

# Gravitational footprints of massive neutrinos and lepton number breaking

Antonino Marcianò

[Fudan University](#)

A.Addazi, A. Marciano, A. Morais, R. Pasechnik, R. Sistrava & Jose W.F. Valle  
arXiv:1909.09740 (PLB 2020)

P. Dona, A. Marciano, Y. Zhang & C. Antolini, arXiv:1509.05824 (PRD 2016)

A.Addazi, A. Marciano & R. Pasechnik, arXiv:1812.07376 (CPC 2019)

A.Addazi, A. Marciano & R. Pasechnik, arXiv:1804.09826 (EPJC 2019)

work in progress...

# Plan of the talk

*The multi-messenger approach & particle physics*

*GW & tests of DM models from binary systems EoS*

*GW generated by FOPT*

*Neutrino physics and the mass-generation*

*See-saw mechanism and GW production*

*Partial conclusions*

*The multi-messenger approach & particle physics*

# The multi-messenger approach

*Electro-magnetism*  
*Neutrinos*  
*Cosmic rays*  
*GW signals*

*Use gravitational waves to probe high and low-scale physics*

Ex. : LISA, U-DECIGO and BBO can test SSB in 10 GeV-10 TeV

Ex. : PTA, SKA, FAST (nHz range) can test in MeV-ish scales

*Cross-checking strategy: meson factories, LHC, CEPC, etc...*



# Recurrent questions

*What is the nature of Dark Matter?*

*Can we use Gravitational Waves to unveil its nature?*

*Can we use a cross-checking multi messenger strategy?*

*How does neutrinos' mass generate?*

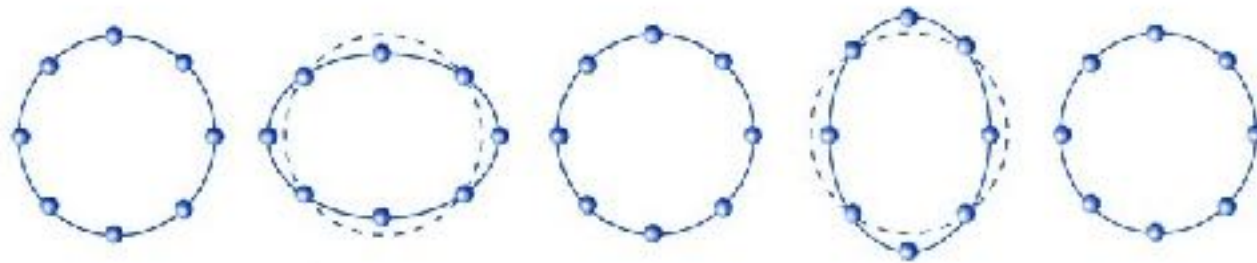
*Can we understand the nature of the inflaton?*

*What can we infer about confinement in QCD?*

***We deploy at the same time informations from different observational channels!***

# Gravitational Waves

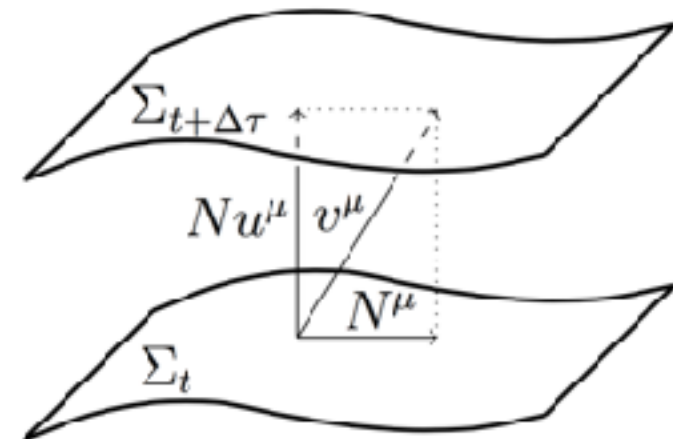
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



*2 propagating d.o.f.*

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 h_{ij}(\mathbf{x}, t) = \frac{16\pi G}{c^4} S_{ij}(\mathbf{x}, t)$$

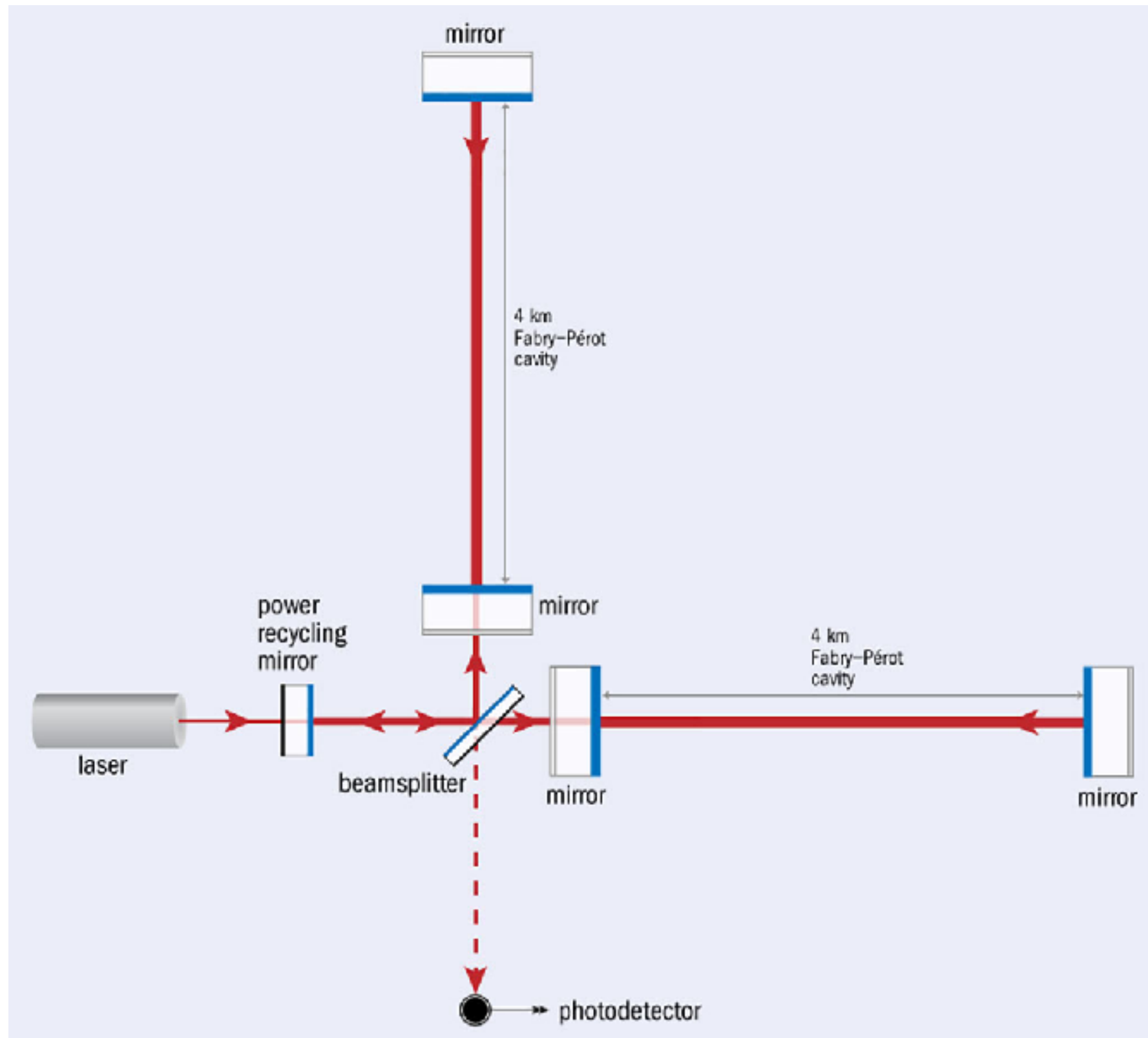
$$S_{ij}(\mathbf{x}, t) \equiv T_{ij}(\mathbf{x}, t) - \frac{1}{3} \delta_{ij} T^k_k(\mathbf{x}, t)$$



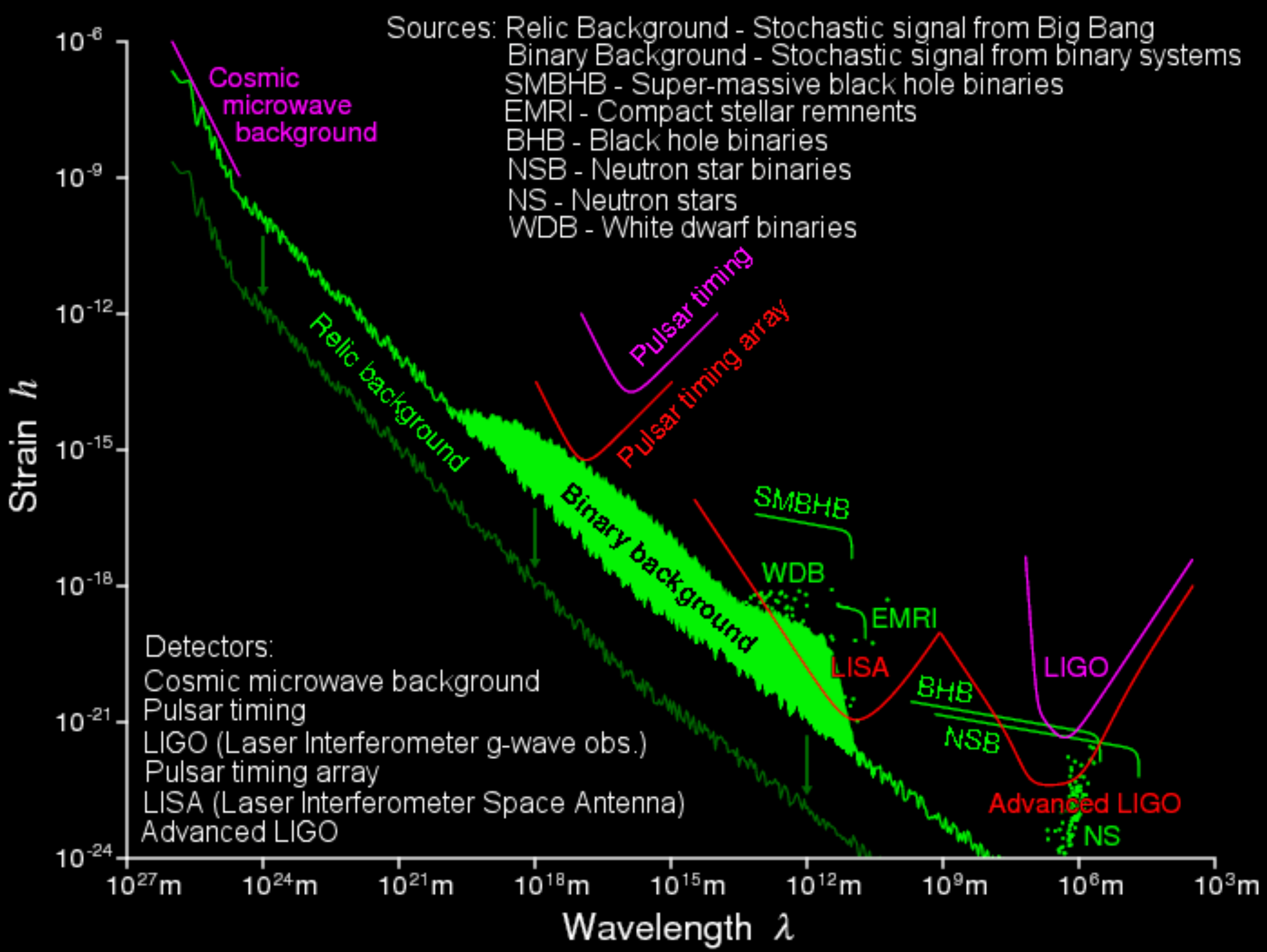
# Gravitational Waves



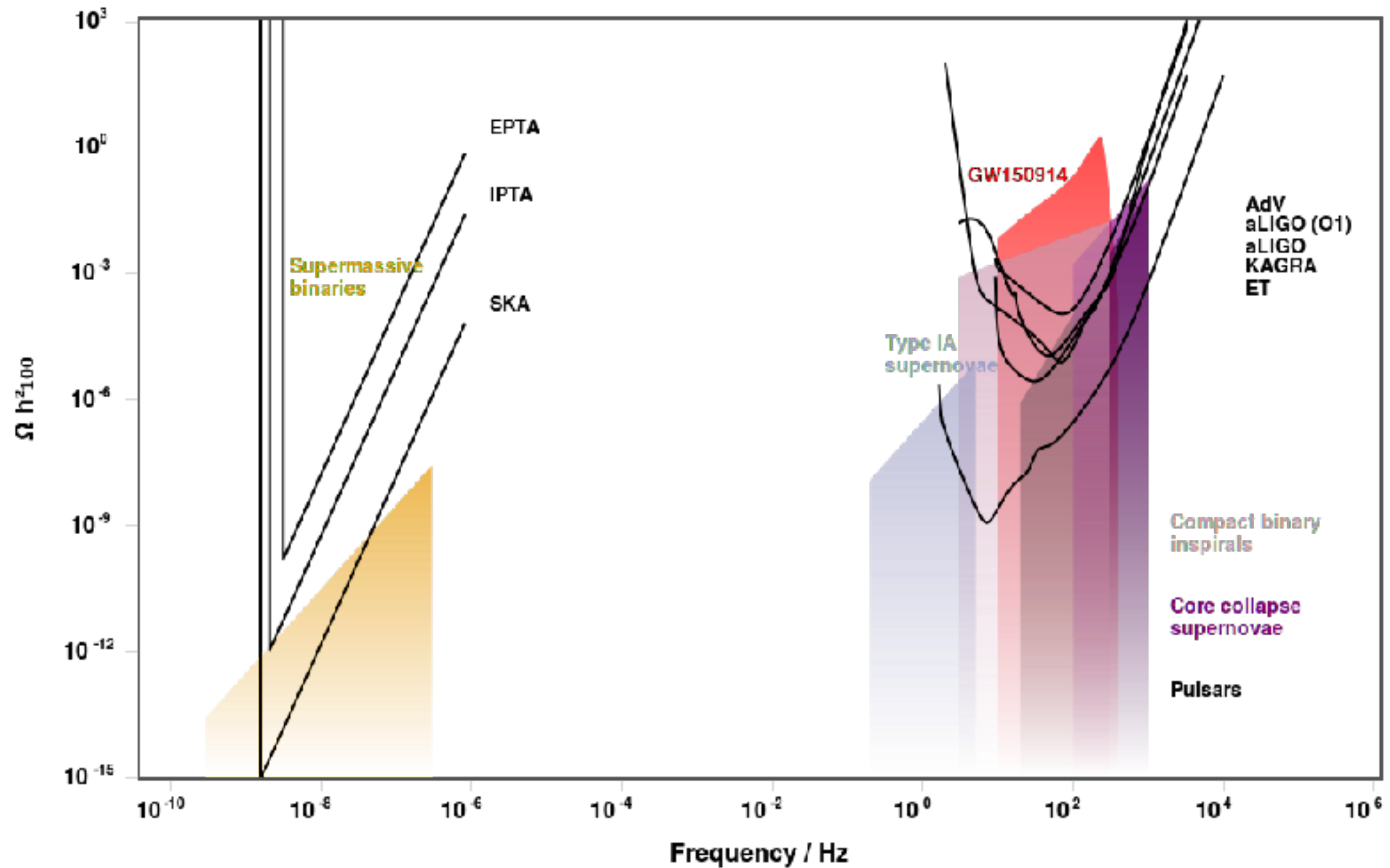
# Gravitational Waves





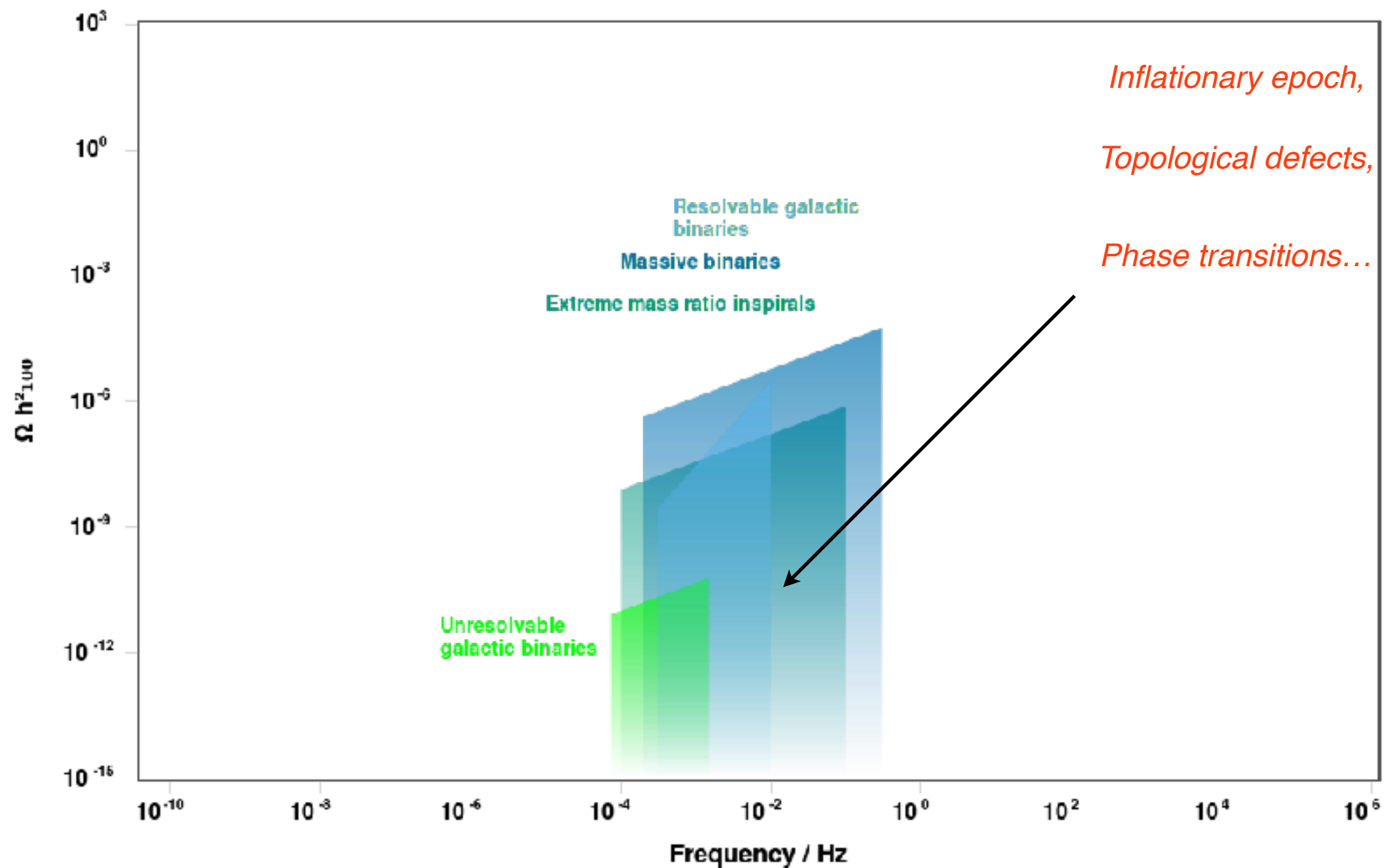


# Gravitational Waves



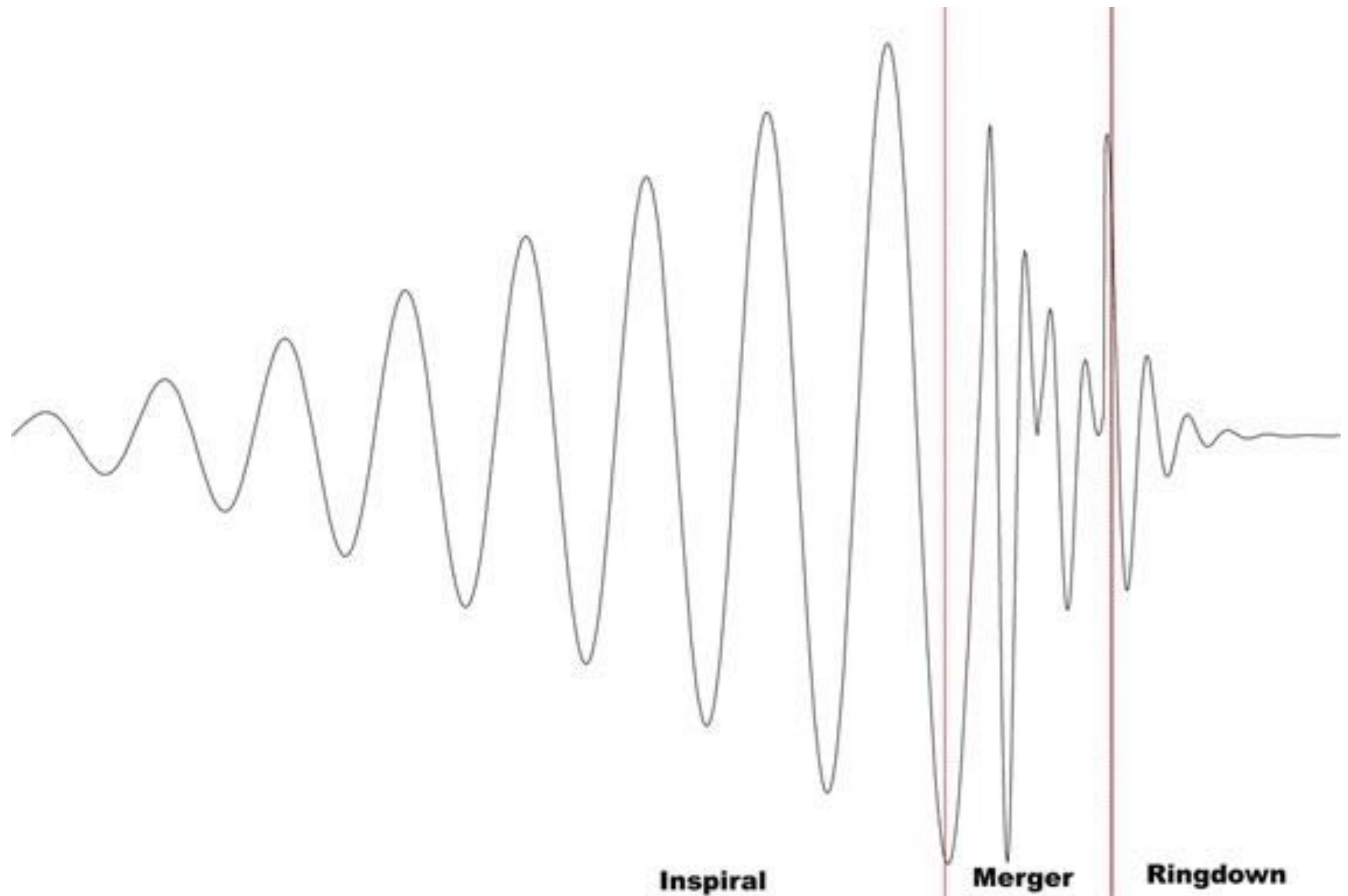
*Credit: G. Nardini (Lisa collaboration), Fudan 2017*

# Gravitational Waves



Credit: G. Nardini (Lisa collaboration), Fudan 2017

# First observations of Gravitational Waves





# Tidal forces and celestial objects

The response of each star to external disturbance is described by the Love numbers

$$Q_{ij} = -k_2 \frac{2R^5}{3G} E_{ij} \equiv -\lambda E_{ij}$$

Induced quadrupole moment

Radius of the star

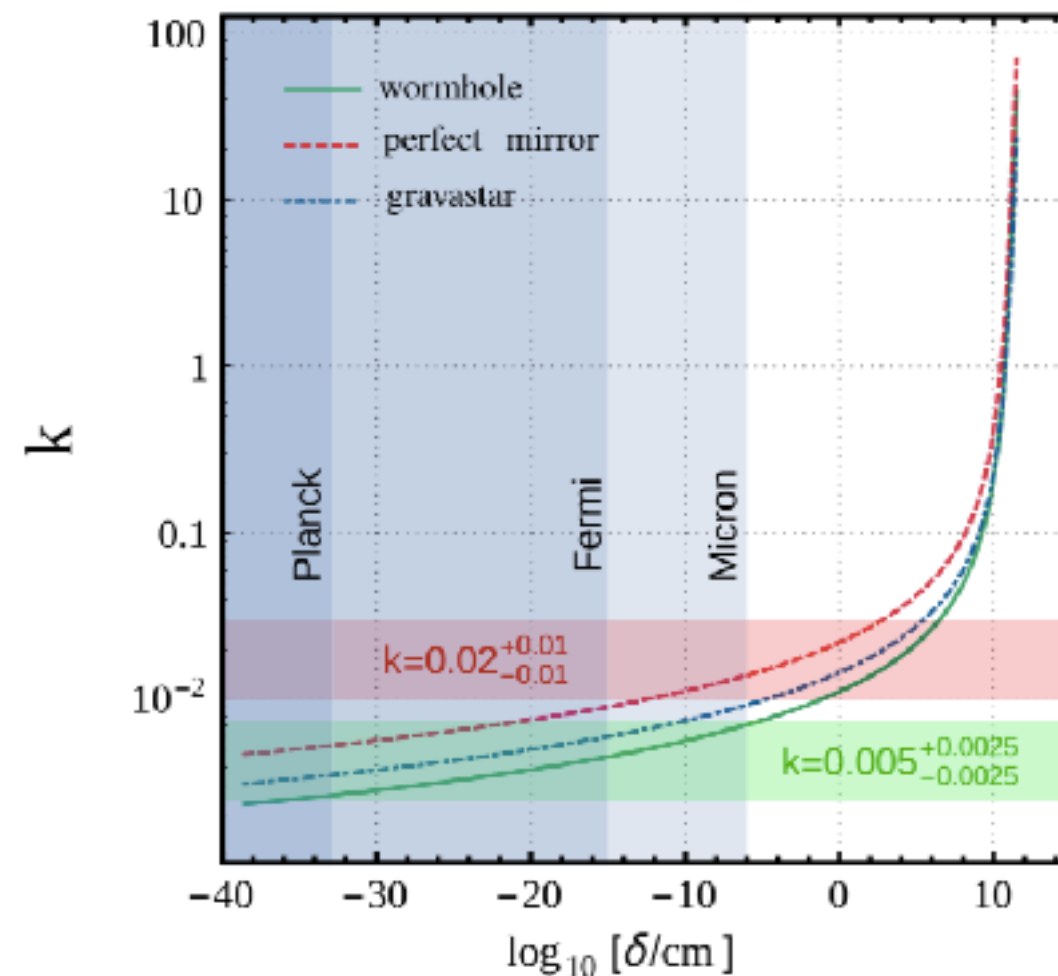
Applied tidal field

# ECOs and their Love numbers

From the Love number of an ECO, can one infer additional near horizon physics?

Maselli et al. , PRL 120, 081101 (2018)

$$\delta = r_0 - r_H = r_H e^{-1/k}$$

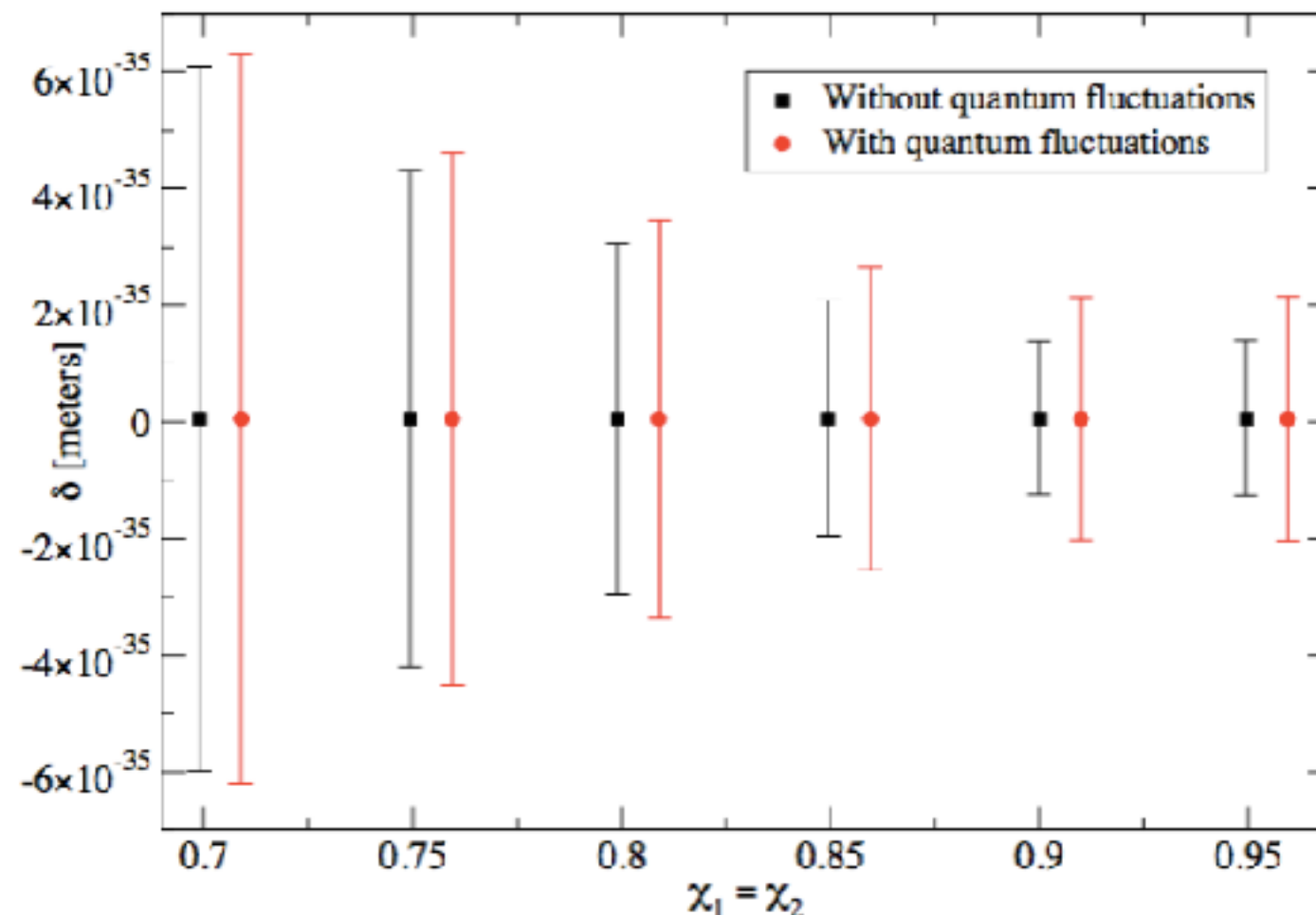


# Encoding quantum fuzziness

Addazi, Marciano & Yunes, arXiv:1810.10417, PRL (2019)

Quantum fluctuations in the signal blur or fuzz out its amplitude and phase

$$\frac{\sigma_{\delta}^{\text{tot}}}{\hat{\delta}} = \sqrt{\left(\frac{\sigma_M^{\text{stat}}}{\hat{M}}\right)^2 + \frac{1}{\hat{k}^2} \left(\frac{\sigma_k^{\text{stat}}}{\hat{k}}\right)^2 + \frac{a^2 \ell_{\text{Pl}}^2}{\hat{\delta}^2}}$$

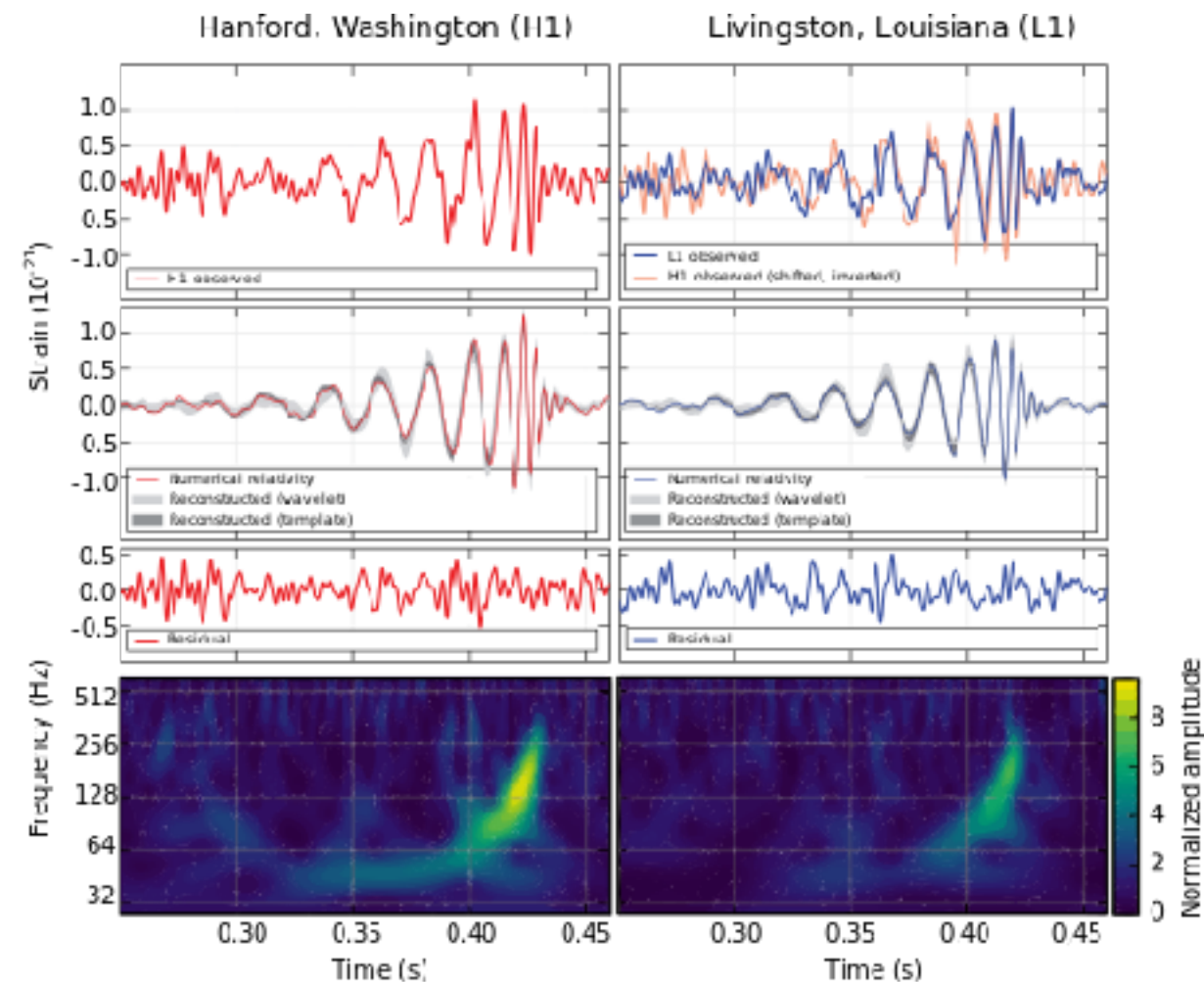


# First observations of Gravitational Waves

GW150914

Distance  $\sim 440$  Mpc

$\sim 3$  solar masses emitted in GW

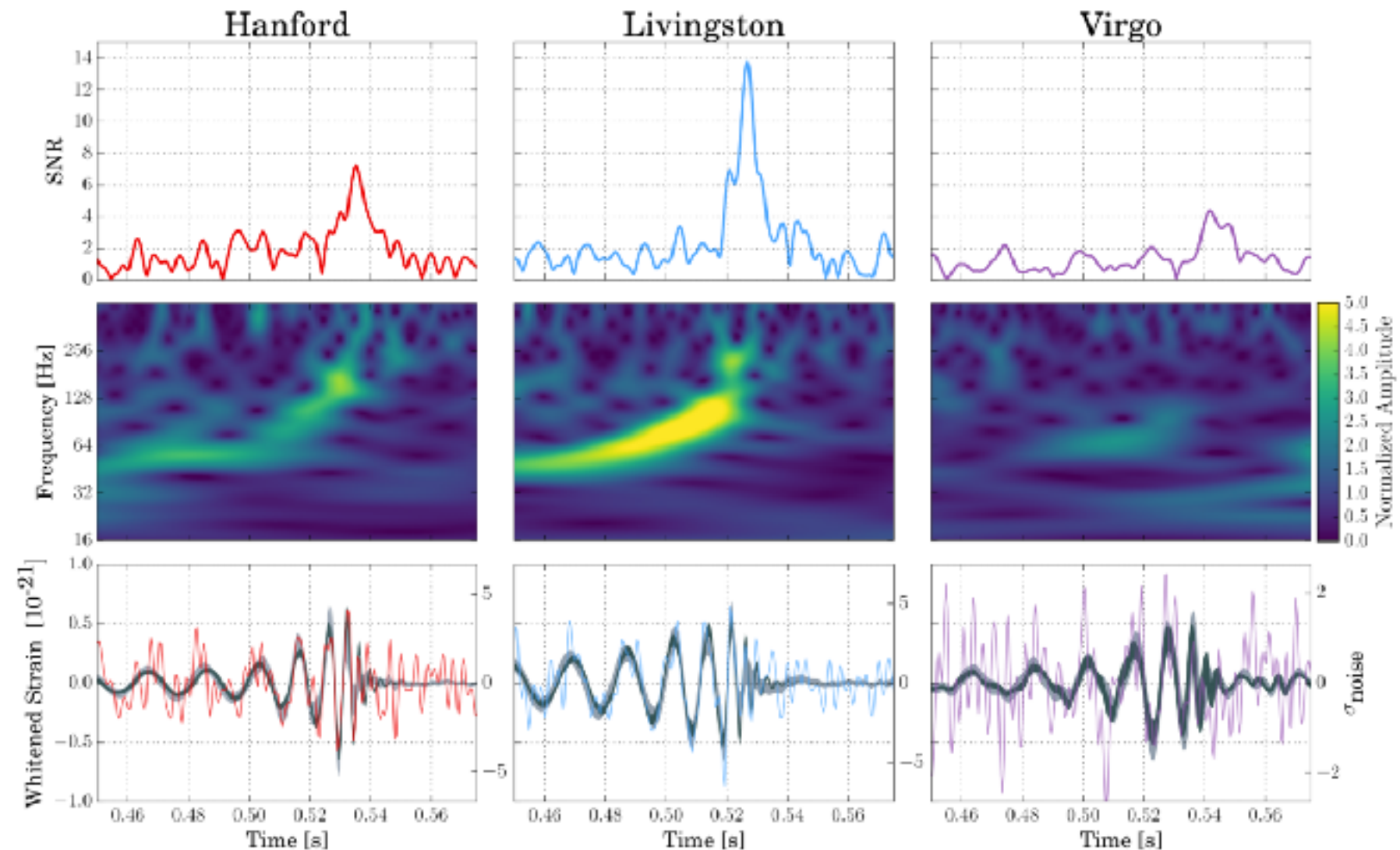
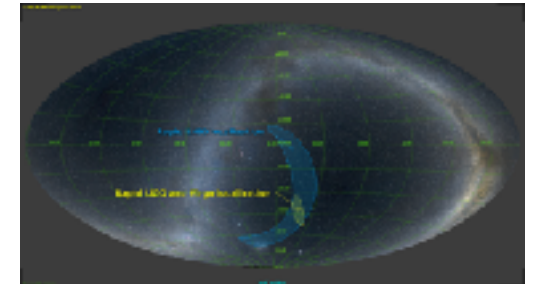


# First observations of Gravitational Waves

GW170814

Distance  $\sim 540$  Mpc

$\sim 3$  solar masses emitted in GW





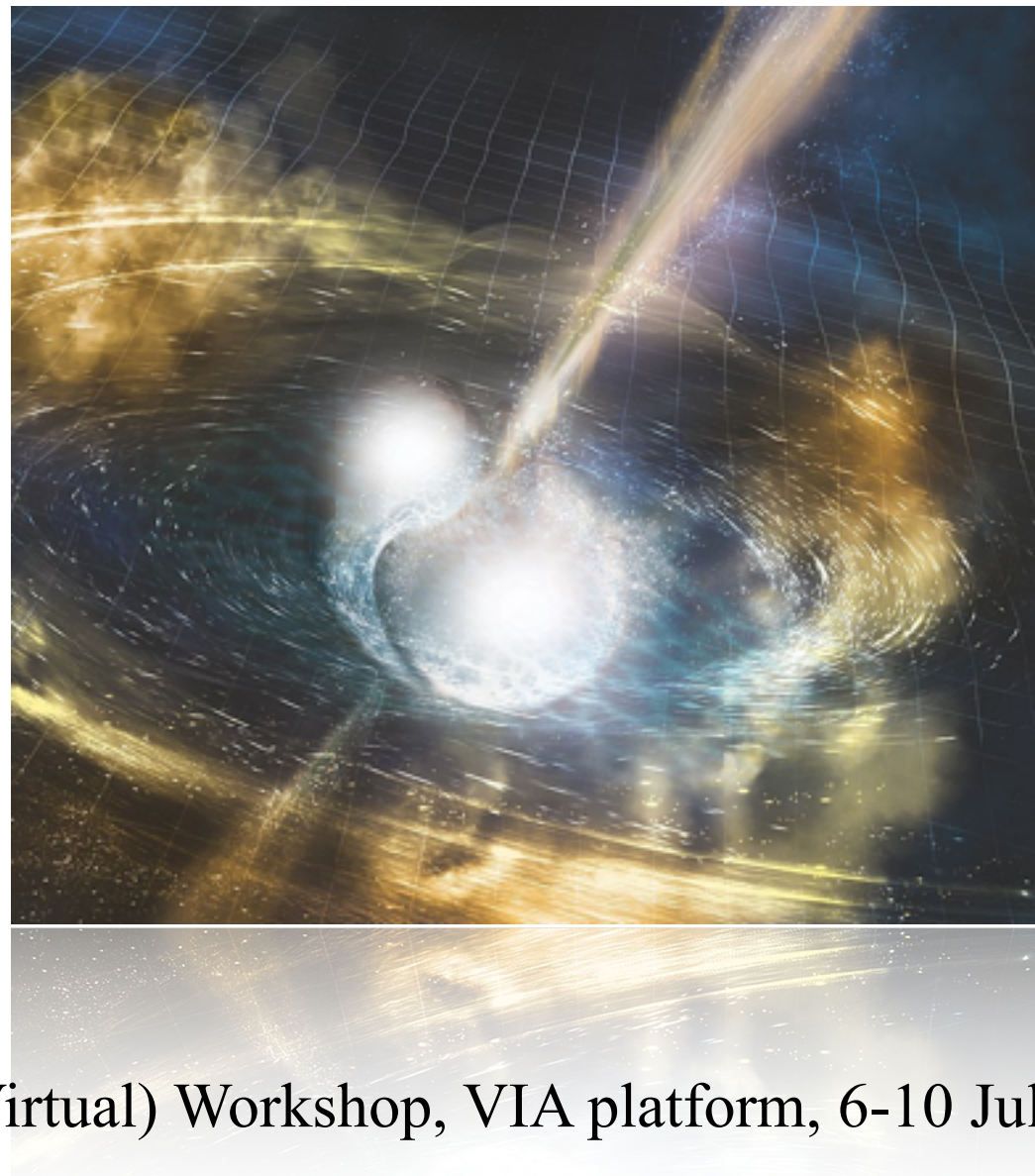
# Multi-messenger perspective for Dark Matter

GW170817

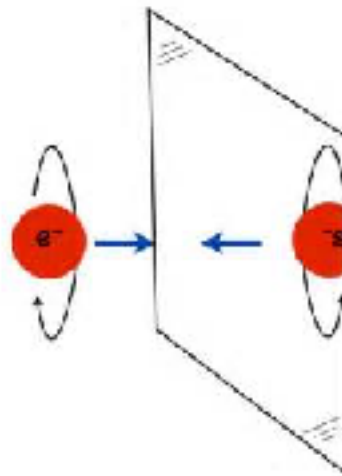
GRB170817A

Distance  $\sim 40$  Mpc

Neutron stars around 1 and 2 solar masses



# Multi-messenger perspective for Dark Matter



A. Addazi & A. Marciano, IJMPA 2018; A. Addazi, R. Ciancarella, F. Pannarale, A. Marciano (in preparation)

XXIII Bled (Virtual) Workshop, VIA platform, 6-10 July 2020

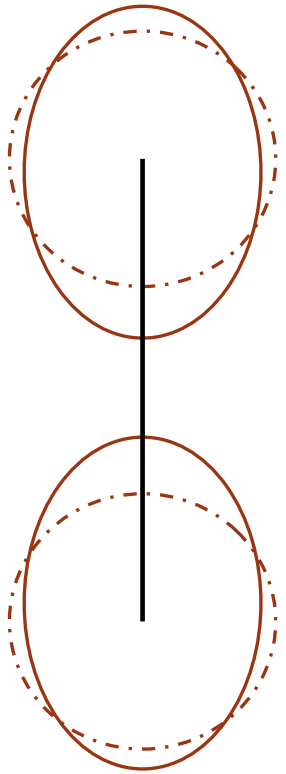
# Equations of State for Neutron Stars

$$Q_{ij} = -\lambda \varepsilon_{ij}$$

*Solve static equilibrium equations (TOV)  
and second order differential equations Hinderer (2010)*

$$\Lambda = \frac{\lambda}{M^5}$$

$$\Lambda_{GW170817} = 190^{+390}_{-120}$$



Measuring NS deformability



matter in density regimes inaccessible on Earth

Pions presence, nontrivial fluidodynamics...



Anisotropic models

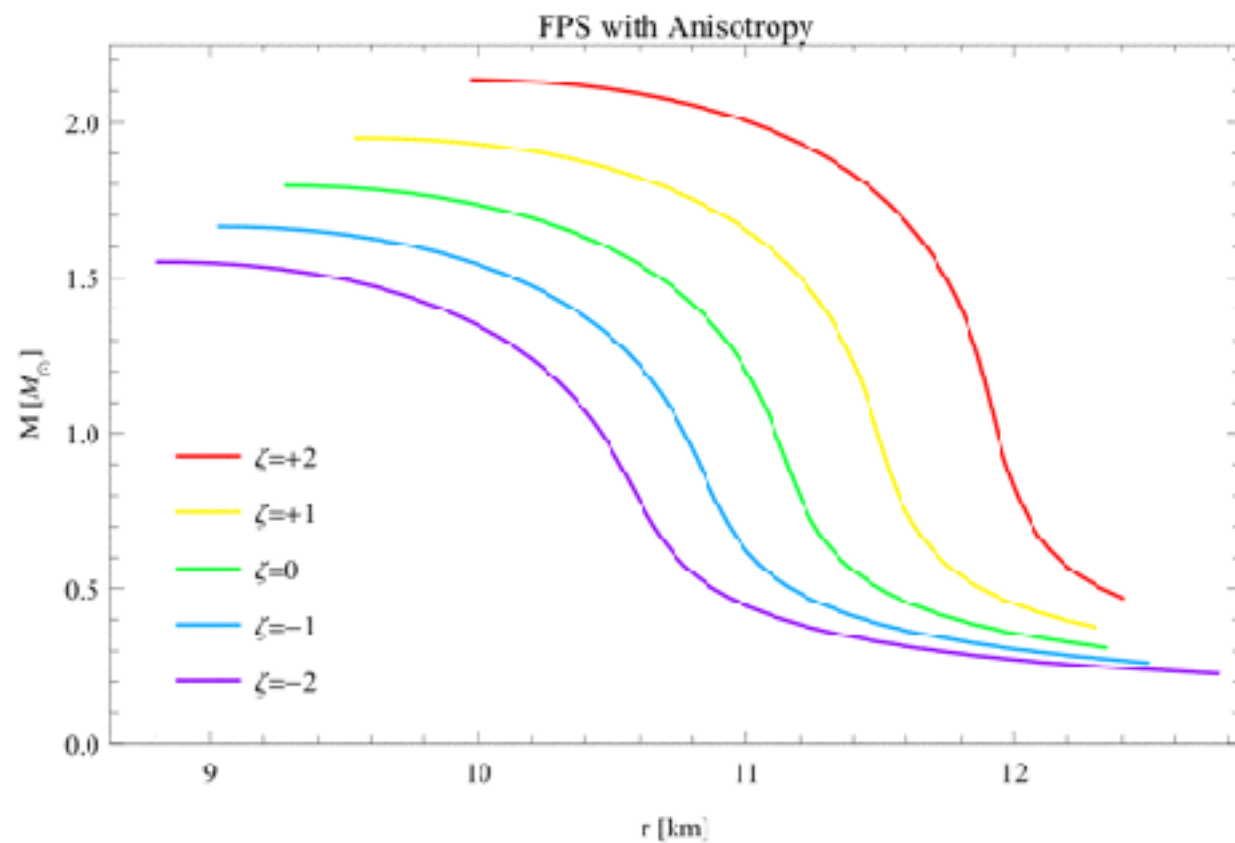
$$p_t(r) = p_r + \frac{\zeta r(\varepsilon - 3p_r)}{3r - 2m(r)} (\varepsilon - p_r)r^2$$

Bowers & Liang 1974

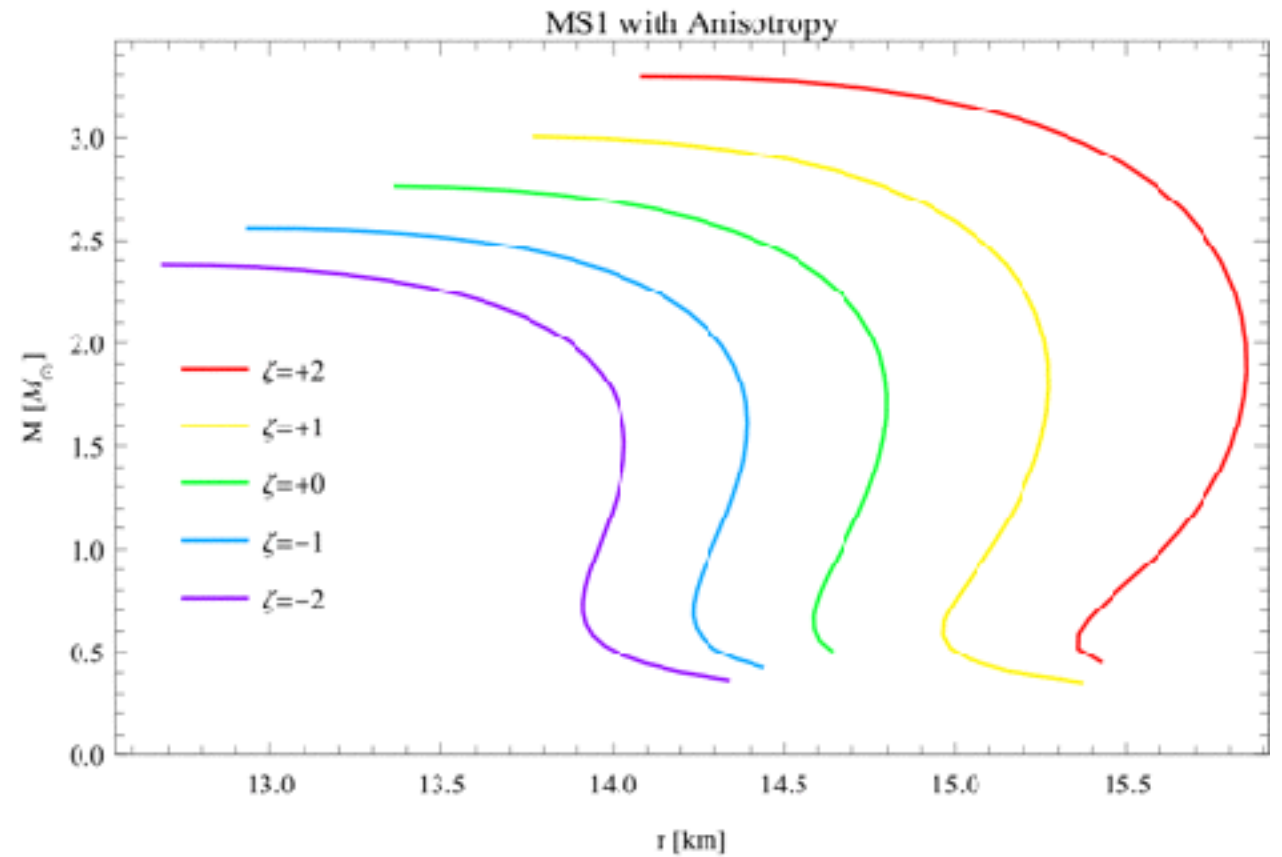


# Role of anisotropies in NS EoS I

A. Addazi, R. Ciancarella, A. Marciano & F. Pannarale

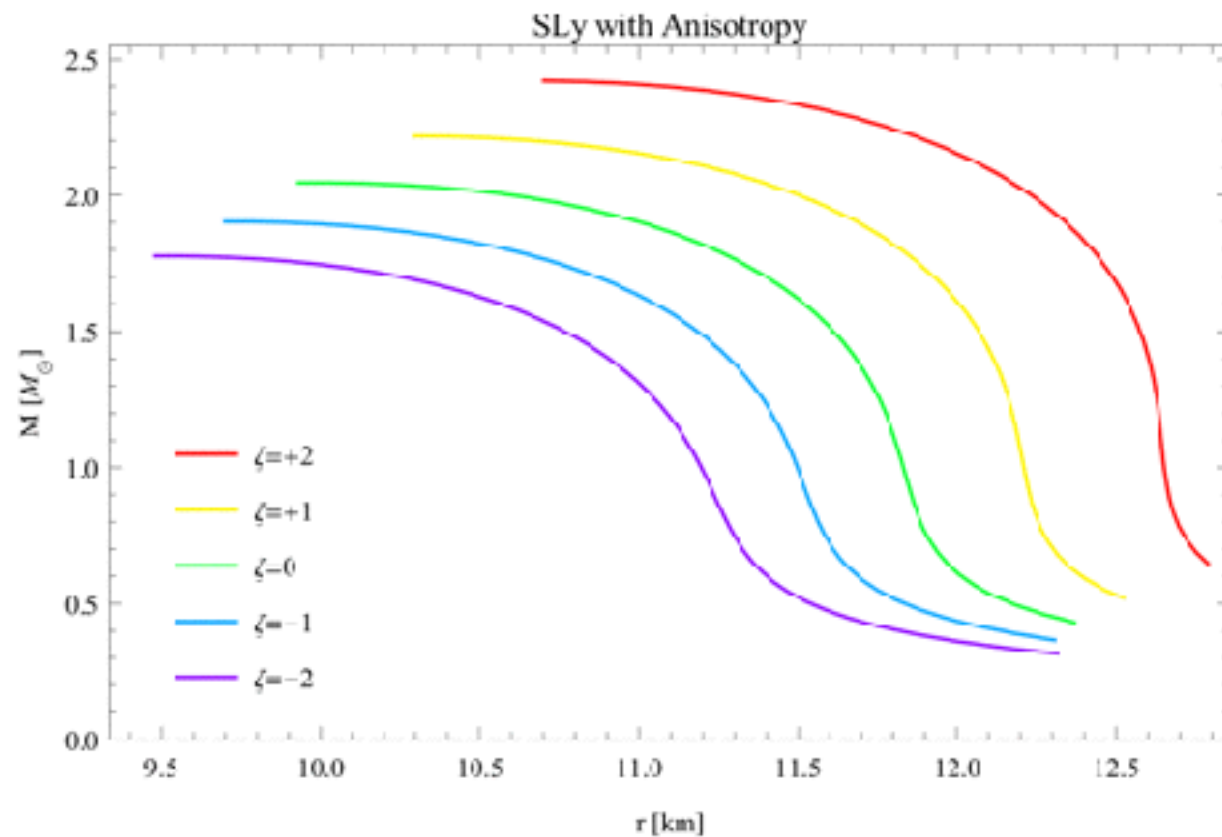


	$\zeta = +2$	$\zeta = +1$	$\zeta = -1$	$\zeta = -2$
R(km)	11.80	11.29	10.36	9.86
C	0.175	0.183	0.200	0.210
$k_2$	0.0863	0.0741	0.0577	0.0511
$\lambda(10^{36} \text{ g cm}^2 \text{ s}^2)$	1.97	1.36	0.69	0.48



	$\zeta = +2$	$\zeta = +1$	$\zeta = -1$	$\zeta = -2$
R(km)	15.78	15.23	14.38	14.03
C	0.131	0.136	0.144	0.147
$k_2$	0.130	0.114	0.0904	0.0842
$\lambda(10^{36} \text{ g cm}^2 \text{ s}^2)$	12.7	9.31	5.55	4.57

# Role of anisotropies in NS EoS II



	SLy			
	$\zeta = +2$	$\zeta = +1$	$\zeta = -1$	$\zeta = -2$
$R(\text{km})$	12.60	12.11	11.28	10.91
$C$	0.164	0.171	0.183	0.190
$k_2$	0.0952	0.0829	0.0663	0.0608
$\lambda(10^{36} \text{ g cm}^2 \text{ s}^2)$	3.02	2.16	1.21	0.93

$$\zeta > 0$$

Maximal mass at fixed central pressure increases.

Tidal deformability increases while compactness decreases.

$$\zeta < 0$$

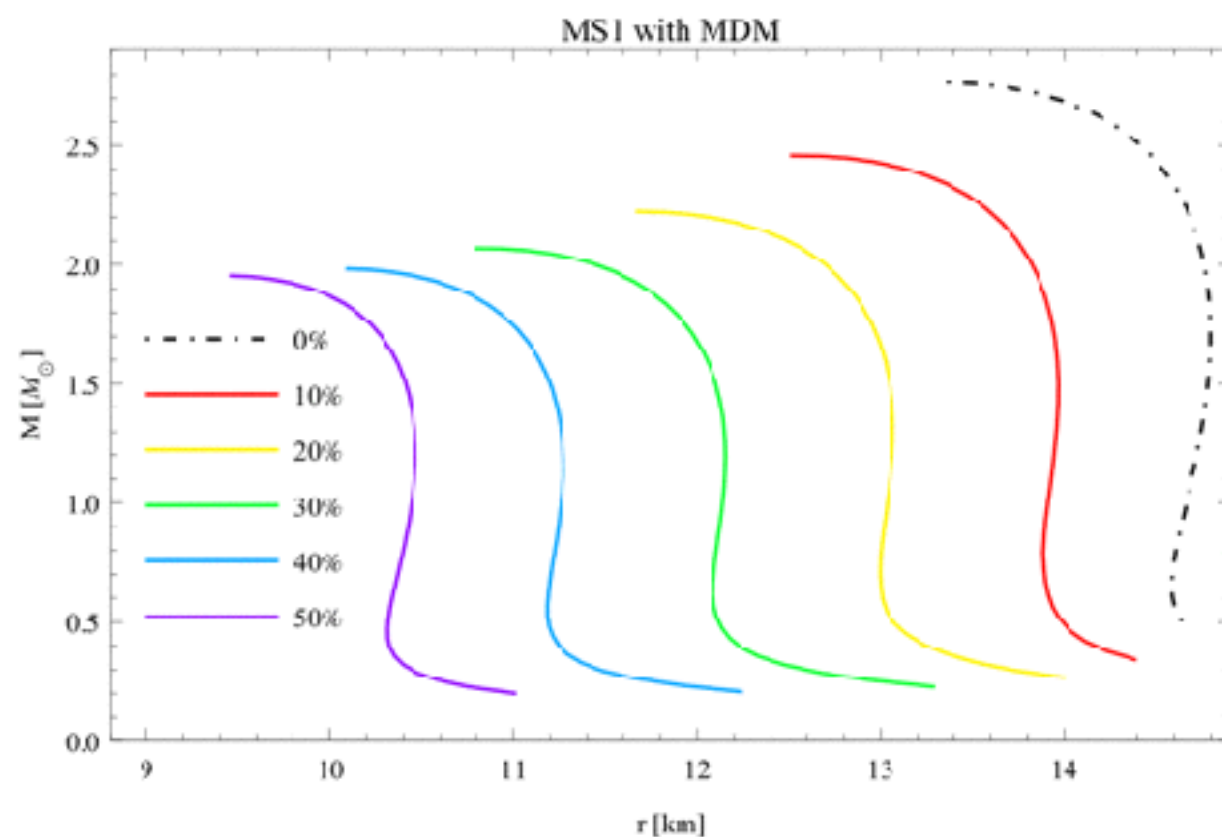
Maximal mass at fixed central pressure decreases.

Tidal deformability decreases while compactness increases.

# Mirror Dark Matter I

Following T.D. Lee & C. Yang (1956), parity, as a global symmetry, might be restored in a dark sector:

- *The Dark Sector as copy of the Standard Model, with opposite chirality*
- *Different nucleosynthesis*
- *Interacting either gravitationally or weakly coupled to EM*



$0% < \text{MDM} < 50\%$

Maximal mass decreases at fixed central pressure.

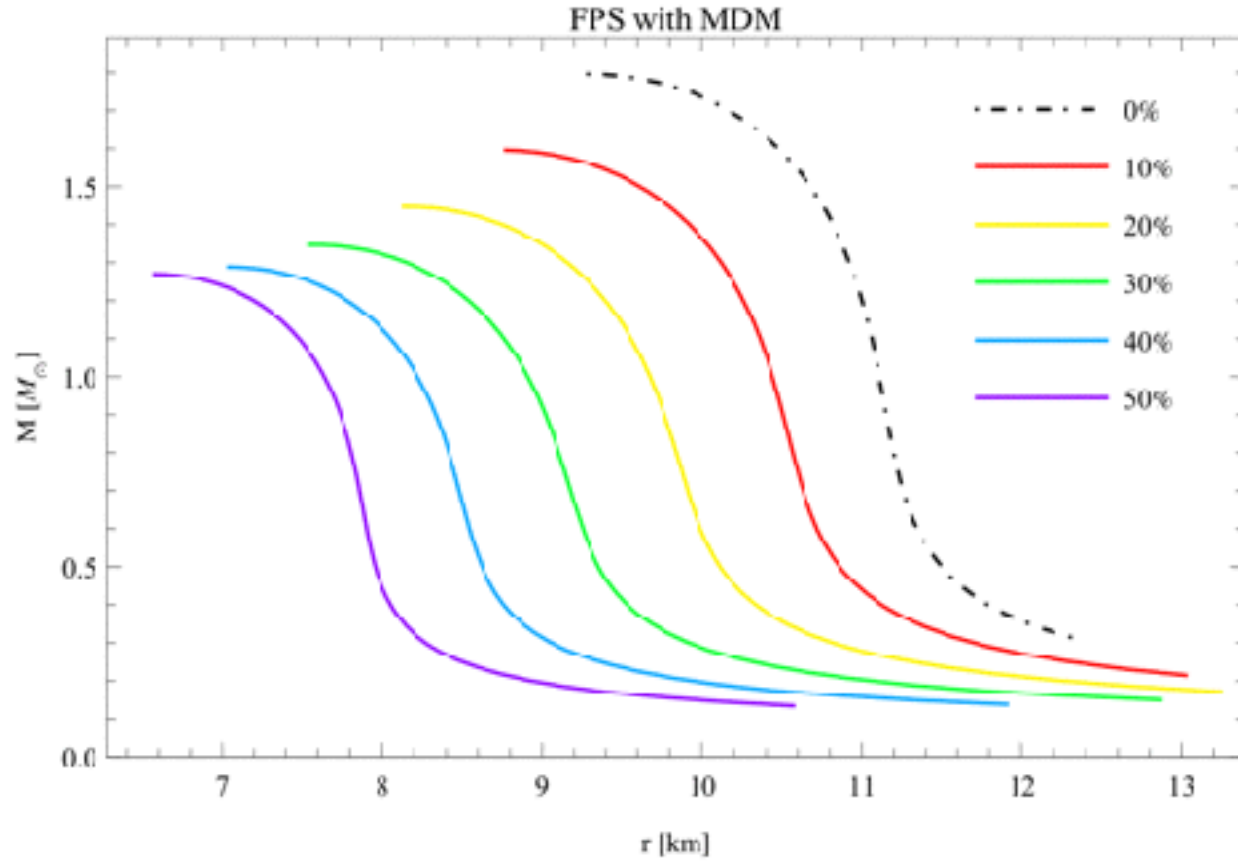
Tidal deformability decreases while compactness increases.

$\text{MDM} > 50\%$

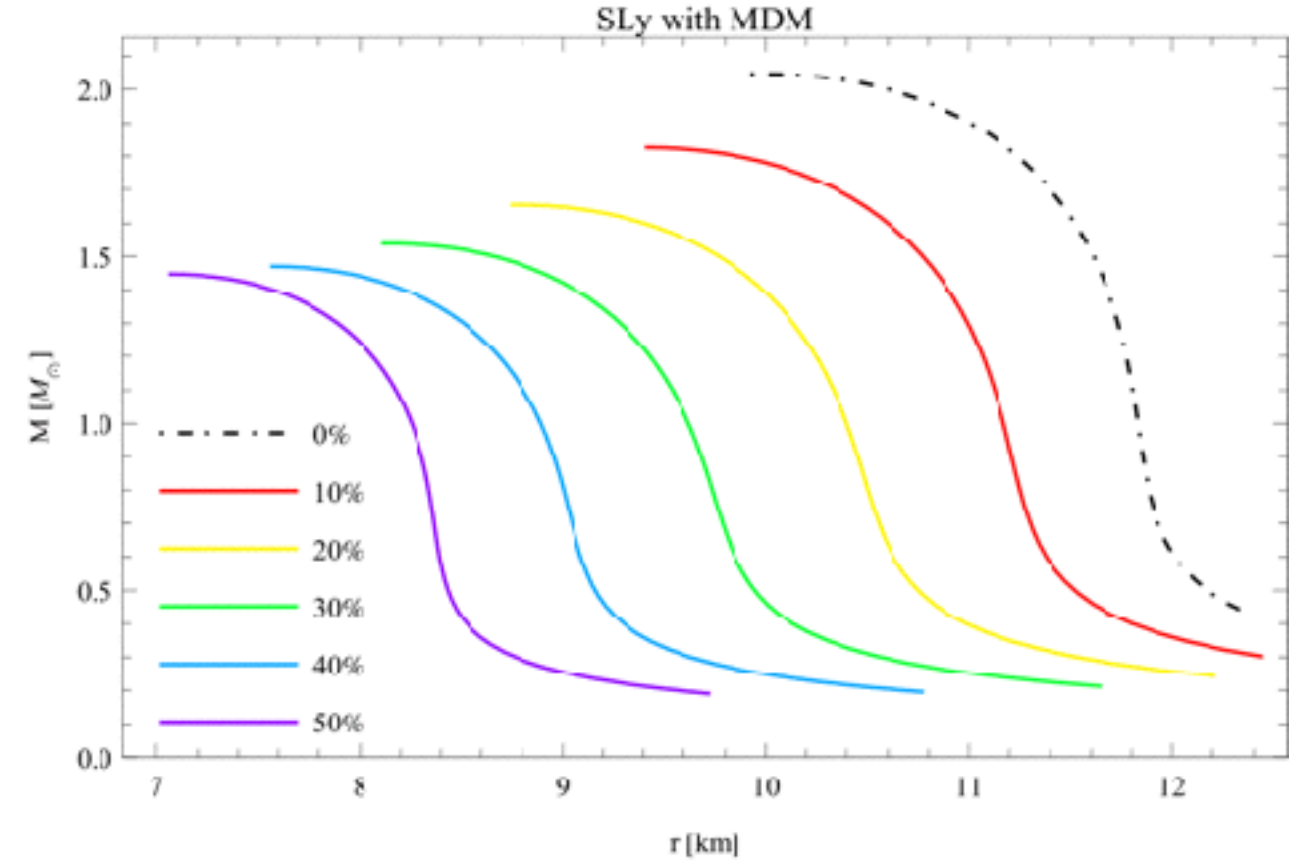
Specular to the case above

	MS1 with MDM				
	10%	20%	30%	40%	50%
R(km)	13.97	13.06	12.13	11.24	10.44
C	0.148	0.158	0.170	0.184	0.198
$k_2$	0.0838	0.0737	0.0647	0.0623	0.0676
$\lambda(10^{36} \text{ g cm}^2 \text{ s}^2)$	4.44	2.80	1.70	1.12	0.839

# Mirror Dark Matter II



FPS with MDM					
	10%	20%	30%	40%	50%
$R$ (km)	10.36	9.56	8.76	8.06	7.47
$C$	0.157	0.170	0.185	0.202	0.217
$k_2$	0.0724	0.0605	0.0535	0.0508	0.0523
$\lambda$ ( $10^{36} \text{ g cm}^2 \text{ s}^2$ )	0.865	0.482	0.276	0.172	0.124



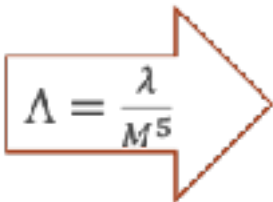
SLy with MDM					
	10%	20%	30%	40%	50%
$R$ (km)	10.90	9.98	9.06	8.22	7.57
$C$	0.190	0.207	0.228	0.251	0.273
$k_2$	0.0600	0.0485	0.0404	0.0352	0.0333
$\lambda$ ( $10^{36} \text{ g cm}^2 \text{ s}^2$ )	0.921	0.481	0.247	0.132	0.0831

# Confronting with GW170817 and PSR J0349+4032

Inferred Mass from PSR  
J0348+4032  
 $M = (1.97 \div 2.05)M_{\odot}$

% of MDM	0%	10%	20%	30%	40%	50%
SLy	<b>282</b>	163	85	44	23	15
MS1	1246	786	<b>495</b>	<b>301</b>	<b>197</b>	148

Tidal deformability from GW170817  
 $\Lambda_{GW170817} = 190^{+390}_{-120}$


$$\Lambda = \frac{\lambda}{M^5}$$

$\zeta$	+2	+1	0	-1	-2
FPS	<b>349</b>	240	172	121	84
SLy	<b>534</b>	<b>381</b>	<b>282</b>	214	165
<del>MS1</del>	<del>2242</del>	<del>1645</del>	<del>1246</del>	<del>979</del>	<del>807</del>

## Necessary condition

Given either the families EoS- $\zeta$  or EoS-MDM, there must be a sequence that satisfies PSR J0348+4032 and a sequence that satisfies GW170817

Within the range assumed:

- a) MS1 is rejected for the anisotropic case
- b) FPS is rejected in the MDM case

# Confronting with GW170817 and PSR J0349+4032

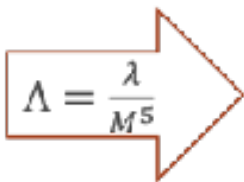
Inferred Mass from PSR  
J0348+4032

$$M = (1.97 \div 2.05)M_{\odot}$$

% of MDM	0%	10%	20%	30%	40%	50%
SLy	<b>282</b>	163	85	44	23	15
MS1	1246	786	<b>495</b>	<b>301</b>	<b>197</b>	148

Tidal deformability from GW170817

$$\Lambda_{GW170817} = 190^{+390}_{-120}$$


$$\Lambda = \frac{\lambda}{M^5}$$

$\zeta$	+2	+1	0	-1	-2
FPS	<b>349</b>	240	172	121	84
SLy	<b>534</b>	<b>381</b>	<b>282</b>	214	165
MS1	2242	1645	1246	979	807

## Sufficient condition

Given either the families EoS- $\zeta$  or EoS-MDM, there must be at least a sequence satisfying at the same time PSR J0348+4032 e GW170817

# Future perspective on NS EoS and (M)DM

## Anisotropies

Several configurations EoS- $\zeta$  satisfy constraints separately. Other satisfy both the constraints

MS1 is rejected in the anisotropic case

FPS must be reconsidered, since it turns out that it can still be valid

## Mirror Dark Matter

FPS is rejected in presence of MDM.

MS1 must be reconsidered, since it turns out that can be still valid.

Recover tidal deformability for different EoS

Implement different model of dark matter

Develop template for wave-forms

Confrontation with the EM channel!

*GW generated by FOPT*



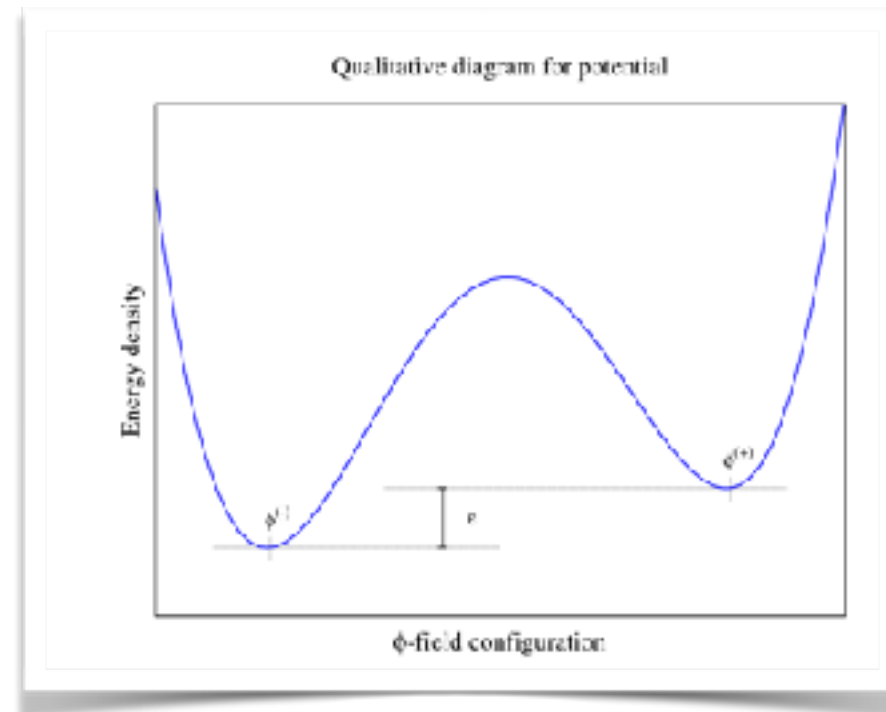
# Gravitational Waves Stochastic Background

Signal from unresolved astrophysical sources

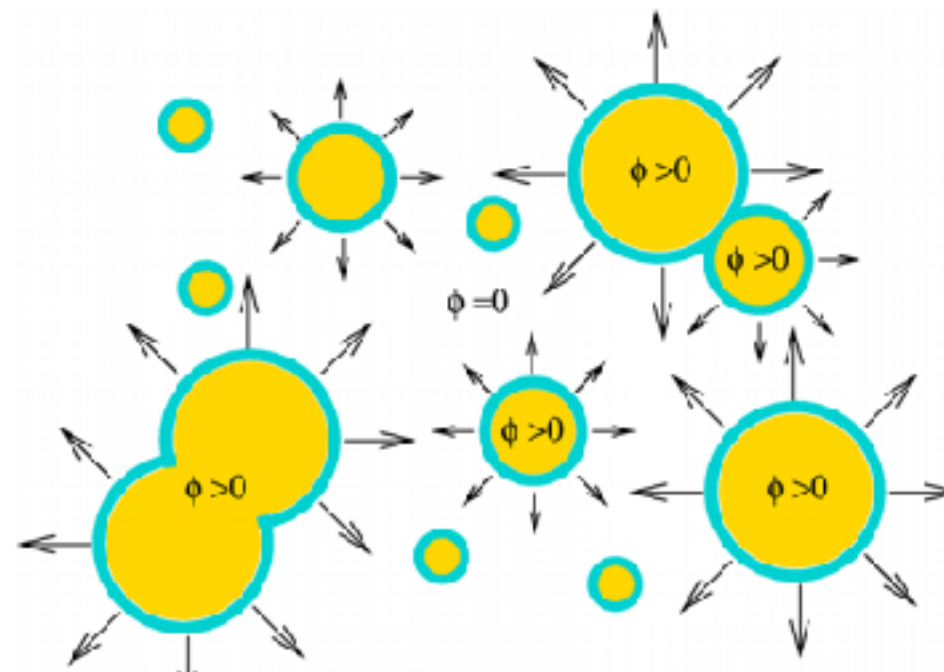
Signal from cosmological events

- i) Early cosmology (inflation, bouncing cosmologies, string gas cosmology etc...)
- ii) Cosmic strings
- iii) Strong Cosmological Phase Transitions

# Tunnelling and bubbles enucleation



Coleman, Frampton etc...



XXIII Bled (Virtual) Workshop, VIA platform, 6-10 July 2020

# Latent energy parameter

Normalized difference between minima

$$\mathcal{E}(\bar{T}) = \left[ T \frac{dV_{eff}}{dT} - V_{eff}(T) \right]_{T=\bar{T}}$$

$$\alpha = \frac{\mathcal{E}(\bar{T})}{\rho_{rad}(\bar{T})} \qquad \rho_{rad} = \frac{\pi^2}{30} g_*(T) T^4$$

**Latent Energy**

# Bubble nucleation parameter

How fast the minimum goes down

$$\beta = - \left[ \frac{dS_E}{dt} \right]_{t=\bar{t}} \simeq \left[ \frac{1}{\Gamma} \frac{d\Gamma}{dt} \right]_{t=\bar{t}},$$

$$S_E(T) \simeq \frac{S_3(T)}{T} \quad \Gamma = \Gamma_0(T) \exp[-S_E(T)]$$

$$\Gamma_0(T) \sim T^4, \quad S_3 \equiv \int d^3r \left( \partial_i s^\dagger \partial_i s + V_{eff}(s, T) \right)$$

*$\beta/H$  provides an inverse time scale*

# Effective action

Relation between size of the bubble wall and bubble velocity

$$d \simeq \frac{V_B}{\beta}$$

Effective potential

$$V_{tree}(s, T = 0) + V_1(s, T)$$

$$V_1(s, T) = V_{CW}(s, T = 0) + \Delta V(s, T)$$

# Bubbles collision

$$\nu_{collision} \simeq 3.5 \times 10^{-4} \left( \frac{\beta}{H_*} \right) \left( \frac{\bar{T}}{10 \text{ GeV}} \right) \left( \frac{g_*(\bar{T})}{10} \right)^{1/6} \text{ mHz}$$

frequency is proportional to temperature

$$\Omega_{collision}(\nu_{collision}) \simeq C \mathcal{E}^2 \left( \frac{\bar{H}}{\beta} \right)^2 \left( \frac{\alpha}{1 + \alpha} \right)^2 \left( \frac{V_B^3}{0.24 + V_B^3} \right) \left( \frac{10}{g_*(\bar{T})} \right)$$

$$C \simeq 2.4 \times 10^{-6}$$

corresponding intensity

$h^2 \Omega_{col}$  dominates for large wall velocities  $v_b \rightarrow 1$

$$h^2 \Omega(f; \alpha, \beta/H, f_{\text{peak}}) \quad f_{\text{peak}}(\alpha, \beta/H, T_n)$$

# Shock waves and turbulence

$$f_{\text{SW}}[\text{Hz}] = 1.9 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6}$$

$$f_{\text{MHD}}[\text{Hz}] = 2.7 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \left( \frac{T_n}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6}$$

frequency is proportional to temperature

$$h^2 \Omega_{\text{SW}}(f) = 2.65 \times 10^{-6} \left( \frac{\beta}{H} \right)^{-1} \left( \frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left( \frac{g_*}{100} \right)^{-1/3} v_b \left( \frac{f}{f_{\text{SW}}} \right)^3 \left( \frac{7}{4 + 3(f/f_{\text{SW}})^2} \right)^{7/2}$$

$$h^2 \Omega_{\text{MHD}}(f) = 3.35 \times 10^{-4} \left( \frac{\beta}{H} \right)^{-1} \left( \frac{\kappa_{\text{turb}} \alpha}{1 + \alpha} \right)^{3/2} \left( \frac{g_*}{100} \right)^{-1/3} v_b \left( \frac{f}{f_{\text{MHD}}} \right)^3 \left( \frac{(1 + f/f_{\text{MHD}})^{-11/3}}{1 + 8\pi f/h_*} \right)$$

corresponding intensity

# Velocity enhancement

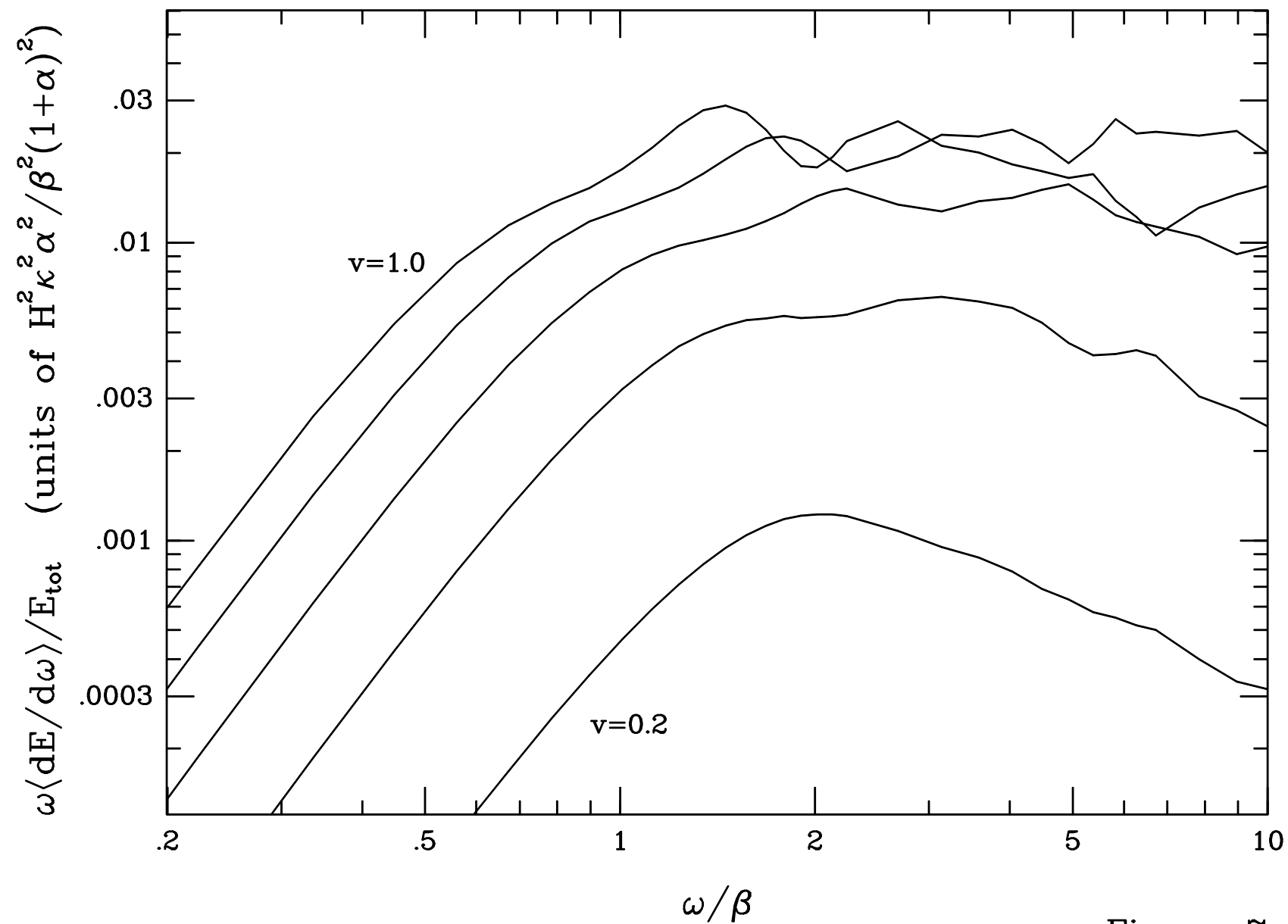


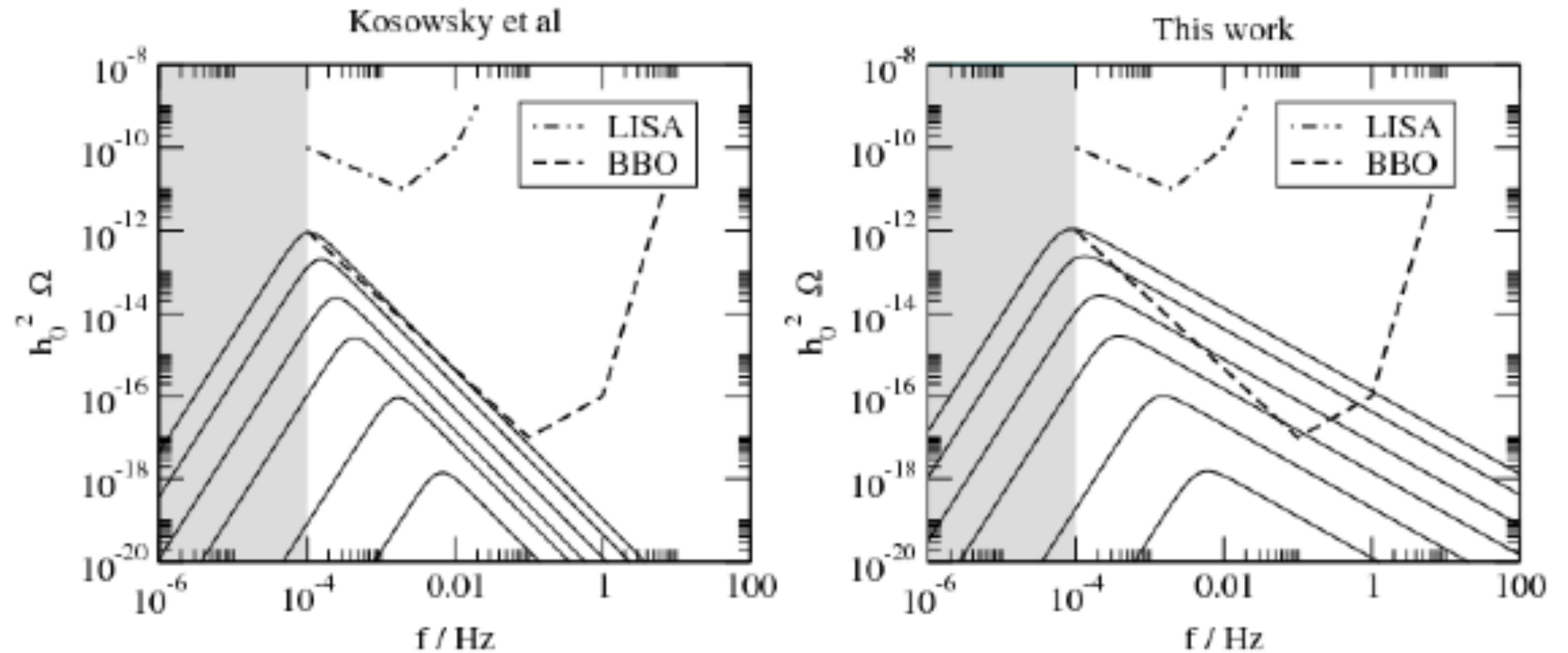
Figure 7

Kamionkowski, Kosowsky, Turner 1994

*Credit: A. Kosowsky, Fudan Spring School of Cosmology 2017*



# Comparison with MHD turbulence



S. Huber, T. Konstandin 2008

# Criteria for phase transitions I

Vacuum bubbles nucleated from first order phase transitions (FOPT)

Three sources of GW production: 1) collision, 2) sound waves and 3) plasma turbulence

$h^2\Omega_{\text{col}}$  dominates for large wall velocities  $v_b \rightarrow 1$

$$h^2\Omega(f; \alpha, \beta/H, f_{\text{peak}}) \quad f_{\text{peak}}(\alpha, \beta/H, T_n)$$

$$\alpha \propto \frac{1}{T_n^4} \left[ V_i - V_f - T \left( \frac{\partial V_i}{\partial T} - \frac{\partial V_f}{\partial T} \right) \right] \quad \frac{\beta}{H} = T_n \frac{\partial}{\partial T} \left( \frac{\hat{S}_3}{T} \right) \Big|_{T_n}$$

# Criteria for phase transitions II

Bubble nucleation arises when the probability to realize 1 transition per cosmological horizon is equal to one:  $\frac{\Gamma}{H^4} \sim 1 \Rightarrow \frac{\hat{S}_3}{T_n} \sim 140$

Strong transition criterion:  $\frac{v_h(T_n)}{T_n} \geq 1 \Rightarrow$  enhances GW production

Classical motion in Euclidean space described by action  $\hat{S}^3$

$$\hat{S}_3 = 4\pi \int_0^\infty dr r^2 \left\{ \frac{1}{2} \left( \frac{d\hat{\phi}}{dr} \right)^2 + V_{\text{eff}}(\hat{\phi}, T) \right\}$$

$$V_{\text{eff}}^{(1)}(\hat{\phi}, T) = V_0 + V_{\text{CW}} + \Delta V^{(1)}(T)$$

solution of the e.o.m. found by the path that minimizes the energy

Implementation via CosmoTransitions [\[Wainwright '12\]](#)

# Dynamics of phase transitions

$$\Gamma \sim T^4 \left( \frac{\hat{S}_3}{2\pi T} \right)^{3/2} \exp \left( -\frac{\hat{S}_3}{T} \right)$$

High  $T \Rightarrow$  classical motion in Euclidean space described by the action

$$\hat{S}_3 = 4\pi \int_0^\infty dr r^2 \left\{ \frac{1}{2} \left( \frac{d\hat{\phi}}{dr} \right)^2 + V_{\text{eff}}(\hat{\phi}) \right\}$$

Field configuration as solutions to the e.o.m. found by the path that minimizes the energy

$$\begin{aligned} V_{\text{eff}}^{(1)}(\hat{\Phi}) &= V_{\text{tree}} + V_{\text{CW}} + \Delta V^{(1)}(T) \\ V_{\text{CW}} &= \sum_i (-1)^F n_i \frac{m_i^4}{64\pi^2} \left( \log \left[ \frac{m_i^2(\hat{\Phi}_\alpha)}{\Lambda^2} \right] - c_i \right) \\ \Delta V^{(1)}(T) &= \frac{T^4}{2\pi^2} \left\{ \sum_b n_b J_B \left[ \frac{m_b^2(\hat{\Phi}_\alpha)}{T^2} \right] - \sum_f n_f J_F \left[ \frac{m_f^2(\hat{\Phi}_\alpha)}{T^2} \right] \right\} \end{aligned}$$

Loop and thermal corrections in then effective potential

*Time-crystal ground state and production of GW from QCD phase transitions*

# Some features of QCD

**Different domains:** from neutron stars equation of states and mergings, to colliders physics (proton-proton and heavy ions collisions)

**SSB & dimensional trans-mutation:** SSB, chiral anomaly and strong CP problem, and anomaly of the conformal symmetry and emergence of the QCD scale


**Un-trivial ground states:** inhomogeneous vacuum state with emergence of structure below the Fermi scale

**Dynamics of the ground state:** the appearance of condensate dynamically breaks the Lorentz symmetry

**Confinement and the ground state dynamics:** can we gain inspiration and suggestion from cosmology to progress towards the resolution of this problem?

# Time-crystals I

**Time-crystals:** non-isolated matter, in non-equilibrium dynamics, shows periodic patterns not only in space, but also in time (Wilczek '12)

PRL 109, 160401 (2012)  Selected for a **Viewpoint** in *Physics*  
PHYSICAL REVIEW LETTERS week ending  
19 OCTOBER 2012

---

## Quantum Time Crystals

Frank Wilczek  
*Center for Theoretical Physics Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*  
(Received 29 March 2012; published 15 October 2012)

Some subtleties and apparent difficulties associated with the notion of spontaneous breaking of time-translation symmetry in quantum mechanics are identified and resolved. A model exhibiting that phenomenon is displayed. The possibility and significance of breaking of imaginary time-translation symmetry is discussed.

DOI: 10.1103/PhysRevLett.109.160401 PACS numbers: 11.30.-j, 03.75.Lm, 05.45.Xt





# Time Crystals II

*“In a delicate balance between strong interactions, weak disorder, and a periodic driving force, a collection of trapped ions qubits has been made to pulsate with a period that is relatively insensitive to the drive. This is a **time crystal**, where the stable pulses emerge and break time symmetry – just like a freezing liquid breaks spatial symmetry and forms a spatial crystal. Trapped ion qubits can pulsate on their own with excellent passive stability, but this observation may guide the stabilization of complex solid-state systems, where true quantum behavior is usually masked by defects and impurities.”*

In 2016 Monroe's group at Maryland University trapped a chain  $171\text{Yb}^+$  (Ytterbium) ions in a Paul trap, a radio frequency device that uses e.m. dynamic to trap charged particles.

Among the two spin states, one is selected by a pair of laser beams. The shape of the lasers' pulse was controlled by an acousto-optic modulator.

A Tukey window was used to avoid that a critical amount of energy would have been peaked at the resonant optical frequency.

# Time Crystals III

The hyperfine electron states in the setup used have very close energy levels, which are separated by 12.642831 GHz.

Finally, ten Doppler-cooled ions are placed in a line 25  $\mu\text{m}$  long, and are coupled together.

Researchers observed subharmonic oscillations of the drive, with “rigidity” of the time crystal, and its oscillation frequency unchanged even under perturbation of the time crystal.

Once the perturbation or the frequency of the vibration grows too strong, the time crystal “melts”.

# Time-crystals and the Dynamical Universe

Time-crystals cannot be in equilibrium

Matter in non-equilibrium in its ground state!

Reminiscent of QCD, but can it be realized in the Universe?

Can a gluon condensate form, because of the space-time backreaction,  
which shows a time-crystal behavior?

This traces back to question about QCD relaxation phenomena that are related

# Coupled YM-Einstein equations I

Incorporate the conformal anomaly via the variational procedure

The gauge coupling  $g_{\text{YM}}$  should acquire a dependence on the quantum fields (RG eqs.)

$$\mathcal{L}_{\text{eff}} = \frac{\mathcal{J}}{4\bar{g}^2}, \quad \bar{g}^2 = \bar{g}^2(\mathcal{J}), \quad \mathcal{J} = -\frac{\mathcal{F}_{\mu\nu}^a \mathcal{F}_a^{\mu\nu}}{\sqrt{-g}}$$

$$g_{\mu\nu} = a(\eta)^2 \text{diag}(1, -1, -1, -1)$$

$$\mathcal{J} = \frac{2}{\sqrt{-g}} \sum_a (\mathbf{E}_a \cdot \mathbf{E}_a - \mathbf{B}_a \cdot \mathbf{B}_a) \equiv \frac{2}{\sqrt{-g}} (\mathbf{E}^2 - \mathbf{B}^2)$$

Chromoelectric (chromomagnetic) contributions contribute with positive (negative) sign to the energy density

# Coupled YM-Einstein equations II

A.Addazi, A. Marciano, R. Pasechnik & G. Prokorov, EPJC '19

The gauge coupling satisfies the RG equation

$$2\mathcal{J} \frac{d\bar{g}^2}{d\mathcal{J}} = \bar{g}^2 \beta$$

Starting from the effective action, one recover at the all-loop the effective YM eom, supplemented by the RG equation

$$\vec{\mathcal{D}}_{\nu}^{ab} \left[ \frac{\mathcal{F}_b^{\mu\nu}}{\bar{g}^2 \sqrt{-g}} \left( 1 - \frac{\beta(\bar{g}^2)}{2} \right) \right] = 0 \qquad \frac{d \ln |\bar{g}^2|}{d \ln |\mathcal{J}|/\mu_0^4} = \frac{\beta(\bar{g}^2)}{2}$$

$$\vec{\mathcal{D}}_{\nu}^{ab} \equiv \left( \delta^{ab} \frac{\vec{\partial}_{\nu} \sqrt{-g}}{\sqrt{-g}} - f^{abc} \mathcal{A}_{\nu}^c \right)$$

# Coupled YM-Einstein equations III

A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC '19

EMT for the Savvidy's theory

$$T_{\mu}^{\nu} = \frac{1}{\bar{g}^2} \left[ \frac{\beta(\bar{g}^2)}{2} - 1 \right] \left( \frac{\mathcal{F}_{\mu\lambda}^a \mathcal{F}_a^{\nu\lambda}}{\sqrt{-g}} + \frac{1}{4} \delta_{\mu}^{\nu} \mathcal{J} \right) - \delta_{\mu}^{\nu} \frac{\beta(\bar{g}^2)}{8\bar{g}^2} \mathcal{J}$$

Chromoelectric vacuum contributions to EMT

$$T_{\mu}^{\mu} = -\frac{\beta(\bar{g}_*^2)}{2\bar{g}_*^2} \mathcal{J}^* = -\frac{1}{\bar{g}_*^2} \mathcal{J}^*$$

Chromomagnetic vacuum contributions to EMT

$$T_{\mu}^{\nu} = \frac{-2}{\bar{g}^2} \left( \frac{\mathcal{F}_{\mu\lambda}^a \mathcal{F}_a^{\nu\lambda}}{\sqrt{-g}} + \frac{1}{4} \delta_{\mu}^{\nu} \mathcal{J}^* \right) - \delta_{\mu}^{\nu} \frac{\beta(\bar{g}_*^2)}{8\bar{g}_*^2} \mathcal{J}^* \longrightarrow T_{\mu}^{\mu} = \frac{1}{\bar{g}_*^2} \mathcal{J}^*$$

# Coupled YM-Einstein equations IV

A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC '19

The coupled YM-Einstein equations read

$$\begin{aligned} \frac{1}{\kappa} \left( R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R \right) &= \bar{\epsilon} \delta^\nu_\mu + \frac{b}{32\pi^2} \frac{1}{\sqrt{-g}} \left[ \left( -\mathcal{F}^a_{\mu\lambda} \mathcal{F}^{\nu\lambda}_a \right. \right. \\ &\quad \left. \left. + \frac{1}{4} \delta^\nu_\mu \mathcal{F}^a_{\sigma\lambda} \mathcal{F}^{\sigma\lambda}_a \right) \ln \frac{e |\mathcal{F}^a_{\alpha\beta} \mathcal{F}^{\alpha\beta}_a|}{\sqrt{-g} \lambda^4} - \frac{1}{4} \delta^\nu_\mu \mathcal{F}^a_{\sigma\lambda} \mathcal{F}^{\sigma\lambda}_a \right], \\ \left( \frac{\delta^{ab}}{\sqrt{-g}} \vec{\partial}_\nu \sqrt{-g} - f^{abc} \mathcal{A}_\nu^c \right) &\left( \frac{\mathcal{F}^{\mu\nu}_b}{\sqrt{-g}} \ln \frac{e |\mathcal{F}^a_{\alpha\beta} \mathcal{F}^{\alpha\beta}_a|}{\sqrt{-g} \lambda^4} \right) = 0 \end{aligned}$$

On FLRW backgrounds

$$\begin{aligned} \frac{3}{\kappa} \frac{(a')^2}{a^4} &= \bar{\epsilon} + T_0^{0,U}, \\ T_0^{0,U} &= \frac{3b}{64\pi^2 a^4} \left( \left[ (U')^2 + \frac{1}{4} U^4 \right] \ln \frac{6e |(U')^2 - \frac{1}{4} U^4|}{a^4 \lambda^4} \right. \\ &\quad \left. + (U')^2 - \frac{1}{4} U^4 \right). \end{aligned}$$

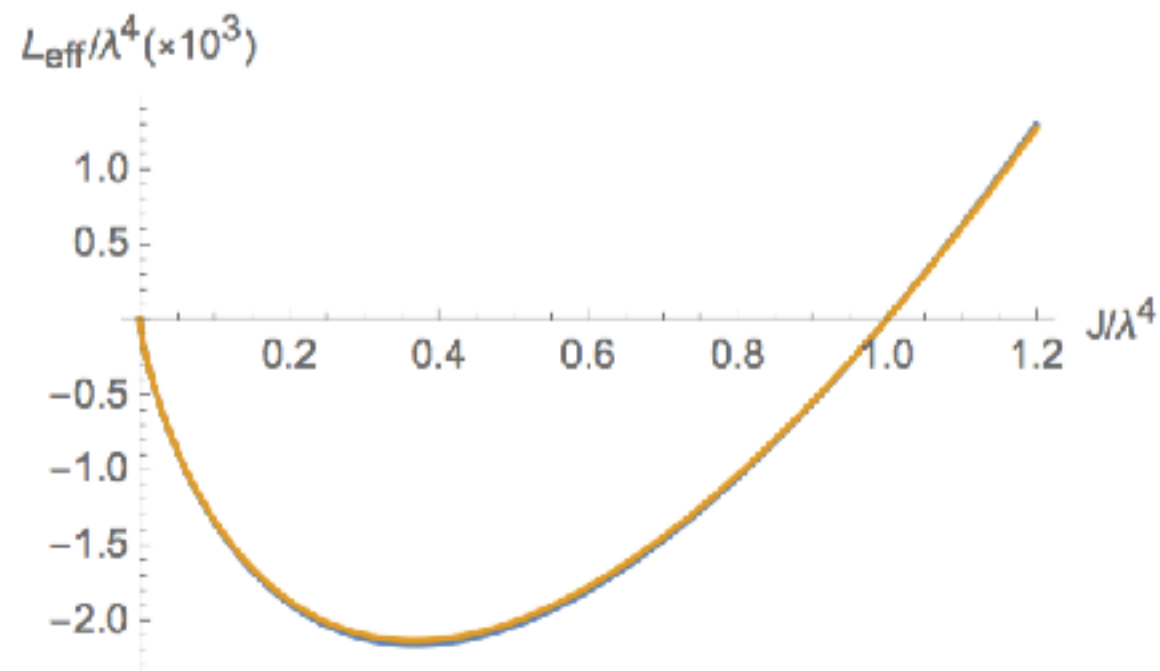
$$\begin{aligned} \frac{6}{\kappa} \frac{a''}{a^3} &= 4\bar{\epsilon} + T_\mu^{\mu,U}, \\ T_\mu^{\mu,U} &= \frac{3b}{16\pi^2 a^4} \left[ (U')^2 - \frac{1}{4} U^4 \right], \\ \frac{\partial}{\partial \eta} \left( U' \ln \frac{6e |(U')^2 - \frac{1}{4} U^4|}{a^4 \lambda^4} \right) &+ \frac{1}{2} U^3 \ln \frac{6e |(U')^2 - \frac{1}{4} U^4|}{a^4 \lambda^4} = 0 \end{aligned}$$



# One-loop effective action

The standard one-loop SU(N) beta-function reads

G. Savvidy, PLB '77



$$\beta_1 = -\frac{bN}{48\pi^2} \bar{g}_1^2, \quad b = 11$$

$$\bar{g}_1^2(\mathcal{J}) = \frac{\bar{g}_1^2(\mu_0^4)}{1 + \frac{bN}{96\pi^2} \bar{g}_1^2(\mu_0^4) \ln(|\mathcal{J}|/\mu_0^4)}$$



$$\mathcal{L}_{\text{eff}}^{(1)} = \frac{bN}{384\pi^2} \mathcal{J} \ln\left(\frac{|\mathcal{J}|}{\lambda^4}\right)$$

# Beyond one-loop effective action I

The all-loops effective action of SU(2) YM theory predicted by the FRG approach

P. Dona, A. Marciano, Y. Zhang & C. Antolini, PRD '16

$$\mathcal{L}_{\text{eff}} = \frac{2\mathcal{J}}{16\pi^2} \int_0^\infty \frac{ds}{s} \left[ e^{-s} \sqrt{\frac{\lambda^4}{\mathcal{J} \tanh(\mathcal{J}/\lambda^4 \varepsilon)}} - e^{-s} \right] \\ \times \left[ \frac{1}{4 \sinh^2 s} - \frac{1}{4s^2} + 1 \right], \quad \varepsilon \ll 1,$$



$$\mathcal{J} \tanh\left(\frac{\mathcal{J}}{\varepsilon \lambda^4}\right) \Big|_{\varepsilon \rightarrow 0} \rightarrow |\mathcal{J}|$$

Corresponding all-loops coupling constant

$$[\bar{g}^2]^{-1} = \frac{2}{4\pi^2} \int_0^\infty \frac{ds}{s} \left[ e^{-s} \sqrt{\frac{\lambda^4}{\mathcal{J} \tanh(\mathcal{J}/\varepsilon \lambda^4)}} - e^{-s} \right] \\ \times \left[ \frac{1}{4 \sinh^2 s} - \frac{1}{4s^2} + 1 \right]$$

# Beyond one-loop effective action II

The all-loops effective action of SU(2) YM theory predicted by the FRG approach

P. Dona, A. Marciano, Y. Zhang & C. Antolini, PRD '16

$$\frac{\beta}{\bar{g}^2} = \frac{-1}{2\pi^2} \sqrt{\frac{\lambda^4}{\mathcal{J} \tanh(\mathcal{J}/\varepsilon\lambda^4)}} \int_0^\infty ds e^{-s} \sqrt{\frac{\lambda^4}{\mathcal{J} \tanh(\mathcal{J}/\varepsilon\lambda^4)}} \times \left[ \frac{1}{4 \sinh^2 s} - \frac{1}{4s^2} + 1 \right],$$

$$\ln A = \int_0^\infty \frac{ds}{s} \left[ e^{-sA} - e^{-s} \right], \quad A > 0$$

$$\left[ \frac{1}{4 \sinh^2 s} - \frac{1}{4s^2} + 1 \right] = \frac{11}{12} + \mathcal{O}(s^2)$$

the exact form of the one-loop effective action is recovered

# Vanishing gravitational effects

The spatially and homogenous isotropic part of A satisfies the classical YM eom

A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC '19

$$(\dot{U})^2 + \bar{g}^2 U^4 = \text{const}$$

integrated analytically in terms of Jacobi Elliptic functions

$$\begin{aligned} U(t) &= U_0 |\text{cd}(\bar{g}U_0 t) - 1|, \\ U(0) &= U_0 > 0, \quad \dot{U}(0) = 0 \end{aligned}$$

oscillatory quasi-harmonic function in time

$$U(nT_U) = U_0, \quad U\left(\left(n + \frac{1}{4}\right)T_U\right) = 0, \quad n = 0, 1, 2, \dots$$

$$U_0 > 0 \quad T_U = B\left(\frac{1}{4}, \frac{1}{2}\right) (\bar{g}U_0)^{-1} \simeq 1.2 \times \pi (\bar{g}U_0)^{-1}$$

# The case of SU(3)

A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC '19

$$A_k = A_{ak} \frac{\lambda_a}{2}, \quad \left[ \frac{\lambda_a}{2}, \frac{\lambda_b}{2} \right] = i f_{abc} \frac{\lambda_c}{2} \quad a = 1, \dots, 8$$

A special minimal  $\mathfrak{su}(2)$  algebra embedding into the  $\mathfrak{su}(3)$  in terms of three Gell-Mann matrices  $\{\lambda_7, -\lambda_5, \lambda_2\}$

$$A_{ak} = (\delta_{a,7}\delta_{k,1} - \delta_{a,5}\delta_{k,2} + \delta_{a,2}\delta_{k,3})U(t) + \tilde{A}_{ak}$$
$$\tilde{A}_{ak} = \tilde{A}_{ak}(t, \mathbf{x})$$

Again, in the absence of gravity, the spatially homogeneous/isotropic gluon condensate  $U(t) > 0$  satisfies the classical YM eom

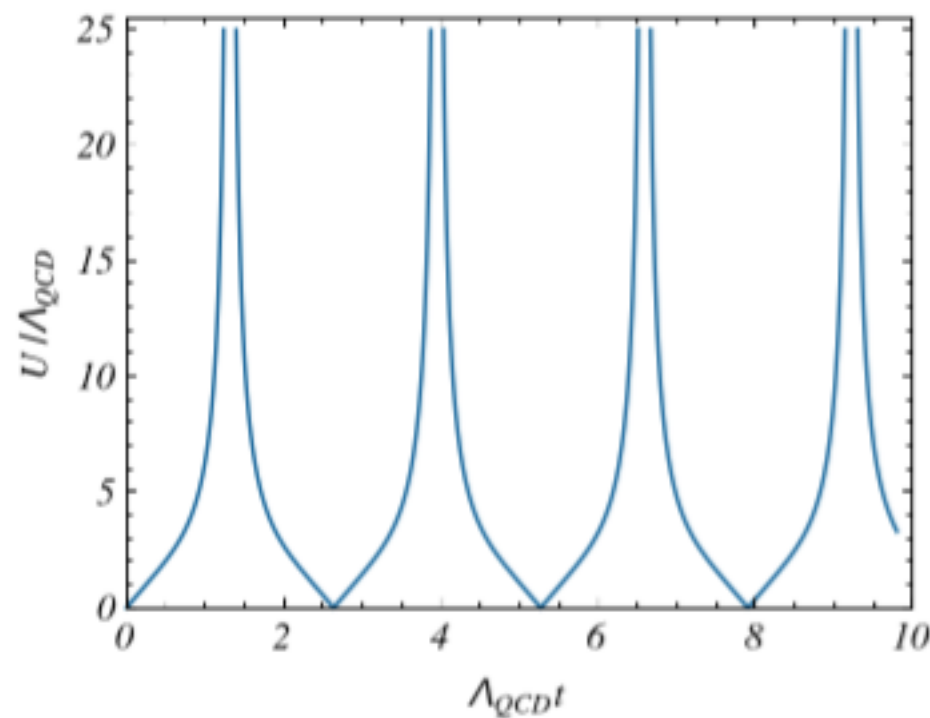
$$(\dot{U})^2 + \frac{1}{4} \bar{g}^2 U^4 = \text{const}$$

# Savvidy vacua

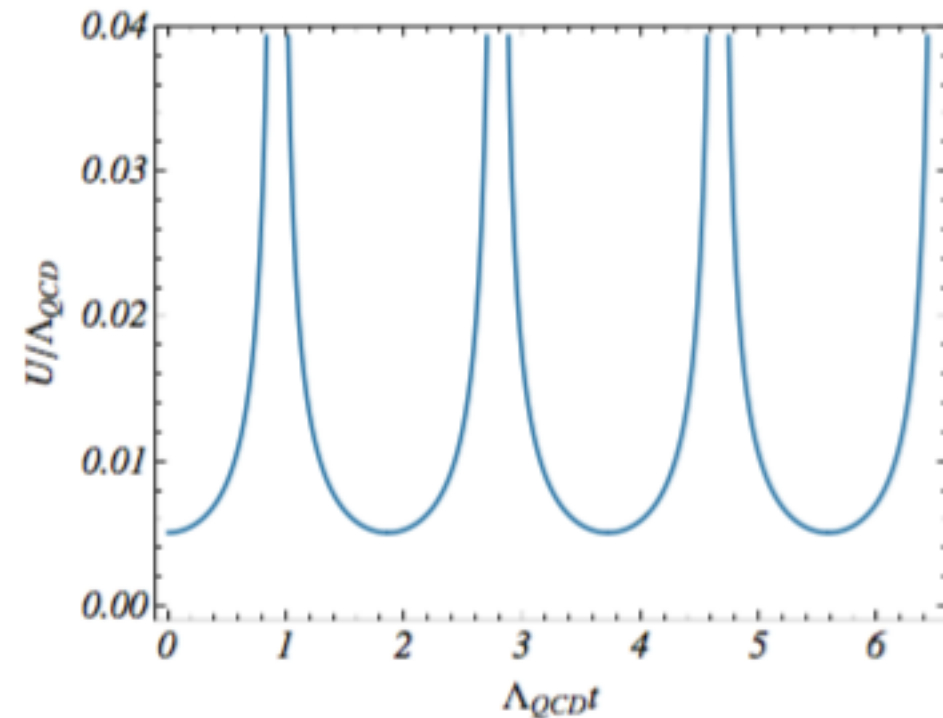
A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC '19

Conventional Fock vacuum, empty of particles and fields, cannot apply to QCD

Solving for the Savvidy's theory for general asymptotic solutions



CE vacua

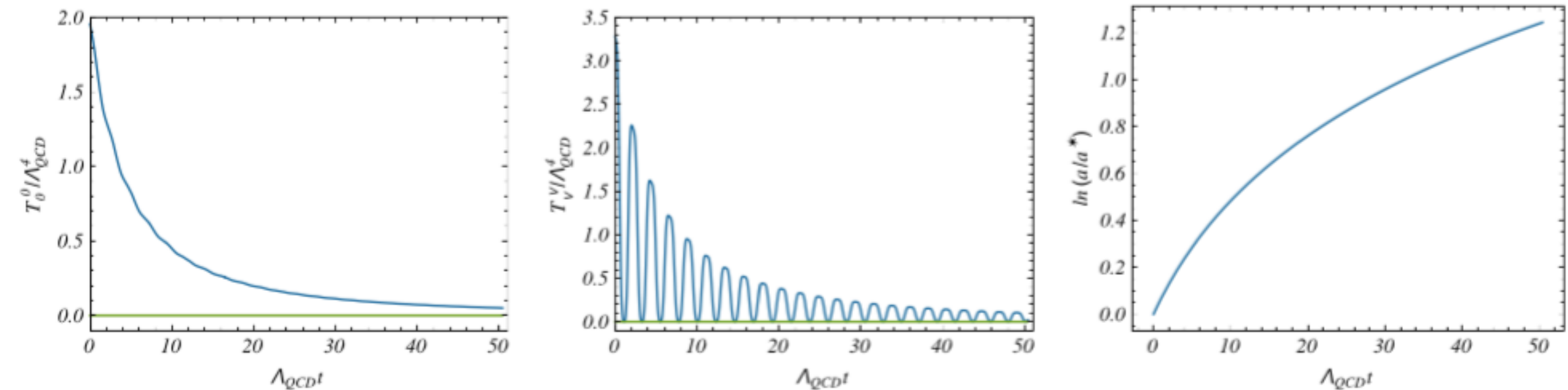


CM vacua

# Time-crystal and pressure fluctuations

A. Addazi, A. Marciano, R. Pasechnik & G. Prokhorov, EPJC '19

Solving numerically the YM-Einstein coupled system



Dissipative system, out of equilibrium: energy transferred to the primordial plasma



# Translational-breaking and Kinks

Standard static domain-walls for a scalar field theory that is  $Z_2$  invariant, and with sombrero-like Higgs potential

**SSB of  $Z_2$ :** when the scalar field rolls down to one of the two possible minima  $\phi_{vac} = \pm v$

$$\phi(z) = v \tanh \left[ \frac{\lambda v}{\sqrt{2}} (z - z_0) \right] \quad z_0 \text{ kink center}$$

The kink profile interpolating the two minima, when internal field configurations are localized in the space direction  $z$

Translational invariance is spontaneously broken, being the barrier localized in a  $z_0$  point

Nambu-Goldstone modulus boson  $z_0(t, x, y)$ , localized on the surface of the domain wall, as a low energy-excitation of its surface in the  $z$ -direction

# T-breaking and space-like domains I

A.Addazi, A. Marciano & R. Pasechnik, CPC '19

Consider a kink profile that, no more localized in a space direction, but in time

$$\phi(t) = v \tanh \left[ \frac{\lambda v}{\sqrt{2}} (t - t_0) \right] \quad t_0 \text{ kink center}$$

interpolating the two vacuum states in the asymptotic time limits

$$\phi(t = -\infty) = -v \quad \phi(t = \infty) = v$$

This instantiates a SSB of the time invariance and triggers the appearance of a Nambu-Goldstone boson localized on the xyz surface  $t_0(x, y, z)$

# T-breaking and space-like domains II

A. Addazi, A. Marciano & R. Pasechnik, CPC '19

Gluon condensate field equation coupled to gravity, in a FLRW background

Let us consider the limit of a static FLRW space time ( $a = \text{const}$ )

$$U'^2 - \frac{1}{4}U^4 = \text{const}$$

A branch of solutions of these equations, obtained by  $U^2 \rightarrow U^2 - U_0^2$  energy density vacuum shift, corresponds to kink (anti-kink) profiles

$$U(\eta) \simeq \frac{v}{\sqrt{2}} \tanh\left[\frac{v}{\sqrt{2}}(\eta - \eta_0)\right] \quad v \simeq \Lambda_{\text{QCD}}$$

A space-like domain wall corresponds to a kink profile of this type

# T-breaking and space-like domains III

A.Addazi, A. Marciano & R. Pasechnik, CPC '19

Time-translation is spontaneously broken, and a  $\eta_0(x, y, z)$  moduli field arises, with  $U$  acquiring the dependence  $U(\eta - \eta_0(x, y, z))$

The coordinate  $x, y, z$  are the domain-walls worldsheet coordinates.

The effective corresponding action reads as

$$\begin{aligned} S &= \int d^4x \frac{1}{2} \left[ \left( \frac{\partial \phi}{\partial \eta_0} \frac{\partial \eta_0}{\partial x^a} \right)^2 - V(U) \right] \\ &= \text{const} + \frac{T_W}{2} \int d^3x \left( \frac{\partial \eta_0(x^a)}{\partial x^a} \right)^2 \end{aligned}$$

The moduli field is massless, according to the Nambu-Goldstone theorem

# T-breaking with gravitational dynamics I

A. Addazi, A. Marciano & R. Pasechnik, CPC '19

When the scale factor time-dependence is considered, a more complicated time pattern for the space-like domain walls arises

Time-translation is spontaneously broken down to a discrete time-translation symmetry:

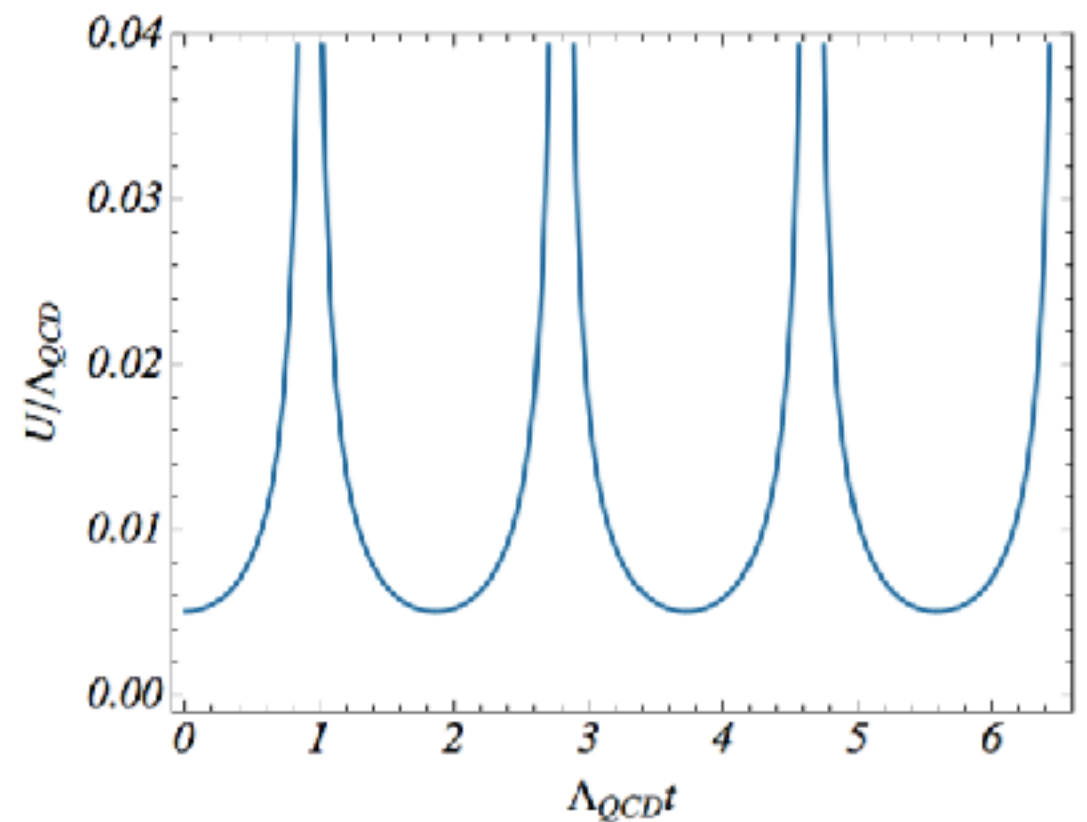
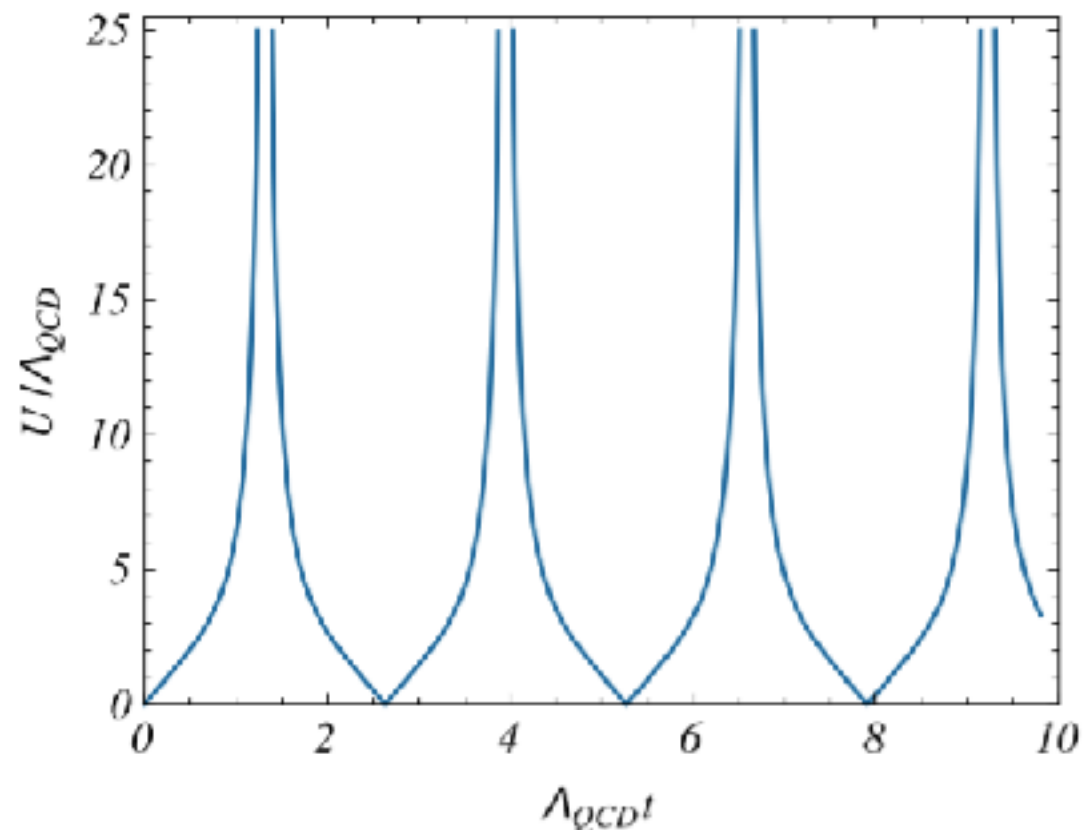
$$T_n : \eta \rightarrow \eta + n\Lambda_{\text{QCD}}^{-1}$$

# T-breaking with gravitational dynamics II

A. Addazi, A. Marciano & R. Pasechnik, CPC '19

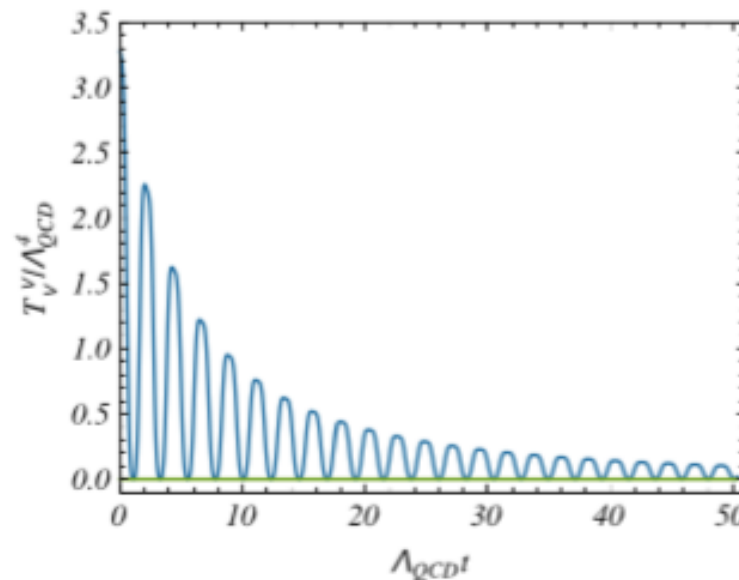
$$T_n : \eta \rightarrow \eta + n\Lambda_{\text{QCD}}^{-1}$$

**The system behaves as a time-crystal!**



# Any way to compare with lattice QCD?

Can Lattice QCD capture this dynamical and non-perturbative regime?



Can we slice FLRW with Minkowski in a time range much smaller than  $\Lambda_{QCD}^{-1}$  ?

Validity of Wick rotation and equilibrium dynamics in the Euclidean!

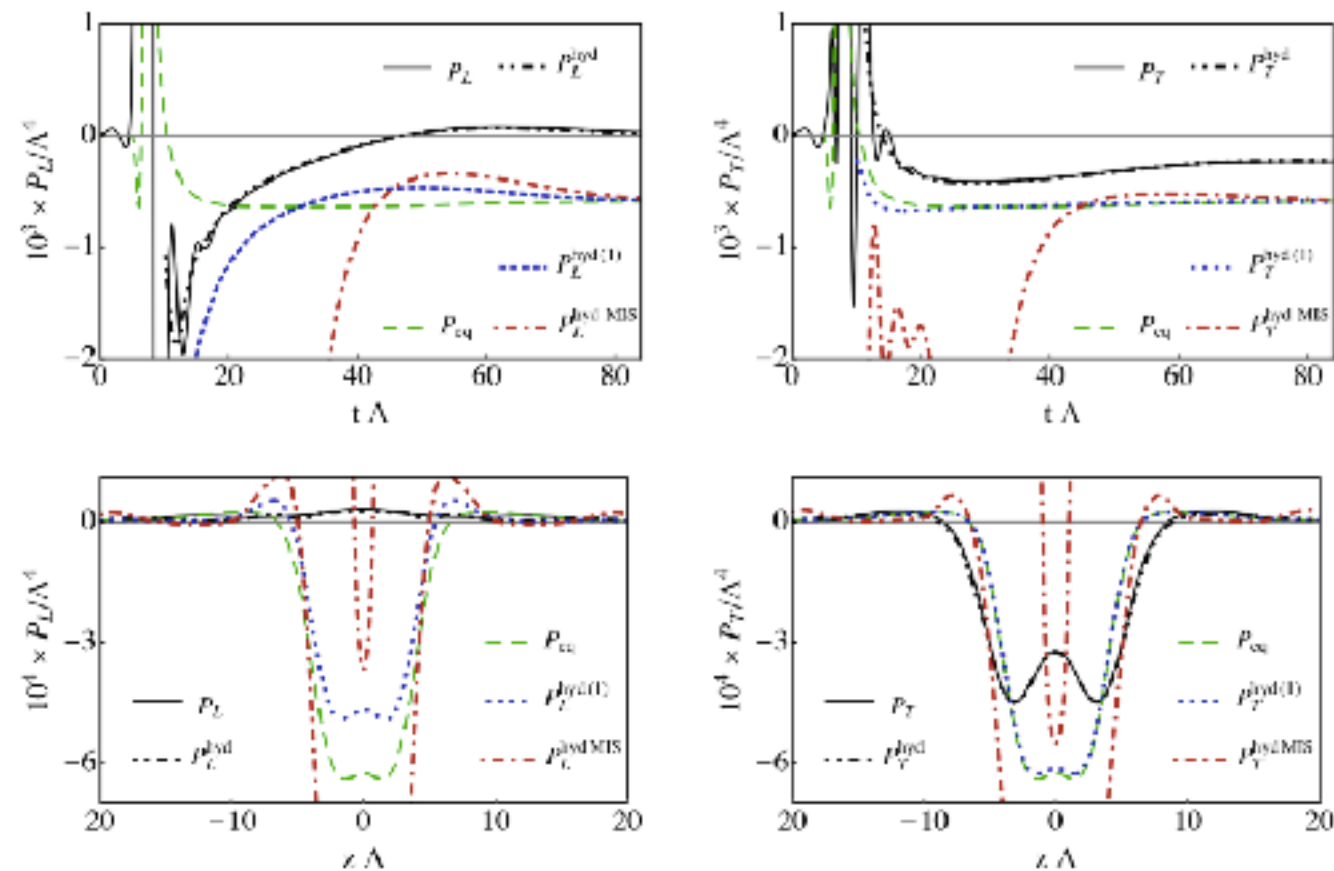
Maiani-Testa theorem (PLB '90): scattering for asymptotic states can be counter-Wick rotated only in the infinite volume limit (the scale being the threshold amplitude) and for time much smaller than the level spacing due to momentum discretization



# Comparison with QCD holography\*

M. Attems, Y. Bea, J. Casalderrey-Solana, D. Mateos, M. Triana & M. Zilhão, PRL '18

“Holography to analyze relativistic collisions in a one-parameter family of strongly coupled gauge theories with thermal phase transitions”



But conclude that “out-of-equilibrium physics smooths out the details of the transition”

# Not only an academic issue: we ask Nature

A.Addazi, A. Marciano & R. Pasechnik, CPC '19

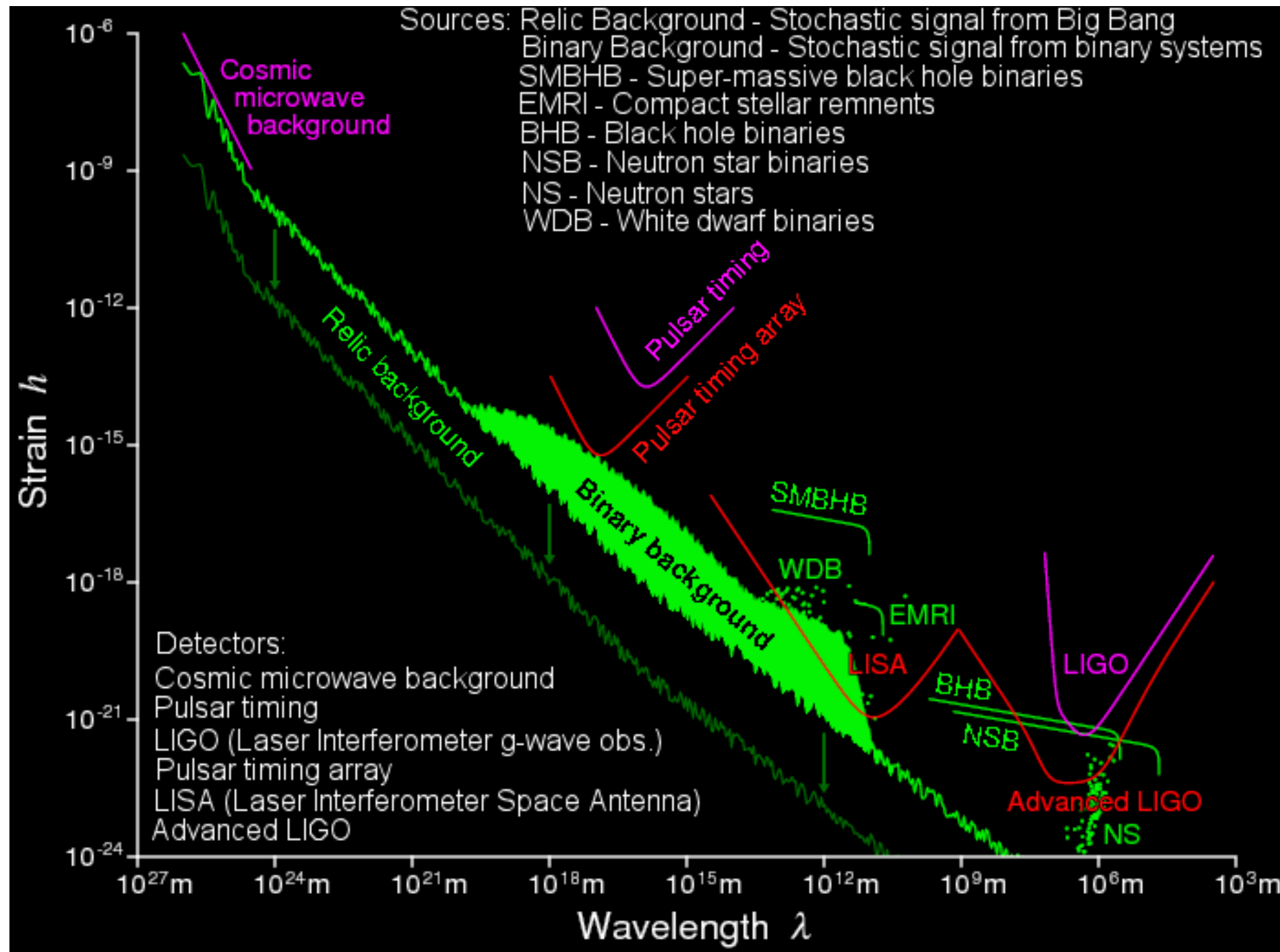
Is there any observable that makes falsifiable this insight?

Pressure oscillations start at a time related to the QCD scale, and are after few cycles suppressed

This effect might leave an imprinting on the primordial plasma, generating GWs in the nHz domains, so within the radio-frequency range of Pulsars Timing

**Turbulence and sounds waves shall be taken into account as GW sources**

# GW frequencies domain



# Primordial plasma and GWs

A. Addazi, A. Marciano & R. Pasechnik, CPC '19

$$\alpha = \frac{\rho_U}{\rho_{\text{rad}}} \quad \text{latent energy released by the gluon condensate into the primordial plasma}$$

$$\beta_i \simeq \Lambda \quad \text{inverse time scale of the peaks, related to the inverse of the QCD scale}$$

Redshifts factors must be taken into account

$$\frac{R_*}{R_0} = 8.0 \times 10^{-14} \left( \frac{100}{g_*} \right)^{\frac{1}{3}} \left( \frac{1 \text{ GeV}}{T_*} \right)$$

$$f_0 = f_* \left( \frac{R_*}{R_0} \right) = 1.65 \times 10^{-7} \text{ Hz} \left( \frac{f_*}{H_*} \right) \left( \frac{T_*}{1 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}}$$

$$H_* = \frac{8\pi G \rho_{\text{rad}}}{3} = \frac{8\pi^3 g_* T_*^4}{90 M_{Pl}^2} \quad \Omega_{\text{GW}} = \Omega_{\text{GW}*} \left( \frac{R_*}{R_0} \right)^4 \left( \frac{H_*}{H_0} \right)^2$$

# Sounds waves and turbulences effects

A.Addazi, A. Marciano & R. Pasechnik, CPC '19

$$h^2\Omega_{\text{turb}} = 3.35 \times 10^{-4} \sum_i^{N_{\text{eff}}} \left( \frac{H_{*,i}}{\beta_i} \right) \left( \frac{\kappa_{\text{turb}}\alpha}{1+\alpha} \right)^{\frac{3}{2}} \left( \frac{100}{g_{*,i}} \right)^{\frac{1}{3}} v,$$

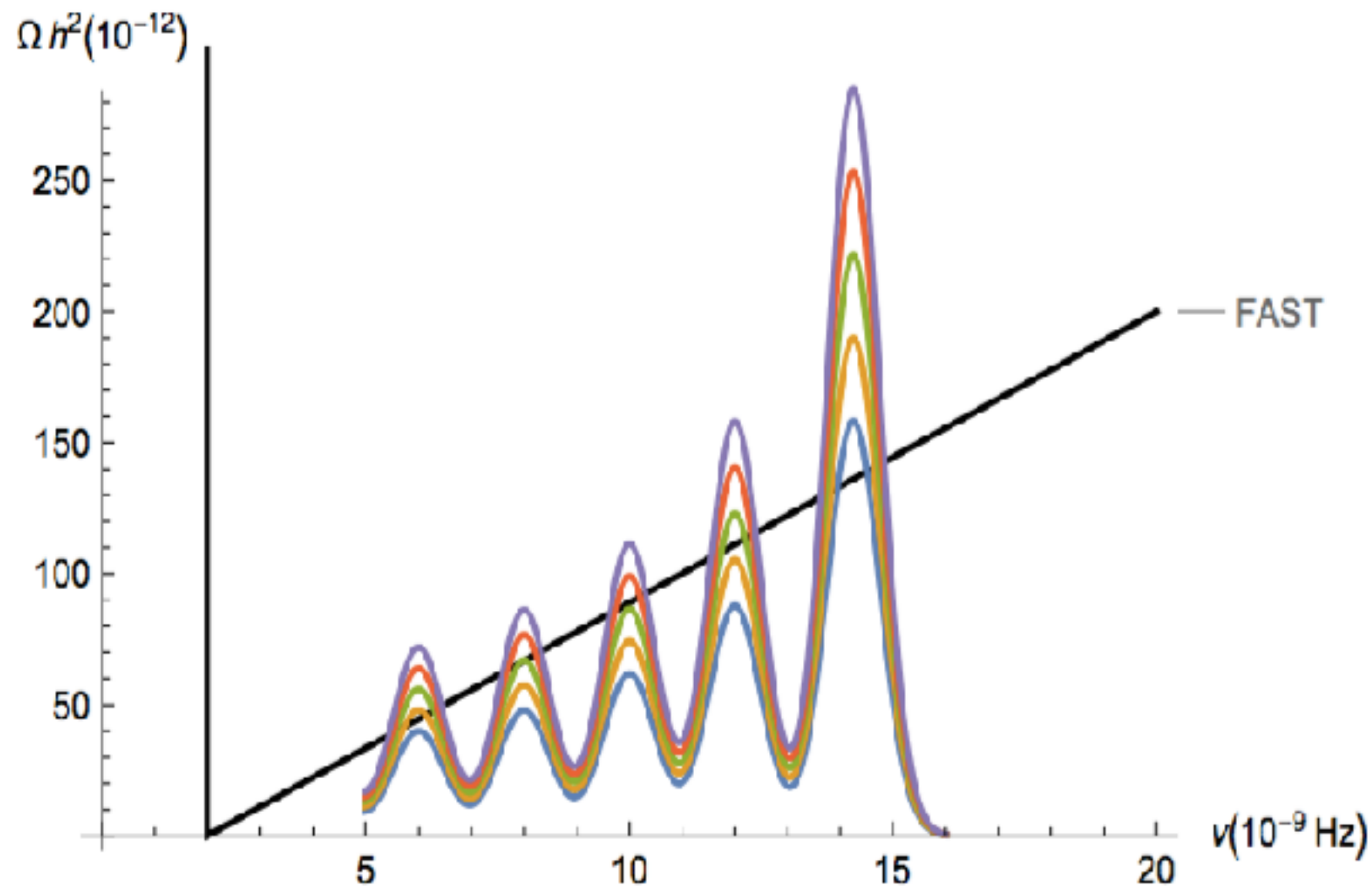
$$f_{\text{turb}} = 2.7 \times 10^{-2} \text{ mHz} \frac{1}{v} \left( \frac{\beta_i}{H_{*,i}} \right) \left( \frac{E_{*,i}}{\text{GeV}} \right) \left( \frac{g_{*,i}}{100} \right)^{\frac{1}{6}}$$

$$h^2\Omega_{\text{sound}} = 2.65 \times 10^{-6} \sum_{i=1}^{N_{\text{eff}}} \left( \frac{H_{*,i}}{\beta_i} \right) \left( \frac{\kappa_v\alpha}{1+\alpha} \right)^2 \left( \frac{100}{g_{*,i}} \right)^{\frac{1}{3}} v_i,$$

$$f_{\text{sw}} = 1.9 \times 10^{-2} \text{ mHz} \sum_{i=1}^{N_{\text{eff}}} \frac{1}{v_i} \left( \frac{\beta_i}{H_{*,i}} \right) \left( \frac{T_{*,i}}{100 \text{ GeV}} \right) \left( \frac{g_{*,i}}{100} \right)^{\frac{1}{6}}$$

# Comparison with FAST sensitivity curves

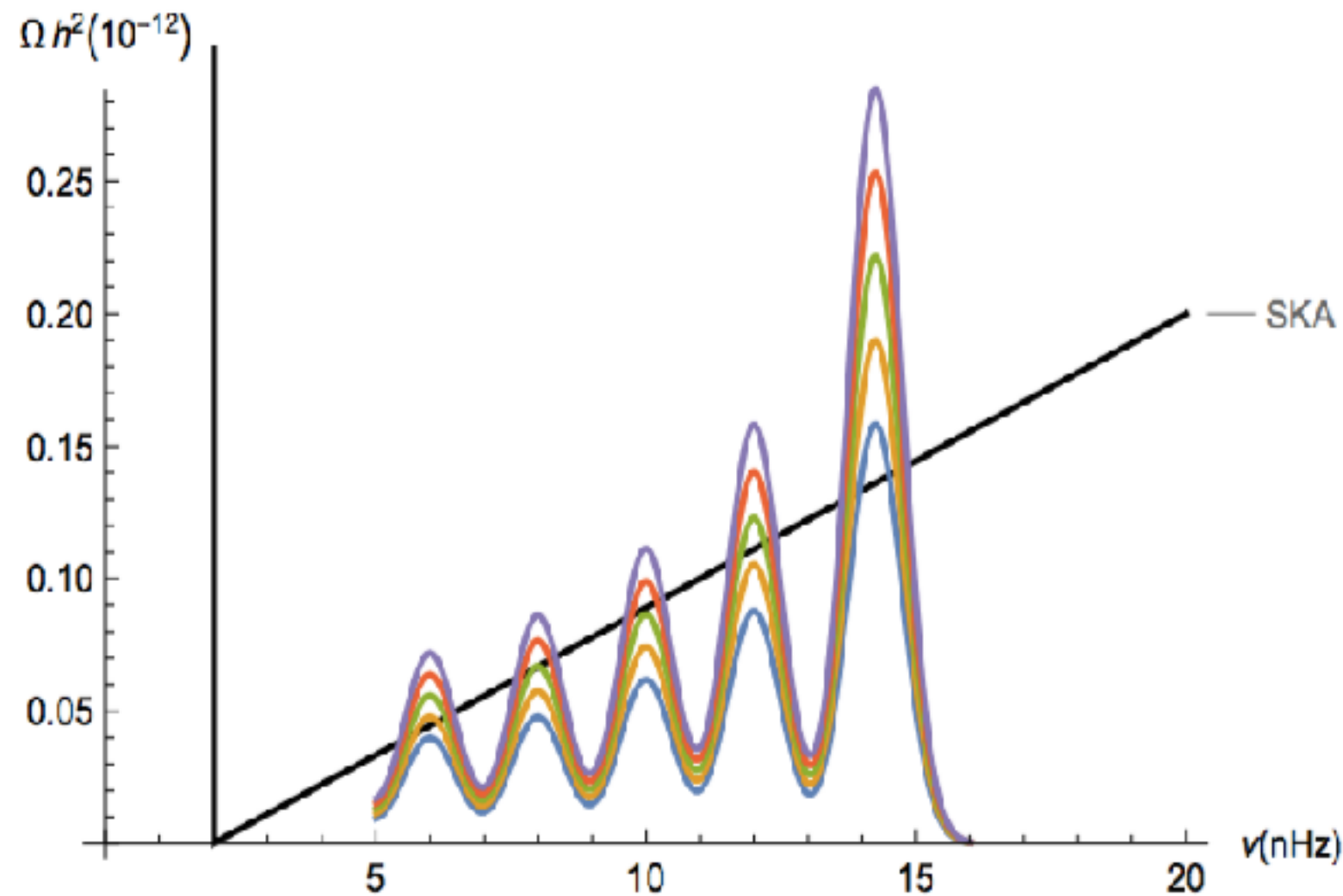
A. Addazi, A. Marciano & R. Pasechnik, CPC '19



The GWs spectrum is displayed for different efficiency factors, in comparison with FAST sensitivity curve. The efficiency factor considered are  $\kappa = 0.03 \div 0.1$

# Comparison with SKA sensitivity curves

A. Addazi, A. Marciano & R. Pasechnik, CPC '19



The GWs spectrum is displayed for different efficiency factors, in comparison with SKA sensitivity curve. The efficiency factor considered are  $\kappa = 0.001 \div 3 \times 0.001$



# *Neutrino physics and the mass-generation*

# Standard Type-I models (high scale)

$$\mathcal{L}_{\text{Yuk}}^{\text{Type-I}} = Y_\nu \bar{L} H \nu^c + M \nu^c \nu^c + h.c.$$

$L = (\nu, l)^T$  and  $\nu^c$  three RH-neutrinos colored as SM-singlet

$Y_\nu$  and  $M$  3 x 3 matrices

$M$  explicitly break lepton number symmetry  $U(1)_L \rightarrow \mathbb{Z}_2$

Mass for light neutrinos generated by EWSB  $m_\nu \sim \mathcal{O}(0.1\text{eV})$

$$m_\nu^{\text{Type-I}} = \frac{v_h^2}{2} Y_\nu^T M^{-1} Y_\nu \quad \langle H \rangle = \frac{v_h}{\sqrt{2}} \quad Y_\nu \sim \mathcal{O}(1) \quad M \sim \mathcal{O}(10^{14}\text{GeV})$$

# Inverse see-saw (low scale)

Two additional gauge singlet fermions, with opposite lepton number charge  $\nu^c, S$

$$\mathcal{L}_{\text{Yuk}}^{\text{Inverse}} = Y_\nu \bar{L} H \nu^c + M \nu^c S + \mu S S + \text{h.c.}$$

The smallness of the neutrino mass is linked to the breaking  $U(1)_L \rightarrow \mathbb{Z}_2$

This is triggered by the  $\mu$ -term  $m_\nu^{\text{Inverse}} = \frac{v_h^2}{2} Y_\nu^T M^{T-1} \mu M^{-1} Y_\nu$

Small neutrino masses are protected by  $U(1)_L$  (restored for  $\mu \rightarrow 0$ )

# Introducing the Majoron

*Global  $B-L$  spontaneously broken by a SM complex scalar singlet  
and generation of LH neutrino mass*

*The NGB associated to the symmetry is the Majoron*

*Possible detection in neutrinoless double-beta decays (GERDA, EXO)*

# The effective action for the Majoron

$$\mathcal{L}_M = f H \bar{L} \nu_R + h \sigma \bar{\nu}_R \nu_R^c + h.c. + V(\sigma, H)$$

A complex singlet scalar  $\sigma$ , **the majoron**, with  $L(\sigma) = -2$

$$V(\sigma, H) = V_0(\sigma, H) + V_1(\sigma) + V_2(h, \sigma)$$

$$V_0(\sigma, H) = \lambda_s \left( |\sigma|^2 - \frac{v_{BL}^2}{2} \right)^2 + \lambda_H \left( |H|^2 - \frac{v^2}{2} \right)^2 \\ + \lambda_{sH} \left( |\sigma|^2 - \frac{v_{BL}^2}{2} \right) \left( |H|^2 - \frac{v^2}{2} \right),$$

$$V_1(\sigma) = \frac{\lambda_1}{\Lambda} \sigma^5 + \frac{\lambda_2}{\Lambda} \sigma^* \sigma^4 + \frac{\lambda_3}{\Lambda} (\sigma^*)^2 \sigma^3 + h.c.$$

$$V_2(H, \sigma) = \beta_1 \frac{(H^\dagger H)^2 \sigma}{\Lambda} + \beta_2 \frac{(H^\dagger H) \sigma^2 \sigma^*}{\Lambda} \\ + \beta_3 \frac{(H^\dagger H) \sigma^3}{\Lambda} + h.c..$$

# The effective action for the Majoron

$$\begin{aligned} V_1^{(6)}(\sigma) &= \frac{\gamma_1}{\Lambda^2} \sigma^6 + \frac{\gamma_2}{\Lambda^2} \sigma^* \sigma^5 + \frac{\gamma_3}{\Lambda^2} (\sigma^*)^2 \sigma^4 \\ &\quad + \frac{\gamma_4}{\Lambda^2} (\sigma^*)^3 \sigma^3 + h.c. , \\ V_2^{(6)}(\sigma, H) &= \frac{\delta_1}{\Lambda^2} (H^\dagger H)^2 \sigma^2 + \frac{\delta_2}{\Lambda^2} (H^\dagger H)^2 \sigma^* \sigma \\ &\quad + \frac{\delta_3}{\Lambda^2} (H^\dagger H) \sigma^3 \sigma^* + \frac{\delta_4}{\Lambda^2} (H^\dagger H) (\sigma \sigma^*)^2 \\ &\quad + \frac{\delta_5}{\Lambda^2} (H^\dagger H) \sigma^4 + h.c. . \end{aligned}$$

[Z.G.Berezghiani, R.N.Mohapatra, G.Senjanovic, PRD 1993](#)

# Missing energy channel and LHC data

$$\Gamma(H \rightarrow \chi\chi) = \frac{C_{h\chi\chi}^2 v^2}{64\pi m_H} \sqrt{1 - \frac{m_\chi^2}{m_H^2}}$$

$$C_{H\chi\chi} = \lambda_{H\chi\chi} + \frac{\beta_2}{\Lambda} v_\sigma.$$

$$Br(H \rightarrow invisible) = \frac{\Gamma_{inv}}{\Gamma_{inv} + \Gamma_{SM}} < 0.51 \text{ (95\% C.L.)}$$

channel	ATLAS	CMS	ATLAS+CMS
$\mu_{\gamma\gamma}$	$1.15^{+0.27}_{-0.25}$	$1.12^{+0.25}_{-0.23}$	$1.16^{+0.20}_{-0.18}$
$\mu_{WW}$	$1.23^{+0.23}_{-0.21}$	$0.91^{+0.24}_{-0.21}$	$1.11^{+0.18}_{-0.17}$
$\mu_{ZZ}$	$1.51^{+0.39}_{-0.34}$	$1.05^{+0.32}_{-0.27}$	$1.31^{+0.27}_{-0.24}$
$\mu_{\tau\tau}$	$1.41^{+0.40}_{-0.35}$	$0.89^{+0.31}_{-0.28}$	$1.12^{+0.25}_{-0.23}$
$\mu_{bb}$	$0.62^{+0.37}_{-0.36}$	$0.81^{+0.45}_{-0.42}$	$0.69^{+0.29}_{-0.27}$

$$\mu_F = \frac{\sigma^{NP}(pp \rightarrow H)}{\sigma^{SM}(pp \rightarrow H)} \frac{BR^{NP}(H \rightarrow F)}{BR^{SM}(H \rightarrow F)}$$



# Majoron phenomenology

*Cosmological limits very stringent on SSB scales beyond EW phase-transition*

*Very open limits on smaller scales!*

*Possibility to say something about the nature of the phase transition:  
violent Majoron, with FOPT*

A. Addazi & A. Marciano, CPC (2018), arXiv:1705.08346

# FOPT at 10 GeV

A. Addazi & A. Marciano, CPC (2018), arXiv:1705.08346

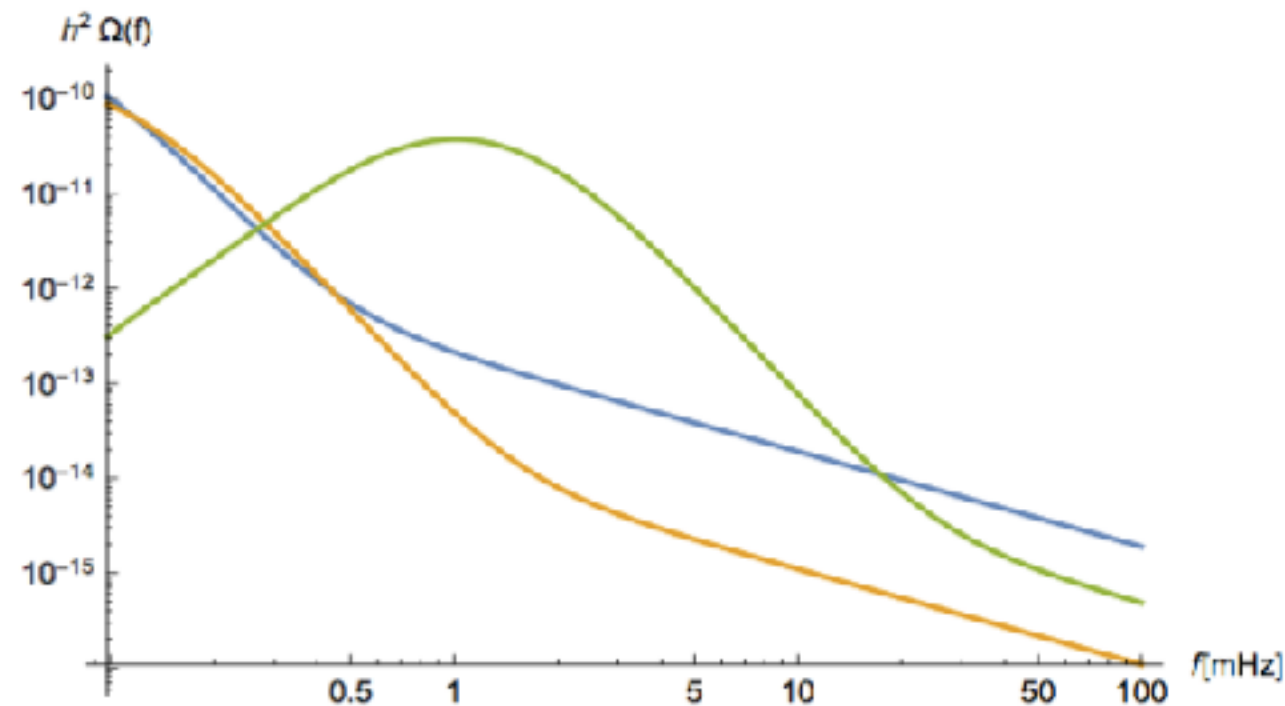


FIG. 1. The gravitational waves energy density as a function of the frequency is displayed. We use the same model independent parametrization of Ref.[18]. We show three *non-runaway* bubbles cases which are compatible with the B-L first order phase transition: In blue, we consider the case of  $\bar{T} = 50 \text{ GeV}$ ,  $\beta/\bar{H} = 100$ ,  $\alpha = 0.5$ ,  $\alpha_\infty = 0.1$ ,  $V_B = 0.95$ ; in green  $\bar{T} = 20 \text{ GeV}$ ,  $\beta/\bar{H} = 10$ ,  $\alpha = 0.5$ ,  $\alpha_\infty = 0.1$ ,  $V_B = 0.95$ . Orange:  $\bar{T} = 10 \text{ GeV}$ ,  $\beta/\bar{H} = 10$ ,  $\alpha = 0.5$ ,  $\alpha_\infty = 0.1$ ,  $V_B = 0.3$ . The three cases lie in the sensitivity range of LISA [18].

# Constrained from GW, colliders and cosmology

A. Addazi & A. Marciano, CPC (2018), arXiv:1705.08346

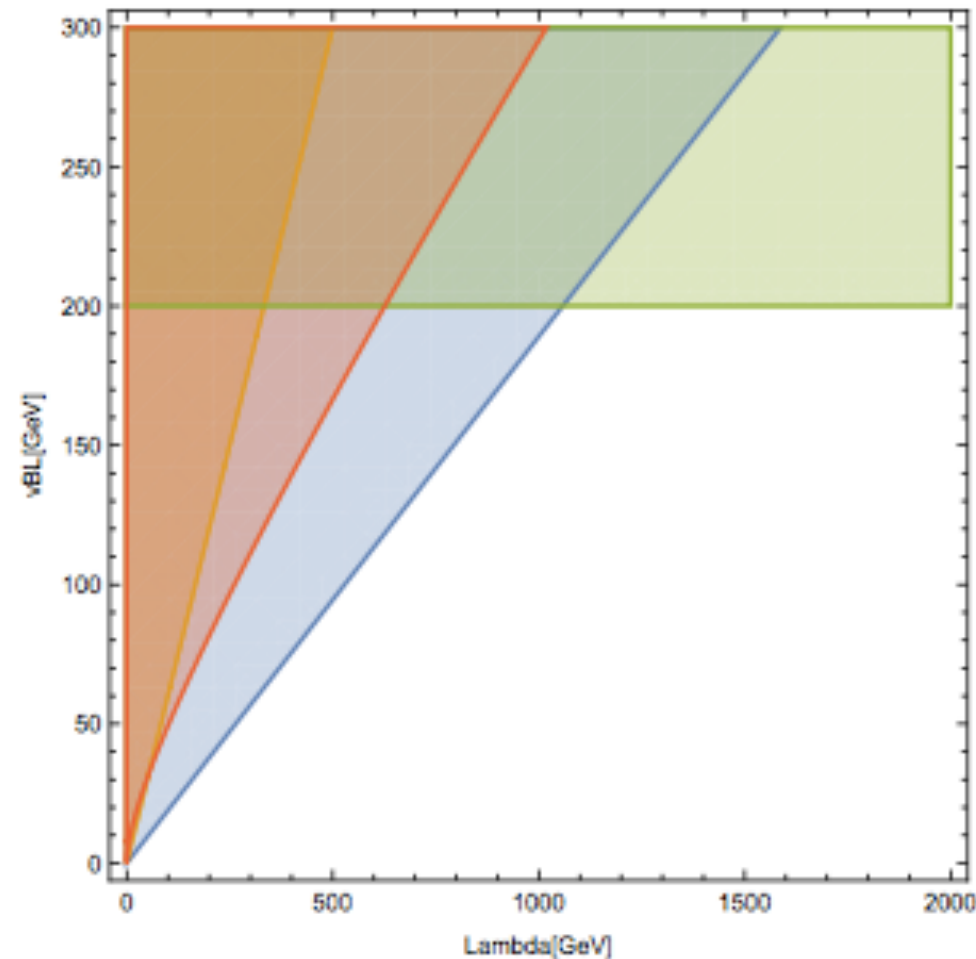
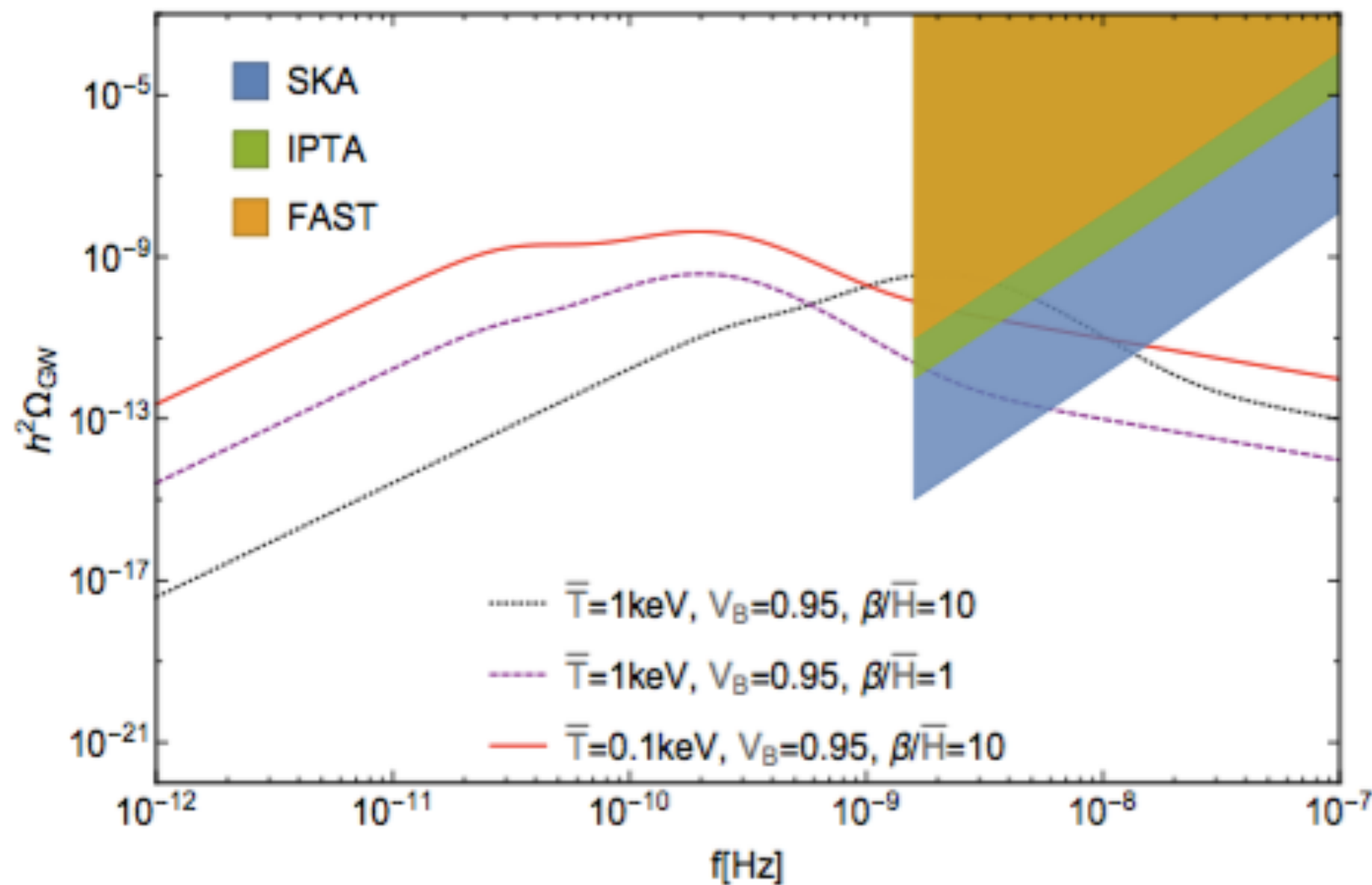


FIG. 2. We report the limits from LHC and future CEPC (in brown and blu respectively), cosmological sphaleron bounds (green) and the region which will be probed by eLISA (red). The case of  $\beta_2 = 1$  is displayed.

# Constrained from radio telescopes at KeV scales



# Type-I and Inverse See-saw with Majoron

For the majoron,  $L(\sigma) = -2$  and mass terms read now:

$$M\nu^c\nu^c \rightarrow Y_\sigma\sigma\nu^c\nu^c \text{ (type-I variant)} \quad \mu SS \rightarrow Y_\sigma\sigma SS \text{ (low-scale inverse variant)}$$

$$\langle\sigma\rangle = \frac{v_\sigma}{\sqrt{2}} \text{ breaks spontaneously } U(1)_L \rightarrow \mathbb{Z}_2$$

$$M \rightarrow Y_\sigma v_\sigma / \sqrt{2} \text{ (type-I variant)} \quad \mu \rightarrow Y_\sigma v_\sigma / \sqrt{2} \text{ (low-scale inverse variant)}$$

*Extended scalar sector:*

$$V_0 = V_{\text{SM}} + \mu_\sigma^2 \sigma^* \sigma + \lambda_\sigma (\sigma^* \sigma)^2 + \lambda_{h\sigma} H^\dagger H \sigma^* \sigma + \left(\frac{1}{2}\mu_b^2 \sigma^2 + \text{c.c.}\right)$$

Tiny  $U(1)_L$  soft breaking term  $\mu_b \sim \mathcal{O}(1\text{KeV})$

Resulting pseudo-Goldstone boson as testable DM candidate (Valle et '93, '07)

*See-saw mechanism and GW production*

# See-saw gravitational footprint

High scale type-I seesaw with explicit  $U(1)_L$  violation

$$\mathcal{L}_{\text{Yuk}}^{\text{Type-I}} = Y_\nu \bar{L} H \nu^c + M \nu^c \nu^c + h.c.$$

Heavy isosinglet neutrinos decouple from EW-scale: no FOPT and thus no GW signal from EWPT!

Low-scale inverse seesaw with explicit  $U(1)_L$  violation

$$\mathcal{L}_{\text{Yuk}}^{\text{Inverse}} = Y_\nu \bar{L} H \nu^c + M \nu^c S + \mu S S + h.c.$$

Singlets closer to EW scale and sizable Higgs coupling

Thermal corrections from heavy neutrinos are not enough to induce FOPT

Fermions affect the potential at loop level, but still no FOPT !



# Gravitational footprint of Lepton number SSB

Spontaneous breaking of  $U(1)_L \rightarrow Z_2$  and inverse see-saw mechanism

Type-I variant

$$M\nu^c\nu^c \rightarrow Y_\sigma\sigma\nu^c\nu^c$$

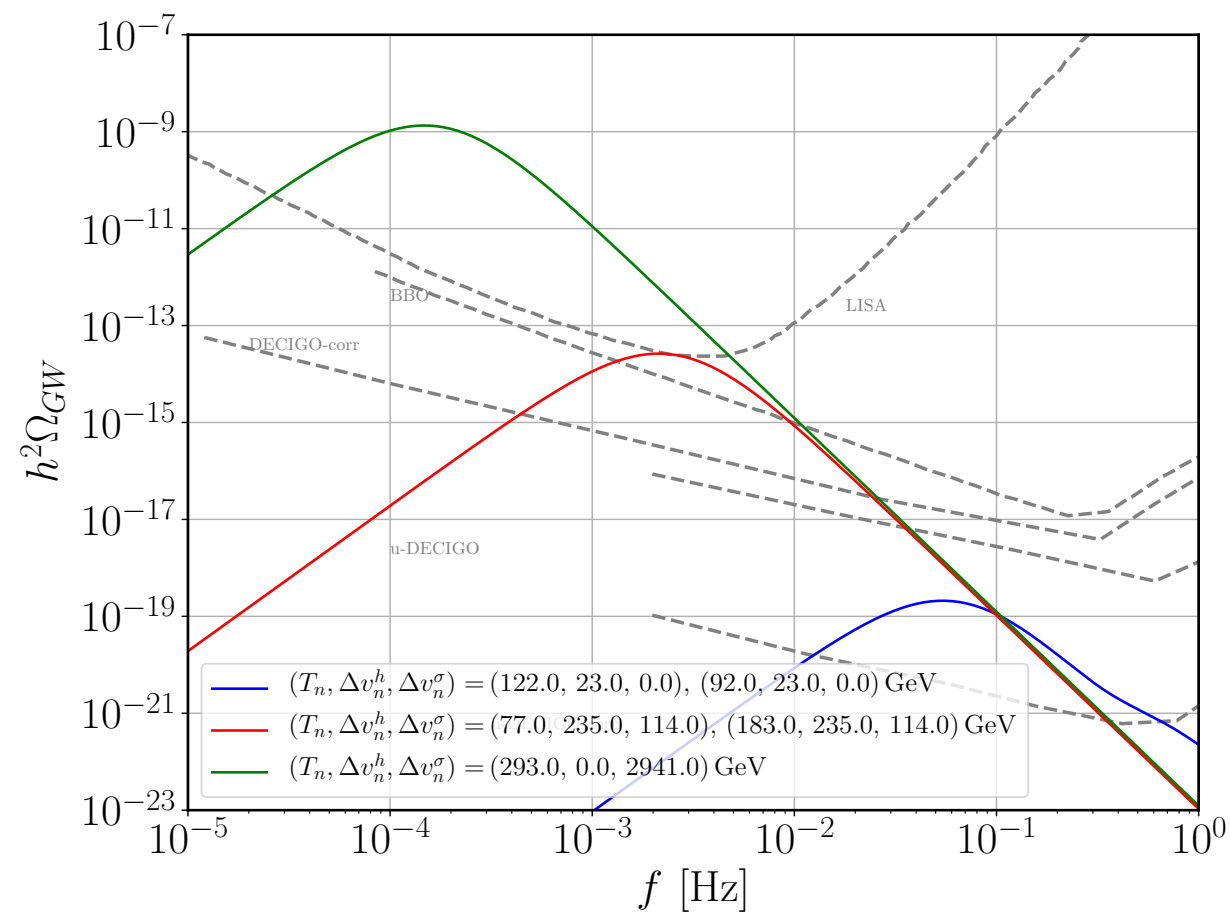
Low-scale inverse variant

$$\mu SS \rightarrow Y_\sigma\sigma SS$$

Majoron scalar  $\sigma$  responsible for a new richer pattern of FOPTs

# Inverse See-Saw with Majoron

Strength of PT enhanced by tree-level contributions



Characteristic signal with multi-peak scenario!

# Richer patterns of FOPTs

Not only heavy isosinglet fermions couple to the Higgs  $\rightarrow$  the Majoron complex scalar that breaks spontaneously  $U(1)_L$  can couple substantially to the Higgs (modulo LHC)



Generation of two or three GW peaks

At the end of any FOPT scalar potential minimization requires non vanishing VEVs to be associated with the generation of EW and neutrino mass scales

**Class I)**  $(0, 0) \rightarrow (v_H, v_\sigma)$

**Class II)**

$(0, 0) \rightarrow (v_H, 0) \rightarrow (v_H, v_\sigma)$  for  $v_\sigma < v_H$

$(0, 0) \rightarrow (0, v_\sigma) \rightarrow (v_H, v_\sigma)$  for  $v_\sigma > v_H$

**Class III)**

$(0, 0) \rightarrow (v_H, 0) \rightarrow (0, v_\sigma) \rightarrow (v_H, v_\sigma)$  for  $v_\sigma < v_H$

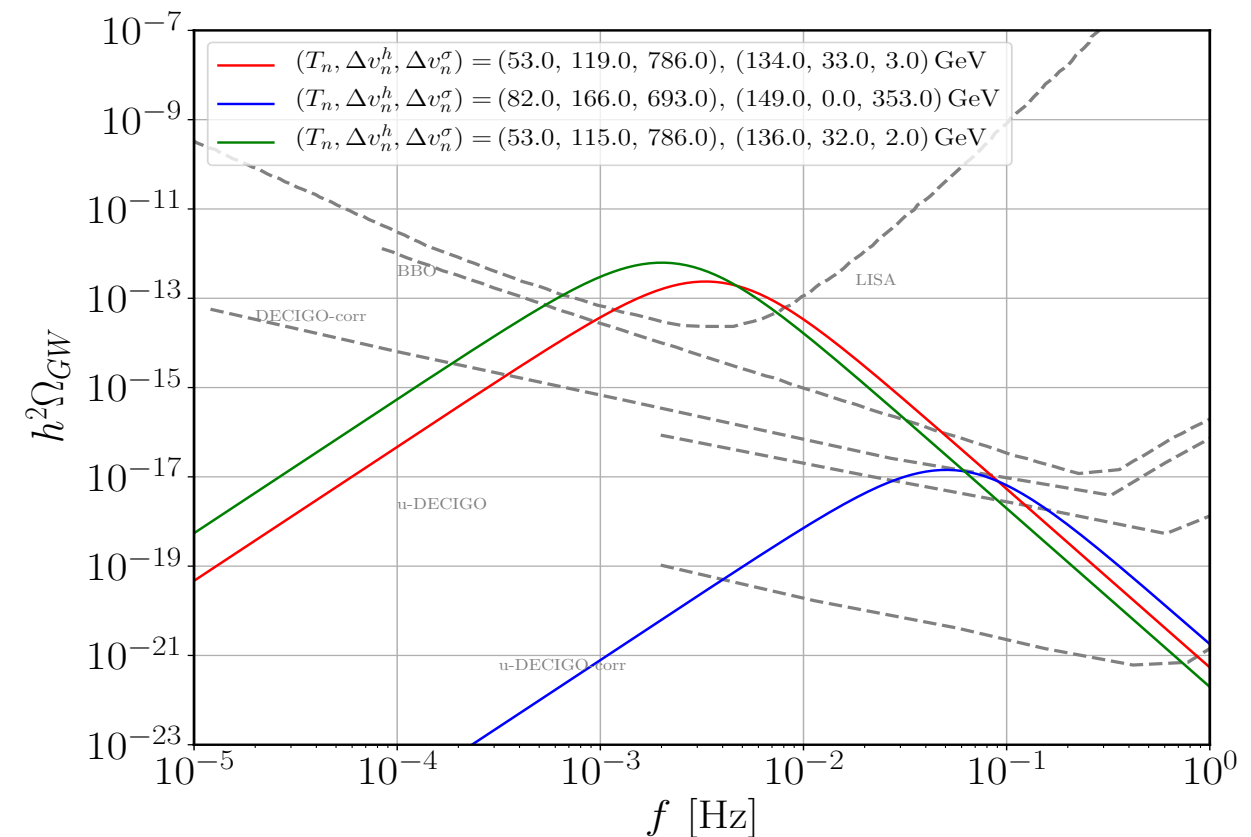
$(0, 0) \rightarrow (0, v_\sigma) \rightarrow (v_H, 0) \rightarrow (v_H, v_\sigma)$  for  $v_\sigma > v_H$

# Three possible scenarios

Three possible scenarios, with nearly preserved U(1)<sub>L</sub>, namely  $\nu_\sigma$  ( $T=0$ )  $\sim$  O (1 keV)

Peak Id	$T_n$	$(v_h^i, v_\sigma^i) \rightarrow (v_h^f, v_\sigma^f)$	$\alpha$	$\beta/H$
Green 1	293	$(0, 0) \rightarrow (0, 2941)$	0.5	4.9
Red 1	183	$(0, 114) \rightarrow (235, 0)$	$7.7 \times 10^{-4}$	$7.2 \times 10^4$
Red 2	77	$(0, 114) \rightarrow (235, 0)$	0.1	231
Blue 1	122	$(193, 0) \rightarrow (216, 0)$	$1.4 \times 10^{-2}$	$3.1 \times 10^3$
Blue 2	92	$(193, 0) \rightarrow (216, 0)$	$9.4 \times 10^{-3}$	$3.0 \times 10^4$

Curve	$m_{h_2}$	$m_A$	$\lambda_{\sigma h}$	$\lambda_\sigma$	$M$	$Y_\sigma$
Green	236	708	1.7	$5 \times 10^{-3}$	380	2
Red	192	970	2.3	1.5	93	0.1
Blue	325	169	4	2.7	158	0.1



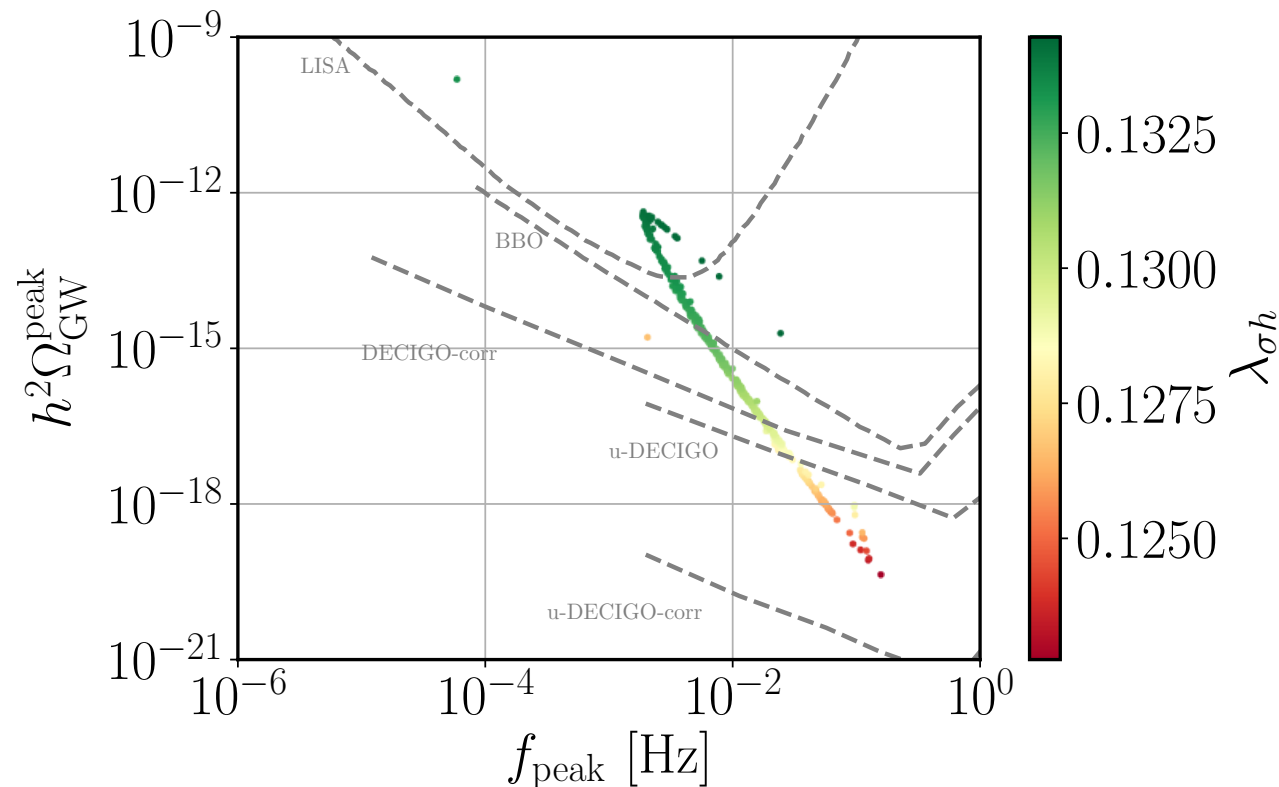
**Detectable by LISA: very strong FOPT with  $\nu_n/T_n = 119$**

(Consistent with invisible Higgs decays LHC bounds [Bonilla, Romão, Valle (2016)])

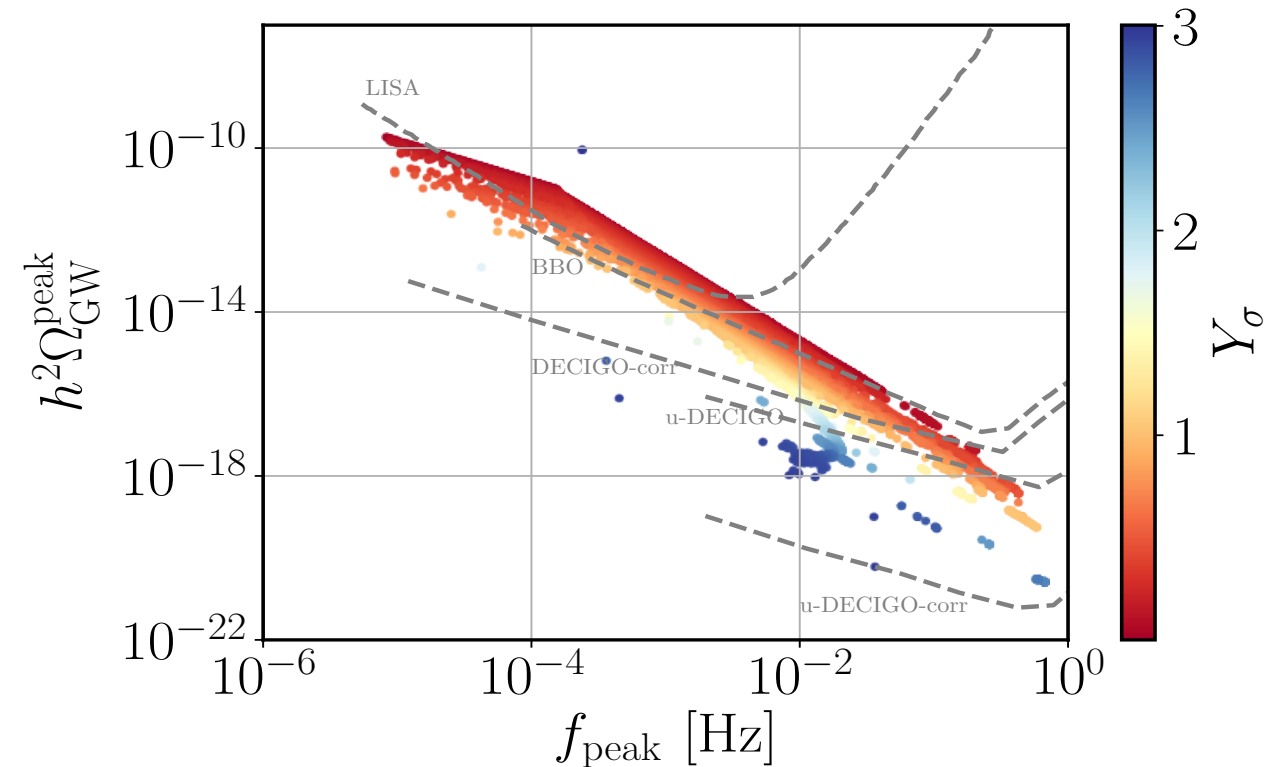
**Two-peak scenarios detectable by DECIGO**

*Large quartic couplings enhance  $m/T$  and facilitate these scenarios: Bosonic  $(m/T)^3$  contributions in  $\Delta V(1)$  ( $T$ ) produce potential barriers*

# Multi-peak scenarios and generic features



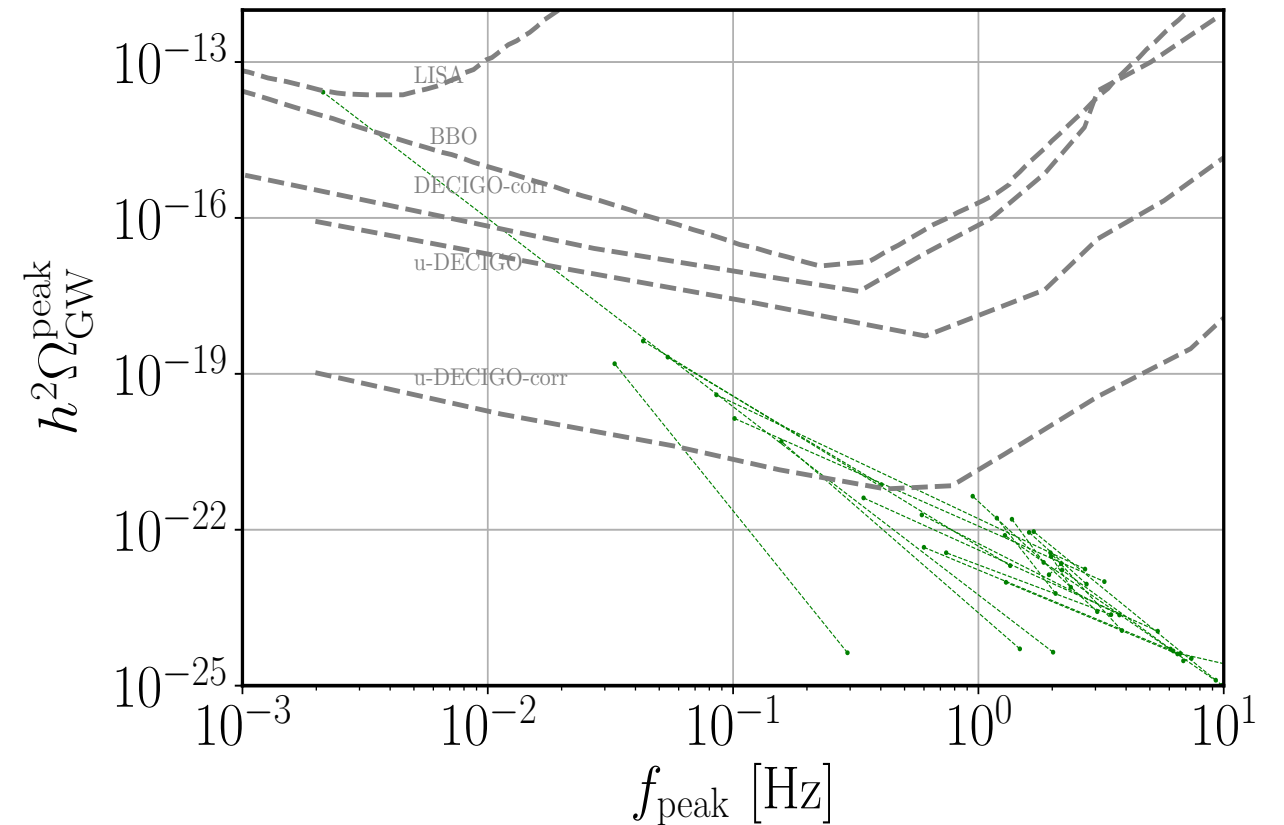
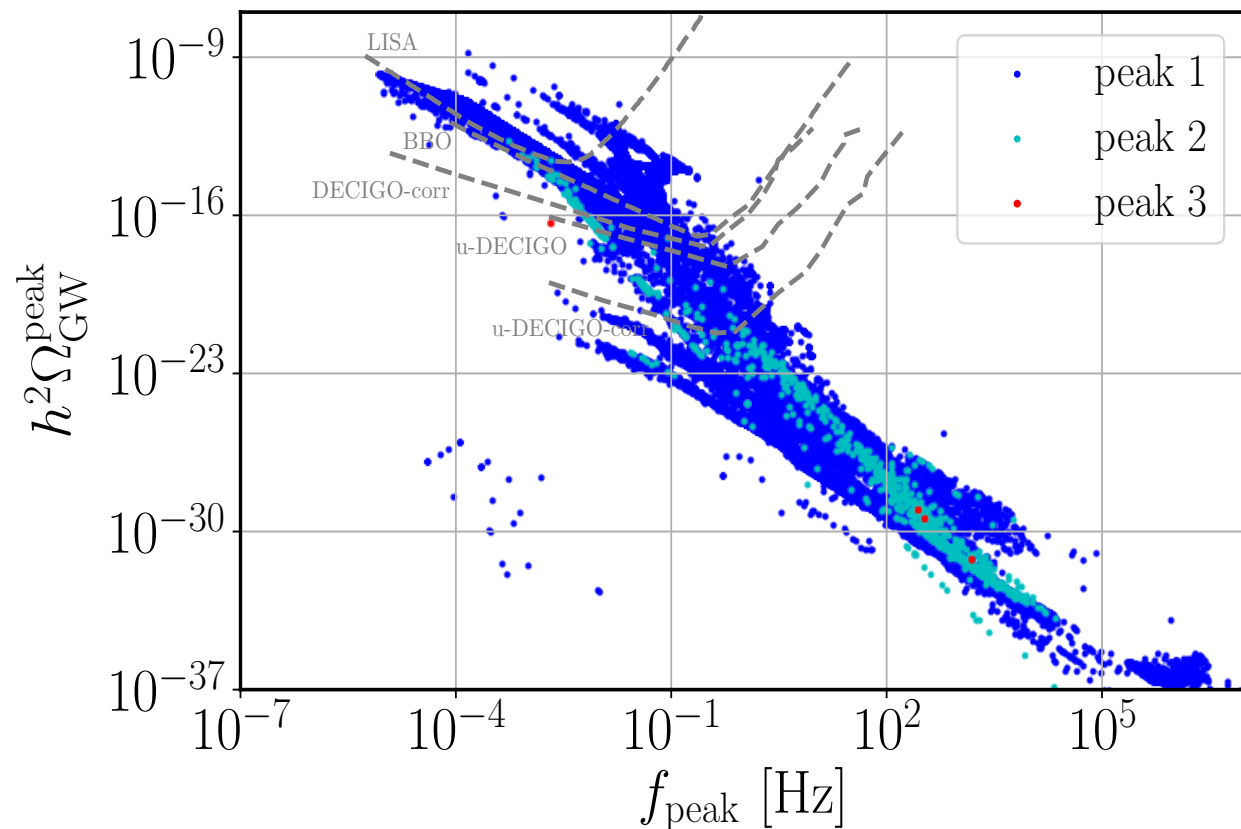
GW spectrum in  $\lambda_{\sigma h}$ , for spontaneously broken  $U(1)_L$ . No solutions consistent with the LHC bound on invisibly decays of Higgs



GW spectrum a in Yukawa coupling  $Y_{\sigma}$ , for softly-broken  $U(1)_L$  (i.e.  $v_{\sigma} = 0$ ). Order one variation of  $Y_{\sigma}$  several order of magnitude variations in the GW spectrum.

An order one variation in the Yukawa coupling reflects into violent variations by several orders of magnitude in the GWs spectrum!

# Multi-peak feature as a prediction of the Inverse seesaw with Majoron



Very hard to resolve the third peak

Multi-peaks only due to distinct phase transitions (no competition effect from the three mechanism, i.e. collision, sound waves, turbulence)

Possibility to distinguish/falsify neutrino mass generation mechanism

# Type-I seesaw with Majoron

High scale variant:

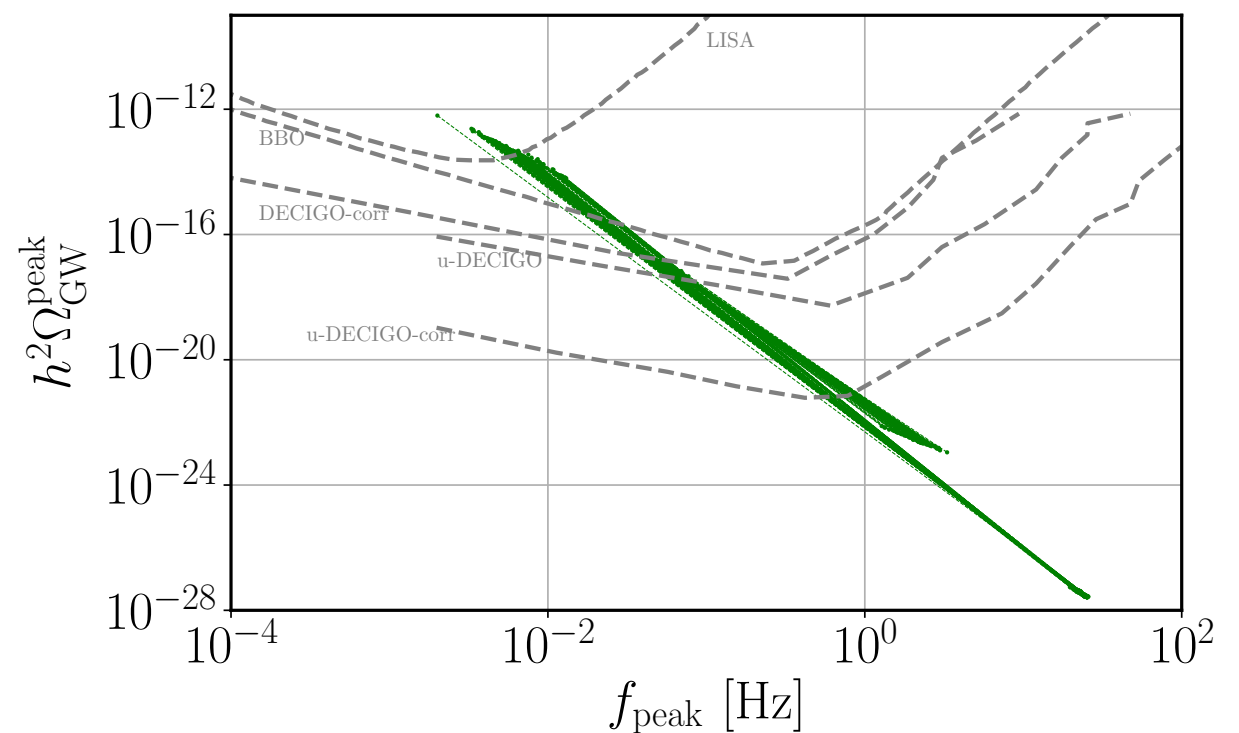
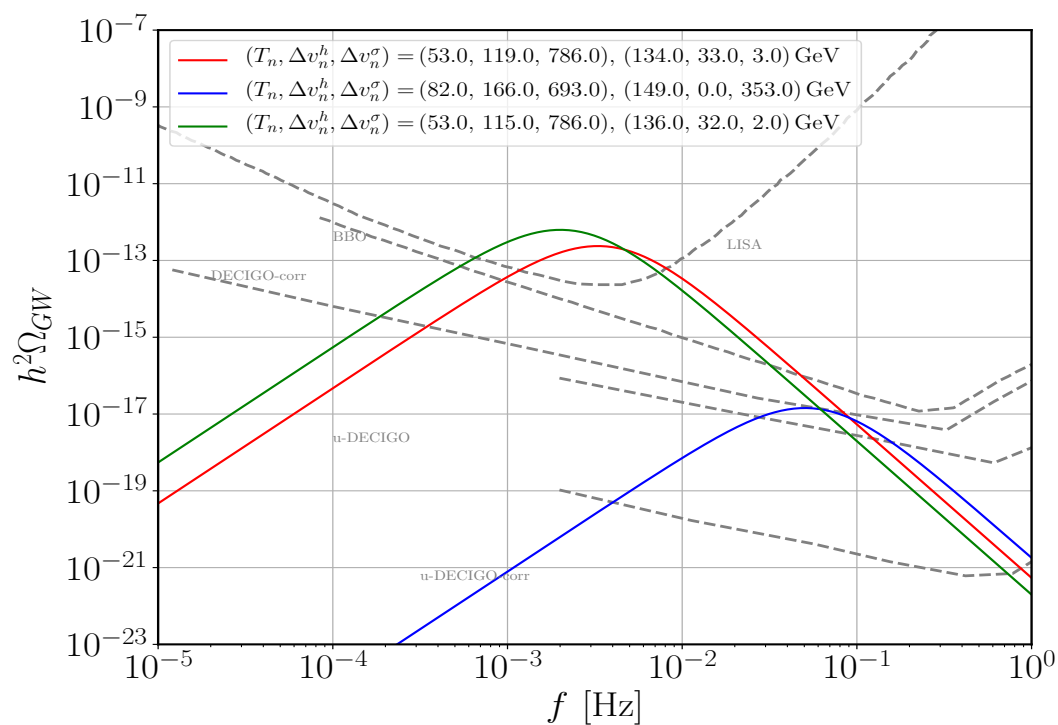
$$Y_\nu \sim \mathcal{O}(1) \rightarrow M = Y_\sigma \nu_\sigma / \sqrt{2} \sim \mathcal{O}(10^{14} \text{ GeV})$$

for  $Y_\sigma \sim \mathcal{O}(1)$  then  $\nu_\sigma \sim \mathcal{O}(10^{14} \text{ GeV}) \rightarrow \text{NO FOPT}$

Low scale variant:

$$Y_\nu \sim \mathcal{O}(10^{-6}) \rightarrow M = Y_\sigma \nu_\sigma / \sqrt{2} \sim \mathcal{O}(100 \text{ GeV})$$

**new states do not decouple: FOPT and GW are found**



**Less double peaks than in the Inverse See-Saw case, and mainly out of reach**



# Double-peak within experimental reach much rarer

**In contrast to inverse seesaw + majoron one, PT is typically much stronger hiding the smaller peak**

Curve	$m_{h_2}$	$m_A$	$\lambda_h$	$\lambda_{\sigma h}$	$\lambda_\sigma$	$\theta$	$v_\sigma$	$Y_{\sigma,1}$	$Y_{\sigma,2}$
Green	203	188	0.14	0.03	0.03	0.26	790	0.07	1.58
Red	206	188	0.14	0.03	0.03	0.26	790	0.08	1.59
Blue	205	188	0.14	-0.02	0.03	-0.18	790	0.08	1.59

Peak Id	$T_n$	$(v_h^i, v_\sigma^i) \rightarrow (v_h^f, v_\sigma^f)$	$\alpha$	$\beta/H$
Green 1	136	$(0, 921) \rightarrow (32, 919)$	$9.4 \times 10^{-5}$	$1.2 \times 10^6$
Green 2	53	$(245, 786) \rightarrow (360, 0)$	0.5	378
Red 1	134	$(0, 922) \rightarrow (33, 919)$	$10^{-4}$	$1.1 \times 10^6$
Red 2	53	$(245, 786) \rightarrow (364, 0)$	0.5	612
Blue 1	149	$(0, 353) \rightarrow (0, 0)$	$1.5 \times 10^{-3}$	$1.0 \times 10^5$
Blue 2	82	$(205, 693) \rightarrow (40, 0)$	0.03	$4.9 \times 10^3$

*Second CP-even Higgs;  $h = \cos \theta h_1 + \sin \theta h_2$*

**Consistency with Higgs invisible decays bounds assured**

# Conclusions

GW & tidal deformability as a probe of DM models and EoS of binary systems

Explicit lepton number violation cannot induce strong FOPT thus testable GW signals

Spontaneous  $U(1)_L$  breaking leads to clearer GW footprints

Type-I seesaw and inverse seesaw (Majoron) models predict GW in the 0.1 -100 mHz

**Different seesaw variants lead to distinct GW spectra!**

**Gravitational wave to shed light on the mystery of DM & neutrino mass generation!**

**Many caveats and issues to be solved: FOPT does not provide precision physics!**

谢谢

Спасибо



Thank you!

Grazie!





# Static equilibrium and TOV

$$T_{\beta}^{\alpha} = \text{Diag}(\varepsilon, p, p, p)$$

$$ds^2 = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\theta^2 + r^2 (\sin \theta)^2 d\phi^2$$



$$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$$

$$\nabla_{\alpha} T^{\alpha\beta} = 0$$

$$\left\{ \begin{array}{l} e^{-2\Lambda} = 1 - \frac{2m(r)}{r} \\ \frac{d\Phi}{dr} = \frac{1}{r} e^{2\Lambda} \left[ 8\pi^2 p(r) + \frac{2m(r)}{r} \right] \\ \frac{dp}{dr} = -(p(r) + \varepsilon(r)) \frac{d\Phi}{dr} \end{array} \right.$$

System is closed by an EoS:  $p = p(\varepsilon)$