Is there really a no-go area?

between unflavored leptogenesis & low-energy CP violation

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OUTLINE:

★ **Seesaw: the Casas-Ibarra parametrization**

★ **A no-go theorem: unflavored leptogenesis**

★ **RGE corrections to the CI parametrization**

★ **Viable example: the no-go area is visitable**

★ **Flavored leptogenesis and low-energy CPV**

Work in collaboration with my PhD student Di Zhang:

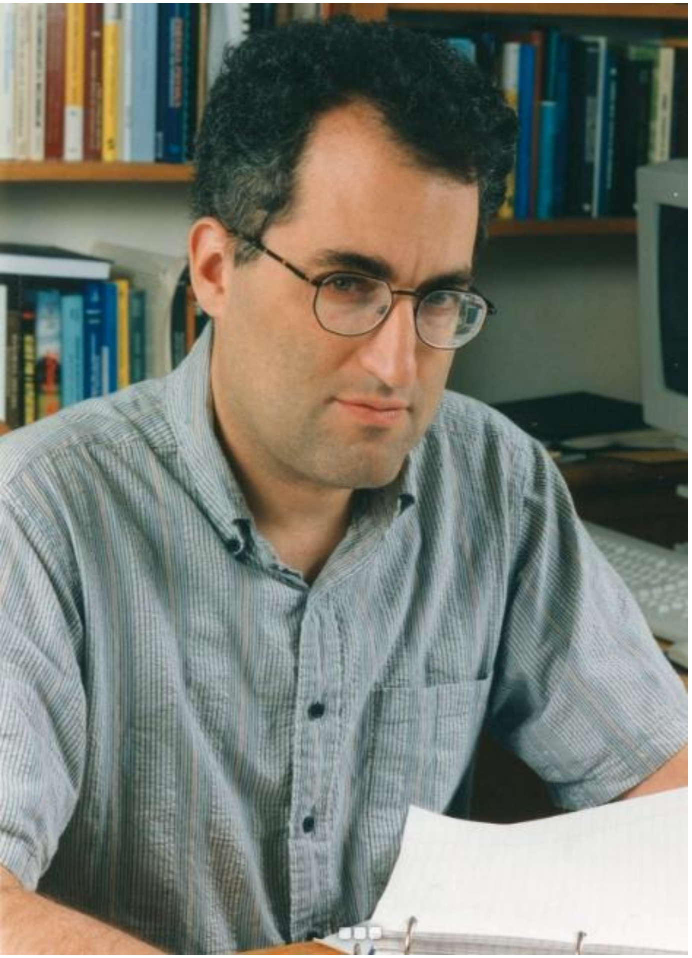
♦ **ZZX, D. Zhang, JHEP 04 (2020) 179, e-print: 2003.00480**

♦ **ZZX, D. Zhang, PLB 804 (2020) 135397, e-print: 2003. 06312**

The 23rd Bled Web-Workshop "What comes beyond the SM", 06 — 10.07.2020

Seesaw: pro and con

Edward Witten's opening talk at the SNO "**Neutrino2000**" conference:



"For neutrino masses, the considerations have always been **qualitative**, and, despite some interesting attempts, there has **never** been a **convincing quantitative** model of the neutrino masses."

- This is still true even today, unfortunately!
- **Qualitatively**, the **seesaw** picture remains most popular, for it is likely to kill two birds with one stone.
- **Leptogenesis** is an interesting mechanism to interpret the baryon-antibaryon number asymmetry of the Universe.
- **Quantitatively**, a **seesaw** mechanism isn't predictive at all unless its flavor structure can be fixed with the help of either flavor symmetries or purely phenomenological assumptions.

How about a factorization?

★ **Inspiration from hadron physics:**

weak part × strong perturbative part × strong non-perturbative part

★ **A factorization of the Yukawa coupling structure in type-I seesaw?**

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H E_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

Dim-5 operator:

$$\mathcal{O}_{\rm Weinberg} = \frac{\kappa_{\alpha\beta}}{2}\left[\overline{\ell_{\alpha\rm L}} \widetilde{H} \widetilde{H}^T \ell^c_{\beta\rm L}\right]$$

$$\kappa = Y_\nu M_{\rm R}^{-1} Y_\nu^T$$

Effective mass matrix:

$$M_\nu = -v^2 \left(Y_\nu M_{\rm R}^{-1} Y_\nu^T\right)$$

seesaw

In the chosen basis, ν-mixing: $U^\dagger M_\nu U^* = D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$

$$M_l = D_l \equiv {\rm Diag}\{m_e, m_\mu, m_\tau\}$$

$$M_{\rm R} = D_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$$

The Casas-Ibarra factorization:

$$Y_\nu = \frac{\rm i}{v} U \sqrt{D_\nu}\, O \sqrt{D_N}$$

Casas-Ibarra

★ **A factorization of the Yukawa coupling structure**

$$Y_\nu = \frac{\mathrm{i}}{v} U \sqrt{D_\nu}\, O \sqrt{D_N}$$

heavy ν-masses

light ν-oscillations

leptogenesis relevant

light ν-masses

The undetermined part is the unknown complex orthogonal matrix *O*, *O O^T* = *I*.

ELSEVIER

NUCLEAR PHYSICS B

Nuclear Physics B 618 (2001) 171–204

Oscillating neutrinos and $\mu \to e, \gamma$

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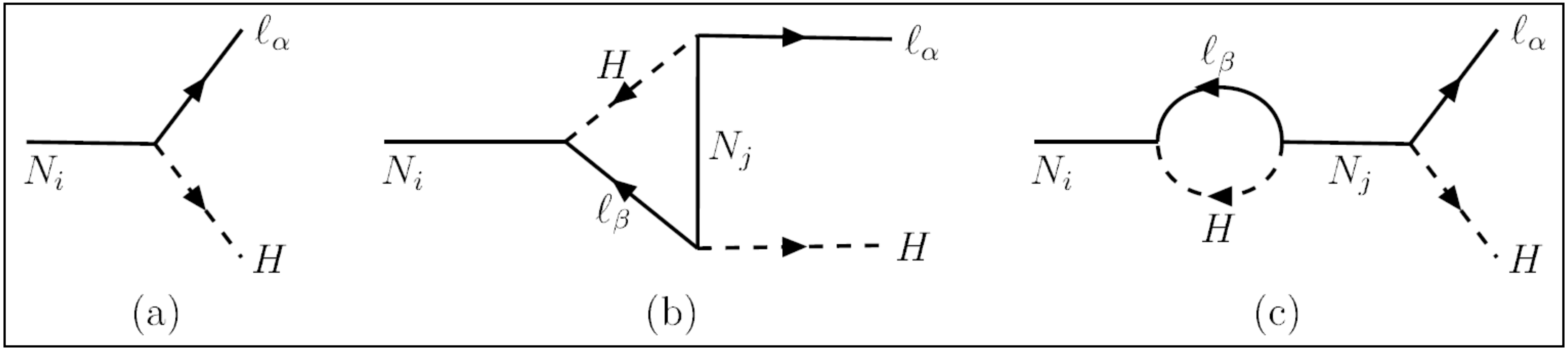
Thermal leptogenesis

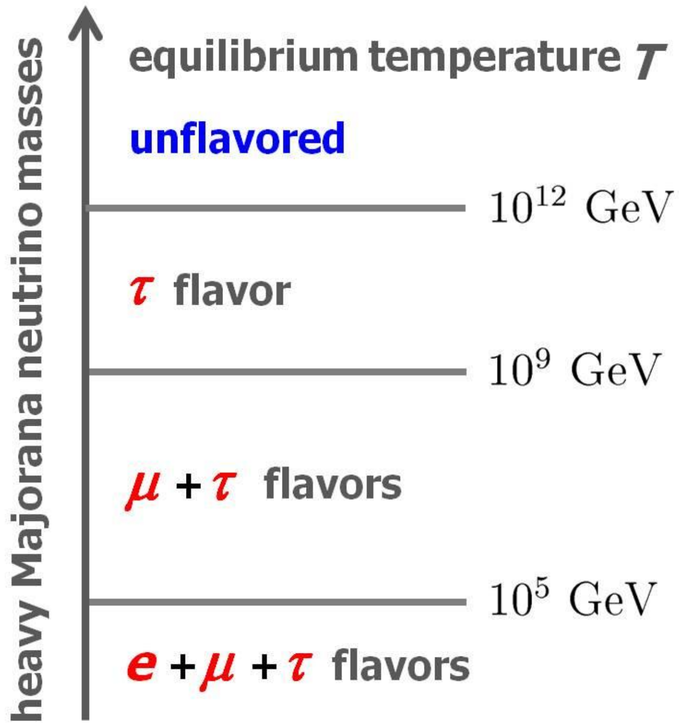
★ **Lepton-number-violating & CP-violating decays of heavy neutrinos:**



★ **Given** $M_3 > M_2 \gg M_1 = T \gtrsim 10^{12}$ GeV **, the CP-violating asymmetry responsible for unflavored leptogenesis is**

$$\varepsilon_1 \equiv \frac{\sum\limits_\alpha \left[\Gamma\left(N_1 \to \ell_\alpha + H\right) - \Gamma\left(N_1 \to \overline{\ell_\alpha} + \overline{H}\right)\right]}{\sum\limits_\alpha \left[\Gamma\left(N_1 \to \ell_\alpha + H\right) + \Gamma\left(N_1 \to \overline{\ell_\alpha} + \overline{H}\right)\right]}$$

$$\simeq -\frac{3M_1}{16\pi\left(Y_\nu^\dagger Y_\nu\right)_{11}} \sum_i \left[\frac{\mathrm{Im}\left(Y_\nu^\dagger Y_\nu\right)_{1i}^2}{M_i}\right]$$



A no-go theorem?

★ **In the Casas-Ibarra parametrization, unflavored leptogenesis turns out to be independent of the PMNS matrix *U* at low energies because**

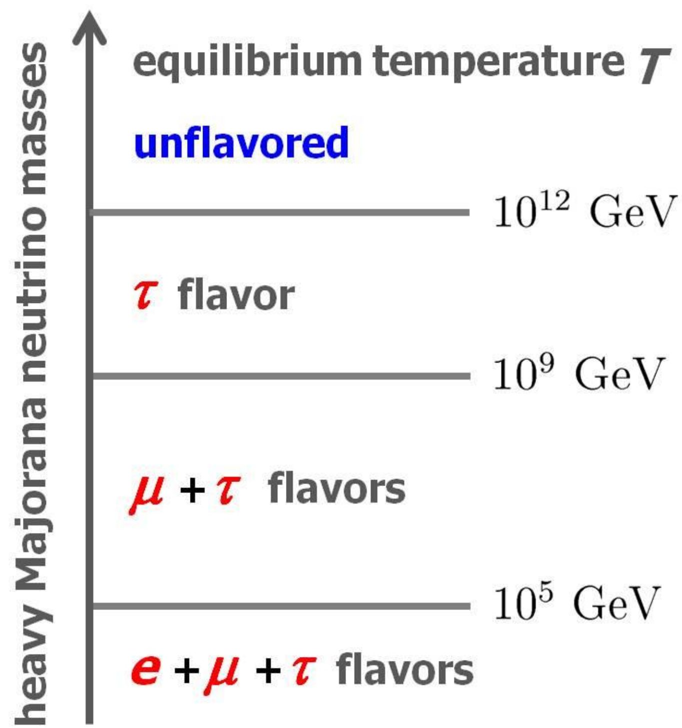
$$\boxed{Y_\nu = \frac{\mathrm{i}}{v} U \sqrt{D_\nu}\, O \sqrt{D_N}} \longrightarrow Y_\nu^\dagger Y_\nu = \frac{1}{v^2}\sqrt{D_N}\, \underline{O^\dagger D_\nu O}\, \sqrt{D_N}$$

no-go

♦ **ZZX, 0902.2469;** ♦ **W. Rodejohann, 0903.4590;**
♦ **S. Antusch et al, 0910.5957;** ♦ **……**

★ **A way out: to realize flavored leptogenesis with *T* below 10^{12} GeV (and even real *O*):**

$$\varepsilon_{i\alpha} \equiv \frac{\Gamma(N_i \to \ell_\alpha + H) - \Gamma(N_i \to \overline{\ell_\alpha} + \overline{H})}{\sum_\alpha \left[\Gamma(N_i \to \ell_\alpha + H) + \Gamma(N_i \to \overline{\ell_\alpha} + \overline{H})\right]}$$
$$= \frac{1}{8\pi (Y_\nu^\dagger Y_\nu)_{ii}} \sum_{j\neq i} \left\{ \mathrm{Im}\left[\underline{(Y_\nu^*)_{\alpha i}(Y_\nu)_{\alpha j}}(Y_\nu^\dagger Y_\nu)_{ij}\right] \mathcal{F}(x_{ji}) + \mathrm{Im}\left[\underline{(Y_\nu^*)_{\alpha i}(Y_\nu)_{\alpha j}}(Y_\nu^\dagger Y_\nu)^*_{ij}\right] \mathcal{G}(x_{ji}) \right\}$$



♦ **S. Pascoli et al, 0302054; 0609125; 0611338;**
♦ **G.C. Branco et al, 1111.5332;**
♦ **K. Moffat et al, 1809.08251;** ♦ **……**

New idea

★ The **Casas-Ibarra** parametrization is done at the seesaw scale, so it is necessary to run light ν-masses and ***U*** down to low energies by use of the renormalization-group equations.

★ From the seesaw scale to the electroweak scale, the one-loop RGE:

$$16\pi^2 \frac{\mathrm{d}\kappa}{\mathrm{d}t} = \alpha_\kappa \kappa + C_\kappa \left[\left(Y_l Y_l^\dagger \right) \kappa + \kappa \left(Y_l Y_l^\dagger \right)^T \right]$$

where $t \equiv \ln\left(\mu/\Lambda_{\rm EW}\right)$, $C_\kappa = -3/2$, $\alpha_\kappa \approx -3g_2^2 + 6y_t^2 + \lambda$ **in the SM.**

$$\kappa\left(\Lambda_{\rm SS}\right) = I_0^2 \left[T_l \cdot \kappa\left(\Lambda_{\rm EW}\right) \cdot T_l \right]$$

where $T_l = {\rm Diag}\{I_e, I_\mu, I_\tau\}$, **and**

$$I_0 = \exp\left[\frac{1}{32\pi^2} \int_0^{\ln\left(\Lambda_{\rm SS}/\Lambda_{\rm EW}\right)} \alpha_\kappa(t)\,\mathrm{d}t \right]$$

$$I_\alpha = \exp\left[\frac{C_\kappa}{16\pi^2} \int_0^{\ln\left(\Lambda_{\rm SS}/\Lambda_{\rm EW}\right)} y_\alpha^2(t)\,\mathrm{d}t \right]$$

Consider $y_e^2 \ll y_\mu^2 \ll y_\tau^2 \ll 1$, **we obtain the approximation:**

$$T_l \simeq {\rm Diag}\{1, 1, 1+\Delta_\tau\}$$

with

$$\Delta_\tau = \frac{C_\kappa}{16\pi^2} \int_0^{\ln\left(\Lambda_{\rm SS}/\Lambda_{\rm EW}\right)} y_\tau^2(t)\,\mathrm{d}t$$

New result

★ **The RGE-assisted Casas-Ibarra parametrization turns out to be**

$$Y_\nu = \frac{\mathrm{i}}{v} U \sqrt{D_\nu}\, O \sqrt{D_N} \Rightarrow Y_\nu(\Lambda_{\rm SS}) = \frac{\mathrm{i}}{v} I_0 T_l U(\Lambda_{\rm EW}) \sqrt{D_\nu(\Lambda_{\rm EW})}\, O \sqrt{D_N(\Lambda_{\rm SS})}$$

In this case unflavored leptogenesis becomes dependent upon *U* as

$$\left(Y_\nu^\dagger Y_\nu\right)_{1i} = \frac{1}{v^2}\left(I_0^2 \sqrt{D_N}\, O^\dagger \sqrt{D_\nu}\, \underline{U^\dagger T_l^2 U} \sqrt{D_\nu}\, O \sqrt{D_N}\right)_{1i}$$

$$\simeq \frac{I_0^2}{v^2}\sqrt{M_1 M_i}\left[\sum_j \left(m_j O_{j1}^* O_{ji}\right) + 2\Delta_\tau \sum_{j,k}\left(\sqrt{m_j m_k}\, O_{j1}^* O_{ki} \underline{U_{\tau j}^* U_{\tau k}}\right)\right]$$

But the *U*-induced contribution at the next-to-leading level. Assuming *O* to be real, a direct link between unflavored leptogenesis and *U* can be established:

$$\varepsilon_1 \simeq -\frac{3\Delta_\tau I_0^2 M_1}{4\pi v^2} \cdot \frac{\sum_{j>k} \sqrt{m_j m_k}\left(m_k - m_j\right) \underline{O_{j1} O_{k1}}\, {\rm Im}\left(\underline{U_{\tau j}^* U_{\tau k}}\right)}{\sum_i m_i O_{i1}^2} + \mathcal{O}\left(\Delta_\tau^2\right)$$

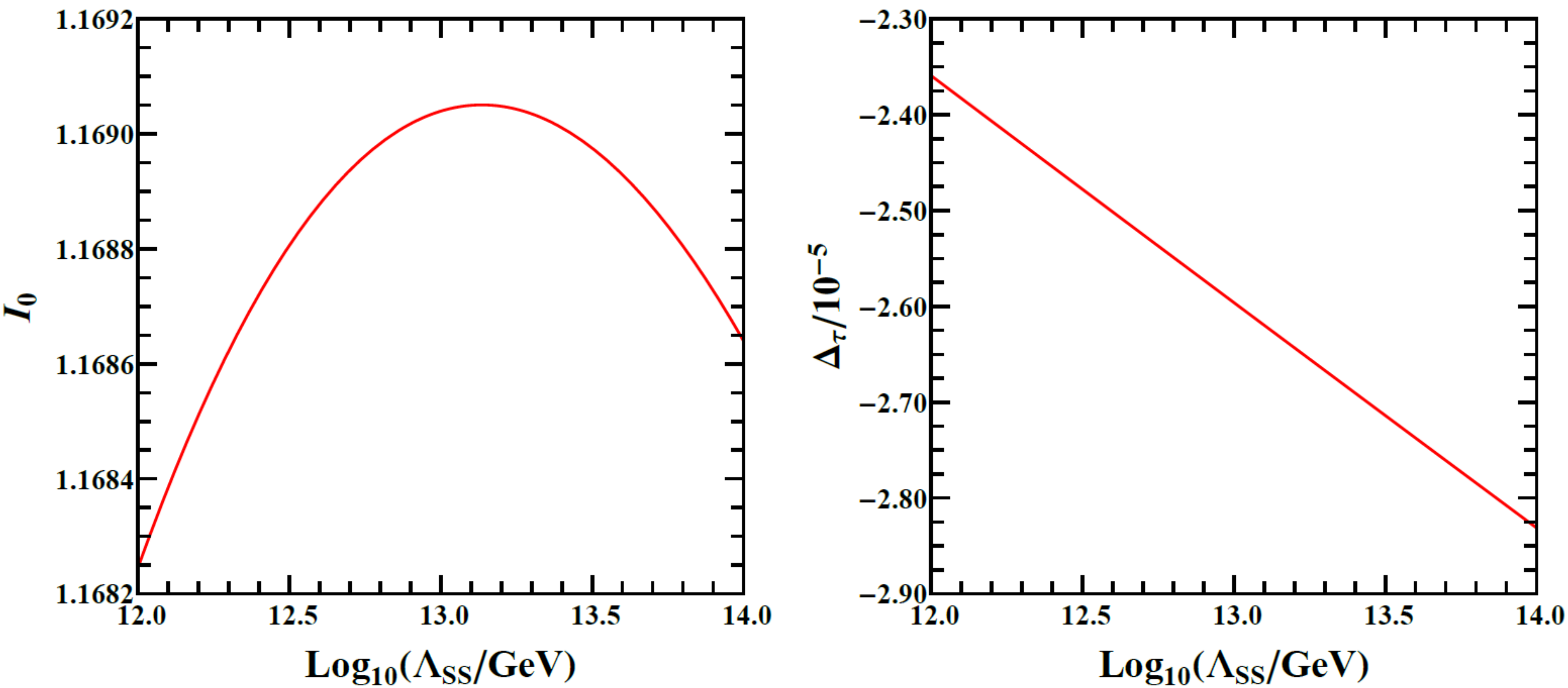
Effectively τ-flavored

Large uncertainties

★ **Even though *O* is real, its values remain arbitrary. In this case what we can do is to find out a viable parameter space for leptogenesis.**

★ **If *O = I* is taken, we are left with CP violation——too small to work.**

$$\varepsilon_1 \simeq -\frac{3\Delta_\tau^2 I_0^2 M_1}{4\pi v^2}\sum_i m_i \mathrm{Im}\left(U_{\tau 1}^* U_{\tau i}\right)^2 + \mathcal{O}\left(\Delta_\tau^3\right)$$



Leptogenesis

★ **Now let us account for the observed baryon-to-photon ratio of the Universe via *unflavored* leptogenesis (W. Buchmueller et al, 2002):**

Fukugita, Yanagida 86

$$\eta \simeq -9.6 \times 10^{-3} \varepsilon_1 \kappa_{\rm f} \quad \Rightarrow \quad \eta \equiv \frac{n_{\rm B}}{n_\gamma} \simeq (6.12 \pm 0.03) \times 10^{-10}$$

CP asymmetry (ε_1) **Efficiency factor** ($\kappa_{\rm f}$)

$$z_{\rm B}(K_1) \simeq 2 + 4K_1^{0.13} \exp\left(-2.5/K_1\right).$$

$$\kappa_{\rm f} \simeq \frac{2}{K_1 z_{\rm B}(K_1)} \left[1 - \exp\left(-\frac{1}{2} K_1 z_{\rm B}(K_1)\right)\right] ,$$

$$K_1 \equiv \frac{\Gamma(N_1)}{H(M_1)} \simeq \frac{I_0^2}{1.08 \times 10^{-3}~{\rm eV}} \left[\sum_i m_i O_{i1}^2 + 2\Delta_\tau \sum_{i,j} \sqrt{m_i m_j} O_{i1} O_{j1} {\rm Re}\left(U_{\tau i}^* U_{\tau j}\right)\right]$$

which determines whether or not the decays of N_1 are in equilibrium. (S. Blanchet, P. Di Bari, 2007; W. Buchmueller et al, 2005)

Some inputs (F. Capozzi et al, 1804.09678; I. Esteban et al, 1811.05487):

$$\sin^2\theta_{12} = 0.310 , \quad \sin^2\theta_{13} = 0.02241 , \quad \sin^2\theta_{23} = 0.558 , \quad \delta = 222^\circ ;$$

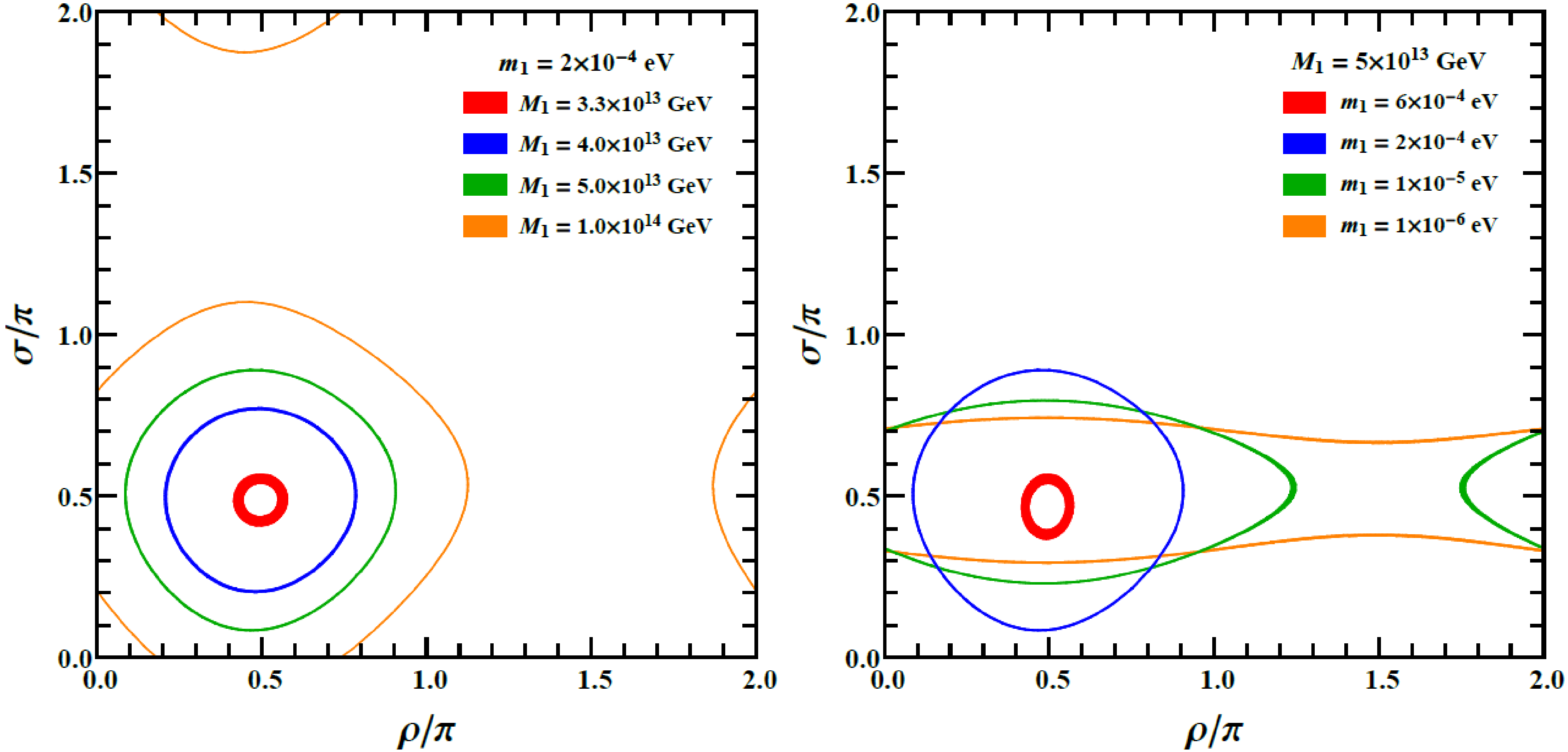
$$\Delta m_{21}^2 = 7.39 \times 10^{-5}~{\rm eV}^2 , \quad \Delta m_{31}^2 = 2.523 \times 10^{-3}~{\rm eV}^2 .$$

Normal ordering

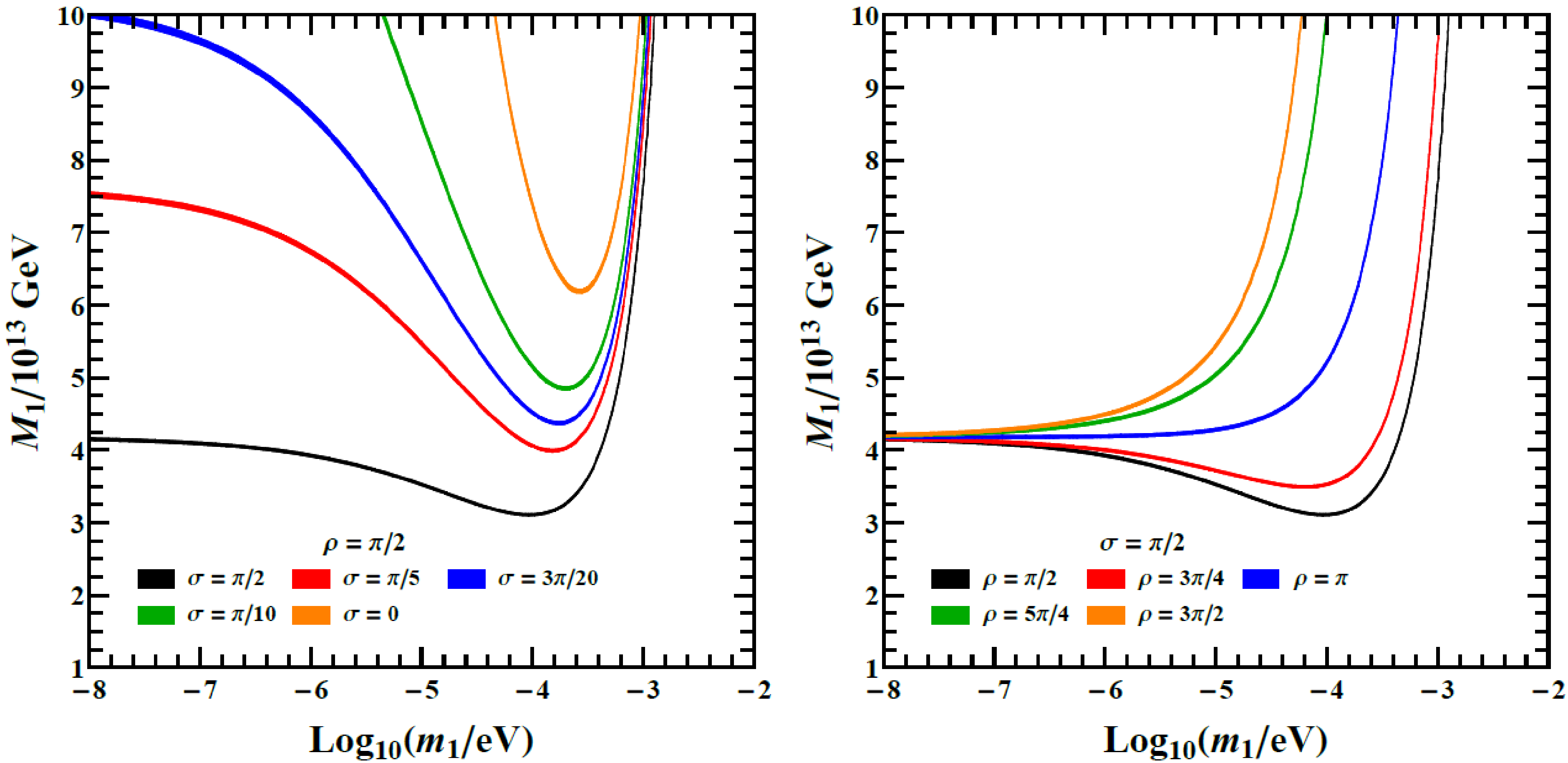
Specific parameter space (1)

★ **One may use two angles to parametrize O_{i1}. To be specific, we take $(\theta, \phi) = (84.9°, 351.1°)$ in our numerical calculation, just for illustration.**

★ **Allowing other unknown parameters to vary in reasonable intervals to reproduce the observed range of η, we output their values and plot the parameter space.**



Specific parameter space (2)



★ **It's possible to interpret the observed baryon number asymmetry of the Universe with CP violation at low energies in our Ansatz.**

★ **But, the arbitrariness of *O* in the CI factorization makes our feeling quite uneasy. We'll show that *O* = *I* works in resonant leptogenesis.**

A special case

★ **We consider the type-I seesaw scenario with three heavy Majorana neutrinos. The RGE-assisted CI parametrization:**

$$Y_\nu(\Lambda_{SS}) = \frac{\mathrm{i}}{v} I_0 T_l U(\Lambda_{EW}) \sqrt{D_\nu(\Lambda_{EW})}\, O \sqrt{D_N(\Lambda_{SS})}$$

O* = *I

$$Y_\nu = \frac{\mathrm{i}}{v} I_0 \left[\begin{pmatrix} \sqrt{m_1 M_1} U_{e1} & \sqrt{m_2 M_2} U_{e2} & \sqrt{m_3 M_3} U_{e3} \\ \sqrt{m_1 M_1} U_{\mu 1} & \sqrt{m_2 M_2} U_{\mu 2} & \sqrt{m_3 M_3} U_{\mu 3} \\ \sqrt{m_1 M_1} U_{\tau 1} & \sqrt{m_2 M_2} U_{\tau 2} & \sqrt{m_3 M_3} U_{\tau 3} \end{pmatrix} + \Delta_\tau \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sqrt{m_1 M_1} U_{\tau 1} & \sqrt{m_2 M_2} U_{\tau 2} & \sqrt{m_3 M_3} U_{\tau 3} \end{pmatrix} \right]$$

In this case the arbitrariness of *O* is removed, but such a model is too special, corresponding to a special Yukawa structure as shown above.

★ **This model works for flavored resonant thermal leptogenesis with a heavy mass spectrum $M_1 \simeq M_2 \ll M_3$ either in $T \simeq M_1 \in (10^9, 10^{12}]~\mathrm{GeV}$ or in $T \simeq M_1 \in (10^5, 10^9]~\mathrm{GeV}$. The light neutrinos have a normal ordering.**

Resonant leptogenesis

★ **The flavored CP-violating asymmetries with resonance are given by**

$$\varepsilon_{i\alpha} \equiv \frac{\Gamma\left(N_i \to \ell_\alpha + H\right) - \Gamma\left(N_i \to \overline{\ell_\alpha} + \overline{H}\right)}{\sum\limits_\alpha \left[\Gamma\left(N_i \to \ell_\alpha + H\right) + \Gamma\left(N_i \to \overline{\ell_\alpha} + \overline{H}\right)\right]}$$

$$= \frac{\mathrm{Im}\left[\left(Y_\nu^*\right)_{\alpha i}\left(Y_\nu\right)_{\alpha j}\left(Y_\nu^\dagger Y_\nu\right)_{ij} + \xi_{ij}\left(Y_\nu^*\right)_{\alpha i}\left(Y_\nu\right)_{\alpha j}\left(Y_\nu^\dagger Y_\nu\right)_{ji}\right]}{\left(Y_\nu^\dagger Y_\nu\right)_{ii}\left(Y_\nu^\dagger Y_\nu\right)_{jj}} \cdot \frac{\xi_{ij}\zeta_j\left(\xi_{ij}^2 - 1\right)}{\left(\xi_{ij}\zeta_j\right)^2 + \left(\xi_{ij}^2 - 1\right)^2}$$

$$\simeq 2\Delta_\tau\left[\mathrm{Im}\left(U_{\tau i}^* U_{\tau j} U_{\alpha i}^* U_{\alpha j}\right) + \xi_{ij}\mathrm{Im}\left(U_{\tau j}^* U_{\tau i} U_{\alpha i}^* U_{\alpha j}\right)\right]\frac{\xi_{ij}\zeta_j\left(\xi_{ij}^2 - 1\right)}{\left(\xi_{ij}\zeta_j\right)^2 + \left(\xi_{ij}^2 - 1\right)^2}$$

where $\xi_{ij} \equiv M_i/M_j$**, and** $\zeta_j \equiv \dfrac{1}{8\pi}\left(Y_\nu^\dagger Y_\nu\right)_{jj} \simeq \dfrac{I_0^2}{8\pi v^2}\left(1 + 2\Delta_\tau |U_{\tau j}|^2\right) m_j M_j$

★ **Given the flavored resonant thermal leptogenesis, one has**

$$\eta \simeq -9.6 \times 10^{-3} \sum_\alpha \left(\varepsilon_{1\alpha}\kappa_{1\alpha} + \varepsilon_{2\alpha}\kappa_{2\alpha}\right)$$

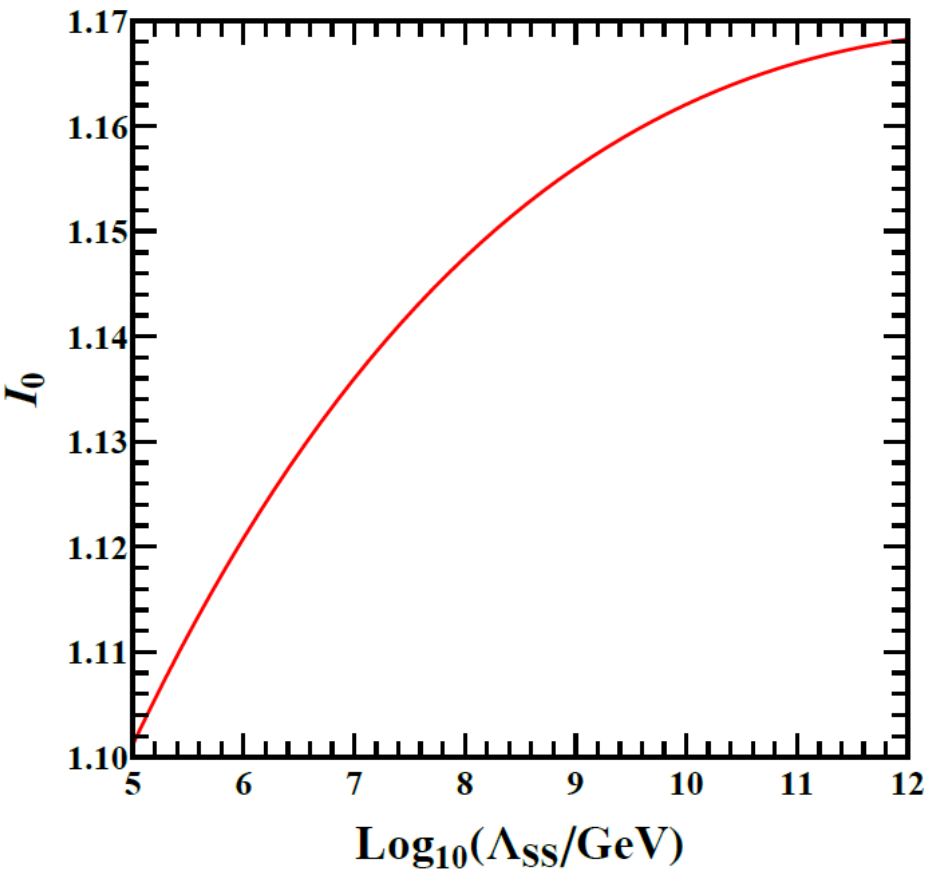
Some references on resonant leptogenesis: ♦ A. Pilaftsis, 9702393; 9707235; ♦ A. Pilaftsis, T. Underwood, 0309343; ♦ A. Anisimov et al, 0511248; ♦ ZZX, S. Zhou, 0607302; ♦ J. Zhang, S. Zhou, 1505.04858; ♦ B. Dev et al, 1711.02863.

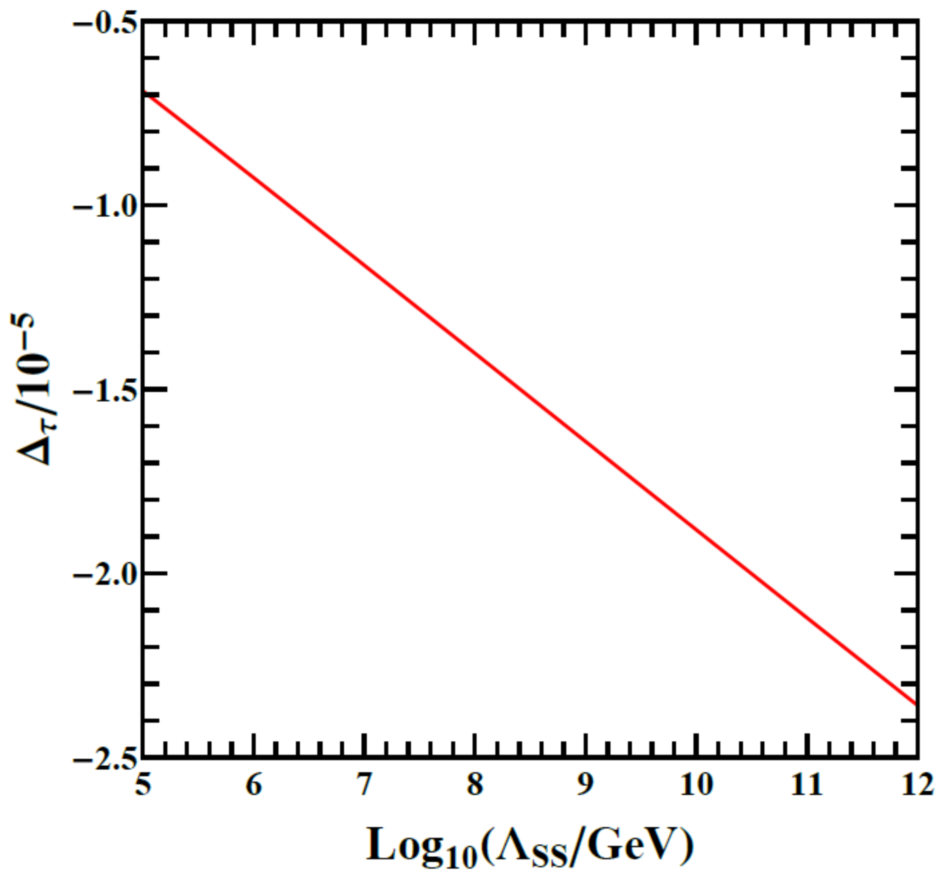
★ **The conversion efficiency factor at** $d \equiv (M_2 - M_1)/M_1 = \xi_{21} - 1 \ll 1$**:**

$$\kappa_{1\alpha} \simeq \kappa_{2\alpha} \equiv \kappa(K_\alpha) \simeq \frac{2}{K_\alpha z_{\rm B}(K_\alpha)}\left[1 - \exp\left(-\frac{1}{2}K_\alpha z_{\rm B}(K_\alpha)\right)\right]$$

where $z_{\rm B}(K_\alpha) \simeq 2 + 4K_\alpha^{0.13}\exp(-2.5/K_\alpha)$**, and the decay parameter is**

$$K_\alpha \simeq \frac{I_0^2}{1.08 \times 10^{-3}\ {\rm eV}}(1 + 2\Delta_\tau \delta_{\alpha\tau})\sum_{i=1}^{2}\left(m_i |U_{\alpha i}|^2\right) + \mathcal{O}(\Delta_\tau^2)$$



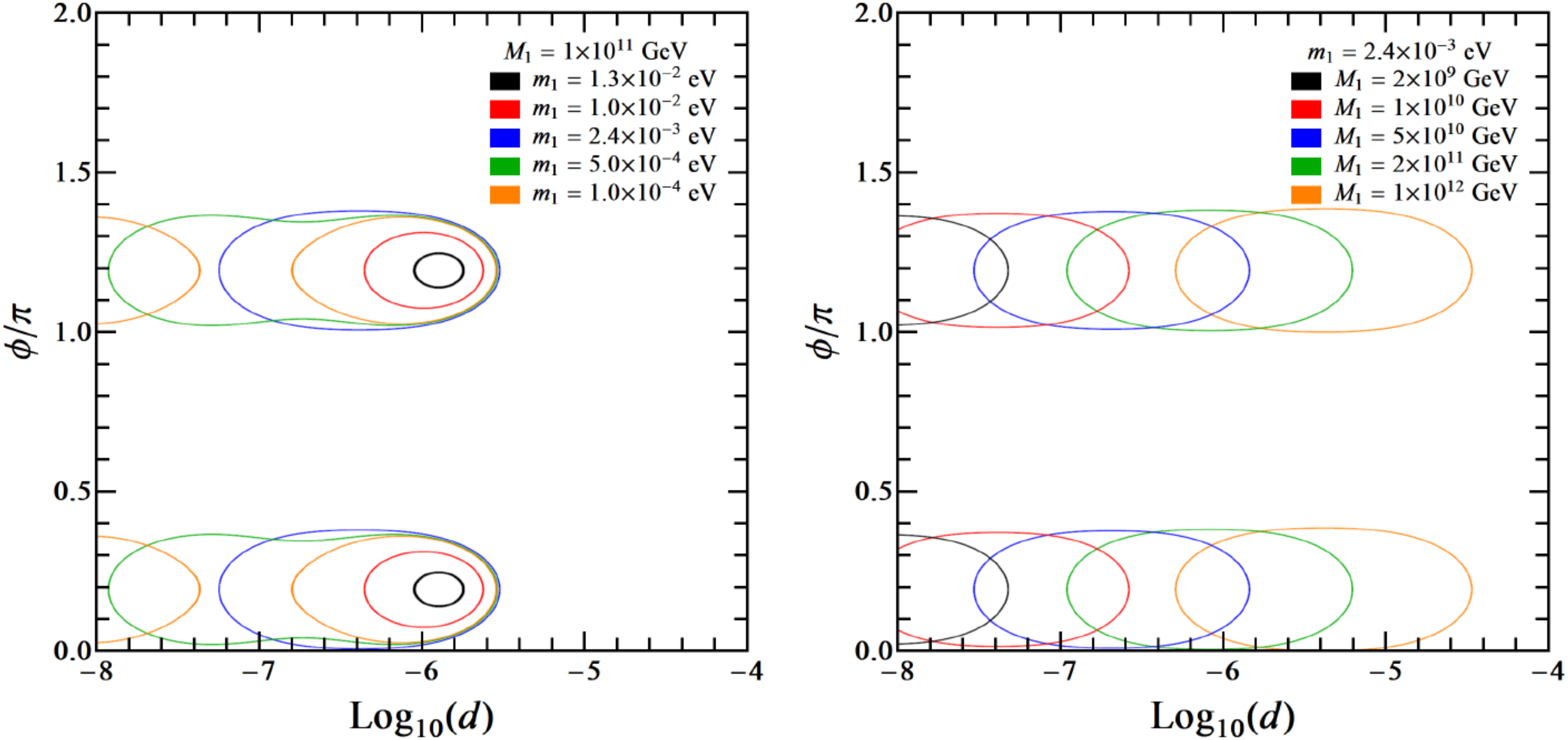


Numerical illustration (1)

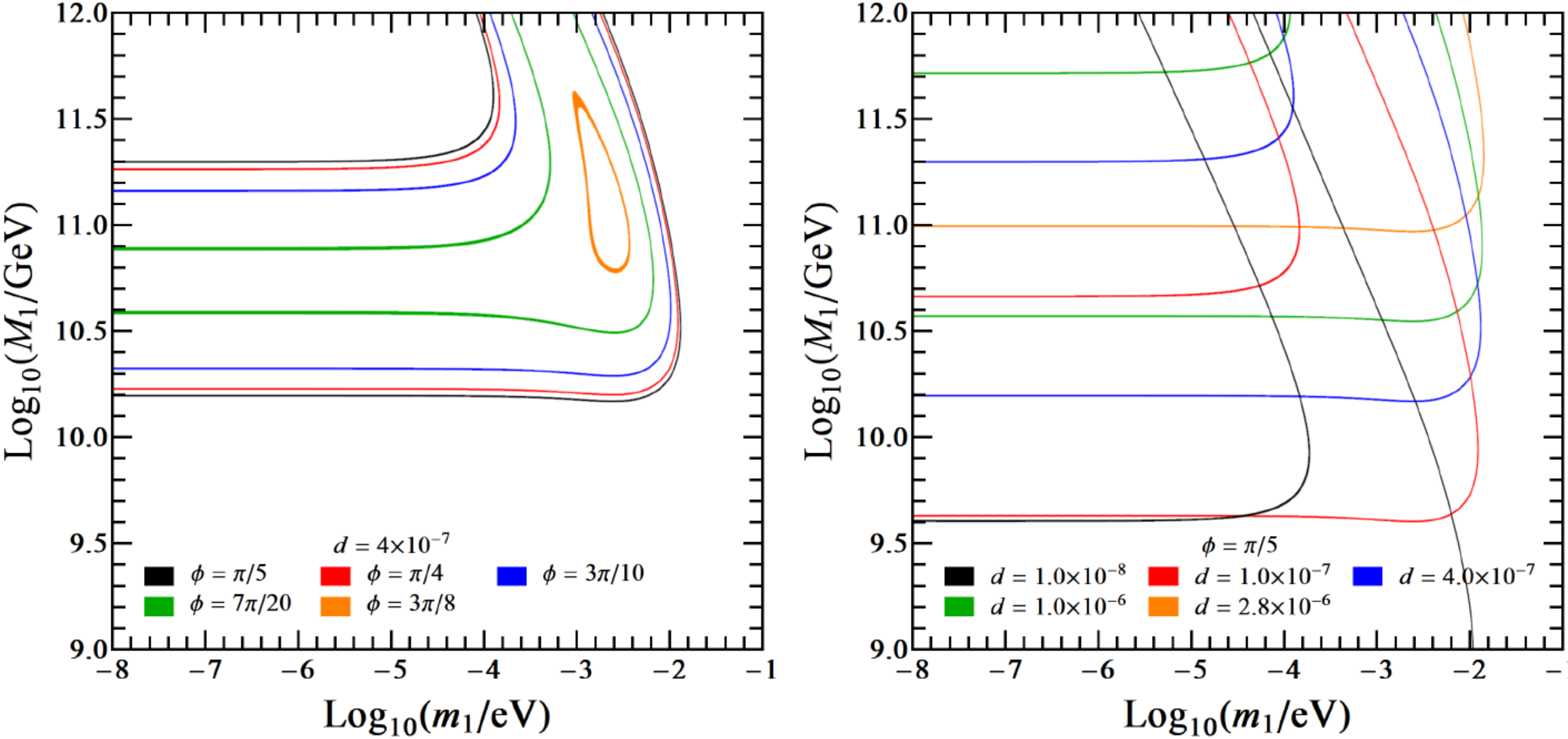
★ With the best-fit values of low-energy flavor parameters, it is found that the wash-out effect is strong ($\phi \equiv \rho - \sigma$ is defined).

★ In the τ-flavored leptogenesis region, the parameter space is constrained as

$$\eta \equiv \frac{n_{\rm B}}{n_\gamma} \simeq (6.12 \pm 0.03) \times 10^{-10}$$



Numerical illustration (2)



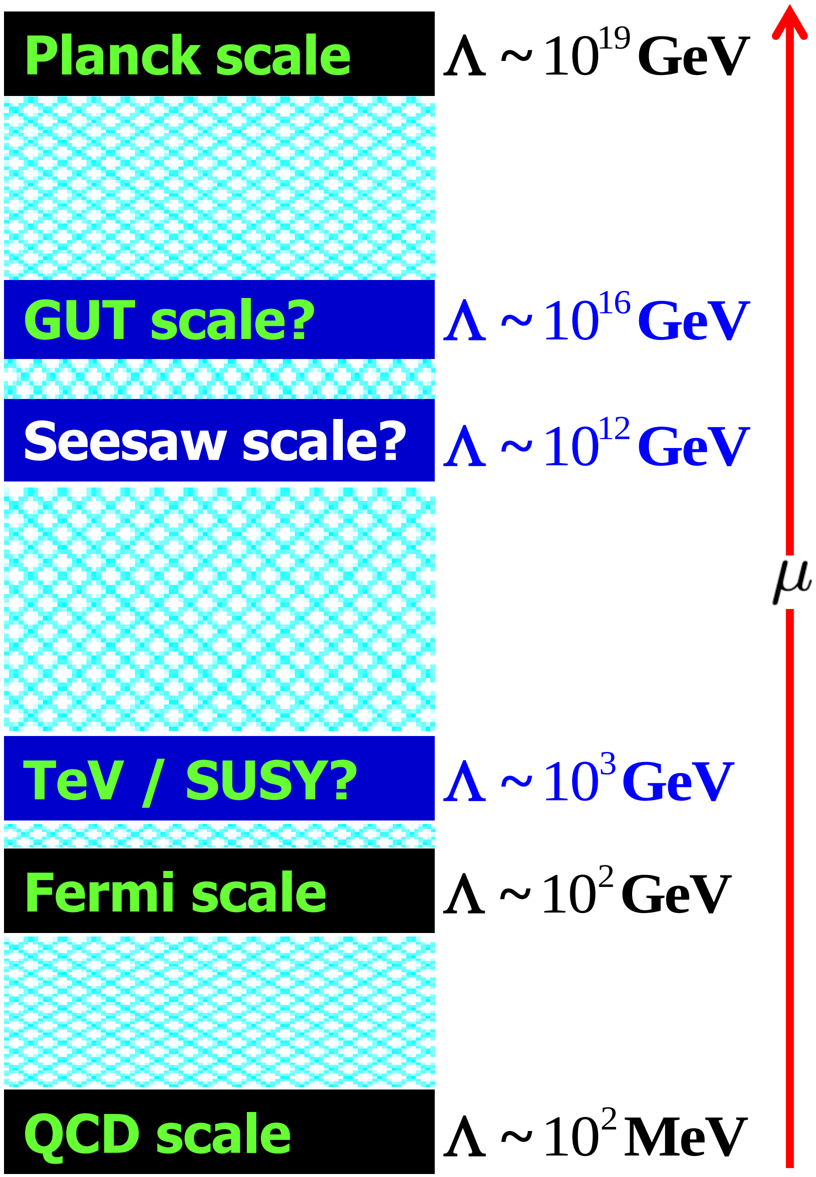
★ **So far so good. A mapping of the whole parameter space will be OK.**

★ **In the ($\mu + \tau$)-flavored leptogenesis region, one may also figure out the allowed parameter space as constrained by the observed η.**

★ **Such discussions can be simplified in the *minimal* seesaw scenario.**

Summary

★ **Seesaw** + **leptogenesis** is a killing-2-birds-with-1-stone framework.



★ But **how to test** this big picture is very challenging. Life would be a bit easier if there is a direct connection between high-scale and low-energy physics.

★ In this connection we point out a **loophole** in some previous works on **unflavored** leptogenesis with the **CI** parametrization——an **RGE-assisted CI** description.

★ Viable examples are given, but it remains unsatisfactory because the seesaw **flavor structure** is unknown to us. Success is still a long way off.

★ **Pauling**: how to get a good idea?