How compact objects challenge our view about dark matter in the Universe

(work with Ilídio Lopes)

(Based on arXiv: 1706.07272, 1709.06643
1904.07191, 1904.07195)

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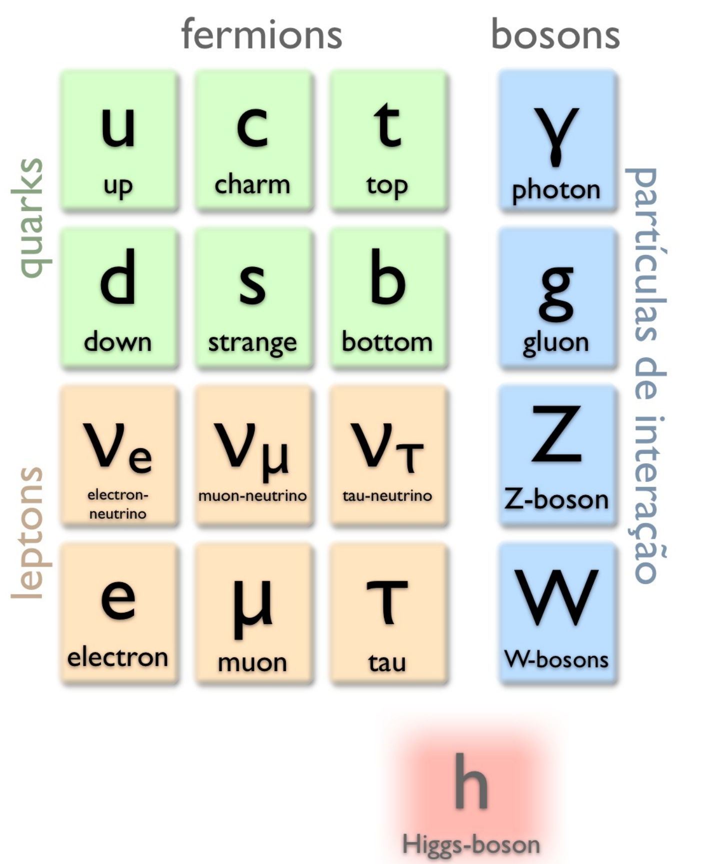
Plan

- Introduction/Motivation
- Relativistic stars in GR (M-R profile, oscillations of pulsating stars)
- Impact of (bosonic) dark matter on strange quark stars
- Radial and nonradial oscillations of BSs made of ULRDM
- Conclusions

Cornerstones of Modern Theoretical Physics

QFT and Group Theory



Einstein's General Relativity

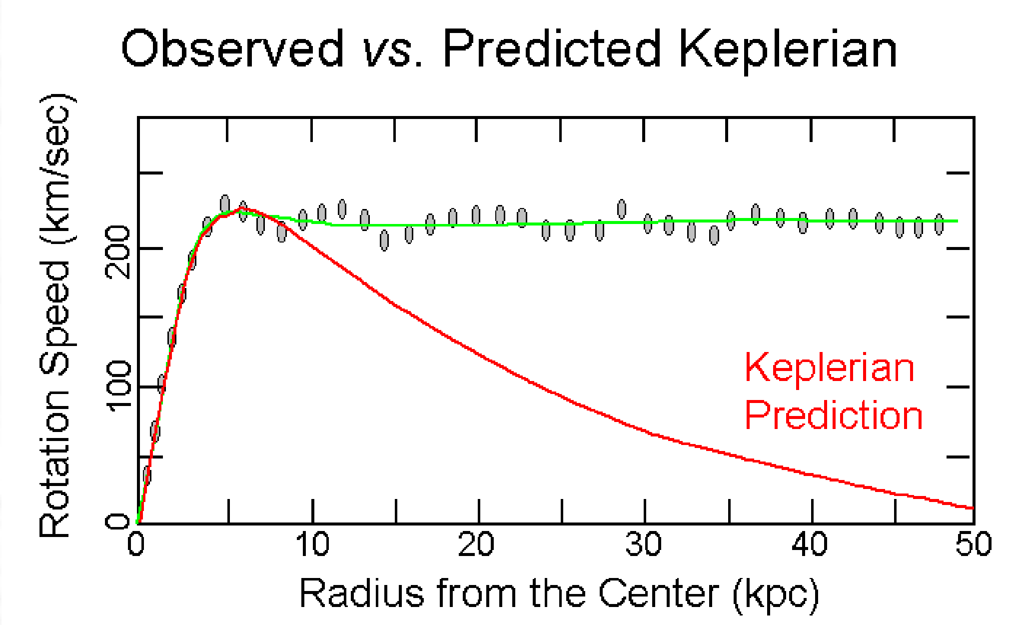
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Dark Matter

Frantz Zwicky 1933

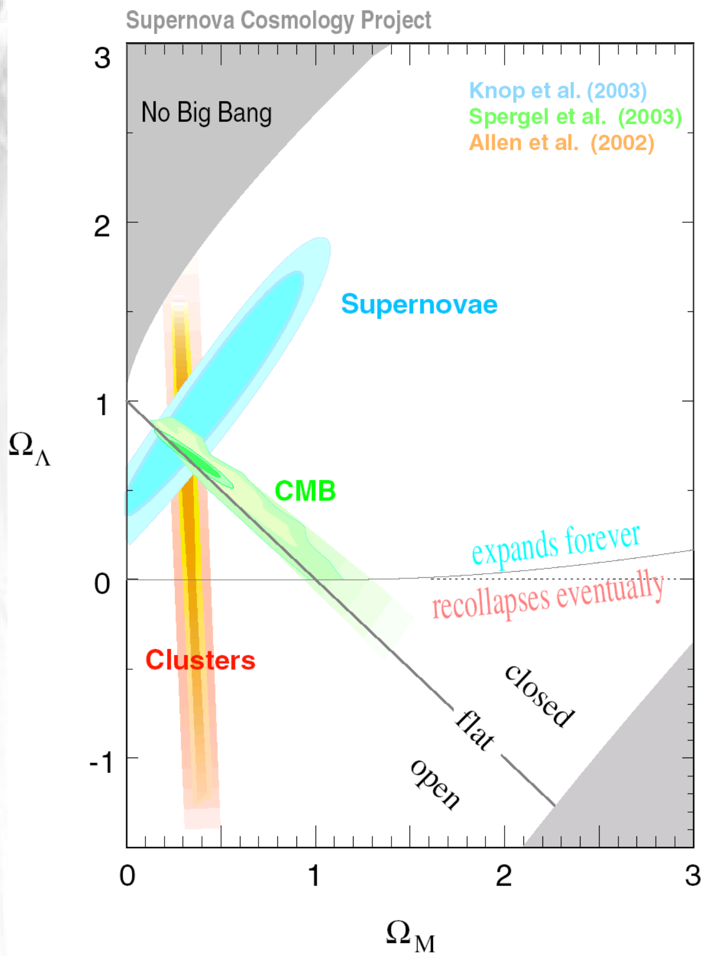
Virial theorem in Coma cluster

Galaxy rotation curves



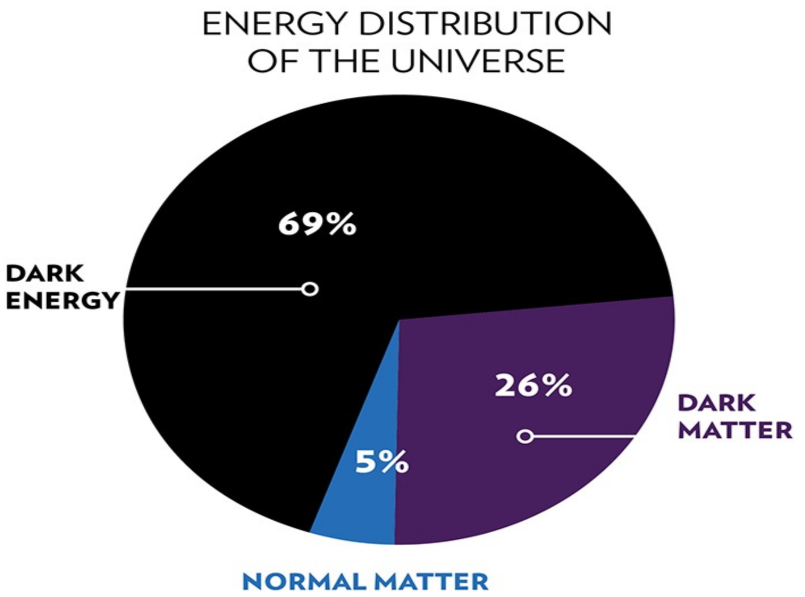
Vera Rubin 1970

Today's picture of the universe



3 independent

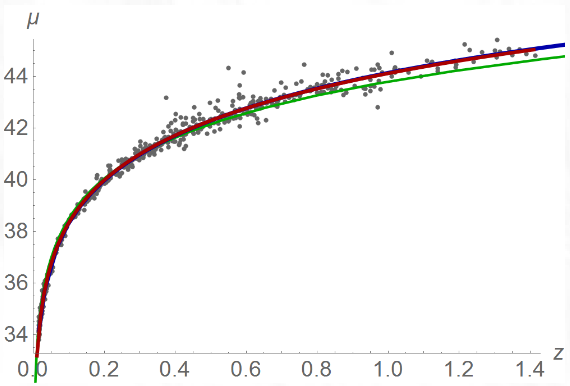
data sets coincide



Concordance cosmological model!

Combination of data

Ia SN

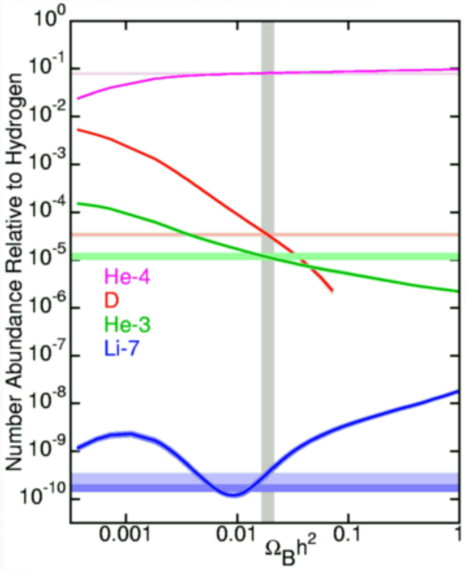


$$\Omega_X = 0.69 \quad \Omega_M = 0.31$$

BBN

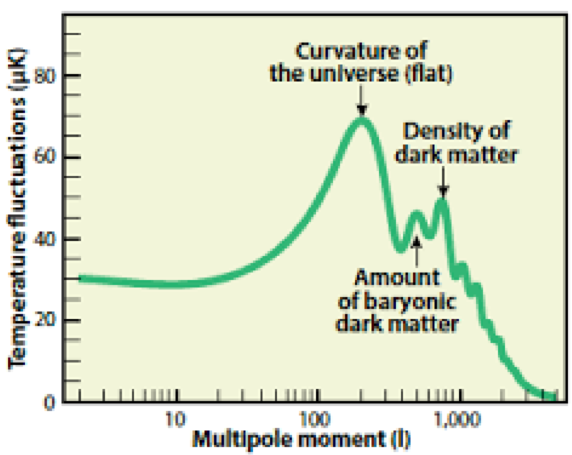
$$\Omega_B = 0.04$$

$$\Omega_{DM} = 0.27$$



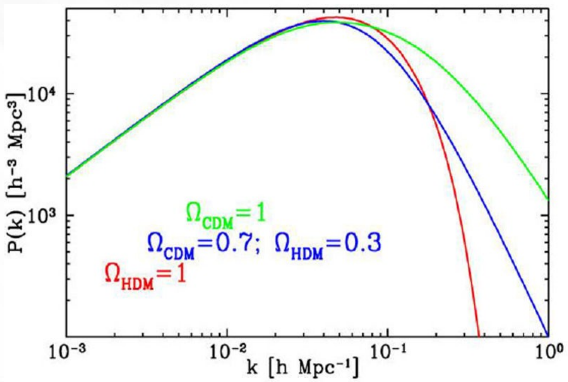
CMB

$$k = 0$$

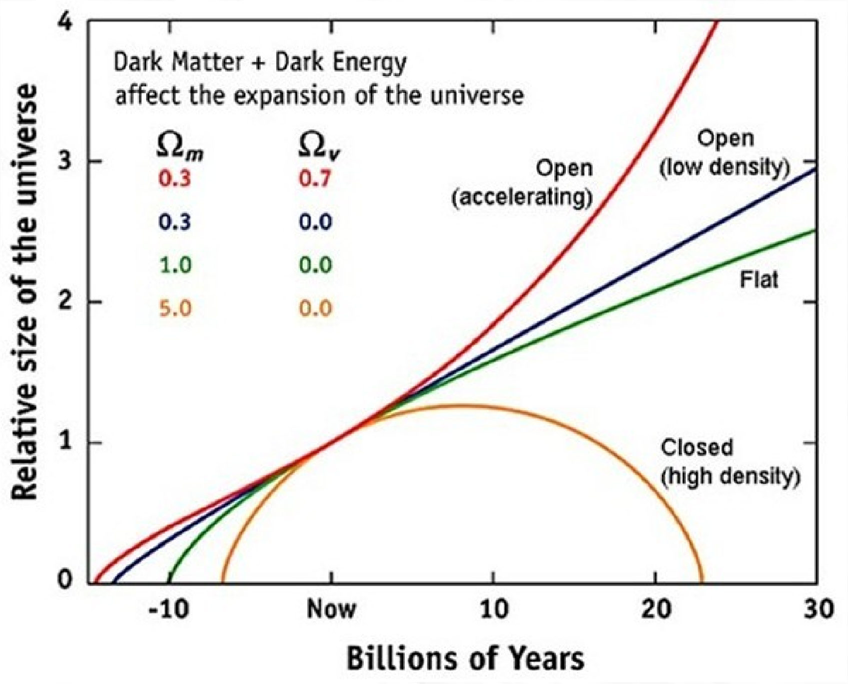


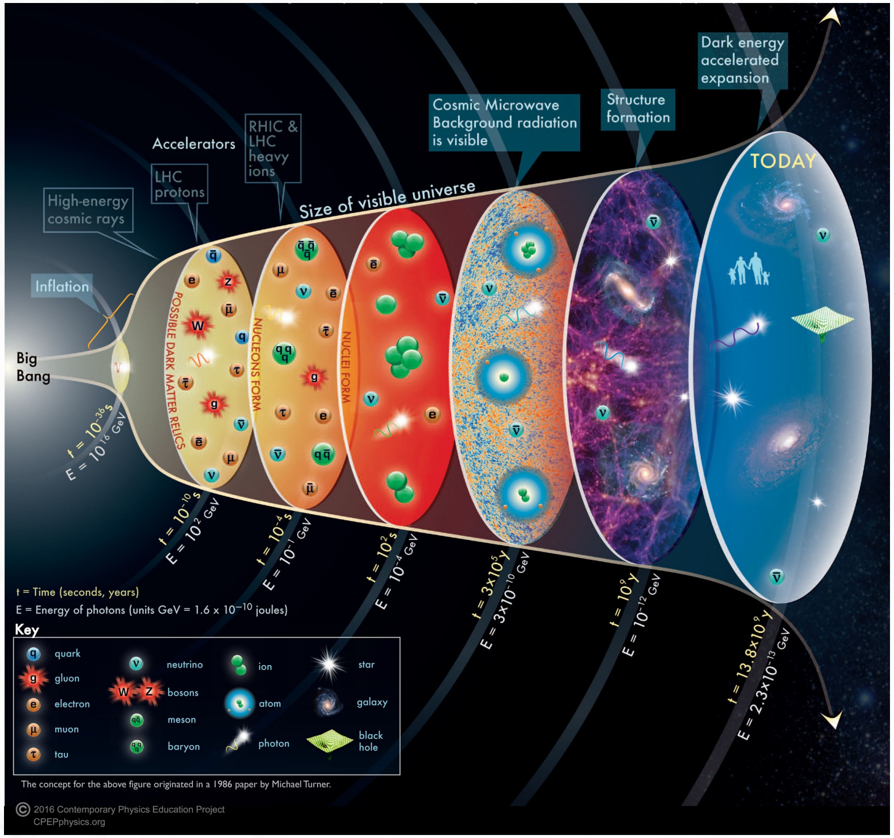
Matter power spectrum

$$\Omega_M = 0.31$$

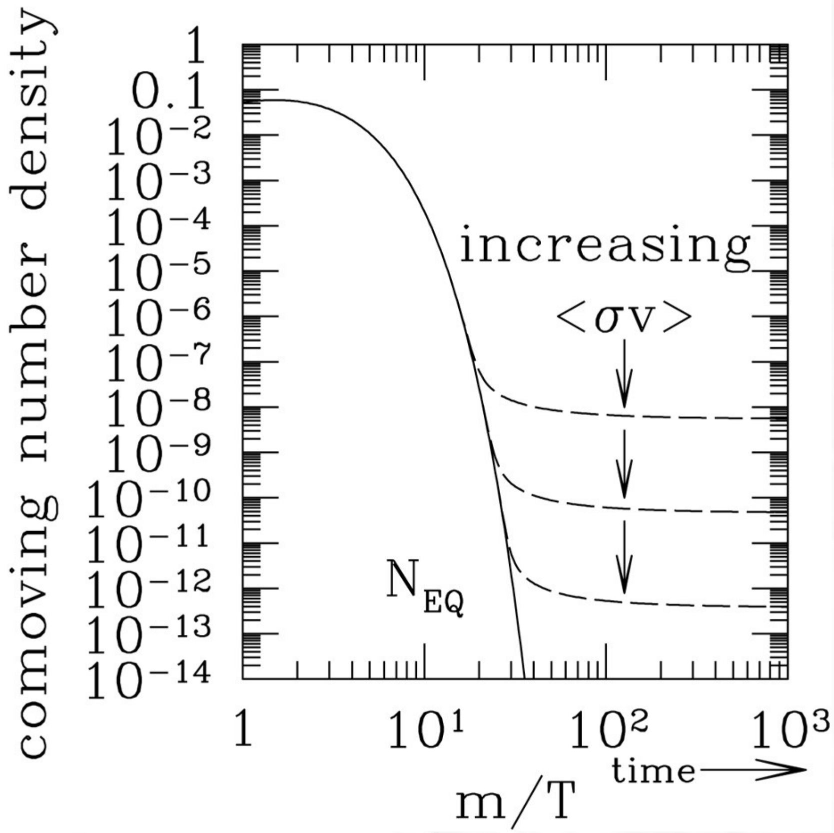


(Accelerating) expansion history





Popular candidate: WIMPs



Today's relic density

$$\Omega_{DM} h^2 = \frac{3 \times 10^{-27} cm^3/s}{\langle \sigma v \rangle}$$

Observations suggest

$$\Omega_{DM} h^2 \sim 0.1$$

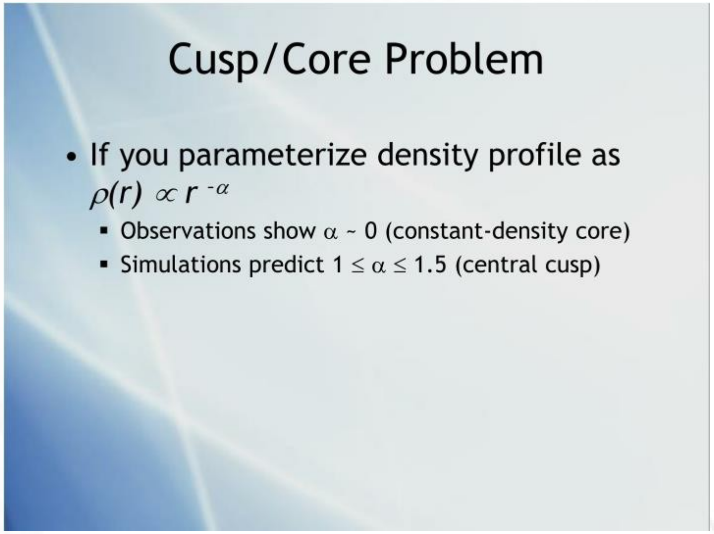
WIMP reaction rate

$$\Gamma = n \langle \sigma v \rangle$$

Expansion rate of the Universe H

Freeze-out $\Gamma = H$

Self-Interacting DM: A solution to the low scale crisis?



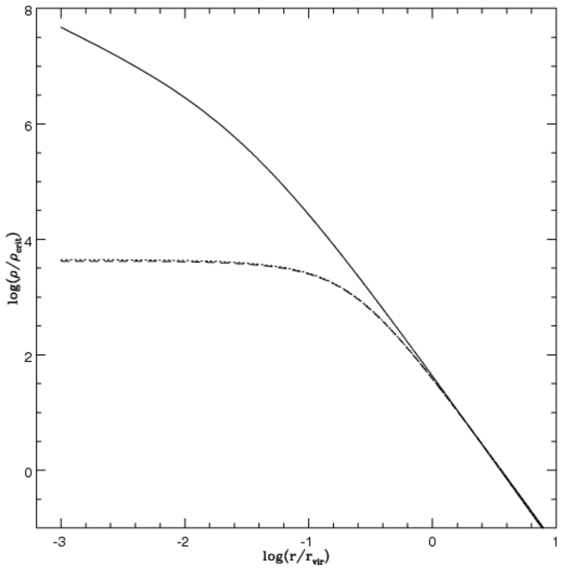
ΛCDM very successful at large scales

BUT: Some problems at low scales

1. Missing satellite problem

Observed MW satellites ~ 10

Simulations based on collisionless DM predict ~ 500 satellites



2. Cusp/Core problem

Ultra-light repulsive scalar DM

Real scalar field with self-interaction potential

(pNGB associated with SSB of global symmetry @ F)

$$\mathcal{L} = (1/2)(\partial\phi)^2 - V(\phi)$$

Expand around minimum

$$V(\phi) \simeq (1/2)m^2\phi^2 + \frac{1}{24}\frac{m^2}{F^2}\phi^4 + ...$$

$$\lambda = \frac{m^2}{F^2} > 0$$

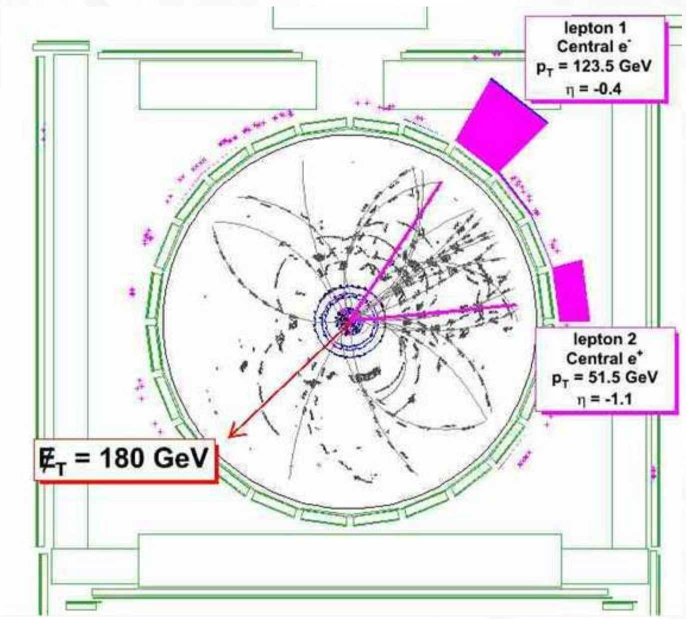
Ignore higher order terms

Repulsive forces

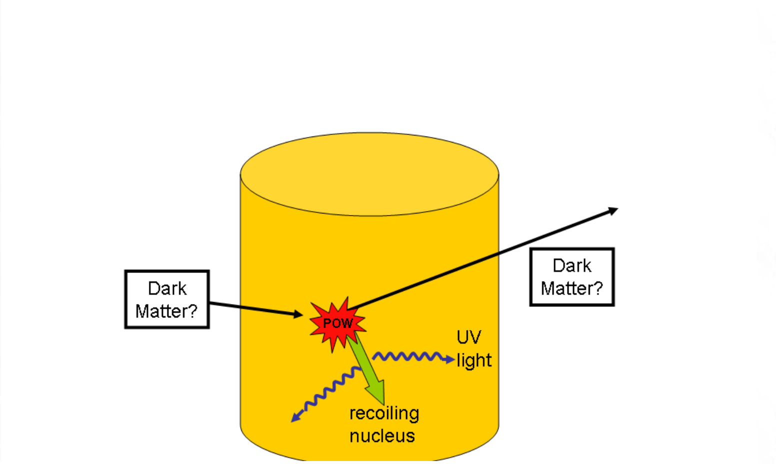
DM searches

Collider searches

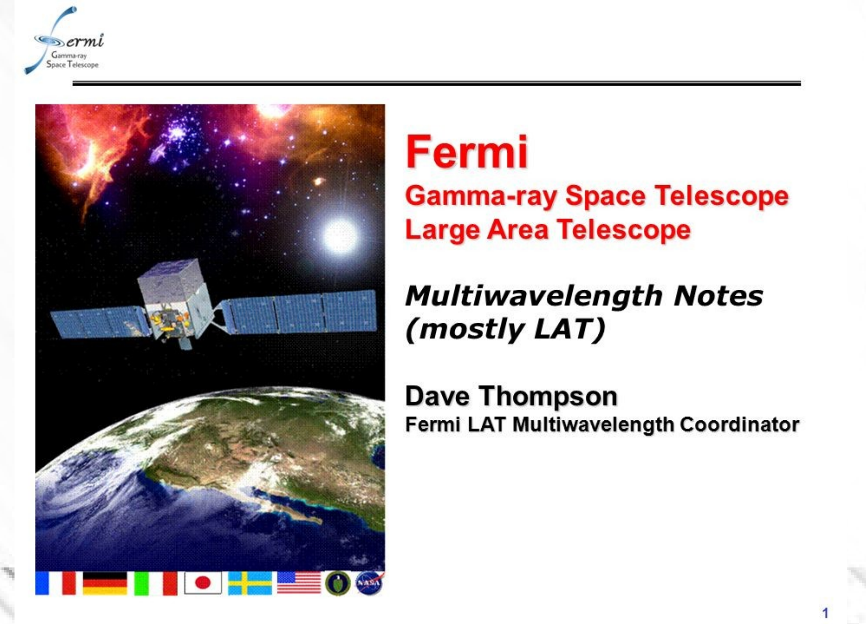
(Tevatron, LHC)



Direct detection searches

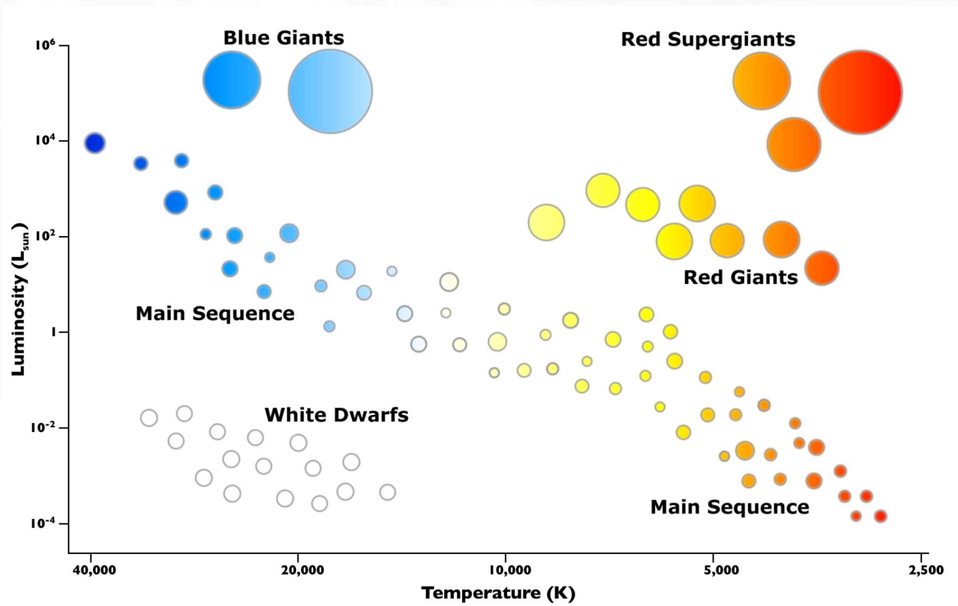


Indirect searches



Compact stars: Ultra high densities

White dwarfs



Neutron stars

1932: Discovery of neutron

1934: NS are predicted to exist

1967: Pulsars are discovered

1968: NS are identified

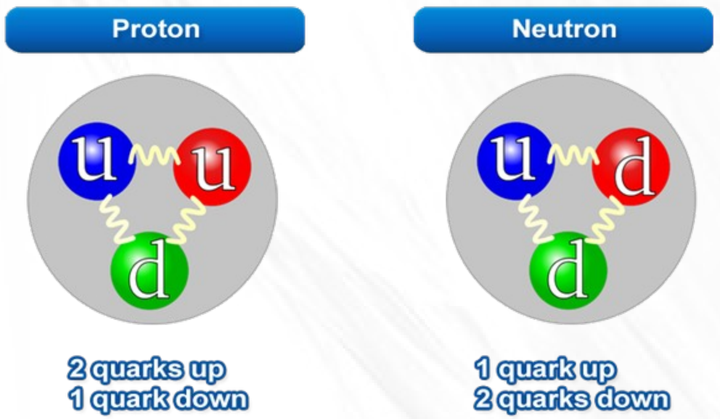
Extreme conditions

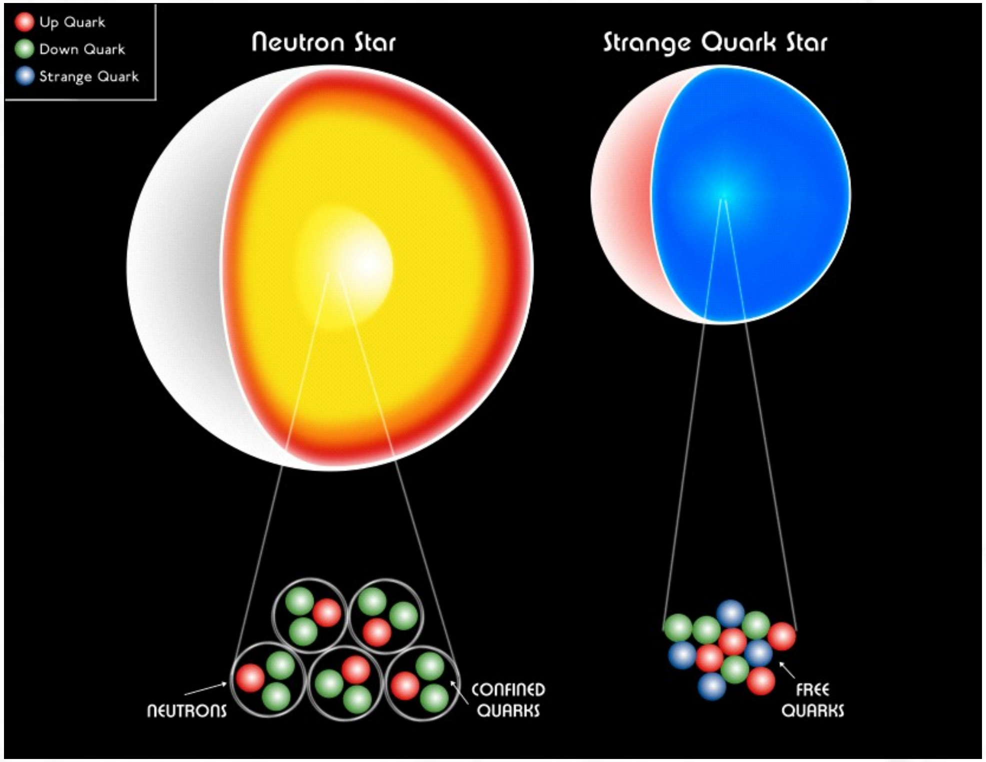
Ideal to constrain new physics

Quark stars vs neutron stars

Made of de-confined quarks

Nucleons are not elementary





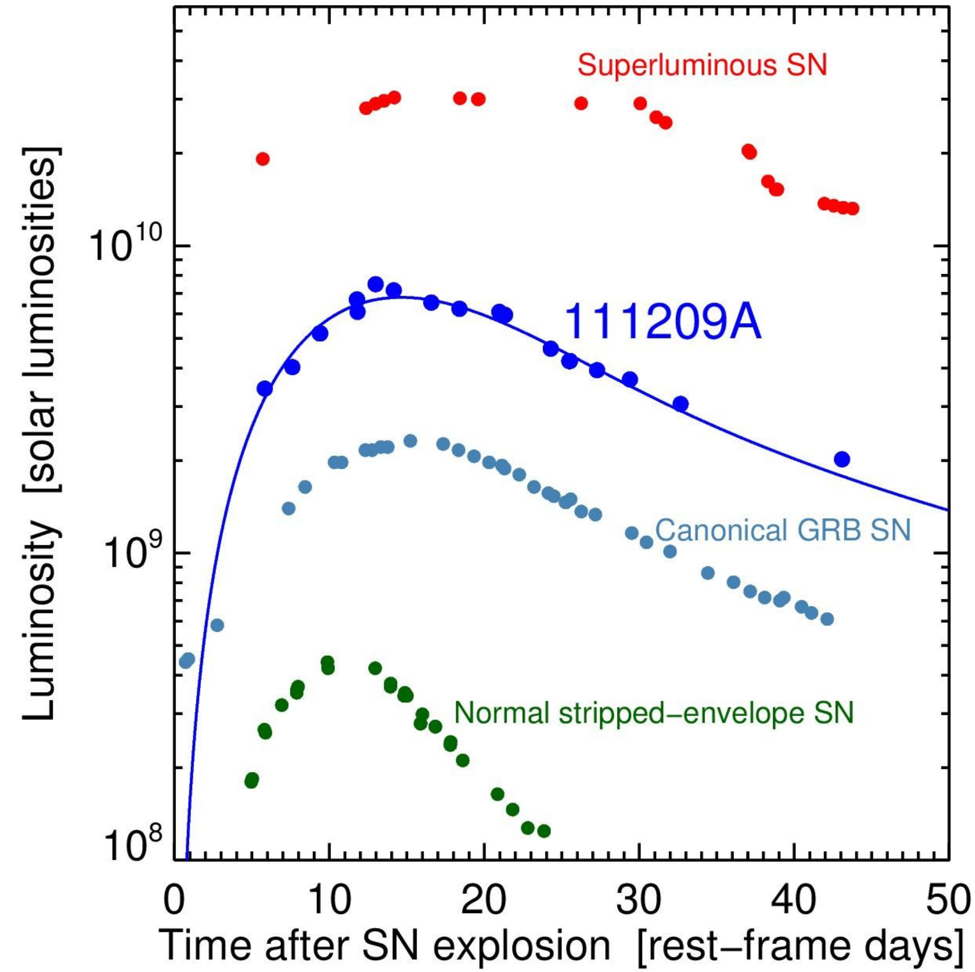
More stable configuration → Less energy

May explain the observed SLSN

SN versus SLSN

100 times brighter

1000 times less frequent



Part I: Impact of SIBDM on SQS

A: Mass-to-radius profile

B: Radial oscillations

Exterior solution

Einstein's field equation in vacuum $\quad r > R$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} = 0 \qquad (\mathrm{G}_N = 1)$$

Make the ansatz for the metric

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega^2$$

Schwarzschild solution

$$f(r) = g(r)^{-1} = 1 - \frac{2M}{r}$$

M is the mass of the star

Interior solution of a star

Einstein's field equations $\quad r < R$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} \qquad (\mathrm{G}_N = 1)$$

$$T^{\mu}_{\nu} = diag(-\rho, p, p, p) \qquad \text{EoS} \quad p(\rho)$$

Make the ansatz for metric

$$ds^2 = -e^{2A(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\Omega^2$$

$$e^{-2\lambda(r)} = 1 - \frac{2m(r)}{r}$$

Structure equations

$$m'(r) = 4\pi r^2 \rho(r)$$

$$p'(r) = -[p(r) + \rho(r)] \frac{m(r) + 4\pi p(r) r^3}{r^2 (1 - \frac{2m(r)}{r})}$$

with initial conditions
$0 < r < R$

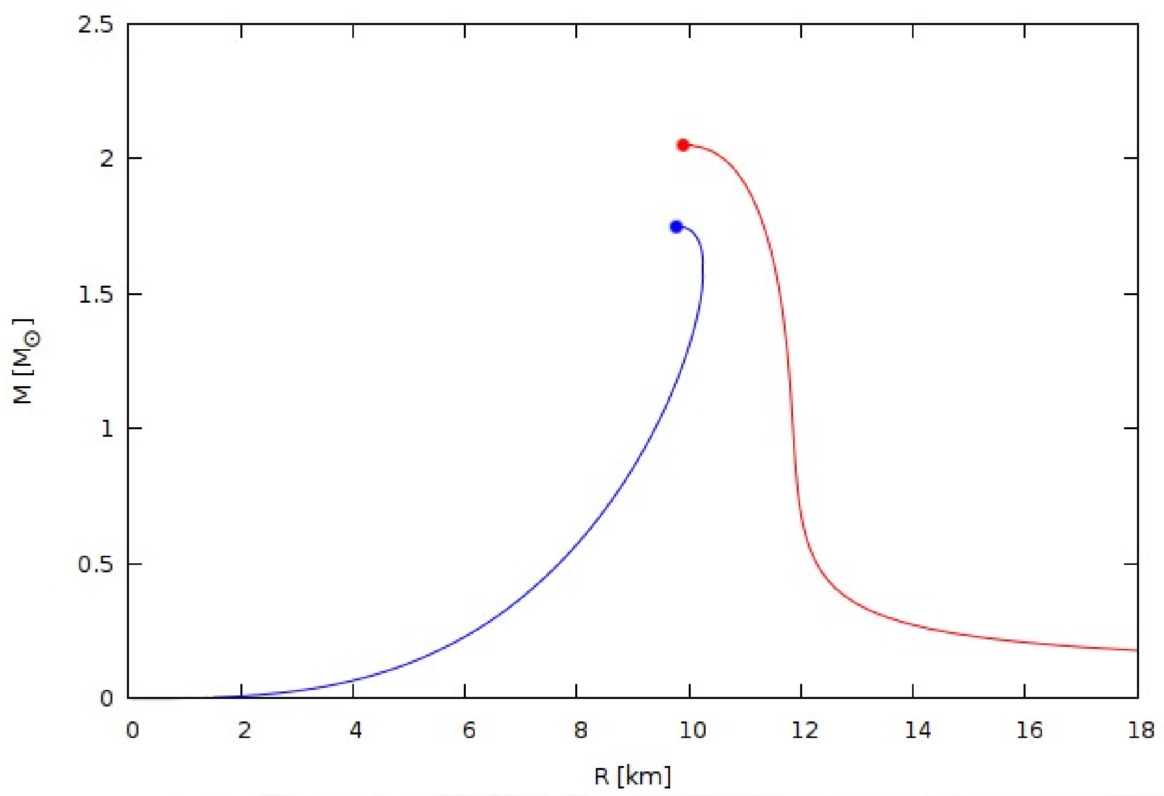
$m(0) = 0$ $\quad p(0) = p_c$

Continouity on the surface

Condition 1 $\quad p(r = R) = 0$
determines the radius

Condition 2 $\quad M = m(r = R)$
determines the mass

Mass-radius diagram



Blue curve: Strange star Red curve: Neutron star

DM effect: two-fluid formalism

ordinary (quark) matter + dark matter

EoSs: $p_s(\rho_s)$ $p_\chi(\rho_\chi)$

$$m'(r) = 4\pi r^2 \rho(r)$$

$$\rho = \rho_s + \rho_\chi$$

$$p_s'(r) = -(p_s(r) + \rho_s(r)) \frac{m(r) + 4\pi p(r) r^3}{r^2(1 - \frac{2m(r)}{r})}$$

$$p = p_s + p_\chi$$

$$p_\chi'(r) = -(p_\chi(r) + \rho_\chi(r)) \frac{m(r) + 4\pi p(r) r^3}{r^2(1 - \frac{2m(r)}{r})}$$

Define the sources

1. Ordinary matter: De-confined u, d, s quarks

$$p_s = \frac{1}{3}(\rho_s - 4B) + \frac{3\alpha}{\pi^2}\,\mu^2$$

$$\alpha = -\frac{m_s^2}{6}$$

$$\mu^2 = -\alpha + \sqrt{\alpha^2 + \frac{4\pi^2}{9}\,(\rho_s - B)}$$

Massless limit: Linear EoS

B: MIT bag constant

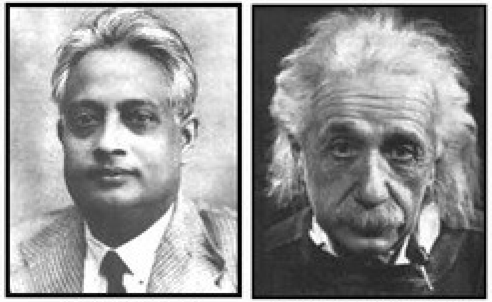
$$m_s \to 0 \quad p_s = \frac{1}{3}(\rho_s - 4B)$$

m_s: mass of s quark

Bose-Einstein Condensate

5th state of matter formed in ultra cold bosonic systems

Predicted in 1924-1925



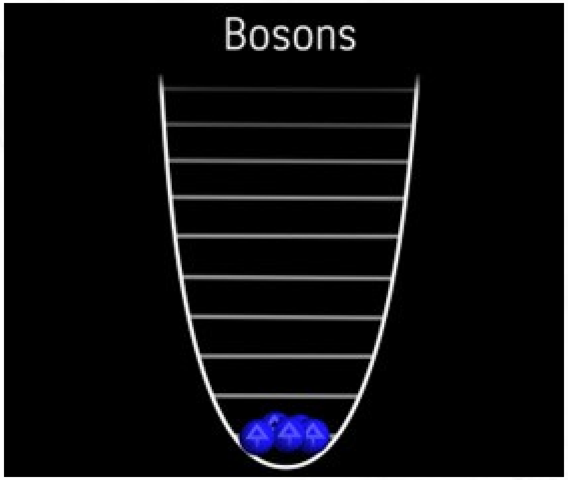
S.N. Bose (1894-1974)   Albert Einstein (1879-1955)

Condition: matter wave overlap

$$d \sim n^{-1/3} \quad \lambda_{DB} = \sqrt{\frac{2\pi}{mT}}$$

$$T \to 0$$

$$N \gg 1$$



No Pauli's principle

No thermal fluctuations

Giant matter wave

Condensed dark matter

Dilute, ultracold gas of bosons: easy to model

1. Repulsive short-range interaction

$$U = \frac{4\pi a}{m}\delta(r - r') = g\delta(r - r') \rightarrow \sigma = 4\pi a^2$$

m: Mass a: Scattering length

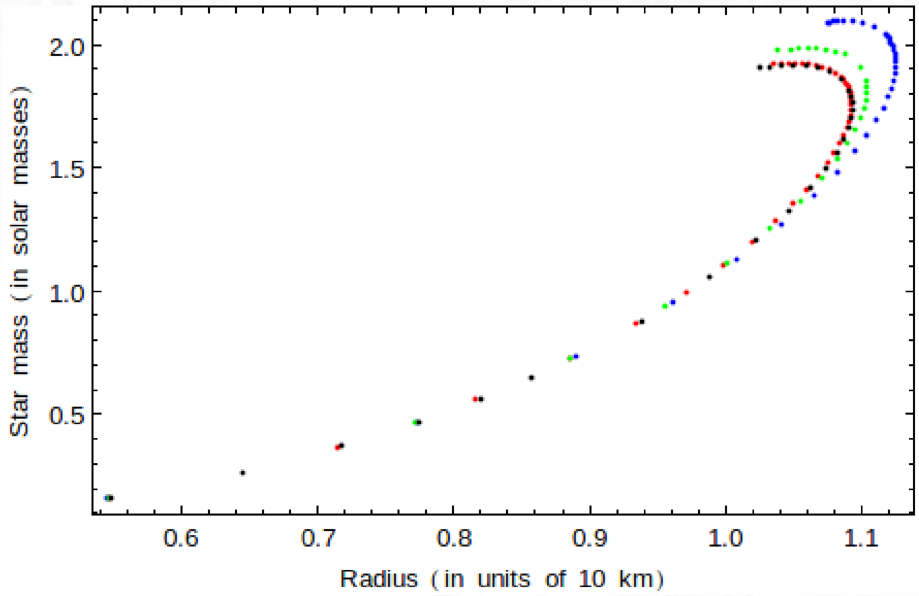
2. BEC is decribed by a single macroscopic wave-function

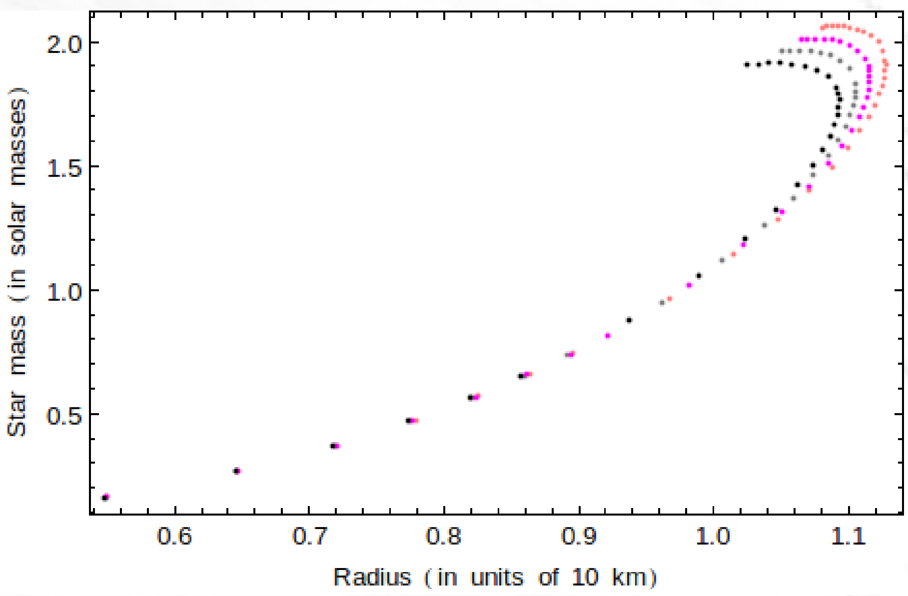
$$i\frac{\partial\Psi(t,r)}{\partial t} = \left(-\frac{\nabla^2}{2m} + V_{ext}(r) + g|\Psi(t,r)|^2\right)\Psi(t,r)$$

Gross-Pitaevskii Equation $\rightarrow$ EoS: $P = \frac{2\pi a}{m^3}\rho^2$

Condensed DM impact on SQSs

Different DM amounts $(5 - 10)\%$

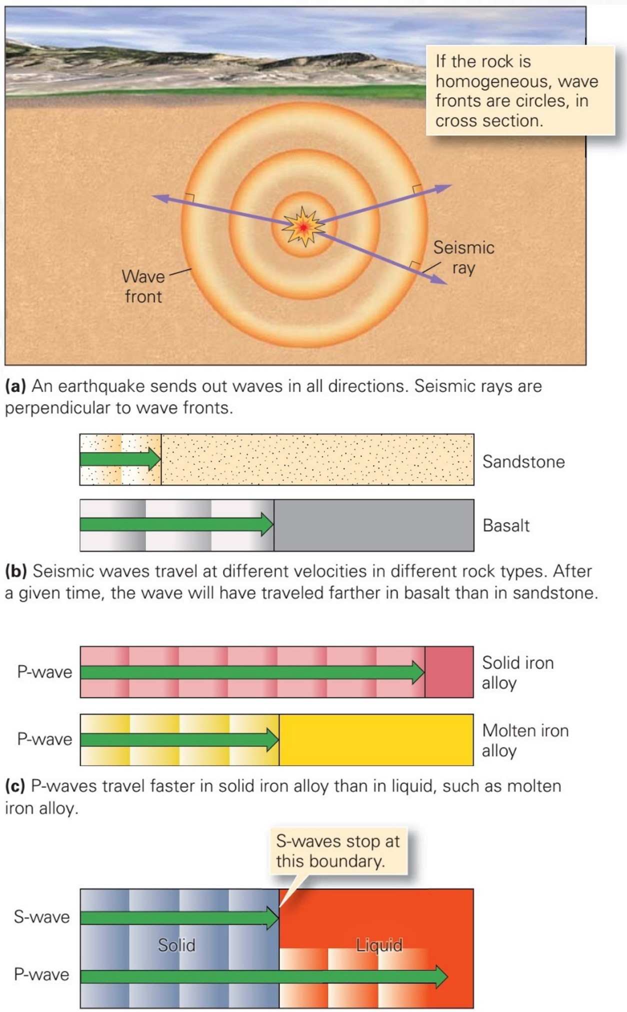




$$K_1 = \frac{4}{B}$$

$$K_2 = \frac{150}{B}$$

Asteroseismology (or sounding the interior of stars)

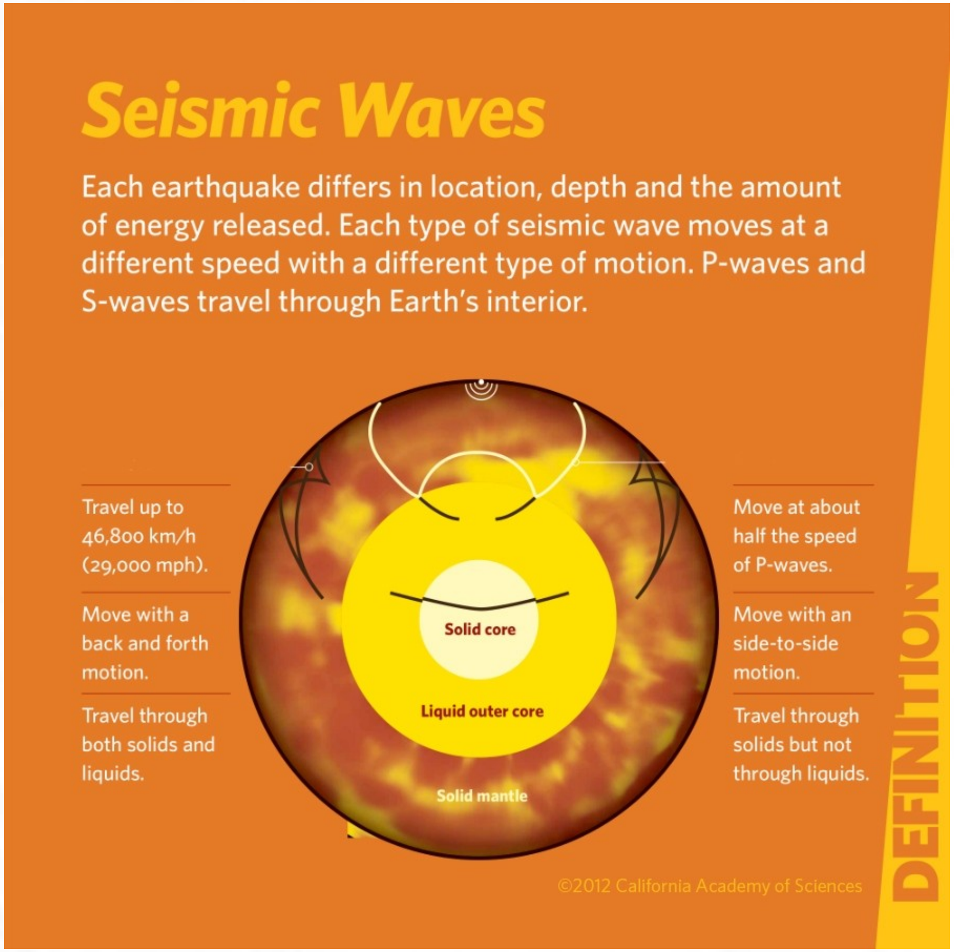


(a) An earthquake sends out waves in all directions. Seismic rays are perpendicular to wave fronts.

(b) Seismic waves travel at different velocities in different rock types. After a given time, the wave will have traveled farther in basalt than in sandstone.

(c) P-waves travel faster in solid iron alloy than in liquid, such as molten iron alloy.

(d) Both P-waves and S-waves can travel through a solid, but only P-waves can travel through a liquid.

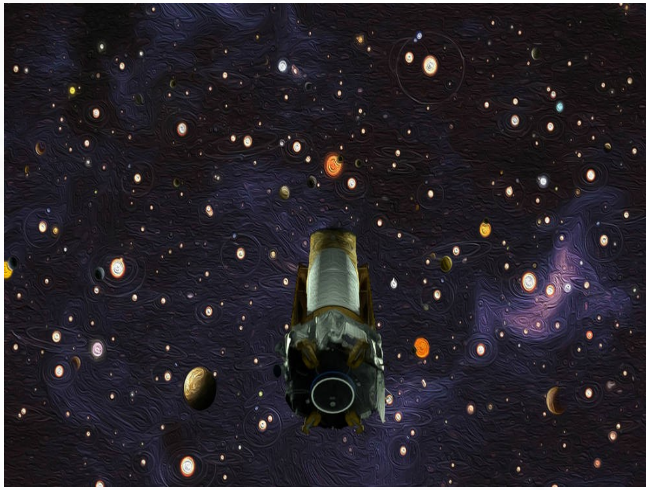


Study the vibrating modes of stars

Powerful tool to probe the interior of stars

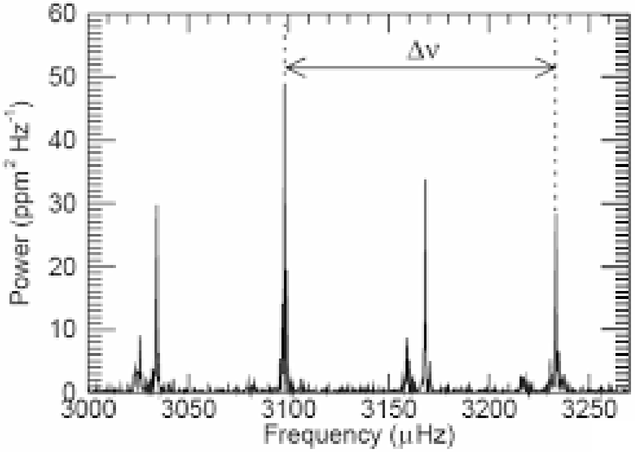
Oscillation frequencies → very sensitive
to internal composition and structure

NASA Kepler launched in 2009



European CoRoT launched in 2006

Oscillation spectra



Seismic parameters

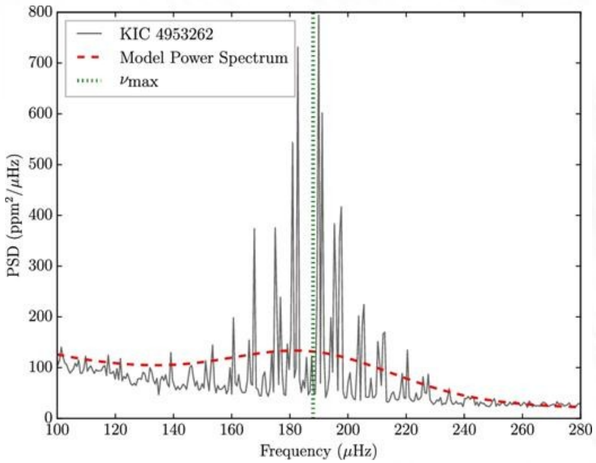
Properties of the star

e.g. Large frequency spacing

$$\Delta\nu_{n,l} = \nu_{n+1,l} - \nu_{n,l} \approx \left[2 \int_0^R \frac{dr}{c_s(r)} \right]^{-1}$$

Mean density of star

(Higher excited modes)



Frequency at the peak

ν_{max}

Perturbation theory

$$g = g_0 + \Delta g \qquad \Delta g \ll g_0$$

Background Perturbation

Perturbe Einstein's equations and linearize

$$\Delta g(t, r, \theta, \phi) = e^{-i\omega t} R(r) Y_l^m(\theta, \phi)$$

Simplest case: Radial oscillations $l = 0 = m$

Do not couple to metric perturbations

$$\Delta g(t, r) = e^{-i\omega t} R(r)$$

Equations for perturbations

$$\eta = \Delta P/P$$

$$\xi'(r) = -\frac{1}{r}\left(3\xi + \frac{\eta}{\gamma}\right) - \frac{P'}{P+\epsilon}\xi$$

$$\xi = \Delta r/r$$

$$\eta'(r) = \xi\left[\omega^2 r\frac{P+\epsilon}{P}e^{\lambda - A} - \frac{4P'}{P} - 8\pi(P+\epsilon)re^{\lambda} + \frac{r(P')^2}{P(P+\epsilon)}\right]$$

$$+\eta\left[-\frac{\epsilon P'}{P(P+\epsilon)} - 4\pi(P+\epsilon)re^{\lambda}\right]$$

$$\gamma = \left(\frac{dP}{d\epsilon}\right)\left(1 + \frac{\epsilon}{P}\right)$$

$\omega \rightarrow$ Eigenvalue

Boundary conditions

At the center

$r = 0$ $\eta(0) = -3\gamma(0)\xi(0)$

At the surface

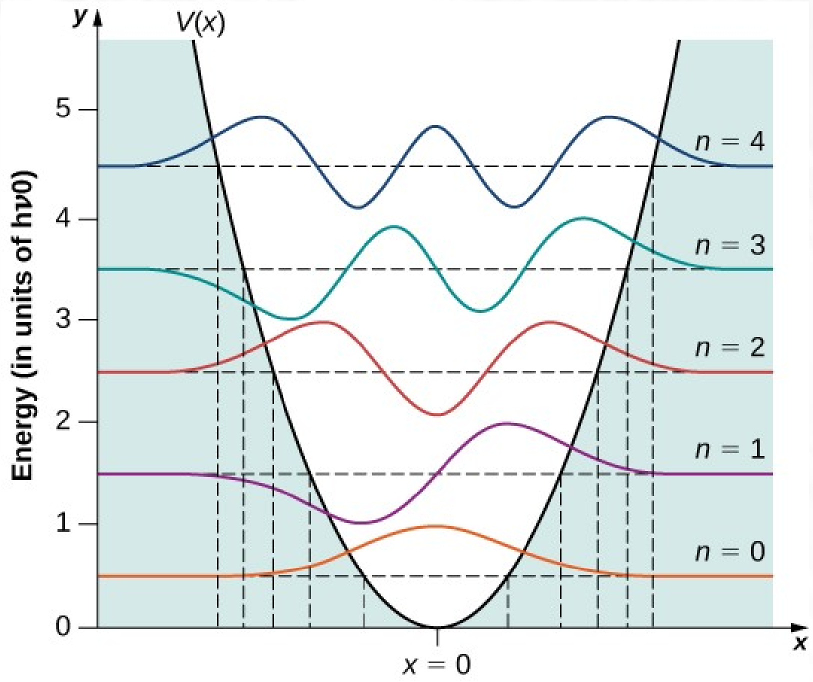
$r = R$

$$\eta(R) = \xi(R)\left[-4 + (1 - 2M/R)^{-1}\left(-\frac{M}{R} - \frac{\omega^2 R^3}{M}\right)\right]$$

Satisfied only for discrete ω_n $n = 0, 1, 2, \ldots$

Number of nodes in a boundary value problem

Recall quantum harmonic oscillator



Ground state: 0 nodes

1st excited state: 1 node

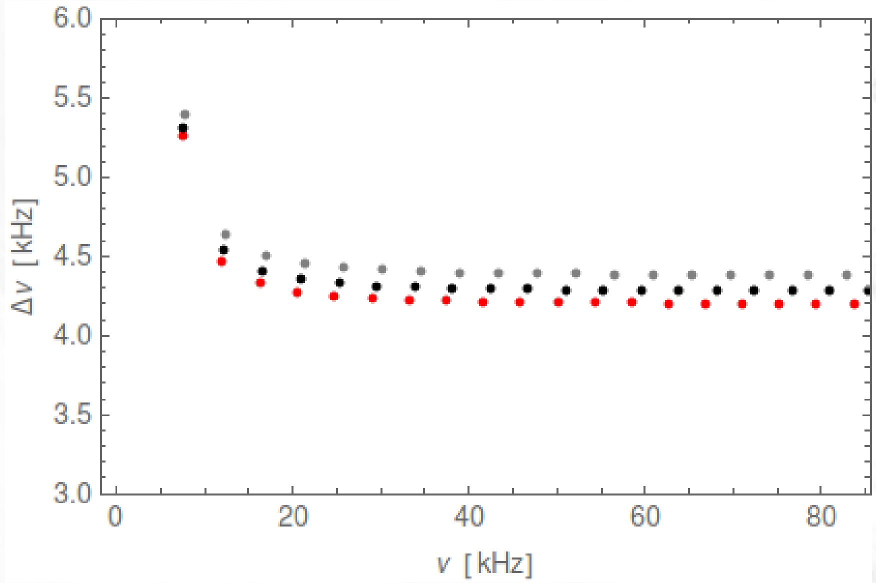
Second excited state: 2 nodes

etc

Radial Oscillations: Frequencies

mode order n	Compact star's models No DM	1st DM	2nd DM
0	2.35	2.47	2.60
1	7.60	7.77	7.99
2	12.06	12.31	12.62
3	16.38	16.71	17.12
4	20.65	21.06	21.57
5	24.89	25.38	25.99
...	...	...	...
10	45.96	46.85	47.96
11	50.17	51.14	52.34
...	...	...	...
18	79.55	81.08	82.99
19	83.74	85.36	87.36

$$K_1 = \frac{1.01}{B} \qquad 5\%\ \ \mathrm{DM}$$

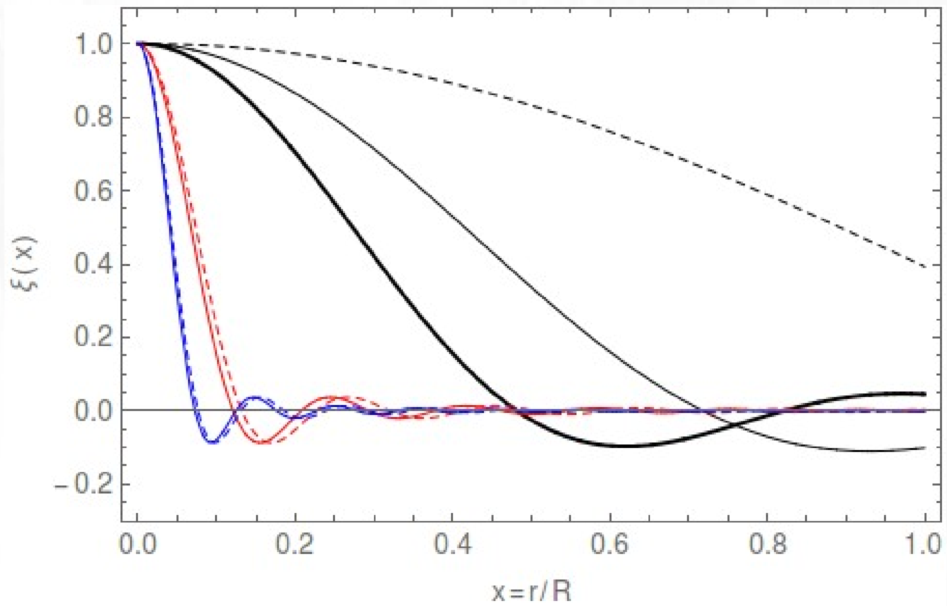


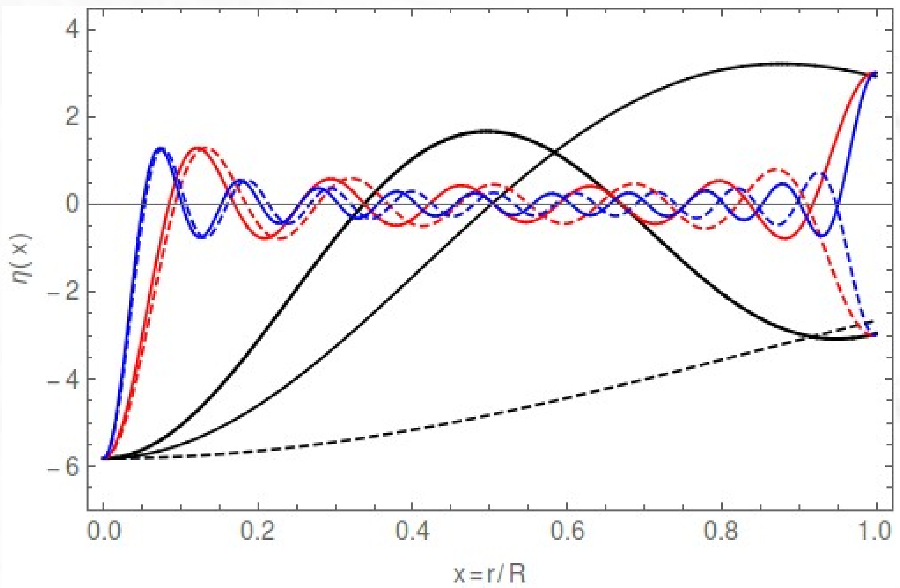
$$M = 1.77\ M_\odot$$

$$R = 10.94\ km$$

$$K_2 = \frac{0.46}{B} \qquad 12\%\ \ \mathrm{DM}$$

Radial oscillations: Eigenfunctions





Part II: Oscillations of pulsating BSs made of ULRDM

A: Radial oscillations

B: Non radial oscillations

Pulsating (Newtonian) boson stars

Structure equations

$$m'(r) = 4\pi r^2 \rho$$

$$p'(r) \simeq -\rho \frac{m(r)}{r^2}$$

EoS:

$$p = K\rho^2$$

$$K = \frac{1}{(2\Lambda)^4}$$

$$\Lambda = \sqrt{mF}$$

$\rightarrow$

Lane-Emden equation

$$\frac{d}{dx}\left(x^2 \frac{d\theta}{dx}\right) = -x^2\theta$$

$$\theta(0) = 1 \qquad \theta'(0) = 0$$

$$x = \frac{r}{a} \qquad \theta = \frac{\rho}{\rho_c}$$

$$a = \sqrt{K/2\pi}$$

Radial oscillations: equation for the perturbation

Replace

$\xi \to \zeta$

$$\zeta'' - \frac{G}{f^2}\zeta' + \left(\frac{\omega^2}{f^2} - \frac{H}{f^2}\right)\zeta = 0$$

$f \simeq c_s$ NR limit

$$\zeta(0) = 0 = \zeta(R)$$

H(r),G(r): Known quantities from the background

Schrödinger-like equation

$$\frac{d^2\psi}{d\tau^2} + \left[\omega^2 - U_l(\tau)\right]\psi = 0$$

Radial oscillations: Frequencies

radial order n	frequency ν_n (mHz)
0	0.8457
1	1.7426
2	2.5456
3	3.3155
4	4.0676
5	4.8110
6	5.5579
7	6.3032
8	7.0359
9	7.7774
10	8.5076

$$F \sim 10^{16}\ GeV$$

GUT scale

$$m \sim 10^{-8}\ eV$$

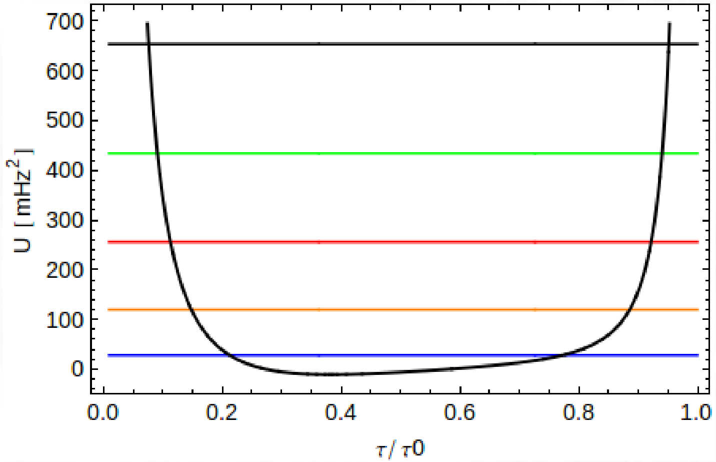
Ultra-light DM

$$M = 1.9 \times 10^{-14}\ M_\odot$$

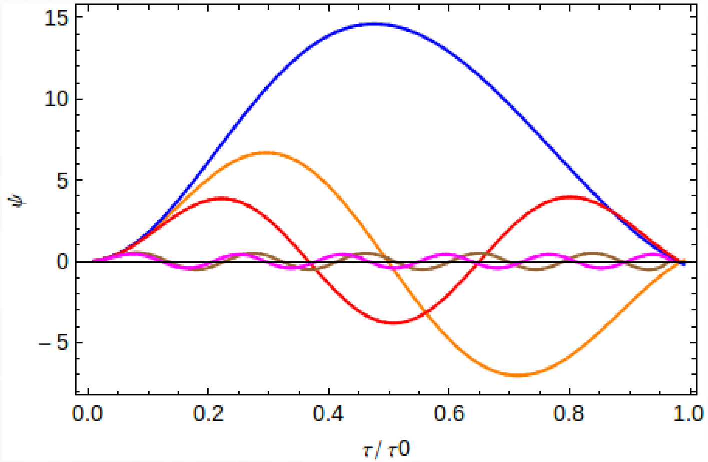
$$R = 6.9\ km$$

$$\omega_0 = \sqrt{M/R^3} = 2.74\ mHz$$

Radial oscillations:Main numerical results

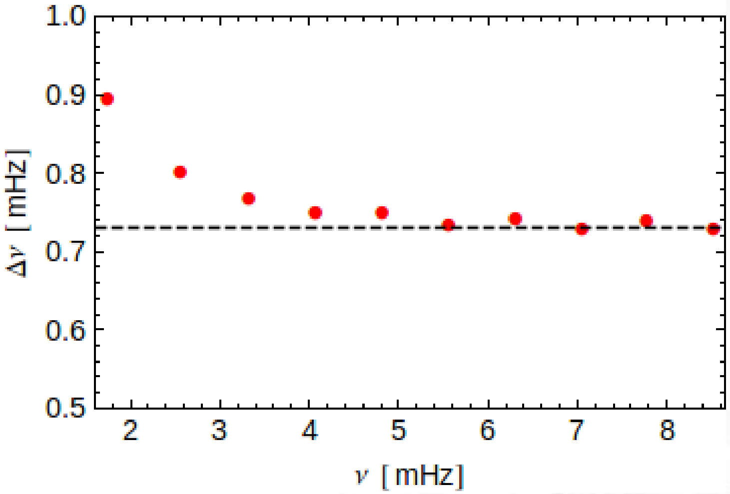


Acoustic potential



Eigenfunctions

Large separation



$$\Delta\nu = \nu_0 = 0.73\ mHz$$

Non-radial oscillations

$$\zeta''(r) + \left(\frac{2}{r} + \frac{2\rho'(r)}{\rho(r)}\right)\zeta'(r) + \left(\frac{\omega_{n,l}^2}{c_s^2} - \frac{l(l+1)}{r^2}\right)\zeta(r) = 0$$

Angular degree $l = 1, 2, 3, \ldots$

Schrödinger-like equation

$$\frac{d^2\psi}{d\tau^2} + \left[\omega^2 - U_l(\tau)\right]\psi = 0$$

BC: $\psi(r = R) = 0$

$\psi(r = 0) = 0$

$F \sim 10^{16}\ GeV$ GUT scale

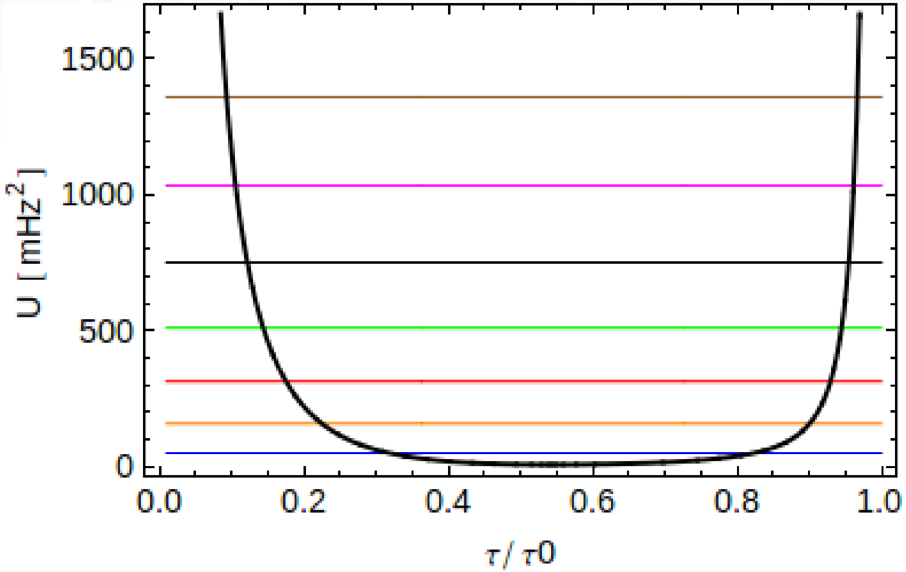
$m \sim 10^{-8}\ eV$ Ultra-light DM

Frequencies

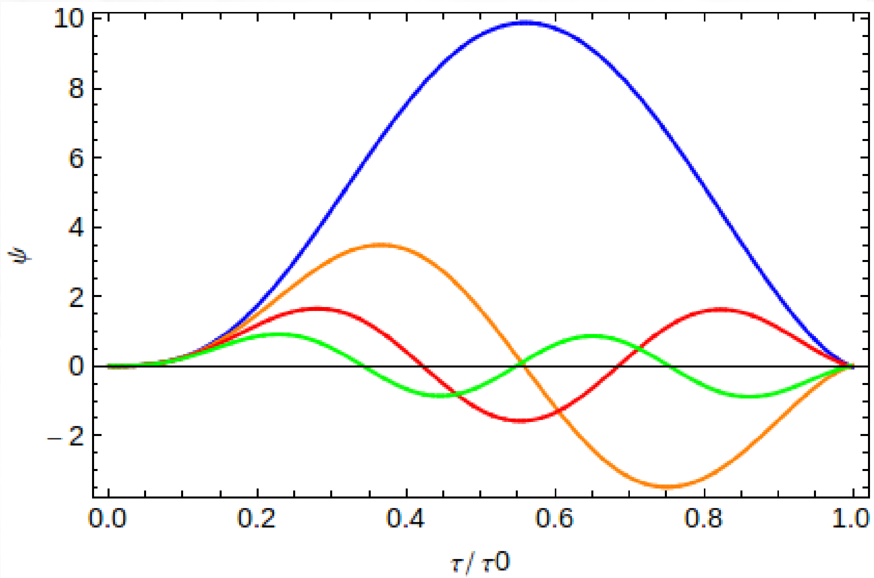
overtones n	$\nu_{1,n}$ (mHz)	$\nu_{2,n}$ (mHz)	$\nu_{3,n}$ (mHz)
0	0.8441	0.8457	1.3274
1	1.7409	1.7426	2.2288
2	2.5397	2.5456	3.0588
3	3.3096	3.3155	3.8555
4	4.0657	4.0676	4.6332
5	4.8140	4.8110	5.3990
6	5.5576	5.5579	6.1568
7	6.2980	6.3032	6.9090
8	7.0362	7.0359	7.6571
9	7.7727	7.7774	8.4021
10	8.5081	8.5076	9.1448
11	...	9.5719	9.8855
12		10.3079	10.6248
13		11.0432	11.3628
14		11.7777	12.0999
15		...	12.8361
16			13.5716
17			14.3066
18			...

Numerical results II

$$l = 2$$



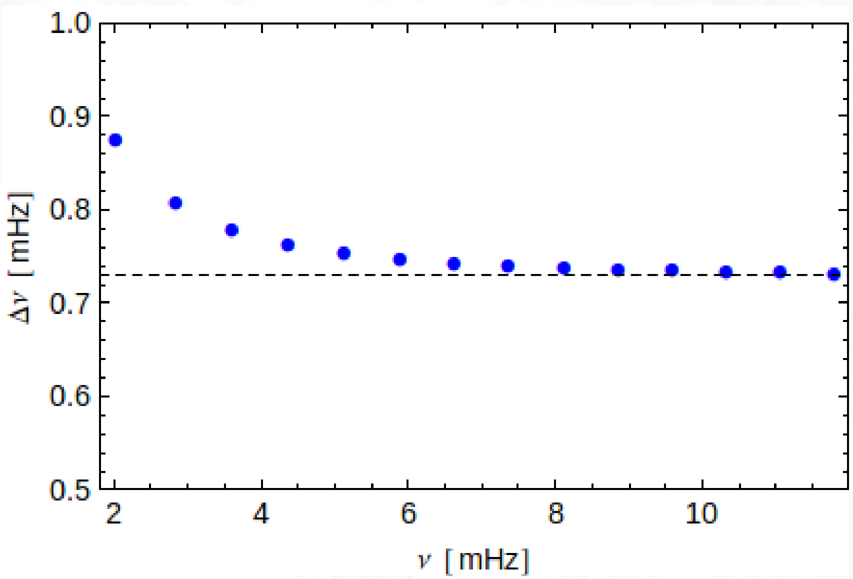
Acoustic potential

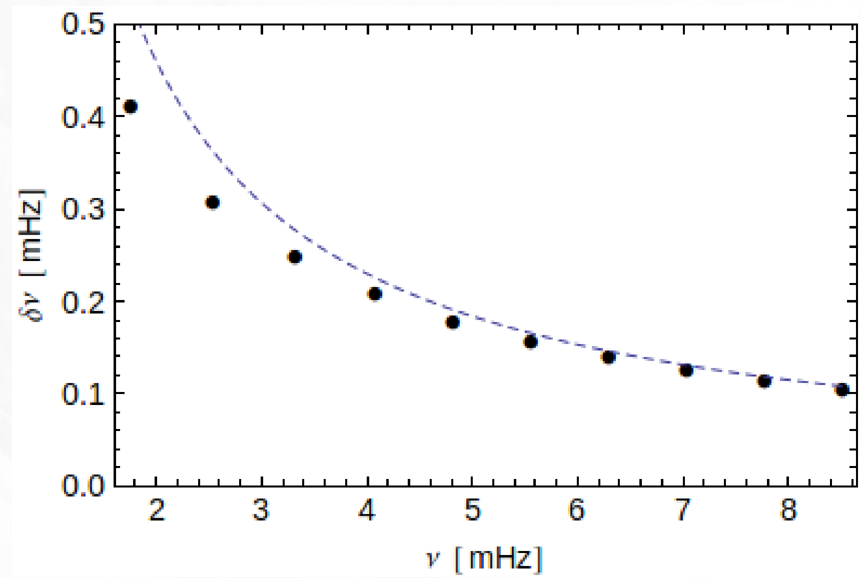


Eigenfunctions

Frequency differences

$$l = 2$$





Large separation

$$\Delta\nu_{n,l} = \nu_{n+1,l} - \nu_{n,l}$$

Small separation

$$\delta\nu_{l,n} = \nu_{l,n} - \nu_{l+2,n-1}$$

$$\nu_0 = \left(\frac{\pi}{2K^3}\right)^{1/4} \frac{\sqrt{M}}{5.9} = 0.73\ mHz$$

Conclusions

- Dark matter is an essential ingredient of standard cosmological model (don't know if it is fermionic or bosonic)
- Strange stars are hypothetical compact stars that can explain super-luminous supernovae
- We have integrated the structure equations assuming a two-fluid system (quarks + bosonic DM) for 2 DM models → M-R profile
- We have integrated the equations for the perturbations to compute the frequencies and the eigenfunctions
- Adding more DM the frequencies increase
- Radial and non radial oscillations of BSs made of ULRDM

Thank you for your attention!