Hierarchy of fermion masses, mixing and FCNC from a gauged $SU(3)$ Family Symmetry

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There are several open questions, motivations, to look for physics ”Beyond the Standard Model” (BSM):

- Neutrino masses and mixing, oscillation
- Hierarchy of masses
- Dart Matter (DM), Dart Energy (DE), Matter-Antimatter Asymmetry.

Main goal of this BSM: To account for the hierarchy of fermion masses:

$$m_t \gg m_c \gg m_u \quad , \quad m_b \gg m_s \gg m_d \quad , \quad m_\tau \gg m_\mu \gg m_e,$$

and quark and lepton mixing matrices: V_{CKM} and U_{PMNS}.

Ordinary Fermions: $Q = T_{3L} + \frac{1}{2} Y$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$q^o_{jL} = \begin{pmatrix} u^o_{jL} \\ d^o_{jL} \end{pmatrix}$	3	2	$\frac{1}{3}$
u^o_{jR}	3	1	$\frac{4}{3}$
d^o_{jR}	3	1	-$\frac{2}{3}$
$l^o_{jL} = \begin{pmatrix} \nu^o_{jL} \\ e^o_{jL} \end{pmatrix}$	1	2	-1
e^o_{jR}	1	1	-2
ν^o_{jR}	1	1	0
$\phi_{SM} = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}$	1	2	-1
$\tilde{\phi}_{SM} = i\,\sigma_2\,\phi^*_{SM}$	1	2	+1

Table: SM fermion content and charges, $j = 1, 2, 3$ family index

Comments:

- Anomalies cancel for each family, $j = 1, 2, 3$.
- $\overline{q^o_{iL}}\,\phi_{SM}\,u^o_{jR}$, $i, j = 1, 2, 3$ is gauge invariant.
- So, the Yukawa couplings $Y_{i,j}\,\overline{q^o_{iL}}\,\phi_{SM}\,u^o_{jR}$, $i, j = 1, 2, 3$ are gauge invariant, with $Y_{i,j}$ completely arbitrary.

The global symmetry in limit of all quarks and leptons massless, including R-handed neutrinos:

$$SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{e_R}$$

$$\supset SU(3)_{q_L+u_R+d_R+l_L+e_R+\nu_R} \equiv SU(3)$$

$SU(3)$: Gauged Family Symmetry

Completely vector-like and universal. That is, couple equally to Left and Right Handed quarks and leptons

$G \equiv SU(3) \otimes G_{SM}$ "GAUGE GROUP"

$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: Standard Model Group

$$i\mathcal{L}_{int,SU(3)_F} =$$

$$\frac{g_H}{2}\left(\bar{f}^o_1\gamma_\mu f^o_1 - \bar{f}^o_2\gamma_\mu f^o_2\right) Z^\mu_1 + \frac{g_H}{2\sqrt{3}}\left(\bar{f}^o_1\gamma_\mu f^o_1 + \bar{f}^o_2\gamma_\mu f^o_2 - 2\bar{f}^o_3\gamma_\mu f^o_3\right) Z^\mu_2$$

$$+ \frac{g_H}{\sqrt{2}}\left(\bar{f}^o_1\gamma_\mu f^o_2 Y^+_1 + \bar{f}^o_1\gamma_\mu f^o_3 Y^+_2 + \bar{f}^o_2\gamma_\mu f^o_3 Y^+_3 + h.c.\right)$$

where g_H is the $SU(3)_F$ coupling constant, Z_1, Z_2 and $Y^{\pm}_i$, $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons.

$$g_H \begin{pmatrix} \bar{f}^o_1 & \bar{f}^o_2 & \bar{f}^o_3 \end{pmatrix} \gamma_\mu \begin{pmatrix} \frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} & \frac{Y^+_1}{\sqrt{2}} & \frac{Y^+_2}{\sqrt{2}} \\ \frac{Y^-_1}{\sqrt{2}} & -\frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} & \frac{Y^+_3}{\sqrt{2}} \\ \frac{Y^-_2}{\sqrt{2}} & \frac{Y^-_3}{\sqrt{2}} & -\frac{Z_2}{\sqrt{3}} \end{pmatrix}^\mu \begin{pmatrix} f^o_1 \\ f^o_2 \\ f^o_3 \end{pmatrix}$$

	$SU(3)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
ψ_q^o	3	3	2	$\frac{1}{3}$
ψ_{uR}^o	3	3	1	$\frac{4}{3}$
ψ_{dR}^o	3	3	1	$-\frac{2}{3}$
ψ_l^o	3	1	2	-1
ψ_{eR}^o	3	1	1	-2
$\psi_{\nu R}^o$	3	1	1	0
$U_{L,R}^o$	1	3	1	$\frac{4}{3}$
$D_{L,R}^o$	1	3	1	$-\frac{2}{3}$
$E_{L,R}^o$	1	1	1	-2
$N_{L,R}^o$	1	1	1	0

Table: Fermion content and charges

	$SU(3)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
η_2, η_3	3	1	1	0
Φ^u	3	1	2	-1
Φ^d	3	1	2	+1

Table: Scalars fields and charges

- η_2, η_3 are introduced to break spontaneously $SU(3)_F$.
- Φ^u, Φ^d are introduced to break spontaneously the $SU(2)_L \times U(1)_Y$.
- u-quarks and neutrinos coupled only to Φ^u
- d-quarks and charged leptons couple only to Φ^d

$\overline{\psi_q^o}\,\Phi^u\,\psi_{uR}^o$ **and** $\overline{\psi_q^o}\,\Phi^d\,\psi_{dR}^o$ **are forbidden by the** $SU(3)_F$ **family symmetry.**

$\bar{3}\times 3 = 1+8\ ,\ 3\times 3 = \bar{3}+6\ ,\ \bar{3}\times 3\times 3 = (1+8)\times 3 = 3+\bar{6}+3+15$

Yukawa couplings:

$$h_u\,\overline{\psi^o_q}\,\Phi^u\,U^o_R \;+\; h_{2u}\,\overline{\psi^o_{uR}}\,\eta_2\,U^o_L \;+\;\; +\,h_{3u}\,\overline{\psi^o_{uR}}\,\eta_3\,U^o_L \;+\; M_U\,\overline{U^o_L}\,U^o_R$$

$$h_d\,\overline{\psi^o_q}\,\Phi^d\,D^o_R \;+\; h_{2d}\,\overline{\psi^o_{dR}}\,\eta_2\,D^o_L \;+\;\; +\,h_{3d}\,\overline{\psi^o_{dR}}\,\eta_3\,D^o_L \;+\; M_D\,\overline{D^o_L}\,D^o_R$$

$$h_\nu\,\overline{\psi^o_l}\,\Phi^u\,N^o_R \;+\; h_{2\nu}\,\overline{\psi^o_{\nu R}}\,\eta_2\,N^o_L \;+\;\; +\,h_{3\nu}\,\overline{\psi^o_{\nu R}}\,\eta_3\,N^o_L \;+\; m_D\,\overline{N^o_L}\,N^o_R$$

$$h_e\,\overline{\psi^o_l}\,\Phi^d\,E^o_R \;+\; h_{2e}\,\overline{\psi^o_{eR}}\,\eta_2\,E^o_L \;+\;\; +\,h_{3e}\,\overline{\psi^o_{eR}}\,\eta_3\,E^o_L \;+\; M_E\,\overline{E^o_L}\,E^o_R$$

$$+h.c$$

Tree level Majorana Yukawa couplings:

$$h_{2R}\,\overline{\psi^o_{\nu R}}\,\eta_2\,(N^o_R)^c \;+\; h_{3R}\,\overline{\psi^o_{\nu R}}\,\eta_3\,(N^o_R)^c \;+\; m_R\,\overline{N^o_R}\,(N^o_R)^c$$

$$h_L\,\overline{\psi^o_l}\,\Phi^u\,(N^o_L)^c \;+\; m_L\,\overline{N^o_L}\,(N^o_L)^c \;+h.c$$

Scalar Potential

$$V(\eta_2, \eta_3, \Phi^u, \Phi^d) =$$

$$\mu_2^2\, \eta_2^\dagger\, \eta_2 + \mu_3^2\, \eta_3^\dagger\, \eta_3 + \mu_{23}^2\, (\eta_2^\dagger\, \eta_3 + h.c) + \mu_u^2\, \Phi^{u\dagger}\Phi^u + \mu_d^2\, \Phi^{d^\dagger}\Phi^d$$

$$\lambda_2\, (\eta_2^\dagger\, \eta_2)^2 + \lambda_3\, (\eta_3^\dagger\, \eta_3)^2 + \lambda_{23}\, (\eta_2^\dagger\, \eta_3)^\dagger\, (\eta_2^\dagger\, \eta_3)$$

$$\lambda_u\, (\Phi^{u\dagger}\, \Phi^u)^2 + \lambda_d\, (\Phi^{d^\dagger}\, \Phi^d)^2 + \lambda_{ud}\, (\Phi^{u\dagger}\, \Phi^u)^\dagger\, \Phi^{d^\dagger}\, \Phi^d$$

$$(\eta_2^\dagger\, \eta_2)\, \left[c_3\, \eta_3^\dagger\, \eta_3 + c_{23}\, (\eta_2^\dagger\, \eta_3 + h.c) + c_u\, \Phi^{u\dagger}\, \Phi^u + c_d\, \Phi^{d^\dagger}\, \Phi^d\right]$$

$$(\eta_3^\dagger\, \eta_3)\, \left[k_{23}\, (\eta_2^\dagger\, \eta_3 + h.c) + k_u\, \Phi^{u\dagger}\, \Phi^u + k_d\, \Phi^{d^\dagger}\, \Phi^d\right]$$

$$(\eta_2^\dagger\, \eta_3 + h.c)\, \left[p_u\, \Phi^{u\dagger}\, \Phi^u + p_d\, \Phi^{d^\dagger}\, \Phi^d\right]$$

	ψ^o_q	ψ^o_{uR}	ψ^o_{dR}	ψ^o_l	ψ^o_{eR}	$\psi^o_{\nu R}$	Φ^u	Φ^d	η_2, η_3	$U^o_{L,R}$	$D^o_{L,R}$	$E^o_{L,R}$	$N^o_{L,R}$
$U(1)_B$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	0	0
$U(1)_Y$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	-1	-2	0	-1	+1	0	$\frac{4}{3}$	$-\frac{2}{3}$	-2	0
$U(1)_\alpha$	1	0	0	1	0	0	1	1	0	0	0	0	0
$U(1)_\beta$	0	1	1	0	1	1	0	0	1	0	0	0	0

Table: Charges under the global symmetry $U(1)_B \times U(1)_Y \times U(1)_\alpha \times U(1)_\beta$

All the Yukawa couplings and the scalar potential are invariant under the global symmetry $U(1)_B \times U(1)_Y \times U(1)_\alpha \times U(1)_\beta$, where B is the baryon number, and Y is the hypercharge.

$$U(1)_\beta = U(1)_{PQ} \text{ ?}$$

Peceei-Quinn symmetry to address the strong CP problem.

We would like to be consistent with low energy Standard Model(SM) and simultaneously generate and account for the hierarchy of quark and lepton masses and mixing

$SU(3)_F \times G_{SM}$	$\rightarrow$	G_{SM}	$\rightarrow$	$SU(3)_C \times U(1)_Q$
SM fermions are massless		SM fermions remain massless		SM fermions become massive (PDG known values)

Previous Basic Assumptions of this BSM, all together, define the required scalars, V.E.V's, and additional fermions, vector-like fermions

Standard Model Singlet Scalars introduced to break $SU(3)_F$ symmetry:

$$\langle \eta_2 \rangle = \begin{pmatrix} 0 \\ \Lambda_2 \\ 0 \end{pmatrix} \quad , \quad \langle \eta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ \Lambda_3 \end{pmatrix}$$

$$SU(3)_F \times G_{SM} \quad \xrightarrow{\langle \eta_3 \rangle} \quad SU(2)_F \, ? \times G_{SM} \quad \xrightarrow{\langle \eta_2 \rangle} \quad SU(2)_g \times G_{SM}$$

FCNC ?

Λ_3: 5 very heavy boson masses ($\geq 100\ TeV's$)
Λ_2: 3 heavy boson masses (a few $TeV's$) , **"approximate $SU(2)_g$ global symmetry"**

To suppress properly FCNC like, for instance: $\mu \to e\gamma$ ($Br < 5.7 \times 10^{-13}$) , $\mu \to e\,e\,e$ ($Br < 1 \times 10^{-12}$) , $K^o - \bar{K}^o$, $D^o - \bar{D}^o$, it is relevant which gauge bosons are heavy, and which ones are very heavy

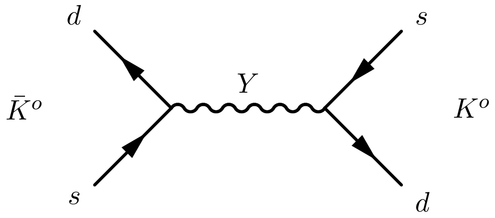

Figure: Generic tree level contribution to $K^o - \bar{K}^o$ from the $SU(3)$ horizontal gauge bosons.

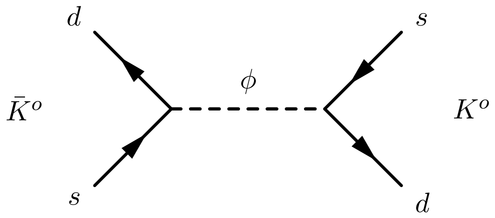

Figure: Generic tree level contribution to $K^o - \bar{K}^o$ from the scalar fields, $\phi = \eta_1 \, , \eta_2 \, , \Phi^u \, , \Phi^d \, ,$

The above scalar fields and VEV's break completely the $SU(3)_F$ family symmetry, generating the mass terms

- $\langle \eta_2 \rangle : \quad \frac{g_{H_2}^2 \Lambda_2^2}{2}(Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2 Z_1 \frac{Z_2}{\sqrt{3}})$
- $\langle \eta_3 \rangle : \quad \frac{g_{H_3}^2 \Lambda_3^2}{2}(Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$

In this scenario we introduce two triplets of $SU(2)_L$ Higgs doublets;
$\Phi^u = (3, 1, 2, -1), \Phi^d = (3, 1, 2, +1)$:

$$\Phi^u = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_1^u \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_2^u \\ \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_3^u \end{pmatrix} \& \langle \Phi^u \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u1} \\ 0 \end{pmatrix} \\ \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u2} \\ 0 \end{pmatrix} \\ \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u3} \\ 0 \end{pmatrix} \end{pmatrix} \ , \ \Phi^d = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_1^d \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_2^d \\ \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_3^d \end{pmatrix} \& \langle \Phi^d \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d1} \end{pmatrix} \\ \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d2} \end{pmatrix} \\ \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d3} \end{pmatrix} \end{pmatrix}$$

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \quad \xrightarrow{\langle \Phi^u \rangle \ , \ \langle \Phi^d \rangle} \quad SU(3)_C \times U(1)_Q$$

Contribute to the W and Z boson masses:

$$\frac{g^2}{4}\,(v_u^2+v_d^2)\,W^+W^- + \frac{(g^2+{g'}^2)}{8}\,(v_u^2+v_d^2)\,Z_o^2$$

$$+\frac{1}{4}\sqrt{g^2+{g'}^2}\,g_H\,Z_o\left[(v_{1u}^2-v_{2u}^2-v_{1d}^2+v_{2d}^2)\,Z_1+(v_{1u}^2+v_{2u}^2-2v_{3u}^2-v_{1d}^2-v_{2d}^2+2v_{3d}^2)\,\frac{Z_2}{\sqrt{3}}\right.$$

$$\left.+2\,(v_{1u}v_{2u}-v_{1d}v_{2d})\,\frac{Y_1^+ + Y_1^-}{\sqrt{2}}+2\,(v_{1u}v_{3u}-v_{1d}v_{3d})\,\frac{Y_2^+ + Y_2^-}{\sqrt{2}}+2\,(v_{2u}v_{3u}-v_{2d}v_{3d})\,\frac{Y_3^+ + Y_3^-}{\sqrt{2}}\right]$$

$+$ **tiny contributions to the $SU(3)_F$ gauge boson masses**

$v_u^2 = v_{u1}^2 + v_{u2}^2 + v_{u3}^2$, $v_d^2 = v_{d1}^2 + v_{d2}^2 + v_{d3}^2$. Hence, if we define $M_W = \frac{1}{2} g\, v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

Tree level Fermion Masses

$\bar{\psi}^o_{SM,L}\, \Phi^{u,d}\, \psi^o_{SM,R}$ **are not** $SU(3)_F$ **invariant**

Allowed Tree Level Yukawa couplings:

$$h_e\, \bar{\psi}^o_l\, \Phi^d\, E^o_R \;+\; h_{2e}\, \bar{\psi}^o_e\, \eta_2\, E^o_L \;+\; h_{3e}\, \bar{\psi}^o_e\, \eta_3\, E^o_L \;+\; M_E\, \bar{E}^o_L\, E^o_R \;+ h.c$$

DIRAC SEE-SAW MECHANISMS

In the gauge basis ${\psi^o_{L,R}}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms read as $\bar{\psi}^o_L \mathcal{M}^o \psi^o_R + h.c$, where

	e^o_R	μ^o_R	τ^o_R	E^o_R
$\overline{e^o_L}$	0	0	0	$h_e\, v_{d1}$
$\overline{\mu^o_L}$	0	0	0	$h_e\, v_{d2}$
$\overline{\tau^o_L}$	0	0	0	$h_e\, v_{d3}$
$\overline{E^o_L}$	0	$h_{2e}\, \Lambda_2$	$h_{3e}\, \Lambda_3$	M_E

Table: Tree level Dirac mass matrix $\mathcal{M}^o$ for u and d quarks, charged leptons and Dirac neutrinos

$$V_L^{o\,T}\mathcal{M}^o\mathcal{M}^{o\,T}\ V_L^o = V_R^{o\,T}\mathcal{M}^{o\,T}\mathcal{M}^o\ V_R^o = Diag(0,0,\lambda_3^2,\lambda_4^2)$$

$$V_L^{o\,T}\mathcal{M}^o\ V_R^o = Diag(0,0,-\lambda_3,\lambda_4)$$

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R}$$

Therefore, in this scenario light fermion masses, including neutrinos, may get extremely small masses from radiative corrections mediated by the $SU(3)_F$ heavy gauge bosons.

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

M_Y is the gauge boson mass, c_Y is coupling constant, $m_3^o = -\lambda_3$, $m_4^o = \lambda_4$, and $f(x,y) = \frac{x^2}{x^2-y^2} \ln \frac{x^2}{y^2}$.

$$\sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i \, b_j \, M}{\lambda_4^2 - \lambda_3^2} \, F(M_Y) \quad i,j = 1,2,3,$$

$$F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2 - \lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$$

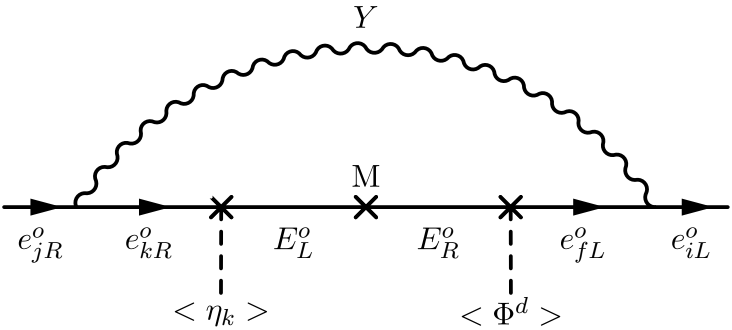

Figure: Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

	e_R^o	μ_R^o	τ_R^o	E_R^o
$\overline{e_L^o}$	D_{11}	D_{12}	D_{13}	0
$\overline{\mu_L^o}$	0	D_{22}	D_{23}	0
$\overline{\tau_L^o}$	0	D_{32}	D_{33}	0
$\overline{E_L^o}$	0	0	0	0

Table: One loop mass matrix $\mathcal{M}_1^o$

$$\bar{\psi}_L^o \mathcal{M}^o \, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \, \psi_R^o = \bar{\chi}_L \, \mathcal{M} \, \chi_R$$

Generic mass matrix up to one loop for quarks and charged leptons.

$$\mathcal{M} = Diag(0, 0, -\lambda_3, \lambda_4) + {V_L^o}^T \mathcal{M}_1^o \, V_R^o$$

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta \, m_{13} & s_\beta \, m_{13} \\ m_{21} & m_{22} & c_\beta \, m_{23} & s_\beta \, m_{23} \\ c_\alpha \, m_{31} & c_\alpha \, m_{32} & -\lambda_3 + c_\alpha c_\beta \, m_{33} & c_\alpha s_\beta \, m_{33} \\ s_\alpha \, m_{31} & s_\alpha \, m_{32} & s_\alpha c_\beta \, m_{33} & \lambda_4 + s_\alpha s_\beta \, m_{33} \end{pmatrix}$$

The diagonalization of $\mathcal{M}$ yields the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\, \Psi_{L,R}$; $\bar{\chi}_L\, \mathcal{M}\, \chi_R = \bar{\Psi}_L\, {V^{(1)}_L}^T \mathcal{M}\, V^{(1)}_R\, \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$ {V^{(1)}_L}^T \mathcal{M}\, \mathcal{M}^T\, V^{(1)}_L = {V^{(1)}_R}^T \mathcal{M}^T\, \mathcal{M}\, V^{(1)}_R = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\,, $$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$ \psi_L^o = V_L^o\, V^{(1)}_L\, \Psi_L \qquad \text{and} \qquad \psi_R^o = V_R^o\, V^{(1)}_R\, \Psi_R $$

and for neutrinos $\Psi_\nu^o = U_\nu^o\, U_\nu\, \Psi_\nu$.

Quark $(V_{CKM})_{4\times4}$ and Lepton $(U_{PMNS})_{4\times8}$ mixing matrices

Vector like quarks are $SU(2)_L$ weak singlets, and hence, the interaction of L-handed up and down quarks; ${f^o_{uL}}^T = (u^o, c^o, t^o)_L$ and ${f^o_{dL}}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}}\,\bar{f^o}_{uL}\gamma_\mu f^o_{dL} W^{+\mu} = \frac{g}{\sqrt{2}}\,\bar{\Psi}_{uL}\,[(V^o_{uL}\,V^{(1)}_{uL})_{3\times4}]^T\,(V^o_{dL}\,V^{(1)}_{dL})_{3\times4}\,\gamma_\mu\Psi_{dL}\,W^{+\mu},$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V^o_{uL}\,V^{(1)}_{uL})_{3\times4}]^T\,(V^o_{dL}\,V^{(1)}_{dL})_{3\times4}$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4\times8} = [(V^o_{eL}\,V^{(1)}_{eL})_{3\times4}]^T\,(U^o_\nu\,U_\nu)_{3\times8}$$

NEUTRINO MASSES
Tree level Dirac Neutrino masses

$$h_D\,\overline{\Psi^o_l}\,\Phi^u\,N^o_R + h_{\nu 2}\,\overline{\Psi^o_{\nu_R}}\,\eta_2\,N^o_L + h_{\nu 3}\,\overline{\Psi^o_{\nu_R}}\,\eta_3\,N^o_L + m_D\,\overline{N^o_L}\,N^o_R + h.c$$

h_D, h_1, h_2, and h_3 are Yukawa couplings, and m_D a Dirac type invariant neutrino mass for the sterile neutrino $N^o_{L,R}$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi^{o\,T}_{\nu L,R} = (\nu^o_e, \nu^o_\mu, \nu^o_\tau, N^o)_{L,R}$, the mass terms

$$h_D\,\left[v_{u1}\,\bar{\nu}^o_{eL} + v_{u2}\,\bar{\nu}^o_{\mu L} + v_{u3}\,\bar{\nu}^o_{\tau L}\right]\,N^o_R + \left[h_{\nu 2}\,\Lambda_2\,\bar{\nu}^o_{\mu R} + h_{\nu 3}\,\Lambda_3\,\bar{\nu}^o_{\tau R}\right]\,N^o_L$$
$$+ m_D\,\bar{N}^o_L\,N^o_R + h.c.$$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_D\, v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_D\, v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_D\, v_{u3}$
$\overline{N^o_L}$	0	$h_{\nu 2}\, \Lambda_2$	$h_{\nu 3}\, \Lambda_3$	m_D

Table: Tree level Dirac mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, \nu^o_{jR}$

Tree level Majorana masses

Since $N^o_{L,R}$ are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L\,\bar{\Psi}^o_l\,\Phi^u(N^o_L)^c \quad + \quad m_L\,\bar{N}^o_L\,(N^o_L)^c + h.c$$

and

$$h_{2R}\,\bar{\Psi}^o_\nu\,\eta_2\,(N^o_R)^c \;+\; h_{3R}\,\bar{\Psi}^o_\nu\,\eta_3\,(N^o_R)^c \;+\; m_R\,\bar{N}^o_R\,(N^o_R)^c + h.c.$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L\,\left[v_{u1}\,\bar{\nu}^o_{eL} + v_{u2}\,\bar{\nu}^o_{\mu L} + v_{u3}\,\bar{\nu}^o_{\tau L}\right]\,(N^o_L)^c \quad + \quad m_L\,\bar{N}^o_L\,(N^o_L)^c$$

$$+\left[h_{2R}\,\Lambda_2\,\bar{\nu}^o_{\mu R} + h_{3R}\,\Lambda_3\,\bar{\nu}^o_{\tau R}\right]\,(N^o_R)^c \;+\; m_R\,\bar{N}^o_R\,(N^o_R)^c + h.c.$$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	0	0	0	$h_L v_{u1}$
$\nu^o_{\mu L}$	0	0	0	$h_L v_{u2}$
$\nu^o_{\tau L}$	0	0	0	$h_L v_{u3}$
N^o_L	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L

Table: Tree level L-handed Majorana mass terms $m_{ij}\,\bar{\nu}^o_{iL}\,(\nu^o_{jL})^c$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	0	0	$h_{2R}\Lambda_2$
$\nu^o_{\tau R}$	0	0	0	$h_{3R}\Lambda_3$
N^o_R	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	m_R

Table: Tree level R-handed Majorana mass terms $m_{ij}\,\bar{\nu}^o_{iR}\,(\nu^o_{jR})^T$

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_L v_{u1}$	0	0	0	$h_D v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_L v_{u2}$	0	0	0	$h_D v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_L v_{u3}$	0	0	0	$h_D v_{u3}$
$\overline{N^o_L}$	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L	0	$h_2\Lambda_2$	$h_3\Lambda_3$	M_D
$\overline{(\nu^o_{eR})^c}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	$h_2\Lambda_2$	0	0	0	$h_{2R}\Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	$h_3\Lambda_3$	0	0	0	$h_{3R}\Lambda_3$
$\overline{(N^o_R)^c}$	$h_D v_{u1}$	$h_D v_{u2}$	$h_D v_{u3}$	M_D	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	m_R

Table: Tree Level Majorana masses

Numerical results I

To illustrate the spectrum of masses and mixing from this scenario, let us consider the following fit of space parameters at the M_Z scale

Using the input values for the $SU(3)_F$ family symmetry:

$$M_2 = 2\,\text{TeV} \quad , \quad M_3 = 2000\,\text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.2$$

with M_2, M_3 the horizontal boson masses, and the coupling constant, respectively,

$$M_2^2 = \frac{g_H^2 \Lambda_2^2}{2} \qquad , \qquad M_3^2 = \frac{g_H^2 \Lambda_3^2}{2}$$

$$g_H = 2.809 \quad , \Lambda_2 = 1.00658\; TeV \quad , \Lambda_3 = 2013.16\; TeV$$

and the tree level mixing angles

$$s_{1d} = s_{1e} = 0.6 \qquad s_{2d} = s_{2e} = 0.1047$$
$$s_{1u} = s_{1\nu} = 0.575341 \qquad s_{2u} = s_{2\nu} = 0.0925127$$

Quark masses and $(V_{CKM})_{4\times 4}$ mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 108921. \\ 0 & 0 & 0 & 17589.5 \\ 0 & 0 & 0 & 154844. \\ 0 & -6.42288\times 10^6 & 462459. & 2.5111\times 10^6 \end{pmatrix} \text{MeV},$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} 7.19764 & -626.533 & -1479.88 & -3792.15 \\ -0.468392 & -81.7707 & -197.807 & -506.875 \\ 5.04103 & 1502.25 & -172425. & 12057.2 \\ 0.0504129 & 15.0233 & 47.0554 & 6.91226\times 10^6 \end{pmatrix} \text{MeV}$$

and the u-quark masses

$$(\, m_u \,,\, m_c \,,\, m_t \,,\, M_U \,) = (\, 1.396 \,,\, 644.835 \,,\, 172438 \,,\, 6.912\times 10^6 \,)\ \text{MeV}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 2860.87 \\ 0 & 0 & 0 & 501.98 \\ 0 & 0 & 0 & 3814.49 \\ 0 & -2.3645\times 10^6 & 323661. & 2.17117\times 10^6 \end{pmatrix} \text{ MeV}$$

$$\mathcal{M}_d = \begin{pmatrix} -4.22954 & 3.26664 & 26.4239 & 29.045 \\ 19.9418 & -41.6 & -57.7027 & -63.4265 \\ -2.27726 & -31.0285 & -2859.26 & 755.343 \\ -0.002277 & -0.031028 & 0.687179 & 3.2264\times 10^6 \end{pmatrix} \text{ MeV}$$

$$(\, m_d \, , \, m_s \, , \, m_b \, , \, M_D \,) = (\, 2.501 \, , \, 45.803 \, , \, 2860.14 \, , \, 3.226{\times}10^6 \,) \text{ MeV}$$

and the quark mixing

$$(V_{CKM})_{4\times 4} = \begin{pmatrix} -0.97445 & 0.224576 & -0.003514 & -0.000021 \\ -0.224523 & -0.973562 & 0.042015 & -0.000010 \\ 0.006011 & 0.041720 & 0.999041 & -0.001233 \\ -0.000219 & -0.0011268 & -0.011702 & 0.000014 \end{pmatrix}$$

Lepton masses and $(U_{PMNS})_{4\times 4}$ mixing:

Charged leptons:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 129165. \\ 0 & 0 & 0 & 22663.9 \\ 0 & 0 & 0 & 172220. \\ 0 & -337398. & 32029.6 & 2.16401 \times 10^7 \end{pmatrix} \text{ MeV}$$

$$\mathcal{M}_e = \begin{pmatrix} 2.2376 & -66.4545 & -394.792 & -6.18239 \\ -0.175708 & -9.01417 & -57.8818 & -0.906422 \\ 1.66889 & 164.714 & -1693.76 & 26.556 \\ 0.016689 & 1.64723 & 16.9589 & 2.16438 \times 10^7 \end{pmatrix} \text{ MeV}$$

fit the charged lepton masses:

$$(m_e\,,\ m_\mu\,,\ m_\tau\,,\ M_E) = (0.486\,,\ 102.702\,,\ 1746.17\,,\ 2.164\times 10^7\,) \text{ MeV}$$

Neglecting for simplicitly the Majorana mass terms, we report the following Dirac neutrino masses and lepton mixing:
Dirac neutrino masses:

$$\mathcal{M}_\nu^o = \begin{pmatrix} 0 & 0 & 0 & 0.076760 \\ 0 & 0 & 0 & 0.012395 \\ 0 & 0 & 0 & 0.109124 \\ 0 & -0.108392 & -0.264395 & 0.854133 \end{pmatrix} \text{ eV}$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0.015703 & -0.004190 & 0.009713 & 0.003179 \\ -0.001021 & -0.01824 & -0.005614 & -0.001837 \\ 0.010890 & 0.004245 & -0.048705 & -0.002164 \\ 0.001539 & 0.000600 & -0.000934 & 0.909297 \end{pmatrix} \text{ eV}$$

fit the light neutrino masses:

$$(m_1\,,\,m_2\,,\,m_3\,,\,m_4) = (\,0.017127\,,\,0.0192\,,\,0.050703\,,\,0.909309\,)\,\text{eV}$$

the squared mass differences:

$$(m_2^2 - m_1^2 = 7.5\times10^{-5},\ m_3^2 - m_2^2 = 2.2\times10^{-3},\ m_4^2 - m_1^2 = 0.826)\,\mathrm{eV}^2$$

and the lepton mixing

$$(U_{PMNS})_{4\times4} = \begin{pmatrix} 0.610887 & -0.786302 & -0.092369 & 0.003816 \\ -0.709911 & -0.595411 & 0.374805 & 0.032066 \\ -0.349473 & -0.164968 & -0.912482 & -0.133926 \\ 0.001821 & 0.000257 & 0.009733 & 0.001377 \end{pmatrix}$$

PREDICTIONS:

- NEW MASSIVE GAUGE BOSONS AND SCALARS, (few-100) TeV
- VECTOR LIKE QUARKS AND CHARGED LEPTONS, M_U, M_D, M_E $\approx$(1 - 100)TeV
- STERILE NEUTRINOS:
  Light (eV), May be two
  Heavy (KeV-GeV),