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Effects of 2HDM in Electroweak Phase Transition

Arnab Chaudhuri, Profesor Maxim Yu. Khlopov, Shiladitya Porey

BLED WORKSHOPS IN PHYSICS

NOVOSIBIRSK STATE UNIVERSITY

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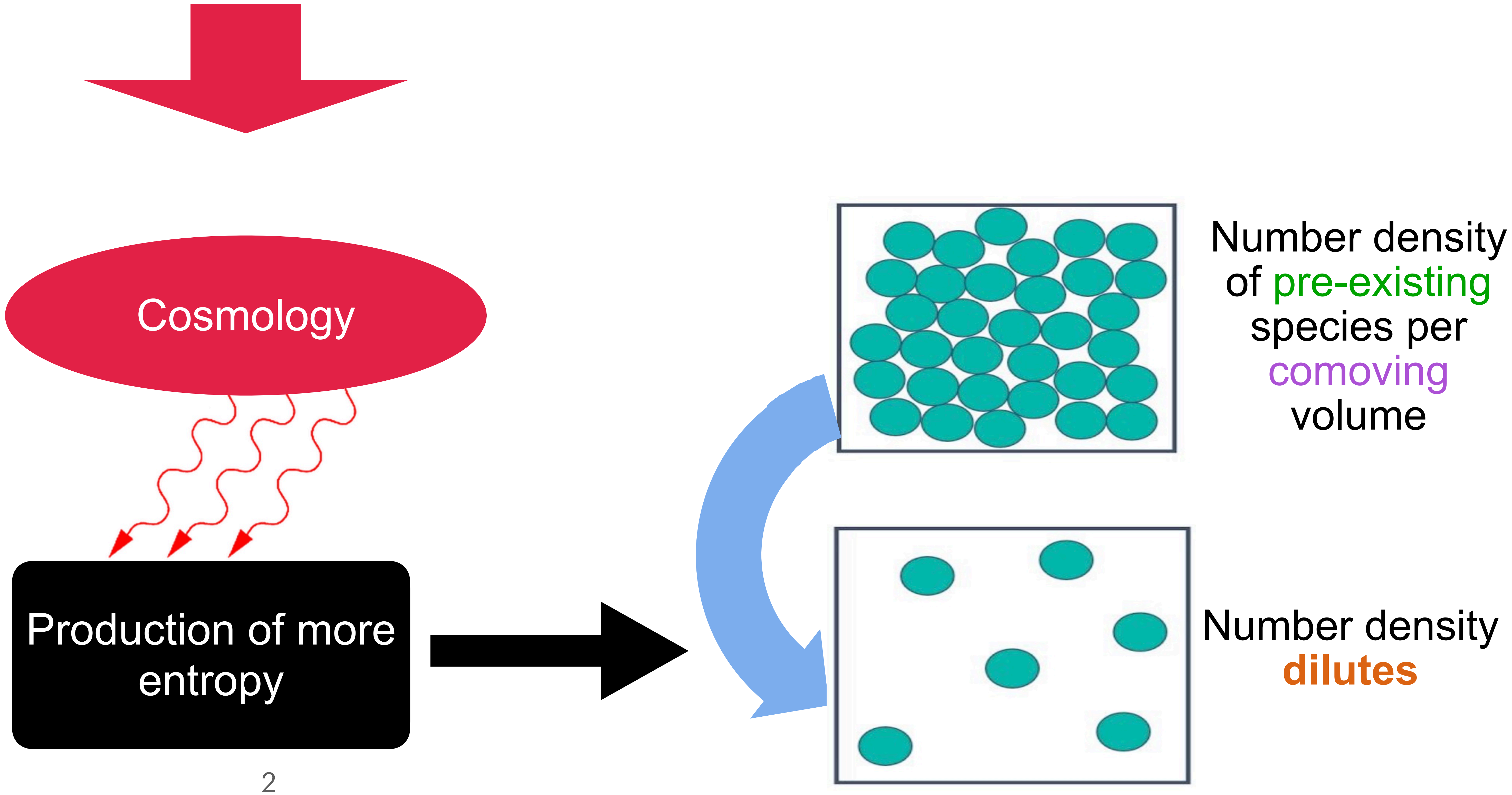
Cosmology | **Particle Physics**

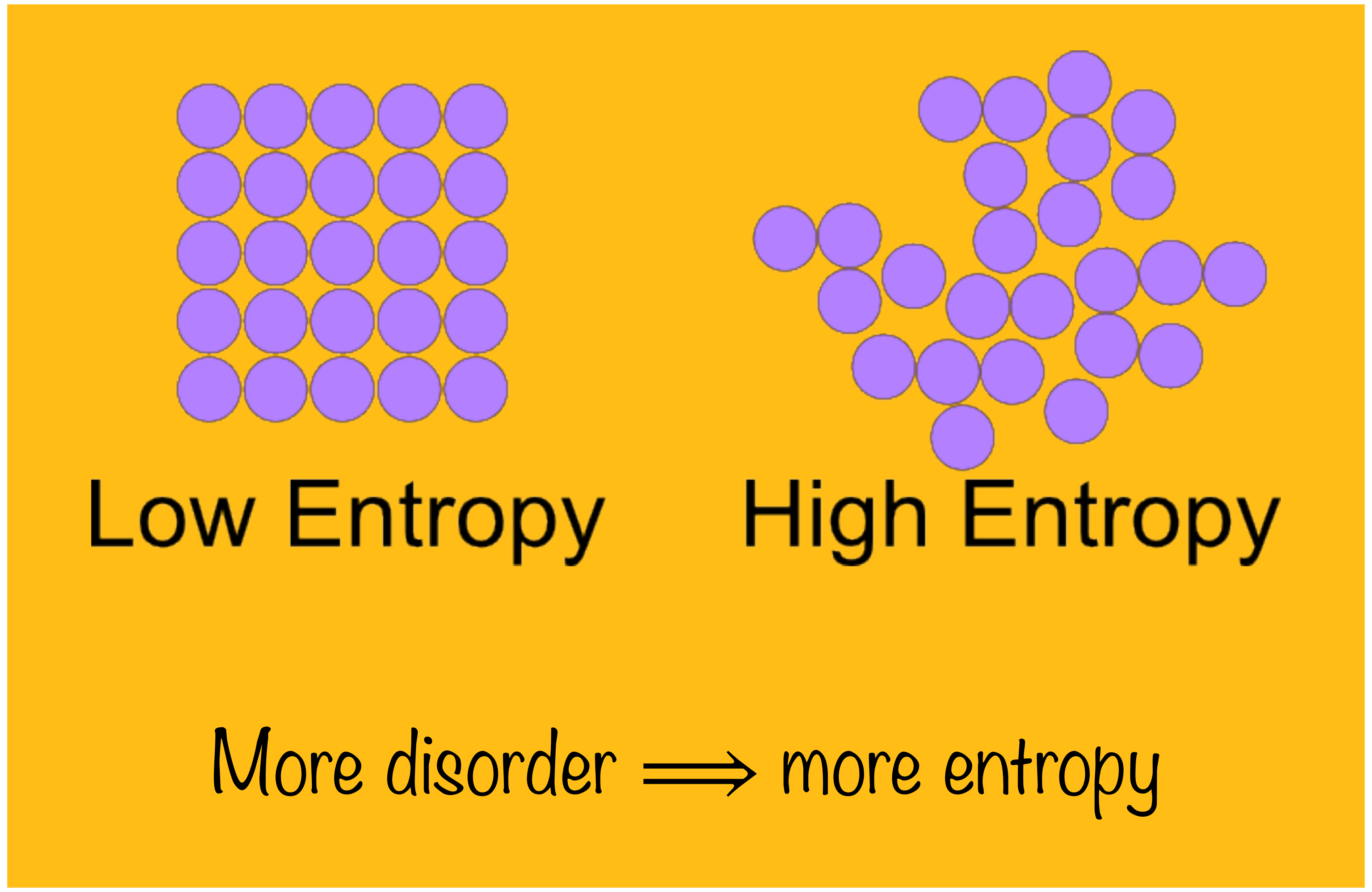
Electroweak Phase Transition

1. Effects of 2HDM in Electroweak Phase Transition
Arnab Chaudhuri, Maxim Yu. Khlopov, Shiladitya Porey
e-Print: 2105.10728 [hep-ph]
DOI: 10.3390/galaxies9020045

2. Entropy production due to electroweak phase transition in the framework of two Higgs doublet model
Arnab Chaudhuri, Maxim Yu. Khlopov
e-Print: 2103.03477 [hep-ph]
DOI: 10.3390/physics3020020



Entropy



- Thermal equilibrium $\implies$ Maximum entropy.
- Entropy never **decreases**.

$$d\mathcal{S} = \frac{\left[d(\rho V) + P dV - \mu_{\text{chm. pot.}} dN \right]}{T}$$

energy density (ρ); Pressure (P); Volume (V); Change in number of particles (dN); Temperature (T); Chemical potential ($\mu_{\text{chm. pot.}}$)

For

A) flat FLRW metric and $T \gg \left(\mu_{\text{chm. pot.}} - m\right)$ and

B) if $V \propto \left(a(t)\right)^3$, $a(t)$ is the scale factor and

C) if energy density is conserved then

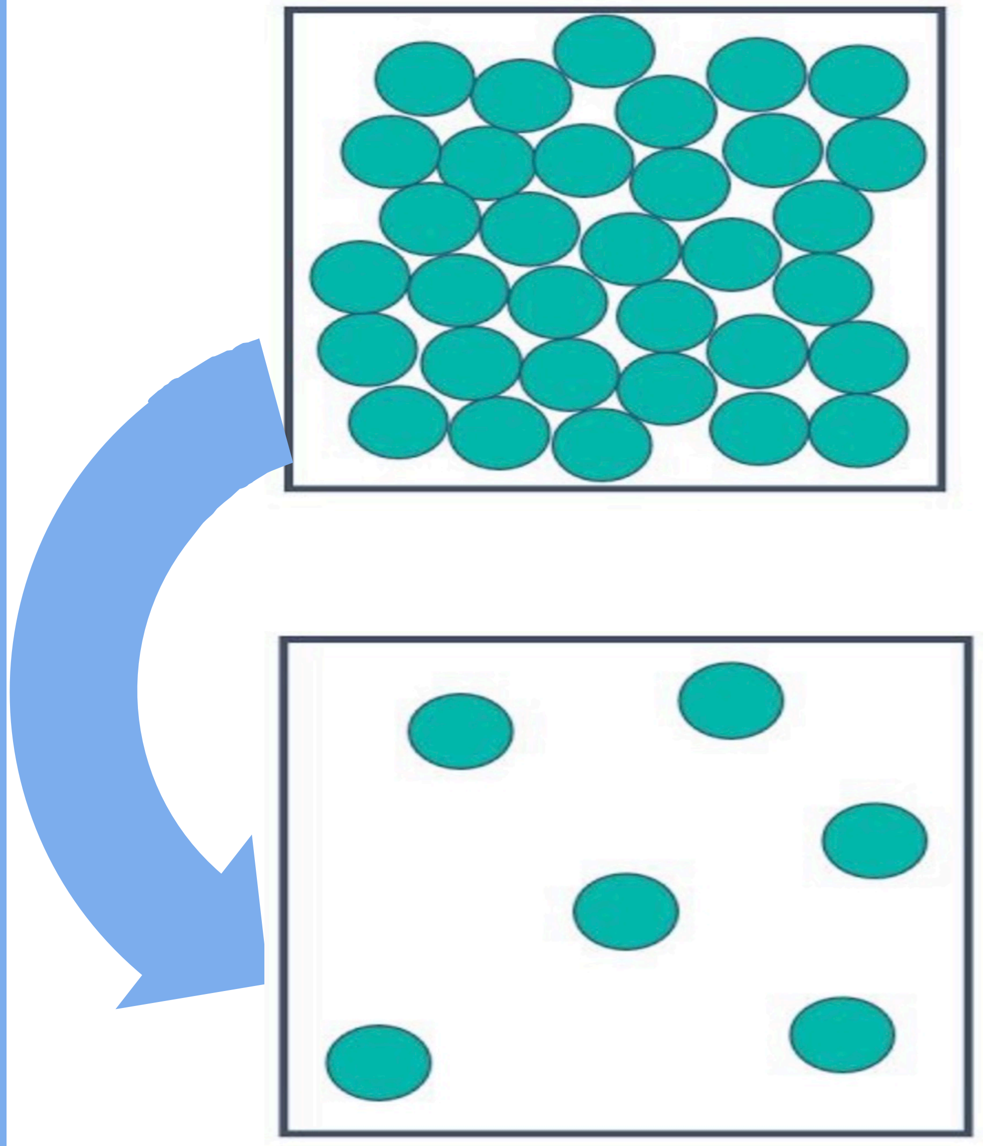
D) if in thermal equilibrium

$$d\mathcal{S} = 0$$

- Entropy density $s \equiv \frac{\mathcal{S}}{V} = \frac{[(\rho + P)]}{T}$ with $T \gg \left(\mu_{\text{chm. pot.}} - m\right)$ (page12-13 of arXiv:0907.0668)

- Conservation of entropy $(\mathcal{S}) \implies s \propto \left(a(t)\right)^{-3}$; $a(t)$ is the scale factor.

- Number of particles in a comoving volume $N_i \equiv \frac{n_i}{s}$



Number density of pre-existing species per comoving volume

Number density dilutes

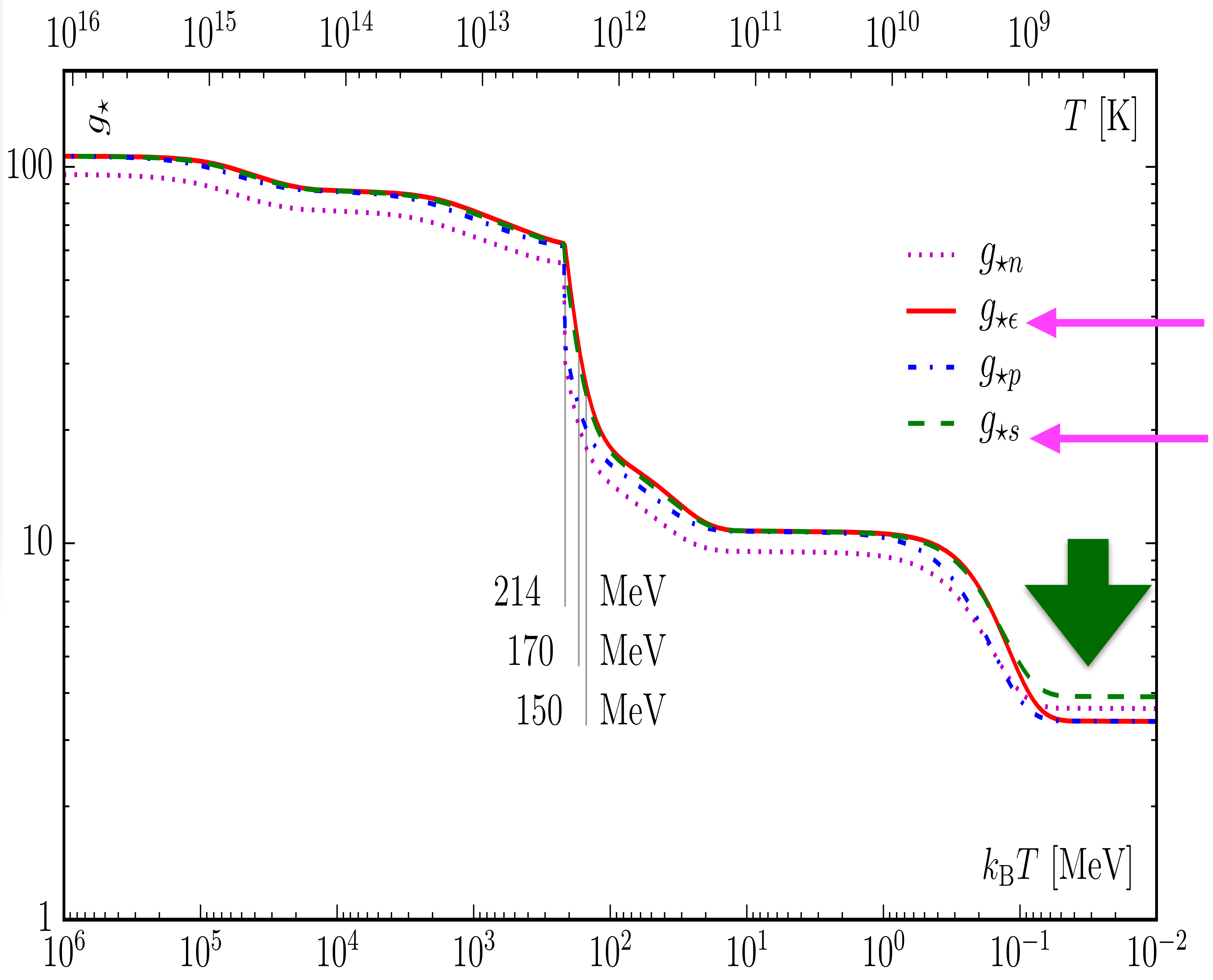
- (In equilibrium) $f(p) = (exp[p/T] \pm 1)^{-1}$ (with $m/T \to 0$)

- For relativistic species, $\mathscr{P}_r = \frac{1}{3}\rho_r$
- $\rho_r + \mathscr{P}_r = \frac{2\pi^2}{45} g_\star \left[T(t)\right]^4$;

- $s \equiv \mathcal{S}/V = \frac{2\pi^2}{45} g_{\star,S} \left[T(t)\right]^3$

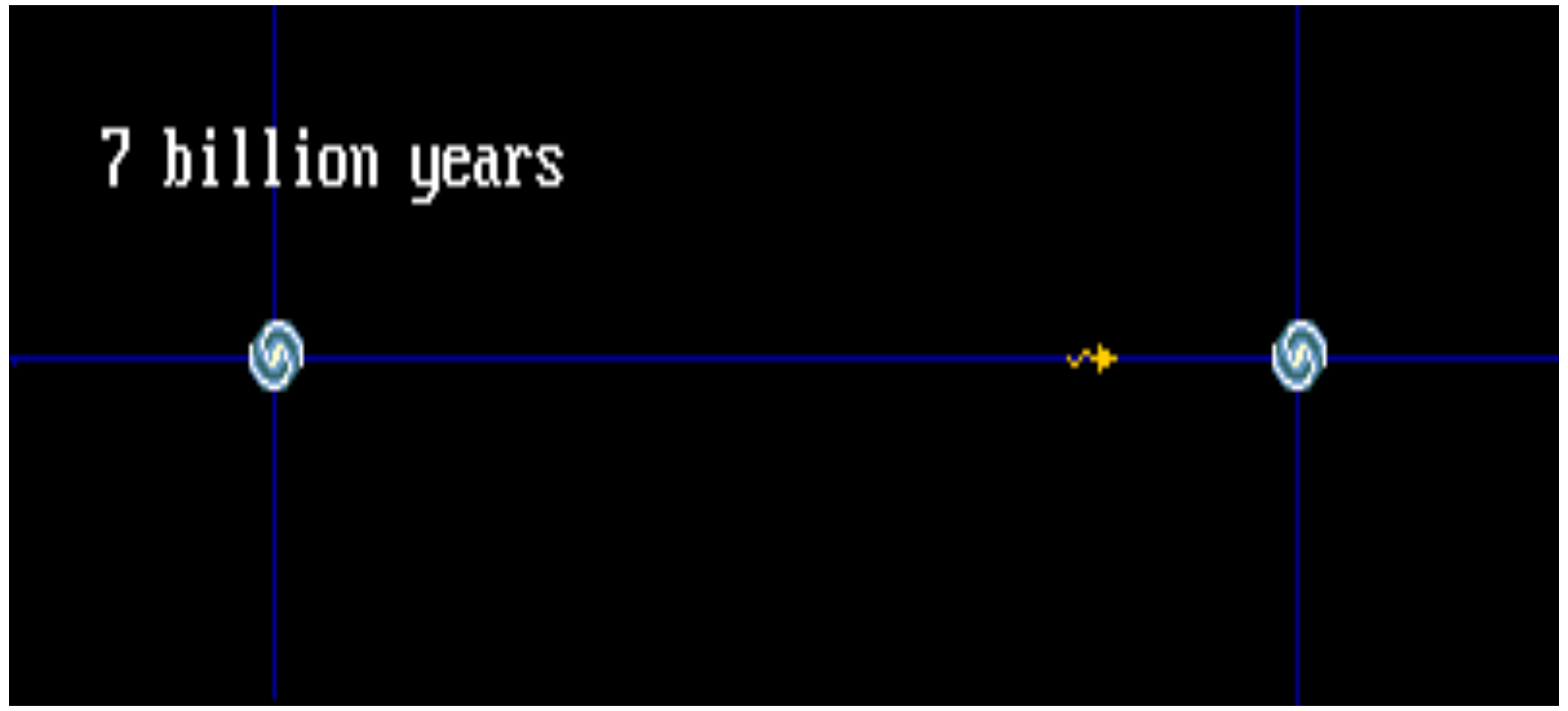
$$g_{\star,S} = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=\text{fermios}} g_i \left(\frac{T_i}{T}\right)^3$$

$g_\star \approx g_{\star,S}$ as long as thermal equilibrium and $t \sim 1$s.



Ref: 10.3390/galaxies4040078

FLRW cosmology & Comoving volume



Wikimedia

- The "size" of Comoving volume in Comoving coordinate system is constant.
- Physical length scale at time t, $R(t) = a(t) \times l_0$, comoving length scale.

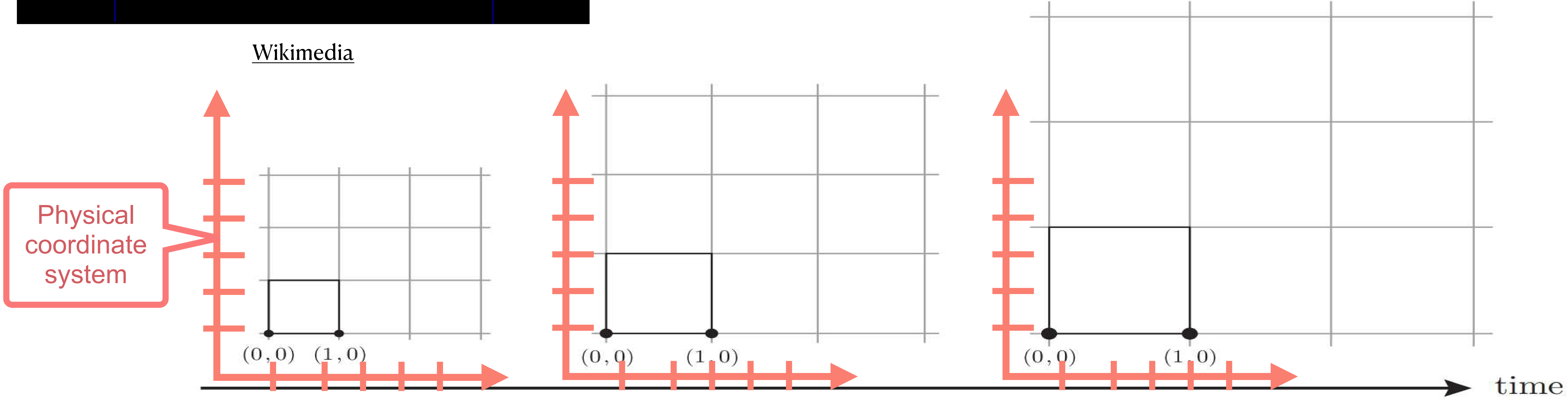
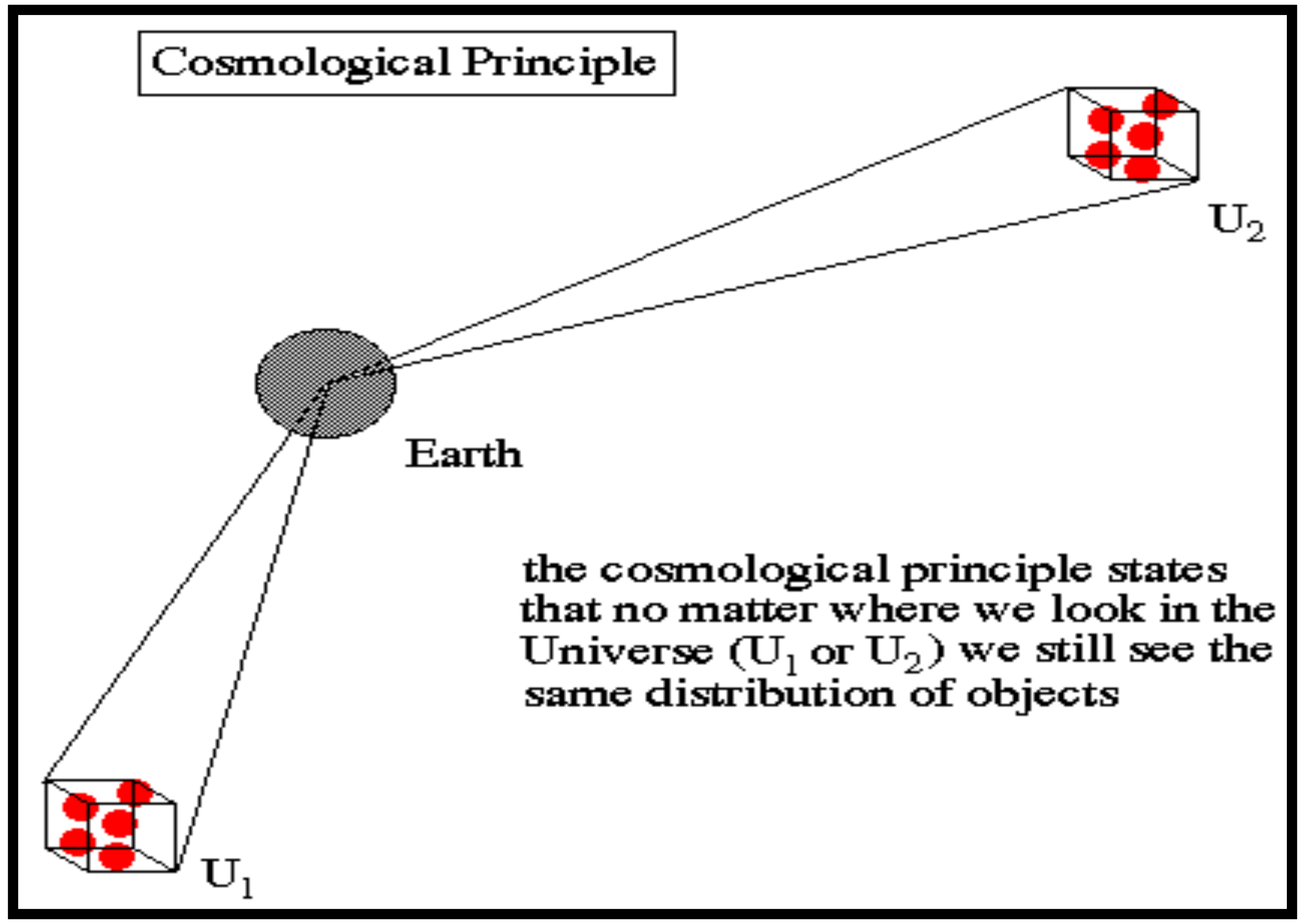
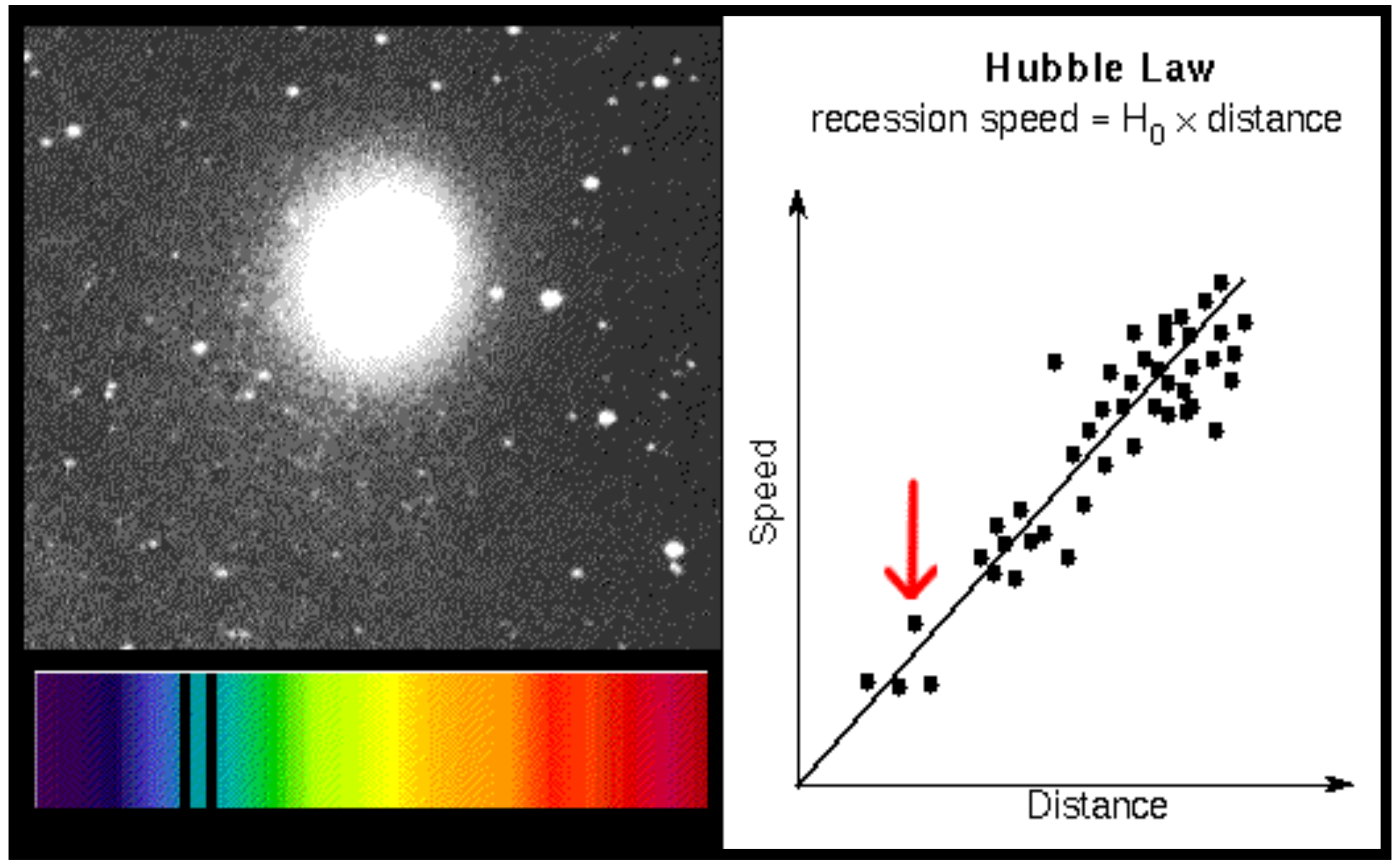


Figure 1.3: *Expansion of the universe. The comoving distance between points on an imaginary coordinate grid remains constant as the universe expands. The physical distance is proportional to the comoving distance times the scale factor $a(t)$ and hence gets larger as time evolves.*

Credit: Professor Daniel Baumann



James Schombert

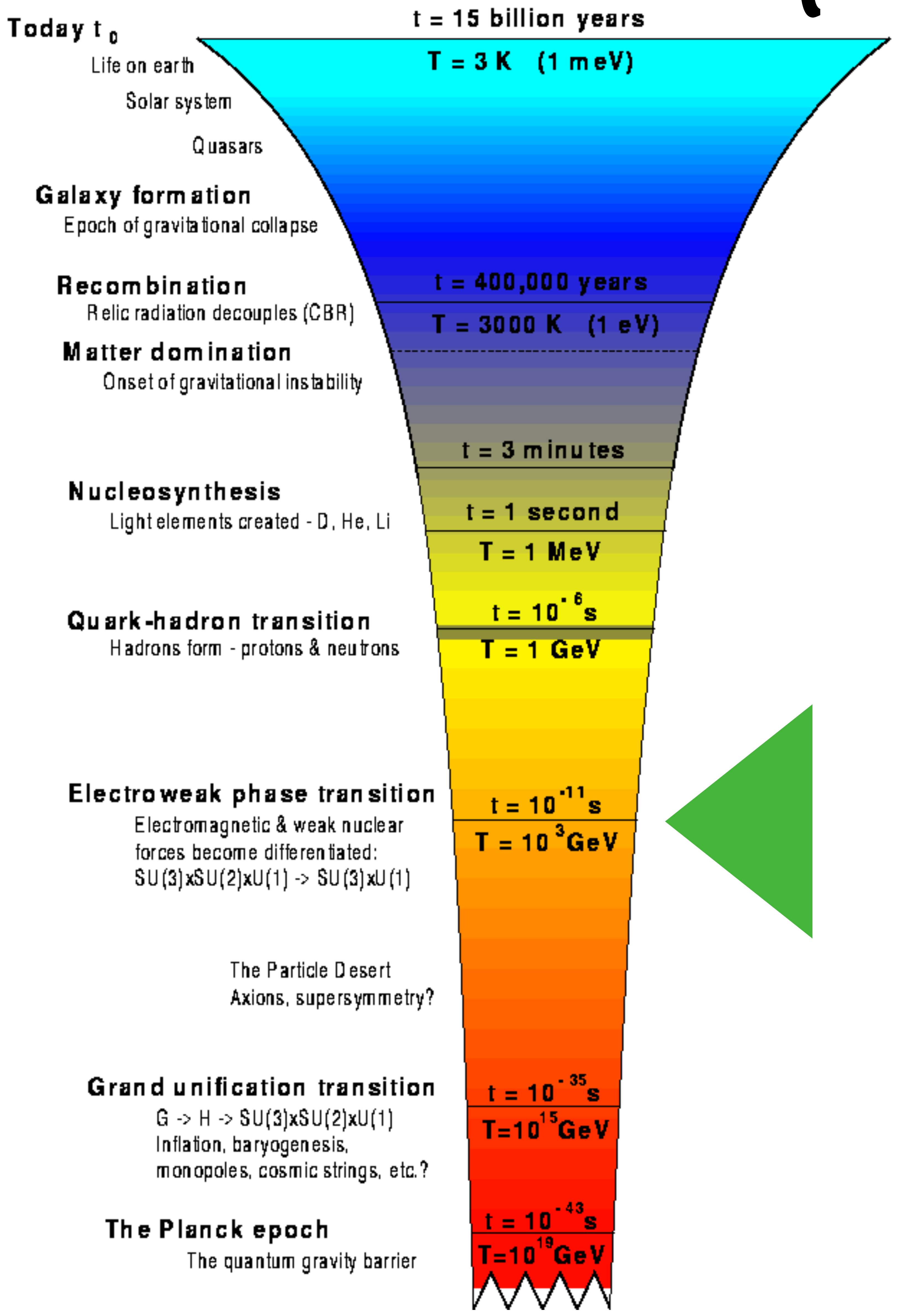


Prof. Richard Gelderman

Universe is homogeneous and isotropic on large scales.

$$v = H(t)\, D$$

Thermal equilibrium in early universe



$$\Gamma = n\sigma v \gg H$$

$$\Gamma \sim \alpha^2 T^5$$
$$H \sim T^2/m_{Pl}$$

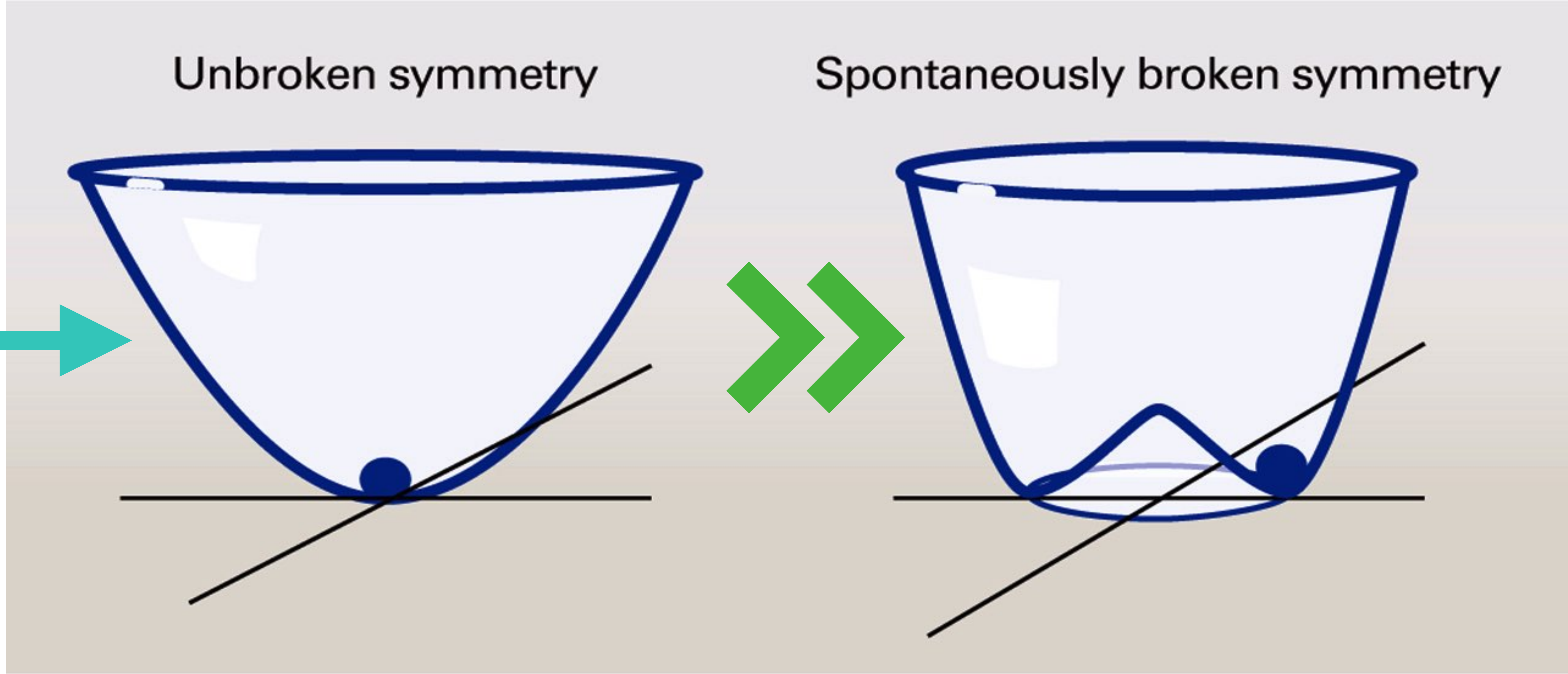
$$\Gamma/H \implies (\alpha^2 T^3 m_{Pl}) \simeq 1$$

Prof. Kandaswamy Subramanian (Semikoz's website)

Electroweak Phase Transition

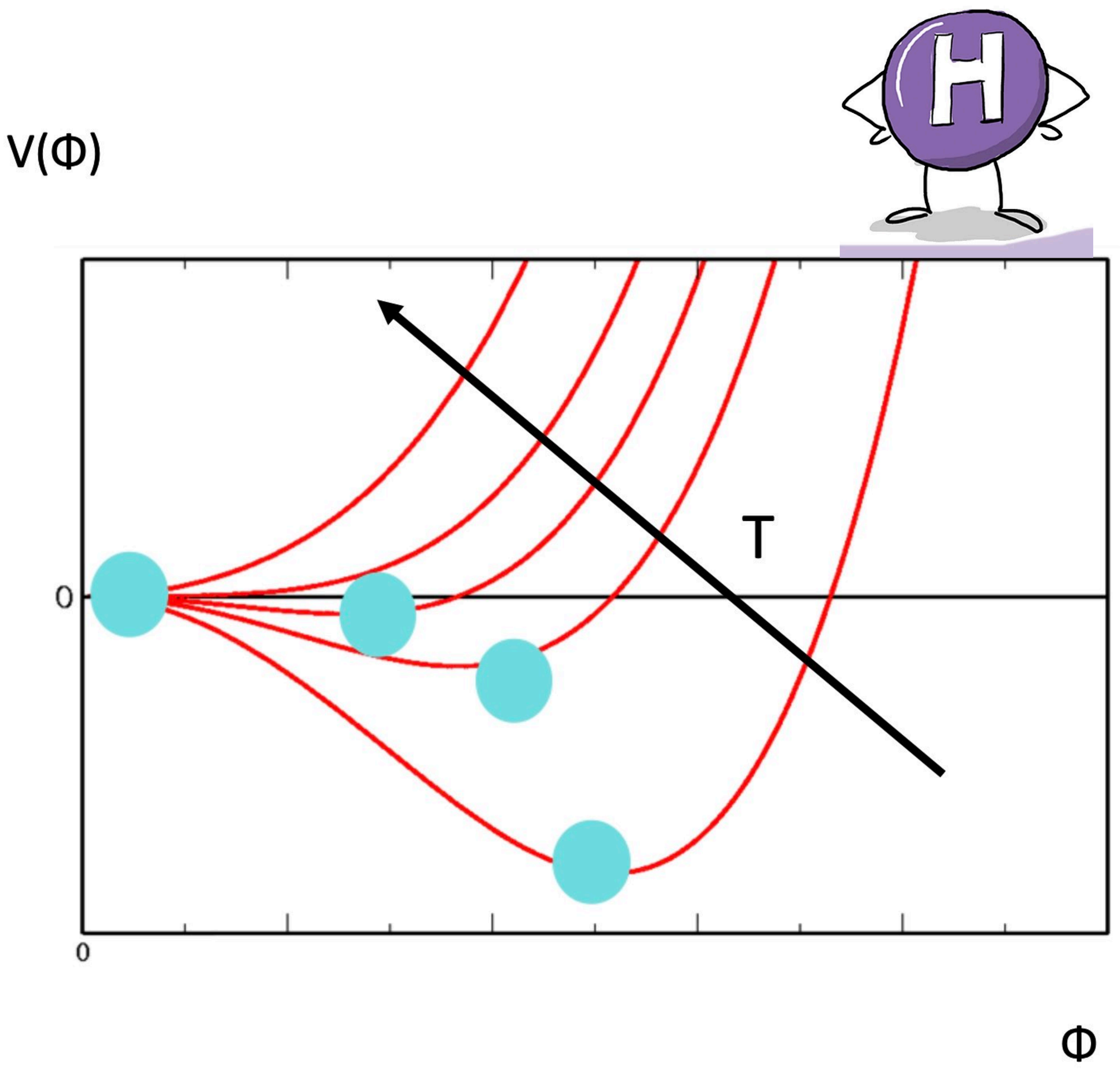
Temperature decreasing

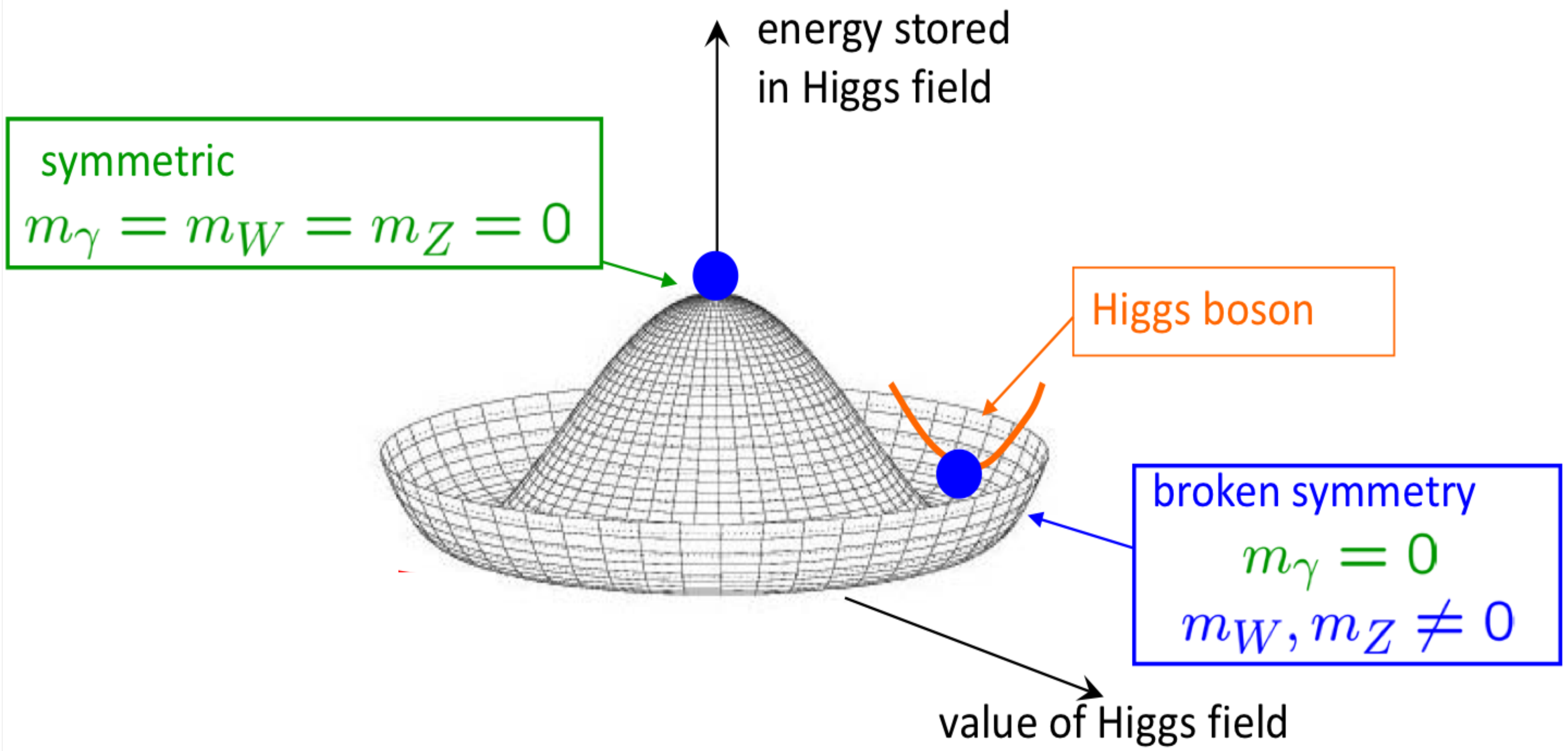
Higgs field
Potential



DOI: 10.1126/science.337.6100.1289

Piled Higher and Deeper





Prof. Mark Neubauer

The particle decouples, when thermal mass $T \lesssim m_f(T)$

$$m_f^2(T) = g_f^2 \Phi_{\min}^2(T)$$

Event	Temperature	$g_{\star s}$
		106.75
Annihilation of $t\bar{t}$ quarks	<173.3 GeV	
		96.25
Annihilation of Higgs boson	<125.6 GeV	
		95.25
Annihilation of Z^0 boson	<91.2 GeV	
		92.25
Annihilation of W^+W^- bosons	<80.4 GeV	
		[illegible]
Annihilation of $b\bar{b}$ quarks	<4190 MeV	
		[illegible]
Annihilation of $\tau^+\tau^-$ leptons	<1777 MeV	
		72.25
Annihilation of $c\bar{c}$ quarks	<1290 MeV	
		61.75
QCD transition †	150–214 MeV	
		17.25
Annihilation of $\pi^+\pi^-$ mesons	<139.6 MeV	
		15.25
Annihilation of π^0 mesons	<135.0 MeV	
		14.25
Annihilation of $\mu^+\mu^-$ leptons	<105.7 MeV	
		10.75
Neutrino decoupling	<800 keV	
		7.409
Annihilation of e^+e^- leptons	<511.0 keV	
		3.909

It will change

† Using lattice QCD, this transition is normally calculated to 150–170 MeV.

Ref: 10.3390/galaxies4040078

Beyond the standard model? 2HDM

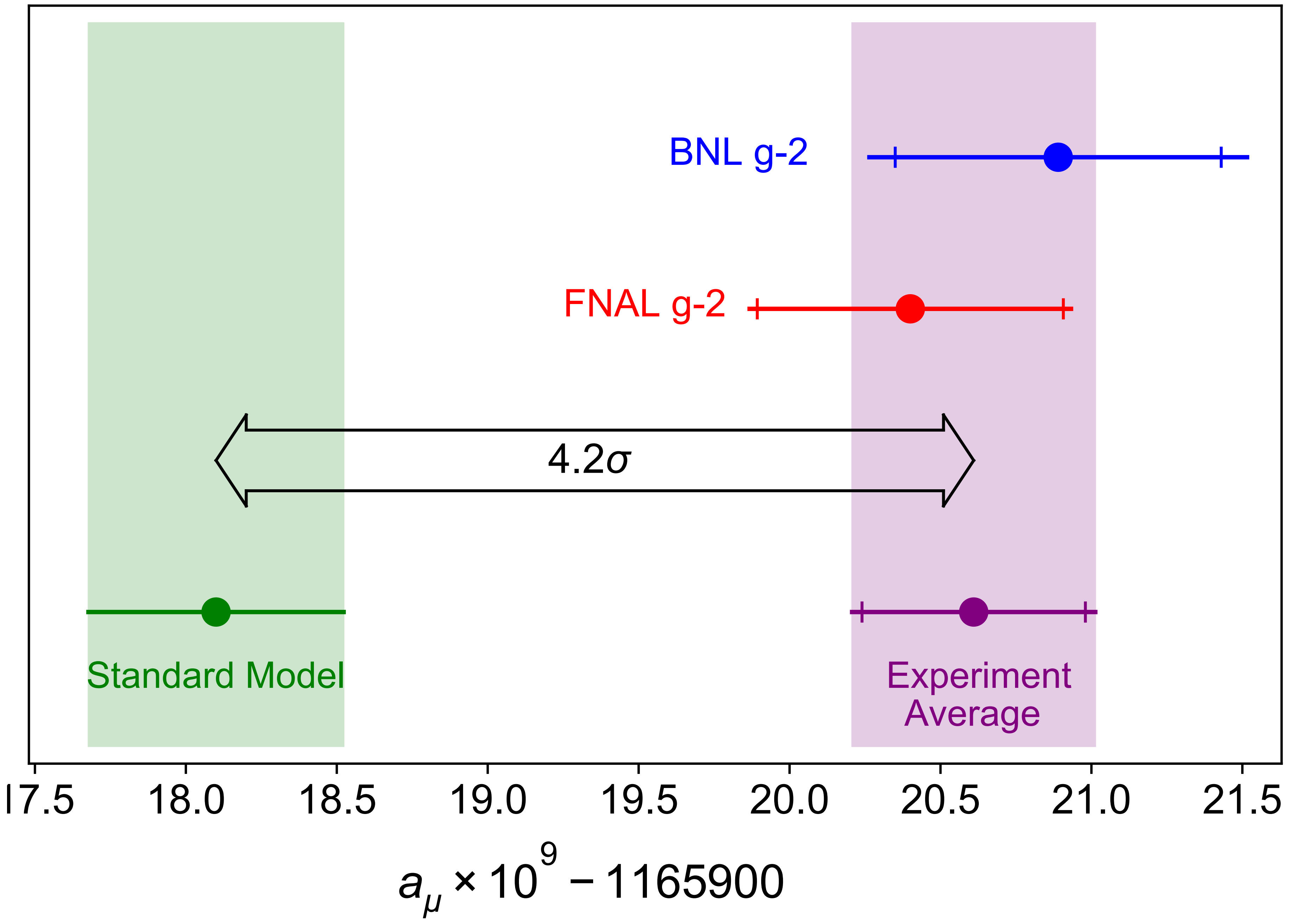
The gyromagnetic ratio of muon, g_μ,

$$g_\mu = \frac{\text{magnetic moment } (e\hbar)/(2m_\mu)}{\text{angular momentum } (\hbar/2)} \approx 2$$

$$a_\mu = \frac{g_\mu - 2}{2}$$

Muon anomalous magnetic moment :

$$a_\mu = \begin{cases} 116{,}591{,}810(43) \ \times \ 10^{-11} \ \text{(SM calculated value)} \\ 16{,}592{,}061(41) \ \times \ 10^{-11} \ \text{(FERMILAB - 2021)} \end{cases}$$



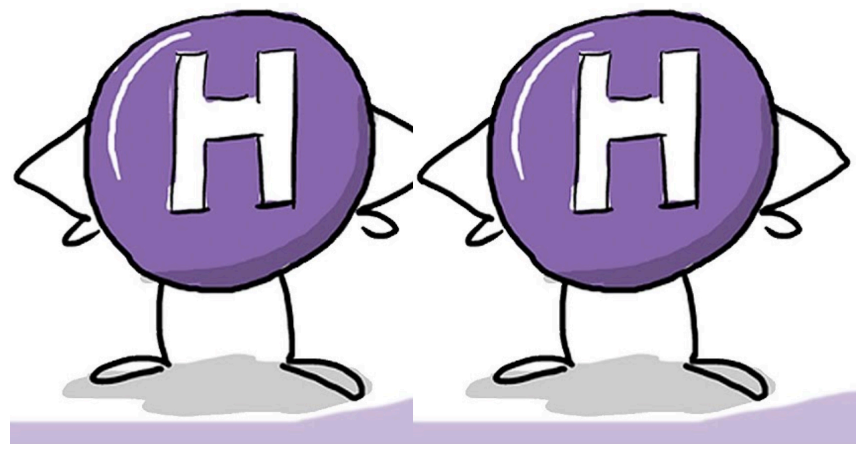
(Muon g−2 Collaboration) Phys. Rev. Lett. **126**, 141801

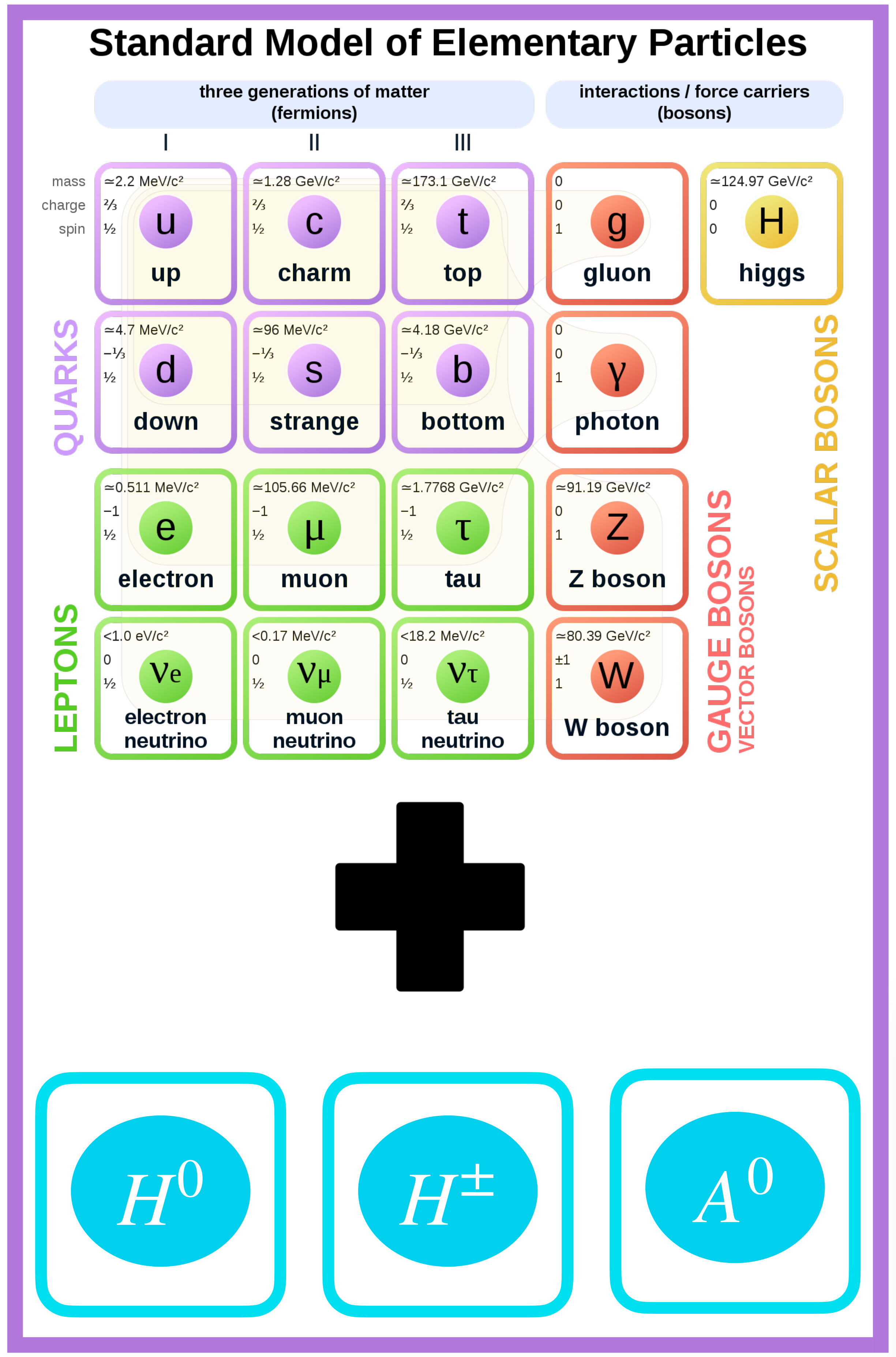
Revisiting lepton-specific 2HDM in light of muon $g−2$ anomaly
Want et al.
DOI: 10.1016/j.physletb.2018.11.045

Fermion masses and mixings, dark matter, leptogenesis and $g-2$ muon anomaly in an extended 2HDM with inverse seesaw
Cárcamo Hernández et al.
e-Print: 2104.02730 [hep-ph]

Light Pseudoscalars, Particle Physics and Cosmology
Jihn E. Kim
DOI: 10.1016/0370-1573(87)90017-2

Lagrangian of EWPT theory in real type-I 2HDM





Original Lagrangian:

$$\mathscr{L} = \mathscr{L}_{\text{gauge,kin}} + \mathscr{L}_f + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yuk}} - V_{\text{tot}}(\Phi_1, \Phi_2, T)$$

Simplified Lagrangian:

$$\mathscr{L} = g^{\mu\nu}\partial_\mu\Phi_a\,\partial_\nu\Phi_a - V_{\text{tot}}(\Phi_1, \Phi_2, T) + \sum_j i\left[g^{\mu\nu}\partial_\mu\chi_j^\dagger\,\partial_\nu\chi_j - U_j(\chi_j)\right] + \mathscr{L}_{int}$$

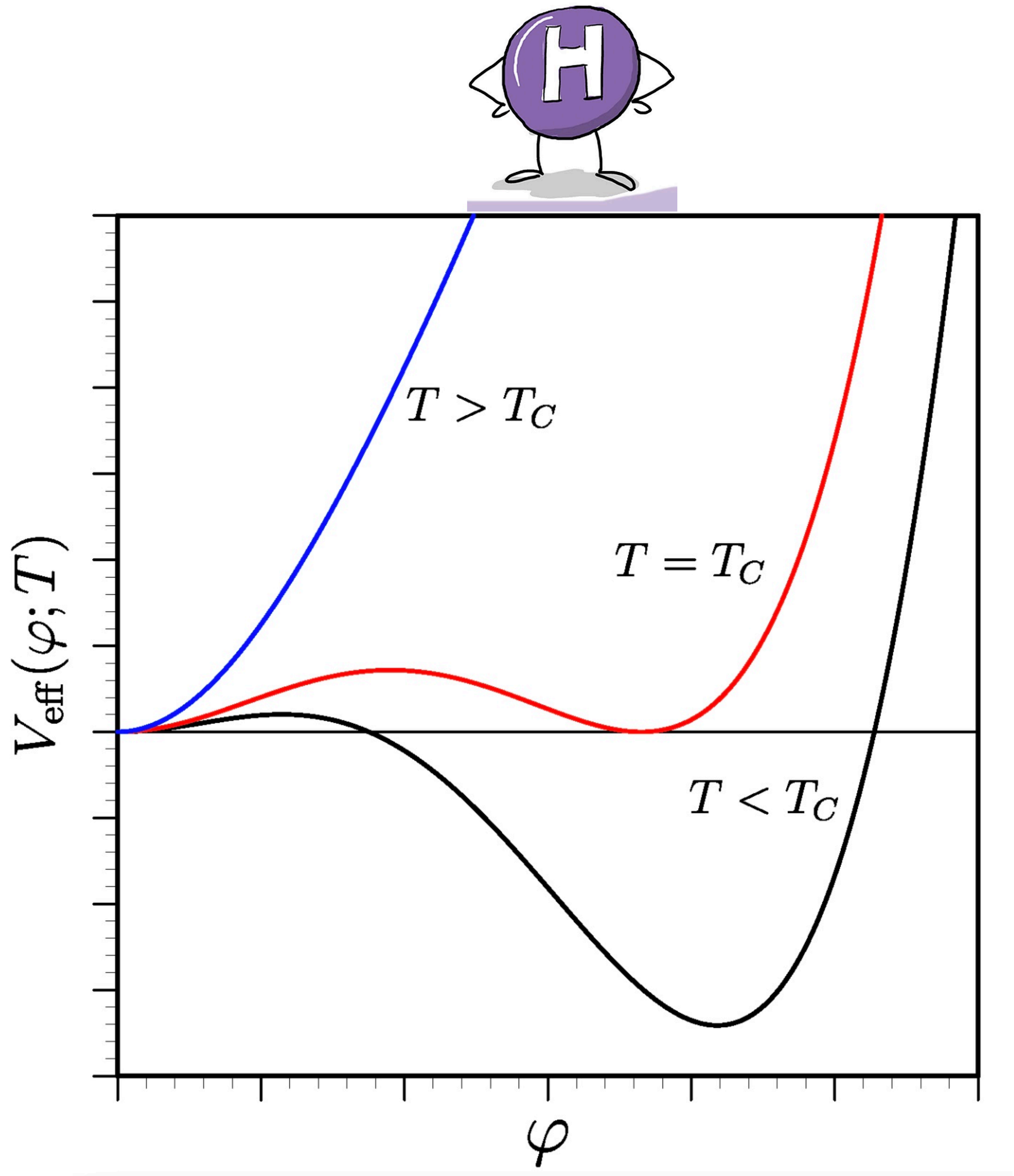
$$\mathscr{L}_{int} = \Phi_a \sum_j g_j \chi_j^\dagger \chi_j\,.$$

$$\rho = \dot{\Phi}_a^2 + V_{\text{tot}}(\Phi_1, \Phi_2, T) + \sum_j \left[\dot{\chi}_j^\dagger\dot{\chi}_j + \partial_l\chi_j^\dagger\,\partial_l\chi_j/a^2 + U_j(\chi_j)\right] - \mathscr{L}_{int}$$

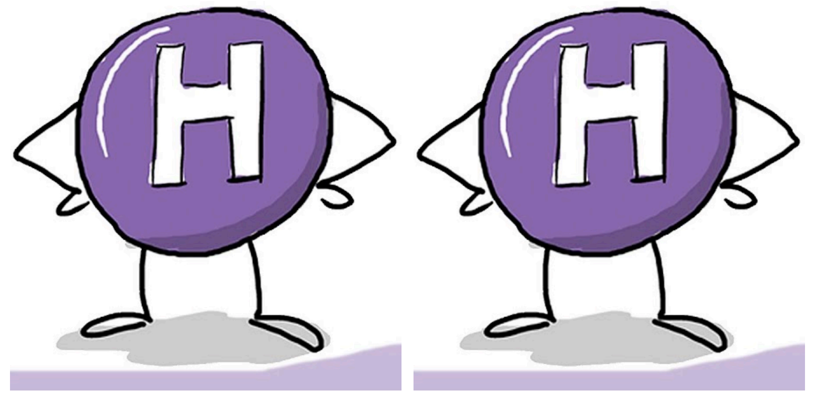
$$\mathscr{P} = \dot{\Phi}_a^2 - V_{\text{tot}}(\Phi_1, \Phi_2, T) + \sum_j \left[\dot{\chi}_j^\dagger\dot{\chi}_j - (1/3)\partial_l\chi_j^\dagger\,\partial_l\chi_j/a^2 - U_j(\chi_j)\right] + \mathscr{L}_{int}$$

$$\rho + \mathscr{P} = 2\dot{\Phi}_a^2 + + \sum_j \left[2\dot{\chi}_j^\dagger\dot{\chi}_j + \frac{2}{3a^2}\partial_l\chi_j^\dagger\,\partial_l\chi_j\right] \approx + \frac{4}{3}\frac{\pi^2 g_*}{30}T^4$$

"The oscillations of Φ_a around $\Phi_{a,\min}$ are quickly damped, so we take $\dot{\Phi}_a = \dot{\Phi}_{a,\min}$ and neglect $\dot{\Phi}_{a'}^2$ because the evolution of $\Phi_{a,\min}$ is induced by the universe expansion which is quite slow. "

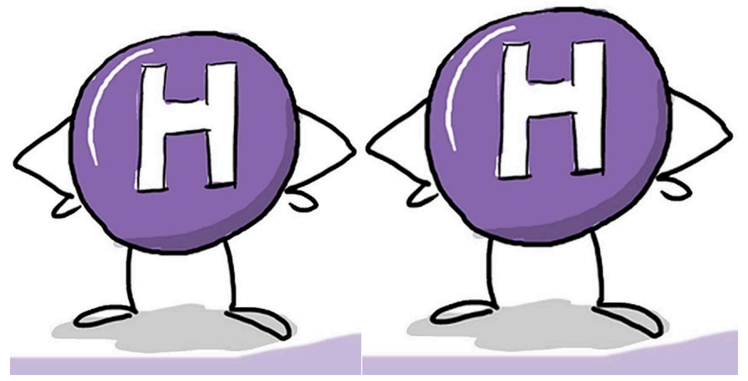


DOI:10.3390/sym12050733



$$V_{\rm tot}(\Phi_1, \Phi_2, T) = V_{tree}(\Phi_1, \Phi_2) + V_{CW}(\Phi_1, \Phi_2) + V_T(T) + V_{\rm daisy}(T)$$

$$V_{\rm tot}\left(\Phi_1 = 0, \Phi_2 = 0, T_c\right) = V_{\rm tot}\left(\Phi_1 = v_1, \Phi_2 = v_2, T_c\right)$$



Piled Higher and Deeper

$$V_{\text{tot}}(\Phi_1, \Phi_2, T)$$

$$V_{\text{tree}}(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \left(m_{12}^* \right)^2 \Phi_2^\dagger \Phi_1 \right]$$
$$+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right)$$
$$+ \left[\frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \frac{1}{2}\lambda_5^* \left(\Phi_2^\dagger \Phi_1 \right)^2 \right]$$

$$\lambda_1 = \frac{(\cos\alpha)^2 m_{H_0}^2 + m_h^2 (\sin\alpha)^2 - \mu^2 \tan\beta}{(\cos\beta)^2 {v_{\text{sm}}}^2}$$

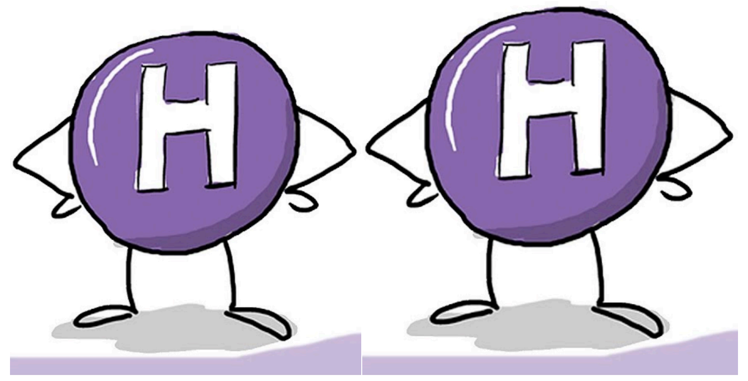
$$\lambda_2 = \frac{(\cos\alpha)^2 {m_h}^2 + {m_{H_0}}^2 (\sin\alpha)^2 - \frac{\mu^2}{\tan\beta}}{(\sin\beta)^2 {v_{\text{sm}}}^2}$$

$$\lambda_3 = \frac{\frac{\sin 2\alpha \left({m_{H_0}}^2 - {m_h}^2 \right)}{\sin 2\beta} + 2{m_{H_\pm}}^2 - \frac{2\mu^2}{\sin 2\beta}}{{v_{\text{sm}}}^2}$$

$$\lambda_4 = \frac{{m_A}^2 - 2{m_{H_\pm}}^2 + \frac{2\mu^2}{\sin 2\beta}}{{v_{\text{sm}}}^2} \qquad \lambda_5 = \frac{\frac{2\mu^2}{\sin 2\beta} - {m_A}^2}{{v_{\text{sm}}}^2}$$

$$v_1 = \sqrt{\frac{{v_{\text{sm}}}^2}{(\tan\beta)^2 + 1}}$$

$$v_2 = \sqrt{\frac{(\tan\beta)^2 {v_{\text{sm}}}^2}{(\tan\beta)^2 + 1}}$$



Piled Higher and Deeper

$$V_{\text{tot}}(\Phi_1, \Phi_2, T)$$

$$\begin{aligned} V_{\text{tree}}(\Phi_1, \Phi_2) = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + m_{12}^* \Phi_2^\dagger \Phi_1 \right] \\ & + \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 \\ & + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \\ & + \left[\frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \frac{1}{2}\lambda_5^* \left(\Phi_2^\dagger \Phi_1 \right)^2 \right] \end{aligned}$$

$$V_{\text{CW}}\left(v_1 + v_2\right) = \sum_j \frac{n_j}{64\pi^2} (-1)^{2s_j} m_j^4\left(v_1, v_2\right) \left[\log\left(\frac{m_j^2\left(v_1, v_2\right)}{\mu^2} \right) - c_j \right]$$

$$V_T = \frac{T^4}{2\pi^2} \left(\sum_{j=\text{bosons}} n_j J_B \left[\frac{m_j^2(v_1, v_2)}{T^2} \right] + \sum_{j=\text{fermions}} n_j J_F \left[\frac{m_j^2(v_1, v_2)}{T^2} \right] \right)$$

$$V_{\text{daisy}}(T) = -\frac{T}{12\pi} \left[\sum_{j=1}^{n_{\text{Higgs}}} \left((\bar{m}_j^2)^{3/2} - (m_j^2)^{3/2} \right) + \sum_{j=1}^{n_{\text{gauge}}} \left((\bar{m}_j^2)^{3/2} - (m_j^2)^{3/2} \right) \right]$$

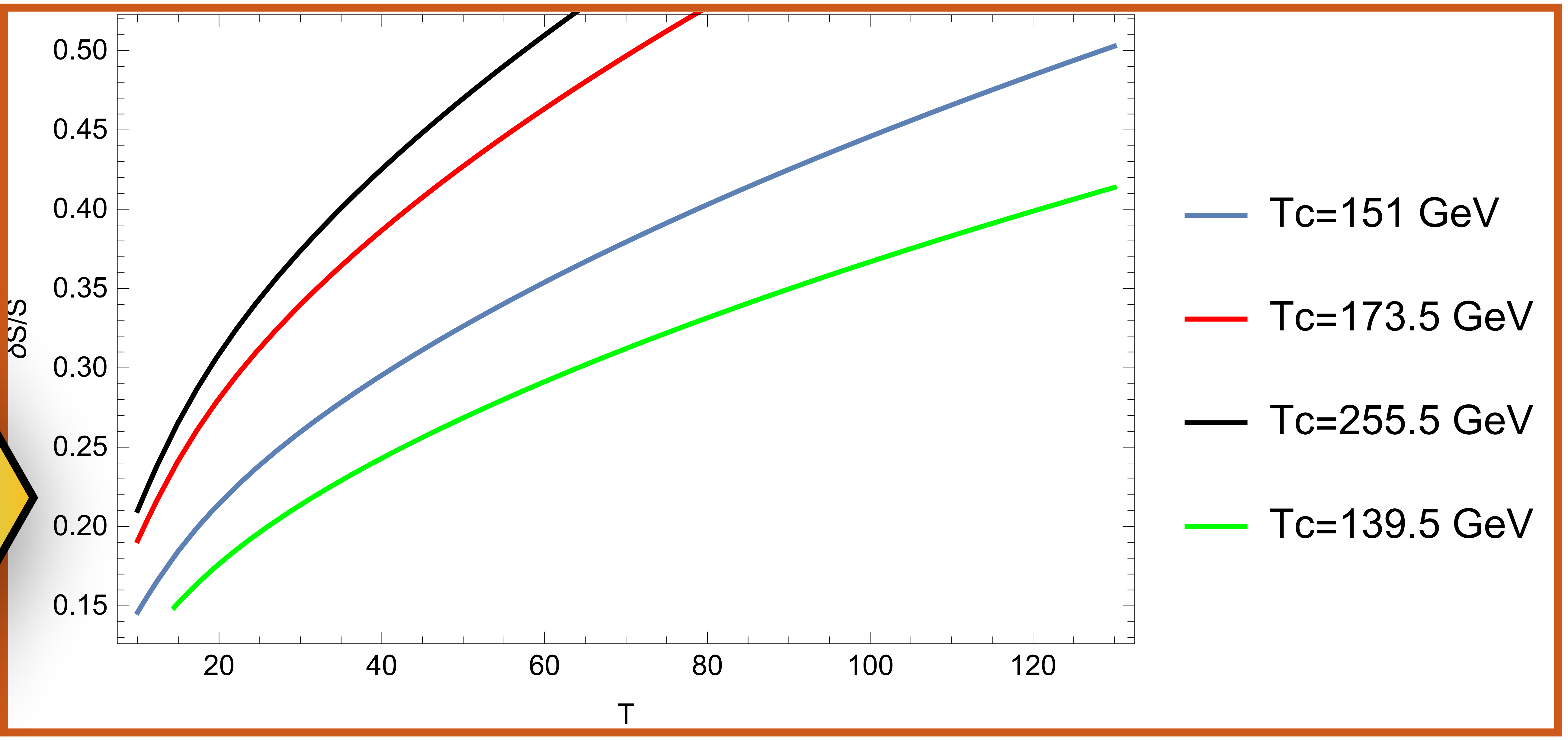
Entropy release

Relaxed Constraints on Masses of New Scalars in 2HDM
Siddhartha Karmakar
DOI: 10.1007/978-981-15-6292-1_23

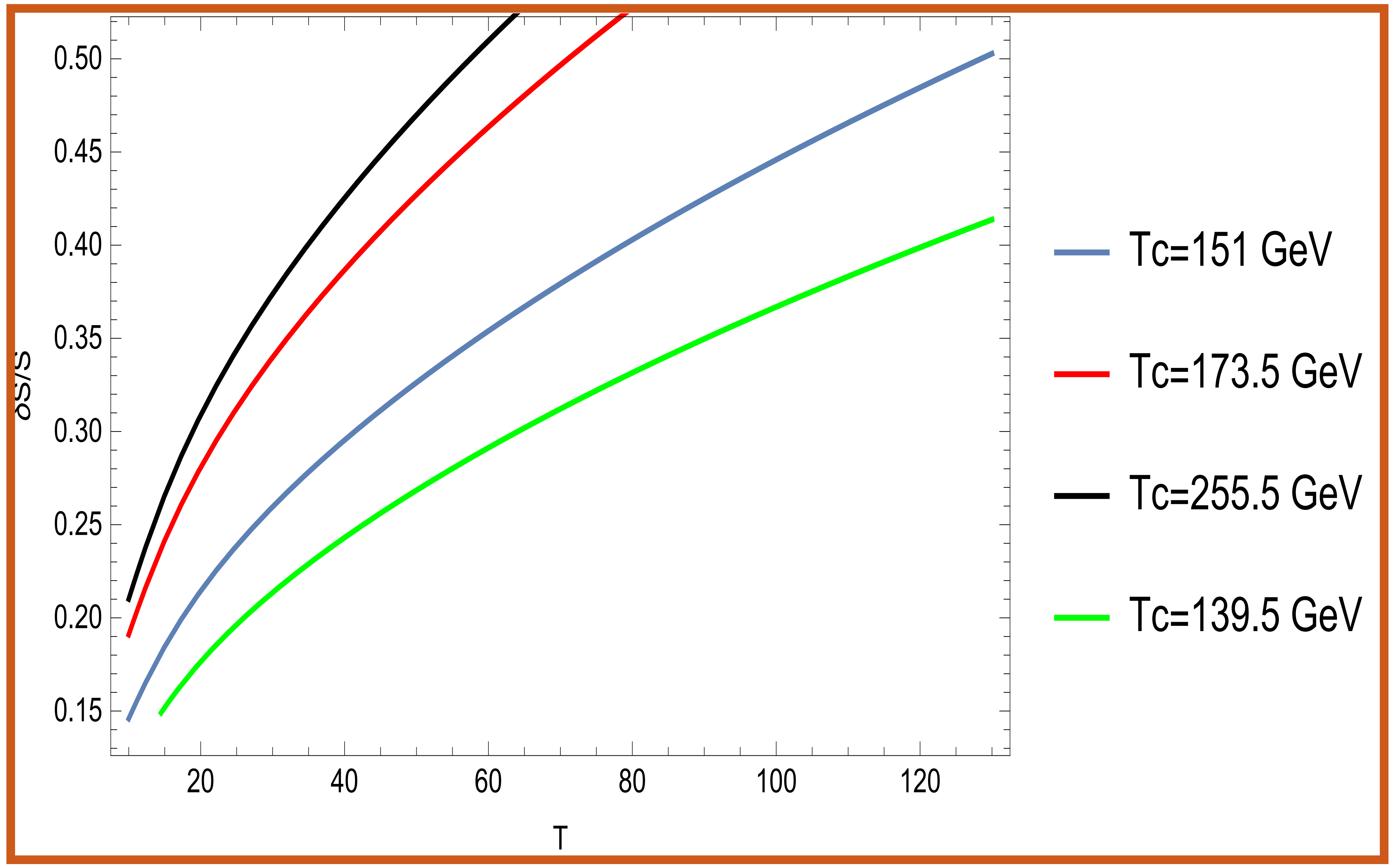
Relaxed constraints on the heavy scalar masses in 2HDM
Karmakar et al.
DOI: 10.1103/PhysRevD.100.055016

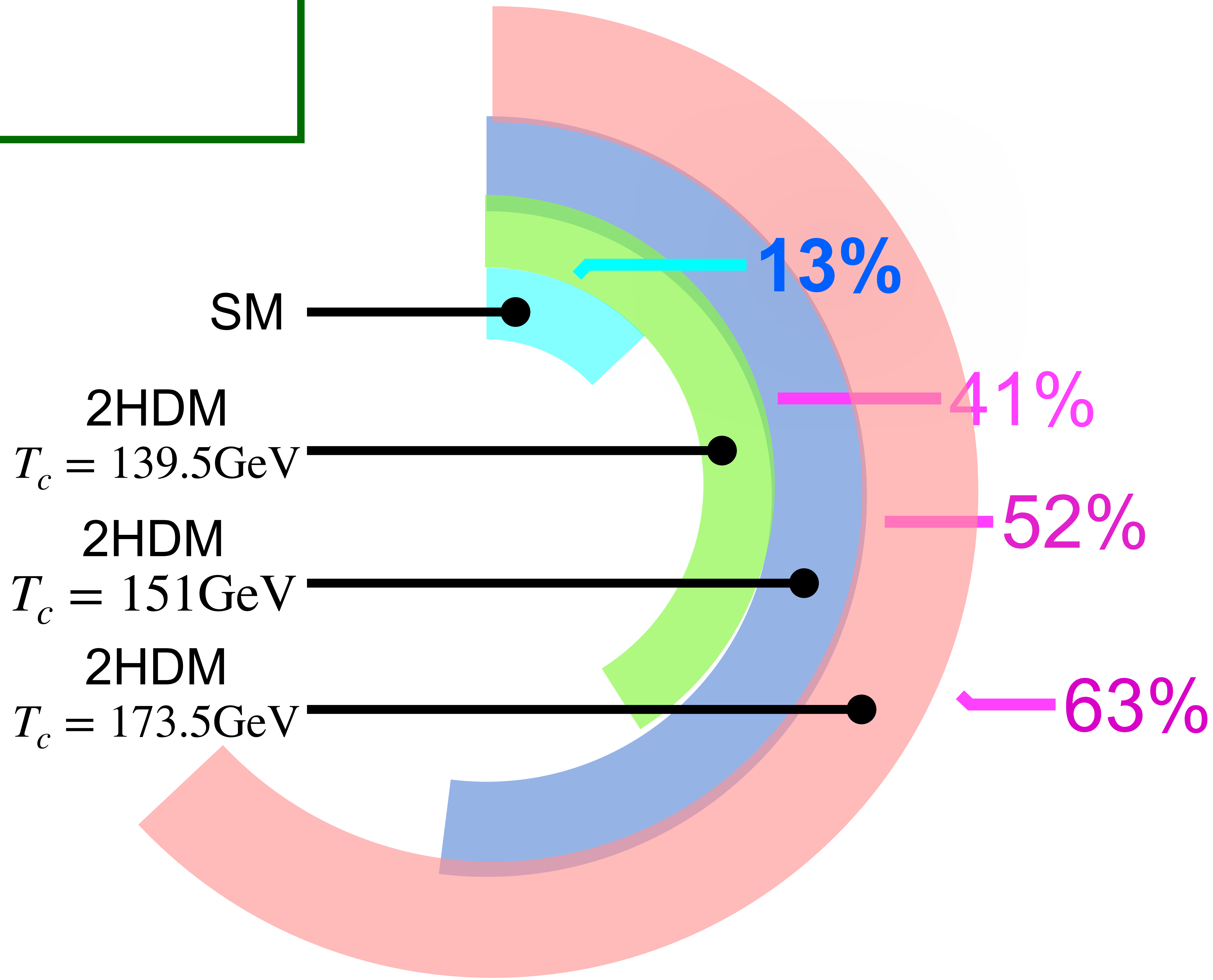
BSMPT (Beyond the Standard Model Phase Transitions): A tool for the electroweak phase transition in extended Higgs sectors
Philipp Basler, Margarete Mühlleitner
DOI: 10.1016/j.cpc.2018.11.006

BSMPT v2 A Tool for the Electroweak Phase Transition and the Baryon Asymmetry of the Universe in Extended Higgs Sectors
P. Basler, M. Muhlleitner, J. Müller
e-Print: 2007.01725 [hep-ph]



Electroweak phase transition and entropy release in the early universe
Arnab Chaudhuri, Alexander Dolgov
DOI: 10.1088/1475-7516/2018/01/032





Conclusion

- More entropy production in 2HDM than the Standard Model (SM).

- Amount of entropy production alters for different choice of parameter space or different models of 2HDM.

- Most entropy production for top-quark. At low temperature, entropy production is approximately same as that of the SM.

- Influx entropy dilutes the preexisting number density of frozen out spices, e.g. dark matter. Also dilutes preexisting baryon asymmetry.

171.2 GeV/c²
2/3
1/2
t
top
1.777 GeV/c²
-1
1/2
τ
tau
THANK YOU :)
126 GeV
0
0
H
Higgs Boson
91.2 GeV/c²
0
1
Z0
Z boson

Simplified Lagrangian:

$$\mathscr{L} = g^{\mu\nu}\partial_\mu \Phi_a \,\partial_\nu \Phi_a - V_{\rm tot}(\Phi_1, \Phi_2, T) + \sum_j i\left[g^{\mu\nu}\partial_\mu \chi_j^\dagger \,\partial_\nu \chi_j - U_j(\chi_j)\right] + \mathscr{L}_{int}$$

Original Lagrangian:

$$\mathscr{L} = \mathscr{L}_{\rm gauge,kin} + \mathscr{L}_f + \mathscr{L}_{\rm Higgs} + \mathscr{L}_{\rm Yuk}$$

$$\mathscr{L}_f = i\bar{\Psi}_L(\partial_\mu + igW_\mu + ig'Y_L B_\mu)\Psi_L + + i\bar{\Psi}_R(\partial_\mu + igW_\mu + ig'Y_R B_\mu)\Psi_R$$

$$\mathscr{L}_{\rm gauge,kin} = -\frac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} - \frac{1}{4}F^B_{\mu\nu}F^{B\mu\nu}; \quad (G^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk}W^j_\mu W^k_\nu \,, F^B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu)$$

$$\mathscr{L}_{\rm Yuk} = -y_e \bar{e}_R \Phi_1^\dagger L_L - y_e^* \bar{L}_L \Phi_1 e_R - y_e \bar{e}_R \Phi_2^\dagger L_L - y_e^* \bar{L}_L \Phi_2 e_R - \cdots$$

$$\mathscr{L}_{\rm Higgs} = (D^\mu \Phi_a)^\dagger (D_\mu \Phi_a) - V_{\rm tot}(\Phi_1, \Phi_2, T)$$

$$= (\partial^\mu {\Phi_a}^\dagger)(\partial_\mu \Phi_a) - i(\mathscr{W}^\mu \Phi_a)^\dagger(\partial_\mu \Phi_a) + i(\partial^\mu {\Phi_a}^\dagger)\mathscr{W}_\mu \Phi_a + (\mathscr{W}^\mu \Phi_a)^\dagger \mathscr{W}_\mu \Phi_a \,; (\mathscr{W}_\mu = gT^i W^i_\mu + g'YB_\mu)$$

Bosons	n_i	s_i	$m(v)^2$	
h	1	1	eigenvalues of 4	Higgs
H	1	1	eigenvalues of 4	Higgs
A	1	1	eigenvalues of 4	Higgs
G^0	1	1	eigenvalues of 4	Goldstone
$H^{\pm}$	2	1	Eq. 2	Charged Higgs
$G^{\pm}$	2	1	Eq. 3	Charged Goldstone
Z_L	1	1	Eq. 1	Higgs
Z_T	2	2	Eq. 1	Higgs
W_L	2	1	Eq. 1	Higgs
W_T	4	2	Eq. 1	Higgs
γ_L	1	2	Eq. 1	
γ_T	2	2	Eq. 1	

$$c_i = \begin{cases} \frac{5}{6}, & (i = W^{\pm}, Z, \gamma) \\ \frac{3}{2}, & \text{otherwise} \end{cases}$$

$$m_W^2 = \frac{g^2}{4} v^2; m_Z^2 = \frac{g^2 + g'^2}{4} v^2; m_\gamma^2 = 0. \cdots (1)$$

Strong First Order Electroweak Phase Transition in the CP-Conserving 2HDM Revisited
P. Basler (Karlsruhe U., ITP), M. Krause (Karlsruhe U., ITP), M. Muhlleitner (Karlsruhe U., ITP), J. Wittbrodt (DESY and Karlsruhe U., ITP), A. Wlotzka (KIT, Karlsruhe, TP)
e-Print: 1612.04086 [hep-ph]
DOI: 10.1007/JHEP02(2017)121

$$\bar{m}^2_{H^\pm} = \frac{1}{2}\left(\mathcal{M}^C_{11} + \mathcal{M}^C_{22}\right) + \frac{1}{2}\sqrt{4\left(\left(\mathcal{M}^C_{12}\right)^2 + \left(\mathcal{M}^C_{13}\right)^2\right) + \left(\mathcal{M}^C_{11} - \mathcal{M}^C_{22}\right)^2} \cdots (2)$$

$$\bar{m}^2_{G^\pm} = \frac{1}{2}\left(\mathcal{M}^C_{11} + \mathcal{M}^C_{22}+\right) - \frac{1}{2}\sqrt{4\left(\left(\mathcal{M}^C_{12}\right)^2 + \left(\mathcal{M}^C_{13}\right)^2\right) + \left(\mathcal{M}^C_{11} - \mathcal{M}^C_{22}\right)^2} \cdots (3)$$

$$\mathcal{M}^C_{11} = m^2_{11} + \lambda_1\frac{v_1^2}{2} + \lambda_3\frac{v_2^2}{2} \qquad \mathcal{M}^C_{22} = m^2_{22} + \lambda_2\frac{v_2^2}{2} + \lambda_3\frac{v_1^2}{2} \qquad \mathcal{M}^C_{12} = \frac{v_1 v_2}{2}\left(\lambda_4 + \lambda_5\right) - m^2_{12} \qquad \mathcal{M}^C_{13} = 0$$

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Masses of h, H, and A are the eigen values of the matrix

$$\mathcal{M}^N \cdots\cdots (4)$$

$$\mathcal{M}^N_{11} = m^2_{11} + \frac{3\lambda_1}{2}v_1^2 + \frac{\lambda_3 + \lambda_4}{2}v_2^2 + \frac{1}{2}\lambda_5 v_2^2$$

$$\mathcal{M}^N_{22} = m^2_{11} + \frac{\lambda_1}{2}v_1^2 + \frac{\lambda_3 + \lambda_4}{2}v_2^2 - \frac{1}{2}\lambda_5 v_2^2$$

$$\mathcal{M}^N_{33} = m^2_{22} + \frac{3\lambda_2}{2}v_2^2 + \frac{1}{2}\left(\lambda_3 + \lambda_4 + \lambda_5\right) v_1^2$$

$$\mathcal{M}^N_{44} = m^2_{22} + \frac{\lambda_2}{2}v_2^2 + \frac{1}{2}\left(\lambda_3 + \lambda_4 - \lambda_5\right) v_1^2$$

$$\mathcal{M}^N_{12} = 0 \qquad \mathcal{M}^N_{34} = 0$$

$$\mathcal{M}^N_{13} = -m^2_{12} + \left(\lambda_3 + \lambda_4 + \lambda_5\right) v_1 v_2$$

$$\mathcal{M}^N_{14} = 0$$

$$\mathcal{M}^N_{23} = 0$$

$$\mathcal{M}^N_{24} = -m^2_{12} + \lambda_5 v_1 v_2$$

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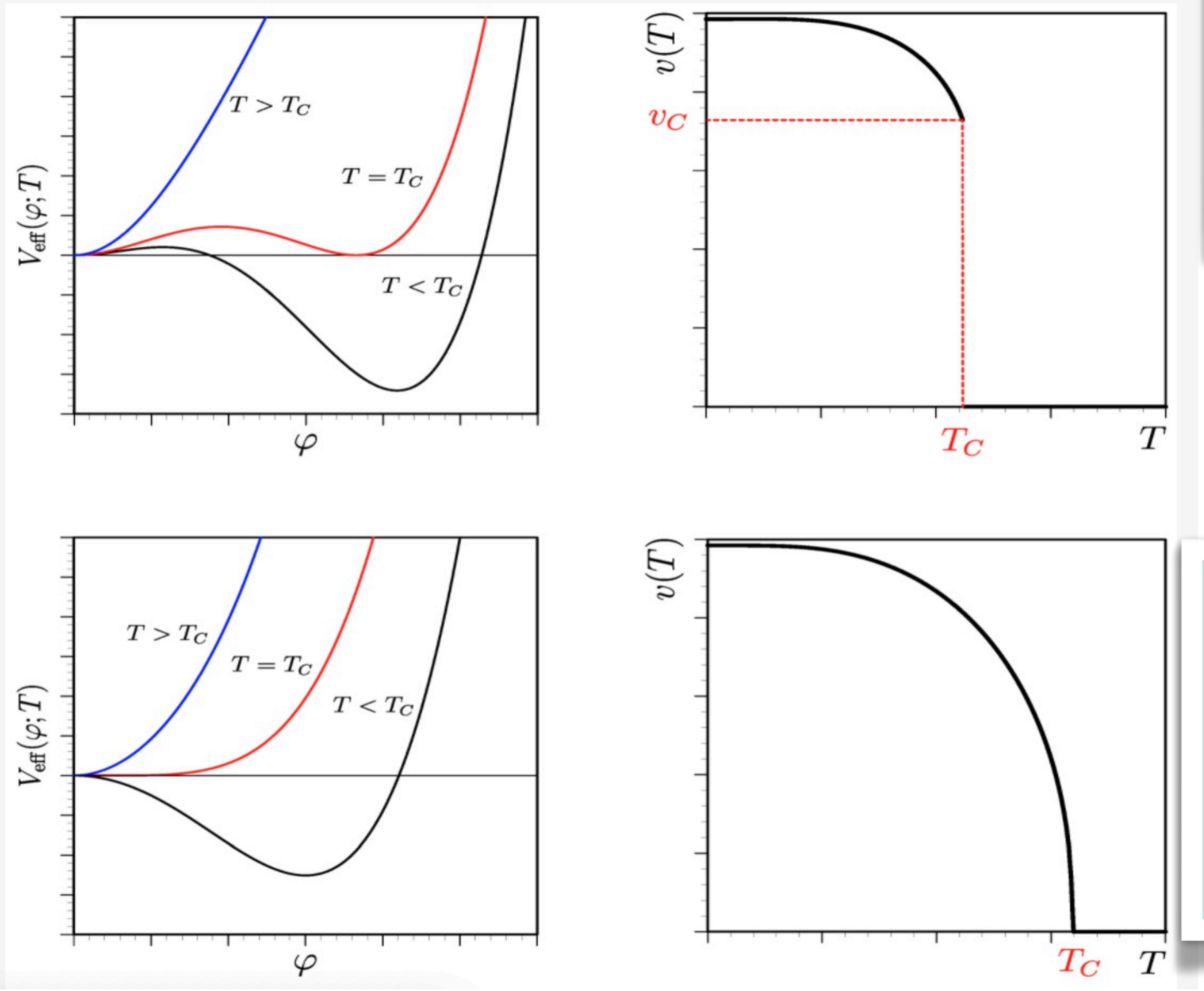
Figure 1. Two types of phase transitions. (Upper) Case of the first-order phase transition; shapes of the effective potential at $T > T_C$, $T = T_C$ and $T < T_C$ [left panel] and the temperature evolution of the VEV of scalar [right panel]. (Lower) Counterparts in the case of the second-order phase transition.



https://www.mdpi.com/2073-8994/12/5/733/htm

First order phase transition

1. **discontinuity** in the evolution of the VEV
2. critical temperature (T_c) $\Longrightarrow$ temperature at which $V_{\rm eff}$ has the degenerate minima separated by the potential barrier.

Second order phase transition

1. **Continuous** in the evolution of the VEV
2. first derivative of VEV with respect to the temperature has the singular behavior at T_c
3. critical temperature (T_c) $\Longrightarrow$ temperature at which the curvature at the origin becomes zero.

A smooth cross over : A crossover is thus not associated with a change of symmetry, or a discontinuity in the free energy functional. *(source)*. i.e. no discontinuity in the order parameter.*(source)*

Free energy in thermodynamics ≡ Effective potential in field theory *(see Sec:1, below eq.[1] of hep-ph/0010275)*

“The equation of motion for the classical field ϕ is often taken as:

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + U'_\phi(\phi) = 0,$$

where Γ is the decay width of the ϕ-boson. Such equation is obtained by **thermal averaging of the interaction** term and, strictly speaking, is valid only for quadratic potential $U_\phi \sim \phi^2$.”

Type	Description	up-type quarks couple to	down-type quarks couple to	charged leptons couple to	remarks
Type I	Fermiophobic	Φ_2	Φ_2	Φ_2	charged fermions only couple to second doublet
Type II	MSSM-like	Φ_2	Φ_1	Φ_1	up- and down-type quarks couple to separate doublets
X	Lepton-specific	Φ_2	Φ_2	Φ_1	
Y	Flipped	Φ_2	Φ_1	Φ_2	
Type III		Φ_1, Φ_2	Φ_1, Φ_2	Φ_1, Φ_2	Flavor-changing neutral currents at tree level
Type FCNC-free		Φ_1, Φ_2	Φ_1, Φ_2	Φ_1, Φ_2	By finding a matrix pair which can be diagonalized simultaneously. [7]

By convention, Φ_2 is the doublet to which up-type quarks couple.

Source: Wikipedia