Phenomenological quark mass matrices

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Flavour states and mass states

Weak interactions occur between flavour states.

Electromagnetic and strong interactions occur between mass eigenstates.

Flavour states are in reality to be understood as weak eigenstates.

This is the root of the weak mixing pheonmena.

The Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \\ V_{ud} & V_{us} & V_{ub} \end{pmatrix}$$

appears in the charged current Lagrangian

$$\mathcal{L}_{cc} = -\frac{g}{2\sqrt{2}}\,\bar{\psi}_L \gamma^\mu V \psi_L' W_\mu + h.\,c.$$

where ψ and ψ' are quark fields with charges Q and $Q-1$, correspondingly.

$\mathcal{L}_{cc}$ is usually interpreted as a (left-handed) up-sector quark interacting with a superposition of (left-handed) down-sector quarks.

From the perspective of weak interactions, it is an interaction between flavour states f, f' which appear in the mass Lagrangian

$$\mathcal{L}_{mass} = \bar{f} M f + \bar{f'} M' f'$$

f, f' are flavour states of charge 2/3 and -1/3, respectively, and the corresponding mass matrices are denoted as $M = M(2/3)$ and $M' = M'(-1/3)$.

Distinguish between flavour states and the mass eigenstates, which are the ``physical particles" taking part in strong and electromagnetic interactions.

The lepton is different.

Neutrino mass eigenstates never appear in interactions - they merely propagate in free space.

When a neutrino of a given flavour interacts, it by definition involves a charged lepton of the same generation: what we call the electron-neutrino ν_e is the superposition of neutrino mass states which interacts with the electron, and likewise for μ and τ; in that sense the conservation of the lepton number is a tautology.

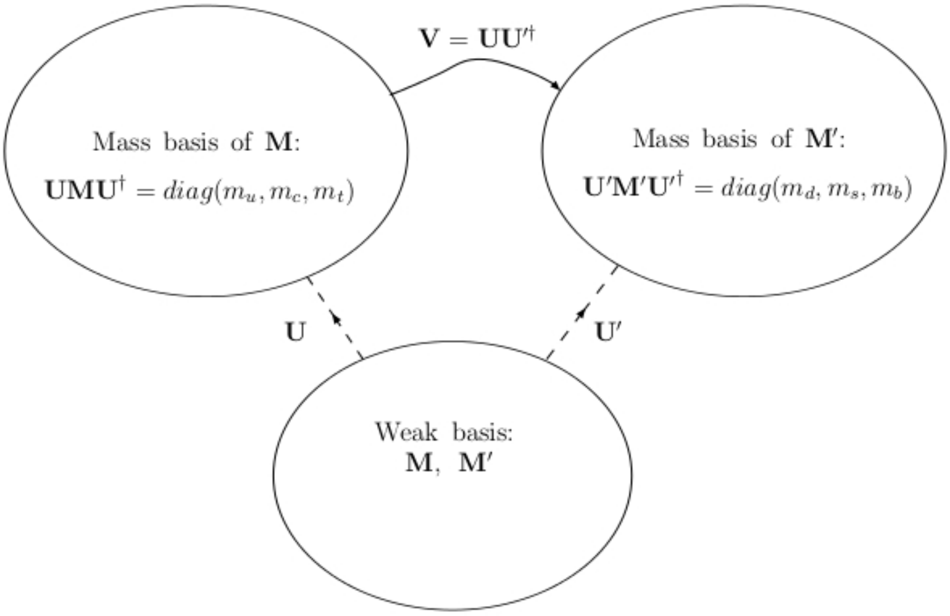
Since the charged leptons interact weakly with flavour states, e, μ, τ are taken to be both weak eigenstates and mass eigenstates. The only mixing matrix that occurs in the lepton sector is the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix U which only operates on neutrino states.

So we go from the interaction basis to the mass bases by rotating the matrices M and M' in flavour space, by the unitary matrices U and U',

$$M \rightarrow UMU^{\dagger} = D = diag(m_u, m_c, m_t)$$

$$M' \rightarrow U'M'U'^{\dagger} = D' = diag(m_d, m_s, m_b)$$

and the mixing matrix $V = UU'^{\dagger}$ acting as bridge between the mass bases,



Factorizing the weak mixing matrix

The most obvious and ``symmetric" factorization of the CKM mixing matrix, following the well-known standard parametrization with three Euler angles $\alpha, \beta, 2\theta$,

$$V = \begin{pmatrix} c_\beta c_{2\theta} & s_\beta c_{2\theta} & s_{2\theta} e^{-i\delta} \\ -c_\beta s_\alpha s_{2\theta} e^{i\delta} - s_\beta c_\alpha & -s_\beta s_\alpha s_{2\theta} e^{i\delta} + c_\beta c_\alpha & s_\alpha c_{2\theta} \\ -c_\beta c_\alpha s_{2\theta} e^{i\delta} + s_\beta s_\alpha & -s_\beta c_\alpha s_{2\theta} e^{i\delta} - c_\beta s_\alpha & c_\alpha c_{2\theta} \end{pmatrix} = U U'^{\dagger}$$

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corresponds to the diagonalizing rotation matrices for the up- and down-sectors

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} W =$$

$$= W \begin{pmatrix} c_\theta e^{-i\gamma} & 0 & s_\theta e^{-i\gamma} \\ -s_\alpha s_\theta e^{i\gamma} & c_\alpha & s_\alpha c_\theta e^{i\gamma} \\ -c_\alpha s_\theta e^{i\gamma} & -s_\alpha & c_\alpha c_\theta e^{i\gamma} \end{pmatrix} W,$$

and

$$U' = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} W =$$

$$W \begin{pmatrix} c_\beta c_\theta e^{-i\gamma} & -s_\beta & -c_\beta s_\theta e^{-i\gamma} \\ s_\beta c_\theta e^{-i\gamma} & c_\beta & -s_\beta s_\theta e^{-i\gamma} \\ s_\theta e^{i\gamma} & 0 & c_\theta e^{i\gamma} \end{pmatrix} W$$

$W = W(\rho)$ is a unitary matrix chosen so the mass matrices of the two charge sectors contain the same phase γ.

$$W(\rho) \sim \begin{pmatrix} 0 & \cos\rho & \pm\sin\rho \\ 1 & 0 & 0 \\ 0 & \mp\sin\rho & \cos\rho \end{pmatrix}, \begin{pmatrix} \cos\rho & 0 & \pm\sin\rho \\ 0 & 1 & 0 \\ \mp\sin\rho & 0 & \cos\rho \end{pmatrix}, \begin{pmatrix} \cos\rho & \pm\sin\rho & 0 \\ 0 & 0 & 1 \\ \mp\sin\rho & \cos\rho & 0 \end{pmatrix}$$

ρ is unknown, whereas α, β, θ and γ correspond to the parameters in the standard parametrization, with

$\gamma = \delta/2, \delta = 1.2 \pm 0.08$ rad, and $2\theta = 0.201 \pm 0.011^\circ$,

$\alpha = 2.38 \pm 0.06^\circ$ and $\beta = 13.04 \pm 0.05^\circ$.

In this scheme, α and β are the rotation angles operating in the up-sector and the down-sector, respectively.

This gives the mass matrices

$$M = U^\dagger diag(m_u, m_c, m_t) U \text{ and } M' = U'^\dagger diag(m_d, m_s, m_b) U',$$

i.e. for the up-sector,

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} = W^\dagger(\rho) \begin{pmatrix} Xc_\theta^2 + Ys_\theta^2 & Zs_\theta\, e^{-i\gamma} & (X-Y)c_\theta s_\theta \\ Zs_\theta\, e^{i\gamma} & Y - 2Z\cot 2\alpha & -Zc_\theta\, e^{i\gamma} \\ (X-Y)c_\theta s_\theta & -Zc_\theta\, e^{-i\gamma} & Xs_\theta^2 + Yc_\theta^2 \end{pmatrix} W(\rho)$$

$X = m_u,$

$Y = m_c \sin^2\alpha + m_t \cos^2\alpha,$

$Z = (m_t - m_c)\sin\alpha\cos\alpha = \sqrt{(m_t - Y)(Y - m_c)},$

m_u, m_c, m_t are the masses of the up-, charm- and top-quark.

The down-sector mass matrix,

$$M' = \begin{pmatrix} M'_{11} & M'_{12} & M'_{13} \\ M'_{21} & M'_{22} & M'_{23} \\ M'_{31} & M'_{32} & M'_{33} \end{pmatrix} = W^\dagger(\rho) \begin{pmatrix} X' s_\theta^2 + Y' c_\theta^2 & Z' c_\theta \, e^{i\gamma} & (X' - Y') c_\theta s_\theta \\ Z' c_\theta \, e^{-i\gamma} & Y' + 2Z' \cot 2\beta & -Z' s_\theta \, e^{-i\gamma} \\ (X' - Y') c_\theta s_\theta & -Z' s_\theta \, e^{i\gamma} & X' c_\theta^2 + Y' s_\theta^2 \end{pmatrix}$$

$$X' = m_b,$$

$$Y' = m_d \cos^2\beta + m_s \sin^2\beta,$$

$$Z' = (m_s - m_d) \sin\beta \cos\beta = \sqrt{(m_s - Y')(Y' - m_d)},$$

m_d, m_s, m_b are the masses of the down-, strange- and bottom-quark, respectively. The two mass matrices thus display similar textures.

With $Y = m_c \sin^2 \alpha + m_t \cos^2 \alpha, Z = (m_t - m_c) \sin \alpha \cos \alpha, Y' = m_d \cos^2 \beta + m_s \sin^2 \beta$, and $Z' = (m_s - m_d) \sin \beta \cos \beta$, we can moreover write

$$m_u = X,\ \ m_c = Y - Z \cot \alpha,\ \ m_t = Y + Z \tan \alpha,$$
$$m_d = Y' - Z' \tan \beta,\ \ m_s = Y' + Z' \cot \beta,\ \ m_b = X',$$

Choose $W(\rho)$ as

$$W(\rho) = \begin{pmatrix} \cos\rho & -\sin\rho & 0 \\ 0 & 0 & 1 \\ \sin\rho & \cos\rho & 0 \end{pmatrix},$$

which gives the up-sector mass matrix

$$M = \begin{pmatrix} X\cos^2\mu + Y\sin^2\mu & (Y-X)\sin\mu\cos\mu & -Z\sin\mu\, e^{-i\gamma} \\ (Y-X)\sin\mu\cos\mu & X\sin^2\mu + Y\cos^2\mu & -Z\cos\mu\, e^{-i\gamma} \\ -Z\sin\mu\, e^{i\gamma} & -Z\cos\mu\, e^{i\gamma} & F \end{pmatrix}$$

$\mu = \rho - \theta.$

Depending on the value of $\mu = \rho - \theta$, we get different matrix textures, e.g. for $\rho - \theta = 0$ or π, we get the simple form

$$M(0,\pi) = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & -Ze^{-i\gamma} \\ 0 & -Ze^{i\gamma} & F \end{pmatrix},$$

and for $\rho - \theta = \pi/2$, equally simple

$$M(\pi/2) = \begin{pmatrix} Y & 0 & -Ze^{-i\gamma} \\ 0 & X & 0 \\ -Ze^{i\gamma} & 0 & F \end{pmatrix}$$

The down-sector mass matrix is

$$M' = \begin{pmatrix} X' \sin^2 \mu' + Y' \cos^2 \mu' & (X' - Y') \sin \mu' \cos \mu' & Z' \cos \mu' \, e^{i\gamma} \\ (X' - Y') \sin \mu' \cos \mu' & X' \cos^2 \mu' + Y' \sin^2 \mu' & -Z' \sin \mu' \, e^{i\gamma} \\ Z' \cos \mu' \, e^{-i\gamma} & -Z' \sin \mu' \, e^{-i\gamma} & F' \end{pmatrix}$$

$\mu' = \rho + \theta.$

Different μ' -values correspond to different matrices.

E.g. for $\mu' = \rho + \theta = 0$ or π, we get

$$M'(0,\pi) = \begin{pmatrix} Y' & 0 & Z'e^{i\gamma} \\ 0 & X' & 0 \\ Z'e^{-i\gamma} & 0 & F' \end{pmatrix}$$

and for $\mu' = \rho + \theta = \pi/2$, we get

$$M'(\pi/2) = \begin{pmatrix} X' & 0 & 0 \\ 0 & Y' & -Z'e^{i\gamma} \\ 0 & -Z'e^{-i\gamma} & F' \end{pmatrix}$$

The texture four matrices

$$\begin{pmatrix} A & 0 & 0 \\ 0 & C & B \\ 0 & B^* & D \end{pmatrix} \text{and} \begin{pmatrix} A & 0 & B \\ 0 & C & 0 \\ B^* & 0 & D \end{pmatrix}$$

are however found to be ruled out.

This gives the constraint

$$\rho \neq \frac{1}{2} N\pi \pm \theta$$

where $N \in \mathcal{Z}$

Reparametrizations

We rewrite the mass matrices using the relations

$$Xc_\mu^2 + Ys_\mu^2 = (Y-X)s_\mu^2 + X, \qquad Xs_\mu^2 + Yc_\mu^2 = (Y-X)c_\mu^2 + X,$$

$$X' s_{\mu'}^2 + Y' c_{\mu'}^2 = (Y'-X')c_{\mu'}^2 + X', \qquad X' c_\mu^2 + Y' s_\mu^2 = (Y'-X')s_{\mu'}^2 + X',$$

$$M = \begin{pmatrix} Xc_\mu^2 + Ys_\mu^2 & (Y-X)s_\mu c_\mu & -Zs_\mu e^{-i\gamma} \\ (Y-X)s_\mu c_\mu & Xs_\mu^2 + Yc_\mu^2 & -Zc_\mu e^{-i\gamma} \\ -Zs_\mu e^{i\gamma} & -Zc_\mu e^{i\gamma} & F \end{pmatrix} =$$

$$= B\begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{i\gamma} \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{-i\gamma} \end{pmatrix} + \begin{pmatrix} X & & \\ & X & \\ & & X+A \end{pmatrix}$$

$$M' = \begin{pmatrix} X' s_{\mu'}^2 + Y' c_{\mu'}^2 & (X' - Y') s_{\mu'} c_{\mu'} & Z' c_{\mu'} \, e^{i\gamma} \\ (X' - Y') s_{\mu'} c_{\mu'} & X' c_{\mu'}^2 + Y' s_{\mu'}^2 & -Z' s_{\mu'} \, e^{i\gamma} \\ Z' c_{\mu'} \, e^{-i\gamma} & -Z' s_{\mu'} \, e^{-i\gamma} & F' \end{pmatrix} =$$

$$= B' \begin{pmatrix} \cos\mu' & & \\ & -\sin\mu' & \\ & & G' e^{-i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \cos\mu' & & \\ & -\sin\mu' & \\ & & G' e^{i\gamma} \end{pmatrix} +$$

$$+ \begin{pmatrix} X' & & \\ & X' & \\ & & X' + A' \end{pmatrix}$$

$$X = m_u, \quad \mu = \rho - \theta, \;\; B = Y - X = m_c s_\alpha^2 + m_t c_\alpha^2 - m_u,$$

$$X' = m_b, \quad ,\mu' = \rho + \theta, \quad B' = Y' - X' = m_s s_\beta^2 + m_d c_\beta^2 - m_b,$$

$$G = -\frac{(m_t - m_c) s_\alpha c_\alpha}{(m_c s_\alpha^2 + m_t c_\alpha^2 - m_u)}, \quad A = \frac{(m_c - m_u)(m_t - m_u)}{(m_c s_\alpha^2 + m_t c_\alpha^2 - m_u)},$$

$$G' == \frac{(m_s - m_d) s_\beta c_\beta}{(m_d c_\beta^2 + m_s s_\beta^2 - m_b)}, \quad A' = \frac{(m_d - m_b)(m_s - m_b)}{(m_d c_\beta^2 + m_s s_\beta^2 - m_b)},$$

$$\alpha = arctan\left(\sqrt{\frac{m_t - Y}{Y - m_c}}\right) = 2.38 \pm 0.06^\circ, \quad \beta = arctan\left(\sqrt{\frac{Y' - m_d}{m_s - Y'}}\right) = 13.04 \pm 0.05^\circ.$$

The matrices of the two charge sectors thus display great similarities, and $A \neq 0$ and $A' \neq 0$ moreover means that $m_c \neq m_u, m_t \neq m_u, m_d \neq m_b, m_s \neq m_b$ and with the additional condition $m_c \neq m_t$ and $m_d \neq m_b$, we almost have the prerequisite for CP-violation - which basically says that CP-violation occurs with the arrival of a third, non-degenerate family (and a complex phase).

Democratic matrices

The Standard model fermions get their masses from the Yukawa couplings by the Higgs mechanism, the model however supplies no information about the form of the Yukawa coupling matrices.

Democratic philosophy: just as the couplings to the gauge bosons of the strong, weak and electromagnetic interactions are identical for all the fermions in a given charge sector, the fermions within each charge sector should also have identical Yukawa couplings.

$$M_0 = k\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \equiv k\mathbf{N}$$

k has dimension mass.

The mass spectrum $(0, 0, 3k)$ reflects the observed fermion mass spectra.

The mass Lagrangian reads

$$\mathcal{L}_{mass} = k\bar{f}\mathbf{N}f = \sum_{i=1,j=1}^{3} k\bar{f}_i f_j,$$

thus the flavours f_j in a given charge sector are initially indistinguishable, with the same Yukawa coupling for all the flavours: it is a totally flavour symmetric situation.

Interpretations

One can perceive a matrix like

$$M = B\begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{i\gamma} \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{-i\gamma} \end{pmatrix} + \begin{pmatrix} X & & \\ & X & \\ & & X + A \end{pmatrix}$$

as starting out totally flavour symmetric, in accordance with the democratic scheme.

Our initial assumption is that the matrices which diagonalize the up-sector and down-sector mass matrices, are given by the factorization of the Cabibbi-Koabayashi-Maskawa matrix.

The parameters of the CKM matrix are well-known, so the only remaining ``steering-parameter" is ρ, in the sense that different values of ρ correspond to mass matrices of different form.

The constraint

$$\rho \neq \frac{1}{2} N\pi \pm \theta$$

exclude certain values of ρ, but we do not know what value(s) ρ actually takes.

Using the numerical quark mass values at M_Z,

$$\begin{array}{lll} m_u(M_Z) = 1.24MeV, & m_c(M_Z) = 624MeV, & m_t(M_Z) = 171550MeV \\ m_d(M_Z) = 2.69MeV, & m_s(M_Z) = 53.8MeV, & m_b(M_Z) = 2850MeV \end{array}$$

and calculating numerical mass matrices for a set of different ρ-values, it (unsurprisningly) turns out that the greatest structural similarity between $M(2/3)$ and $M'(-1/3)$ occurs for $\rho \sim \pi/4$, i.e.

$$M(\rho = \pi/4) = \begin{pmatrix} 85327.85 & 85626.47 & -5005.93e^{-i\gamma} \\ 85626.47 & 85928.63 & -5023.52e^{-i\gamma} \\ -5005.93e^{i\gamma} & -5023.52e^{i\gamma} & 918.76 \end{pmatrix} MeV$$

$$M'(\rho = \pi/4) = \begin{pmatrix} 1432.71 & 1422.27 & 8.15e^{i\gamma} \\ 1422.27 & 1422.73 & -8.18e^{i\gamma} \\ 8.15e^{-i\gamma} & -8.18e^{-i\gamma} & 51.06 \end{pmatrix} MeV,$$

Disregarding signs, these matrices display the same approximate texture

$$M, M' \sim \begin{pmatrix} A & A & Be^{\pm i\gamma} \\ A & A & Be^{\pm i\gamma} \\ Be^{\mp i\gamma} & Be^{\mp i\gamma} & C \end{pmatrix} + \Lambda$$

with

$$\Lambda = \begin{pmatrix} 0 & \varepsilon & 0 \\ \varepsilon & 0 & \varepsilon' \\ 0 & \varepsilon' & 0 \end{pmatrix},$$

where $\varepsilon, \varepsilon'$ are tiny, $A_{(2/3)}/A_{(-1/3)} \sim m_t/m_b$ and $B < A$.

This schematic matrix can be interpreted as emerging from an initial form

$$M_{(initial)} = \begin{pmatrix} A & A & B \\ A & A & B \\ B & B & C \end{pmatrix} = \begin{pmatrix} \eta & & \\ & \eta & \\ & & \xi \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \eta & & \\ & \eta & \\ & & \xi \end{pmatrix} + \begin{pmatrix} 0 & & \\ & 0 & \\ & & \Sigma \end{pmatrix}$$

such that for the up-sector, $\eta = 292.6MeV, \xi = 17.1MeV$ and $\Sigma = 625.1MeV \approx m_c/c_\alpha^2$; and for the down-sector, $\eta' = 37.8MeV, \xi' = 174.8MeV$ and $\Sigma' = 51.01MeV \approx m_s c_\beta^2$.

So by factorizing the mixing matrix, we end up with a specific form of quark mass matrices with traces of an initial democratic texture, which generically be written as

$$M = k\begin{pmatrix} s_\eta^2 + \Sigma & s_\eta c_\eta & s_\eta e^{-i\gamma} \\ s_\eta c_\eta & c_\eta^2 + \Sigma & c_\eta e^{-i\gamma} \\ s_\eta e^{-i\gamma} & c_\eta e^{-i\gamma} & 1 + \Omega \end{pmatrix} =$$

$$= k\left[\begin{pmatrix} \sin\eta & & \\ & \cos\eta & \\ & & e^{i\gamma} \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \sin\eta & & \\ & \cos\eta & \\ & & e^{-i\gamma} \end{pmatrix} + \begin{pmatrix} \Sigma & & \\ & \Sigma & \\ & & \Omega \end{pmatrix}\right]$$

where only k is dimensional.