

**Spin-Charge-Family Theory explains all the
assumptions of the Standard Model, the
matter-antimatter asymmetry, the
appearance of the Dark Matter, the.. ..,
making several predictions**

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workshop (20th - anniversary) entitled "What Comes Beyond
the Standard Models?", Bled 2017, Slovenia

Bled 9 - 17 July

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More than **40 years ago** the **standard model** offered an **elegant new step** in **understanding the origin of fermions and bosons** by **postulating**:

- ▶ The existence of the **massless family members** with the **charges** in the **fundamental representation of the groups** -
 - o the **coloured triplet quarks and colourless leptons**,
 - o the **left handed members as the weak charged doublets**
 - o the **right handed weak chargeless members** ,
 - o the **left handed quarks distinguishing in the hyper charge from the left handed leptons**,
 - o **each right handed member having a different hyper charge**.
- ▶ The existence of **massless families to each of a family member**.

α name	hand- edness $-4iS^{03}S^{12}$	weak charge τ^{13}	hyper charge Y	colour charge	elm charge Q
u_L^i	-1	$\frac{1}{2}$	$\frac{1}{6}$	colour triplet	$\frac{2}{3}$
d_L^i	-1	$-\frac{1}{2}$	$\frac{1}{6}$	colour triplet	$-\frac{1}{3}$
ν_L^i	-1	$\frac{1}{2}$	$-\frac{1}{2}$	colourless	0
e_L^i	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	colourless	-1
u_R^i	1	weakless	$\frac{2}{3}$	colour triplet	$\frac{2}{3}$
d_R^i	1	weakless	$-\frac{1}{3}$	colour triplet	$-\frac{1}{3}$
ν_R^i	1	weakless	0	colourless	0
e_R^i	1	weakless	-1	colourless	-1

Members of each of the $i = 1, 2, 3$ massless families before the electroweak break. Each family contains the left handed weak charged quarks and the right handed weak chargeless quarks, belonging to the colour triplet $(1/2, 1/(2\sqrt{3})), (-1/2, 1/(2\sqrt{3})), (0, -1/(\sqrt{3}))$.

And the anti-fermions to each family and family member.

- ▶ The existence of the **massless vector gauge fields** to the observed **charges** of the **family members**, **carrying charges** in the **adjoint representation of the charge groups**.

Gauge fields before the electroweak break

- ▶ Three massless vector fields, the gauge fields of the three charges.

name	hand- edness	weak charge	hyper charge	colour charge	elm charge
hyper photon	0	0	0	colourless	0
weak bosons	0	triplet	0	colourless	triplet
gluons	0	0	0	colour octet	0

They all are vectors in $d = (3 + 1)$, in the adjoint representations with respect to the weak, colour and hyper charges.

- ▶ The **existence of a massive scalar field - the higgs**,
 - o carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$ as it would be in the **fundamental representation of the groups**,
 - o gaining at some step a **"nonzero vacuum expectation values"**, breaking the weak and the hyper charge and correspondingly breaking the **mass protection**.
- ▶ The **existence** of the **Yukawa couplings**,
 - o taking care of the properties of **fermions** and
 - o the masses of the **heavy bosons**.

- The **Higgs's field**, **the scalar** in $d = (3 + 1)$, a **doublet** with respect to the weak charge. $P_R = (-1)^{2s+3B+L} = 1$.

name	hand- edness	weak charge	hyper charge	colour charge	elm charge
0. Higgs_u	0	$\frac{1}{2}$	$\frac{1}{2}$	colourless	1
$\langle \text{Higgs}_d \rangle$	0	$-\frac{1}{2}$	$\frac{1}{2}$	colourless	0

name	hand- edness	weak charge	hyper charge	colour charge	elm charge
$\langle \text{Higgs}_u \rangle$	0	$\frac{1}{2}$	$-\frac{1}{2}$	colourless	0
0. Higgs_d	0	$-\frac{1}{2}$	$-\frac{1}{2}$	colourless	-1

- ▶ There is the **gravitational field** in **$d=(3+1)$** .

- ▶ **The *standard model* assumptions have been confirmed without offering surprises.**
- ▶ The last unobserved field as a field, the **Higgs's scalar**, detected in June 2012, was confirmed in March 2013.
- ▶ The waves of the **gravitational field** were detected in February 2016.

There are many phenomena

- ▶ the **dark matter**,
- ▶ the **matter-antimatter** asymmetry,
- ▶ the **dark energy**,
- ▶ the **observed dimension of space time**,
- ▶ **many other phenomena**,

not yet understood.

Obviously it is the time to make **a next steps beyond both standard models.**

What questions should one ask to find **next steps** beyond the *standard model* and to understand not yet understood phenomena?

- ▶ ○ Where do **family members** originate?
 - Where do **charges** of **family members** originate?
 - Why are the **charges** of **family members** so different?
 - Why have the **left handed family members** so different charges from the **right handed** ones?
- ▶ ○ Where do **families** of **family members** originate?
 - How **many different families** exist?
 - Why do **family members – quarks and leptons** – manifest so different properties if they all start as massless?

- ▶ **o** How is the **origin** of the **scalar field** (the Higgs's scalar) and the **Yukawa couplings connected** with the origin of **families**?
o How many scalar fields determine properties of the so far (and others possibly be) **observed fermions** and masses of **weak bosons**? (The Yukawa couplings certainly speak for the existence of several scalar fields with the properties of Higgs's scalar.)
- ▶ Why is the **Higgs's scalar**, or are all **scalar fields**, if there are several, **doublets** with respect to the weak and the hyper charge?
- ▶ Do **exist** also **scalar fields** with the **colour charge in the fundamental representation** and where, if they are, **do they manifest**?

- ▶ Where do the **charges** and correspondingly the so far (and others possibly be) **observed vector gauge fields** originate?
- ▶ **Where** does the **dark matter** originate?
- ▶ **Where** does the "ordinary" **matter-antimatter asymmetry** **originate?**
- ▶ **Where** does the **dark energy** originate?
- ▶ What is the dimension of space? $(3 + 1)?$, $((d - 1) + 1)?$, $\infty?$
- ▶ **What** is the role of the **symmetries**– discrete, continuous, global and gauge – in our **universe, in Nature?**
- ▶ And many others.

My statement:

- ▶ **An elegant trustworthy next step** must offer answers to **several** open questions, explaining:
 - o The **origin of the family members and the charges.**
 - o The **origin of the families and their properties.**
 - o The **origin of the scalar fields and their properties.**
 - o The **origin of the vector fields and their properties.**
 - o The **origin of the dark matter.**
 - o The **origin of the "ordinary" matter-antimatter asymmetry.**

My statement continues:

- ▶ There exist not yet observed families, gauge vector and scalar gauge fields.
- ▶ **Dimension of space is larger than 4** (very probably infinite).
- ▶ Inventing a next step which covers one of the open questions, might be of a help **but can hardly show the right next step in understanding nature.**

In the literature **NO explanation for the existence of the families can be found**, which would not just assume the family groups.

Several extensions of the **standard model** are, however, proposed, like:

- ▶ The $SU(3)$ group is assumed to describe – not explain – the existence of three families.

Like the **Higgs's** scalar charges are in the **fundamental** representations of the groups, also the **Yukawas** are assumed to emerge from the scalar fields, in the **fundamental** representation of the $SU(3)$ group.

- ▶ **SU(5) and SO(10)** grand unified theories are proposed, unifying all the charges. But the **spin** (the handedness) is obviously connected with the (weak and the hyper) charges, what these theories do "by hand" as it does the *standard model*, and the appearance of families is not explained.
- ▶ **Supersymmetric theories**, assuming the existence of bosons with the charges of quarks and leptons and fermions with the charges of the gauge vector fields, **although having several nice properties** but not explaining the appearance of families (except again by assuming larger groups), are not, to my understanding, the right next step beyond the *standard model*.

o The **Spin-Charge-Family** theory does offer the **explanation for all the assumptions of the standard model**, answering up to now several of the above cited open questions!

o The **more effort** is put into this theory, the more answers to the open questions in elementary particle physics and cosmology is the theory offering.

- ▶ A brief introduction into the **spin-charge-family theory**.

- ▶ **Spinors** carry in $d \geq (13 + 1)$ **two kinds of spin**, no charges.
 - The **Dirac spin** (γ^a) in $d = (13 + 1)$ describes in $d = (3 + 1)$ **spin** and **ALL the charges of quarks and leptons, left and right handed**.
 - The **second kind** of **spin** ($\tilde{\gamma}^a$) describes **FAMILIES**.
 - There is **NO third kind of spin**.
- ▶ **C,P,T** symmetries in $d = (3 + 1)$ follow from the **C,P,T** symmetry in $d \geq (13 + 1)$. (*JHEP* 04 (2014) 165)

- ▶ All **vector** and **scalar gauge fields origin in gravity**:
o in two spin connection fields,
 the **gauge fields** of γ^a and $\tilde{\gamma}^a$, and in
o vielbeins
 (*Eur. Phys. J. C*, DOI: 10.1140/epjc/s10052-017-4804-y)
 .
- ▶ If there are **no spinor sources present**, then either vector
 (\vec{A}_m^A , $m = 0, 1, 2, 3$) or scalar (\vec{A}_s^A , $s = 5, 6, \dots, d$) gauge
 fields are determined by **vielbeins** uniquely.

- ▶ **Spinors** interact correspondingly with
 - o the **vielbeins** and
 - o the **two kinds of the spin connection fields**.
- ▶ In $d = (3 + 1)$ the **spin-connection fields**, together with the **vielbeins**, manifest either as
 - o **vector** gauge fields with all the **charges** in the **adjoint** representations or as
 - o **scalar** gauge fields with the **charges** with respect to the **space index** in the **"fundamental"** representations and all the other **charges** in the **adjoint** representations or as
 - o **tensor** gravitational field.

There are two kinds of **scalar fields** with respect to the space index s :

- ▶ Those with ($s = 5, 6, 7, 8$) (they carry zero "**spinor charge**") are **doublets** with respect to the $SU(2)_I$ (the weak) charge and the **second $SU(2)_{II}$ charge** (determining the hyper charge). They are in the **adjoint** representations with respect to the **family** and the **family members charges**.
- These **scalars** explain the **Higgs's scalar** and the **Yukawa couplings**.

- ▶ **Those** with twice the "spinor charge" of a quark and ($s = 9, 10, \dots d$) are **colour triplets**. **Also they are in the adjoint representations** with respect to the **family** and the **family members charges**.
- These **scalars** transform **antileptons** into **quarks**, and **antiquarks** into **quarks** and back and correspondingly **contribute to matter-antimatter asymmetry** of our universe and to **proton decay**.
- ▶ There are **no additional scalar fields** in the **spin-charge-family theory**.

Condensate

- ▶ The (assumed) **scalar condensate** of **two right handed neutrinos** with the **family** quantum numbers of the upper four families (there are two four family groups in the theory), appearing $\approx 10^{16}$ GeV or higher,
 - o **breaks the CP** symmetry, causing the **matter-antimatter asymmetry** and the proton decay,
 - o couples to all the **scalar fields**, making them massive,
 - o couples to all the phenomenologically **unobserved vector gauge fields**, making them massive.

- ▶ The **vector fields**, which do not couple to the condensate and remain massless, are:
 - the **hyper charge vector field**.
 - the **weak vector fields**,
 - the **colour vector fields**,
 - the **gravity fields**.

The $SU(2)_{//}$ symmetry breaks due to the **condensate**, leaving the **hyper charge unbroken**.

Nonzero vacuum expectation values of scalars

- ▶ The scalar fields with the **space index $(7, 8)$** , gaining **nonzero vacuum expectation values**, cause the **electroweak break**,
 - breaking the weak and the hyper charge,
 - changing their own masses,
 - bringing masses to the **weak bosons**,
 - bringing masses to the **families of quarks and leptons**.

- ▶ The only gauge fields which do not couple to these scalars and remain massless are
 - o the **electromagnetic**,
 - o **colour vector gauge fields**,
 - o and **gravity**.
- ▶ There are two times four decoupled massive **families** of **quarks and leptons** after the electroweak break:
 - o There are the observed **three families** among the **lower four, the fourth to be observed**.
 - o The stable among the **upper four families** form the **dark matter**.

- All the families are **singlets with respect to $\widetilde{SU}(3)$ group, originating in the second kind of the Clifford algebra object $\tilde{\gamma}^a$.**

- ▶ It is **extremely encouraging** for the **spin-charge-family theory**, that a **simple starting action** contains **all the degrees of freedom observed at low energies**, directly or indirectly, and that only the

- **condensate** and

- **nonzero vacuum expectation values of all the scalar fields with $s = (7, 8)$**

are needed that the **theory explains**

- **all the assumptions** of the standard model,
- **explaining also the dark matter**,
- **the matter/antimatter asymmetry**,
- **and...**

The **spin-charge-family** theory is a kind of a **Kaluza-Klein-like** theory, but with **two kinds of spins**.

In d -dimensional space there are **fermions** with **two kinds of spins** and **gravity**, represented by two **spin connection** and **vielbein gauge fields**.

J. of Mod. Phys. **4** (2013) 823,
Phys. Rev. **D 91** (2015) 6, 065004, doi:10.1103,
J. of Mod. Phys. **6** (2015) 2244-2274,
<http://dx.doi.org/10.4236/jmp.2015.615230>
[<http://arxiv.org/abs/1409.4981>],
J. Phys.: Conf. Ser. 845 012017
(<http://iopscience.iop.org/1742-6596/845/1/012017>)

A short look "inside" the **spin-charge-family** theory.

There are **only two kinds of the Clifford algebra objects in any d**:

- ▶ The **Dirac γ^a operators** (used by Dirac 90 years ago).
- ▶ The **second one: $\tilde{\gamma}^a$** , which I recognized in the Grassmann space.

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+,$$

$$\{\gamma^a, \tilde{\gamma}^b\}_+ = 0,$$

$$(\tilde{\gamma}^a \mathbf{B} : = \mathbf{i}(-)^{n_B} \mathbf{B} \gamma^a) |\psi_0 >,$$

$$(\mathbf{B} = a_0 + a_a \gamma^a + a_{ab} \gamma^a \gamma^b + \cdots + a_{a_1 \dots a_d} \gamma^{a_1} \dots \gamma^{a_d}) |\psi_0 >$$

$(-)^{n_B} = +1, -1$, when the object B has a Clifford even or odd character, respectively.

$|\psi_0 >$ is a vacuum state on which the operators γ^a apply.

$$\mathbf{S}^{ab} := (\mathbf{i}/4)(\gamma^a \gamma^b - \gamma^b \gamma^a),$$

$$\tilde{\mathbf{S}}^{ab} := (\mathbf{i}/4)(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a),$$

$$\{\mathbf{S}^{ab}, \tilde{\mathbf{S}}^{cd}\}_- = \mathbf{0}.$$

- $\tilde{\mathbf{S}}^{ab}$ define the equivalent representations with respect to \mathbf{S}^{ab} .

My recognition:

- If γ^a are used to describe **the spin and the charges** of spinors,
 $\tilde{\gamma}^a$ - since it must be used or show why it does not manifest - it **must be used to describe families of spinors**

.

A simple action for a **spinor** which carries in $d = (13 + 1)$ only **two kinds of spins** (no charges) and for **gauge fields**:

$$\begin{aligned} \mathbf{S} = & \int d^d x \, E \, \mathcal{L}_f + \\ & \int d^d x \, E \, (\alpha R + \tilde{\alpha} \tilde{R}) \end{aligned}$$



$$\begin{aligned} \mathcal{L}_f &= \frac{1}{2} (\bar{\psi} \gamma^a p_{0a} \psi) + h.c. \\ p_{0a} &= f^\alpha{}_a p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^\alpha{}_a\} - \\ \mathbf{p}_{0\alpha} &= \mathbf{p}_\alpha - \frac{1}{2} \mathbf{S}^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{\mathbf{S}}^{ab} \tilde{\omega}_{ab\alpha} \end{aligned}$$

- The Einstein action for a free gravitational field is assumed to be linear in the curvature

$$\begin{aligned}
 \mathcal{L}_g &= E (\alpha R + \tilde{\alpha} \tilde{R}), \\
 R &= f^\alpha [a f^{\beta b}] (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \omega^c_{b\beta}), \\
 \tilde{R} &= f^\alpha [a f^{\beta b}] (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^c_{b\beta}),
 \end{aligned}$$

with $E = \det(e^a_\alpha)$

and $f^\alpha [a f^{\beta b}] = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$.

- ▶ The only internal degrees of freedom of **spinors** (**fermions**) are the **two kinds of the spin**.
- ▶ The only **gauge fields** are the **gravitational** ones – **vielbeins** and the **two kinds of spin connections**.
- ▶ Either γ^a or $\tilde{\gamma}^a$ transform as vectors in **d**,

$$\gamma'^a = \Lambda^a_b \gamma^b, \quad \tilde{\gamma}'^a = \Lambda^a_b \tilde{\gamma}^b,$$

$$\delta \gamma^c = -\frac{i}{2} \alpha_{ab} S^{ab} \gamma^c = \alpha^c_a \gamma^a,$$

$$\delta \tilde{\gamma}^c = -\frac{i}{2} \alpha_{ab} \tilde{S}^{ab} \tilde{\gamma}^c = \alpha^c_a \tilde{\gamma}^a,$$

$$\delta A^{c\dots ef} = -\frac{i}{2} \alpha_{ab} S^{ab} A^{c\dots ef} = \alpha^e_a A^{c\dots af},$$

$$S^{ab} A^{c\dots e\dots f} = i (\eta^{ae} A^{c\dots b\dots f} - \eta^{be} A^{c\dots a\dots f})$$

and correspondingly also $f^\alpha_a \omega_{bc\alpha}$ and $f^\alpha_a \tilde{\omega}_{bc\alpha}$ transform as tensors with respect to the flat index a .

Variation of the action brings for $\omega_{ab\alpha}$

$$\begin{aligned}
 \omega_{ab\alpha} = & -\frac{1}{2E} \left\{ e_{e\alpha} e_{b\gamma} \partial_\beta (E f^{\gamma[e} f^{\beta}_{a]}) + e_{e\alpha} e_{a\gamma} \partial_\beta (E f^{\gamma}_{[b} f^{\beta e]}) \right. \\
 & \left. - e_{e\alpha} e^e{}_\gamma \partial_\beta (E f^{\gamma}_{[a} f^{\beta}_{b]}) \right\} \\
 & - \frac{e_{e\alpha}}{4} \left\{ \bar{\Psi} \left(\gamma_e S_{ab} + \frac{3i}{2} (\delta_b^e \gamma_a - \delta_a^e \gamma_b) \right) \Psi \right\} \\
 & - \frac{1}{d-2} \left\{ e_{a\alpha} \left[\frac{1}{E} e^d{}_\gamma \partial_\beta (E f^{\gamma}_{[d} f^{\beta}_{b]}) + \frac{1}{2} \bar{\Psi} \gamma^d S_{db} \Psi \right] \right. \\
 & \left. - e_{b\alpha} \left[\frac{1}{E} e^d{}_\gamma \partial_\beta (E f^{\gamma}_{[d} f^{\beta}_{a]}) + \frac{1}{2} \bar{\Psi} \gamma^d S_{da} \Psi \right] \right\}
 \end{aligned}$$

IARD, J. Phys.: Conf. Ser. 845 012017

(<http://iopscience.iop.org/1742-6596/845/1/012017>)

and for $\tilde{\omega}_{ab\alpha}$,

$$\begin{aligned}\tilde{\omega}_{ab\alpha} = & -\frac{1}{2E} \left\{ e_{e\alpha} e_{b\gamma} \partial_{\beta} (E f^{\gamma[e} f^{\beta]}_{a]}) + e_{e\alpha} e_{a\gamma} \partial_{\beta} (E f^{\gamma}_{[b} f^{\beta e]}_{])} \right. \\ & \left. - e_{e\alpha} e^e_{\gamma} \partial_{\beta} (E f^{\gamma}_{[a} f^{\beta]}_{b]}) \right\} \\ & - \frac{e_{e\alpha}}{4} \left\{ \bar{\Psi} \left(\gamma_e \tilde{S}_{ab} + \frac{3i}{2} (\delta^e_b \gamma_a - \delta^e_a \gamma_b) \right) \Psi \right\} \\ & - \frac{1}{d-2} \left\{ e_{a\alpha} \left[\frac{1}{E} e^d_{\gamma} \partial_{\beta} (E f^{\gamma}_{[d} f^{\beta]}_{b]}) + \frac{1}{2} \bar{\Psi} \gamma^d \tilde{S}_{db} \Psi \right] \right. \\ & \left. - e_{b\alpha} \left[\frac{1}{E} e^d_{\gamma} \partial_{\beta} (E f^{\gamma}_{[d} f^{\beta]}_{a]}) + \frac{1}{2} \bar{\Psi} \gamma^d \tilde{S}_{da} \Psi \right] \right\}\end{aligned}$$

Eur. Phys. J. C, DOI: 10.1140/epjc/s10052-017-4804-y)

Fermions

- The action for **spinors** **seen** from $d = (3 + 1)$ and **analyzed** with respect to the standard model groups as subgroups of $SO(1 + 13)$, J. of Mod. Phys. **4** (2013) 823:

$$\begin{aligned}\mathcal{L}_f = & \bar{\psi} \gamma^m (p_m - \sum_{A,i} g^A \tau^{Ai} A_m^{Ai}) \psi + \\ & \{ \sum_{s=[7],[8]} \bar{\psi} \gamma^s p_{0s} \psi \} + \\ & \{ \sum_{s=[5],[6]} \bar{\psi} \gamma^s p_{0s} \psi + \sum_{t=[9],\dots,[14]} \bar{\psi} \gamma^t p_{0t} \psi \} .\end{aligned}$$

$$p_{0m} = \{p_m - \sum_A g^A \vec{\tau}^A \vec{A}_m^A\}$$

$$m \in (0, 1, 2, 3),$$

$$p_{0s} = f_s^\sigma [p_\sigma - \sum_A g^A \vec{\tau}^A \vec{A}_\sigma^A - \sum_A \tilde{g}^A \vec{\tilde{\tau}}^A \vec{\tilde{A}}_\sigma^A],$$

$$s \in (7, 8),$$

$$p_{0s} = f_s^\sigma [p_\sigma - \sum_A g^A \vec{\tau}^A \vec{A}_\sigma^A - \sum_A \tilde{g}^A \vec{\tilde{\tau}}^A \vec{\tilde{A}}_\sigma^A],$$

$$s \in (5, 6),$$

$$p_{0t} = f_t^{\sigma'} (p_{\sigma'} - \sum_A g^A \vec{\tau}^A \vec{A}_{\sigma'}^A - \sum_A \tilde{g}^A \vec{\tilde{\tau}}^A \vec{\tilde{A}}_{\sigma'}^A),$$

$$t \in (9, 10, 11, \dots, 14),$$

$$\mathbf{A}_s^{\text{Ai}} = \sum_{\mathbf{a}, \mathbf{b}} \mathbf{c}_{\mathbf{ab}}^{\text{Ai}} \omega_{\mathbf{abs}} ,$$

$$\mathbf{A}_t^{\text{Ai}} = \sum_{\mathbf{a}, \mathbf{b}} \mathbf{c}_{\mathbf{ab}}^{\text{Ai}} \omega_{\mathbf{abt}} ,$$

$$\tilde{\mathbf{A}}_s^{\text{Ai}} = \sum_{\mathbf{a}, \mathbf{b}} \tilde{\mathbf{c}}_{\mathbf{ab}}^{\text{Ai}} \tilde{\omega}_{\mathbf{abs}} ,$$

$$\tilde{\mathbf{A}}_t^{\text{Ai}} = \sum_{\mathbf{a}, \mathbf{b}} \tilde{\mathbf{c}}_{\mathbf{ab}}^{\text{Ai}} \tilde{\omega}_{\mathbf{abt}} .$$

$$\tau^{\mathbf{Ai}} = \sum_{a,b} c^{Ai}_{ab} \mathbf{S}^{ab},$$

$$\tilde{\tau}^{\mathbf{Ai}} = \sum_{a,b} \tilde{c}^{Ai}_{ab} \tilde{\mathbf{S}}^{ab},$$

$$\{\tau^{\mathbf{Ai}}, \tau^{\mathbf{Bj}}\}_- = i\delta^{AB} f^{Aijk} \tau^{\mathbf{Ak}},$$

$$\{\tilde{\tau}^{\mathbf{Ai}}, \tilde{\tau}^{\mathbf{Bj}}\}_- = i\delta^{AB} f^{Aijk} \tilde{\tau}^{\mathbf{Ak}},$$

$$\{\tau^{\mathbf{Ai}}, \tilde{\tau}^{\mathbf{Bj}}\}_- = 0.$$

- o $\tau^{\mathbf{Ai}}$ stay for the standard model charge groups,
for the second $SU(2)_{II}$,
for the "spinor" charge,
- o $\tilde{\tau}^{\mathbf{Ai}}$ denote the family quantum numbers.

$$\begin{aligned}
\vec{N}_{(L,R)} : &= \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03}), \\
\vec{\tau}^{(1,2)} : &= \frac{1}{2}(S^{58} \mp S^{67}, S^{57} \pm S^{68}, S^{56} \mp S^{78}), \\
\vec{\tau}^3 := &\frac{1}{2}\{S^{9\ 12} - S^{10\ 11}, S^{9\ 11} + S^{10\ 12}, S^{9\ 10} - S^{11\ 12}, \\
&S^{9\ 14} - S^{10\ 13}, S^{9\ 13} + S^{10\ 14}, S^{11\ 14} - S^{12\ 13}, \\
&S^{11\ 13} + S^{12\ 14}, \frac{1}{\sqrt{3}}(S^{9\ 10} + S^{11\ 12} - 2S^{13\ 14})\}, \\
\tau^4 := &-\frac{1}{3}(S^{9\ 10} + S^{11\ 12} + S^{13\ 14}), \quad Y := \tau^4 + \tau^{23}, \\
Y' := &-\tau^4 \tan^2 \vartheta_2 + \tau^{23}, \quad Q := \tau^{13} + Y, \quad Q' := -Y \tan^2 \vartheta_1 + \tau^{13},
\end{aligned}$$

and equivalently for \tilde{S}^{ab} and S^{ab} .

Breaks of symmetries when starting with the **massless spinors** (**fermions**), **vielbeins and two kinds of spin connection fields**

$$SO(1, 13) \times \widetilde{SO(1, 13)} \times$$

BREAK I



$$SO(1, 7) \times \widetilde{SO(1, 7)}$$

$$U(1) \times$$

$$SU(3)$$



eight massless families



$$SO(1, 3) \times SO(4) \times U(1) \times$$

$$(\widetilde{SU}(2)_{I_{\widetilde{SO}(1,3)}} \times \widetilde{SU}(2)_{I_{\widetilde{SO}(4)}}) \times$$

(divided into two groups)

$$(\widetilde{SU}(2)_{II_{\widetilde{SO}(1,3)}} \times \widetilde{SU}(2)_{II_{\widetilde{SO}(4)}}) \times SU(3)$$

BREAK II



The Standard Model like way of breaking



$$SO(1, 3) \times U(1) \times SU(3)$$

× (two groups of four massive families)

Breaking the starting symmetry from:

- ▶ $SO(1, 13) \times \widetilde{SO}(1, 13)$ to
 $SO(1, 7) \times \widetilde{SO}(1, 7) \times U(1)_{II} \times SU(3)$ (at $E \geq 10^{16}$ GeV)
 - o makes the **spin** (the handedness) in $d = (1 + 3)$ of two massless groups of four families of spinors connected with the **weak** and the **hyper** charge ,
- ▶ $SO(1, 7) \times \widetilde{SO}(1, 7) \times U(1)_{II} \times SU(3)$ to
 $SO(1, 3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$
 $\widetilde{SO}(1, 3) \times \widetilde{SU}(2)_I \times \widetilde{SU}(2)_{II}$
 - o makes that each member of the two groups of four massless families manifests in $d = (1 + 3)$ the **weak** ($SU(2)_I$), the **hyper** ($SU(2)_{II}$), the **colour** ($SU(3)$) and the "spin charge" ($U(1) = \tau^4$).

- ▶ Both breaks leave **eight families** ($2^{8/2-1} = 8$, determined by the symmetry of $SO(1,7)$) massless. All the families are $\widetilde{SU}(3)$ **chargeless**.
- ▶ The appearance of the **condensate of the two right handed neutrinos**, coupled to **spin 0**, makes the boson gauge fields, with which the condensate interacts, massive. These gauge fields are:
 - All the scalar gauge fields with the space index $s \geq 5$.
 - The vector ($m \leq 3$) gauge fields with the charges, which are the superposition of $SU(2)_{II}$ and $U(1)_{II}$ charges.

- ▶ The **colour, elm, weak and hyper** vector gauge fields do not interact with the condensate and remain massless.

- ▶ **At the electroweak break** from $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ to $SO(1,3) \times U(1) \times SU(3)$
 - scalar fields with the space index $s = (7, 8)$ obtain nonzero vacuum expectation values,
 - break correspondingly the weak and the hyper charge and change their own masses.
 - They leave massless only the **colour, elm** and **gravity gauge fields**.
- ▶ All the eight massless families gain masses.

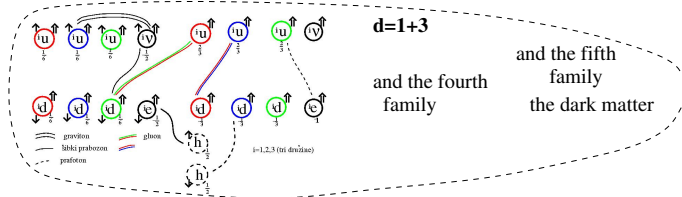
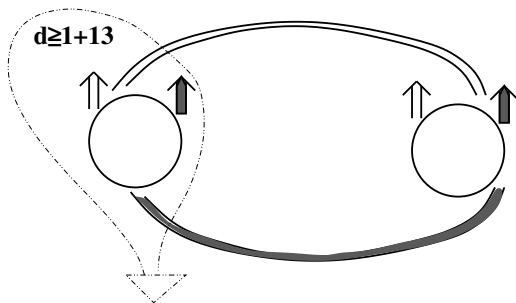
- ▶ The mass matrices of each family member manifest the $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$ symmetry, which remains unchanged in all loop corrections.

- ▶ To the **electroweak break** several scalar fields, the gauge fields of **twice** $\widetilde{SU}(2) \times \widetilde{SU}(2)$ **and three** $\times U(1)$, contribute, all with the **weak and the hyper charge** of the *standard model* Higgs.
- ▶ They carry besides the **weak** and the **hyper charge** either
 - o the **family members** quantum numbers originating in (Q, Q', Y') or
 - o the **family** quantum numbers originating in **twice** $\widetilde{SU}(2) \times \widetilde{SU}(2)$.

- ▶ Both, $SO(n)$ and $\widetilde{SO}(n)$ (are assumed) to break simultaneously.
- ▶ We studied (with H.B. Nielsen, T. Troha and D. Lukman) on a toy model of $d = (1 + 5)$ conditions which lead after breaking symmetries to massless spinors chirally coupled to the Kaluza-Klein-like gauge fields,

New J. Phys. **13** (2011) 103027, 1-25,

Int. J Mod. Phys. A **29**, 1450124 (2014), 21 pages, DOI:
10.1142/S0217751X14501243.



Families of quarks and leptons and antiquarks and antileptons

Our technique to represent spinors is elegant.

- ▶ N. S. Mankoč Borštnik, *J. of Math. Phys.* **34**, 3731 (1993),
- ▶ N.S.M.B. *Int. J. of Modern Phys.* **A 9**, 1731 (1994),
- ▶ N.S.M.B. *J. of Math. Phys.* **36**, 1593 (1995),
- ▶ *J. of Math. Phys.* **43**, 5782-5803 (2002), hep-th/0111257,
- ▶ *J. of Math. Phys.* **44** 4817 (2003), hep-th/0303224,
- ▶ *J. of High Ener. Phys.* **04** (2014) 165, arxiv:1212.2362v2,
the last three with H.B. Nielsen.

The **spinors** states are created out of
nilpotents $(\pm i)^{ab}$ and **projectors** $[\pm i]^{ab}$

$$\begin{aligned}
 (\pm i)^{ab} &:= \frac{1}{2}(\gamma^a \mp \gamma^b), \quad [\pm i]^{ab} := \frac{1}{2}(1 \pm \gamma^a \gamma^b) \\
 &\text{for } \eta^{aa} \eta^{bb} = -1, \\
 (\pm)^{ab} &:= \frac{1}{2}(\gamma^a \pm i \gamma^b), \quad [\pm]^{ab} := \frac{1}{2}(1 \pm i \gamma^a \gamma^b), \\
 &\text{for } \eta^{aa} \eta^{bb} = 1
 \end{aligned}$$

with γ^a which are usual **Dirac operators** in d -dimensional space.

Nilpotents $(\pm i)^{ab}$ and **projectors** $[\pm i]^{ab}$ are eigensates of S^{ab} and \tilde{S}^{ab} .

$$\begin{aligned} S^{ab}(\mathbf{k}) &= \frac{k^{ab}}{2}(\mathbf{k}), & S^{ab}[\mathbf{k}] &= \frac{k^{ab}}{2}[\mathbf{k}], \\ \tilde{S}^{ab}(\mathbf{k}) &= \frac{k^{ab}}{2}(\mathbf{k}), & \tilde{S}^{ab}[\mathbf{k}] &= -\frac{k^{ab}}{2}[\mathbf{k}]. \end{aligned}$$

γ^a transforms $\binom{ab}{k}$ into $\binom{ab}{-k}$, **never** to $\binom{ab}{k}$.

$\tilde{\gamma}^a$ transforms $\binom{ab}{k}$ into $\binom{ab}{k}$, **never** to $\binom{ab}{-k}$.

- ▶ One **Weyl representation** of one **family contains** all the **family members** with the **right handed neutrinos included**. It includes also **antimembers**, reachable by $\mathbb{C}_N \mathcal{P}_N$ on a **family member**.
- ▶ There are $2^{(7+1)/2-1} = 8$ **families**, which decouple into **twice four families**, with the quantum numbers $(\tilde{\tau}^{2i}, \tilde{N}_R^i)$ and $(\tilde{\tau}^{1i}, \tilde{N}_L^i)$, respectively.

S^{ab} generate **all the members of one family**. The **eightplet** (represent. of $SO(7,1)$) of quarks of a particular colour charge

i		$ \psi_i\rangle$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Octet, $\Gamma^{(7,1)} = 1$, $\Gamma^{(6)} = -1$, of quarks							
1	u_R^c	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ (+i)(+) & & (+)(+) & & (+)(-) & (-) \end{smallmatrix}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
2	u_R^c	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ [-i][-] & & (+)(+) & & (+)(-) & (-) \end{smallmatrix}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
3	d_R^c	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ (+i)(+) & & [-][-] & & (+)(-) & (-) \end{smallmatrix}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
4	d_R^c	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ [-i][-] & & [-][-] & & (+)(-) & (-) \end{smallmatrix}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
5	d_L^c	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ [-i](+) & & (+)(+) & & (+)(-) & (-) \end{smallmatrix}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	d_L^c	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ (+i)[-] & & [-](+) & & (+)(-) & (-) \end{smallmatrix}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	u_L^c	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ [-i](+) & & (+)[-] & & (+)(-) & (-) \end{smallmatrix}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^c	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ (+i)[-] & & (+)[-] & & (+)(-) & (-) \end{smallmatrix}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

$\gamma^0 \gamma^7$ and $\gamma^0 \gamma^8$ transform u_R of the 1st row into u_L of the 7th row, and d_R of the 4th row into d_L of the 6th row, doing what the Higgs scalars and γ^0 do in the Stan. model.

The **anti-eightplet** (repres. of $SO(7,1)$) of **anti-quarks** of the anti-colour charge, reachable by either S^{ab} or $\mathbb{C}_N \mathcal{P}_N^{(d-1)}$:

i		$ ^a \psi_i \rangle$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Antioctet, $\Gamma^{(7,1)} = -1$, $\Gamma^{(6)} = 1$, of antiquarks							
33	$\bar{d}_L^{\bar{c}1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ [-i](+) & & (+)(+) & & [-] & [+] & [+] \end{smallmatrix}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
34	$\bar{d}_L^{\bar{c}1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ (+i)[-] & & (+)(+) & & [-] & [+] & [+] \end{smallmatrix}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
35	$\bar{u}_L^{\bar{c}1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ [-i](+) & & - & & [-] & [+] & [+] \end{smallmatrix}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
36	$\bar{u}_L^{\bar{c}1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ (+i)[-] & & - & & [-] & [+] & [+] \end{smallmatrix}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
37	$\bar{d}_R^{\bar{c}1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ (+i)(+) & & (+)[-] & & [-] & [+] & [+] \end{smallmatrix}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
38	$\bar{d}_R^{\bar{c}1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ [-i][-] & & (+)[-] & & [-] & [+] & [+] \end{smallmatrix}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
39	$\bar{u}_R^{\bar{c}1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ (+i)(+) & & [-](+) & & [-] & [+] & [+] \end{smallmatrix}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
40	$\bar{u}_R^{\bar{c}1}$	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 1011 & 1213 & 14 \\ [-i][-] & & [-](+) & & [-] & [+] & [+] \end{smallmatrix}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$

$\gamma^0 \gamma^7$ and $\gamma^0 \gamma^8$ transform \bar{d}_L of the 1st row into \bar{d}_R of the 5th row, and \bar{u}_L of the 4th row into \bar{u}_R of the 8th row.

Vector gauge fields

- ▶ **All the vector gauge fields**, the gauge fields of the observed charges $\tau^{Ai} = \sum_{s,t} c^{Ai}_{st} S^{st}$ manifesting at the observable energies, **have all the properties as assumed by the standard model**.
- ▶ They carry with respect to the space index $m \in (0, 1, 2, 3)$ the vector degrees of freedom, while they have additional **internal degrees of freedom** (τ^{Ai}) in the adjoint representations.
- ▶ They origin as spin conection gauge fields of S^{ab} :
 $A^{Ai}_m = c^{Aist} \omega_{stm}$.
- ▶ S^{ab} applies on indexes (s, t, m) as follows

$$S^{ab} \omega_{stm\dots g} = i (\delta_s^a \omega_{tm\dots g}^b - \delta_s^b \omega_{tm\dots g}^a).$$

The action for vectors with respect to the space index $m = (0, 1, 2, 3)$ origin in gravity

$$\int E d^4x d^{(d-4)}x_\alpha R^{(d)} = \int d^4x \left\{ -\frac{1}{4} F^{Ai}{}_{\mu\nu} F^{Ai\mu\nu} \right\}.$$

Scalar fields - doublets and triplets with respect to the space index $s \geq 5$

- ▶ There are several **scalar gauge fields** with the space index $(s,t,s') = (7,8)$, all origin in the spin connection fields, $\tilde{\omega}_{abs}$ or $\omega_{s'ts}$:
 - o Twice **three triplets**, the scalar gauge fields of the **family** quantum numbers ($\tilde{\tau}^{Ai} = \sum_{a,b} \tilde{c}^{Ai}_{ab} \tilde{S}^{ab}$) and
 - o **three singlets** with the quantum numbers (Q,Q',Y') , the gauge fields of S^{st} .
- ▶ They are all doublets with respect to the space index **(5,6,7,8)**.
- ▶ They have all the rest quantum numbers **determined by the adjoint representations**.
- ▶ They explain at the so far observable energies the **Higgs's scalar** and the **Yukawa couplings**.

- ▶ There are besides **doublets**, with the space index $s = (5, 6, 7, 8)$, as well **triplets** and **anti-triplets**, with respect to the space index $s = (9, \dots, 14)$.
- ▶ **There are no additional scalars** in the theory.
- ▶ **All are massless.**
- ▶ All the scalars have the family and the family members quantum numbers in the **adjoint** representation.
- ▶ The properties of scalars are to be analyzed with respect to the generators of the corresponding subgroups, expressible with S^{ab} , as it is the case of the vector gauge fields.
- ▶ It is the (assumed) **condensate**, which makes those gauge fields, with which it interacts, massive.
 - o **The condensate breaks the CP symmetry.**

- ▶ The **scalar condensate** of two **right handed neutrinos** couple to
 - o all the **scalar and vector** gauge fields, making them massive,
 - o It does not interact with the **weak charge $SU(2)_I$** , the **hyper charge $U(1)$** , and the **colour $SU(3)$ charge gauge fields**, as well as the **gravity**, leaving them **massless**.

J. of Mod.Phys.**4** (2013) 823-847, J. of Mod.Phys. **6** (2015) 2244-2247, Phys Rev.**D 91**(2015)6,065004.

- The **condensate** has spin $S^{12} = 0$, $S^{03} = 0$

state	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	\tilde{Y}	\tilde{Q}	\tilde{N}_R^3	\tilde{N}_L^3	$\tilde{\tau}^4$
$ \nu_{1R}^{VIII} > 1 \mid \nu_{2R}^{VIII} > 2$	0	1	-1	0	0	0	1	0	0	1	0	-1
$ \nu_{1R}^{VIII} > 1 \mid e_{2R}^{VIII} > 2$	0	0	-1	-1	-1	0	1	0	0	1	0	-1
$ e_{1R}^{VIII} > 1 \mid e_{2R}^{VIII} > 2$	0	-1	-1	-2	-2	0	1	0	0	1	0	-1

The two doublets, determining the properties of the Higgs's scalar and the Yukawa couplings, are:

	state	τ^{13}	$\tau^{23} = Y$	spin	τ^4	Q
A_{78}^{Ai} (-)	$A_7^{Ai} + iA_8^{Ai}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
A_{56}^{Ai} (-)	$A_5^{Ai} + iA_6^{Ai}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1
A_{78}^{Ai} (+)	$A_7^{Ai} - iA_8^{Ai}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0
A_{56}^{Ai} (+)	$A_5^{Ai} - iA_6^{Ai}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	+1

There are A_{78}^{Ai} and A_{56}^{Ai} which gain **nonzero vacuum expectation values** at the **electroweak break**.

Index Ai determines the **family** ($\tilde{\tau}^{Ai}$) and the **family members** (Q, Q', Y') quantum numbers, both in adjoint representations.

Scalars with $s=(7,8)$ gain nonzero vacuum expectation values breaking the weak and the hyper symmetry, and conserving the electromagnetic and colour charge.

$$\begin{aligned} \mathbf{A}_s^{\mathbf{Ai}} &\supset (\mathbf{A}_s^Q, \mathbf{A}_s^{Q'}, \mathbf{A}_s^{Y'}, \tilde{\tilde{\mathbf{A}}}_s^{\tilde{\mathbf{I}}}, \tilde{\tilde{\mathbf{A}}}_s^{\tilde{\mathbf{N}}_{\tilde{\mathbf{L}}}}, \tilde{\tilde{\mathbf{A}}}_s^{\tilde{\mathbf{2}}}, \tilde{\tilde{\mathbf{A}}}_s^{\tilde{\mathbf{N}}_{\tilde{\mathbf{R}}}}), \\ \tau^{\mathbf{Ai}} &\supset (\mathbf{Q}, \mathbf{Q}', \mathbf{Y}', \tilde{\tilde{\tau}}^1, \tilde{\tilde{\mathbf{N}}}_L, \tilde{\tilde{\tau}}^2, \tilde{\tilde{\mathbf{N}}}_R), \\ s &= (7, 8). \end{aligned}$$

\mathbf{Ai} denotes family quantum numbers and (Q, Q', Y') , $(\tilde{\tilde{\tau}}^1, \tilde{\tilde{\mathbf{N}}}_L)$ quantum numbers of the first group of four families and $(\tilde{\tilde{\tau}}^2, \tilde{\tilde{\mathbf{N}}}_R)$ quantum numbers of the second group of four families.

A_s^{Ai} are expressible with either $\omega_{sts'}$ or $\tilde{\omega}_{abs'}$.

$$\vec{A}_s^1 = (\tilde{\omega}_{58s} - \tilde{\omega}_{67s}, \tilde{\omega}_{57s} + \tilde{\omega}_{68s}, \tilde{\omega}_{56s} - \tilde{\omega}_{78s}),$$

$$\vec{A}_s^2 = (\tilde{\omega}_{58s} + \tilde{\omega}_{67s}, \tilde{\omega}_{57s} - \tilde{\omega}_{68s}, \tilde{\omega}_{56s} + \tilde{\omega}_{78s}),$$

$$\vec{A}_{Ls}^N = (\tilde{\omega}_{23s} + i\tilde{\omega}_{01s}, \tilde{\omega}_{31s} + i\tilde{\omega}_{02s}, \tilde{\omega}_{12s} + \tilde{\omega}_{03s}),$$

$$\vec{A}_{Rs}^N = (\tilde{\omega}_{23s} - i\tilde{\omega}_{01s}, \tilde{\omega}_{31s} - i\tilde{\omega}_{02s}, \tilde{\omega}_{12s} - i\tilde{\omega}_{03s}),$$

$$A_s^Q = \omega_{56s} - (\omega_{910s} + \omega_{1112s} + \omega_{1314s}),$$

$$A_s^Y = (\omega_{56s} + \omega_{78s}) - (\omega_{910s} + \omega_{1112s} + \omega_{1314s})$$

$$A_s^4 = -(\omega_{910s} + \omega_{1112s} + \omega_{1314s}).$$

The **mass term** - from the **starting action** - is (p_s , when treating the lowest energy solutions, is left out)

$$\mathcal{L}_M = \sum_{s=(7,8),Ai} \bar{\psi} \gamma^s (-\tau^{Ai} A_s^{Ai}) \psi =$$

$$-\bar{\psi} \{ \overset{78}{(+)} \tau^{Ai} (A_7^{Ai} - i A_8^{Ai}) + \overset{78}{(-)} \tau^{Ai} (A_7^{Ai} + i A_8^{Ai}) \} \psi ,$$

$$\overset{78}{(\pm)} = \frac{1}{2} (\gamma^7 \pm i \gamma^8), \quad A_{\underset{(\pm)}{78}}^{Ai} := (A_7^{Ai} \mp i A_8^{Ai}).$$

Operators Y , Q and τ^{13} , applied on $(A_7^{Ai} \mp i A_8^{Ai})$

$$\tau^{13} (A_7^{Ai} \mp i A_8^{Ai}) = \pm \frac{1}{2} (A_7^{Ai} \mp i A_8^{Ai}),$$

$$Y (A_7^{Ai} \mp i A_8^{Ai}) = \mp \frac{1}{2} (A_7^{Ai} \mp i A_8^{Ai}),$$

$$Q (A_7^{Ai} \mp i A_8^{Ai}) = 0,$$

manifest that **all** $(A_7^{Ai} \mp i A_8^{Ai})$ have quantum numbers of the **Higgs's scalar of the standard model**, "dressing", after **gaining nonzero expectation values**, the right handed members of a family with appropriate charges, so that they gain charges of the left handed partners:

$(A_7^{Ai} + i A_8^{Ai})$ "dresses" u_R, ν_R and $(A_7^{Ai} - i A_8^{Ai})$ "dresses" d_R, e_R , with quantum numbers of their left handed partners, just as required by the "standard model".

Ai either measures:

- o the **Q,Q',Y'** charges of the family members or
- o **transforms a family member of one family into the same family member of another family, within each of the two groups of four families,**

manifesting in each group of four families the $\widetilde{SU}(2) \times \widetilde{SU}(2)$ symmetry.

Eight families of u_R (spin 1/2, colour $(\frac{1}{2}, \frac{1}{2\sqrt{3}})$) and of colourless ν_R (spin 1/2). All have "tilde spinor charge"

$\tilde{\tau}^4 = -\frac{1}{2}$, the weak charge $\tau^{13} = 0$, $\tau^{23} = \frac{1}{2}$. Quarks have "spinor" q.no. $\tau^4 = \frac{1}{6}$ and leptons $\tau^4 = -\frac{1}{2}$. The

first four families have $\tilde{\tau}^{23} = 0$, $\tilde{N}_R^3 = 0$, the second four families have $\tilde{\tau}^{13} = 0$, $\tilde{N}_L^3 = 0$.

$\tilde{N}_R^3 = 0, \tilde{\tau}^{23} = 0$							$\tilde{N}_R^3 = 0, \tilde{\tau}^{23} = 0$							$\tilde{\tau}^{13}$	\tilde{N}_L^3
u_{R1}^{c1}	03	12	56	78	9 10	11 12 13 14	ν_{R1}	03	12	56	78	9 10	11 12 13 14	$-\frac{1}{2}$	$-\frac{1}{2}$
	(+i)	[+]		[+]	(+)	(+) [-] [-]		(+i)	[+]		[+]	(+)	(+) (+) (+)		
u_{R2}^{c1}	03	12	56	78	9 10	11 12 13 14	ν_{R2}	03	12	56	78	9 10	11 12 13 14	$-\frac{1}{2}$	$\frac{1}{2}$
	[+i]	(+)		[+]	(+)	(+) [-] [-]		[+i]	(+)		[+]	(+)	(+) (+) (+)		
u_{R3}^{c1}	03	12	56	78	9 10	11 12 13 14	ν_{R3}	03	12	56	78	9 10	11 12 13 14	$\frac{1}{2}$	$-\frac{1}{2}$
	(+i)	[+]		(+)	[+]	(+) [-] [-]		(+i)	[+]		(+)	[+]	(+) (+) (+)		
u_{R4}^{c1}	03	12	56	78	9 10	11 12 13 14	ν_{R4}	03	12	56	78	9 10	11 12 13 14	$\frac{1}{2}$	$\frac{1}{2}$
	[+i]	(+)		(+)	[+]	(+) [-] [-]		[+i]	(+)		(+)	[+]	(+) (+) (+)		
$\tilde{N}_L^3 = 0, \tilde{\tau}^{13} = 0$							$\tilde{N}_L^3 = 0, \tilde{\tau}^{13} = 0$							$\tilde{\tau}^{23}$	\tilde{N}_R^3
u_{R5}^{c1}	03	12	56	78	9 10	11 12 13 14	ν_{R5}	03	12	56	78	9 10	11 12 13 14	$-\frac{1}{2}$	$-\frac{1}{2}$
	(+i)	(+)		(+)	(+)	(+) [-] [-]		(+i)	(+)		(+)	(+)	(+) (+) (+)		
u_{R6}^{c1}	03	12	56	78	9 10	11 12 13 14	ν_{R6}	03	12	56	78	9 10	11 12 13 14	$-\frac{1}{2}$	$\frac{1}{2}$
	(+i)	(+)		[+]	[+]	(+) [-] [-]		(+i)	(+)		[+]	[+]	(+) (+) (+)		
u_{R7}^{c1}	03	12	56	78	9 10	11 12 13 14	ν_{R7}	03	12	56	78	9 10	11 12 13 14	$\frac{1}{2}$	$-\frac{1}{2}$
	[+i]	[+]		(+)	(+)	(+) [-] [-]		[+i]	[+]		(+)	(+)	(+) (+) (+)		
u_{R8}^{c1}	03	12	56	78	9 10	11 12 13 14	ν_{R8}	03	12	56	78	9 10	11 12 13 14	$\frac{1}{2}$	$\frac{1}{2}$
	[+i]	[+]		[+]	[+]	(+) [-] [-]		[+i]	[+]		[+]	[+]	(+) (+) (+)		

Before the **electroweak break** all the **families** are **mass protected** and correspondingly **massless**.

- Scalars with the weak and the hyper charge ($\mp\frac{1}{2}, \pm\frac{1}{2}$) determine masses of **all** the members α of the **lower four families**, ν_R **have nonzero** $Y' := -\tau^4 + \tau^{23}$, and (together with the **condensate**) also the masses of the **upper four families**.

The group of the lower four families manifest the $\widetilde{SU}(2)_{\widetilde{SO}(1,3)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)} \times U(1)$ **symmetry** (after all loop corrections).

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e^* & -a_2 - a & b & d \\ d^* & b^* & a_2 - a & e \\ b^* & d^* & e^* & a_1 - a \end{pmatrix}^{\alpha}.$$

We **made calculations**, treating **quarks** and **leptons** in equivalent way, as required by the "spin-charge-family" theory. Although

- ▶ any **$(n-1) \times (n-1)$** submatrix of an unitary **$n \times n$** matrix determines the **$n \times n$** matrix for **$n \geq 4$** uniquely,
- ▶ the **measured mixing matrix elements** of the **3×3** submatrix **are not yet accurate enough even for quarks to predict the masses m_4 of the fourth family members.**
 - We can say, taking into account the data for the mixing matrices and masses, that **m_4 quark masses might be any in the interval $(300 < m_4 < 1000)$ GeV or even **above**.**
- ▶ **Assuming** masses **m_4** we can predict mixing matrices.

Results are presented for two choices of $m_{u_4} = m_{d_4}$, [arxiv:1412.5866]:

- ▶ 1. $m_{u_4} = 700 \text{ GeV}$, $m_{d_4} = 700 \text{ GeV}$new₁
- ▶ 2. $m_{u_4} = 1\,200 \text{ GeV}$, $m_{d_4} = 1\,200 \text{ GeV}$new₂

$ V_{ud} =$	exp_n	0.97425 ± 0.00022	0.2253 ± 0.0008	0.00413 ± 0.00049	
	new_1	0.97423(4)	0.22539(7)	0.00299	0.00776(1)
	new_2	0.97423[5]	0.22538[42]	0.00299	0.00793[466]
	exp_n	0.225 ± 0.008	0.986 ± 0.016	0.0411 ± 0.0013	
	new_1	0.22534(3)	0.97335	0.04245(6)	0.00349(60)
	new_2	0.22531[5]	0.97336[5]	0.04248	0.00002[216]
	exp_n	0.0084 ± 0.0006	0.0400 ± 0.0027	1.021 ± 0.032	
	new_1	0.00667(6)	0.04203(4)	0.99909	0.00038
	new_2	0.00667	0.04206[5]	0.99909	0.00024[21]
	new_1	0.00677(60)	0.00517(26)	0.00020	0.99996
new_2	0.00773	0.00178	0.00022	0.99997[9]	

- ▶ o The **matrix elements V_{CKM}** depend strongly on the **accuracy** of the experimental **3 x 3 submatrix**.
- o Calculated **3 x 3 submatrix** of 4 x 4 **V_{CKM}** depends on the **m_{4th} family masses**, but not much.
- o **$V_{u_i d_4}$, $V_{d_i u_4}$** do not depend strongly on the **m_{4th} family masses** and are obviously **very small**.
- ▶ The higher are the fourth family members masses, the closer are the mass matrices to the **democratic matrices** for either quarks or leptons, as expected.

- ▶ The **stable family** of the **upper four families** group is the candidate to form the **Dark Matter**.
- ▶ Masses of the upper four families are influenced :
 - by the $\widetilde{SU}(2)_{II\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II\widetilde{SO}(4)}$ **scalar fields** with the corresponding family quantum numbers,
 - by the **scalars** $(A_{78(\mp)}^Q, A_{78(\mp)}^{Q'}, A_{78(\mp)}^{Y'})$, and
 - by the **condensate** of the two ν_R of the **upper four families**.

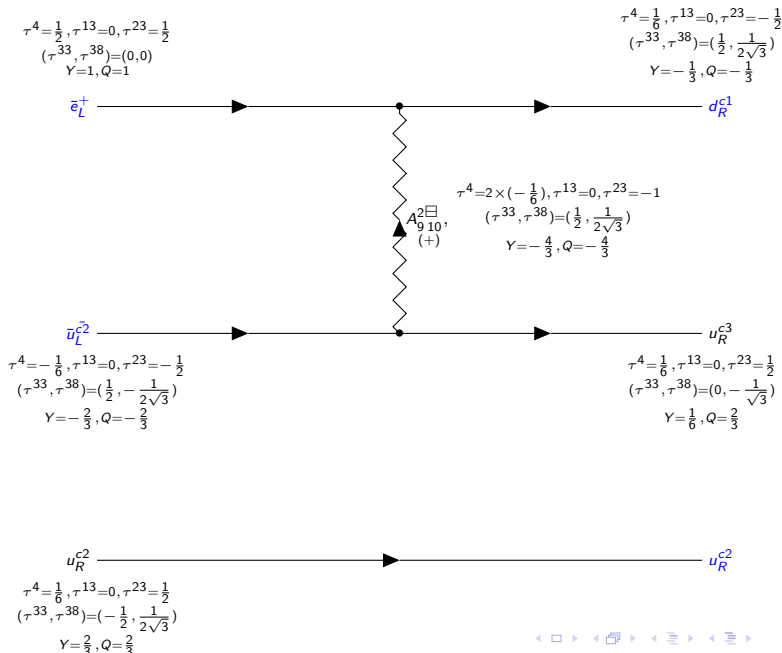
Matter-antimatter asymmetry

There are also **triplet** and **anti-triplet** scalars, $s = (9, \dots, d)$:

	state	τ^{33}	τ^{38}	spin	τ^4	Q
$A_{9\ 10}^{Ai}$ (+)	$A_9^{Ai} - iA_{10}^{Ai}$	$+\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A_{11\ 12}^{Ai}$ (+)	$A_{11}^{Ai} - iA_{12}^{Ai}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A_{13\ 14}^{Ai}$ (+)	$A_{13}^{Ai} - iA_{14}^{Ai}$	0	$-\frac{1}{\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A_{9\ 10}^{Ai}$ (-)	$A_9^{Ai} + iA_{10}^{Ai}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A_{11\ 12}^{Ai}$ (-)	$A_{11}^{Ai} + iA_{12}^{Ai}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A_{13\ 14}^{Ai}$ (-)	$A_{13}^{Ai} + iA_{14}^{Ai}$	0	$\frac{1}{\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$

They cause transitions from **anti-leptons** into **quarks** and **anti-quarks** into **quarks** and back, **transforming matter into antimatter and back**. The **condensate breaks CP symmetry**, offering the explanation for the **matter-antimatter asymmetry in the universe**.

Let us look at scalar triplets, causing the birth of a proton from the left handed **positron**, **antiquark** and **quark**:



These two quarks, d_R^{c1} and u_R^{c3} can bind (at low enough energy) together with u_R^{c2} into the colour **chargeless baryon - a proton**.

After the appearance of the **condensate** the **CP is broken**.

In the expanding universe, fulfilling the Sakharov request for appropriate non-thermal equilibrium, **these triplet scalars have a chance to explain the matter-antimatter asymmetry**.

The opposite transition makes the **proton decay**.

Dark matter

$d \rightarrow (d - 4) + (3 + 1)$ before (or at least at) the electroweak break.

- ▶ We follow the **evolution of the universe**, in particular the **abundance of the fifth family members** - the **candidates** for the **dark matter** in the universe.
- ▶ We estimate the behaviour of our stable heavy family quarks and anti-quarks in the expanding universe by solving the system of **Boltzmann equations**.
- ▶ We follow the **clustering** of the **fifth family** quarks and antiquarks into the **fifth family baryons** through the **colour** phase transition.
- ▶ The **mass** of the fifth family members is determined from the today **dark matter density**.

Phys. Rev. D (2009) 80.083534

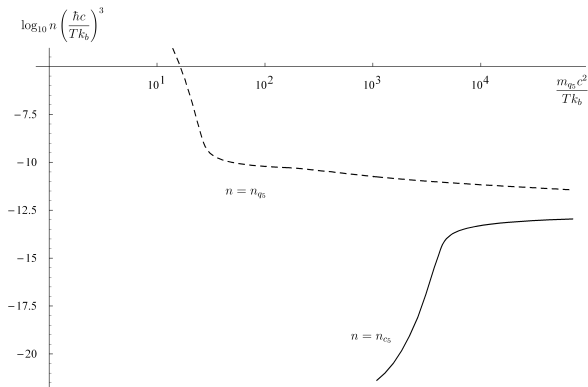


Figure: The dependence of the two number densities n_{q_5} (of the fifth family quarks) and n_{c_5} (of the fifth family clusters) as the function of $\frac{m_{q_5} c^2}{T k_b}$ is presented for the values $m_{q_5} c^2 = 71$ TeV, $\eta_{c_5} = \frac{1}{50}$ and $\eta_{(q\bar{q})_b} = 1$. We take $g^* = 91.5$.

We estimated from following the fifth family members in the expanding universe:



$$10 \text{ TeV} < m_{q_5} c^2 < 4 \cdot 10^2 \text{ TeV} .$$



$$10^{-8} \text{ fm}^2 < \sigma_{c_5} < 10^{-6} \text{ fm}^2 .$$

(It is at least $10^{-6} \times$ smaller than the cross section for the first family neutrons.)

We estimate from the scattering of the fifth family members on the ordinary matter on our Earth, on the direct measurements - DAMA, CDMS,...- ...



$$200 \text{ TeV} < m_{q_5} c^2 < 10^5 \text{ TeV} .$$

► Due to

$$\tau^{1+} \tau^{1-} \mathbf{A}_{78(+)}^{\mathbf{Ai}} = \mathbf{A}_{78(+)}^{\mathbf{Ai}},$$

$$\tau^{1-} \tau^{1+} \mathbf{A}_{78(-)}^{\mathbf{Ai}} = \mathbf{A}_{78(-)}^{\mathbf{Ai}},$$

$$Q \mathbf{A}_{78(\mp)}^{\mathbf{Ai}} = 0,$$

$$Q' \mathbf{A}_{78(\mp)}^{\mathbf{Ai}} = \pm \frac{1}{2 \cos^2 \theta_1} \mathbf{A}_{78(\mp)}^{\mathbf{Ai}},$$

the **vector gauge fields** $A_m^{1\pm} (= W_m^{\pm})$ and $A_m^{Q'} (= Z_m)$
 $= \cos \theta_1 A_m^{13} - \sin \theta_1 A_m^Y$ become massive, while $A_m^Q (= A_m)$
 $= \sin \theta_2 A_m^{13} + \cos \theta_1 A_m^Y$ remain massless, if $\frac{g^1}{g^Y} \tan \theta_1 = 1$.

- Correspondingly the mass term of the **vector gauge bosons** is

$$(p_{0m} A_{\mp}^{Ai})^\dagger (p_0^m A_{\mp}^{Ai}) \rightarrow$$

$$\left(\frac{1}{2}\right)^2 (g^1)^2 v^2 \left(\frac{1}{(\cos \theta_1)^2} Z_m^{Q'} Z^{Q' m} + 2 W_m^+ W^{-m} \right),$$

$$Tr(< A_{\mp}^{Ai\dagger} > < A_{\mp}^{vAi} >) = \frac{v^2}{2}.$$

- ▶ In the *standard model* the **family members**, the **families**, the **gauge vector fields**, the **scalar Higgs**, the **Yukawa couplings**, exist by the **assumption**.
- ▶ ** In the **spin-charge-family theory** all these properties follow from the simple starting action with **two kinds of spins** and with **gravity only** .
 - ** The theory offers the explanation for the **dark matter**.
 - ** The theory offers the explanation for the **matter-antimatter asymmetry**.
 - ** All the **scalar** and all the **vector** gauge fields are **directly or indirectly observable**.

The *spin-charge-family theory* explains also many other properties, which are not explainable in the *standard model*, like "miraculous" non-anomalous triangle Feynman diagrams.

The more work is put into the *spin-charge-family theory* the more explanations for the phenomena follow.

Concrete predictions:

- ▶ There are several scalar fields;
 - **two triplets** , ◦ **three singlets** ,explaining **higgs** and **Yukawa couplings**, some of them will be observed at the LHC, JMP 6 (2015) 2244, Phys. Rev. D 91 (2015) 6, 065004.
- ▶ There is the **fourth family**, (weakly) coupled to the observed **three**, which will be observed at the LHC, New J. of Phys. 10 (2008) 093002.
- ▶ There is the **dark matter** with the predicted properties, Phys. Rev. D (2009) 80.083534.
- ▶ There is the ordinary **matter/antimatter asymmetry** explained and the **proton decay** predicted and explained, Phys. Rev. D 91 (2015) 6, 065004.

We recognize that:

- ▶ The last **data for mixing matrix of quarks** are in better agreement with our prediction for the **3×3 submatrix** elements of the **4×4 mixing matrix** than the previous ones.
- ▶ Our **fit** to the last data predicts how will the **3×3 submatrix elements change** in the next more accurate measurements.
- ▶ Masses of the **fourth family** lie **much above** the known three, masses of quarks are close to each other.

- ▶ Masses of the **fifth family** lie **much above** the known three and the **predicted fourth family** masses.
- ▶ **Baryons** of the **fifth family** are heavy, forming small enough clusters with small enough scattering amplitude among themselves and with the ordinary matter to be the candidate for the **dark matter**.
- ▶ The "nuclear" force among them is different from the force among ordinary nucleons.

- ▶ The **spin-charge-family** theory is offering an explanation for the **hierarchy problem**:
The mass matrices of the **two four families groups** are almost democratic, causing spreading of the **fermion masses** from 10^{16} **GeV** to 10^{-8} **MeV**.

To summarize:

- ▶ I hope that I managed to convince you that I can answer many open questions of particle physics and cosmology.
- ▶ **The collaborators are very welcome!**

Let me present in a little more details, what was published since last year Bled workshop, supporting the **spin-charge-family theory** as a promising theory showing the way beyond the **standard models**. The ideas and most of proof were developed before.

- ▶ **Vector and scalar gauge fields with respect to $d = (3 + 1)$ in Kaluza-Klein theories and in the spin-charge-family theory**, Eur. Phys. J. C, DOI: 10.1140/epjc/s10052-017-4804-y).
- ▶ **The spin-charge-family theory offers understanding of the triangle anomalies cancellation in the standard model**, Fortschritte der Physik, Progress of Physics, www.fp-journal.org, DOI: 10.1002/prop.201700046 <http://arxiv.org/abs/1607.01618>.

- ▶ **Spin-charge-family theory is offering next step in understanding elementary particles and fields and correspondingly universe**, Proceedings to the IARD conferences, Ljubljana, 6-9 June 2016, [<http://arxiv.org/abs/1409.4981>], [arXiv:1607.01618]/v2] **J. Phys.: Conf. Ser. 845 012017** (<http://iopscience.iop.org/1742-6596/845/1/012017>) doi:10.1088/1742-6596/845/1/012017.

In this contribution besides the upper two topics also other solved problems, and as well as the discussions on the arguments, that the fourth family, coupled to the measured three, predicted by the spin-charge-family theory might exist, although the elementary particle physicists do not believe that.

Let me start with the proof that **both – vector and scalar gauge fields with respect to $d = (3 + 1)$ in Kaluza-Klein theories and in the spin-charge-family theory** can be represented by either spin connections or vielbeins.

Coauthor: Dragan Lukman

A simple action for a **spinor** which carries in $d = (13 + 1)$ only **two kinds of spins** (no charges) and for **gauge fields**:

$$\mathbf{S} = \int d^d x E \mathcal{L}_f + \int d^d x E (\alpha R + \tilde{\alpha} \tilde{R})$$



$$\mathcal{L}_f = \frac{1}{2}(\bar{\psi} \gamma^a p_{0a} \psi) + h.c.$$

$$p_{0a} = f^\alpha{}_a p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^\alpha{}_a\} -$$

$$\mathbf{p}_{0\alpha} = \mathbf{p}_\alpha - \frac{1}{2} \mathbf{S}^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{\mathbf{S}}^{ab} \tilde{\omega}_{ab\alpha}$$

- The Einstein action for a free gravitational field is assumed to be linear in the curvature

$$\begin{aligned}
 \mathcal{L}_g &= E (\alpha R + \tilde{\alpha} \tilde{R}), \\
 R &= f^{\alpha[a} f^{\beta b]} (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \omega^c_{b\beta}), \\
 \tilde{R} &= f^{\alpha[a} f^{\beta b]} (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^c_{b\beta}),
 \end{aligned}$$

with $E = \det(e^a_{\alpha})$
 and $f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$.

- $g^{\alpha\beta} = f^{\alpha}_a f^{\beta a}$, $g_{\alpha\beta} = e^a_{\alpha} e_{a\beta}$

Variation of the action brings for $\omega_{ab\alpha}$

$$\begin{aligned}
 \omega_{ab\alpha} = & -\frac{1}{2E} \left\{ e_{e\alpha} e_{b\gamma} \partial_\beta (E f^{\gamma[e} f^{\beta}_{a]}) + e_{e\alpha} e_{a\gamma} \partial_\beta (E f^{\gamma}_{[b} f^{\beta e]}) \right. \\
 & \left. - e_{e\alpha} e^e{}_\gamma \partial_\beta (E f^{\gamma}_{[a} f^{\beta}_{b]}) \right\} \\
 & - \frac{e_{e\alpha}}{4} \left\{ \bar{\Psi} \left(\gamma_e S_{ab} + \frac{3i}{2} (\delta_b^e \gamma_a - \delta_a^e \gamma_b) \right) \Psi \right\} \\
 & - \frac{1}{d-2} \left\{ e_{a\alpha} \left[\frac{1}{E} e^d{}_\gamma \partial_\beta (E f^{\gamma}_{[d} f^{\beta}_{b]}) + \frac{1}{2} \bar{\Psi} \gamma^d S_{db} \Psi \right] \right. \\
 & \left. - e_{b\alpha} \left[\frac{1}{E} e^d{}_\gamma \partial_\beta (E f^{\gamma}_{[d} f^{\beta}_{a]}) + \frac{1}{2} \bar{\Psi} \gamma^d S_{da} \Psi \right] \right\}
 \end{aligned}$$

IARD, J. Phys.: Conf. Ser. 845 012017

(<http://iopscience.iop.org/1742-6596/845/1/012017>)

Let $(d - 4)$ space manifest the rotational symmetry

$$x'^{\mu} = x^{\mu},$$

$$x'^{\sigma} = x^{\sigma} + \varepsilon^{st}(x^{\mu}) E_{st}^{\sigma}(x^{\tau}) = x^{\sigma} - i\varepsilon^{st}(x^{\mu}) M_{st} x^{\sigma},$$

$M^{st} = S^{st} + L^{st}$, $L^{st} = x^s p^t - x^t p^s$, S^{st} concern internal degrees of freedom of boson and fermion fields,

$$\{M^{st}, M^{s't'}\}_{-} = i(\eta^{st'} M^{ts'} + \eta^{ts'} M^{st'} - \eta^{ss'} M^{tt'} - \eta^{tt'} M^{ss'}).$$

It follows

$$-i M_{st} x^{\sigma} = E_{st}^{\sigma} = x_s f^{\sigma}_t - x_t f^{\sigma}_s,$$

$$E_{st}^{\sigma} = (e_{s\tau} f^{\sigma}_t - e_{t\tau} f^{\sigma}_s) x^{\tau},$$

$$M_{st}^{\sigma} : = iE_{st}^{\sigma},$$

and correspondingly: $M_{st} = E_{st}^{\sigma} p_{\sigma}$.

Let the corresponding background field ($g_{\alpha\beta} = e^a{}_\alpha e_{a\beta}$) be

$$e^a{}_\alpha = \begin{pmatrix} \delta^m{}_\mu & e^m{}_\sigma = 0 \\ e^s{}_\mu & e^s{}_\sigma \end{pmatrix}, \quad f^\alpha{}_a = \begin{pmatrix} \delta^\mu{}_m & f^\sigma{}_m \\ 0 = f^\mu{}_s & f^\sigma{}_s \end{pmatrix},$$

This leads to

$$g_{\alpha\beta} = \begin{pmatrix} \eta_{mn} + f^\sigma{}_m f^\tau{}_n e^s{}_\sigma e_{s\tau} & -f^\tau{}_m e^s{}_\tau e_{s\sigma} \\ -f^\tau{}_n e^s{}_\tau e_{s\sigma} & e^s{}_\sigma e_{s\tau} \end{pmatrix},$$

and

$$g^{\alpha\beta} = \begin{pmatrix} \eta^{mn} & f^{\sigma m} \\ f^{\sigma m} & f^\sigma{}_s f^{\tau s} + f^\sigma{}_m f^{\tau m} \end{pmatrix}.$$

Statement: Let the space with $s \geq 5$ have the symmetry allowing the infinitesimal transformations of the kind

$$x'^{\mu} = x^{\mu}, \quad x'^{\sigma} = x^{\sigma} - i \sum_{A,i,s,t} \varepsilon^{Ai}(x^{\mu}) c_{Ai}{}^{st} M_{st} x^{\sigma},$$

then the vielbeins f^{σ}_m manifest in $d = (3 + 1)$ the vector gauge fields A_m^{Ai}

$$f^{\sigma}_m = \sum_A \vec{\tau}^{A\sigma} \vec{A}_m^A,$$

where

$$\begin{aligned} \tau^{Ai} &= \sum_{s,t} c^{Ai}{}_{st} M^{st}, \\ \tau^{Ai\sigma} &= \sum_{s,t} -i c^{Ai}{}_{st} M^{st\sigma} \\ &= \sum_{s,t} c^{Ai}{}_{st} (e_{s\tau} f^{\sigma}_t - e_{t\tau} f^{\sigma}_s) x^{\tau} = E^{\sigma}_{Ai}, \\ A_m^{Ai} &= \sum_{s,t} c^{Ai}{}_{st} \omega^{st}_m. \end{aligned}$$

$$f^\sigma{}_m = \sum_A \vec{\tau}^{A\sigma} \vec{A}_m.$$

When $(d - 4)$ space manifests the symmetry $x'^{\mu} = x^{\mu}$, $x'^{\sigma} = x^{\sigma} - i \sum_{A,i,s,t} \varepsilon^{Ai} (x^{\mu}) c_{Ai}^{st} M_{st} x^{\sigma}$, and $d = (3 + 1)$ is a flat space, the curvature $R^{(d)}$ becomes equal to

$$\begin{aligned}
 R^{(d)} &= R^{(d-4)} \\
 &\quad - \frac{1}{4} \sum_{\substack{A,i,A',i', \\ \sigma,\tau,\mu,\nu}} g_{\sigma\tau} E^{\sigma}_{Ai} E^{\tau}_{A'i'} F^{Ai}_{mn} F^{A'i'}{}^{mn}, \\
 F^{Ai}_{mn} &= \partial_m A_n^{Ai} - \partial_n A_m^{Ai} - if^{Aijk} A_m^j A_n^k, \\
 A_m^{Ai} &= \sum_{s,t} c^{Ai}_{st} \omega^{st}_m, \\
 \tau^{Ai} &= \sum_{s,t} c^{Aist} M_{st}.
 \end{aligned}$$

The integration of the action $\int E d^4x d^{(d-4)}x R^{(d)}$ over an even dimensional $(d - 4)$ space leads to the well known effective action for the vector gauge fields in $d = (3 + 1)$ space:

$\int E' d^4x \{ -\frac{1}{4} \sum_{A,i,m,n} F^{Ai}_{mn} F_{Ai}{}^{mn}$, where E' is determined by the gravitational field in $(3 + 1)$ space ($E' = 1$, if $(3 + 1)$ space is flat).

Let me present the proof that **the spin-charge-family theory offers understanding of the triangle anomalies cancellation in the standard model.**

The **standard model** has for massless quarks and leptons "miraculously" no triangle anomalies due to the fact that the sum of all possible traces $Tr[\tau^{Ai}\tau^{Bj}\tau^{Ck}]$ — where τ^{Ai} , τ^{Bi} and τ^{Ck} are the generators of one, of two or of three of the groups $SU(3)$, $SU(2)$ and $U(1)$ — over the representations of one family of the left handed fermions and anti-fermions (and separately of the right handed fermions and anti-fermions), contributing to the triangle currents, is equal to zero. This triangle anomaly cancellation follows straightforwardly if the $SO(3,1)$, $SU(2)$, $U(1)$ and $SU(3)$ are the subgroups of the orthogonal group $SO(13,1)$, as it is in the *spin-charge-family* theory. It is not difficult to see that also the $SO(10)$ anomaly cancellation works, provided that handedness and charges are related "by hand".

The triangle anomaly of the *standard model* occurs if the traces $Tr[\tau^{Ai}\tau^{Bj}\tau^{Ck}]$ are not zero for either the left handed quarks and leptons and anti-quarks and anti-leptons or the right handed quarks and leptons and anti-quarks and anti-leptons for the Feynman triangle diagrams in which the gauge vector fields of the type

$$\begin{aligned} &U(1) \times U(1) \times U(1), \\ &SU(2) \times SU(2) \times U(1), \\ &SU(3) \times SU(3) \times SU(3), \\ &SU(3) \times SU(3) \times U(1), \\ &U(1) \times \text{gravitational} \end{aligned}$$

contribute to the triangle anomaly.

From only the *standard model* assumptions point of view the cancellation of the triangle anomalies does look miraculously.

For $\sum_{i_{L,R}} (Y_{i_{L,R}})^3$ one obtains:

- ▶ For the left handed members:

$$3 \cdot 2 \cdot \left(\frac{1}{6}\right)^3 + 2 \cdot \left(-\frac{1}{2}\right)^3 + 3 \cdot \left(\left(-\frac{2}{3}\right)^3 + \left(\frac{1}{3}\right)^3\right) + 1^3, \text{ and}$$

- ▶ For the right handed members:

$$3 \cdot \left(\left(\frac{2}{3}\right)^3 + \left(-\frac{1}{3}\right)^3\right) + (-1)^3 + 3 \cdot 2 \cdot \left(-\frac{1}{6}\right)^3 + 2 \cdot \left(\frac{1}{3}\right)^3$$

Properties of the left handed quarks and leptons and of the left handed anti-quarks and anti-leptons in the first table and of the right handed quarks and leptons and the right handed anti-quarks and anti-leptons in the second table, as assumed by the *standard model*, are presented in the first eight columns. In the last two columns the two quantum numbers are added, which fermions and anti-fermions would have if the *standard model* groups $SO(3,1)$, $SU(2)$, $SU(3)$ and $U(1)$ are embedded into the $SO(13,1)$ group. The whole quark part appears, due to the colour charges, three times. These quantum numbers are the same for all the families.

Left handed spinors and antispinors

i_L	name	hand- edness $\Gamma^{(3,1)}$	weak charge τ^{13}	hyper charge Y	colour τ^{33}	charge τ^{38}	elm charge Q	$SU(2)_H$ charge τ^{23}	$U(1)_H$ charge τ^4
1_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
2_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
3_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
4_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
5_L	u_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$	0	$\frac{1}{6}$
6_L	d_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$	0	$\frac{1}{6}$
7_L	ν_L	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$-\frac{1}{2}$
8_L	e^L	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	0	$-\frac{1}{2}$
9_L	\bar{u}_L	-1	0	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
10_L	\bar{d}_L	-1	0	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
11_L	\bar{u}_L	-1	0	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
12_L	\bar{d}_L	-1	0	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
13_L	\bar{u}_L	-1	0	$-\frac{2}{3}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$
14_L	\bar{d}_L	-1	0	$\frac{1}{3}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{6}$
15_L	$\bar{\nu}_L$	-1	0	0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
16_L	\bar{e}_L	-1	0	1	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$

Right handed spinors and antispinors

i_L	name	hand- edness $\Gamma^{(3,1)}$	weak charge τ^{13}	hyper charge Y	colour τ^{33}	charge τ^{38}	elm charge Q	$SU(2)_{II}$ charge τ^{23}	$U(1)_{II}$ charge τ^4
1_R	u_R	1	0	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
2_R	d_R	1	0	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
3_R	u_R	1	0	$\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
4_R	d_R	1	0	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
5_R	u_R	1	0	$\frac{2}{3}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
6_R	d_R	1	0	$-\frac{1}{3}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{6}$
7_R	ν_R	1	0	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
8_R	e_R	1	0	-1	0	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
9_R	\bar{u}_R	1	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
10_R	\bar{d}_R	1	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
11_R	\bar{u}_R	1	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
12_R	\bar{d}_R	1	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
13_R	\bar{u}_R	1	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$	0	$-\frac{1}{6}$
14_R	\bar{d}_R	1	$\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
15_R	$\bar{\nu}_R$	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$
16_R	\bar{e}_R	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	0	$\frac{1}{2}$

i		$ \mathbf{^a\psi_i} >$	$\Gamma^{(3,1)}$	S^{12}	τ^{13}	τ^{23}	τ^4	Y
		(Anti)octet, $\Gamma^{(7,1)} = (-1)1$, $\Gamma^{(6)} = (1) - 1$ of (anti)quarks and (anti)leptons						
1	u_R^c1	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & (+) & & (+) & (+) & & (+) & [-] & [-] \end{smallmatrix}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
2	u_R^c1	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & [-] & & (+) & (+) & & (+) & [-] & [-] \end{smallmatrix}$	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
3	d_R^c1	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & (+) & & [-] & [-] & & (+) & [-] & [-] \end{smallmatrix}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
4	d_R^c1	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & [-] & & [-] & [-] & & (+) & [-] & [-] \end{smallmatrix}$	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
5	d_L^c1	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & (+) & & [-] & (+) & & (+) & [-] & [-] \end{smallmatrix}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	d_L^c1	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ - & (+i) & [-] & & [-] & (+) & & (+) & [-] & [-] \end{smallmatrix}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	u_L^c1	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ - & [-i] & (+) & & (+) & [-] & & (+) & [-] & [-] \end{smallmatrix}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^c1	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & [-] & & (+) & [-] & & (+) & [-] & [-] \end{smallmatrix}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
9	u_R^c2	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & (+) & & (+) & (+) & & [-] & (+) & [-] \end{smallmatrix}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
10	u_R^c2	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & [-] & & (+) & (+) & & [-] & (+) & [-] \end{smallmatrix}$	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
11	d_R^c2	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & (+) & & [-] & [-] & & [-] & (+) & [-] \end{smallmatrix}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
12	d_R^c2	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & [-] & & [-] & [-] & & [-] & (+) & [-] \end{smallmatrix}$	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
13	d_L^c2	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ [-i] & (+) & & [-] & (+) & & [-] & (+) & [-] \end{smallmatrix}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
14	d_L^c2	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ - & (+i) & [-] & & [-] & (+) & & [-] & (+) & [-] \end{smallmatrix}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
15	u_L^c2	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ - & [-i] & (+) & & (+) & [-] & & [-] & (+) & [-] \end{smallmatrix}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
16	u_L^c2	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9 & 10 & 11 & 12 & 13 & 14 \\ (+i) & [-] & & (+) & [-] & & [-] & (+) & [-] \end{smallmatrix}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

In the case of embedding the *standard model* groups into $SO(13, 1)$ we have

$$\begin{aligned} \sum_{i_{L,R}} (Y_{i_{L,R}})^3 &= \sum_{i_{L,R}} (\tau_{i_{L,R}}^4 + \tau_{i_{L,R}}^{23})^3 = \sum_{i_{L,R}} (\tau_{i_{L,R}}^4)^3 + \sum_{i_{L,R}} (\tau_{i_{L,R}}^{23})^3 \\ &+ \sum_{i_{L,R}} 3 \cdot (\tau_{i_{L,R}}^4)^2 \cdot \tau_{i_{L,R}}^{23} + \sum_{i_{L,R}} 3 \cdot \tau_{i_{L,R}}^4 \cdot (\tau_{i_{L,R}}^{23})^2, \end{aligned}$$

for either the left, i_L , or the right, i_R , handed members. Table of one Weyl representation demonstrates clearly that $(Y_{i_{L,R}})^3 = 0$ without really making any algebraic evaluation. Namely, the last column of Table ?? manifests that $\sum_{i_L} (\tau_{i_L}^4)^3 = 0$ [in details: $\sum_{i_L} (\tau_{i_L}^4)^3 = 2 \cdot 3 \cdot (\frac{1}{6})^3 + 2 \cdot 3 \cdot (-\frac{1}{6})^3 + 2 \cdot (-\frac{1}{2})^3 + 2 \cdot (\frac{1}{2})^3 = 0$]. Table ?? also demonstrates (the last but one column) that $\sum_{i_L} (\tau_{i_L}^{23})^3 = 0$ [$= (3 + 1) \cdot ((-\frac{1}{2})^3 + (\frac{1}{2})^3)$], and that also $\sum_{i_R} (\tau_{i_R}^{23})^3 = 0$ [$= (3 + 1) \cdot ((\frac{1}{2})^3 + (-\frac{1}{2})^3)$].