Lorentz Violation in the Neutrino Sector: New Stringent Contraints due to Additional Decay Channels Squeeze the Parameter Space

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Abstract

The hypothesis of Lorentz violation in the neutrino sector has intrigued scientists for the last two to three decades. A number of theoretical arguments support the emergence of such violations first and foremost for neutrinos, which constitute the "most elusive" and "least interacting" particles known to mankind. It is of obvious interest to place strigent bounds on the Lorentz-violating parameters in the neutrino sector. In the past, the most stringent bounds have been placed by calculating the probability of neutrino decay into a lepton pair, a process made kinematically feasible by Lorentz violation in the neutrino sector, above a certain threshold. However, even more stringent bounds can be placed on the Lorentz-violating parameters if one takes into account, additionally, the possibility of neutrino splitting, i.e., of neutrino decay into a neutrino of lower energy, accompanied by "neutrino-pair Cerenkov radiation". This process has negligible threshold and can be used to improve the bounds on Lorentz-violating parameters in the neutrino sector. Finally, we take the opportunity to discuss the relation of Lorentz and gauge symmetry breaking, with a special emphasis on the theoretical models employed in our calculations.

Motivation

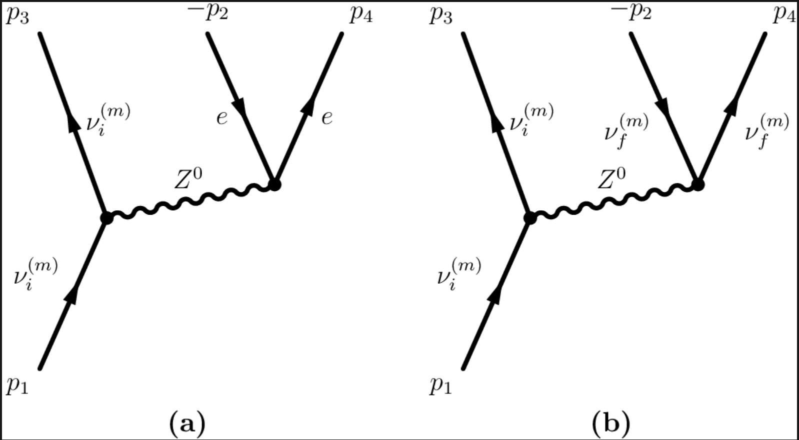
- Neutrinos are very elusive particles.
- Speculation about tachyonic nature [Chodos, Hauser, Kostelecky, PLB 1985]
- Speculation about Lorentz violation $E = \sqrt{\vec{p}^2\, v^2 + m^2}$ with $v > 1$. Since 1998 [Colladay and Kostelecky].
- Lorentz–Violating Extension of Standard Model (SME) developed with strong inspiration from neutrinos.
- Anyway, decay among neutrino mass eigenstates kinematically allowed due to their mass differences.
- However, decay rates for “ordinary” neutrinos (both Dirac as well as Majorana) exceed lifetime of Universe by orders of magnitude.
- Lorentz-violating neutrinos undergo stronger decay and energy loss mechanims than “ordinary” neutrinos because of their dispersion relation $E \approx |\vec{p}|\, v$ (at high energy), which makes a number of decays kinematically possible.

Looking at Neutrinos

- Early arrival of the 1987A neutrinos from the supernova.
- Consistent (statistically insignificant) experimental results $\delta_\nu \gtrsim 0$ by various groups. ($v_\nu = \sqrt{1 + \delta_\nu}$.)
- Neutrinos cannot be used to transmit information (at least not easily) because of their small interaction cross sections. Superluminality of neutrinos would not necessarily lead to violation of causality.
- Cutoff in the cosmic spectrum seen by IceCUBE at about 2 PeV.

LPCR and NPCR

(a) LPCR=Lepton–Pair Cerenkov Radiation
(b) NPCR=Neutrino–Pair Cerenkov Radiation



Double–Bind Situation and Subtle Points

- On one hand, it seems natural to assume that $\delta_\nu > 0$, while $\delta_e = 0$, i.e., that Lorentz violation only occurs in the neutrino, but not in the charged-lepton (electron-positron) sector.
- On the other hand, once we make this assuption, then, strictly speaking, we break $SU(2)_L$ gauge invariance, because neutrinos and charged leptons are in the same $SU(2)_L$ doublet, and thus, both convariantly coupled to the electroweak gauge sector (same multiplet).
- The SME is constructed so that strict gauge invariance is maintained.
- Pragmatic approach used by Cohen and Glashow, by Bezrukov and Lee, and by us: Stick with the "natural assumption" $\delta_e \approx 0$, while $\delta_\nu > 0$, and allow for a small violation of gauge invariance. (Finally, we also break Lorentz invariance.)
- Double-bind situation: Either break gauge invariance or accept that $\delta_\nu = \delta_e$ is bound by any limit set for electrons, defeating the purpose of looking at the neutrino sector.
- (The relation of gauge invariance and Lorentz invariance breaking will be discussed later.)
- The way in which electroweak gauge invariance is broken, influences the functional form of the interaction Lagrangian. The model dependence should be studied (Bezrukov and Lee).

Three Papers

- A. G. Cohen and S. L. Glashow, *Pair Creation Con- strains Superluminal Neutrino Propagation*, Phys. Rev. Lett. 107, 181803 (2011).
  Refuted the OPERA experiment based on LPCR.
- F. Bezrukov and H. M. Lee, *Model dependence of the bremsstrahlung effects from the superluminal neutrino at OPERA*, Phys. Rev. D 85, 031901(R) (2012).
  Model dependence of LPCR.
- G. Somogyi, I. Nandori and U. D. Jentschura, *Neutrino Splitting for Lorentz–Violating Neutrinos: Detailed Analysis*, Phys. Rev. D 100, 035036 (2019).
  Model dependence, gauge invariance and addition of NPCR.

Additional remarks on the gauge invariance:
U. D. Jentschura, I. Nándori, and G. Somogyi, *Lorentz Breaking and* $SU(2)_L \times U(1)_Y$ *Gauge Invariance for Neutrinos*, Int. J. Mod. Phys. E **28**, 1950072 (2019).

Threshold (I)

- Parameterize the deviation from the speed of light as:

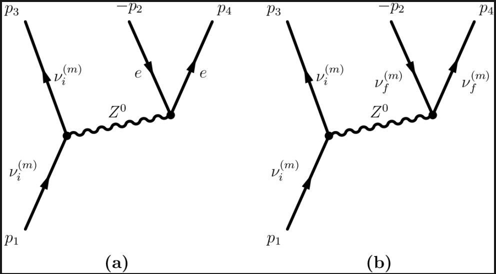
$$v = \sqrt{1+\delta} \approx 1 + \tfrac{1}{2}\delta\,.$$

- Initial state/particle: δ_i, four-momentum p_1:

$$v_i = \sqrt{1+\delta_i}\,.$$

- Emitted particles: δ_f, four-momenta p_2 and p_4:

$$v_f = \sqrt{1+\delta_f},.$$



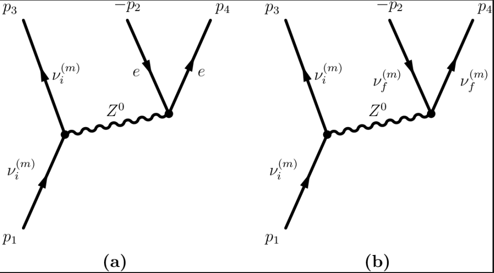
Threshold (II)

- Threshold for LPCR ($\delta_f = 0$):
  [minimum energy of particle 1 to undergo LPCR]

$$E_{\text{th}} = \frac{2m_e}{\sqrt{\delta_i}}\,,$$

- Threshold for NPCR ($\delta_f = 0$):

$$E_{\text{th}} = \frac{2m_\nu}{\sqrt{\delta_i}}\,, \qquad m_\nu \ll m_e\,.$$



Mass eigenstate Basis

- Mass and flavor basis:

$$\nu_k^{(f)} = \sum U_{k\ell}\, \nu_\ell^{(m)} \,.$$

- Interaction with Z^0 boson in flavor basis:

$$\mathcal{L} = -\frac{g_w}{4\,\cos\theta_W} \sum_{k,\ell,\ell'} U^{+}_{\ell k}\, U_{k\ell'}\, \overline{\nu}_\ell^{(m)}\, \gamma^\mu (1-\gamma^5)\, \nu_{\ell'}^{(m)}\, Z_\mu \,.$$

- Unitary transformation:

$$\mathcal{L} = -\frac{g_w}{4\,\cos\theta_W} \sum_{k,\ell,\ell'} U^{+}_{\ell k}\, U_{k\ell'}\, \overline{\nu}_\ell^{(m)}\, \gamma^\mu (1-\gamma^5)\, \nu_{\ell'}^{(m)}\, Z_\mu \,.$$

- Interaction with Z^0 boson in mass eigenstate basis:

$$\mathcal{L} = -\frac{g_w}{4\,\cos\theta_W} \sum_{\ell} \overline{\nu}_\ell^{(m)}\, \gamma^\mu (1-\gamma^5)\, \nu_{\ell}^{(m)}\, Z_\mu \,.$$

Lagrangian of Superluminal Particle

- Introduce metric with tilde:

$$\mathcal{L} = \sum_{\ell} \mathrm{i}\, \overline{\nu}_{\ell}^{(m)}\, \gamma^{\mu}\, (1 - \gamma^{5})\, \tilde{g}_{\mu\nu}(v_{\ell})\, \partial^{\nu}\, \nu_{\ell}^{(m)} \,.$$

- Define:

$$\tilde{g}_{\mu\nu}(v_{\ell}) = \mathrm{diag}(1, -v_{\ell}, -v_{\ell}, -v_{\ell}) \,.$$

- Dispersion relation:

$$E_{\ell} = |\vec{p}|\, v_{\ell} \,.$$

- Ignore mass in

$$E_{\ell} = \sqrt{(|\vec{p}|\, v_{\ell})^2 + m_{\ell}^2} \,.$$

- For neutrinos: We know that the m_{ℓ} are different. So, there is reason to assume that the δ_{ℓ} are also different among mass (flavor) eigenstates, if they are different from zero.

Interaction Lagrangian and Model Dependence

- Define parameter $v_{\rm int}$ for unified description of LPCR and NPCR:

$$\mathcal{L}_{\rm int} = f_e \, \frac{G_F}{2\sqrt{2}} \, \overline{\nu}_i^{(m)} \, \gamma^\lambda \, (1 - \gamma^5) \, \nu_i^{(m)}$$
$$\times \, \tilde{g}_{\lambda\sigma}(v_{\rm int}) \, \bar{\psi}_f \, \gamma^\sigma \, (c_V - c_A \, \gamma^5) \, \psi_f \, .$$

- Cohen and Glashow: $v_{\rm int} = 1$. Bezrukov and Lee: $v_{\rm int} = 1$ ("model I") and $v_{\rm int} = v_i$ ("model II"). Somogyi, Nandori and Jentschura: $v_{\rm int}$ is kept as a variable. "Gauge invariance" (to be clarified later): $v_{\rm int} = v_i \, v_f$. Both Cohen and Glashow, as well as Bezrukov and Lee, assume that $\delta_f = 0$ for LPCR.
- Parameter f_e:

$$f_e = \begin{cases} 1, & \psi_f = \nu_f^{(m)} \\ 2, & \psi_f = e \end{cases} \, .$$

- Approximately, one has

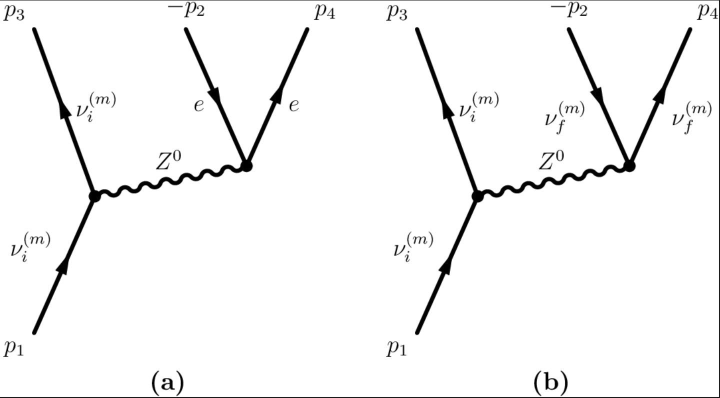
$$(c_V \, , c_A) = \begin{cases} (1, 1) & \psi_f = \nu_f^{(m)} \\ (0, -\frac{1}{2}), & \psi_f = e \end{cases} \, .$$

Calculation of the Decay Rate

Consider:

$$\nu_i(p_1) \to \nu_i(p_3) + \bar{\psi}_f(p_2) + \psi_f(p_4)\,.$$

Recall Feynman diagrams:



Calculation of the Decay Rate

Matrix element:

$$\mathcal{M} = f_e \frac{G_F}{2\sqrt{2}} \left[\bar{u}_i(p_3)\gamma^\lambda(1-\gamma^5)u_i(p_1)\right] \tilde{g}_{\lambda\sigma}(v_{\text{int}}) \times \left[\bar{u}_f(p_4)(c_V\gamma^\sigma - c_A\gamma^\sigma\gamma^5)v_f(p_2)\right] .$$

It gets complicated:

$$\frac{1}{n_s}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{1}{n_s} f_e^2 \frac{G_F^2}{8} \text{Tr}[(v_i \not{p}_3 + (1-v_i)(p_3\cdot t)\not{t}) \times \gamma^\lambda(1-\gamma^5)(v_i\not{p}_1 + (1-v_i)(p_1\cdot t)\not{t})\gamma^\sigma(1-\gamma^5)] \times [v_{\text{int}}g_{\lambda\rho} + (1-v_{\text{int}})t_\lambda t_\rho][v_{\text{int}}g_{\sigma\tau} + (1-v_{\text{int}})t_\sigma t_\tau] \times \text{Tr}[(v_f\not{p}_4 + (1-v_f)(p_4\cdot t)\not{t})(c_V\gamma^\rho - c_A\gamma^\rho\gamma^5) \times (v_f\not{p}_2 + (1-v_f)(p_2\cdot t)\not{t})(c_V\gamma^\tau - c_A\gamma^\tau\gamma^5)] .$$

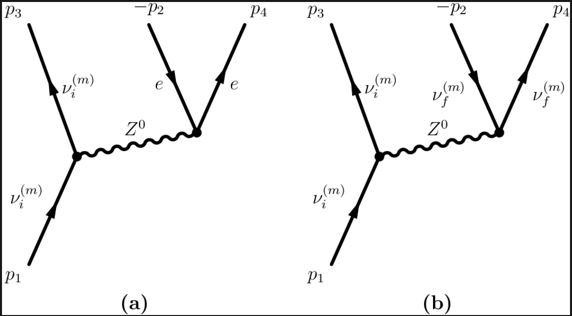
Unified Treatment and Splitting of Phase Space

We have:

$$\Gamma = \frac{1}{2E_1}\int \mathrm{d}\phi_3(p_2,p_3,p_4;p_1)\frac{1}{n_s}\sum_{\text{spins}}|\mathcal{M}|^2\,.$$

We want to write:

$$\Gamma = \frac{1}{2E_1}\int_{M^2_{\min}}^{M^2_{\max}}\frac{\mathrm{d}M^2}{2\pi}\mathrm{d}\phi_2(p_3,p_{24};p_1)\,\mathrm{d}\phi_2(p_2,p_4;p_{24})\frac{1}{n_s}\sum_{\text{spins}}|\mathcal{M}|^2\,.$$



Key to the Calculation

Formalism described in
E. Byckling and K. Kajantie, *Particle Kinematics* (1973):

$$
\begin{aligned}
\mathrm{d}\phi_3(p_2, p_3, p_4; p_1) = \int \frac{\mathrm{d}M^2}{2\pi} \frac{\mathrm{d}^4 p_3}{(2\pi)^3} \delta_+(p_3^2 - \delta_i k_3^2) \\
\times \frac{\mathrm{d}^4 p_{24}}{(2\pi)^3} \delta_+(p_{24}^2 - M^2)(2\pi)^4 \delta^{(4)}(p_1 - p_3 - p_{24}) \\
\times \frac{\mathrm{d}^4 p_2}{(2\pi)^3} \delta_+(p_2^2 - \delta_f k_2^2) \frac{\mathrm{d}^4 p_4}{(2\pi)^3} \delta_+(p_4^2 - \delta_f k_4^2) \\
\times (2\pi)^4 \delta^{(4)}(p_{24} - p_2 - p_4) \\
= \int \frac{\mathrm{d}M^2}{2\pi} \mathrm{d}\phi_2(p_3, p_{24}; p_1) \mathrm{d}\phi_2(p_2, p_4; p_{24}) \,.
\end{aligned}
$$

General Result for the Decay Rate

$$\Gamma_{\nu_i \to \nu_i \psi_f \bar{\psi}_f} = \frac{G_F^2 k_1^5}{192\pi^3} f_e^2 \frac{c_V^2 + c_A^2}{420 n_s} (\delta_i - \delta_f) \Big[(60 - 43\sigma_i)(\delta_i - \delta_f)^2$$
$$+ 2(50 - 32\sigma_i - 25\sigma_f + 7\sigma_i\sigma_f)(\delta_i - \delta_f)\delta_f$$
$$+ 7(4 - 3\sigma_i - 3\sigma_f + 2\sigma_i\sigma_f)\delta_f^2 + 7\delta_{\rm int}^2 \Big] .$$

(Result vanishes for $\delta_i = \delta_f$.) Cohen and Glashow have $n_s = 2$ active spin states for the (initial) neutrino, Bezrukov and Lee calculate with $n_s = 1$. Parameter σ_i:

$$\sigma_i = \begin{cases} 0, & \text{CG spin sum for } \nu_i \\ 1, & \text{BL spin sum for } \nu_i \end{cases}, \quad \sigma_f = \begin{cases} 0, & \text{CG spin sum for } \psi_f \\ 1, & \text{BL spin sum for } \psi_f \end{cases} .$$

CG spin sum (“polarization sum”):

$$\sum_s \nu_{\ell,s} \otimes \bar{\nu}_{\ell,s} = p^\mu g_{\mu\nu} \gamma^\nu .$$

BL spin sum (quite frankly, to be preferred):

$$\sum_s \nu_{\ell,s} \otimes \bar{\nu}_{\ell,s} = p^\mu \tilde{g}_{\mu\nu}(v_\ell) \gamma^\nu .$$

General Result for the Energy Loss Rate

$$\frac{\mathrm{d}E_{\nu_i \to \nu_i \psi_f \bar{\psi}_f}}{\mathrm{d}x} = -\frac{G_F^2 k_1^6}{192\pi^3} f_e^2 \frac{c_V^2 + c_A^2}{672 n_s} (\delta_i - \delta_f)$$
$$\times \Big[(75 - 53\sigma_i)(\delta_i - \delta_f)^2 + (122 - 77\sigma_i - 61\sigma_f + 16\sigma_i\sigma_f)(\delta_i - \delta_f)\delta_f$$
$$+ 8(4 - 3\sigma_i - 3\sigma_f + 2\sigma_i\sigma_f)\delta_f^2 + 8\delta_{\mathrm{int}}^2 \Big].$$

(Result vanishes for $\delta_i = \delta_f$.) Cohen and Glashow have $n_s = 2$ active spin states for the (initial) neutrino, Bezrukov and Lee calculate with $n_s = 1$. Parameter σ_i:

$$\sigma_i = \begin{cases} 0, & \text{CG spin sum for } \nu_i \\ 1, & \text{BL spin sum for } \nu_i \end{cases}, \quad \sigma_f = \begin{cases} 0, & \text{CG spin sum for } \psi_f \\ 1, & \text{BL spin sum for } \psi_f \end{cases}.$$

(We have verified and checked compatibility with all formulas contained in CG and BL.)
(This is important because it confirms that the model dependence of the results is only contained in the numerical prefactors, but not in the overall scaling of the results.)

Parameterization of the Results

Write b coefficients:

$$\Gamma_{\nu_i \to \nu_i \nu_f \bar{\nu}_f} = b \frac{G_F^2}{192\pi^3} k_1^5 \,,$$
$$\frac{\mathrm{d}E_{\nu_i \to \nu_i \nu_f \bar{\nu}_f}}{\mathrm{d}x} = -b' \frac{G_F^2}{192\pi^3} k_1^6 \,.$$

For the CG spin sum:

$$b_{\mathrm{CG}} = \frac{1}{7}(\delta_i - \delta_f)\left[(\delta_i - \delta_f)^2 + \frac{5}{3}\delta_f(\delta_i - \delta_f) + \frac{7}{15}\delta_f^2\right] \,,$$
$$b'_{\mathrm{CG}} = \frac{25}{224}(\delta_i - \delta_f)\left[(\delta_i - \delta_f)^2 + \frac{112}{75}\delta_f(\delta_i - \delta_f) + \frac{32}{75}\delta_f^2\right] \,.$$

For the BL spin sum:

$$b_{\mathrm{BL}} = \frac{17}{210}(\delta_i - \delta_f)\left[(\delta_i - \delta_f)^2 + \frac{7}{17}\delta_{\mathrm{int}}^2\right] \,,$$
$$b'_{\mathrm{BL}} = \frac{11}{168}(\delta_i - \delta_f)\left[(\delta_i - \delta_f)^2 + \frac{4}{11}\delta_{\mathrm{int}}^2\right] \,.$$

(Numerical prefactors are larger than those for LPCR by a factor of four or five. Also, NPCR has negligible threshold.)

Comparison to Astrophysical Data I

In papers of Stecker and Scully (Astropart. Phys., 2014 and Phys. Rev. D, 2014), the following bound is derived for the Lorentz-violating parameter of the electron-positron field alone (watch out for a difference in the conventions used for defining the δ_e parameter):

$$\delta_e \leq 1.04 \times 10^{-20} .$$

The observation of very-high-energy neutrinos by IceCube, taking into consideration the LPCR process (but not NPCR!), implies that the Lorentz-violating parameter for neutrinos cannot be larger than

$$\delta_\nu \leq 2.0 \times 10^{-20}$$

(Stecker, Scully, Liberati and Mattingly, Phys. Rev. D, 2015). This bound is based on the assumption that δ_e and δ_ν are different. Colloquially speaking, we can say that, if δ_ν were larger, then "Big Bird" (the 2 PeV specimen) would have already decayed before it arrived at the IceCube detector. However, the full analysis requires Monte Carlo simulations and is much more involved. (We have not performed it separately.)

The proponents of the SME might object that within the gauge-invariant theory, one has $\delta_\nu = \delta_e$, and so, the bound $\delta_\nu \leq 2.0 \times 10^{-20}$ is not applicable, because the LPCR process does not exist. But then, they have to acknowledge that the bound $\delta_e \leq 1.04 \times 10^{-20}$, which is derived for electrons, based on other physical processes, applies to the neutrino sector.

Comparison to Astrophysical Data II

- In view of the existence of the NPCR process, the proponents of Lorentz violation in the neutrino sector are in even more trouble.
- There is a negligible threshold for NPCR, and so, if the Lorentz-violating parameters for the different neutrino mass eigenstates are different, then the decay and energy loss processes connected with NPCR affect low-energy neutrinos. There is no discussion about a threshold.
- Numerical coefficients for NPCR and typically a factor of four or five larger than for LPCR, depending on the model used for the spin sums. This enhances the importance of the effect.
- We conjecture(!) that a full analysis of astrophysical data, using the NPCR process as a limiting factor for the observation of high-energy neutrinos, should yield a bound on the order of

$$|\delta_i - \delta_f| \leq \frac{1}{5^{1/5}} \times 2.0 \times 10^{-20} \,.$$

It would be great if interested astrophysicists could confirm this conjecture.

Comparison to Astrophysical Data III

On the other hand...

- If there is a tiny Lorentz violation in the neutrino sector and if the Lorentz-violating parameters of neutrino mass eigenstates are different, then, a signature of the NPCR process would be that only one mass eigenstate (with a defined flavor composition) would be able to propagate at very high energy.
- Thus, above a certain energy, detectors should observe a defined ratio of neutrino flavors, commensurate with the arrival of one, and only one, mass eigenstate (which of course has a defined flavor decomposition).
- (Some more detailed speculation on this point is contained in Phys. Rev. D 100, 035036 (2019).)

Gauge Invariance (Detour I)

There are subtle connections of Lorentz violation and gauge invariance.
A wide field with an open discussion…

- Chkareuli, Froggatt and Nielsen (PRL, 2001): "We argue that, generally, Lorentz invariance can be imposed only in the sense that all Lorentz noninvariant effects caused by the spontaneous breakdown of Lorentz symmetry are physically unobservable. The application of this principle to the most general relativistically invariant Lagrangian, with arbitrary couplings for all the fields involved, leads to the appearance of a symmetry and, what is more, to the massless vector fields gauging this symmetry in both Abelian and non-Abelian cases."
- The photon could potentially be formulated as the Nambu–Goldstone boson linked to spontaneous Lorentz invariance violation. This was formulated in papers of Nambu, and Jona Lasinio.

Gauge Invariance (Detour II)

- Chkareuli and Jejeleva [Phys. Lett. B **659**, 754 (2008)] consider a model where

$$\langle A_\mu \rangle = n_\mu M \,, \qquad A_\mu = a_\mu + \frac{n_\mu}{n^2}(n \cdot A) \,.$$

They finally arrive at the Lagrangian

$$\begin{aligned} L(a, \psi) = & -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{2}\delta(n \cdot a)^2 \\ & + \bar{\psi}(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi - e a_\mu \bar{\psi}\gamma^\mu \psi \\ & - \frac{1}{4} f_{\mu\nu} h^{\mu\nu} \frac{n^2 a_\rho a^\rho}{M} + \frac{e n^2 a_\rho a^\rho}{2M} \bar{\psi}(\gamma \cdot n)\psi \,. \end{aligned}$$

Some terms are gauge invariant, others break gauge invariance.

Gauge Invariance (Detour III)

- Gauge invariance and Lorentz symmetry are interconnected. A model, still gauge invariant with respect to a restricted subclass of electroweak gauge transformations, which justifies the *ansatz* $v_{\rm int} = v_i \, v_f$ in the context of the NPCR calculation, has been discussed in [U. D. Jentschura, I. Nándori, and G. Somogyi, *Lorentz Breaking and $SU(2)_L \times U(1)_Y$ Gauge Invariance for Neutrinos*, Int. J. Mod. Phys. E **28**, 1950072 (2019)].
- A gauge-invariant model, with different Lorentz-violating parameters for different neutrino flavor eigenstates, has also been discussed in [U. D. Jentschura, I. Nándori, and G. Somogyi, *Lorentz Breaking and $SU(2)_L \times U(1)_Y$ Gauge Invariance for Neutrinos*, Int. J. Mod. Phys. E **28**, 1950072 (2019)]. In this model, the mass and flavor eigenstates become identical in the high-energy limit ($E_i = |\vec{p}_i| v_i$), with different v_i. One then defines generalized Dirac matrices in the Lagrangians, and couples to the electroweak gauge sector. Finally, one realizes that inter-generation decay processes (NPCR and LPCR) are allowed provided the Lorentz-violating parameters are different for the generations. This consideration highlights the importance of NPCR in such processes.

Conclusions

- LPCR results (Cohen and Glashow, and Bezrukov and Lee) have been verified and generalized.
- NPCR results have been obtained.
- Results pressure the parameter space of Lorentz-violating theories in the neutrino sector (most attractive candidate for such theories).
- Full electroweak gauge invariance would imply that all bounds on Lorentz-violating parameters, originally obtained for electrons, should also apply to the neutrino sector, defeating the purpose of considering the neutrinos as attractive candidates.
- Conversely, assuming that $\delta_\nu > \delta_e \approx 0$, or, that $\delta_i \neq \delta_f$ for neutrinos, one arrives at stringent bounds for the Lorentz-violating parameters.
- Current astrophysical data, combined with LPCR and NPCR, limit neutrino Lorentz violation, and differential neutrino Lorentz violation in the neutrino sector, to parameters $\delta \lesssim 10^{-20}$, which $v = \sqrt{1+\delta}$.