The Eightfold Way Revisited: SU(8) GUT

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Abstract

It is now almost clear that there is no meaningful symmetry higher than the one family GUTs like as SU(5), SO(10), or E(6) for classification of all observed quarks and leptons. Any attempt to describe all three quark-families leads to higher symmetries with enormously extended representations which contain lots of other states, normal or exotical, that never been detected in the experiment. This may motivate us to seek solution in some subparticle or preon models for quark and leptons just like as in the nineteen-sixties the spectroscopy of hadrons required to seek solution in the quark model. By that time there was very popular some concept introduced by Murray Gell-Mann and called the Eightfold Way according to which all low-lying baryons and mesons are grouped into octets. However, there are also decuplets of the baryon resonances so that the Eightfold Way appeared not to be absolutely exact, though very beautiful. We find now that this concept looks much more adequate and successful when it is applied to elementary preons and composite quarks and leptons. Remarkably, just the eight preons and their generic SU(8) symmetry may determine the fundamental entities of the World and its total internal symmetry. We also show that some independent indication in favor of SU(8) GUT may follow from a solution to gauge hierarchy problem that provides decoupling of electroweak scale from a scale of grand unification.
PART 1

SU(8) from PREONS
I PREAMBLE

The Eightfold Way or Noble Eightfold Path is a summary of the path of Buddhist practices leading to liberation.

1. The eight spoke Dharma wheel symbolizes the Noble Eightfold Path

Eight Preon Superfields $\mathcal{P}_i (i=1...8)$ with a basic symmetry $SU(8)$

2. The Noble Eightfold Path is usually divided into three basic divisions

<table>
<thead>
<tr>
<th>Division</th>
<th>Eightfold Path factors</th>
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<tbody>
<tr>
<td>Insight (Sanskrit: prajñā)</td>
<td>1. Right view     2. Right resolve</td>
</tr>
<tr>
<td>Moral virtue (śīla)</td>
<td>3. Right speech  4. Right action</td>
</tr>
<tr>
<td>Meditation (samādhī)</td>
<td>5. Right livelihood  6. Right effort</td>
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<tr>
<td></td>
<td>7. Right mindfulness  8. Right concentration</td>
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Eight Preons $\mathcal{P}_i (i=1...8)$ are divided into three basic groups

Weakons $\mathcal{P}_w (w=1,2)$, Colorons $\mathcal{P}_c (c=1,2,3)$, Familons $\mathcal{P}_f (f=1,2,3)$

with a split symmetry $SU(8) \rightarrow SU(2)_w \times SU(3)_c \times SU(3)_f$
II  PREONS AND COMPOSITES

• Preons are proposed massless with unbroken chiral symmetry $SU(N)_L \otimes SU(N)_R$ depending on their number and metaflavors they carry out.

• Preons possess metaflavors which are known physical colors, weak isospin etc and metacolors which binding them into composites - quarks and leptons.

• Preons do not form condensate $< \bar{L}R >$ to provide masslessness of composites.

• Composites typically have a minimal 3-preon configuration.

• There only appear those massless composites which remain a chiral symmetry of preons that is controlled by 't Hooft's anomaly matching condition.
III ANOMALY MATCHING CONDITION

- 't Hooft's anomaly matching condition: triangle anomalies of metaflavor currents for elementary preons have to match those for massless composite quarks and leptons being composed by forces arranged by some metacolor symmetry $G_{MC}$

- This puts powerful constraints on the classification of massless composite fermions with respect to the chiral metaflavor group $K(N) = SU(N)_L \otimes SU(N)_R$ with anomalies given by $A = \text{Tr} \{ T^i T^j T^k \}$

$$\sum_q I_q A(q) = 3A(N)$$

according to 't Hooft. Here $A(N)$ and $A(q)$ are the group coefficients of the triangle anomalies with respect to one of the groups, $SU(N)_L$ or $SU(N)_R$, of the chiral symmetry of the preons, $K(N) = SU(N)_L \otimes SU(N)_R$ ($N$ is the number of preons); $A(N)$ corresponds to a fundamental representation of $N$ for preons, and $A(q)$ to a representation for massless composite fermions. The values of $|I_q|$ are the numbers of times the representation $q$ appears in the spectrum of the composite fermions; for $I_q > 0$, these values correspond to left-hand chiral states, and at $I_q < 0$ they correspond to right-hand chiral states.

- Too many solutions in general
IV STRIGHTENING AM CONDITION: \( N = 8 \)

1. Only some single representation of the massless composites, rather than their combination, has to satisfy the ’t Hooft condition

\[
A(q) = 3
\]

2. Quarks and leptons are all possible 3-preon massless composites with a radius of compositeness \( R_{MC} \sim 1/\Lambda_{MC} \)

3. Metacolor symmetry of preons is anomaly-free: \( SO(3)_{MC} \)

4. Left-Right symmetry: left-handed and right-handed preons

\[
\begin{align*}
L_{i\alpha} & \quad \text{with symmetry } SU(N)_{L} \otimes SO(3)_{MC,L} \\
R_{i\alpha} & \quad \text{with symmetry } SU(N)_{R} \otimes SO(3)_{MC,R}
\end{align*}
\]

5. L-preon and R-preon composites:

\[
\Psi^{i}_{[jk]LR} \left( A = \frac{N^2}{2} - \frac{7N}{2} - 1 \right), \quad \Psi_{[ijk]LR} \left( A = \frac{N^2}{2} - \frac{9N}{2} - 9 \right), \ldots \ldots
\]

6. Unique Eightfold Solution to AM conditions:

\[
\frac{N^2}{2} - \frac{7N}{2} - 1 = 3 \quad \Rightarrow \quad N = 8
\]
V Symmetries of preons and composites

1. Small distances (PREONS)

\{8_i^L, 8_i^R\} - global chiral symmetry $SU(8)_L \otimes SU(8)_R$
- local symmetry $SO(3)_{MC,L} \otimes SO(3)_{MC,R} \otimes SU(8)_{MF}$

2. Large distances (COMPOSITES)

\{216^i_{[jk]}L, 216^i_{[jk]}R\}
- global chiral symmetry $SU(8)_L \otimes SU(8)_R$
- local symmetry $SU(8)_{MF}$

Is it right multiplet?

$216 = (\bar{5}+10, \bar{3}) + (45, 1) + (5, 8) + (24, 3) + (1, 3) + (1, \bar{6})$

[ $SU(5) \otimes SU(3)$ decomposition
  shows $SU(5)$ GUT $\otimes$ Family Symmetry]

However, theory is still vectorlike
  until L-R symmetry is broken
3. Left-Right symmetry partial breaking

\[ SU(8)_L \otimes SU(8)_R \rightarrow SU(8)_L \otimes [SU(5)_R \otimes SU(3)_R] \]

a. Through the unique 3-preon superfield condensate

\[ \langle L_i L_j L_k \rangle = 0 \quad \langle R_i R_j R_k \rangle = f_{ijk} M^4 \]

b. Through the 3-index scalar field potential

\[ U = m^2 (\varphi^*_L \varphi_L + \varphi^*_R \varphi_R) + h(\varphi^*_L \varphi_L + \varphi^*_R \varphi_R)^2 \]
\[ + h'(\varphi^*_L \varphi_L)(\varphi^*_R \varphi_R) \quad (h, h' > 0, \quad m^2 < 0) \]

Natural minimum corresponds to a total L-R asymmetry

\[ \langle \varphi^{[ijk]}_L \rangle = 0 \quad , \quad \langle \varphi^{[ijk]}_R \rangle = f^{\alpha\beta\gamma} M \quad \{\alpha, \beta, \gamma \in SU(3)\} \]

c. Yukawa couplings through the superpotential

\[ L_i L_j L_k \varphi^L_L [ijk] + R_i R_j R_k \varphi^R_R [ijk] \]

change AM conditions for R-preons
d. Eventual chiral symmetry $SU(8)_L \otimes [SU(5)_R \otimes SU(3)_R]$

- **L-preon composites** $[SU(8)_L, N_{L8} = 1]$ –

  $216_L \to (\bar{5}+10, \bar{3})$

  $+ (45, 1) + (5, 8) + (24, 3) + (1, 3) + (1, \bar{6})$

- **R-preon composites** $[SU(5)_R \otimes SU(3)_R, N_{R5}=1, N_{R3}=1]$ –

  $216_R \to (45, 1) + (5, 8) + (1, 3)$

*L-preon composites fill total $216_L$- multiplet*

(with a common preon number $N_{L8} = 1$)

*R-preon composites fill only few $SU(5) \otimes SU(3)$- multiplets of $216_R$*

(with particular preon numbers $N_{R5}=1, N_{R3}=1$; composites with “mixed” preon numbers can not appear)
VI  Physical sector - quarks & leptons

1. After chiral symmetry breaking - metaflavor symmetry for composites is reduced to

\[ \text{SU}(8) \text{ GUT} \rightarrow \text{SU}(5) \text{ GUT} \otimes \text{SU}(3)_F \]

with matter multiplets

(due to pairing of identical L- and R-composites and decoupling from low-energy spectra)

\[ (\bar{5}+10, \bar{3}) + (24, 3) + (1, \bar{6}) \]

Just 3 families of quarks and leptons + massive (on family scale) multiplets

2. Tiny radius of compositeness for universal preons

- \( u + d \) quark pair contains same preons as \( \bar{u} + e^+ \) pair
- \( u + d \rightarrow \bar{u} + e^+ \) leads to proton decay \( p \rightarrow \pi^0 + e^+ \) due to simple rearrangement of preons

- \( R_{MC} \sim 1/\Lambda_{MC}, \quad \Lambda_{MC} \geq M_{\text{GUT}}, \quad R_{MC} \leq 10^{-30} \text{ cm} \)
3. Family or Horizontal symmetry $SU(3)_F$ – main bonus

- It provides a natural explanation of 3 observed quark-lepton families
- Its local nature conforms with other local symmetries of SM
- Its chiral nature leads to hierarchical pattern of quark and lepton masses and mixings as a result of spontaneous symmetry breaking
- It provides natural suppression of dangerous flavor-changing transitions
- It suggests a new type of topological defects - flavored cosmic strings / monopoles as possible candidates for a cold dark matter in the Universe


https://en.wikipedia.org/wiki/Family_symmetries
PART 2

SU(8) from SUSY / SUGRA
Invoking SUSY Scenario

- **Geometry:** \( x^\mu \rightarrow x^\mu + i\sigma^\mu \bar{\theta} \)

- **Matter:**

\[ \begin{array}{c}
\text{Existing particles} \\
\begin{array}{c}
\text{(u)} \\
\text{d_L} \\
\text{u_R} \\
\text{c_L} \\
\text{c_R} \\
\text{t_L} \\
\text{t_R} \\
\text{e_L} \\
\text{e_R} \\
\text{\mu_L} \\
\text{\mu_R} \\
\text{\tau_L} \\
\text{\tau_R} \\
\end{array} \\
\begin{array}{c}
\text{g} \\
\text{\gamma} \\
\text{Z^0} \\
\text{W^\pm} \\
\text{h^0} \\
\text{H^0} \\
\text{A^0} \\
\text{H^\pm} \\
\end{array} \\
\end{array} \]

\[ \begin{array}{c}
\text{SUSY particles (MSSM model)} \\
\begin{array}{c}
\text{(\bar{u})} \\
\text{\bar{d}_L} \\
\text{\bar{u}_R} \\
\text{(\bar{c})} \\
\text{(\bar{c})_R} \\
\text{(\bar{t}_L)} \\
\text{(\bar{t}_R)} \\
\text{(\bar{e})_L} \\
\text{(\bar{e})_R} \\
\text{(\bar{\mu})_L} \\
\text{(\bar{\mu})_R} \\
\text{(\bar{\tau})_L} \\
\text{(\bar{\tau})_R} \\
\end{array} \\
\begin{array}{c}
\text{\tilde{g}} \\
\text{\tilde{\gamma}} \\
\text{\tilde{Z}^0} \\
\text{\tilde{W}^\pm} \\
\text{\tilde{h}^0} \\
\text{\tilde{H}^0} \\
\text{\tilde{A}^0} \\
\end{array} \\
\end{array} \]

**Gauge hierarchies**
1 WHAT IS GAUGE HIERARCHY?

- Two-scale or many-scale physics stability
- SUSY GUTs suggest a desired stabilization of the Higgs masses against the radiative corrections
- $2/3$ problem - light EW doublet for SM and heavy triplet (not to mediate proton decay) from the same GUT multiplet

2 MISSING VEV CONJECTURE

- This might normally appear through a missing VEV conjecture for doublet-triplet (or $2/(N-2)$ in general) splitting according to which heavy adjoint scalar $\Phi^i_j (i, j = 1...N)$ of $SU(N)$ might not develop a VEV in the weak $SU(2)$ direction and through its direct coupling with fundamental chiral pairs $H_i$ and $\mathcal{H}$ (containing the ordinary Higgs doublets) could hierarchically split their masses in the desired $2/(N-2)$ way.

3 MOTIVATION

- True solution to a doublet-triplet splitting problem in SUSY GUTs might choose itself the total starting symmetry of GUT

4 MISSING VEV SOLUTION

- We have found a stable missing VEV solution which naturally stems from general reflection-invariant

\[ \Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow - \Phi_2, \quad H\mathcal{H} \rightarrow -H\mathcal{H} \]

- two-adjoint scalar superpotential

\[ W = \frac{M_1}{2} \Phi_1^2 + \frac{M_2}{2} \Phi_2^2 + \frac{h}{2} \Phi_1 \Phi_2^* + \frac{\lambda}{3} \Phi_1^3 + \delta \Phi_2 H\mathcal{H} \]

- generating preferably orthogonal VEVS ($Tr(\Phi_1 \Phi_2) = 0$)
\[
\Phi_1 = V_1 \text{diag}(m \quad N-m, \quad -m \quad N-m)
\]
\[
\Phi_2 = V_2 \text{diag}(m/2 \quad m/2 \quad N-m)
\]

5 \text{ \textbf{COLOR-FLAVOR INTERRELATION}}

- While an ”ordinary” adjoint \( \Phi_1 \) having a cubic term in \( W_\Phi \) develops in all cases only a ”standard” VEV which breaks the starting symmetry

\[
SU(N) \rightarrow SU(m) \otimes SU(N-m) \otimes U(I)
\]

the second adjoint \( \Phi_2 \) having direct coupling with fundamental chiral pairs \( H_i \) and \( \bar{H}^i \) possesses, among others, a new solution with \( SU(N) \) broken along the channel

\[
SU(N) \rightarrow SU(m/2) \otimes SU(m/2) \otimes SU(N-m) \otimes U(I)_1 \otimes U(I)_2
\]

requiring a strict equality of numbers of fundamental colors and flavors in \( SU(N) \) and for any even-order groups

6 \text{ \textbf{SUGRA CHOOSES } SU(8)}

- There is vacuum degeneracy for all vacuum configurations in \( SU(N) \) SUSY GUTs so that SUSY only admits the MISSING VEV SOLUTION with color-flavor number equality but leaves free the weak symmetry part in \( SU(N) \)

- Remarkably, a supergravity-induced lifting the vacuum degeneracy modifies the form of the effective potential at low energies

\[
U_{adj} \simeq -3k^2|W_{adj}|^2, \quad W_{adj} = \alpha N(a+1)(r+a)
\]

where

\[
\alpha = \frac{\lambda M_2^3}{3h^3}, \quad r = \frac{3h M_1}{2\lambda M_2}, \quad a = \frac{-N-2m}{N-m}
\]

that gives in a finite parameter area (depending on group parameters)

\[
-3(N-2)^{1/3} < r < 3(N-2)^{1/3}
\]
the predominant breaking channel for any even-order groups \( N = 2n + 2, \ n = 1, 2, ... \)

\[
SU(N) \rightarrow SU(n)_C \otimes SU(n)_F \otimes SU(2)_W \otimes U(1)_1 \otimes U(1)_2
\]

which definitely favors just ab initio three-color \( SU(8) \) case among the other \( SU(N) \) GUTs

\[
SU(8) \rightarrow SU(3)_C \otimes SU(3)_F \otimes SU(2)_W \otimes U(1)
\]

7 SUMMARY

- Missing VEV solutions in \( SU(N) \) SUSY/SUGRA theories appear only when the numbers of colors and families are happened to be equal and weak part has a generic dimension \( n_W = 2 \) that for the three-color even-order \( SU(N) \) groups unavoidably leads to

\[
SU(8) \text{ GUT}
\]

8 SOME PAPERS


Both – the observed quark-lepton spectroscopy and requirement of two-scale physics stability - single out SU(8) as a viable family unified Grand Unification Theory
1982 WINTER SCHOOL
ON PARTICLE PHYSICS
AND COSMOLOGY

(Leningrad NPI)