Deriving physics from a very general theory with diffeomorphism symmetry

Holger Bech Nielsen and Astri Kleppe

(within the Random Dynamics framework).

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Plan

We want to derive "all of physics" from a very general theory with diffeomorphism symmetry (as gauge symmetry).

In constructing a desireable, non-superlocal theory, we assume that we have:

- An action S that is analytic or at least smooth.
- An action that obeys diffeomorphism symmetry and certain analyticity requirements is effectively local, but in order to avoid superlocality it is necessary that the theory contains a field transforming like the metric tensor with upper indices, $g^{\mu\nu}$. The resulting theory will contain gravity, and will as a whole be like a usual quantum field theory.

The Lagrangian for a superlocal theory is of the form

$$\mathcal{L} = \varphi(x)^2 + \ldots$$

without any terms with ∂_μ, ∂_μ, ... the equation of motion $\frac{\partial \mathcal{L}}{\partial \varphi} = 0$ has no terms of the form

$$\partial_\nu \frac{\partial \mathcal{L}}{\partial(\partial_\nu \varphi)}$$

thus there is no connection between the different points on the manifold. In order to get diffeomorphism invariance but no superlocality, we want a Lagrangian that contains terms with $\partial_\mu \varphi$. We therefore need terms with $g^{\mu\nu}$ or some field that transforms like $g^{\mu\nu}$, like A^μ, which gives a Lagrangian of the form

$$\mathcal{L} = A^\mu \partial_\mu \varphi + V(\varphi(x)),$$

so $\partial_\mu A^\mu + V'(\varphi(x)) = 0$.

So we assume

- A smooth/analytic action.
- Diffeomorphism symmetry.
- Minimizing of energy.
- In order to avoid superlocality, a contravariant metric $g^{\mu\nu}$, or some field that transforms like $g^{\mu\nu}$.

Diffeomorphisms

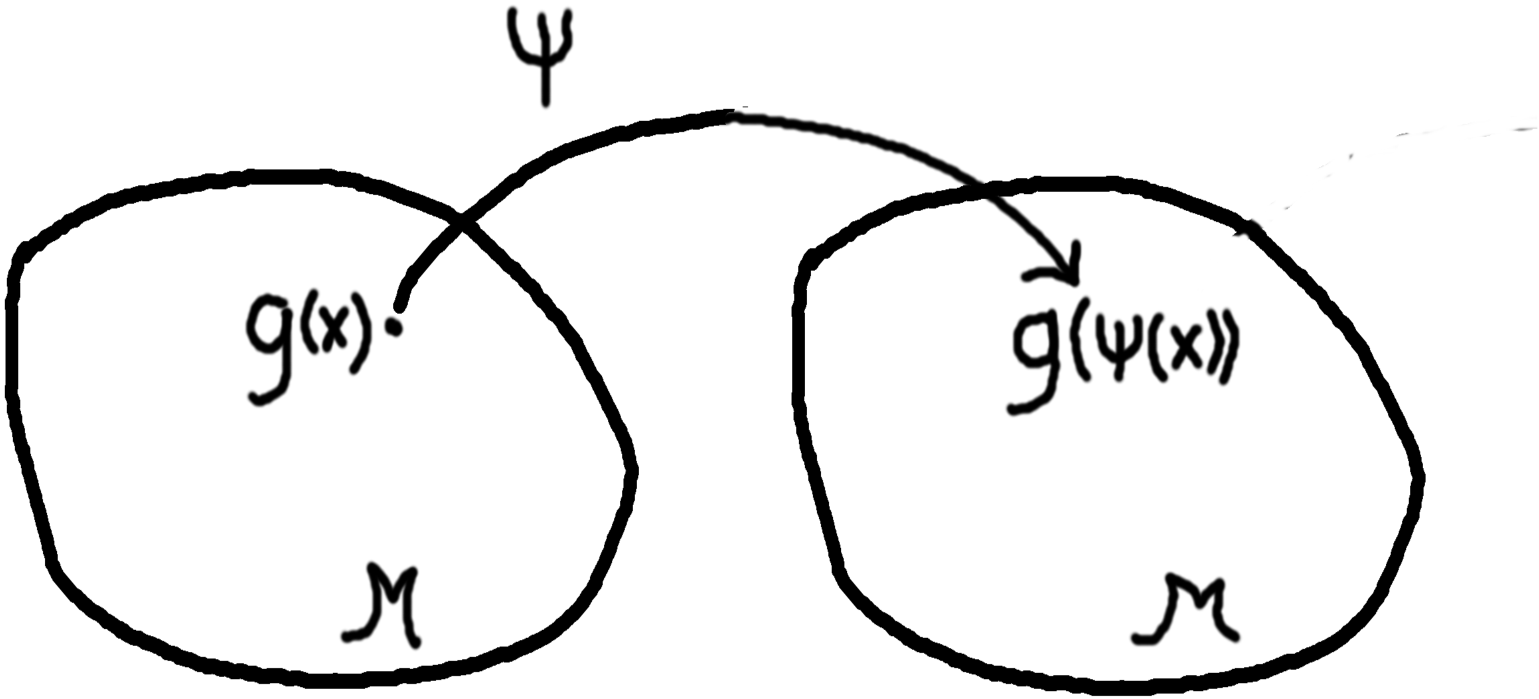
A diffeomorphism is a mapping ψ on a manifold $\mathcal{M}$ which preserves the stucture of $\mathcal{M}$, and the group of diffeomorphisms on $\mathcal{M}$ are the set of such mappings $\{\psi : \mathcal{M} \to \mathcal{M}\}$. Diffeomorphisms are often perceived as synonymous with reparametrizations, which in local coordinates x^μ are analytic or at least smooth maps,

$$\psi : x^\mu \to \psi^\mu(x)$$

When we have diffeomorphism invariance, we ought to hav a rule for how the the fields of the theory transform under diffeomorphism, e.g. for the metric,

$$\tilde{g}_{\mu\nu}(x) = \frac{\partial \psi^\eta}{\partial x^\mu} \frac{\partial \psi^\rho}{\partial x^\nu} \, g_{\eta\rho}(\psi(x))$$

where $\psi^\eta = \eta$th coordinate of $\left(\psi^1, \psi^2, \ldots \psi^\eta . . \right)$.



As straightforward as it seems, this relation can be interpreted in different ways, namely as a $passive$ diffeomorphism, or an $active$ diffeomorphism.

- Passive diffeomorphisms describe invariance under change of coordinates. From this perspective, $\tilde{g}_{\mu\nu}(\psi(x))$ and $g_{\mu\nu}(x)$ are the same metric on $\mathcal{M}$, written in different ways. This corresponds to a change of coordinate system in $\mathcal{M}$, but if a diffeomorphism transformation is nothing but a rewriting of the formalism, in itself, diffeomorphism is empty.

- Active diffeomorphisms, on the other hand, relate different entities in $\mathcal{M}$ within the same coordinate system - ψ thus relates different points in $\mathcal{M}$. From this perspective, $\tilde{g}_{\mu\nu}$ and $g_{\mu\nu}$ are not the same, but different physical entities related by a diffeomorphism transformation.

In the active interpretation the two metrics are not the same.

Therefore a distance expressed in terms of $\tilde{g}_{\mu\nu}$ is not the same as a distance expressed in terms of $g_{\mu\nu}$. If P_1 and P_2 are two points on $\mathcal{M}$, the distance $dist_g(P_1, P_2)$ measured in terms of $g_{\mu\nu}$ is not the same as the distance $dist_{\tilde{g}}(P_1, P_2)$ measured in terms of $\tilde{g}_{\mu\nu}$, but the two distances are related by the diffeomorphism symmetry. So while

$$dist_{\tilde{g}}(P_1, P_2) \neq dist_g(P_1, P_2),$$

we have

$$dist_{\tilde{g}}(P_1, P_2) = dist_g\left(\psi^{-1}(P_1), \psi^{-1}(P_2)\right)$$

where ψ is a diffeomorphism on $\mathcal{M}$.

It seems that the idea of passive diffeomorphisms is not very interesting, since it is nothing but a reformulation of the theory.

Active diffeomorphisms which map the points of a manifold onto other points of that manifold seem more interesting. In this case a metric is mapped on a different metric.

In reality, the difference between passive and active diffeomorphisms is only about our interpretation of the diffeomorphism, and in any case we have an action

$$S(\tilde{g}) = S(g)$$

which is the same in both the passive and the active case.

We should have a contravariant field $g^{\mu\nu}$ that transforms as

$$\tilde{g}_{\mu\nu}(x) = \frac{\partial x^\mu}{\partial \psi^\eta} \, \partial x^\nu \frac{\partial \psi^{\rho\eta\rho}}{g} \, (\psi(x))$$

where the partial derivatives

$$\frac{\partial x^\mu}{\partial \psi^\mu}$$

of course are the partial derivative of the inverse ψ^{-1}.

Diffeomorphism invariance is part of the concept of manifold. The diffeomorphism ψ on the manifold $\mathcal{M}$ is at least piecewise invertible, thus $\partial x^\mu / \partial \psi^\mu$ is well-defined. So if there is a $g^{\mu\nu}$ it obeys

$$\tilde{g}^{\mu\nu}(x) = \frac{\partial x^\mu}{\partial \psi^\eta} \, \frac{\partial x^\nu}{\partial \psi^\rho} \, g^{\eta\rho}(\psi(x))$$

Construction of a contravariant metric

We assume that we in our model have fields like $V_a^{\ \mu}$,

then we can construct

$$g^{\mu\nu} = V_a^{\ \mu} V_b^{\ \nu} \eta^{ab}$$

And we can construct a $g^{\mu\nu}$ from $g_{\mu\nu}$ provided it is invertible, which is not possible if it is degenerate.

Suppose we have $V_a^{\ \mu}$:s with four $a's$. Then we get

$$g^{\mu\nu} = V_a^{\ \mu} V_b^{\ \nu} \eta^{ab}$$

Because a and b take the values 1,..,4, this $g^{\mu\nu}$ is of rank 4 - meaning that the metric is "degenerate" w r t the dimension of the manifold,

dim $\mathcal{M}$=N > 4.

Claim: each $g^{\mu\nu}$ of rank 4 defines a D=4 layer within the manifold, so we get a slicing of our N-dimensional manifold into 4-dimensional sheets.

These metrics will be proven to constitute a metric over the whole manifold, a metric which has the covariant counterpart $g_{\mu\nu}$, such that they transform under diffeomorfisms on $\mathcal{M}$ as

$$\tilde{g}^{\mu\nu}(x) = \frac{\partial x^\mu}{\partial \psi^\eta} \frac{\partial x^\nu}{\partial \psi^\rho} g^{\eta\rho}(\psi(x))$$

and

$$\tilde{g}_{\mu\nu}(x) = \frac{\partial \psi^\eta}{\partial x^\mu} \frac{\partial \psi^\rho}{\partial x^\nu} g_{\eta\rho}(\psi(x))$$

In this sense we have ensured that our model is local but not superlocal.

It remains to break the diffeomorphism symmetry until Poincaré is the remaining subgroup.