Theoretical investigation of possibility to suppress FSR in specific dark matter models explaining cosmic positron data

Airat Kamaletdinov, Ekaterina Shlepkina, Konstantin Belotsky

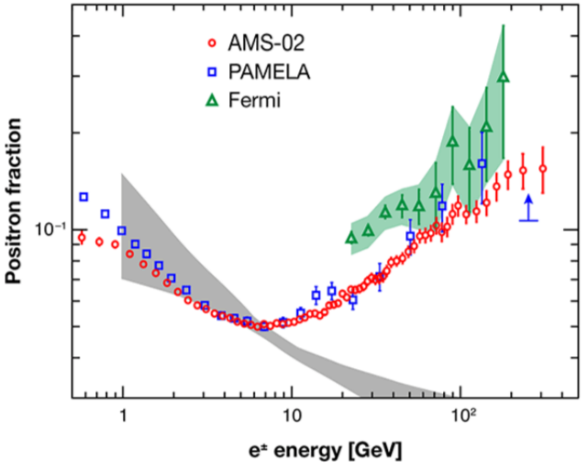
NRNU MEPhI

10 July 2020

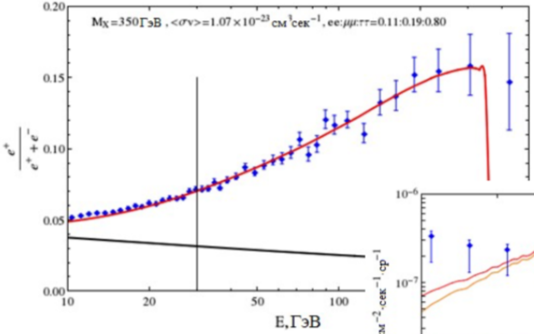
Positron Anomaly

Pamela
Спутник: Ресурс ДК1
Since: 15 june 2006

АМС-02
Установлен на МКС
Since: 16 may 2011

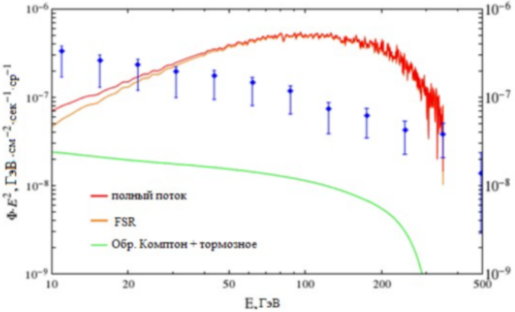


Contradiction with IGRB



Satisfactory description of the positron fraction

Strong contradiction (excess) in IGRB

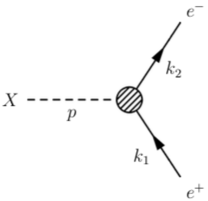


Interaction vertex parametrization

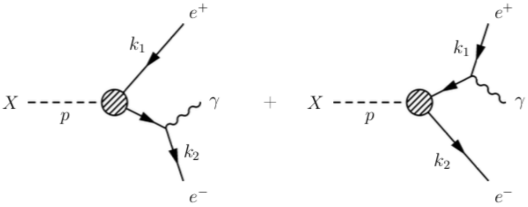
We started by choosing the simplest decay vertices:

$$\mathcal{L} = X\bar{\Psi}(a + b\gamma^5)\Psi \qquad \text{and} \qquad \mathcal{L} = X_\mu\bar{\Psi}\gamma^\mu(a + b\gamma^5)\Psi$$

Two-body decay



Three-body decay



Suppression of the photon yield is achieved by

$$\frac{\sigma(X \to e^- e^+ \gamma)}{\sigma(X \to e^- e^+)} \to min \qquad \text{where a and b are fixed parameters.}$$

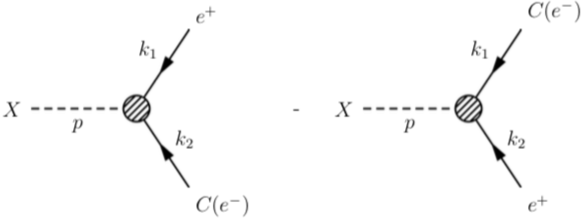
Decay into identical positrons

Double charged Dark Matter particles model was also considered

$$\mathcal{L}_C = X\overline{\psi^C}(a + b\gamma^5)\psi + X^*\overline{\psi}(a + b\gamma^5)\psi^C.$$

$$X \to e^+e^+ \qquad X^* \to e^-e^-$$

We assume that there are no particles X^* in the DM sector.



Similar models of heavy double charged DM particles are proposed, for example, in arXiv:1411.365 and arXiv:astro-ph/0511789

Independence of photon yield on model parameters

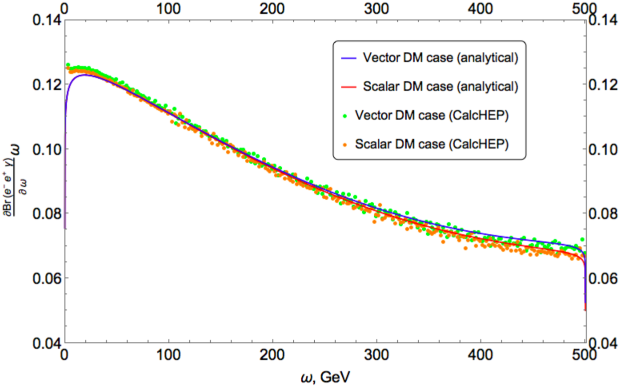
For $X \to e^- e^+ (\gamma)$

	$\mathcal{L} = X\bar{\Psi}(a + b\gamma^5)\Psi$	$\mathcal{L} = X_\mu \bar{\Psi}\gamma^\mu(a + b\gamma^5)\Psi$
$\lvert M\rvert^2_{2\ body}$	$2\ (a^2 + b^2)m_X^2$	$4\ (a^2 + b^2)m_X^2$
$\lvert M\rvert^2_{(3\ body)}$	$(a^2 + b^2)F(k_1, k_2, l)$	$(a^2 + b^2)G(k_1, k_2, l)$
$\frac{\sigma(e^- e^+ \gamma)}{\sigma(e^- e^+)}$	$\frac{F(k_1,k_2,l)}{2\ m_X^2}$	$\frac{G(k_1,k_2,l)}{4\ m_X^2}$

For $X \to e^+ e^+ (\gamma)$ $\quad \mathcal{L} = X\overline{\Psi^C}\hat{O}\Psi + X^*\overline{\Psi}\hat{O}\Psi^C \quad |in\rangle \equiv \hat{X}|0\rangle$

	$\mathcal{L} = X\bar{\Psi}^C(a + b\gamma^5)\Psi$	$\mathcal{L} = X_\mu \bar{\Psi}^C\gamma^\mu(a + b\gamma^5)\Psi$
$\lvert M\rvert^2_{(2\ body)}$	$8\ (a^2 + b^2)m_X^2$	$16\ b^2 m_X^2$
$\lvert M\rvert^2_{(3\ body)}$	$(a^2 + b^2)F(k_1, k_2, l)$	$b^2 G(k_1, k_2, l)$
$\frac{\sigma(e^+ e^+ \gamma)}{\sigma(e^+ e^+)}$	$\frac{F(k_1,k_2,l)}{8\ m_X^2}$	$\frac{G(k_1,k_2,l)}{16\ m_X^2}$

Difference of **scalar** coupling $\mathcal{L} = X\overline{\Psi}(a + b\gamma^5)\Psi$ in comparison with **vector** one $\mathcal{L} = X_\mu\overline{\Psi}\gamma^\mu(a + b\gamma^5)\Psi$



$$\left.\frac{\partial\sigma(e^-e^+\gamma)/\partial\omega}{\sigma(e^-e^+)}\right|_{scalar} = -e^2\left.\frac{(m^2 - 2m\omega + 2\omega^2)\log(|\frac{m-2E_1}{m-2(E_1+\omega)}|)}{4\pi^2 m^2\omega}\right|_{E_1^-}^{E_1^+}$$

$$\left.\frac{\partial\sigma(e^-e^+\gamma)/\partial\omega}{\sigma(e^-e^+)}\right|_{vector} = -e^2\left.\frac{(m^2 - 2m\omega + 2\omega^2)\log(|\frac{m-2E_1}{m-2(E_1+\omega)}|) - 4E_1\omega}{4\pi^2 m^2\omega}\right|_{E_1^-}^{E_1^+}$$

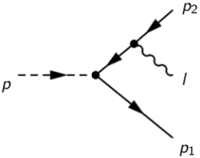
Derivative in the interaction vertex

Class of interaction vertices which depend on the decaying particle momentum was considered.

$$\mathcal{L} = \overline{\Psi}\gamma^{\mu}(a + b\frac{\gamma^{\nu}\partial_{\nu}}{m})X_{\mu}\Psi \qquad \mathcal{L} = \overline{\Psi}\gamma^{\mu}(a + b\frac{(\gamma^{\nu}\partial_{\nu})(\gamma^{\rho}\partial_{\rho})...}{m^{n}})X_{\mu}\Psi$$

$$\mathcal{L} = \overline{\Psi}\gamma^{\mu}(a\gamma^{5} + b\frac{(\gamma^{\nu}\partial_{\nu})}{m})X_{\mu}\Psi \qquad ...$$

Such approach makes it possible to achieve an effect on the photon yield by the parametrization of interaction Lagrangian.



$$(X \to e^{+}e^{-}) \;\Rightarrow\; \bar{u}(p_1)\gamma^{\mu}\left(a + b\frac{\hat{p_1} + \not{\hat{p_2}}}{m}\right)v(p_2) = \bar{u}(p_1)\gamma^{\mu}\left(a + b\frac{\hat{p_1}}{m}\right)v(p_2)$$

$$(X \to e^{+}e^{-}\gamma) \;\Rightarrow\; \bar{u}(p_1)\gamma^{\mu}\left(a + b\frac{\hat{p_1} + \hat{p_2} + \hat{l}}{m}\right)\left[\frac{\hat{p_2} + \hat{l}}{(p_2 + l)^2}\hat{\epsilon}(l)\right]v(p_2)$$

For example, for vertex $\mathcal{L} = \overline{\Psi}\gamma^{\mu}(a + \frac{b(\gamma^{\nu}\partial_{\nu})}{m})X_{\mu}\Psi$:

$$\frac{\partial Br(e^+e^-\gamma)}{\partial\omega} = -e^2 \frac{(2a^2+b^2)m(m^2-2m\omega+2\omega^2)\log(|\frac{m-2E_1}{m-2(E_1+\omega)}|) - 8E_1\omega(a^2m+2b^2\omega)}{4\pi^2 m^3\omega(2a^2+b^2)}\Bigg|_{E_1^-}^{E_1^+}$$

However, this vertex does not lead to a significant result. Moreover, the class of such vertices is **bounded** and their extension to arbitrary polynomials $f(\hat{p})$ is impossible since:

$$\hat{p}\hat{p} \equiv p^2 = m^2$$

$$f(\hat{p}) = a + b\frac{\hat{p}}{m} + c\frac{\hat{p}\hat{p}}{m^2} + d\frac{\hat{p}\hat{p}\hat{p}}{m^3} + ... + A\gamma^5 + B\gamma^5\frac{\hat{p}}{m} + C\gamma^5\frac{\hat{p}\hat{p}}{m^2} + D\gamma^5\frac{\hat{p}\hat{p}\hat{p}}{m^3} + ... =$$

$$= a + b\frac{\hat{p}}{m} + c + d\frac{\hat{p}}{m} + ... + A\gamma^5 + B\gamma^5\frac{\hat{p}}{m} + C\gamma^5 + D\gamma^5\frac{\hat{p}}{m} =$$

$$= (a + c + ...) + (b + d + ...)\frac{\hat{p}}{m} + (A + C + ...)\gamma^5 + (B + D + ...)\gamma^5\frac{\hat{p}}{m}$$

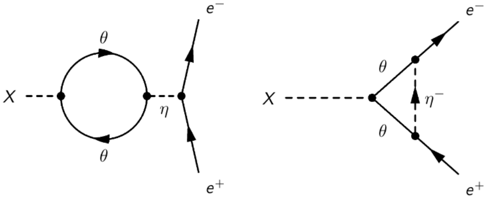
Thus $f(\hat{p})$ can only be a linear function of $\hat{p}$.

Consideration of loop contributions

The dependences of the coefficients a and b on the decay energies can also be achieved by considering the loop processes.

$$a \to F_1(\sqrt{s}), \qquad b \to F_2(\sqrt{s})$$

The following processes were considered



Corresponding interaction Lagrangians of such models are follows:

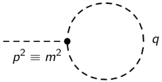
$$\mathcal{L}_{\bigcirc} = X\bar{\theta}(a + \mathrm{i}\, b\gamma^5)\theta + \eta\, \bar{\theta}(c + \mathrm{i}\, d\gamma^5)\theta + \eta\bar{\Psi}\Psi$$

$$\mathcal{L}_{\triangle} = X\bar{\theta}(a + \mathrm{i}\, b\gamma^5)\theta + \eta\, \bar{\theta}(c + \mathrm{i}\, d\gamma^5)\Psi + \eta^*\, \bar{\Psi}(c + \mathrm{i}\, d\gamma^5)\theta$$

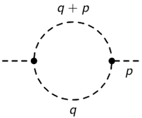
The Passarino and Veltman reduction procedure was used for one-loop integrals, described in detail in https://arxiv.org/abs/1105.4319.

such procedure
This procedure consists in reducing single-loop integrals to a linear combination of standard scalar integrals:

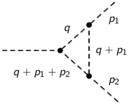
$$A_0(m) = \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q^2 - m^2)}$$



$$B_0(p;\ m_1, m_2) = \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q^2 - m_1^2)((q+p)^2 - m_2^2)}$$

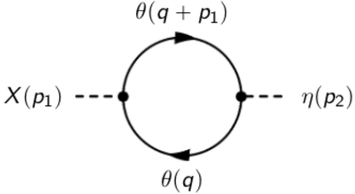


$$C_0(p_1, p_2;\ m_1, m_2, m_3) = \int \frac{d^D q}{(2\pi)^D} \frac{1}{d_1 d_2 d_3}$$



$$d_i \equiv \left((q + \sum_{k=1}^{i-1} p_k)^2 - m_i^2 \right)$$

Bubble vertex



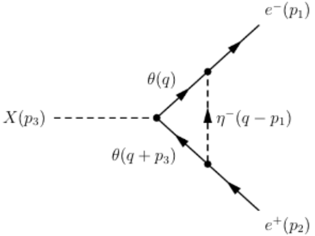
The functions A_0, B_0, C_0 depend quadratically on their arguments. Hence the loop contribution to $(X \to e^+e^-)$ and $(X \to e^+e^-\gamma)$ turns out to be the same.

$$\hat{O} = \int \frac{d^D q}{(2\pi)^D} \frac{Tr\Big((a+ib\gamma^5)(\hat{q}+m)(c+id\gamma^5)(\hat{q}+\hat{p_1}+m)\Big)}{(q^2-m^2)((q+p_1)^2-m^2)} =$$

$$= \int \frac{d^D q}{(2\pi)^D} \frac{4m^2(ac-bd)}{(q^2-m^2)((q+p_1)^2-m^2)} + \int \frac{d^D q}{(2\pi)^D} \frac{4(q^2-p_1\cdot q)(ac+bd))}{(q^2-m^2)((q+p_1)^2-m^2)} =$$

$$= 4m^2(ac-bd)B_0(p_1,m,m) + 4(ac+bd)\Big(A_0(m) + m^2 B_0(p_1,m,m) - \frac{p_1^2}{2}B_0(p_1,m,m)\Big)$$

Triangle diagram (two-body decay)



$$i\,\mathcal{M} = \bar{u}\Bigg[(c+id\gamma^5)\int\frac{d^Dq}{(2\pi)^D}\frac{i(\hat{q}+m_1)(a+ib\gamma^5)i(\hat{q}-\hat{p_1}-\hat{p_2}+m_3)(-i)}{(q^2-m_1^2)((q-p_1)^2-m_2^2)((q-p_1-p_2)^2-m_3^2)}(c+id\gamma^5)\Bigg]v =$$

$$= i\ \bar{u}(p_1)\Bigg[\int\frac{d^Dq}{(2\pi)^D}\frac{\hat{f}_1(q)-i\hat{f}_2(q)\gamma^5}{d_1d_2d_3}\Bigg]v(p_2) \quad ; \qquad d_i \equiv (q-\sum_{k=1}^{i-1}p_k)^2-m_i^2$$

$$\hat{f}_1(q) = a(c^2+d^2)\Big(m_1(\hat{q}-\hat{p_1}-\hat{p_2})+m_3\hat{q}\Big)+a(c^2-d^2)\Big(\hat{q}(\hat{q}-\hat{p_1}-\hat{p_2})+m_1m_3\Big)+$$
$$+2bcd\Big(\hat{q}(\hat{q}-\hat{p_1}-\hat{p_2})-m_1m_3\Big)$$

$$\hat{f}_2(q) = b(c^2+d^2)\Big(m_1(\hat{q}-\hat{p_1}-\hat{p_2})-m_3\hat{q}\Big)+b(c^2-d^2)\Big(\hat{q}(\hat{q}-\hat{p_1}-\hat{p_2})-m_1m_3\Big)-$$
$$-2acd\Big(\hat{q}(\hat{q}-\hat{p_1}-\hat{p_2})+m_1m_3\Big)$$

Calculations ($X \to e^+e^-$)

The following vertex factors should be integrated:

$$\hat{f}_\pm(q) = H_{(\pm)}(c^2+d^2)\Big(m_1(\hat{q}-\hat{p_1}-\hat{p_2}) \pm m_3\hat{q}\Big) + H_{(\pm)}(c^2-d^2)\Big(\hat{q}(\hat{q}-\hat{p_1}-\hat{p_2}) \pm m_1 m_3\Big) \pm$$
$$\pm 2\ H_{(\mp)}\ cd\Big(\hat{q}(\hat{q}-\hat{p_1}-\hat{p_2}) \mp m_1 m_3\Big) \quad \text{где} \quad \{H_{(+)}, H_{(-)}\} \equiv \{a, b\}$$

Let's define the following vector integral

$$C^\mu(p_1,p_2;\ m_1,m_2,m_3) = \int \frac{d^D q}{(2\pi)^D} \frac{q^\mu}{(q^2-m_1^2)((q+p_1)^2-m_2^2)((q+p_1+p_2)^2-m_3^2)}$$

From Lorentz-invariance of this integral it follows that

$$C^\mu(p_1,p_2;\ m_1,m_2,m_3) = p_1^\mu C_1(p_1,p_2;\ m_1,m_2,m_3) + p_2^\mu C_2(p_1,p_2;\ m_1,m_2,m_3)$$
$$\Rightarrow \quad \int \frac{d^D q}{(2\pi)^D} \frac{\hat{q}}{d_1 d_2 d_3} = \gamma_\mu C^\mu = -\hat{p_1} C_1 - \hat{p_2} C_2 \qquad \Rightarrow \qquad \bar{u}(p_1)\gamma_\mu C^\mu v(p_2) = 0$$

Thus the first term of vertex factors does not contribute to the two-body decay

$$\hat{f}_{\pm}(q) = H_{(\pm)}(c^2 - d^2)\Big(\hat{q}(\hat{q} - \hat{p_1} - \hat{p_2}) \pm m_1 m_3\Big) \pm 2\, H_{(\mp)} cd \Big(\hat{q}(\hat{q} - \hat{p_1} - \hat{p_2}) \mp m_1 m_3\Big)$$

$$1\Big) \int \frac{d^D q}{(2\pi)^D} \frac{\hat{q}\hat{q} \equiv q^2}{d_1 d_2 d_3} = \int \frac{d^D q}{(2\pi)^D} \frac{q^2}{d_1 d_2 d_3} = d_1 C_0(p_1, p_2;\ m_1, m_2, m_3) +$$

$$+ m_1^2 C_0(p_1, p_2;\ m_1, m_2, m_3) = B_0(p_2;\ m_2, m_3) + m_1^2 C_0(p_1, p_2;\ m_1, m_2, m_3)$$

$$\left(d_1 C_0(p_1, p_2;\ m_1, m_2, m_3) = \int \frac{d^D q}{(2\pi)^D} \frac{1}{d_2 d_3} \overset{q \to q + p_1}{=\!=\!=} B_0(p_2;\ m_2, m_3) \right)$$

$$2\Big) \bar{u}(p_1) \int \frac{d^D q}{(2\pi)^D} \frac{\hat{q}(\hat{q} - \hat{p_1} - \not{\hat{p_2}})}{d_1 d_2 d_3} v(p_2) = \Big/ p_{1,2}^2 = 0 \Big/ = \bar{u}(p_1)\Big(B_0(p_2;\ m_2, m_3) +$$

$$+ m_1^2 C_0(p_1, p_2;\ m_1, m_2, m_3) + 2(p_1 \cdot p_2) C_2(p_1, p_2;\ m_1.m_2.m_3)\Big) v(p_2) =$$

$$= \bar{u}(p_1)\Big(B_0(p_1 + p_2;\ m_1, m_3) + m_2^2 C_0(p_1, p_2;\ m_1, m_2, m_3)\Big) v(p_2)$$

$$3\Big) C_2 = \frac{1}{2(p_1 \cdot p_2)} \Big((m_2^2 - m_1^2 - \not{p_1^2}) C_0 + B_0(p_1 + p_2;\ m_1, m_3) - B_0(p_2;\ m_2, m_3)\Big)$$

$$\Rightarrow F_{\pm} = H_{(\pm)}(c^2 - d^2)\Big(B_0(\sqrt{s};\ m_1, m_3) + (m_2^2 \pm m_1 m_3) C_0(p_1, p_2;\ m_1, m_2, m_3)\Big) \pm$$

$$\pm 2 H_{(\mp)}\Big(B_0(\sqrt{s};\ m_1, m_3) + (m_2^2 \mp m_1 m_3) C_0(p_1, p_2;\ m_1, m_2, m_3)\Big)$$

$$\Rightarrow i\,\mathcal{M} = i\bar{u}(p_1)\Big(F_1(\sqrt{s}) - iF_2(\sqrt{s})\gamma^5\Big)v(p_2) \qquad C_0(p_1,p_2;\ m_1,m_2,m_3) \sim F(s)$$

Loop vertices leads to a complex dependence of the decay width on the decay energy $\sqrt{s}$. Corresponding vertex factors are follows:

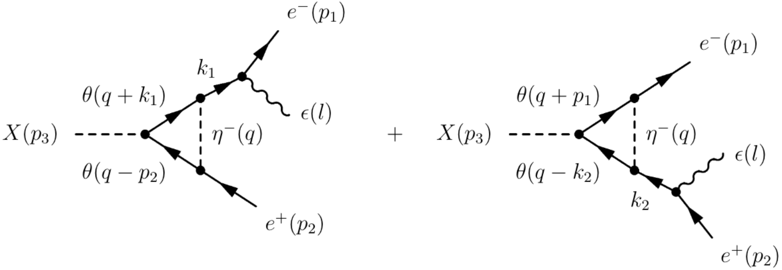
$$F_1(\sqrt{s}) = a(c^2 - d^2)\Big(B_0(\sqrt{s};\ m_1,m_3) + (m_2^2 + m_1 m_3)C_0(p_1,p_2;\ m_1,m_2,m_3)\Big) +$$
$$+2bcd\Big(B_0(\sqrt{s};\ m_1,m_3) + (m_2^2 - m_1 m_3)C_0(p_1,p_2;\ m_1,m_2,m_3)\Big)$$
$$F_2(\sqrt{s}) = b(c^2 - d^2)\Big(B_0(\sqrt{s};\ m_1,m_3) + (m_2^2 - m_1 m_3)C_0(p_1,p_2;\ m_1,m_2,m_3)\Big) -$$
$$-2acd\Big(B_0(\sqrt{s};\ m_1,m_3) + (m_2^2 + m_1 m_3)C_0(p_1,p_2;\ m_1,m_2,m_3)\Big)$$

The amplitude's square averaged over the final state polarizations is:

$$\frac{1}{4}\sum_{\lambda}\mathcal{M}\mathcal{M}^* = \frac{m_X^2}{2}\Big(F_1(\sqrt{s})^2 + F_2(\sqrt{s})^2\Big)$$

$$\frac{1}{4}\sum_{\lambda}\mathcal{M}\mathcal{M}^* = (c^2+d^2)^2 m_X^2\,\frac{a^2\Big(B_0 + (m_2^2 + m_1 m_3)C_0\Big)^2 + b^2\Big(B_0 + (m_2^2 - m_1 m_3)C_0\Big)^2}{2}$$

Triangle diagram (three-body decay)



$$i\,\mathcal{M} = i\,\bar{u}(p_1)\left[\gamma^\mu \frac{\hat{p}_1+\hat{l}}{(p_1+l)^2}\right]\left[\int \frac{d^D q}{(2\pi)^D}\frac{\hat{f}_{11}(q) - i\hat{f}_{12}(q)\gamma^5}{b_1 b_2 b_3}\right]v(p_2) +$$
$$+\, i\,\bar{u}(p_1)\left[\int \frac{d^D q}{(2\pi)^D}\frac{\hat{f}_{21}(q) - i\hat{f}_{22}(q)\gamma^5}{b_1 b_2 b_3}\right]\left[\frac{\hat{p}_2+\hat{l}}{(p_2+l)^2}\gamma^\mu\right]v(p_2),$$

$$\int \frac{d^4q}{(2\pi)^4}\frac{\hat{f_{11}}}{b_1 b_2 b_3} = a(c^2+d^2)\Big(\hat{p_1}(m_1+m_3)C_1(k_1,p_2)+\hat{k_1}m_3C_0(k_1,p_2)\Big)+$$
$$+a(c^2-d^2)\Big(B_0(\sqrt{s})+(m_2^2+m_1m_3)C_0(k_1,p_2)+2(p_1\cdot l)C_1(k_1,p_2)\Big)\hat{1}+$$
$$+2bcd\Big(B_0(\sqrt{s})+(m_2^2-m_1m_3)C_0(k_1,p_2)+2(p_1\cdot l)C_1(k_1,p_2)\Big)\hat{1},$$
$$\int \frac{d^4q}{(2\pi)^4}\frac{\hat{f_{12}}}{b_1 b_2 b_3} = a(c^2+d^2)\Big(\hat{p_1}(m_1+m_3)C_1(k_1,p_2)+\hat{k_1}m_3C_0(k_1,p_2)\Big)+$$
$$+a(c^2-d^2)\Big(B_0(\sqrt{s})+(m_2^2-m_1m_3)C_0(k_1,p_2)+2(p_1\cdot l)C_1(k_1,p_2)\Big)\hat{1}+$$
$$+2bcd\Big(B_0(\sqrt{s})+(m_2^2+m_1m_3)C_0(k_1,p_2)+2(p_1\cdot l)C_1(k_1,p_2)\Big)\hat{1},$$
$$\int \frac{d^4q}{(2\pi)^4}\frac{\hat{f_{21}}}{b_1 b_2 b_3} = a(c^2+d^2)\Big(\hat{p_2}(m_1+m_3)C_2(p_1,k_2)-\hat{k_2}m_1C_0(p_1,k_2)\Big)+$$
$$+a(c^2-d^2)\Big(B_0(\sqrt{s})+(m_2^2+m_1m_3)C_0(p_1,k_2)+2(p_2\cdot l)C_2(p_1,k_2)\Big)\hat{1}+$$
$$+2bcd\Big(B_0(\sqrt{s})+(m_2^2-m_1m_3)C_0(p_1,k2)+2(p_2\cdot l)C_2(p_1,k_2)\Big)\hat{1},$$
$$\int \frac{d^4q}{(2\pi)^4}\frac{\hat{f_{22}}}{b_1 b_2 b_3} = a(c^2+d^2)\Big(\hat{p_2}(m_1+m_3)C_2(p_1,k_2)-\hat{k_2}m_1C_0(p_1,k_2)\Big)+$$
$$+a(c^2-d^2)\Big(B_0(\sqrt{s})+(m_2^2-m_1m_3)C_0(p_1,k_2)+2(p_2\cdot l)C_2(p_1,k_2)\Big)\hat{1}+$$
$$+2bcd\Big(B_0(\sqrt{s})+(m_2^2+m_1m_3)C_0(p_1,k2)+2(p_2\cdot l)C_2(p_1,k_2)\Big)\hat{1},$$

An analytical expression of the three-body decay's square of matrix element was found

$$\frac{1}{4}\sum_{\lambda}\mathcal{M}\mathcal{M}^* = (c^2+d^2)^2\Big(|M_1|^2 - M_1 M_2^* - M_2 M_1^* + |M_2|^2\Big),$$

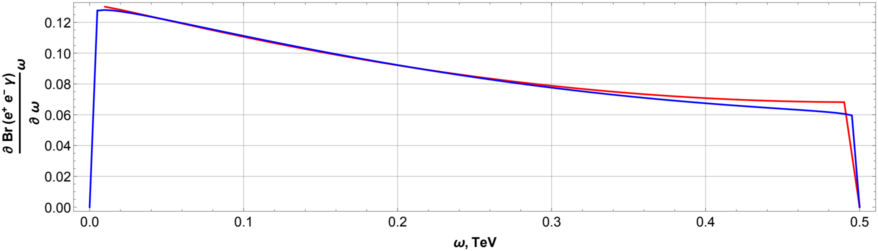
$$|M_1|^2 = a^2\frac{|X_1^+|^2 + 2m_1^2(l\cdot p_1)^2(p_1\cdot p_2)|Y_1|^2}{(p_1+l)^4} + b^2\frac{|X_1^-|^2 + 2m_1^2(l\cdot p_1)^2(p_1\cdot p_2)|C_0(k_1,p_2)|^2}{(p_1+l)^4},$$

$$|M_2|^2 = a^2\frac{|X_2^+|^2 + 2m_1^2(l\cdot p_2)^2(p_1\cdot p_2)|Y_2|^2}{(p_2+l)^4} + b^2\frac{|X_2^-|^2 + 2m_1^2(l\cdot p_2)^2(p_1\cdot p_2)|C_0(p_1,k_2)|^2}{(p_2+l)^4},$$

$$\begin{aligned}
M_1 M_2^* = {}& a^2\frac{2m_1^2(p_1\cdot p_2)(l\cdot p_1)(l\cdot p_2)\Big(Y_1Y_2^* + Y_2Y_1^* - 4C_1C_2^* - 4C_2C_1^*\Big)}{(p_1+l)^2(p_2+l)^2} - \\
& -a^2\frac{\Big((p_1\cdot p_2)^2 + (l\cdot p_1)(l\cdot p_2) + (p_1\cdot p_2)(l\cdot(p_1+p_2))\Big)(X_1^+X_2^{+*} + X_2^+X_1^{+*})}{(p_1+l)^2(p_2+l)^2} + \\
& +b^2\frac{2m_1^2(p_1\cdot p_2)(l\cdot p_1)(l\cdot p_2)\Big(C_0(k_1,p_2)C_0(p_1,k_2)^* + C_0(p_1,k_2)C_0(k_1,p_2)^*\Big)}{(p_1+l)^2(p_2+l)^2} - \\
& -b^2\frac{\Big((p_1\cdot p_2)^2 + (l\cdot p_1)(l\cdot p_2) + (p_1\cdot p_2)(l\cdot(p_1+p_2))\Big)(X_1^-X_2^{-*} + X_2^-X_1^{-*})}{(p_1+l)^2(p_2+l)^2},
\end{aligned}$$

$$X_1^{\pm} = 2(l \cdot p_1)C_1 + B_0(\sqrt{s}) + C_0(k_1, p_2)(m_2^2 \pm m_1^2),$$
$$X_2^{\pm} = 2(l \cdot p_2)C_2 + B_0(\sqrt{s}) + C_0(p_1, k_2)(m_2^2 \pm m_1^2),$$
$$Y_1 = 2C_1 + C_0(k_1, p_2) \qquad Y_2 = 2C_2 + C_0(p_1, k_2),$$
$$C_1 = C_1(k_1, p_2) \qquad C_2 = C_2(p_1, k_2).$$

An integration over phase volume was performed numerically using the Wolfram Mathematica software environment. The PackageX was used to calculate the Passarino-Veltman functions.



Obtained result

A lot of work has been done in search of a model for suppressing the γ yield, the results of which are follows:

Model	Result
$X^0 \to e^+e^-$, $X^0 \to e^+e^-\gamma$	—
$X^0_\mu \to e^+e^-$, $X^0_\mu \to e^+e^-\gamma$	—
$X^{2+} \to e^+e^+$, $X^{2+} \to e^+e^+\gamma$	— /+
$X^{2+}_\mu \to e^+e^+$, $X^{2+}_\mu \to e^+e^+\gamma$	— /+
Comparision of $X_\mu \to e^+e^-(\gamma)$ with $X \to e^+e^-(\gamma)$	— /+
Linear on $\hat{p}$ vertex $a + b\hat{p}/m$	—
Bubble loop	—
Triangle loop	—

Thanks for attention

Сопутствующие вычисления ("Пузарьковая"диаграмма)

$$\int \frac{d^D q}{(2\pi)^D} \frac{4(q^2 - p_1 \cdot q)(ac + bd))}{(q^2 - m^2)((q + p_1)^2 - m^2)} = ?$$

$$1\Big) \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q+p)^2 - m^2} \overset{q \to q-p}{=\!=\!=} \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q^2 - m^2)} = A_0(m)$$

$$2\Big) \int \frac{d^D q}{(2\pi)^D} \frac{q^2}{(q^2 - m_1^2)((q + p_1)^2 - m_2^2)} = \int \frac{d^D q}{(2\pi)^D} \frac{\cancel{(q^2 - m_1^2)}}{\cancel{(q^2 - m_1^2)}((q + p_1)^2 - m_2^2)} +$$

$$+ m_1^2 B_0(p_1, m_1, m_2) = A_0(m_2) + m_1^2 B_0(p_1, m_1, m_2)$$

$$3\Big) \int \frac{d^D q}{(2\pi)^D} \frac{p_1 \cdot q}{(q^2 - m_1^2)((q + p_1)^2 - m_2^2)} = \int \frac{d^D q}{(2\pi)^D} \frac{q^2 + \cancel{2\, p_1 \cdot q + p_1^2 - m_2^2}}{(q^2 - m_1^2)\cancel{((q + p_1)^2 - m_2^2)}} -$$

$$- \int \frac{d^D q}{(2\pi)^D} \frac{p_1 \cdot q}{(q^2 - m_1^2)((q + p_1)^2 - m_2^2)} - \int \frac{d^D q}{(2\pi)^D} \frac{q^2 + p_1^2 - m_2^2}{(q^2 - m_1^2)((q + p_1)^2 - m_2^2)} =$$

$$= A_0(m_1) - A_0(m_2) - m_1^2 B_0(p_1, m_1, m_2) - p_1^2 B_0(p_1, m_1, m_2) + m_2^2 B_0(p_1, m_1, m_2) -$$

$$- \int \frac{d^D q}{(2\pi)^D} \frac{p_1 \cdot q}{(q^2 - m_1^2)((q + p_1)^2 - m_2^2)} \quad \Rightarrow \int \frac{d^D q}{(2\pi)^D} \frac{p_1 \cdot q}{(q^2 - m_1^2)((q + p_1)^2 - m_2^2)} =$$

$$= \frac{1}{2} A_0(m_1) - \frac{1}{2} A_0(m_2) + \Big(\frac{m_2^2}{2} - \frac{m_1^2}{2} - \frac{p_1^2}{2}\Big) B_0(p_1, m_1, m_2)$$