Damping oscillations of a domain wall in the early Universe

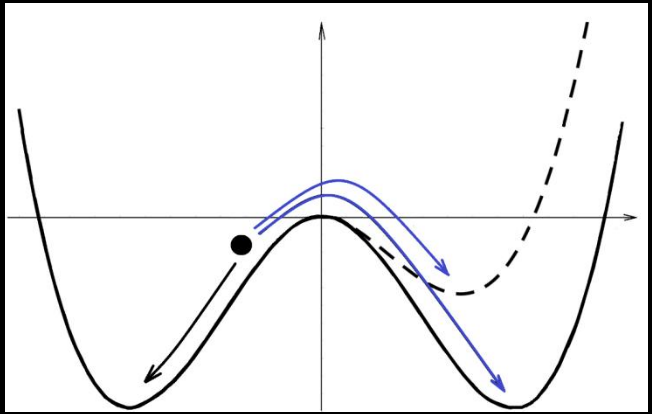
Andrey Kurakin NRNU MEPHI
Rubin S.G.

Possible evidence of the considered effect

Heating effect:

- Observation of Early Hot Regions (see 1903.04424)
- CMB distortion (see 1804.10059)

PBH formation or primordial large density fluctuation (inhomogeneities)

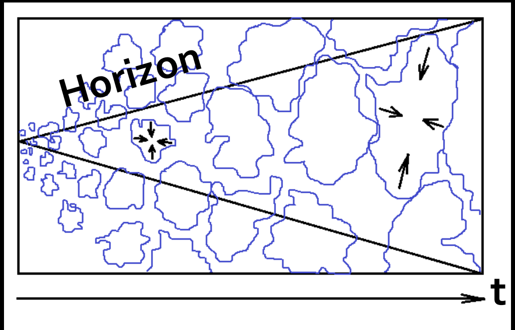


- One of the reasons for primordial black holes formation is phase transitions during and after inflation. This is three-step process:

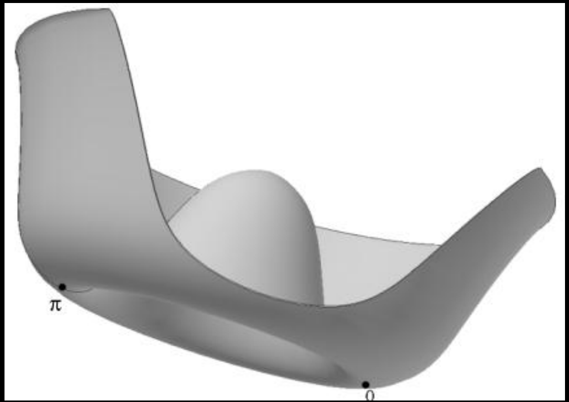
- At the first step closed walls separating two vacuum states appear after the inflation is terminated. The shape of most of them is **non spherical**.

- At the second step the walls evolve in the relativistic plasma.

- The third step consists of wall shrinking due to its internal tension. It can be finished by a black hole creation.



Domain wall description



- Consider a complex scalar field with the lagrangian:

$$\mathcal{L}_{wall} = \partial_\mu\phi^\dagger\partial^\mu\phi - \frac{1}{4}(\phi^\dagger\phi - f^2/2)^2 - \Lambda^4(1 - \cos\theta), \quad (1)$$

ϕ - complex scalar field; θ – phase of the field ϕ; Λ, f – lagrangian parameters.

- Substitution: $\phi = \frac{f}{\sqrt{2}} exp(i\theta) = \frac{f}{\sqrt{2}} exp(i\chi/f),$ (2)

- As a result, for the phase we obtain the Lagrangian sine of Gordon:

$\mathcal{L}_{wall} = \frac{1}{2}(\partial_\mu\chi)^2 - \Lambda^4(1 - \cos(\chi/f))$ (3) $\longrightarrow$ solution: $\chi(z) = 4f\,\mathrm{arctg}\left(\exp\left[\frac{\Lambda^2}{f}z\right]\right) = 4f\,\mathrm{arctg}\left(\exp\left[\frac{2z}{d}\right]\right)$ (4)

- where d – domain wall width: $d = \frac{2f}{\Lambda^2}$

Scalar-fermion coupling

- Interaction between the wall field and fermion particles described by the Yukawa coupling:

$$\mathcal{L}_f = i\bar{\psi}\gamma^\mu\partial_\mu\psi + g_0(\phi\bar{\psi}\psi + h.c.) - m\bar{\psi}\psi = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \sqrt{2}g_0 f\bar{\psi}\psi\cos(\chi/f) - m\bar{\psi}\psi \quad (5)$$

- Rewrite the interaction term in the form:

$$\mathcal{L}_{int} = m_0\cos(\chi/f)\bar{\psi}\psi = m_0\left(1 - \frac{2}{\mathrm{ch}^2(2z/d)}\right)\bar{\psi}\psi \quad (6)$$

- As a result, the fermionic part of the Lagrangian takes the form:

$$\mathcal{L}_f = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m_0\frac{2}{\mathrm{ch}^2(2z/d)}\bar{\psi}\psi - m_f\bar{\psi}\psi \quad (7)$$

, where $m_f = m - m_0$ - fermionic mass

Dirac equation

- The field equation for ψ can be written as: $0 = (\gamma^\mu \partial_\mu - g(x))\, \psi$ (8)

  where $g(x) = \dfrac{2m_0}{\mathrm{ch}^2(2x/d)} + m_f$ (9)
- anzatz: $\psi(x) = \begin{pmatrix} u_1(x) & u_2(x) & u_3(x) & u_4(x) \end{pmatrix}^T e^{-iEt + ip_t x_t}$ (10)
- introduce the following combinations: $$\phi_+(x) = u_1(x) + iu_3(x)$$ $$\phi_-(x) = u_1(x) - iu_3(x).$$ (11)
- We obtain the folowing system:

$$0 = iE\phi_-(x) + \phi'_+(x) - g(x)\phi_+(x)$$
$$0 = iE\phi_+(x) + \phi'_-(x) + g(x)\phi_-(x).$$ (12)

- The equation (8) is reduced to solving the scalar equation:

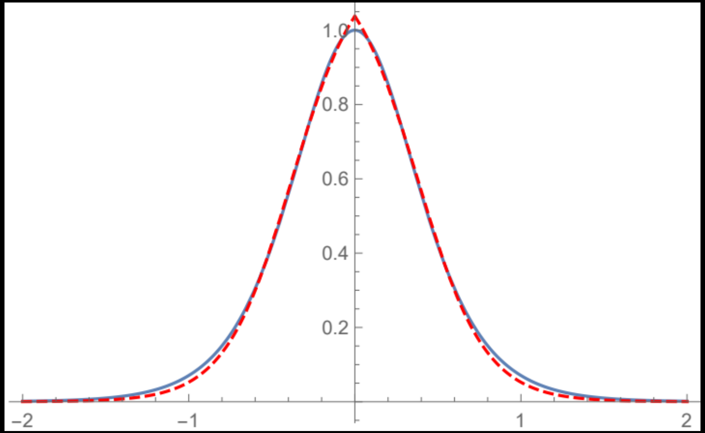
$$0 = \left(\frac{d^2}{dx^2} \mp g'(x) + E^2 - g^2(x) \right) \phi_\pm(x)$$ (13)

Approximation

- We approximate the function g (x) by a potential of the form (Woods-Saxon potential):

(14) $$g(x) = \frac{A\theta(x)}{1+exp(a(x-x_0))} + \frac{A\theta(-x)}{1+exp(-a(x+x_0))} + m_f.$$

A, a, x_0 - approximation parameters.



- Then the equation (12) is reduced to hypergeometrical form
  - We solve the equation in the regions x <0, x> 0 and carry out the matching of solutions at the point x = 0

Solution of Dirac equation

- Solution of hypergeometrical equation for x<o region:

$$\phi_+^L(\xi) = C_1\xi^{-\alpha}(1-\xi)^{-\beta}\ {}_2F_1(-\alpha-\nu-\beta, -\alpha+\nu-\beta, 1-2\alpha;\xi) + C_2\xi^{\alpha}(1-\xi)^{-\beta}\ {}_2F_1(\alpha-\nu-\beta, \alpha+\nu-\beta, 1+2\alpha;\xi). \quad (15)$$

where $\xi = -\exp(-a(x+x_0))$, parameters:

$$\alpha = \frac{1}{a}\sqrt{(m_f+A)^2-E^2} = \frac{ip}{a}$$
$$\beta = -\frac{A}{a} \quad (16)$$
$$\nu = \frac{1}{a}\sqrt{m_f^2-E^2} = \frac{i}{a}\sqrt{E^2-m_f^2} = \frac{ik}{a}$$

- This asymptotic solution gives two waves: incident and reflection:

$$\phi_+^L \xrightarrow{x\to-\infty} D_1 e^{ik(x+x_0)} + D_2 e^{-ik(x+x_0)}$$ (D_1, D_2 = const)

$$\phi_-^L(x) \xrightarrow{x\to-\infty} -\frac{k+im}{E}D_1 e^{ik(x+x_0)} + \frac{k-im}{E}D_2 e^{-ik(x+x_0)} \quad (17)$$

Solution of Dirac equation

- For region x>o in asymptotic we have:

$$\phi_+^R(x) = d_1 e^{ik(x-x_0)}$$

$$\phi_-^R(x) \xrightarrow{x\to+\infty} -\frac{k+im}{E} d_1 e^{ik(x-x_0)} \qquad (18)$$

there is only a transmitted wave, the coefficient for a wave running opposite the x axis $d_2 = 0$.

- these two wave functions (for region x>o and x<o) must be matched at x :

$$\phi_+^R|_{x=0} = \phi_+^L|_{x=0}$$

$$(\phi_+^R)'|_{x=0} = (\phi_+^L)'|_{x=0} \qquad (19)$$

Reflection coefficient

- The current density for the function (10) is defined as:

$$j = \bar{\psi}(x)\gamma^3\psi(x) = -|u_1(x)|^2 + |u_2(x)|^2 + |u_3(x)|^2 - |u_4(x)|^2 = \\ = -\phi_+^*\phi_- - \phi_-^*\phi_+. \quad (20)$$

- Current in region x<0

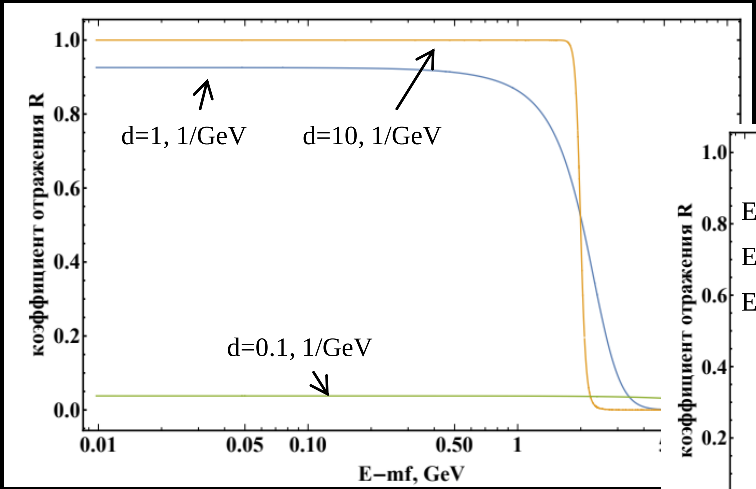
$$j = \frac{k}{E}(|D_1|^2 - |D_2|^2) = j_{inc} - j_{ref}. \quad (21)$$

- Reflection coefficient:

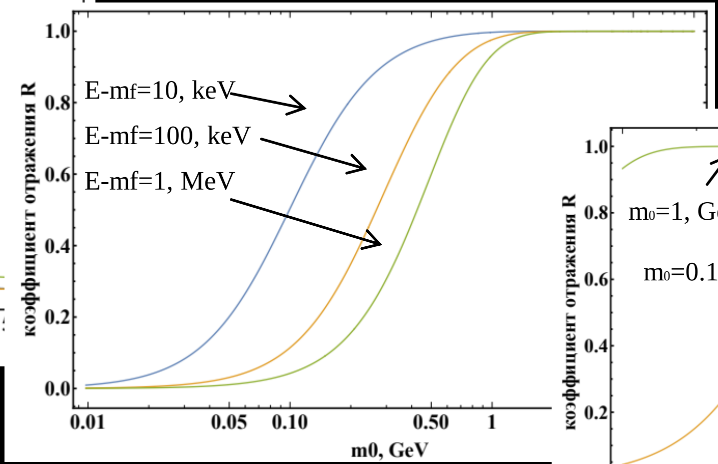
$$R = \frac{j_{ref}}{j_{inc}} = \frac{|D_2|^2}{|D_1|^2}. \quad (22)$$

Reflection coefficient

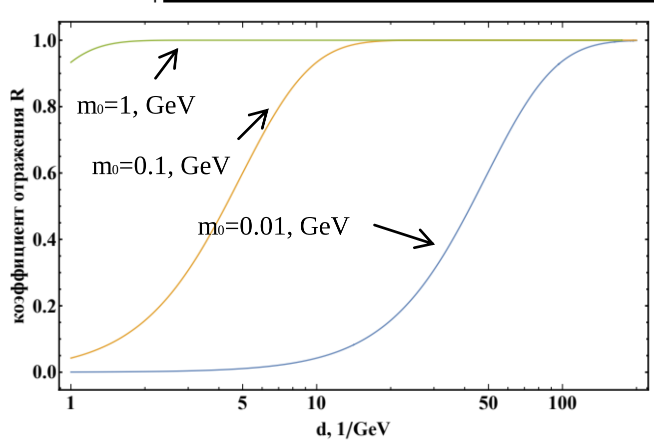
Reflection coef. R



Reflection coef. R for varying kinetic energy, m_0 = 1 GeV



Reflection coef. R for varying coupling constan m_0, d= 1 , 1/GeV



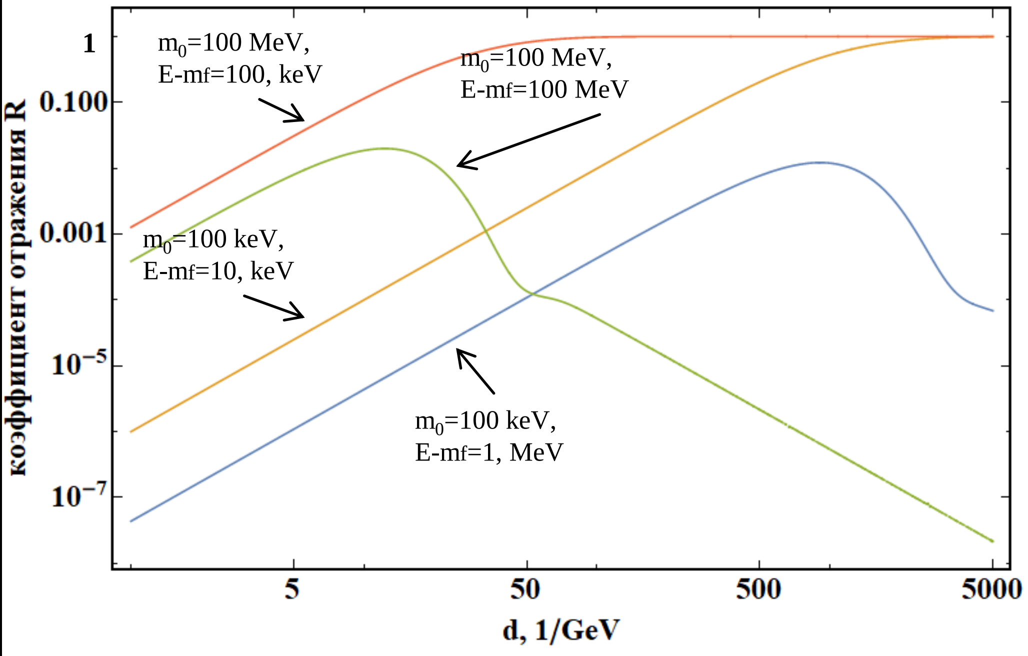
Reflection coef. R for varying wall width, E-m_f = 1 MeV

d- wall width

m_0 - coupling constant

Reflection coefficient

Reflection coef. R



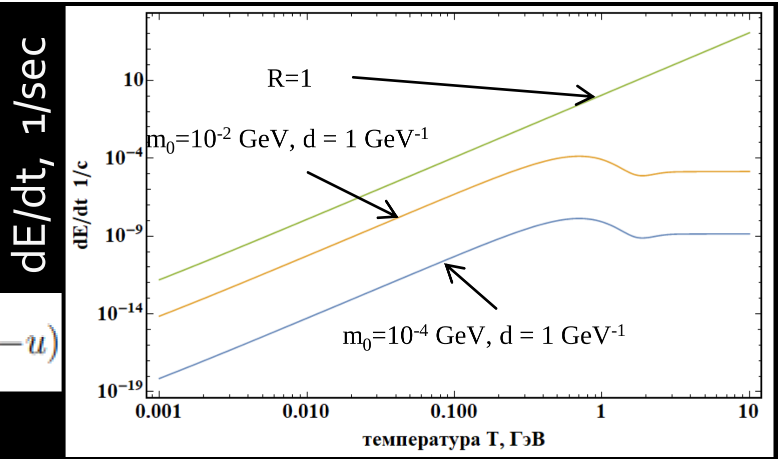
d – domain wall width

Energy transfer

- Rate of energy transfer (estimation):

(23) $$d\tilde{E}/dt \approx [\tilde{E}'(T) - \tilde{E}(T)] \cdot R(\tilde{E}(T)) \cdot \tilde{n}_{tot} \cdot v_{rel},$$

(24) $$d\tilde{E}/dt \approx \frac{2u^2\tilde{E} + 2u\tilde{k}_z}{1-u^2} \cdot 5T^3 \cdot R(T) \cdot \frac{\tilde{v}(T) + u}{1 + u\tilde{v}(T)} + (u \to -u)$$



- Kinetic energy per unit wall area

(25) $$\sigma = (\gamma - 1)M_{sol} = 8(\gamma - 1)\Lambda^2 f = 4(\gamma - 1)\Lambda^4 d$$

Energy transfer rate per unit wall area, per unit σ as a function of plasma temperature

Conclusion

- We consider model of interaction between fermionic particles and domain wall described by complex scalar field with potential

- It can be used to estimate the possible heating effect of areas that are detached from the Hubble flow due to domain walls in early Universe. It can provide check of this model, deriving it on the effects of the existence of heated regions, accumulations of PBH, linking this with the parameters of the physics of the early Universe