

Neutrino masses within a $SU(3)$ family symmetry and a 3+5 scenario

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Open questions:

There are several open questions, motivations, to look for physics "Beyond the Standard Model" (BSM):

- Neutrino masses and mixing, oscillation
- Hierarchy of masses
- Dark Matter (DM), Dark Energy (DE), Matter-Antimatter Asymmetry.

Main goal of this BSM: To account for the hierarchy of fermion masses:

$$m_t \gg m_c \gg m_u \quad , \quad m_b \gg m_s \gg m_d \quad , \quad m_\tau \gg m_\mu \gg m_e,$$

and quark and lepton mixing matrices: V_{CKM} and U_{PMNS} .

Standard Model "SM"

Ordinary Fermions: $Q = T_{3L} + \frac{1}{2}Y$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$q_{jL}^o = \begin{pmatrix} u_{jL}^o \\ d_{jL}^o \end{pmatrix}$	3	2	$\frac{1}{3}$
u_{jR}^o	3	1	$\frac{4}{3}$
d_{jR}^o	3	1	$-\frac{2}{3}$
$l_{jL}^o = \begin{pmatrix} \nu_{jL}^o \\ e_{jL}^o \end{pmatrix}$	1	2	-1
e_{jR}^o	1	1	-2
ν_{jR}^o	1	1	0
$\phi_{SM} = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}$	1	2	+1
$\tilde{\phi}_{SM} = i\sigma_2 \phi_{SM}^*$	1	2	-1

Table: SM fermion content and charges, $j = 1, 2, 3$ family index

Comments:

- $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$: Standard Model Group
- Neutrinos are massless
- Anomalies cancel for each family, $j = 1, 2, 3$.
- $\bar{q}_{iL}^o \phi_{SM} u_{jR}^o$, $i, j = 1, 2, 3$ is gauge invariant. So, the Yukawa couplings $Y_{i,j} \bar{q}_{iL}^o \phi_{SM} u_{jR}^o$, $i, j = 1, 2, 3$ are gauge invariant, with $Y_{i,j}$ completely arbitrary.
- SM gauge bosons: Z, A(photon), and H(Higgs) couplings to fermions do not change flavor.
- $\frac{g}{\sqrt{2}} \bar{\Psi}_{uL} V_{CKM} \gamma_\mu \Psi_{dL} W^{+\mu}$

MODEL WITH LOCAL $SU(3)$ VECTOR LIKE FAMILY SYMMETRY

	$SU(3)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
ψ_q^o	3	3	2	$\frac{1}{3}$
ψ_{uR}^o	3	3	1	$\frac{4}{3}$
ψ_{dR}^o	3	3	1	$-\frac{2}{3}$
ψ_l^o	3	1	2	-1
ψ_{eR}^o	3	1	1	-2
$\psi_{\nu R}^o$	3	1	1	0
$U_{L,R}^o$	1	3	1	$\frac{4}{3}$
$D_{L,R}^o$	1	3	1	$-\frac{2}{3}$
$E_{L,R}^o$	1	1	1	-2
$N_{L,R}^o$	1	1	1	0

Table: Fermion content and charges. Right handed neutrinos are needed to cancel anomalies

3+5 Neutrino scenario

3 Active L-handed SM neutrinos

5 Sterile neutrinos: (3 Right handed & N_L, N_R vector like neutrinos)

	$SU(3)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
η_1, η_2	3	1	1	0
Φ^u	3	1	2	-1
Φ^d	3	1	2	+1

Table: Scalars fields and charges

- η_1, η_2 are introduced to break spontaneously $SU(3)$.
- Φ^u, Φ^d are introduced to break spontaneously the $SU(2)_L \times U(1)_Y$.
- u-quarks and neutrinos coupled only to Φ^u
- d-quarks and charged leptons couple only to Φ^d

$\bar{\psi}_q^o \Phi^u \psi_{uR}^o$ and $\bar{\psi}_q^o \Phi^d \psi_{dR}^o$ are forbidden by the $SU(3)$ family symmetry.

$$\bar{3} \times 3 = 1 + 8, \quad 3 \times 3 = \bar{3} + 6, \quad \bar{3} \times 3 \times 3 = (1+8) \times 3 = 3 + \bar{6} + 3 + 15$$

$$i\mathcal{L}_{int, SU(3)} =$$

$$\begin{aligned} \frac{g_H}{2} (\bar{f}_1^o \gamma_\mu f_1^o - \bar{f}_3^o \gamma_\mu f_3^o) Z_1^\mu + \frac{g_H}{2\sqrt{3}} (\bar{f}_1^o \gamma_\mu f_1^o + \bar{f}_3^o \gamma_\mu f_3^o - 2\bar{f}_2^o \gamma_\mu f_2^o) Z_2^\mu \\ + \frac{g_H}{\sqrt{2}} (\bar{f}_1^o \gamma_\mu f_2^o Y_1^+ + \bar{f}_1^o \gamma_\mu f_3^o Y_2^+ + \bar{f}_2^o \gamma_\mu f_3^o Y_3^+ + h.c.) \end{aligned}$$

g_H is the $SU(3)$ coupling constant, Z_1 , Z_2 and Y_j^\pm , $i = 1, 2, 3$, $j = 1, 2$ are the eight gauge bosons, which have **flavor changing couplings to both left and right handed SM fermions**

$$g_H \begin{pmatrix} \bar{f}_1^o & \bar{f}_2^o & \bar{f}_3^o \end{pmatrix} \gamma_\mu \begin{pmatrix} \frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} & \frac{Y_1^+}{\sqrt{2}} & \frac{Y_2^+}{\sqrt{2}} \\ \frac{Y_1^-}{\sqrt{2}} & -\frac{Z_2}{\sqrt{3}} & \frac{Y_3^+}{\sqrt{2}} \\ \frac{Y_2^-}{\sqrt{2}} & \frac{Y_3^-}{\sqrt{2}} & -\frac{Z_1}{2} + \frac{Z_2}{2\sqrt{3}} \end{pmatrix}^\mu \begin{pmatrix} f_1^o \\ f_2^o \\ f_3^o \end{pmatrix}$$

Yukawa couplings:

Dirac Yukawa couplings:

$$\begin{aligned} & h_u \overline{\psi}_q^o \Phi^u U_R^o + h_{1u} \overline{\psi}_{uR}^o \eta_1 U_L^o + h_{2u} \overline{\psi}_{uR}^o \eta_2 U_L^o + M_U \overline{U}_L^o U_R^o \\ & h_d \overline{\psi}_q^o \Phi^d D_R^o + h_{1d} \overline{\psi}_{dR}^o \eta_1 D_L^o + h_{2d} \overline{\psi}_{dR}^o \eta_2 D_L^o + M_D \overline{D}_L^o D_R^o \\ & h_\nu \overline{\psi}_l^o \Phi^u N_R^o + h_{1\nu} \overline{\psi}_{\nu R}^o \eta_1 N_L^o + h_{2\nu} \overline{\psi}_{\nu R}^o \eta_2 N_L^o + m_D \overline{N}_L^o N_R^o \\ & h_e \overline{\psi}_l^o \Phi^d E_R^o + h_{1e} \overline{\psi}_{eR}^o \eta_1 E_L^o + h_{2e} \overline{\psi}_{eR}^o \eta_2 E_L^o + M_E \overline{E}_L^o E_R^o \\ & \hspace{20em} + h.c \end{aligned}$$

Majorana Yukawa couplings:

$$\begin{aligned} & h_{1R} \overline{\psi}_{\nu R}^o \eta_1 (N_R^o)^c + h_{2R} \overline{\psi}_{\nu R}^o \eta_2 (N_R^o)^c + m_R \overline{N}_R^o (N_R^o)^c \\ & h_L \overline{\psi}_l^o \Phi^u (N_L^o)^c + m_L \overline{N}_L^o (N_L^o)^c + h.c \end{aligned}$$

Spontaneous Symmetry breaking (SSB)

We would like to be consistent with low energy Standard Model(SM) and simultaneously generate and account for the hierarchy of quark and lepton masses and mixing

$$SU(3) \times G_{SM} \xrightarrow{\langle \eta_2 \rangle, \langle \eta_1 \rangle} G_{SM} \xrightarrow{\langle \Phi^u \rangle, \langle \Phi^d \rangle} SU(3)_C \times U(1)_Q$$

SM fermions are massless SM fermions still massless SM fermions become massive (PDG known values)

$SU(3)$ family symmetry breaking

Two SM singlet scalars are introduced in the fundamental representation of $SU(3)$:

$$\langle \eta_1 \rangle = \begin{pmatrix} \Lambda_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \eta_2 \rangle = \begin{pmatrix} 0 \\ \Lambda_2 \\ 0 \end{pmatrix}$$

$$SU(3) \times G_{SM} \xrightarrow{\langle \eta_2 \rangle} SU(2) ? \times G_{SM} \xrightarrow{\langle \eta_1 \rangle} G_{SM}$$

FCNC ?

Λ_2 : 5 very heavy boson masses of order M_2 ($M_2^2 = \frac{g_H^2 \Lambda_2^2}{2}$) $\gtrsim 10^6$ TeV

Λ_1 : 3 boson masses of order M_1 ($M_1^2 = \frac{g_H^2 \Lambda_1^2}{2}$) $\gtrsim 10^3$ TeV

To suppress properly FCNC like, for instance: $\mu \rightarrow e\gamma$
($Br < 5.7 \times 10^{-13}$), $\mu \rightarrow eee$ ($Br < 1 \times 10^{-12}$), $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$,
 $M_1 \gtrsim 10^3$ TeV's

The above scalar fields and VEV's break completely the $SU(3)$ family symmetry, generating the mass terms

- $\langle \eta_1 \rangle : \frac{g_H^2 \Lambda_1^2}{2} (Y_1^+ Y_1^- + Y_2^+ Y_2^-) + \frac{g_H^2 \Lambda_1^2}{4} (Z_1^2 + \frac{Z_2^2}{3} + 2Z_1 \frac{Z_2}{\sqrt{3}})$

- $\langle \eta_2 \rangle : \frac{g_H^2 \Lambda_2^2}{2} (Y_1^+ Y_1^- + Y_3^+ Y_3^-) + g_H^2 \Lambda_2^2 \frac{Z_2^2}{3}$

Electroweak Symmetry Breaking

In this scenario we introduce two triplets of $SU(2)_L$ Higgs doublets;

$$\Phi^u = (3, 1, 2, -1), \Phi^d = (3, 1, 2, +1):$$

$$\Phi^u = \begin{pmatrix} \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}_1^u \\ \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}_2^u \\ \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}_3^u \end{pmatrix}, \langle \Phi^u \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u1} \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u2} \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} v_{u3} \\ 0 \end{pmatrix} \end{pmatrix}; \Phi^d = \begin{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_1^d \\ \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_2^d \\ \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_3^d \end{pmatrix}, \langle \Phi^d \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d1} \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d2} \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{d3} \end{pmatrix} \end{pmatrix}$$

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{\langle \Phi^u \rangle, \langle \Phi^d \rangle} SU(3)_C \times U(1)_Q$$

Contribute to the W and Z boson masses:

$$\frac{g^2}{4} (v_u^2 + v_d^2) W^+ W^- + \frac{(g^2 + g'^2)}{8} (v_u^2 + v_d^2) Z_0^2 +$$

$$\frac{1}{4} \sqrt{g^2 + g'^2} g_H Z_0 \left[(v_{1u}^2 - v_{2u}^2 - v_{1d}^2 + v_{2d}^2) Z_1 + (v_{1u}^2 + v_{2u}^2 - 2v_{3u}^2 - v_{1d}^2 - v_{2d}^2 + 2v_{3d}^2) \frac{Z_2}{\sqrt{3}} \right. \\ \left. + 2(v_{1u}v_{2u} - v_{1d}v_{2d}) \frac{Y_1^+ + Y_1^-}{\sqrt{2}} + 2(v_{1u}v_{3u} - v_{1d}v_{3d}) \frac{Y_2^+ + Y_2^-}{\sqrt{2}} + 2(v_{2u}v_{3u} - v_{2d}v_{3d}) \frac{Y_3^+ + Y_3^-}{\sqrt{2}} \right]$$

+ tiny contributions to the $SU(3)$ gauge boson masses

$v_u^2 = v_{u1}^2 + v_{u2}^2 + v_{u3}^2$, $v_d^2 = v_{d1}^2 + v_{d2}^2 + v_{d3}^2$. Hence, if we define $M_W = \frac{1}{2} g v$, we may write $v = \sqrt{v_u^2 + v_d^2} \approx 246$ GeV.

Tree level Fermion Masses

$\bar{\psi}_{SM,L}^o \Phi^{u,d} \psi_{SM,R}^o$ are not $SU(3)$ invariant

Allowed Tree Level Yukawa couplings:

$$h_e \bar{\psi}_1^o \Phi^d E_R^o + h_{1e} \bar{\psi}_e^o \eta_1 E_L^o + h_{2e} \bar{\psi}_e^o \eta_2 E_L^o + M_E \bar{E}_L^o E_R^o + h.c$$

Dirac See-Saw Mechanisms

In the gauge basis $\psi_{L,R}^o T = (e^o, \mu^o, \tau^o, E^o)_{L,R}$, the mass terms read as $\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + h.c$, where

\bar{e}_L^o	e_R^o	μ_R^o	τ_R^o	E_R^o	\equiv	\bar{e}_L^o	e_R^o	μ_R^o	τ_R^o	E_R^o
$\bar{\mu}_L^o$	0	0	0	$h_e v_{d1}$		$\bar{\mu}_L^o$	0	0	0	a_1
$\bar{\tau}_L^o$	0	0	0	$h_e v_{d2}$		$\bar{\tau}_L^o$	0	0	0	a_2
\bar{E}_L^o	0	0	0	$h_e v_{d3}$		\bar{E}_L^o	0	0	0	a_3
	$h_{1e} \Lambda_1$	$h_{2e} \Lambda_2$	0	M_E			b_1	b_2	0	M

Table: Generic tree level Dirac mass matrix \mathcal{M}^o for u and d quarks, charged leptons and Dirac neutrinos

$$V_L^{oT} \mathcal{M}^o \mathcal{M}^{oT} V_L^o = V_R^{oT} \mathcal{M}^{oT} \mathcal{M}^o V_R^o = \text{Diag}(0, 0, \lambda_3^2, \lambda_4^2)$$

$$V_L^{oT} \mathcal{M}^o V_R^o = \text{Diag}(0, 0, -\lambda_3, \lambda_4)$$

The non-zero eigenvalues may be obtained from diagonalization of the 2x2 matrix

$$m_o = \begin{pmatrix} 0 & a \\ b & M \end{pmatrix}, \quad a = \sqrt{a_1^2 + a_2^2 + a_3^2}, \quad b = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

$$v_L = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix}, \quad v_R = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}$$

$$\lambda_3 c_\alpha c_\beta = \lambda_4 s_\alpha s_\beta$$

$$v_L^T m_o v_R = \text{Diag}(0, 0, -\lambda_3, \lambda_4)$$

$$v_L^T m_o m_o^T v_L = v_R^T m_o^T m_o v_R = \text{Diag}(\lambda_3^2, \lambda_4^2)$$

One loop contribution to fermion masses

After tree level contributions the fermion global symmetry is broken down to:

$$SU(2)_{q_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{\nu_R} \otimes SU(2)_{e_R}$$

Therefore, in this scenario light fermion masses, including neutrinos, may get extremely small masses from radiative corrections mediated by the $SU(3)$ heavy gauge bosons.

$$c_Y \frac{\alpha_H}{\pi} \sum_{k=3,4} m_k^0 (V_L^0)_{ik} (V_R^0)_{jk} f(M_Y, m_k^0) \quad , \quad \alpha_H \equiv \frac{g_H^2}{4\pi}$$

M_Y is the gauge boson mass, c_Y is coupling constant, $m_3^0 = -\lambda_3$, $m_4^0 = \lambda_4$, and $f(x, y) = \frac{x^2}{x^2 - y^2} \ln \frac{x^2}{y^2}$.

$$\sum_{k=3,4} m_k^0 (V_L^0)_{ik} (V_R^0)_{jk} f(M_Y, m_k^0) = \frac{a_i b_j M}{\lambda_4^2 - \lambda_3^2} F(M_Y) \quad i, j = 1, 2, 3,$$

$$F(M_Y) \equiv \frac{M_Y^2}{M_Y^2 - \lambda_4^2} \ln \frac{M_Y^2}{\lambda_4^2} - \frac{M_Y^2}{M_Y^2 - \lambda_3^2} \ln \frac{M_Y^2}{\lambda_3^2}$$

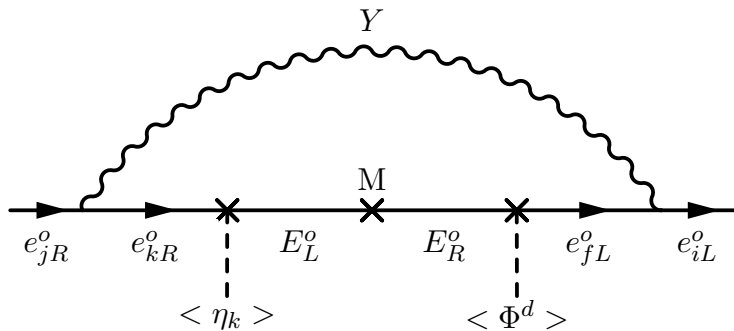


Figure: Generic one loop diagram contribution to the mass term $m_{ij} \bar{e}_{iL}^o e_{jR}^o$

	e_R^o	μ_R^o	τ_R^o	E_R^o
$\overline{e_L^o}$	D_{11}	D_{12}	0	0
$\overline{\mu_L^o}$	D_{21}	D_{22}	0	0
$\overline{\tau_L^o}$	D_{31}	D_{32}	D_{33}	0
$\overline{E_L^o}$	0	0	0	0

Table: One loop mass matrix \mathcal{M}_1^o

$$\bar{\psi}_L^o \mathcal{M}^o \psi_R^o + \bar{\psi}_L^o \mathcal{M}_1^o \psi_R^o = \bar{\chi}_L \mathcal{M} \chi_R$$

Generic mass matrix up to one loop for quarks and charged leptons.

$$\mathcal{M} = \text{Diag}(0, 0, -\lambda_3, \lambda_4) + V_L^{oT} \mathcal{M}_1^o V_R^o$$

$$\mathcal{M} = \begin{pmatrix} m_{11} & m_{12} & c_\beta m_{13} & s_\beta m_{13} \\ m_{21} & m_{22} & c_\beta m_{23} & s_\beta m_{23} \\ c_\alpha m_{31} & c_\alpha m_{32} & -\lambda_3 + c_\alpha c_\beta m_{33} & c_\alpha s_\beta m_{33} \\ s_\alpha m_{31} & s_\alpha m_{32} & s_\alpha c_\beta m_{33} & \lambda_4 + s_\alpha s_\beta m_{33} \end{pmatrix}$$

The diagonalization of \mathcal{M} yields the physical masses for u, d, e and ν fermions. Using a new biunitary transformation

$\chi_{L,R} = V_{L,R}^{(1)} \Psi_{L,R}$; $\bar{\chi}_L \mathcal{M} \chi_R = \bar{\Psi}_L V_L^{(1)T} \mathcal{M} V_R^{(1)} \Psi_R$, with $\Psi_{L,R}^T = (f_1, f_2, f_3, F)_{L,R}$ the mass eigenfields, that is

$$V_L^{(1)T} \mathcal{M} \mathcal{M}^T V_L^{(1)} = V_R^{(1)T} \mathcal{M}^T \mathcal{M} V_R^{(1)} = \text{Diag}(m_1^2, m_2^2, m_3^2, M_F^2),$$

$m_1^2 = m_e^2$, $m_2^2 = m_\mu^2$, $m_3^2 = m_\tau^2$ and $M_F^2 = M_E^2$ for charged leptons.

Therefore, the transformation from massless to mass fermions eigenfields reads

$$\psi_L^o = V_L^o V_L^{(1)} \Psi_L \quad \text{and} \quad \psi_R^o = V_R^o V_R^{(1)} \Psi_R$$

and for neutrinos $\Psi_\nu^o = U_\nu^o U_\nu \Psi_\nu$.

Quark (V_{CKM}) $_{4 \times 4}$ and Lepton (U_{PMNS}) $_{4 \times 8}$ mixing matrices

Vector like quarks are $SU(2)_L$ weak singlets, and hence, the interaction of L-handed up and down quarks; $f_{uL}^o T = (u^o, c^o, t^o)_L$ and $f_{dL}^o T = (d^o, s^o, b^o)_L$, to the W charged gauge boson is

$$\frac{g}{\sqrt{2}} \bar{f}_{uL}^o \gamma_\mu f_{dL}^o W^{+\mu} = \frac{g}{\sqrt{2}} \bar{\Psi}_{uL} [(V_{uL}^o V_{uL}^{(1)})_{3 \times 4}]^T (V_{dL}^o V_{dL}^{(1)})_{3 \times 4} \gamma_\mu \Psi_{dL} W^{+\mu},$$

g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary V_{CKM} of dimension 4×4 is identified as

$$(V_{CKM})_{4 \times 4} = [(V_{uL}^o V_{uL}^{(1)})_{3 \times 4}]^T (V_{dL}^o V_{dL}^{(1)})_{3 \times 4}$$

Similar analysis of the couplings of active L-handed neutrinos and L-handed charged leptons to W boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4 \times 8} = [(V_{eL}^o V_{eL}^{(1)})_{3 \times 4}]^T (U_\nu^o U_\nu)_{3 \times 8}$$

Neutrino masses: 3 + 5 Scenario

Tree level Dirac Neutrino masses

$$h_D \overline{\Psi}_l^o \phi^u N_R^o + h_{\nu 1} \overline{\Psi}_{\nu R}^o \eta_1 N_L^o + h_{\nu 2} \overline{\Psi}_{\nu R}^o \eta_2 N_L^o + m_D \overline{N}_L^o N_R^o + h.c$$

h_D , h_1 , h_2 , and h_3 are Yukawa couplings, and m_D a Dirac type invariant neutrino mass for the sterile neutrino $N_{L,R}^o$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi_{\nu L,R}^{oT} = (\nu_e^o, \nu_\mu^o, \nu_\tau^o, N^o)_{L,R}$, the mass terms

$$h_D [v_{u1} \bar{\nu}_{eL}^o + v_{u2} \bar{\nu}_{\mu L}^o + v_{u3} \bar{\nu}_{\tau L}^o] N_R^o + [h_{\nu 1} \Lambda_1 \bar{\nu}_{eR}^o + h_{\nu 2} \Lambda_2 \bar{\nu}_{\mu R}^o] N_L^o + m_D \bar{N}_L^o N_R^o + h.c.$$

	ν_{eR}^o	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	N_R^o
$\overline{\nu_{eL}^o}$	0	0	0	$h_D v_{u1}$
$\overline{\nu_{\mu L}^o}$	0	0	0	$h_D v_{u2}$
$\overline{\nu_{\tau L}^o}$	0	0	0	$h_D v_{u3}$
$\overline{N_L^o}$	$h_{\nu 1} \Lambda_1$	$h_{\nu 2} \Lambda_2$	0	m_D

Table: Tree level Dirac mass terms $m_{ij} \bar{\nu}_{iL}^o \nu_{jR}^o$

Tree level Majorana masses

Since $N_{L,R}^o$ are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

$$h_L \bar{\Psi}_l^o \Phi^u (N_L^o)^c + m_L \bar{N}_L^o (N_L^o)^c + h.c.$$

and

$$h_{1R} \bar{\Psi}_{\nu R}^o \eta_1 (N_R^o)^c + h_{2R} \bar{\Psi}_{\nu R}^o \eta_2 (N_R^o)^c + m_R \bar{N}_R^o (N_R^o)^c + h.c.$$

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

$$h_L \left[v_{u1} \bar{\nu}_{eL}^o + v_{u2} \bar{\nu}_{\mu L}^o + v_{u3} \bar{\nu}_{\tau L}^o \right] (N_L^o)^c + m_L \bar{N}_L^o (N_L^o)^c$$

$$+ \left[h_{1R} \Lambda_1 \bar{\nu}_{eR}^o + h_{2R} \Lambda_2 \bar{\nu}_{\mu R}^o \right] (N_R^o)^c + m_R \bar{N}_R^o (N_R^o)^c + h.c.$$

	ν_{eL}^o	$\nu_{\mu L}^o$	$\nu_{\tau L}^o$	N_L^o
ν_{eL}^o	0	0	0	$h_L v_{u1}$
$\nu_{\mu L}^o$	0	0	0	$h_L v_{u2}$
$\nu_{\tau L}^o$	0	0	0	$h_L v_{u3}$
N_L^o	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L

Table: Tree level L-handed Majorana mass terms $m_{ij} \bar{\nu}_{iL}^o (\nu_{jL}^o)^c$

	ν_{eR}^o	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	N_R^o
ν_{eR}^o	0	0	0	$h_{1R} \Lambda_1$
$\nu_{\mu R}^o$	0	0	0	$h_{2R} \Lambda_2$
$\nu_{\tau R}^o$	0	0	0	0
N_R^o	$h_{1R} \Lambda_1$	$h_{2R} \Lambda_2$	0	m_R

Table: Tree level R-handed Majorana mass terms $m_{ij} \bar{\nu}_{iR}^o (\nu_{jR}^o)^T$

	$(\nu_{eL}^o)^c$	$(\nu_{\mu L}^o)^c$	$(\nu_{\tau L}^o)^c$	$(N_L^o)^c$	ν_{eR}^o	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	N_R^o
$\overline{\nu_{eL}^o}$	0	0	0	$h_L v_{u1}$	0	0	0	$h_D v_{u1}$
$\overline{\nu_{\mu L}^o}$	0	0	0	$h_L v_{u2}$	0	0	0	$h_D v_{u2}$
$\overline{\nu_{\tau L}^o}$	0	0	0	$h_L v_{u3}$	0	0	0	$h_D v_{u3}$
$\overline{N_L^o}$	$h_L v_{u1}$	$h_L v_{u2}$	$h_L v_{u3}$	m_L	$h_1 \Lambda_1$	$h_2 \Lambda_2$	0	M_D
$\overline{(\nu_{eR}^o)^c}$	0	0	0	$h_1 \Lambda_1$	0	0	0	$h_{1R} \Lambda_1$
$\overline{(\nu_{\mu R}^o)^c}$	0	0	0	$h_2 \Lambda_2$	0	0	0	$h_{2R} \Lambda_2$
$\overline{(\nu_{\tau R}^o)^c}$	0	0	0	0	0	0	0	0
$\overline{(N_R^o)^c}$	$h_D v_{u1}$	$h_D v_{u2}$	$h_D v_{u3}$	M_D	$h_{1R} \Lambda_1$	$h_{2R} \Lambda_2$	0	m_R

Table: Tree Level Majorana mass matrix

**Diagonalization yields 4 zero neutrino mass eigenvalues
from tree level**

$$U_\nu^{oT} \mathcal{M}_\nu^o U_\nu^o = \text{Diag}(0, 0, 0, 0, \lambda_5, \lambda_6, \lambda_7, \lambda_8)$$

One loop Dirac neutrino masses

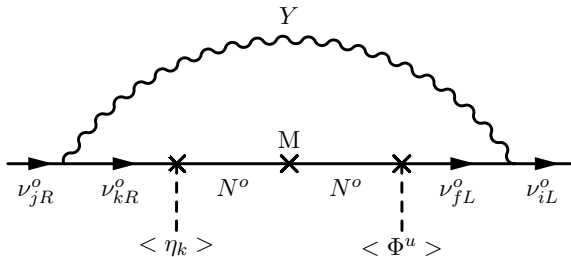


Figure: Generic one loop diagram contribution to the Dirac mass term $m_{ij} \bar{\nu}_{iL}^o \nu_{jR}^o$. $M = M_D, m_L, m_R$

One loop Majorana neutrino masses

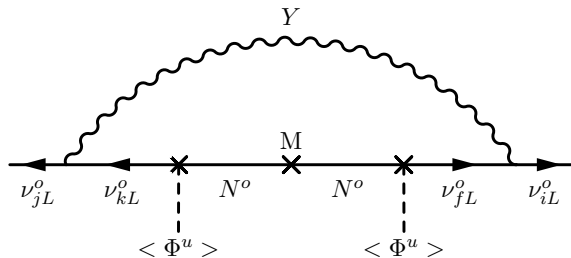


Figure: Generic one loop diagram contribution to the L-handed Majorana mass term $m_{ij} \bar{\nu}_{iL}^o (\nu_{jL}^o)^T$. $M = M_D, m_L, m_R$

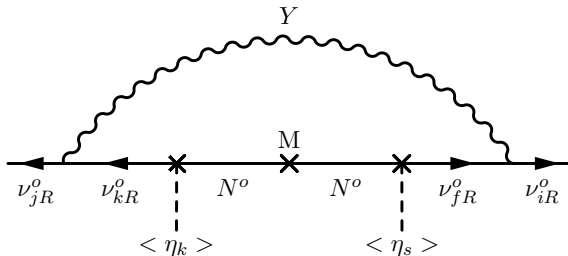


Figure: Generic one loop diagram contribution to the R-handed Majorana mass term $m_{ij} \bar{\nu}_{iR}^o (\nu_{jR}^o)^T$. $M = M_D, m_L, m_R$

Numerical results

Particular parameter space region at the M_Z scale:

Input values for the $SU(3)$ family symmetry:

$$M_1 = 5300 \text{ TeV's} \quad , \quad M_2 = 10^3 M_1 \quad , \quad \frac{\alpha_H}{\pi} = 0.1266$$

with M_1, M_2 the horizontal boson masses, and the coupling constant, respectively,

$$M_1^2 = \frac{g_H^2 \Lambda_1^2}{2} \quad , \quad M_2^2 = \frac{g_H^2 \Lambda_2^2}{2}$$

$$g_H = 2.23561 \quad , \quad \Lambda_1 = 3352.7 \text{ TeV} \quad , \quad \Lambda_2 = 10^3 \Lambda_1$$

Quark masses and $(V_{CKM})_{4 \times 4}$ mixing

u-quarks:

Tree level see-saw mass matrix:

$$\mathcal{M}_u^o = \begin{pmatrix} 0 & 0 & 0 & 32445.2 \\ 0 & 0 & 0 & 153081 \\ 0 & 0 & 0 & 188414. \\ 7.09 \times 10^9 & -1.3905 \times 10^9 & 0 & 5.04313 \times 10^9 \end{pmatrix} \text{ MeV},$$

the mass matrix up to one loop corrections:

$$\mathcal{M}_u = \begin{pmatrix} 0 & -164.78 & -450.207 & -644.992 \\ 2.05472 & 600.381 & 1627.36 & 2331.45 \\ -2.47402 & 1593.14 & -173000. & 39878.4 \\ -0.0000393617 & 0.0253469 & 0.442859 & 8.81106 \times 10^9 \end{pmatrix} \text{ MeV}$$

the u-quark masses

$$(m_u, m_c, m_t, M_U) = (1.38049, 638.077, 173016, 8.81 \times 10^9) \text{ MeV}$$

the mixing matrices:

$$V_{uL} = V_{uL}^o V_{uL}^{(1)}:$$

$$\begin{pmatrix} 0.985383 & 0.105814 & -0.133508 & 2.67779 \times 10^{-6} \\ 0.000823743 & -0.78665 & -0.617398 & 0.0000129872 \\ -0.170353 & 0.608264 & -0.775239 & 0.000015552 \\ 0 & 4.73287 \times 10^{-7} & 0.0000204323 & 1. \end{pmatrix}$$

$$V_{uR} = V_{uR}^o V_{uR}^{(1)}:$$

$$\begin{pmatrix} -0.000293496 & 0.187295 & -0.563405 & 0.804671. \\ -0.00149523 & 0.982274 & 0.101149 & -0.157814 \\ 0.999999 & 0.00152369 & -0.0000144046 & 0. \\ 3.50745 \times 10^{-7} & 0.00752158 & 0.819966 & 0.572364 \end{pmatrix}$$

d-quarks:

$$\mathcal{M}_d^o = \begin{pmatrix} 0 & 0 & 0 & 478.509. \\ 0 & 0 & 0 & 2132.96. \\ 0 & 0 & 0 & 2914.63. \\ 3.19851 \times 10^6 & -501085. & 0 & 1.97236 \times 10^6 \end{pmatrix} \text{ MeV}$$

$$\mathcal{M}_d = \begin{pmatrix} 0 & -3.71878 & -29.5924 & -48.5743 \\ 34.2 & 45.5029 & -1.54773 & -2.54051 \\ -45.6 & 34.1272 & -2860. & 412.606. \\ -0.0228 & 0.0170636 & 0.125684 & 3.79101 \times 10^6 \end{pmatrix} \text{ MeV}$$

$$(m_d, m_s, m_b, M_D) = (2.82069, 57., 2860.72, 3.79 \times 10^6) \text{ MeV}$$

Mixing matrices:

$$V_{dL} = V_{dL}^o V_{dL}^{(1)}:$$

$$\begin{pmatrix} -0.982152 & -0.123914 & -0.141504 & 0.0000673452 \\ 0.183872 & -0.790966 & -0.583578 & 0.000358725 \\ 0.0396112 & 0.599181 & -0.799633 & 0.000487473 \\ -0.0000191258 & 0 & 0.000608673 & 1. \end{pmatrix}$$

$$V_{dR} = V_{dR}^o V_{dR}^{(1)}:$$

$$\begin{pmatrix} 0.0824904 & -0.123915 & -0.515747 & 0.843709 \\ 0.593446 & -0.790966 & 0.0687286 & -0.132177 \\ 0.800455 & 0.59918 & -0.0159327 & -7.32943 \times 10^{-9} \\ 0.0169952 & -3.18402 \times 10^{-8} & 0.853831 & 0.520273 \end{pmatrix}$$

Quark mixing matrix

$$V_{CKM} = \begin{pmatrix} -0.974392 & -0.224827 & -0.003696 & -0.0000163 \\ -0.224475 & 0.973562 & -0.042288 & 0.0000214 \\ -0.013106 & 0.0403763 & 0.999098 & -0.0006083 \\ 3.74 \times 10^{-7} & -1.28 \times 10^{-6} & -0.000020 & 1.24 \times 10^{-8} \end{pmatrix}$$

Lepton masses and $(U_{PMNS})_{4 \times 8}$ mixing

Charged leptons:

Tree level:

$$\mathcal{M}_e^o = \begin{pmatrix} 0 & 0 & 0 & 2670.25 \\ 0 & 0 & 0 & 11902.6 \\ 0 & 0 & 0 & 16264.7 \\ 1.21882 \times 10^{10} & -2.32202 \times 10^9 & 0 & 6.07835 \times 10^{10} \end{pmatrix} \text{ MeV,}$$

up to one loop corrections:

$$\mathcal{M}_e = \begin{pmatrix} 0 & -19.9797 & -83.226 & -16.9884 \\ 0.6408 & 71.9782 & 293.027 & 59.814 \\ -0.8544 & 168.853 & -1712.54 & 480.432 \\ -2.74 \times 10^{-7} & 0.000054 & 0.000755 & 6.20 \times 10^{10} \end{pmatrix} \text{ MeV}$$

the charged lepton masses

$$(m_e, m_\mu, m_\tau, M_E) = (0.486031, 102.717, 1746.17, 6.20 \times 10^{10}) \text{ MeV}$$

Mixing matrices:

$$V_{eL} = V_{eL}^o V_{eL}^{(1)}:$$

$$\begin{pmatrix} 0.986458 & 0.0744614 & -0.146138 & 4.30921 \times 10^{-8} \\ 0.00276675 & -0.898433 & -0.439101 & 1.93334 \times 10^{-7} \\ -0.163991 & 0.43275 & -0.886473 & 2.62497 \times 10^{-7} \\ 0 & 5.68933 \times 10^{-8} & 3.23887 \times 10^{-7} & 1 \end{pmatrix}$$

$$V_{eR} = V_{eR}^o V_{eR}^{(1)}:$$

$$\begin{pmatrix} -0.00110967 & 0.258469 & -0.945829 & 0.196466 \\ -0.0055735 & 0.965887 & 0.256182 & -0.0374297 \\ 0.999984 & 0.00567042 & 0.00037637 & 0 \\ 9.59174 \times 10^{-6} & -0.0149293 & 0.199442 & 0.979796 \end{pmatrix}$$

	$(\nu_{eL}^o)^c$	$(\nu_{\mu L}^o)^c$	$(\nu_{\tau L}^o)^c$	$(N_L^o)^c$	ν_{eR}^o	$\nu_{\mu R}^o$	$\nu_{\tau R}^o$	N_R^o
$\overline{\nu_{eL}^o}$	0	0	0	30.9559	0	0	0	13.2472
$\overline{\nu_{\mu L}^o}$	0	0	0	434.898	0	0	0	62.502
$\overline{\nu_{\tau L}^o}$	0	0	0	1980.48	0	0	0	76.9286
$\overline{N_L^o}$	30.9559	434.898	1980.48	40	790642.	114364	0	4000
$\overline{(\nu_{eR}^o)^c}$	0	0	0	790642.	0	0	0	9.8860×10^6
$\overline{(\nu_{\mu R}^o)^c}$	0	0	0	114364.	0	0	0	1.4086×10^6
$\overline{(\nu_{\tau R}^o)^c}$	0	0	0	0	0	0	0	0
$\overline{(N_R^o)^c}$	13.2472	62.502	76.9286	4000.	9.8860×10^6	1.4086×10^6	0	100000

Table: Tree Level Majorana masses

Tree level neutrino masses:

\mathcal{M}_ν^0 :

$$\begin{pmatrix} 0 & 0 & 0 & 30.9559 & 0 & 0 & 0 & 13.2472 \\ 0 & 0 & 0 & 434.898 & 0 & 0 & 0 & 62.502 \\ 0 & 0 & 0 & 1980.48 & 0 & 0 & 0 & 76.9286 \\ 30.9559 & 434.898 & 1980.48 & 40 & 790642. & 114364 & 0 & 4000 \\ 0 & 0 & 0 & 790642. & 0 & 0 & 0 & 9.8860 \times 10^6 \\ 0 & 0 & 0 & 114364. & 0 & 0 & 0 & 1.4086 \times 10^6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13.2472 & 62.502 & 76.9286 & 4000. & 9.8860 \times 10^6 & 1.4086 \times 10^6 & 0 & 100000 \end{pmatrix}$$

Neutrino Majorana mass matrix up to one loop:

\mathcal{M}_ν :

$$\begin{pmatrix} 4.89118 \times 10^{-7} & 0.00534 & 0.02683 & -0.02718 & 0.02291 & 0.02261 & 0.16167 & 0.16087 \\ 0.00534 & -0.00830 & 0.04847 & -0.03714 & -0.02119 & -0.02098 & -0.05213 & -0.05187 \\ 0.02683 & 0.04847 & -0.62927 & 0.62907 & -0.00373 & -0.00353 & -0.20960 & -0.20855 \\ -0.02718 & -0.03714 & 0.62907 & -0.53094 & -0.22186 & -0.22037 & 0.22869 & 0.22756 \\ 0.02291 & -0.02119 & -0.00373 & -0.22186 & -2604.12 & -0.25395 & -0.94008 & -0.93540 \\ 0.02261 & -0.02098 & -0.00353 & -0.22037 & -0.25395 & 2643.36 & 0.74848 & 0.74476 \\ 0.16167 & -0.05213 & -0.20960 & 0.22869 & -0.94008 & 0.74848 & -9.97002 \times 10^6 & 2109.94 \\ 0.16087 & -0.05187 & -0.20855 & 0.22756 & -0.93540 & 0.74476 & -2109.94 & 1.00658 \times 10^7 \end{pmatrix}$$

Neutrino masses: eV

$$(m_1 = 0.005847, m_2 = 0.010488, m_3 = 0.051461, m_4 = 1.21534,$$

$$m_5 = 2604.12, m_6 = 2643.36, m_7 = 9.97002 \times 10^6, m_8 = 1.00658 \times 10^7)$$

Squared neutrino mass differences:

$$m_2^2 - m_1^2 = 7.58162 \times 10^{-5} \text{ eV}^2$$

$$m_3^2 - m_2^2 = 2.53822 \times 10^{-3} \text{ eV}^2$$

$$m_4^2 - m_1^2 = 1.47441 \text{ eV}^2$$

Neutrino mixing:

$$U_\nu = U_\nu^o U_\nu^{(1)}:$$

$$\begin{pmatrix} -0.817815 & -0.573736 & -0.030985 & -0.030274 \\ 0.398121 & -0.525222 & -0.73405 & 0.011549 \\ 0.009873 & 0.027107 & 0.142716 & -0.645085 \\ 0.000011 & 5.86044 \times 10^{-6} & -0.000089 & -0.000079 \\ 0.013900 & -0.015841 & -0.002901 & -0.106192 \\ -0.097564 & 0.111201 & 0.020387 & 0.74528 \\ -0.40357 & 0.617751 & -0.662886 & -0.126871 \\ -9.34653 \times 10^{-7} & -4.8869 \times 10^{-7} & 7.12916 \times 10^{-6} & 6.29816 \times 10^{-6} \\ 0.008056 & -0.008006 & 1.12646 \times 10^{-6} & -1.08644 \times 10^{-6} \\ 0.115983 & -0.115003 & 6.87784 \times 10^{-6} & -6.81358 \times 10^{-6} \\ 0.532465 & -0.528342 & 0.000016 & -0.000016 \\ -0.702146 & -0.707553 & -0.056523 & -0.056254 \\ -0.064448 & 0.063411 & 0.701853 & -0.698211 \\ 0.450162 & -0.446888 & 0.100018 & -0.099499 \\ 0.000048 & 0.000047 & 0 & 0 \\ 0.056171 & 0.056604 & -0.702997 & -0.706708 \end{pmatrix}$$

U_{PMNS} lepton mixing matrix :

$$\begin{pmatrix} -0.807257 & -0.571865 & -0.0560008 & 0.0759557 \\ -0.414308 & 0.440887 & 0.718949 & -0.291791 \\ -0.0640545 & 0.29044 & 0.200336 & 0.571204 \\ 4.43209 \times 10^{-8} & -1.19151 \times 10^{-7} & -1.05789 \times 10^{-7} & -1.68405 \times 10^{-7} \\ -0.0790509 & 0.0784275 & -1.6032 \times 10^{-6} & 1.60766 \times 10^{-6} \\ 0.126822 & -0.125914 & 1.11777 \times 10^{-6} & -1.07963 \times 10^{-6} \\ -0.524122 & 0.520029 & -0.0000179606 & 0.0000177362 \\ 1.62541 \times 10^{-7} & -1.61267 \times 10^{-7} & 0 & 0 \end{pmatrix}$$

FCNC in Neutral Mesons

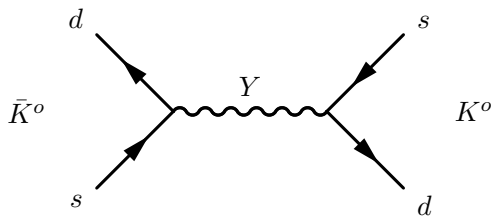


Figure: Generic tree level contribution to $K^0 - \bar{K}^0$ from the $SU(3)$ horizontal gauge bosons and scalar fields

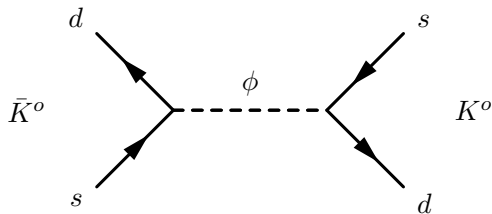


Figure: Generic tree level contribution to $K^0 - \bar{K}^0$ from the scalar

FCNC in Neutral Mesons

PDG2018: CKM quark-mixing matrix

To illustrate the level of suppression required for BSM contributions, consider a class of models in which the unitarity of the CKM is maintained, and the dominant effect of the new physics is to modify the neutral meson amplitudes by

$$\frac{z_{ij}}{\Lambda^2} (\bar{q}_i \gamma^\mu P_L q_j)^2$$

The existent data imply that

$$\frac{\Lambda}{\sqrt{|z_{ij}|}} \gtrsim \begin{cases} 10^4 \text{ TeV for } K^0 - \bar{K}^0 \\ 10^3 \text{ TeV for } D^0 - \bar{D}^0 \\ 500 \text{ TeV for } B^0 - \bar{B}^0 \\ 100 \text{ TeV for } B_s^0 - \bar{B}_s^0 \end{cases}$$

$K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ meson mixing

$K^0 - \bar{K}^0$:

$$\delta_L = 0.145437 \quad , \quad \frac{M_1}{\frac{g_H}{2} |\delta_L|} = 32601.3 \text{ TeV's}$$

$$\delta_R = 0.469396 \quad , \quad \frac{M_1}{\frac{g_H}{2} |\delta_R|} = 10101.1 \text{ TeV's}$$

$$\sqrt{|\delta_{LR}|} = 0.369507 \quad , \quad \frac{M_1}{\frac{g_H}{2} \sqrt{|\delta_{LR}|}} = 12831.8 \text{ TeV's}$$

$D^0 - \bar{D}^0$:

$$\delta_L = 0.000647997 \quad , \quad \frac{M_1}{\frac{g_H}{2} |\delta_L|} = 7.31705 \times 10^6 \text{ TeV's}$$

$$\delta_R = 0.00146872 \quad , \quad \frac{M_1}{\frac{g_H}{2} |\delta_R|} = 3.22828 \times 10^6 \text{ TeV's}$$

$$\sqrt{|\delta_{LR}|} = 0.669613 \quad , \quad \frac{M_1}{\frac{g_H}{2} \sqrt{|\delta_{LR}|}} = 7080.85 \text{ TeV's}$$

Conclusions:

- Particle model with local $SU(3)$ family symmetry can account for the hierarchical spectrum of quark masses and mixing, and charged lepton masses
- FCNC in $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ Neutral Mesons may be properly suppressed
- The mass of the $SU(2)_L$ weak singlet vector-like D quark, M_D , may lie within a few TeV's region
- Simultaneously this scenario can fit the squared neutrino mass differences: $m_2^2 - m_1^2 \approx 7.58 \times 10^{-5} \text{ eV}^2$, $m_3^2 - m_2^2 \approx 2.53 \times 10^{-3} \text{ eV}^2$, $m_4^2 - m_1^2 \approx 1.47 \text{ eV}^2$, as well as the majority of the corresponding entries of the U_{PMNS} lepton mixing matrix.