Sub-Planckian evolution of the Universe

Polina M. Petriakova, Arkady A. Popov, Sergey G. Rubin

National Research Nuclear University MEPhI (Moscow Engineering Physics Institute),

N. I. Lobachevsky Institute of Mathematics and Mechanics, Kazan Federal University

23rd International Bled Workshop "What Comes Beyond the Standard Models?"

July 10 2020

Research Objectives

- The idea of extra dimensions has long been known and is currently able to clarify many questions of physics. However, it is still not clear why we live in expanding and large dimensions while others are strictly compactified.
- Can we find a model that reproduces the dynamics of two sub-spaces: one of which asymptotically tends to an infinitely large size while the other to a small one?
- There are many works that indicate only infinitely expanding of sub-spaces. In this work exact solutions in the framework of $f(R)$–gravity are presented and allowing to obtain spaces of the required sizes for various dimensions. The influence of the parameter values on the behavior of the solution also is discussed.
- We elaborate the method which is applied to the maximally symmetric spaces. In the following, it will be widen to several factor spaces.

The case of a three-dimensional sphere

Consider a theory of $f(R)$–gravity acting in $D = 4$ described by an action

$$S[g_{\mu\nu}] = \frac{1}{2} M_{Pl}^2 \int d^4x \sqrt{|\mathrm{det} g_{\mu\nu}|}\, f(R) \ , \ f(R) = aR^2 + bR + c \tag{1}$$

and in the case of space-time with positive curvature where the interval

$$ds^2 = dt^2 - \mathrm{e}^{2\alpha(t)} \left(dx^2 + \sin^2 x\, dy^2 + \sin^2 x\, \sin^2 y\, dz^2 \right), \tag{2}$$

is defined by the metric of the three-dimensional sphere.
The equations of motion for this theory (1) have the form

$$f_R'(R) R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} + \left[\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box \right] f_R'(R) = 0 \,, \quad \Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \,, \tag{3}$$

We use the conventions $R^{\nu}_{\alpha\beta\mu} = \partial_\mu \Gamma^{\nu}_{\alpha\beta} - \partial_\beta \Gamma^{\nu}_{\alpha\mu} + \Gamma^{\nu}_{\sigma\mu} \Gamma^{\sigma}_{\beta\alpha} - \Gamma^{\nu}_{\sigma\beta} \Gamma^{\sigma}_{\alpha\mu}$ and the Ricci curvature tensor is forming as $R_{\mu\nu} = R^{\sigma}_{\mu\sigma\nu}$

The equations of motion (3) taking into account the chosen metric (2) are reduced to the system

$$\begin{cases} \ddot{\alpha}(t) = -2\,\dot{\alpha}^2(t) - \mathrm{e}^{-2\alpha(t)} + \dfrac{1}{6}R(t), \\ \ddot{R}(t) = -2\,\dot{\alpha}(t)\dot{R}(t) - \dfrac{1}{12}R^2(t) + \dot{\alpha}^2(t)R(t) + \mathrm{e}^{-2\alpha(t)}R(t) - \dfrac{b}{6a}R(t) + \\ \qquad + \dot{\alpha}^2(t)\dfrac{b}{2a} + \mathrm{e}^{-2\alpha(t)}\dfrac{b}{2a} - \dfrac{c}{4a}, \end{cases}$$

here $R(t)$ - is the scalar curvature (Ricci scalar) and set as

$$R(t) \equiv g^{\mu\nu}R_{\mu\nu} = 12\dot{\alpha}^2(t) + 6\ddot{\alpha}(t) + 6\mathrm{e}^{-2\alpha(t)}\,. \tag{4}$$

Let us choose the initial conditions of the system by constant

$$\alpha(0) = \alpha_0\,, \qquad \dot{\alpha}(0) = \alpha_1\,, \qquad \dot{R}(0) = R_1\,. \tag{5}$$

Using one of the equations of the system, we obtain the quadratic equation for the scalar curvature depending on other initial conditions (5) and as a result we have

$$R(0) = 6\left(\alpha_1^2 + \mathrm{e}^{-2\alpha_0}\right) \pm \sqrt{36\left(\alpha_1^2 + \mathrm{e}^{-2\alpha_0}\right)^2 + \left(12\alpha_1 R_1 + \frac{6b}{a}\left(\alpha_1^2 + \mathrm{e}^{-2\alpha_0}\right) + \frac{c}{a}\right)}$$

The coefficients a, c in front of the degrees of scalar of curvature R in action (1) ($b = 1$ without the loss of generality) significantly affect the behavior of the solution of the system:

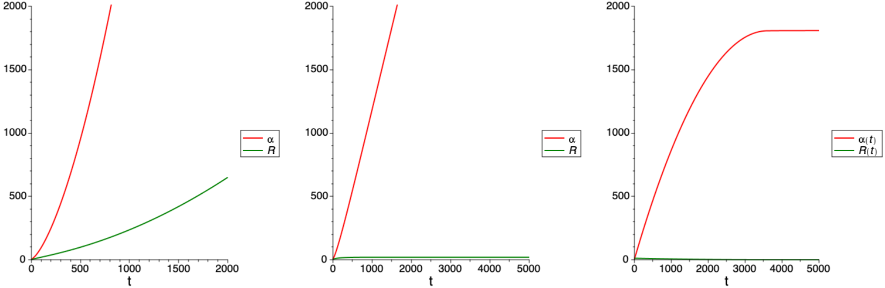

Figure: The results of solving the system in the case of a three-dimensional sphere with coefficients $a = -10, c = 10$; $a = 10, c = -10$; $a = 100, c = 0$ under the initial conditions $\alpha(0) = 5$, $\dot{\alpha}(0) = 0.5$, $\dot{R}(0) = 0$, $R(0) = 0.14$; $R(0) = 11.97$; $R(0) = 12.00$.

We are interested in the possibility of obtaining a sphere no smaller size than the size of the visible part of the Universe.
Therefore, we have a restriction on the asymptotic value of the function $\alpha(t)$ for large t : $L \sim 10^{61}\ l_{Pl} \Rightarrow \alpha(t) \sim \ln 10^{61} \sim 140$. By selecting the parameters it is possible to obtain the necessary class of solutions:

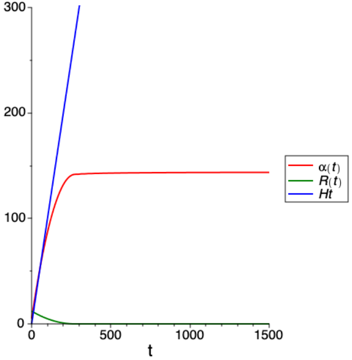

Figure: The result of solving the equations of the system with choosen parameters $a = 7.5$, $b = 1$, $c = 10^{-6}$ and initial conditions (they are all represented in Planck units) $\alpha(0) = 5$, $\dot{\alpha}(0) = 0.5$, $R(0) = 12.07$ $\dot{R}(0) = 0$.

The dependence of the obtained solution $\alpha(t)$ on the values of the initial conditions α_0, α_1 for fixed Lagrangian (1) coefficients $a = 7.5$, $b = 1$, $c = 10^{-6}$ is

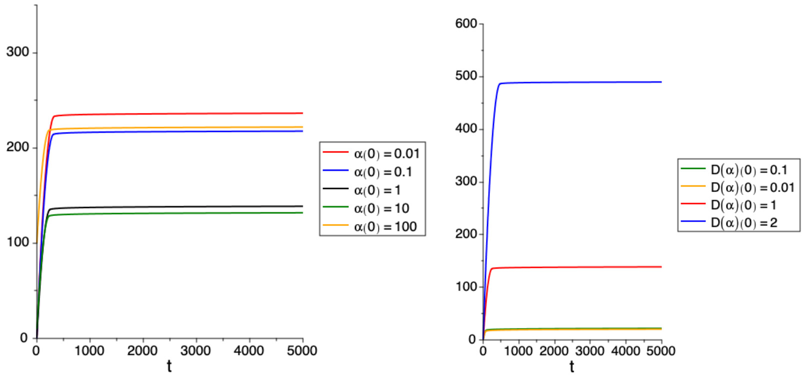

Figure: On the left with a fixed value of $\dot{\alpha}(0) \equiv \alpha_1 = 1$, on the right with a fixed value of $\alpha(0) \equiv \alpha_0 = 5$.

The choice of parameters actually does not limit the possibility of obtaining large spheres of finite size under the following conditions $a > 0$, $b = 1$, $c \ll 1$, $\alpha(0) \geq 2.5$, $\dot{\alpha}(0) \geq 0$, $\dot{R}(0) = 0$ and $R(0)$ defined by (5).

Generalization to the case of other dimensions

When performing similar calculations in the case of $D = 3$ for two-dimensional sphere with fixed coefficients that provide a large three-dimensional sphere

$$S = \frac{1}{2} M_{Pl}^{D-2} \int d^D x \sqrt{|\det g_{\mu\nu}|}\; f(R)\,, \;\; ds^2 = dt^2 - e^{2\alpha(t)} \left(dx^2 + \sin^2 x\, dy^2 \right) \quad (6)$$

we get a solution containing a singularity:

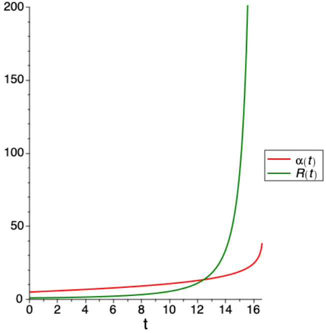

Figure: The result of solving the equations of the system for a two-dimensional sphere with choosen parameters $a = 7.5,\ b = 1,\ c = 10^{-6}$ and initial conditions $\alpha(0) = 5$, $\dot{\alpha}(0) = 0.5,\ R(0) = 1.06\ \dot{R}(0) = 0.$

Calculations for a four-dimensional sphere ($D = 5$) with a metric

$$ds^2 = dt^2 - e^{2\alpha(t)}\left(dx^2 + \sin^2 x\, dy^2 + \sin^2 x\, \sin^2 y\, dz^2 + \sin^2 x\, \sin^2 y\, \sin^2 z\, d\phi^2\right) \quad (7)$$

lead again to a sphere of finite size at large time:

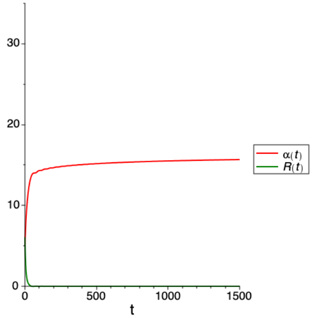

Figure: The result of solving the equations of the system for a four-dimensional sphere with choosen parameters $a = 7.5,\ b = 1,\ c = 10^{-6}$ and initial conditions $\alpha(0) = 5$, $\dot{\alpha}(0) = 0.5,\ R(0) = 6.07\ \dot{R}(0) = 0$.

Conclusion and Future Plans

- It is shown that when considering $f(R)$–gravity starting from sub-Planck scales it seems possible to obtain three-dimensional space of large and finite size which can be identified with the visible part of the Universe. Regions of parameters were found for which the dynamics are significantly different.
- Carrying out similar calculations for spaces of other dimensions as a result of the solution we obtain in the case $D = 3$ unstable configurations or more compact sizes in $D = 5$.
- In the future we will study more complex metrics in this way. The aim is to find conditions which lead to the large size of main space and small size of an extra dimensions.

Thank you for attention!