Quark masses, mixing and $\Delta F = 2$ in Neutral Mesons from a gauged $SU(3)_F$ family symmetry

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Open questions:

There are several open questions, motivations, to look for physics "Beyond the Standard Model" (BSM):

- Neutrino masses and mixing, oscillation
- Hierarchy of masses
- Dart Matter (DM), Dart Energy (DE), Matter-Antimatter Asymmetry.

Main goal of this BSM: To account for the hierarchy of fermion masses:

$$m_t \gg m_c \gg m_u \quad , \quad m_b \gg m_s \gg m_d \quad , \quad m_\tau \gg m_\mu \gg m_e,$$

and quark and lepton mixing matrices: V_{CKM} and U_{PMNS}.

Standard Model "SM"

Ordinary Fermions: $Q = T_{3L} + \frac{1}{2} Y$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$q^o_{jL} = \begin{pmatrix} u^o_{jL} \\ d^o_{jL} \end{pmatrix}$	3	2	$\frac{1}{3}$
u^o_{jR}	3	1	$\frac{4}{3}$
d^o_{jR}	3	1	$-\frac{2}{3}$
$l^o_{jL} = \begin{pmatrix} \nu^o_{jL} \\ e^o_{jL} \end{pmatrix}$	1	2	-1
e^o_{jR}	1	1	-2
ν^o_{jR}	1	1	0
$\phi_{SM} = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}$	1	2	+1
$\tilde{\phi}_{SM} = i\,\sigma_2\,\phi^*_{SM}$	1	2	-1

Table: SM fermion content and charges, $j = 1, 2, 3$ family index

Comments:

- $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: Standard Model Group
- Neutrinos are massless
- Anomalies cancel for each family, $j = 1, 2, 3$.
- $\overline{q^o_{iL}}\, \phi_{SM}\, u^o_{jR}$, $i, j = 1, 2, 3$ is gauge invariant. So, the Yukawa couplings $Y_{i,j}\ \overline{q^o_{iL}}\, \phi_{SM}\, u^o_{jR}$, $i, j = 1, 2, 3$ are gauge invariant, with $Y_{i,j}$ completely arbitrary.
- SM gauge bosons: Z, A(photon), and H(Higgs) couplings to fermions do not change flavor.
- $\frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\ V_{CKM}\, \gamma_\mu \Psi_{dL}\ W^{+\mu}$

MODEL WITH LOCAL $SU(3)_F$ FAMILY SYMMETRY

	$SU(3)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
ψ_q^o	3	3	2	$\frac{1}{3}$
ψ_{uR}^o	3	3	1	$\frac{4}{3}$
ψ_{dR}^o	3	3	1	$-\frac{2}{3}$
ψ_l^o	3	1	2	-1
ψ_{eR}^o	3	1	1	-2
$\psi_{\nu R}^o$	3	1	1	0
$U_{L,R}^o$	1	3	1	$\frac{4}{3}$
$D_{L,R}^o$	1	3	1	$-\frac{2}{3}$
$E_{L,R}^o$	1	1	1	-2
$N_{L,R}^o$	1	1	1	0

Table: Fermion content and charges

	$SU(3)_F$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
η_2, η_3	3	1	1	0
Φ^u	3	1	2	-1
Φ^d	3	1	2	+1

Table: Scalars fields and charges

- η_2, η_3 are introduced to break spontaneously $SU(3)_F$.
- Φ^u, Φ^d are introduced to break spontaneously the $SU(2)_L \times U(1)_Y$.
- u-quarks and neutrinos coupled only to Φ^u
- d-quarks and charged leptons couple only to Φ^d

$\overline{\psi_q^o}\, \Phi^u\, \psi_{uR}^o$ **and** $\overline{\psi_q^o}\, \Phi^d\, \psi_{dR}^o$ **are forbidden by the** $SU(3)_F$ **family symmetry.**

$\bar{3} \times 3 = 1 + 8\ ,\ 3 \times 3 = \bar{3} + 6\ ,\ \bar{3} \times 3 \times 3 = (1+8) \times 3 = 3 + \bar{6} + 3 + 15$

$$i\mathcal{L}_{int,SU(3)_F} =$$
$$\frac{g_H}{2}\left(\bar{f}_1^o\gamma_\mu f_1^o - \bar{f}_2^o\gamma_\mu f_2^o\right) Z_1^\mu + \frac{g_H}{2\sqrt{3}}\left(\bar{f}_1^o\gamma_\mu f_1^o + \bar{f}_2^o\gamma_\mu f_2^o - 2\bar{f}_3^o\gamma_\mu f_3^o\right) Z_2^\mu$$
$$+ \frac{g_H}{\sqrt{2}}\left(\bar{f}_1^o\gamma_\mu f_2^o Y_1^+ + \bar{f}_1^o\gamma_\mu f_3^o Y_2^+ + \bar{f}_2^o\gamma_\mu f_3^o Y_3^+ + h.c.\right)$$

g_H is the $SU(3)_F$ coupling constant, Z_1, Z_2 and $Y_i^{\pm}$, $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons, which have **flavor changing couplings to both left and right handed SM fermions**

$$g_H \begin{pmatrix} \bar{f}_1^o & \bar{f}_2^o & \bar{f}_3^o \end{pmatrix} \gamma_\mu \begin{pmatrix} \frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} & \frac{Y_1^+}{\sqrt{2}} & \frac{Y_2^+}{\sqrt{2}} \\ \frac{Y_1^-}{\sqrt{2}} & -\frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} & \frac{Y_3^+}{\sqrt{2}} \\ \frac{Y_2^-}{\sqrt{2}} & \frac{Y_3^-}{\sqrt{2}} & -\frac{Z_2}{\sqrt{3}} \end{pmatrix}^\mu \begin{pmatrix} f_1^o \\ f_2^o \\ f_3^o \end{pmatrix}$$

Yukawa couplings:

Dirac Yukawa couplings:

$$
\begin{aligned}
&h_u\,\overline{\psi^o_q}\,\Phi^u\,U^o_R \;+\; h_{2u}\,\overline{\psi^o_{uR}}\,\eta_2\,U^o_L \;+\; h_{3u}\,\overline{\psi^o_{uR}}\,\eta_3\,U^o_L \;+\; M_U\,\overline{U^o_L}\,U^o_R\\
&h_d\,\overline{\psi^o_q}\,\Phi^d\,D^o_R \;+\; h_{2d}\,\overline{\psi^o_{dR}}\,\eta_2\,D^o_L \;+\; h_{3d}\,\overline{\psi^o_{dR}}\,\eta_3\,D^o_L \;+\; M_D\,\overline{D^o_L}\,D^o_R\\
&h_\nu\,\overline{\psi^o_l}\,\Phi^u\,N^o_R \;+\; h_{2\nu}\,\overline{\psi^o_{\nu R}}\,\eta_2\,N^o_L \;+\; h_{3\nu}\,\overline{\psi^o_{\nu R}}\,\eta_3\,N^o_L \;+\; m_D\,\overline{N^o_L}\,N^o_R\\
&h_e\,\overline{\psi^o_l}\,\Phi^d\,E^o_R \;+\; h_{2e}\,\overline{\psi^o_{eR}}\,\eta_2\,E^o_L \;+\; h_{3e}\,\overline{\psi^o_{eR}}\,\eta_3\,E^o_L \;+\; M_E\,\overline{E^o_L}\,E^o_R\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad +h.c
\end{aligned}
$$

Majorana Yukawa couplings:

$$
h_{2R}\,\overline{\psi^o_{\nu R}}\,\eta_2\,(N^o_R)^c \;+\; h_{3R}\,\overline{\psi^o_{\nu R}}\,\eta_3\,(N^o_R)^c \;+\; m_R\,\overline{N^o_R}\,(N^o_R)^c
$$

$$
h_L\,\overline{\psi^o_l}\,\Phi^u\,(N^o_L)^c \;+\; m_L\,\overline{N^o_L}\,(N^o_L)^c \;+h.c
$$

Scalar Potential

$$V(\eta_2, \eta_3, \Phi^u, \Phi^d) =$$

$$-\mu_2^2\, \eta_2^\dagger\, \eta_2 - \mu_3^2\, \eta_3^\dagger\, \eta_3 - \mu_{23}^2\, (\eta_2^\dagger\, \eta_3 + h.c) - \mu_u^2\, \Phi^{u\dagger}\Phi^u - \mu_d^2\, \Phi^{d\dagger}\Phi^d$$

$$\lambda_2\, (\eta_2^\dagger\, \eta_2)^2 + \lambda_3\, (\eta_3^\dagger\, \eta_3)^2 + \lambda_{23}\, (\eta_2^\dagger\, \eta_3)^\dagger\, (\eta_2^\dagger\, \eta_3)$$

$$\lambda_u\, (\Phi^{u\dagger}\, \Phi^u)^2 + \lambda_d\, (\Phi^{d\dagger}\, \Phi^d)^2 + \lambda_{ud}\, (\Phi^{u\dagger}\, \Phi^u)^\dagger\, \Phi^{d\dagger}\, \Phi^d$$

$$(\eta_2^\dagger\, \eta_2)\, \left[c_3\, \eta_3^\dagger\, \eta_3 + c_{23}\, (\eta_2^\dagger\, \eta_3 + h.c) + c_u\, \Phi^{u\dagger}\, \Phi^u + c_d\, \Phi^{d\dagger}\, \Phi^d\right]$$

$$(\eta_3^\dagger\, \eta_3)\, \left[k_{23}\, (\eta_2^\dagger\, \eta_3 + h.c) + k_u\, \Phi^{u\dagger}\, \Phi^u + k_d\, \Phi^{d\dagger}\, \Phi^d\right]$$

$$(\eta_2^\dagger\, \eta_3 + h.c)\, \left[p_u\, \Phi^{u\dagger}\, \Phi^u + p_d\, \Phi^{d\dagger}\, \Phi^d\right]$$

Spontaneous Symmetry breaking (SSB)

We would like to be consistent with low energy Standard Model(SM) and simultaneously generate and account for the hierarchy of quark and lepton masses and mixing

$SU(3)_F \times G_{SM}$	$\rightarrow$	G_{SM}	$\rightarrow$	$SU(3)_C \times U(1)_Q$
SM fermions are massless		SM fermions remain massless		SM fermions become massive (PDG known values)

$SU(3)_F$ family symmetry breaking

Two SM singlet scalars are introduced in the fundamental representation of $SU(3)_F$:

$$\langle \eta_2 \rangle = \begin{pmatrix} 0 \\ \Lambda_2 \\ 0 \end{pmatrix} \quad , \quad \langle \eta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ \Lambda_3 \end{pmatrix}$$

$$SU(3)_F \times G_{SM} \quad \xrightarrow{\langle \eta_3 \rangle} \quad SU(2)_F \, ? \times G_{SM} \quad \xrightarrow{\langle \eta_2 \rangle} \quad SU(2)_g \times G_{SM}$$

FCNC ?

Λ_3: 5 very heavy boson masses ($\gtrsim 10^{11}\ GeV$)
Λ_2: 3 heavy boson masses (a few $TeV's$) , **"approximate $SU(2)_g$ global symmetry"**

To suppress properly FCNC like, for instance: $\mu \to e\gamma$ ($Br < 5.7 \times 10^{-13}$) , $\mu \to e\,e\,e$ ($Br < 1 \times 10^{-12}$) , $K^o - \bar{K}^o$, $D^o - \bar{D}^o$, it is relevant which gauge bosons are heavy, and which ones are very heavy

The above scalar fields and VEV's break completely the $SU(3)_F$ family symmetry, generating the mass terms

- $\langle \eta_2 \rangle$: $\quad \frac{g_{H_2}^2 \Lambda_2^2}{2}(Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2}^2 \Lambda_2^2}{4}(Z_1^2 + \frac{Z_2^2}{3} - 2Z_1 \frac{Z_2}{\sqrt{3}})$
- $\langle \eta_3 \rangle$: $\quad \frac{g_{H_3}^2 \Lambda_3^2}{2}(Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda_3^2 \frac{Z_2^2}{3}$

FCNC in Neutral Mesons

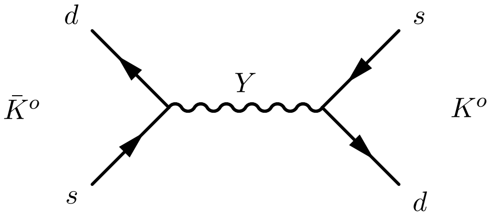
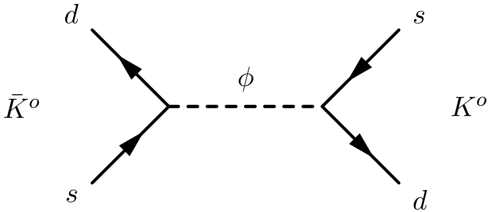


Figure: Generic tree level contribution to $K^o - \bar{K}^o$ from the $SU(3)_F$ horizontal gauge bosons and scalar fields



PDG2016: CKM quark-mixing matrix

To illustrate the level of suppression required for BSM contributions, consider a class of models in which the unitarity of the CKM is maintained, and the dominant effect of the new physics is to modify the neutral meson amplitudes by

$$\frac{z_{ij}}{\Lambda^2}\,(\bar{q}_i\gamma^\mu P_L q_j)^2$$

The existent data imply that

$$\frac{\Lambda}{\sqrt{|z_{ij}|}} \gtrsim \begin{cases} 10^4\ \text{TeV for } K^o - \bar{K}^o \\ 10^3\ \text{TeV for } D^o - \bar{D}^o \\ 500\ \text{TeV for } B^o - \bar{B}^o \\ 100\ \text{TeV for } B_s^o - \bar{B}_s^o \end{cases}$$

Electroweak Symmetry Breaking

In this scenario we introduce two triplets of $SU(2)_L$ Higgs doublets; $\Phi^u = (3, 1, 2, -1), \Phi^d = (3, 1, 2, +1)$:

$$\Phi^u = \begin{pmatrix} \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_1^u \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_2^u \\ \begin{pmatrix} \phi^o \\ \phi^- \end{pmatrix}_3^u \end{pmatrix} \& \langle \Phi^u \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u1} \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u2} \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u3} \\ 0 \end{pmatrix} \end{pmatrix} , \; \Phi^d = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_1^d \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_2^d \\ \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}_3^d \end{pmatrix} \& \langle \Phi^d \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d1} \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d2} \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d3} \end{pmatrix} \end{pmatrix}$$

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \quad \xrightarrow{\langle \Phi^u \rangle \,,\, \langle \Phi^d \rangle} \quad SU(3)_C \times U(1)_Q$$

Contribute to the W and Z boson masses:

$$\frac{g^2}{4}\,(v_u^2+v_d^2)\,W^+W^- + \frac{(g^2+g'^2)}{8}\,(v_u^2+v_d^2)\,Z_o^2$$
$$+\frac{1}{4}\sqrt{g^2+g'^2}\,g_H\,Z_o\left[(v_{1u}^2-v_{2u}^2-v_{1d}^2+v_{2d}^2)\,Z_1+(v_{1u}^2+v_{2u}^2-2v_{3u}^2-v_{1d}^2-v_{2d}^2+2v_{3d}^2)\,\frac{Z_2}{\sqrt{3}}\right.$$
$$\left.+2\,(v_{1u}v_{2u}-v_{1d}v_{2d})\,\frac{Y_1^+ + Y_1^-}{\sqrt{2}}+2\,(v_{1u}v_{3u}-v_{1d}v_{3d})\,\frac{Y_2^+ + Y_2^-}{\sqrt{2}}+2\,(v_{2u}v_{3u}-v_{2d}v_{3d})\,\frac{Y_3^+ + Y_3^-}{\sqrt{2}}\right]$$
$$+\ \textbf{tiny contributions to the } SU(3)_F \textbf{ gauge boson masses}$$

$v_u^2 = v_{u1}^2 + v_{u2}^2 + v_{u3}^2$, $v_d^2 = v_{d1}^2 + v_{d2}^2 + v_{d3}^2$. Hence, if we define $M_W = \frac{1}{2} g\, v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

Tree level Fermion Masses

$\bar{\psi}^o_{SM,L}\, \Phi^{u,d}\, \psi^o_{SM,R}$ **are not $SU(3)_F$ invariant**
Allowed Tree Level Yukawa couplings:

$$h_e\, \bar{\psi}^o_l\, \Phi^d\, E^o_R \;+\; h_{2e}\, \bar{\psi}^o_e\, \eta_2\, E^o_L \;+\; h_{3e}\, \bar{\psi}^o_e\, \eta_3\, E^o_L \;+\; M_E\, \bar{E}^o_L\, E^o_R \;+ h.c$$

DIRAC SEE-SAW MECHANISMS
In the gauge basis ${\psi^o_{L,R}}^T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms read as $\bar{\psi}^o_L \mathcal{M}^o \psi^o_R + h.c$, where

	e^o_R	μ^o_R	τ^o_R	E^o_R
$\overline{e^o_L}$	0	0	0	$h_e\, v_{d1}$
$\overline{\mu^o_L}$	0	0	0	$h_e\, v_{d2}$
$\overline{\tau^o_L}$	0	0	0	$h_e\, v_{d3}$
$\overline{E^o_L}$	0	$h_{2e}\, \Lambda_2$	$h_{3e}\, \Lambda_3$	M_E

$$V_L^{o\,T}\mathcal{M}^o\mathcal{M}^{o\,T}\ V_L^o = V_R^{o\,T}\mathcal{M}^{o\,T}\mathcal{M}^o\ V_R^o = Diag(0,0,\lambda_3^2,\lambda_4^2)$$

$$V_L^{o\,T}\mathcal{M}^o\ V_R^o = Diag(0,0,-\lambda_3,\lambda_4)$$

One loop contribution to fermion masses

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R}$$

Therefore, in this scenario light fermion masses, including neutrinos, may get extremely small masses from radiative corrections mediated by the $SU(3)_F$ heavy gauge bosons.

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) \qquad , \qquad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

M_Y is the gauge boson mass, c_Y is coupling constant, $m_3^o = -\lambda_3$, $m_4^o = \lambda_4$, and $f(x,y) = \frac{x^2}{x^2 - y^2} \ln \frac{x^2}{y^2}$.

$$\sum_{k=3,4} m_k^o \, (V_L^o)_{ik} (V_R^o)_{jk} f(M_Y, m_k^o) = \frac{a_i \, b_j \, M}{\lambda_4^2 - \lambda_3^2} \, F(M_Y) \quad i,j = 1,2,3,$$

$$F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2 - \lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$$

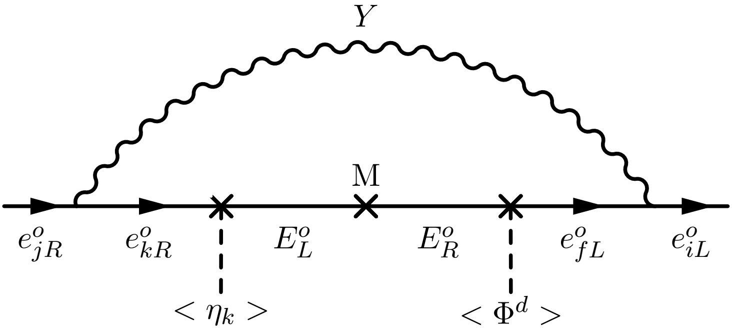


Figure: Generic one loop diagram contribution to the mass term $m_{ij}\ \bar{e}^o_{iL} e^o_{jR}$

	e_R^o	μ_R^o	τ_R^o	E_R^o
$\overline{e_L^o}$	D_{11}	D_{12}	D_{13}	0
$\overline{\mu_L^o}$	0	D_{22}	D_{23}	0
$\overline{\tau_L^o}$	0	D_{32}	D_{33}	0
$\overline{E_L^o}$	0	0	0	0

Table: One loop mass matrix $\mathcal{M}_1^o$

$$\bar{\psi}_L^o \mathcal{M}^o \, \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \, \psi_R^o = \bar{\chi}_L \, \mathcal{M} \, \chi_R$$

Generic mass matrix up to one loop for quarks and charged leptons.

$$\mathcal{M} = Diag(0, 0, -\lambda_3, \lambda_4) + {V_L^o}^T \mathcal{M}_1^o \, V_R^o$$

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta \, m_{13} & s_\beta \, m_{13} \\ m_{21} & m_{22} & c_\beta \, m_{23} & s_\beta \, m_{23} \\ c_\alpha \, m_{31} & c_\alpha \, m_{32} & -\lambda_3 + c_\alpha c_\beta \, m_{33} & c_\alpha s_\beta \, m_{33} \\ s_\alpha \, m_{31} & s_\alpha \, m_{32} & s_\alpha c_\beta \, m_{33} & \lambda_4 + s_\alpha s_\beta \, m_{33} \end{pmatrix}$$

The diagonalization of $\mathcal{M}$ yields the physical masses for u, d, e and ν fermions. Using a new biunitary transformation $\chi_{L,R} = V^{(1)}_{L,R}\, \Psi_{L,R};\ \bar{\chi}_L\, \mathcal{M}\, \chi_R = \bar{\Psi}_L\, {V^{(1)}_L}^T \mathcal{M}\, V^{(1)}_R\, \Psi_R$, with ${\Psi_{L,R}}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$${V^{(1)}_L}^T \mathcal{M}\, \mathcal{M}^T\, V^{(1)}_L = {V^{(1)}_R}^T \mathcal{M}^T\, \mathcal{M}\, V^{(1)}_R = Diag(m_1^2, m_2^2, m_3^2, M_F^2)\,,$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons. Therefore, the transformation from massless to mass fermions eigenfields in this scenario reads

$$\psi^o_L = V^o_L\, V^{(1)}_L\, \Psi_L \qquad \text{and} \qquad \psi^o_R = V^o_R\, V^{(1)}_R\, \Psi_R$$

and for neutrinos $\Psi^o_\nu = U^o_\nu\, U_\nu\, \Psi_\nu$.

Quark $(V_{CKM})_{4\times4}$ and Lepton $(U_{PMNS})_{4\times8}$ mixing matrices

Vector like quarks are $SU(2)_L$ weak singlets, and hence, the interaction of L-handed up and down quarks; ${f^o_{uL}}^T = (u^o, c^o, t^o)_L$ and ${f^o_{dL}}^T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}}\, \bar{f^o}_{uL}\gamma_\mu f^o_{dL} W^{+\mu} = \frac{g}{\sqrt{2}}\, \bar{\Psi}_{uL}\, [(V^o_{uL}\, V^{(1)}_{uL})_{3\times4}]^T\, (V^o_{dL}\, V^{(1)}_{dL})_{3\times4}\, \gamma_\mu \Psi_{dL}\, W^{+\mu},$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4\times4} = [(V^o_{uL}\, V^{(1)}_{uL})_{3\times4}]^T\, (V^o_{dL}\, V^{(1)}_{dL})_{3\times4}$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4\times8} = [(V^o_{eL}\, V^{(1)}_{eL})_{3\times4}]^T\, (U^o_\nu\, U_\nu)_{3\times8}$$

Neutrino masses

Tree level Dirac Neutrino masses

$$h_D\,\overline{\Psi^o_l}\,\Phi^u\,N^o_R \;+\; h_{\nu 2}\,\overline{\Psi^o_{\nu_R}}\,\eta_2\,N^o_L \;+\; h_{\nu 3}\,\overline{\Psi^o_{\nu_R}}\,\eta_3\,N^o_L \;+\; m_D\,\overline{N^o_L}\,N^o_R \;+ h.c$$

h_D, h_1, h_2, and h_3 are Yukawa couplings, and m_D a Dirac type invariant neutrino mass for the sterile neutrino $N^o_{L,R}$. After electroweak symmetry breaking, we obtain in the interaction basis ${\Psi^o_{\nu}}^T_{L,R} = (\nu^o_e, \nu^o_\mu, \nu^o_\tau, N^o)_{L,R}$, the mass terms

$$h_D\,\left[v_{u1}\,\bar{\nu}^o_{eL} + v_{u2}\,\bar{\nu}^o_{\mu L} + v_{u3}\,\bar{\nu}^o_{\tau L}\right]\,N^o_R + \left[h_{\nu 2}\,\Lambda_2\,\bar{\nu}^o_{\mu R} + h_{\nu 3}\,\Lambda_3\,\bar{\nu}^o_{\tau R}\right]\,N^o_L$$
$$+ m_D\,\bar{N}^o_L\,N^o_R + h.c.$$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_D\, v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_D\, v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_D\, v_{u3}$
$\overline{N^o_L}$	0	$h_{\nu 2}\, \Lambda_2$	$h_{\nu 3}\, \Lambda_3$	m_D

Table: Tree level Dirac mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, \nu^o_{jR}$

Tree level Majorana masses

Since $N^o_{L,R}$ are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L\,\bar{\Psi}^o_l\,\Phi^u (N^o_L)^c \quad + \quad m_L\,\bar{N}^o_L\,(N^o_L)^c + h.c$$

and

$$h_{2R}\,\bar{\Psi}^o_\nu\,\eta_2\,(N^o_R)^c \;+\; h_{3R}\,\bar{\Psi}^o_\nu\,\eta_3\,(N^o_R)^c \;+\; m_R\,\bar{N}^o_R\,(N^o_R)^c + h.c.$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L\,\left[v_{u1}\,\bar{\nu}^o_{eL} + v_{u2}\,\bar{\nu}^o_{\mu L} + v_{u3}\,\bar{\nu}^o_{\tau L}\right]\,(N^o_L)^c \quad + \quad m_L\,\bar{N}^o_L\,(N^o_L)^c$$

$$+\left[h_{2R}\,\Lambda_2\,\bar{\nu}^o_{\mu R} + h_{3R}\,\Lambda_3\,\bar{\nu}^o_{\tau R}\right]\,(N^o_R)^c \;+\; m_R\,\bar{N}^o_R\,(N^o_R)^c + h.c.$$

	ν^o_{eL}	$\nu^o_{\mu L}$	$\nu^o_{\tau L}$	N^o_L
ν^o_{eL}	0	0	0	$h_L v_{u1}$
$\nu^o_{\mu L}$	0	0	0	$h_L v_{u2}$
$\nu^o_{\tau L}$	0	0	0	$h_L v_{u3}$
N^o_L	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L

Table: Tree level L-handed Majorana mass terms $m_{ij}\, \bar{\nu}^o_{iL}\, (\nu^o_{jL})^c$

	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
ν^o_{eR}	0	0	0	0
$\nu^o_{\mu R}$	0	0	0	$h_{2R}\Lambda_2$
$\nu^o_{\tau R}$	0	0	0	$h_{3R}\Lambda_3$
N^o_R	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	m_R

	$(\nu^o_{eL})^c$	$(\nu^o_{\mu L})^c$	$(\nu^o_{\tau L})^c$	$(N^o_L)^c$	ν^o_{eR}	$\nu^o_{\mu R}$	$\nu^o_{\tau R}$	N^o_R
$\overline{\nu^o_{eL}}$	0	0	0	$h_L v_{u1}$	0	0	0	$h_D v_{u1}$
$\overline{\nu^o_{\mu L}}$	0	0	0	$h_L v_{u2}$	0	0	0	$h_D v_{u2}$
$\overline{\nu^o_{\tau L}}$	0	0	0	$h_L v_{u3}$	0	0	0	$h_D v_{u3}$
$\overline{N^o_L}$	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L	0	$h_2\Lambda_2$	$h_3\Lambda_3$	M_D
$\overline{(\nu^o_{eR})^c}$	0	0	0	0	0	0	0	0
$\overline{(\nu^o_{\mu R})^c}$	0	0	0	$h_2\Lambda_2$	0	0	0	$h_{2R}\Lambda_2$
$\overline{(\nu^o_{\tau R})^c}$	0	0	0	$h_3\Lambda_3$	0	0	0	$h_{3R}\Lambda_3$
$\overline{(N^o_R)^c}$	$h_D v_{u1}$	$h_D v_{u2}$	$h_D v_{u3}$	M_D	0	$h_{2R}\Lambda_2$	$h_{3R}\Lambda_3$	m_R

Table: Tree Level Majorana masses

SSB and $U(1)_{PQ}$ global symmetry

$$[SU(3)_F \otimes G_{SM}]_{\text{gauge}} \times [U(1)_B \otimes U(1)_L \otimes U(1)_{PQ}]_{\text{global}}$$

$$\downarrow \langle \eta_3 \rangle \sim \Lambda_3 \geq 10^8 \; TeV$$

$$[SU(2)_F \otimes G_{SM}]_{\text{gauge}} \times [U(1)_B \otimes U(1)_L]_{\text{global}}$$

$$\downarrow \langle \eta_2 \rangle \sim \Lambda_2 \approx \text{ few } TeV$$

$$[G_{SM}]_{\text{gauge}} \times [U(1)_B \otimes U(1)_L]_{\text{global}}$$

$$\downarrow \langle \Phi_u \rangle, \langle \Phi_d \rangle \approx \; 250 \; GeV$$

$$[SU(3)_C \otimes U(1)_{QED}]_{\text{gauge}} \times [U(1)_B]_{\text{global}}$$

	$U(1)_B$	$U(1)_L$	$U(1)_{PQ}$
ψ_q^o	$\frac{1}{3}$	0	1
ψ_{uR}^o	$\frac{1}{3}$	0	2
ψ_{dR}^o	$\frac{1}{3}$	0	2
ψ_l^o	0	1	0
ψ_{eR}^o	0	1	1
$\psi_{\nu R}^o$	0	1	1
$U_{L,R}^o$	$\frac{1}{3}$	0	1
$D_{L,R}^o$	$\frac{1}{3}$	0	1
$E_{L,R}^o$	0	1	0
$N_{L,R}^o$	0	1	0

Table: All the Yukawa couplings and the scalar potential are invariant under the global symmetry $U(1)_B \times U(1)_L \times U(1)_{PQ}$, where B is the baryon number, L is the lepton number, and $U(1)_{PQ}$ the candidate for the Peceei-Quinn symmetry to address the strong CP problem

	$U(1)_B$	$U(1)_L$	$U(1)_{PQ}$
η_2, η_3	0	0	1
Φ^u	0	0	0
Φ^d	0	0	0

the best parameter space region?

As an example of the possible spectrum of quark masses and mixing from this scenario, we consider the following set of parameters at the M_Z scale

Input values for the $SU(3)_F$ family symmetry:

$$M_2 = 6\,\text{TeV} \quad , \quad M_3 = 1.5 \times 10^8\,\text{TeV} \quad , \quad \frac{\alpha_H}{\pi} = 0.2$$

with M_2, M_3 the horizontal boson masses, and the coupling constant, respectively,

$$M_2^2 = \frac{g_H^2 \Lambda_2^2}{2} \qquad , \qquad M_3^2 = \frac{g_H^2 \Lambda_3^2}{2}$$

$$g_H = 2.809 \quad , \Lambda_2 = 3.019\ TeV \quad , \Lambda_3 = 7.549 \times 10^7\ TeV$$

Quark masses and $(V_{CKM})_{4\times4}$ mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 5573.43 \\ 0 & 0 & 0 & 23883.8 \\ 0 & 0 & 0 & 397346. \\ 0 & -1.931\times10^8 & 5.193\times10^6 & 2.470\times10^8 \end{pmatrix} \text{MeV}\,,$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_{u\,1}^o = \begin{pmatrix} 1.42 & -220.786 & 34.6742 & 0 \\ 0 & -944.713 & 148.589 & 0 \\ 0 & -91921.3 & 11631.6 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{MeV}$$

the u-quark masses

$$(\, m_u\,,\ m_c\,,\ m_t\,,\ M_U\,) = (\,1.382\,,\ 633.289\,,\ 172968\,,\ 313.606\times10^6\,)\ \text{MeV}$$

the mixing matrices:
$V_{u\,L} = V^o_{u\,L}\, V^{(1)}_{u\,L}$:

$$\begin{pmatrix} 0.973838 & -0.226464 & -0.0188217 & 0.0000144353 \\ -0.227244 & -0.970491 & -0.080663 & 0.0000618569 \\ 9.34208 \times 10^{-7} & 0.0828299 & -0.996563 & 0.0011792 \\ -2.14837 \times 10^{-9} & -0.0000343726 & 0.00118041 & 0.999999 \end{pmatrix}$$

$V_{u\,R} = V^o_{u\,R}\, V^{(1)}_{u\,R}$:

$$\begin{pmatrix} 1. & 0.000507791 & 1.54519 \times 10^{-7} & 0 \\ -7.6088 \times 10^{-6} & 0.0147444 & 0.787788 & -0.61577 \\ 0.000507462 & -0.999362 & 0.0316481 & 0.0165598 \\ -0.0000166153 & 0.0325336 & 0.615132 & 0.787752 \end{pmatrix}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3102.75 \\ 0 & 0 & 0 & 61977.5 \\ 0 & -9.805\times 10^7 & 2.837\times 10^6 & 6.046\times 10^8 \end{pmatrix} \text{ MeV}$$

$$\mathcal{M}_{d\,1}^o = \begin{pmatrix} 2.82 & 0 & 0 & 0 & 0 \\ 0 & -130.851 & 10.43 & 0 & \\ 0 & -7200.83 & 664.801 & 0 & \\ 0 & 0 & 0 & 0 & \end{pmatrix} \text{ MeV}$$

$$(\, m_d\, ,\, m_s\, ,\, m_b\, ,\, M_D\,) = (\, 2.82\, ,\, 52.087\, ,\, 2861.96\, ,\, 612.541\times 10^6\,) \text{ MeV}$$

Mixing matrices:

$V_{d\,L} = V^{o}_{d\,L}\, V^{(1)}_{d\,L}$:

$$\begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.991883 & -0.127155 & 5.03428 \times 10^{-6} \\ 0 & -0.127155 & -0.991883 & 0.000101762 \\ 0 & 7.94612 \times 10^{-6} & 0.000101576 & 1. \end{pmatrix}$$

$V_{d\,R} = V^{o}_{d\,R}\, V^{(1)}_{d\,R}$:

$$\begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & -0.127762 & 0.9788 & -0.160083 \\ 0 & 0.99148 & 0.130175 & 0.00463162 \\ 0 & -0.0253722 & 0.158128 & 0.987093 \end{pmatrix}$$

Quark mixing matrix

$$V_{CKM} = \begin{pmatrix} 0.97383 & 0.2254 & 0.02889 & -1.14 \times 10^{-6} \\ -0.22646 & 0.97314 & 0.04124 & 3.54 \times 10^{-6} \\ -0.01882 & -0.04670 & 0.99873 & -0.00010 \\ 1.44 \times 10^{-5} & 8.85 \times 10^{-5} & -0.00117 & 1.20 \times 10^{-7} \end{pmatrix}$$

$\Delta F = 2$ Processes in Neutral Mesons

Horizontal gauge bosons from $SU(3)_F$ introduce flavor changing couplings, and in particular they mediate $\Delta F = 2$ processes at tree level.

Here we study the leading contribution to $K^o - \bar{K}^o$, $D^o - \bar{D}^o$ meson mixing mediated by the $SU(2)$ horizontal gauge boson which become massive at the scale of few TeV.

The interaction Lagrangian

$$i\mathcal{L}_{int,SU(2)_F} = g_H \begin{pmatrix} \bar{f}_1^o & \bar{f}_2^o & \bar{f}_3^o \end{pmatrix} \gamma_\mu \begin{pmatrix} \frac{Z_1^\mu}{2} & \frac{Y_1^{+\mu}}{\sqrt{2}} & 0 \\ \frac{Y_1^{-\mu}}{\sqrt{2}} & -\frac{Z_1^\mu}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_1^o \\ f_2^o \\ f_3^o \end{pmatrix}$$

contributing to the $\Delta S = 2$ effective operators

$$\mathcal{O}_{LL} = (\bar{d}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu s_L) \quad , \quad \mathcal{O}_{RR} = (\bar{d}_R \gamma_\mu s_R)(\bar{d}_R \gamma^\mu s_R)$$

$$\mathcal{O}_{LR} = (\bar{d}_L \gamma_\mu s_L)(\bar{d}_R \gamma^\mu s_R)$$

may be written in the mass basis as

$$\mathcal{L}_{int,Z_1} = \frac{g_H}{2} \left(C_{L\,Z_1}\, \bar{d}_L \gamma_\mu s_L + C_{R\,Z_1}\, \bar{d}_R \gamma_\mu s_R \right) Z_1^\mu$$

$$\mathcal{L}_{int,Y_1^1} = \frac{g_H}{2} \left(C_{L\,Y_1^1}\, \bar{d}_L \gamma_\mu s_L + C_{R\,Y_1^1}\, \bar{d}_R \gamma_\mu s_R \right) Y_1^{1\,\mu}$$

$$\mathcal{L}_{int,Y_1^2} = \frac{g_H}{2} \left(C_{L\,Y_1^2}\, \bar{d}_L \gamma_\mu s_L + C_{R\,Y_1^2}\, \bar{d}_R \gamma_\mu s_R \right) i\, Y_1^{2\,\mu}$$

with the flavor changing couplings in both left- and right-handed quarks

$$C_{L\,Z_1} = L_{11}\,L_{12} - L_{21}\,L_{22} \quad , \quad C_{R\,Z_1} = R_{11}\,R_{12} - R_{21}\,R_{22}$$

$$C_{L\,Y_1^1} = L_{12}\,L_{21} + L_{11}\,L_{22} \quad , \quad C_{R\,Y_1^1} = R_{12}\,R_{21} + R_{11}\,R_{22}$$

$$C_{L\,Y_1^2} = (L_{12}\,L_{21} - L_{11}\,L_{22}) \quad , \quad C_{R\,Y_1^2} = (R_{12}\,R_{21} - R_{11}\,R_{22})$$

using the mixing angles

$$V_{L\,ij} \equiv L_{ij} \quad , \quad V_{R\,ij} \equiv R_{ij}$$

The effective four-fermion hamiltonian just below the scale Λ_2, (**??**) of the $SU(3)_F$ SSB induced from the generic tree level diagram of Fig. (**??**) is

$$\mathcal{H}_{Z_1} = \frac{g_H^2}{4M_{Z_1}^2}\left(C_{L\,Z_1}^2\,\mathcal{O}_{LL} + 2\,C_{L\,Z_1}C_{R\,Z_1}\mathcal{O}_{LR} + C_{R\,Z_1}^2\,\mathcal{O}_{RR}\right)$$

$$\mathcal{H}_{Y_1^1} = \frac{g_H^2}{4M_{Y_1^1}^2}\left(C_{L\,Y_1^1}^2\,\mathcal{O}_{LL} + 2\,C_{L\,Y_1^1}C_{R\,Y_1^1}\mathcal{O}_{LR} + C_{R\,Y_1^1}^2\,\mathcal{O}_{RR}\right)$$

$$\mathcal{H}_{Y_1^2} = -\frac{g_H^2}{4M_{Y_1^2}^2}\left(C_{L\,Y_1^2}^2\,\mathcal{O}_{LL} + 2\,C_{L\,Y_1^2}C_{R\,Y_1^1}\mathcal{O}_{LR} + C_{R\,Y_1^2}^2\,\mathcal{O}_{RR}\right)$$

Therefore, the total four-fermion hamiltonian $\mathcal{H}_{\mathrm{SU(2)}} = \mathcal{H}_{Z_1} + \mathcal{H}_{Y_1^1} + \mathcal{H}_{Y_1^2}$ can be written as

$$\mathcal{H}_{\mathrm{SU(2)}} = \frac{g_H^2}{4M_1^2} \Big[(C^2_{L\,Z_1} + C^2_{L\,Y_1^1} - C^2_{L\,Y_1^2})\mathcal{O}_{LL} + (C^2_{R\,Z_1} + C^2_{R\,Y_1^1} + C^2_{R\,Y_1^2}$$

$$+2(C_{L\,Z_1} C_{R\,Z_1} + C_{L\,Y_1^1} C_{R\,Y_1^1} - C_{L\,Y_1^2} C_{R\,Y_1^2})\mathcal{O}_{LR} \Big]$$

$$+ \frac{g_H^2}{4} \left(\frac{1}{M_{Z_1}^2} - \frac{1}{M_1^2} \right) \Big[C^2_{L\,Z_1}\mathcal{O}_{LL} + C^2_{R\,Z_1}\mathcal{O}_{RR} + 2C_{L\,Z_1} C_{R\,Z_1}\mathcal{O}_{LR}) \Big]$$

From eq.(41)

$$C^2_{L\,Z_1} + C^2_{L\,Y_1^1} - C^2_{L\,Y_1^2} = \delta_L^2 \quad , \quad C^2_{R\,Z_1} + C^2_{R\,Y_1^1} - C^2_{R\,Y_1^2} = \delta_R^2 \quad ,$$

and

$$\begin{aligned} C_{L,Z_1}\,C_{R,Z_1} + C_{L,Y_1^1}\,C_{R,Y_1^1} - C_{L,Y_1^2}\,C_{R,Y_1^2} &= \delta_L\,\delta_R \\ &+ 2(L_{11}\,R_{21} - L_{21}\,R_{11})(L_{22}\,R_{12} - L_{12}\,R_{22}) \end{aligned}$$

$$\delta_L = L_{11}\,L_{12} + L_{21}\,L_{22} \quad , \quad \delta_R = R_{11}\,R_{12} + R_{21}\,R_{22}$$

Effective Hamiltonian

and finally

$$\mathcal{H}_{\rm Efe} = \frac{g_H^2}{4M_1^2}\ \Big[\delta_L^2\ \mathcal{O}_{LL} + \delta_R^2\ \mathcal{O}_{RR}$$

$$+2(\delta_L\,\delta_R + 2(L_{11}\,R_{21}\ - L_{21}\,R_{11})(L_{22}\,R_{12}\ - L_{12}\,R_{22}))\ \mathcal{O}_{LR}]$$

$$+\frac{g_H^2}{4}(\frac{1}{M_{Z_1}^2} - \frac{1}{M_1^2})\ \Big[(L_{11}L_{12} - L_{21}L_{22})^2\mathcal{O}_{LL} + (R_{11}R_{12} - R_{21}R_{22})^2\mathcal{O}_{RR}$$

$$+2(L_{11}L_{12} - L_{21}L_{22})(R_{11}R_{12} - R_{21}R_{22})\ \mathcal{O}_{LR})]$$

$K^o - \bar{K^o}$ meson mixing

The parameter space region shown in previous section for d-quarks yield the mixing angles $V_{d\,12} = V_{d\,21} = 0$ for left- and right-handed d-quarks, and then all the contributions to the effective operators, eq.(40)-(40), for $\Delta S = 2$ vanish.

So, $K^o - \bar{K^o}$ is completely suppressed

$D^o - \bar{D}^o$ meson mixing

In the limit $M_{Z_1} = M_1$ only the four-fermion Hamiltonian in eq.**??** contribute. For this case we compute from the parameter space region reported in section

$$|\delta_{L\,D}| = 7.73804 \times 10^{-8} \quad , \quad \frac{M_2}{\frac{g_H}{2}\,|\delta_{L\,D}|} = 5.51894 \times 10^{7}\ TeV$$

$$|\delta_{R\,D}| = 5.07679 \times 10^{-4} \quad , \quad \frac{M_2}{\frac{g_H}{2}\,|\delta_{R\,D}|} = 8411.97\ TeV$$

$$|\delta_{LR\,D}| = 0.0508 \quad , \quad \frac{M_2}{\frac{g_H}{2}\,|\delta_{LR\,D}|} = 83.96\ TeV$$

FCNC in Neutral Mesons

PDG2016: CKM quark-mixing matrix
To illustrate the level of suppression required for BSM contributions, consider a class of models in which the unitarity of the CKM is maintained, and the dominant effect of the new physics is to modify the neutral meson amplitudes by

$$\frac{z_{ij}}{\Lambda^2}\,(\bar{q}_i\gamma^\mu P_L q_j)^2$$

The existent data imply that

$$\frac{\Lambda}{\sqrt{|z_{ij}|}} \gtrsim \begin{cases} 10^4 \text{ TeV for } K^o - \bar{K}^o \\ 10^3 \text{ TeV for } D^o - \bar{D}^o \\ 500 \text{ TeV for } B^o - \bar{B}^o \\ 100 \text{ TeV for } B^o_s - \bar{B}^o_s \end{cases}$$

Conclusions and Predictions:

- Particle model with local $SU(3)_F$ family symmetry can account for the hierarchical spectrum of quark masses and mixing
- FCNC in $K^o - \bar{K^o}$ and $D^o - \bar{D^o}$ Neutral Mesons may be properly suppressed in particular parameter space regions
- NEW MASSIVE GAUGE BOSONS AND SCALARS, (few-100) TeV
- VECTOR LIKE QUARKS AND CHARGED LEPTONS, M_U, M_D, M_E $\approx$(1 - 100)TeV
- STERILE NEUTRINOS:
  Light (eV), May be two
  Heavy (KeV-GeV),