Phenomenological mass matrices

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Bled Workshop, July 2016

Phenomenology

The Standard Model of particle physics is not a fundamental model.

It is a phenomenological model, and however successful, it is flawed by the great number of free parameters,

12 of which are fermion masses.

The only obvious structure displayed by the masses, is the hierarchical structure of the charged fermion mass spectra.

In the hope of unveiling a dymanics underlying the mass spectra, an industry of mass matrix models has developed.

Until now, the phenomenological the Standard Model has been amazingly successful

$\Rightarrow$ Pursue the phenomenlogical track: scrutinize all available data that are relevant for the particle masses:

- Numerical mass values.
- Mixing matrices (quark sector as well as the lepton sector).

The information content of a matrix

The information content of a $N \times N$ matrix M is contained in its N matrix invariants, which are the sums and products of the eigenvalues λ_j, such as $traceM$, $detM$, i.e. more generally

$$
\begin{aligned}
I_1 &= \sum_j \lambda_j = \lambda_1 + \lambda_2 + \lambda_3 \ldots \\
I_2 &= \sum_{jk} \lambda_j \lambda_k = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \ldots \\
I_3 &= \sum_{jkl} \lambda_j \lambda_k \lambda_l = \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \ldots
\end{aligned}
$$

$\therefore$ the information content of a maxtrix in principle contained in its invariants.

The form that the mass matrices take in the weak interaction basis is however also important, because - this is the hope - it could give a hint about the origin of the fermion masses.

M and M': quark mass matrices of the fermions with charge Q and charge Q-1

A flavour space basis corresponds to a specific form of M and M'.

Assumption: the form of the mass matrices M and M' in the "weak interaction basis" is a key to the origin of the fermion masses.

The matrices

The fermion mass matrices appear in the mass Lagragian,

$$\mathcal{L}_{mass} = \bar{f}\,M f \quad \text{and} \quad \mathcal{L}_{mass'} = \bar{f}'\,M'\,f'.$$

where f and f' denote fermion fields of charge Q and $Q-1$, respectively.

The unitary mixing matrix V appears in the flavour changing charged current Lagrangian

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\,\bar{f}_L \gamma^\mu V f'_L W_\mu + h.c.$$

where in the case of quarks f and f' are up-fields and down-fields with charges $2/3$ and $-1/3$,

V is the Cabbibo-Kobayashi-Maskawa (CKM), relating the mass matrices of the two sectors.

Quark mass matrices in a flavour space basis

What do the quark mass matrices look like in different flavour space bases?

In the mass basis for the up-sector:

$M = D = diag(m_u, m_c, m_t)$ and $M' = VD'V^\dagger$,
$D' = diag(m_d, m_s, m_b)$,

In the mass basis for the down-sector:
$M' = D'$ and $M = V^\dagger DV$.

The question is what M and M' look like in the weak basis.

The assumption is that they are similar - that they have similar texture.

The charged current Lagrangian

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\, \bar{f}_L \gamma^\mu V f'_L W_\mu + h.c.$$

can be seen as describing the interaction between the physical particles f with the mixed f'-states, χ',

$$\begin{pmatrix} \chi'_1 \\ \chi'_2 \\ \chi'_3 \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \begin{pmatrix} f'_1 \\ f'_2 \\ f'_3 \end{pmatrix}$$

It can equivalently be interpreted as the interaction between physical particles f' and mixed f-states.

The mass eigenstates of the quarks however appear both in electromagnetic and strong interactions, we thus recognize the mass states as "physical".

Therefore natural to perceive the charged current interactions as taking place between mixed up-sector states and mixed down-sector states,

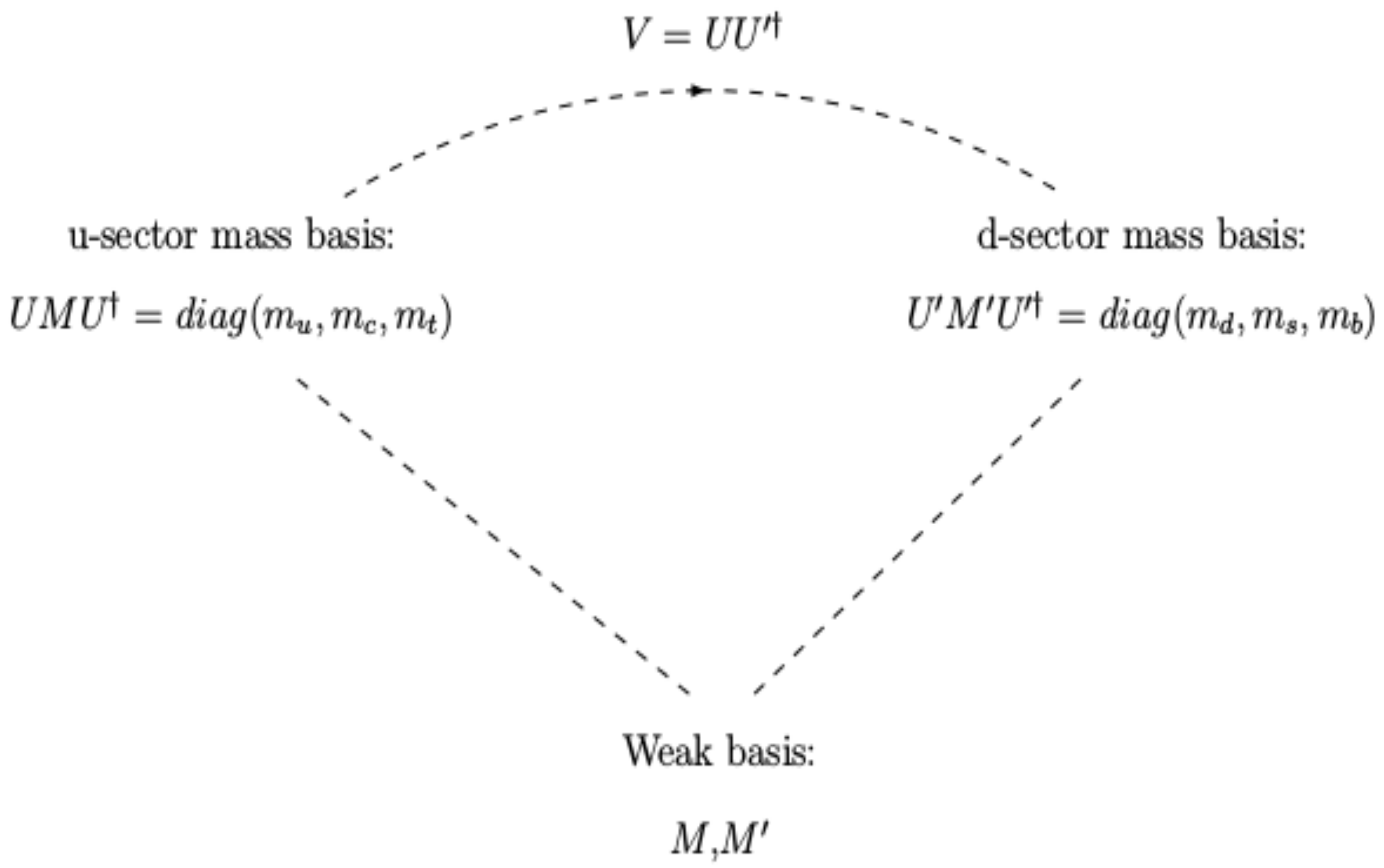
$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\,\bar{\varphi}_L \gamma^\mu \varphi'_L W_\mu + h.c.$$

where

$$\varphi = U^{\dagger} \begin{pmatrix} u \\ c \\ t \end{pmatrix} \quad \text{and} \quad \varphi' = U'^{\dagger} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

and $V = UU'^{\dagger}$.

Here φ and φ' are understood as fermion fields living in the weak basis.



$$M \Rightarrow UMU^{\dagger} = D = diag(m_u, m_c, m_t)$$

$$M' \Rightarrow U'M'U'^{\dagger} = D' = diag(m_d, m_s, m_b)$$

Factorizing the mixing matrix

The Cabbibo-Kobayashi-Maskawa mixing matrix can of course be parametrized - and factorized - in many different ways.

With the standard parametrization of the CKM mixing matrix, with three Euler angles α, β, 2θ,

$$V = \begin{pmatrix} c_\beta c_{2\theta} & s_\beta c_{2\theta} & s_{2\theta} e^{-i\delta} \\ -c_\beta s_\alpha s_{2\theta} e^{i\delta} - s_\beta c_\alpha & -s_\beta s_\alpha s_{2\theta} e^{i\delta} + c_\beta c_\alpha & s_\alpha c_{2\theta} \\ -c_\beta c_\alpha s_{2\theta} e^{i\delta} + s_\beta s_\alpha & -s_\beta c_\alpha s_{2\theta} e^{i\delta} - c_\beta s_\alpha & c_\alpha c_{2\theta} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} c_{2\theta} & 0 & s_{2\theta} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{2\theta} e^{i\delta} & 0 & c_{2\theta} \end{pmatrix} \begin{pmatrix} c_\beta & s_\beta & 0 \\ -s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Here the most "symmetric" rotation matrices for the up- and down-sectors are

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{pmatrix}$$

and

$$U' = \begin{pmatrix} c_\beta & -s_\beta & 0 \\ s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} c_\theta & 0 & -s_\theta \\ 0 & 1 & 0 \\ s_\theta & 0 & c_\theta \end{pmatrix},$$

where α, β, θ and γ correspond to the parameters in the standard parametrization in such a way that

$\gamma = \delta/2$, $\delta = 1.2 \pm 0.08$ rad, and $2\theta = 0.201 \pm 0.011^\circ$,
$\alpha = 2.38 \pm 0.06^\circ$ and $\beta = 13.04 \pm 0.05^\circ$.

In this factorization scheme, α and β are perceived as rotation angles operating in the up-sector and the down-sector, respectively.

From the rotation matrices U and U', we then obtain the up- and down-sector mass matrices

$$M = U^\dagger diag(m_u, m_c, m_t)U \quad \text{and} \quad \mathrm{M' = U'^\dagger diag(m_d, m_s, m}$$

parametrized as

$$M = \begin{pmatrix} Xc_\theta^2 + Ys_\theta^2 & -Zs_\theta\, e^{-i\gamma} & (X-Y)c_\theta s_\theta \\ -Zs_\theta\, e^{i\gamma} & m_c c_\alpha^2 + m_t s_\alpha^2 & Zc_\theta\, e^{i\gamma} \\ (X-Y)c_\theta s_\theta & Zc_\theta\, e^{-i\gamma} & Xs_\theta^2 + Yc_\theta^2 \end{pmatrix}$$

where $X = m_u$, $Y = m_c \sin^2\alpha + m_t \cos^2\alpha$,
$Z = (m_c - m_t)\sin\alpha\cos\alpha$.

$$M' = \begin{pmatrix} X's_\theta^2 + Y'c_\theta^2 & Z'c_\theta\, e^{i\gamma} & (X'-Y')c_\theta s_\theta \\ Z'c_\theta\, e^{-i\gamma} & m_s c_\beta^2 + m_d s_\beta^2 & -Z's_\theta\, e^{-i\gamma} \\ (X'-Y')c_\theta s_\theta & -Z's_\theta\, e^{i\gamma} & X'c_\theta^2 + Y's_\theta^2 \end{pmatrix}$$

where $X' = m_b$, $Y' = m_s \sin^2\beta + m_d \cos^2\beta$,
$Z' = (m_s - m_d)\sin\beta\cos\beta$.

Thus, the matrices M and M' have similar textures, and even a relational equality,

$$M_{32}\,/\,M_{12} = M'_{12}\,/\,M'_{32} = -\cot\theta$$

which is independent of the quark masses.

Numerical matrices

To get a feeling for what the mass matrices actually look like, we insert numerical mass values.

With $\beta = 13.04^o$, $\alpha = 2.38^o$, and the mass values for the up- and down-sectors (Jamin 2014)

$$m_u(M_Z) = 1.24 MeV, m_c(M_Z) = 624 MeV, m_t(M_Z) = 1715$$

$$m_d(M_G) = 2.69 MeV, m_s(M_G) = 53.8 MeV, m_b(M_G) = 285$$

we get the numerical mass matrices M and M':

$$M = \begin{pmatrix} 1.7669 & 12.44e^{-i\gamma} & -300.39 \\ 12.44e^{i\gamma} & 918.7589 & -7091.89e^{i\gamma} \\ -300.39 & -7091.89e^{-i\gamma} & 171254.71 \end{pmatrix} MeV$$

where $M_{11} = m_u + \sigma$, $M_{22} = m_c + X - \sigma$, $M_{33} = m_t - X$,

$M_{32}/M_{12} = cot\,\theta$ and $\sigma M_{33} = M_{13}^2$, $\sigma \simeq 0.5269 MeV$, $X \simeq 295.286 MeV$.

For the down-sector

$$M' = \begin{pmatrix} 5.299 & 11.22e^{i\gamma} & 4.99 \\ 11.22e^{-i\gamma} & 51.2 & -0.0197e^{-i\gamma} \\ 4.99 & -0.0197e^{i\gamma} & 2849.99 \end{pmatrix} MeV$$

where $M'_{11} = m_d + Y + \varepsilon$, $M'_{22} = m_s + Y$, and $M'_{33} = m_b - \varepsilon$,

$M'_{12}/M'_{32} = cot\,\theta$ and $\varepsilon M'_{33} = M'^2_{13}$, $Y \simeq 2.59 MeV$ and $\varepsilon \simeq 0.0087 MeV$.

Alternative factorization schemes

We can always sandwich a unit matrix $\mathbf{1} = \Omega\Omega^\dagger$ between the factors of the mixing matrix,

$$V = UU'^\dagger = V = U\Omega\Omega^\dagger U'^\dagger$$

The question is *which* unitary matrix Ω.

And what symmetries do we expect, if any?

In the hunt for the right mass matrices, the lodestar is the family hierarchy, with two lighter particles in the first and second family, and a much heavier particle in the third family.

This hierarchy is present in all the charged sectors, with fermions in different families exhibiting very different mass values, ranging from the electron mass to the about 10^5

times larger top mass.

Still an open question whether the neutrino masses also follow this pattern.

Democracy

In the democratic approach the family hierarchy is taken very seriously.

Assumption: in the weak basis the fermion mass matrices have a form close to the $S(3)_L \times S(3)_R$ symmetric "democratic" matrix

$$N = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

with the eigenvalues $(0, 0, 3k)$, reflecting the steep family hierarchy.

Underlying philosophy: in the Standard Model, where the fermions get their masses from the Yukawa couplings by the Higgs mechanism, there is no reason why there should be a different Yukawa coupling for each fermion.

The couplings to the gauge bosons of the strong, weak and electromagnetic interactions are identical for all the fermions in a given charge sector.

Natural that they should also have identical Yukawa couplings.

The difference is that the weak interactions take place in a specific flavour space basis, while the other interactions are flavour independent.

With three families, the (zeroth order) democratic quark mass matrices in the weak interaction basis are:

$$M^{(0)} = k_u \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad M'^{(0)} = k_d \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

where k_u and k_d have dimension mass.

The corresponding mass spectra $(m_1, m_2, m_3) \sim (0, 0, 3k_j)$ reflect the family hierarchy.

The lepton sector

The democratic scheme applies to the lepton sector as well, postulating democratic (zeroth order) mass matrices for the charged leptons and the neutrinos, whether they are Fermi-Dirac or Majorana:

$$M_l^{(0)} = k_l \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad M_\nu^{(0)} = k_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Relative to the quark ratio $k_u/k_d \sim m_t/m_b \sim 40 - 60$, the leptonic ratio $k_\nu/k_l < 10^{-8}$ is so extremely small that it seems "unnatural".

One way out: assume that k_ν vanishes $\Rightarrow$ the neutrinos get no mass contribution in the democratic limit.

Then no reason for a hierarchical pattern à la the one observed in the charged sectors; the neutrino masses could even be of the same order of magnitude.

Data are indeed compatible with a much weaker hierarchical structure for the neutrino masses than the hierarchy displayed by the charged fermion masses.

Supposing that the neutrino masses do not emerge from a democratic scheme, a (relatively) flat neutrino mass spectrum could be taken as a support for the idea that the masses in the charged sectors emerge from a democratic scheme.

The democratic basis

In the case that both the up- and down-sector mass matrices have a purely democratic texture, they are diagonalized by the unitary matrix

$$U_{dem} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

We use this to define the democratic basis, meaning the flavour space basis where the mass matrices are diagonalized by U_{dem} and the mass Lagrangian is symmetric under permutations of the fermion fields $(\varphi_1, \varphi_2, \varphi_3)$ of a given charge sector.

In the democratic basis the mass Lagrangian

$$\mathcal{L}_m = \bar{\varphi} M_{(dem)} \varphi = k \sum_{j,k=1}^{3} \bar{\varphi}_j \varphi_k$$

is symmetric under permutations of the fermion fields $(\varphi_1, \varphi_2, \varphi_3)$, while in the mass basis with

$$M_{(mass)} = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}$$

the mass Lagrangian has the form

$$\mathcal{L}_m = \lambda_1 \bar{\psi}_1 \psi_1 + \lambda_2 \bar{\psi}_2 \psi_2 + \lambda_3 \bar{\psi}_3 \psi_3$$

which is clearly not invariant under permutations of (ψ_1, ψ_3, ψ_3).

Breaking the democratic symmetry

In order to obtain non-degenerate, non-vanishing masses for the physical flavours, we need to break the permutation symmetry in the democratic basis.

The hope is to perform a symmetry breaking in such a way that the matrices retain an almost democratic form.

We can perform a shift of the democratic matrix, by just adding a unit matrix $diag(a, a, a)$, $M_0 \Rightarrow M_1$,

$$M_1 = k\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} a & & \\ & a & \\ & & a \end{pmatrix} = \left(\begin{matrix} k+a & k & \\ k & k+a & \\ k & k & k+ \end{matrix}\right.$$

corresponding to the mass spectrum $(a, a, a + 3k)$. It is clear that the matrix M_1 has a democratic texture, both because it is diagonalized by U_{dem}, and because the mass Lagrangian

$$\mathcal{L}_{M_1} = (k + a)\sum \bar{\varphi}_j \varphi_j + k \sum_{j \neq k} \bar{\varphi}_j \varphi_j$$

is invariant under permutations of the quark fields.

If M_1 and M_1' both have this kind of texture, there is no CP-violation. This is independent of how many families there are, because of the degeneracy of the mass values.
CP-violation only occurs once there are three or more non-degenerate families, because only then the phases can no longer be defined away.

To break the democratic symmetry, the standard scheme is to add a breaking term to the democratic matrix.

Breaking democracy by "perturbing" the mixing matrix

We however break the symmetry not by explicitely perturbing the zeroth order democratic mass matrix, but by embedding the assumption of democratic symmetry into the Standard Model mixing matrix, by expressing the mixing matrix as

$$V_{new} = UU_{dem}U_{dem}^{\dagger}U'^{\dagger} = V.$$

V thus remains unchanged, but the rotation matrices fro the up- and down-sectors are modified.

The new U and U' constitute the "perturbations" on the democratic symmetry,

$$U_{new} = U_{old}U_{dem}, U'_{new} = U'_{old}U_{dem}.$$

In this way, the weak interaction basis remains close to the democratic basis.

Democratic mass matrices

The new rotation matrices are

$$U_{new} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{pmatrix} \iota$$

and

$$U'_{new} = \begin{pmatrix} c_\beta & -s_\beta & 0 \\ s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} c_\theta & 0 & -s_\theta \\ 0 & 1 & 0 \\ s_\theta & 0 & c_\theta \end{pmatrix} \iota$$

and the corresponding numerical mass matrices
$M = U^\dagger_{new} diag(m_u, m_c, m_t) U_{new}$ and
$M' = U'^\dagger_{new} diag(m_d, m_s, m_b) U'_{new}$, numerically;

$$M_{(dem)} = C_u \begin{pmatrix} 1 & 1.0044e^{-i\mu} & 1.0711e^{-i\rho} \\ 1.0044e^{i\mu} & 1.0083 & 1.076e^{-i\eta} \\ 1.0711^{i\rho} & 1.076e^{i\eta} & 1.1655 \end{pmatrix} MeV$$

where $\mu = \rho - \eta$; which can be written as

$$M = C_u \left[\begin{pmatrix} 1 & & \\ & k\, e^{i(\rho-\eta)} & \\ & & kp \quad e^{i\rho} \end{pmatrix} \mathbf{N} \begin{pmatrix} 1 & & \\ & k\, e^{-i(\rho-\eta)} & \\ & & kp \quad \end{pmatrix} \right.$$

where $\mathbf{N}$ is the democratic matrix, and the "small" matrix

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon & \varepsilon' e^{-i\eta} \\ 0 & \varepsilon' e^{i\eta} & \chi \end{pmatrix},$$

supplies the two lighter families with non-zero masses and breaks the democratic symmetry.

$C_u \approx m_t/3$, $k = 1.0043$, $p = 1.0665$.

$\varepsilon \sim \varepsilon' \sim 0.00012$, $\chi = 0.018 \approx \frac{1}{2}\,(m_t m_c)/C_u^2$.

We repeat the procedure for the down-sector, $U' = U'_{(symm)}D$, which gives

$$M'_{(dem)} = C_d \begin{pmatrix} 1.044 & 1.028e^{i\mu'} & e^{-i\rho'} \\ 1.028e^{-i\mu'} & 1.0238 & 1.0071e^{i\eta'} \\ e^{i\rho'} & 1.0071e^{-i\eta'} & 1.059 \end{pmatrix} MeV$$

where $C_d \approx m_b/3$ and $\mu' \sim \rho' \sim \eta' = 0.0039$ rad.

The relations between the matrix elements, $M_{(dem)ij}/M'_{(dem)ij} \sim m_t/m_b \simeq 60$

Conclusion

Instead of "guessing" the form of the mass matrices, we obtain mass matrices by factorizing the mixing matrix in a specific way.

With the most "symmetric" factorization based on the standard parametrization of the Cabbibo-Kobayashi-Maskawa matrix, mass matrices are obtained, which in both charge sectors display certain internal relations, i.e. structures which are not identical but similar.

By including the democratic rotation matrix in the parametrization of the weak mixing matrix, we moreover obtain mass matrices with specific democratic textures.

In this way we make contact between the democratic hypothesis and the experimentally derived parameters of the CKM mixing matrix, avoiding the introduction of additional concepts.