

	e_R^o	μ_R^o	τ_R^o	E_R^o
$\overline{e_L^o}$	D_{11}	D_{12}	D_{13}	0
$\overline{\mu_L^o}$	0	D_{22}	D_{23}	0
$\overline{\tau_L^o}$	0	D_{32}	D_{33}	0
$\overline{E_L^o}$	0	0	0	0

Table: Generic one loop diagram contribution to the mass term $m_{ij}\, \bar{e}_{iL}^o e_{jR}^o$

ONE LOOP CONTRIBUTION TO FERMION MASSES

$$\mathcal{M}_1^o = \begin{pmatrix} D_{11} & D_{12} & D_{13} & 0 \\ 0 & D_{22} & D_{23} & 0 \\ 0 & D_{32} & D_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} ,$$

$$\bar{\psi}_L^o \mathcal{M}^o \, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \, \psi_R^o = \bar{\chi}_L \, \mathcal{M} \, \chi_R \, ,$$

$$\mathcal{M} \equiv \left[Diag(0, 0, -\lambda_3, \lambda_4) + {V_L^o}^T \mathcal{M}_1^o \, V_R^o \right]$$

Generic mass matrix up to one loop for quarks and charged leptons.

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta \; m_{13} & s_\beta \; m_{13} \\ m_{21} & m_{22} & c_\beta \; m_{23} & s_\beta \; m_{23} \\ c_\alpha \; m_{31} & c_\alpha \; m_{32} & -\lambda_3 + c_\alpha c_\beta \; m_{33} & c_\alpha s_\beta \; m_{33} \\ s_\alpha \; m_{31} & s_\alpha \; m_{32} & s_\alpha c_\beta \; m_{33} & \lambda_4 + s_\alpha s_\beta \; m_{33} \end{pmatrix}$$

The diagonalization of $\mathcal{M}$ yields the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\, \Psi_{L,R}$; $\bar{\chi}_L\, \mathcal{M}\, \chi_R = \bar{\Psi}_L\, {V^{(1)}_L}^T \mathcal{M}\, V^{(1)}_R\, \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$ {V^{(1)}_L}^T \mathcal{M}\, \mathcal{M}^T\, V^{(1)}_L = {V^{(1)}_R}^T \mathcal{M}^T\, \mathcal{M}\, V^{(1)}_R = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\,, $$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$ \psi_L^o = V_L^o\, V_L^{(1)}\, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V_R^{(1)}\, \Psi_R $$

Quark $(V_{CKM})_{4\times4}$ and Lepton $(U_{PMNS})_{4\times8}$ mixing matrices

The transformation from massless to physical mass fermion eigenfields for quarks and charged leptons is

$$\psi_L^o = V_L^o \, V_L^{(1)} \, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o \, V_R^{(1)} \, \Psi_R \,,$$

and for neutrinos $\Psi_\nu^o = U_\nu^o \, U_\nu \, \Psi_\nu$. Vector like quarks are $SU(2)_L$ weak singlets, and hence, the interaction of L-handed up and down quarks; ${f_{uL}^o}^T = (u^o, c^o, t^o)_L$ and ${f_{dL}^o}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}} \, \bar{f^o}_{uL} \gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}} \, \bar{\Psi}_{uL} \, [(V_{uL}^o \, V_{uL}^{(1)})_{3\times4}]^T \, (V_{dL}^o \, V_{dL}^{(1)})_{3\times4} \, \gamma_\mu \Psi_{dL} \, W^{+\mu} \,, \quad (3)$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V_{uL}^o \, V_{uL}^{(1)})_{3\times4}]^T \, (V_{dL}^o \, V_{dL}^{(1)})_{3\times4} \quad (4)$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4\times8} = [(V_{eL}^o \, V_{eL}^{(1)})_{3\times4}]^T \, (U_\nu^o \, U_\nu)_{3\times8} \quad (5)$$

It is important to comment here that the scalar fields introduced to break the symmetries in the model: Φ^u, Φ^d, η_2 and η_3, do not couple ordinary fermions directly. So, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which as is shown in $(V_{CKM})_{4\times 4}$, may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.

One loop Dirac Neutrino masses

Light neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 2, as well as L-handed and R-handed Majorana masses from Fig. 3 and Fig. 4, respectively. The contribution from these diagrams read

$$c_Y \frac{\alpha_H}{\pi} m_\nu(M_Y)_{ij} \quad , \quad \alpha_H = \frac{g_H^2}{4\pi} \, ,$$

$$m_\nu(M_Y)_{ij} \equiv \sum_{k=5,6,7,8} m_k^o \, U_{ik}^o U_{jk}^o \, f(M_Y, m_k^o)$$

$$f(M_Y, m_k^o) = \frac{M_Y^2}{M_Y^2 - m_k^{o\,2}} \, ln \frac{M_Y^2}{m_k^{o\,2}} \; , \; m_\nu(M_Y)_{i,4+j} = \frac{a_i \, b_j}{a \, b} \mathcal{F}_\nu(M_Y)$$

$$\mathcal{F}_\nu(M_Y) = u_{11} u_{31} \, m_5^o \, f(M_Y, m_5^o) + u_{12} u_{32} \, m_6^o \, f(M_Y, m_6^o) + u_{13} u_{33} \, m_7^o \, f(M_Y, m_7^o) + u_{14} u_{34} \, m_8^o \, f(M_Y, m_8^o)$$

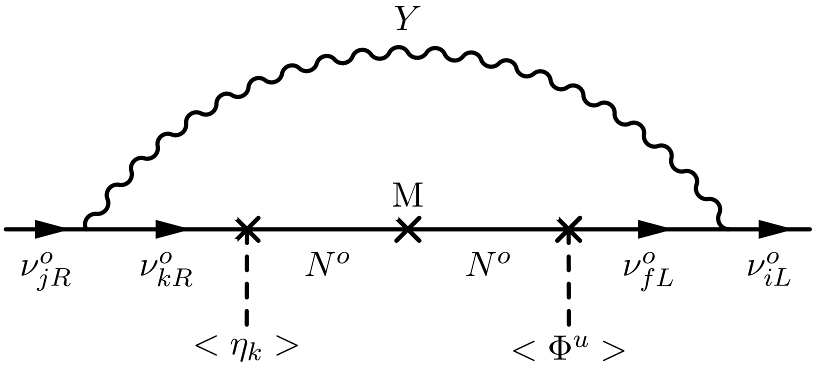

Figure: Generic one loop diagram contribution to the Dirac mass term $m_{ij}\, \bar{\nu}^o_{iL} \nu^o_{jR}$. M $= M_N, m_L, m_R$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	$D_{\nu\, 11}$	$D_{\nu\, 12}$	$D_{\nu\, 13}$	0
$\overline{\nu^o_{\mu L}}$	0	$D_{\nu\, 22}$	$D_{\nu\, 23}$	0
$\overline{\nu^o_{\tau L}}$	0	$D_{\nu\, 32}$	$D_{\nu\, 33}$	0
$\overline{N^o_L}$	0	0	0	0

Table: One loop Dirac neutrino mass terms $m^{\nu}_{ij}\ \bar{\nu}^o_{iL}\ \nu^o_{jR}$

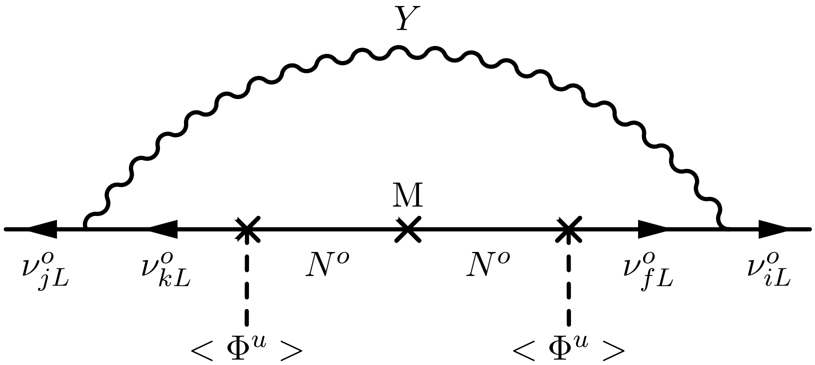

Figure: Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij}\, \bar{\nu}^o_{iL} (\nu^o_{jL})^T$. M $= M_N, m_L, m_R$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	0
$\nu^o_{\mu L}$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0
$\nu^o_{\tau L}$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0
N^o_L	0	0	0	0

Table: One loop L-handed neutrino Majorana mass terms
$m_{ij}\,\bar{\nu}^o_{iL}\,(\nu^o_{jL})^T$

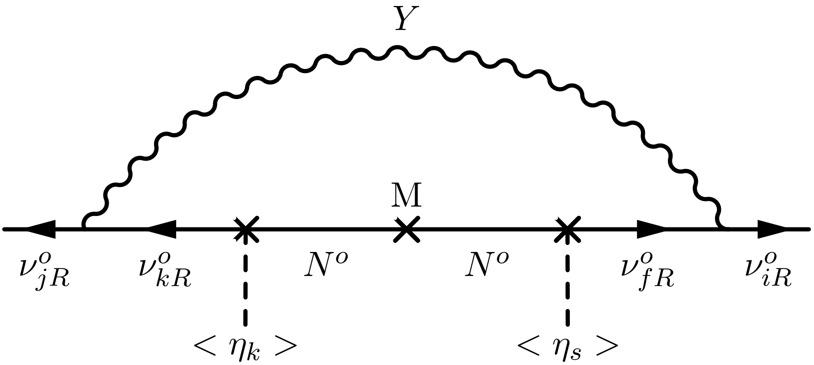

Figure: Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij}\, \bar{\nu}^o_{iR} (\nu^o_{jR})^T$. $\mathrm{M} = M_N, m_L, m_R$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	$R_{\nu\,11}$	$R_{\nu\,12}$	$R_{\nu\,13}$	0
$\nu^o_{\mu R}$	$R_{\nu\,12}$	$R_{\nu\,22}$	$R_{\nu\,23}$	0
$\nu^o_{\tau R}$	$R_{\nu\,13}$	$R_{\nu\,23}$	$R_{\nu\,33}$	0
N^o_R	0	0	0	0

Table: One loop R-handed neutrino Majorana mass terms $m_{ij}\,\bar{\nu}^o_{iR}\,(\nu^o_{jR})^T$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	$L_{\nu\,11}$	$L_{\nu\,12}$	$L_{\nu\,13}$	0	$D_{\nu\,11}$	$D_{\nu\,12}$	$D_{\nu\,13}$	0
$\overline{\nu^o_{\mu L}}$	$L_{\nu\,12}$	$L_{\nu\,22}$	$L_{\nu\,23}$	0	$D_{\nu\,21}$	$D_{\nu\,22}$	$D_{\nu\,23}$	0
$\overline{\nu^o_{\tau L}}$	$L_{\nu\,13}$	$L_{\nu\,23}$	$L_{\nu\,33}$	0	$D_{\nu\,31}$	$D_{\nu\,32}$	$D_{\nu\,33}$	0
$\overline{N^o_L}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{eR})^c}$	$D_{\nu\,11}$	$D_{\nu\,21}$	$D_{\nu\,31}$	0	$R_{\nu\,11}$	$R_{\nu\,12}$	$R_{\nu\,13}$	0
$\overline{(\nu^o_{\mu R})^c}$	$D_{\nu\,12}$	$D_{\nu\,22}$	$D_{\nu\,32}$	0	$R_{\nu\,12}$	$R_{\nu\,22}$	$R_{\nu\,23}$	0
$\overline{(\nu^o_{\tau R})^c}$	$D_{\nu\,13}$	$D_{\nu\,23}$	$D_{\nu\,33}$	0	$R_{\nu\,13}$	$R_{\nu\,23}$	$R_{\nu\,33}$	0
$\overline{(N^o_R)^c}$	0	0	0	0	0	0	0	0

Table: One Loop Majorana mass matrix $\mathcal{M}^o_{1\,\nu}$ in the Ψ^o_ν basis

NEUTRINO MASS MATRIX UP TO ONE LOOP

$$\mathcal{M}_\nu = (U_\nu^o)^T \, \mathcal{M}^o_{1\,\nu} \, U_\nu^o + Diag(0,0,0,0,m_5^o,m_6^o,m_7^o,m_8^o)$$

$$\mathcal{M}_\nu = \begin{pmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\ N_{12} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\ N_{13} & N_{23} & 0 & 0 & N_{35} & N_{36} & N_{37} & N_{38} \\ N_{14} & N_{24} & 0 & N_{44} & N_{45} & N_{46} & N_{47} & N_{48} \\ N_{15} & N_{25} & N_{35} & N_{45} & N_{55}+m_5^o & N_{56} & N_{57} & N_{58} \\ N_{16} & N_{26} & N_{36} & N_{46} & N_{56} & N_{66}+m_6^o & N_{67} & N_{68} \\ N_{17} & N_{27} & N_{37} & N_{47} & N_{57} & N_{67} & N_{77}+m_7^o & N_{78} \\ N_{18} & N_{28} & N_{38} & N_{48} & N_{58} & N_{68} & N_{78} & N_{88}+m_8^o \end{pmatrix}$$

How much consistent is this scenario?

Preliminary numerical results

Performing a numerical analysis of the free space parameter at the M_Z scale: Zhi-zhong Xing,He Zhang and Shun Zhou, Phys. Rev. D 86, 013013 (2012).

SU(3) PARAMETERS: $\frac{\alpha_H}{\pi} = 0.2$, $M_2 = 2$ TeV , $M_3 = 2000$ TeV

$$\frac{v_{1d}}{v_{2d}} \simeq 0.362085 \quad , \quad \frac{\sqrt{v_{1d}^2 + v_{2d}^2}}{v_{3d}} \simeq 4.75972$$

$$\frac{v_{1u}}{v_{2u}} \simeq 0.37808 \quad , \quad \frac{\sqrt{v_{1u}^2 + v_{2u}^2}}{v_{3u}} \simeq 4.95379$$

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 114787. \\ 0 & 0 & 0 & 23171.6 \\ 0 & 0 & 0 & 309729. \\ 0 & -3.17479 \times 10^6 & 156759. & 4.24124 \times 10^6 \end{pmatrix} \text{MeV}\,,$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} 6.43247 & -577.395 & -4630.47 & -3456.86 \\ -0.16682 & -108.687 & -879.805 & -656.816 \\ 2.37508 & 2422.47 & -172176. & 19530.5 \\ 0.118903 & 121.275 & 1309.7 & 5.3078 \times 10^6 \end{pmatrix} \text{MeV}$$

and the u-quark masses

$$(\, m_u \,,\; m_c \,,\; m_t \,,\; M_U \,) = (\, 1.35499 \,,\; 653.465 \,,\; 172261 \,,\; 5.30784 \times 10^6 \,)\,\text{MeV}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 1046.42 \\ 0 & 0 & 0 & 219.849 \\ 0 & 0 & 0 & 2953.08 \\ 0 & -1.85135 \times 10^6 & 221156. & 613328. \end{pmatrix} \text{ MeV}$$

$$\mathcal{M}_d = \begin{pmatrix} -3.10297 & 1.33734 & 16.9541 & 51.5401 \\ 18.553 & -53.9152 & -3.79617 & -11.5403 \\ -47.3173 & -19.0579 & -2859.57 & 376.526 \\ -0.0236586 & -0.00952897 & 0.0619291 & 1.9628 \times 10^6 \end{pmatrix} \text{ MeV}$$

$$(\, m_d \,,\; m_s \,,\; m_b \,,\; M_D \,) = (\, 2.79562 \,,\; 57.0595 \,,\; 2860.08 \,,\; 1.9628 \times 10^6 \,) \text{ MeV}$$

non-unitary $(V_{CKM})_{4\times4}$

and the quark mixing

$$(V_{CKM})_{4\times4} = \begin{pmatrix} -0.974353 & 0.225 & -0.003419 & -0.000028 \\ -0.224674 & -0.973565 & -0.041082 & 0.000029 \\ -0.012580 & -0.039314 & 0.997707 & -0.000690 \\ 0.000193 & 0.000038 & -0.053674 & 0.000037 \end{pmatrix}$$

Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 221060. \\ 0 & 0 & 0 & 46443.8 \\ 0 & 0 & 0 & 623847. \\ 0 & -422644. & 19927.4 & 6.63423 \times 10^7 \end{pmatrix} \text{MeV}$$

$$\mathcal{M}_e = \begin{pmatrix} 2.20939 & -47.9938 & -467.564 & -2.9817 \\ -0.0548027 & -9.37083 & -94.7006 & -0.603915 \\ 0.780936 & 204.812 & -1669.71 & 16.335 \\ 0.00780976 & 2.04822 & 25.6163 & 6.63469 \times 10^7 \end{pmatrix} \text{MeV}$$

fit the charged lepton masses:

$$(m_e\,,\,m_\mu\,,\,m_\tau\,,\,M_E) = (0.4860\,,\,102.727\,,\,1746.23\,,\,66.346 \times 10^6\,)\,\text{MeV}$$

Conclusions:

Solutions for quark masses and mixing, show parameter space regions where some of the extra particles; scalars, gauge bosons and vector-like fermions may lie within a few TeV's region, and hence, within current LHC energies.