A YM-like theory with infinite many fields

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We have indications that quantum gravitational effects need a theory with infinite many fields (Camanho-Edelstein-Maldacena-Zhiboedov, 2014). See also H.Steinacker, 2019

Are string theories the only ones that can satisfy all the consistency requirements or are there others?
And, in case, how are they related to string theories?

Can they exhibit a critical dimension?

Or the good UV convergence properties of string theories?

This talk is about an attempt to construct theories with infinite many massless local fields that comprise matter and gravity

- Worldline quantization
- HS Yang-Mills-like theories
- Absence of ghosts
- Locality and non-locality
- No-go theorems: comments

How can we construct other candidate theories?

For HS theories, see:

Fradlin, Vasiliev, Prokushkin, Metsaev, Ponomarev, Skvortsov, Bekaert, Young, Mourad, Francia, Iazeolla, Sagnotti, Campoleoni, Fredenhagen, Fotopoulos, Tsulaia, Taronna,…

Their ambition is to construct sensible HS theories and, to a certain extent, they have succeeded (3d, 4d AdS…)

Our approach: worldline quantization

In simple words: quantize a particle worldline X^m with the Weyl quantization method and interpret a field as a function of the quantum X^m (in analogy with SFT where string fields are functions of first quantized string)

Strassler, Segal, Bastianelli, Bonezzi, Boulanger, Corradini, Latini, Bekaert, Joung, Mourad ...

Formulas in Weyl-Wigner quantization

In the Weyl-Wigner quantization a quantum operator $\widehat{O}$ can be represented with a symbol $O(x,p)$ through the Weyl map

$$\widehat{O} = \int d^d x\, d^d y \frac{d^d k}{(2\pi)^d} \frac{d^d p}{(2\pi)^d}\, O(x,p)\, e^{ik\cdot(x-\widehat{X})-iy\cdot(p-\widehat{P})}$$

where $\widehat{X}\,(\widehat{P})$ is the position (momentum) operator.

Product of operators corresponds to Moyal product of symbols:

$$\widehat{O}_1\widehat{O}_2 \longleftrightarrow O_1(x,p) * O_2(x,p)$$

The Moyal product is defined by

$$a(x,p) * b(x,p) = a(x,p)\, \exp\left[\frac{i}{2}\left(\overleftarrow{\partial}_x \cdot \overrightarrow{\partial}_p - \overrightarrow{\partial}_x \cdot \overleftarrow{\partial}_p\right)\right] b(x,p),$$

it is associative and has the trace property

$$\int d^p x\, \frac{d^d p}{(2\pi)^d}\, a(x,p) * b(x,p) = \int d^p x\, \frac{d^d p}{(2\pi)^d} a(x,p)\, b(x,p) = \int d^p x\, \frac{d^d p}{(2\pi)^d} b(x,p) * a(x,p)$$

Master fields

We imitate string field theory, where string fields are function of $x^\mu(\sigma, \tau)$, and introduce master fields, i.e. fields on the phase space that dipend both on x^μ and the conjugate momentum u_μ

$$[x^\mu, u_\nu] = i\delta^\mu_\nu$$

so that we can fully exploit the Weyl-Wigner formalism.

In particular we introduce a gauge master field $h_a(x, u)$:

$$h_a(x, u) = \sum_{n=0}^{\infty} \frac{1}{n!} h_a^{\mu_1 \dots \mu_n}(x) u_{\mu_1} \dots u_{\mu_n}$$

and endow it with a HS gauge symmetry.

The dependence on u is a counting device. The dynamics is in the x spacetime alone.

Higher spin symmetry

The HS transformation is

$$\delta_\varepsilon h_a(x,u) = \partial_a^x \varepsilon(x,u) - i[h_a(x,u) \stackrel{*}{,} \varepsilon(x,u)] \equiv \mathcal{D}_a^{x*}\varepsilon(x,u)$$

Let us expand

$$h_a(x,u) = A_a(x) + \chi_a^\nu(x)u_\nu + \frac{1}{2}B_a^{\nu\lambda}u_\nu u_\lambda + \dots$$

and

$$\varepsilon(x,u) = \epsilon(x) + \xi^\mu u_\mu + \frac{1}{2}\Lambda^{\mu\nu}u_\mu u_\nu + \dots$$

Then, in components

$$\begin{aligned}
\delta A_a &= \partial_a\epsilon + \xi\cdot\partial A_a - \partial_\rho\epsilon\,\chi_a^\rho + \dots \\
\delta\chi_a^\nu &= \partial_a\xi^\nu + \xi\cdot\partial\chi_a^\nu - \partial_\rho\xi_\nu\chi_a^\rho + \partial^\rho A_a\Lambda_\rho{}^\nu - \partial_\lambda\epsilon\,B_a{}^{\lambda\nu} + \dots \\
\delta B_a{}^{\nu\lambda} &= \partial_a\Lambda^{\nu\lambda} + \xi\cdot\partial B_a{}^{\nu\lambda} - \partial_\rho\xi^\nu B_a{}^{\rho\lambda} \\
&\quad -\partial_\rho\xi^\lambda B_a{}^{\rho\nu} + \partial_\rho\chi_a^\nu\Lambda_\rho{}^\lambda + \partial_\rho\chi_a^\lambda\Lambda_\rho{}^\nu - \chi_a^\rho\partial_\rho\Lambda_{\nu\lambda} + \dots
\end{aligned}$$

Higher spin symmetry

Denoting with a tilde the standard fields and restricting to gauge and diff transformations, we have

$$\begin{aligned}\delta\tilde{A}_a &\equiv \delta\left(\tilde{e}^\mu_a\tilde{A}_\mu\right)\equiv\delta\left((\delta^\mu_a-\tilde{\chi}^\mu_a)\tilde{A}_\mu\right)\\ &= \left(-\xi\cdot\partial\tilde{\chi}^\mu_a+\partial_\lambda\xi^\mu\tilde{\chi}^\lambda_a\right)\tilde{A}_\mu+(\delta^\mu_a-\tilde{\chi}^\mu_a)\left(\partial_\mu\epsilon+\xi\cdot\partial\tilde{A}_\mu\right)\approx\partial_a\epsilon+\xi\cdot\partial\tilde{A}_a-\tilde{\chi}^\mu_a\partial_\mu\epsilon\end{aligned}$$

and

$$\delta\tilde{e}^\mu_a\equiv\delta(\delta^\mu_a-\tilde{\chi}^\mu_a)=\xi\cdot\partial\tilde{e}^\mu_a-\partial_\lambda\xi^\mu\tilde{e}^\lambda_a=-\xi\cdot\partial\tilde{\chi}^\mu_a-\partial_a\xi^\mu+\partial_\lambda\xi^\mu\tilde{\chi}^\lambda_a$$

so that

$$\delta\tilde{\chi}^\mu_a=\xi\cdot\partial\tilde{\chi}^\mu_a+\partial_a\xi^\mu-\partial_\lambda\xi^\mu\tilde{\chi}^\lambda_a$$

where $\tilde{A}_\mu, \tilde{e}^\mu_a$ represent the standard vector field and vielbein and we have retained only the terms at most linear in the fields. From the above we see that we can make the identifictions

$$A_a=\tilde{A}_a,\qquad \chi^\mu_a=\tilde{\chi}^\mu_a$$

HS YM eom's. Remark

In the usual approach to gravity one assume the fluctuation is $h_{\mu\nu}$ with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.
Here instead we have

$$\begin{aligned} e^{\mu}_{a} &= \delta^{\mu}_{a} - \chi^{\mu}_{a} \\ e^{a}_{\mu} &= \delta^{a}_{\mu} + \chi^{\mu}_{a} + \chi^{a}_{b}\chi^{b}_{\mu} + \ldots \\ g^{\mu\nu} &= \eta^{\mu\nu} - 2\chi^{\mu\nu} + \chi^{\nu}_{a}\chi^{a\nu} \\ g_{\mu\nu} &= \eta_{\mu\nu} + 2\chi_{\mu\nu} + 3\chi^{a}_{\mu}\chi_{a\nu} + \ldots \end{aligned}$$

so that

$$h_{\mu\nu} = 2\chi_{\mu\nu} + 3\chi^{a}_{\mu}\chi_{a\nu} + \ldots \quad (1)$$

$$\chi^{\mu}_{a} = h^{\mu}_{a} - \frac{3}{4}h^{\nu}_{a}h^{\mu}_{\nu} + \ldots \quad (2)$$

What is at most quartic in χ^{μ}_{a} is any order in $h_{\mu\nu}$!

HS YM

Let us introduce the ‘field strenght’ components

$$G_{ab}(x,u) = \partial^x_a h_b(x,u) - \partial^x_b h_a(x,u) - i[h_a(x,u) \stackrel{*}{,} h_b(x,u)]$$

Its transformation rule is

$$\delta_\varepsilon G_{ab} = -i[G_{ab} \stackrel{*}{,} h_b]$$

Now define

$$\mathcal{YM}(h) = \int d^d x \frac{d^d u}{(2\pi)^d} G^{ab}(x,u) * G_{ab}(x,u) \equiv \langle\langle G^{ab} * G_{ab} \rangle\rangle$$

It follows that

$$\delta_\varepsilon \langle\langle G^{ab} * G_{ab} \rangle\rangle = 0$$

HS YM eom's

Taking the variation of $\mathcal{YM}(h)$ we get the following eom's

$$\partial_b G^{ab} - i[h_b \stackrel{*}{,} G^{ab}] \equiv \mathcal{D}^*_b G^{ab} = 0$$

Let us expand G_{ab} in component fields: $G_{ab} = F_{ab} + X^{\mu}_{ab} u_\mu + \frac{1}{2} B^{\mu\nu}_{ab} u_\mu u_\nu + \ldots$, where

$$\begin{aligned}
F_{ab} &= \partial_a A_b - \partial_b A_a + \partial_\sigma A_a \chi^\sigma_b - \partial_\sigma A_b \chi^\sigma_a + \ldots \\
X^{\mu}_{ab} &= \partial_a \chi^\mu_b - \partial_b \chi^\mu_a + \partial_\sigma A_a\, b_b{}^{\sigma\mu} - \partial_\sigma A_b\, b_a{}^{\sigma\mu} - \chi^\sigma_a\, \partial_\sigma \chi^\mu_b + \chi^\sigma_b\, \partial_\sigma \chi^\mu_a + \ldots \\
B_{ab}{}^{\mu\nu} &= \partial_a b_b{}^{\mu\nu} - \partial_b b_a{}^{\mu\nu} + 2\partial_\sigma \chi^{(\mu}_a b_b{}^{\nu)\sigma} - 2\partial_\sigma \chi^{(\mu}_b b_a{}^{\nu)\sigma} + \partial_\sigma b_a{}^{\mu\nu} \chi^\sigma_b - \partial_\sigma b_b{}^{\mu\nu} \chi^\sigma_a + \ldots \\
& \ldots\ldots
\end{aligned}$$

The explicit eom's are

$$\begin{aligned}
0 &= \partial_a F^{ab} + \partial_\nu A_a\, X^{ab\nu} - \chi^\nu_a \partial_\nu F^{ab} + \ldots \\
0 &= \partial_a X^{ab\lambda} + \partial_\nu A_a\, B^{ab\nu\lambda} - b^{\nu\lambda}_a \partial_\nu F^{ab} + \partial_\nu \chi^\lambda_a X^{ab\nu} - \chi^\nu_a \partial_\nu X^{ab\lambda} + \ldots \\
0 &= \partial_a B^{ab\mu\lambda} + \ldots\ldots \\
& \ldots\ldots
\end{aligned}$$

HS YM eom's. Examples

- Case of a pure U(1) gauge field A alone. The eom is

$$\partial_a F^{ab} = \Box A^b - \partial_b \partial\cdot A = 0$$

In the Landau gauge, $\partial\cdot A = 0$, this reduces to $\Box A^b = 0$.

- Only gravity is present. The eom becomes

$$\partial_a X^{ab\mu} = \Box \chi_b^\mu - \partial_b \partial\cdot \chi^\mu = 0$$

In the 'Landau gauge', $\partial\cdot\chi^\mu = 0$, it reduces to $\Box\chi_b^\mu = 0$.

- Keeping only the spin 3 field the eom becomes

$$\partial_a B^{ab\mu\nu} = \Box {b_b}^{\mu\nu} - \partial_b \partial^a b_a^{\mu\nu} = 0 \qquad (1)$$

Again in the 'Landau gauge', $\partial^a b_a^{\mu\nu} = 0$, we get $\Box {b_b}^{\mu\nu} = 0$.

Remark: The interaction is at most quartic!

Local Lorentz symmetry

The action is apparently not invariant under local Lorentz transformations (LLT's). Under $A_a \rightarrow A_a + \Lambda_a{}^b A_b$, terms $\left((\partial_a \Lambda_b{}^c)\, A_c - (\partial_b \Lambda_a{}^c)\, A_c\right) F^{ab}$ are generated in $F_{ab}F^{ab}$, that do not vanish. This is because the formalism automatically fixes the LL gauge. However LL invariance can be easily recovered.

Let us introduce a trivial frame $e_b{}^\mu(x)$ via

$$O_a{}^b(x) e_b{}^\mu(x) = \delta_a^\mu$$

where $O_a{}^b(x)$ is an LLT. A full gravitational (dynamical) frame is the sum of a trivial frame and a nontrivial piece

$$\tilde{E}_a^\mu(x) = e_a{}^\mu(x) - \tilde{\chi}_a^\mu(x)$$

With a LLT we can take it to the form

$$E_a^\mu(x) = \delta_a{}^\mu(x) - \chi_a^\mu(x)$$

In a similar way we can define a trivial connection

$$\mathcal{A}^a{}_{b\mu} = \left(O^{-1}(x)\partial_\mu O(x)\right)^a{}_b$$

whose curvature vanishes.

Local Lorentz symmetry (cnt.)

Local Lorentz symmetry is recovered by interpreting any flat index as follows:

$$O_a(x) \quad \text{as} \quad e_a{}^\mu(x) O_\mu(x),$$

and replacing the ordinary derivative with the inertial covariant derivative

$$\partial_\mu \quad \text{by} \quad D_\mu = \partial_\mu - \frac{i}{2}\mathcal{A}_\mu$$

everywhere (even in the Moyal product). Then, for instance

$$\delta_\Lambda G_{ab} = \Lambda_a{}^c G_{cb} + \Lambda_b{}^c G_{ac}$$

which implies the local Lorentz invariance of $G_{ab}G^{ab}$.

The separation between inertial and gravitational parts in frames and connections is characteristic of teleparallelism.

Matter fields

It is possible to introduce scalar master fields of the type

$$\Phi(x,u) = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi^{\mu_1\mu_2\ldots\mu_n}(x) u_{\mu_1} u_{\mu_2} \ldots u_{\mu_n}$$

which under gauge transformations behaves like

$$\delta_\varepsilon \Phi = i\varepsilon * \Phi, \qquad \delta_\varepsilon \Phi^\dagger = -i\Phi^\dagger * \varepsilon,$$

The covariant derivative is $\mathcal{D}^*_a \Phi = \partial_a \Phi - i h_a * \Phi$

In the same way one can introduce spinor master fields

$$\Psi(x,u) = \sum_{n=0}^{\infty} \frac{1}{n!} \Psi^{\mu_1\ldots\mu_n}_{(n)}(x) u_{\mu_1} \ldots u_{\mu_n},$$

Under HS gauge transformations it transforms according to

$$\delta_\varepsilon \Psi = i\varepsilon * \Psi, \qquad \delta_\varepsilon \Psi^\dagger = -i\Psi^\dagger * \varepsilon,$$

and let us define the covariant derivative: $\mathcal{D}^*_a \Psi = \partial_a \Psi - i h_a * \Psi$.

BRST quantization

One can introduce FP ghost c and antighost $\bar{c}$ master fields together with an auxiliary master field B and write down the action

$$\mathcal{YM}(h_a, c, B) = \frac{1}{g^2}\langle\langle -\frac{1}{4}G_{ab} * G^{ab} - h^a * \partial_a B - i\partial^a \bar{c} * \mathcal{D}^*_a c + \frac{\alpha}{2} B * B \rangle\rangle$$

which is invariant under the nilpotent BRST transformations

$$\begin{aligned} sh_a &= \mathcal{D}^*_a c \\ sc &= ic * c = \frac{i}{2}[c \overset{*}{,} c] \\ s\bar{c} &= iB \\ sB &= 0 \end{aligned}$$

Integrating out B one gets the gauge fixed action

$$\mathcal{YM}(h_a, c) = \frac{1}{g^2}\langle\langle -\frac{1}{4}G_{ab} * G^{ab} - \frac{1}{2\alpha}\partial_a h^a * \partial_b h^b - i\partial^a \bar{c} * \mathcal{D}^*_a c \rangle\rangle$$

Higgs mechanism

Add a scalar master field Φ with action

$$\mathcal{S}(\Phi, h) = \frac{1}{2}\langle\langle(\mathcal{D}^a_*\Phi)^\dagger * \mathcal{D}^*_a\Phi + \frac{\mu^2}{2}\Phi^\dagger * \Phi - \frac{\lambda}{4!}(\Phi^\dagger * \Phi)^2_*\rangle\rangle$$

and suppose that only φ_0 takes on a nonvanishing vacuum expectation value, say v, so that

$$\varphi_0(x) = v + \phi_0(x), \qquad v = \mu\frac{6}{\sqrt{\lambda}}$$

Next it is convenient to use finite gauge transformations:

$$\Phi \longrightarrow e^{i\varepsilon}_* * \Phi, \qquad \Phi^\dagger \longrightarrow \Phi^\dagger * e^{-i\varepsilon}_*, \qquad h_a \longrightarrow i\, e^{i\varepsilon}_* * \mathcal{D}^*_a e^{-i\varepsilon}_*$$

Then one can parametrize Φ as $e^{i\omega}_* * \varphi_0$, where$\omega = \omega^\mu_1(x)u_\mu + \frac{1}{2}\omega^{\mu\nu}_2(x)u_\mu u_\nu + \dots$. Then the potential becomes

$$V(\phi_0) \quad = \quad \langle\langle\frac{\mu^4}{4\lambda} - \mu^2\phi_0^2 - \frac{3}{4}\mu\sqrt{\lambda}\phi_0^3 - \frac{\lambda}{4}\phi_0^4\rangle\rangle$$

and, defining the new gauge master fields

$$h'_a = e^{i\omega}_* * \mathcal{D}^*_a e^{-i\omega}_*,$$

the kinetic term for the scalars becomes

$$K(\varphi_0, h') = \frac{1}{2}\langle\langle(\mathcal{D}'^a_*\phi_0)^\dagger * \mathcal{D}'^*_a\phi_0 + v^2\, h'^a h'_a\rangle\rangle$$

The scalars, except ϕ_0, have been eaten and the gauge fields have become massive.

Perturbative quantization

One has to get rid of the u integration. This is done extracting a mass scale from u_μ: $\sqrt{u^2} = \mathrm{m}\mathrm{u}$ and by redefining $h_a^{\mu_1\dots\mu_{s-1}} \to h_a^{\prime\,\mu_1\dots\mu_{s-1}} = \mathrm{u}^{s-1} h_a^{\mu_1\dots\mu_{s-1}}$. One must also redefine the coordinates as follows: $x^\mu \to \mathrm{u} x^\mu$, as well as the coupling as $g \to \frac{g}{\mathrm{u}}$. The integrand becomes u-independent and the measure becomes

$$\int d^d x d^d u \equiv \mathrm{m}^d \int d^d x d^d \mathrm{u} \longrightarrow \mathrm{m}^d \int d^d x d^d \mathrm{u}\, \mathrm{u}^{d-2}$$

so one can factor out the u integration.

For instance the cubic part of the action becomes

$$\begin{aligned} \mathcal{S}_3 \;=\; & -g \int d^d x \Big\{ \partial^a A^b (\partial_\sigma A_a \chi_b^\sigma - \partial_\sigma A_b \chi_a^\sigma) \\ & -\frac{1}{24} (\partial^a A^b - \partial^b A^a)(\partial_{\sigma_1}\partial_{\sigma_2}\partial_{\sigma_3} A_a\, c_b^{\sigma_1\sigma_2\sigma_3} + 3\partial_{\sigma_3} b_a^{\sigma_1\sigma_2} \partial_{\sigma_1}\partial_{\sigma_2}\chi_b^{\sigma_3}) \\ & +\frac{\mathrm{m}^2}{2d}\Big(\partial^a A^b \partial_\sigma b_{a\mu}{}^{\mu} \chi_b^\sigma - \partial^a A^b \partial_\sigma b_{b\mu}{}^{\mu}\chi_a^\sigma \\ & +2\partial^a A^b \partial_\sigma \chi_a^\mu b_{b\mu}{}^{\sigma} - 2\partial^a A^b \partial_\sigma \chi_b^\mu b_{a\mu}{}^{\sigma} + \partial^a A^b \partial_\sigma A_a c_{b\mu}{}^{\mu\sigma} - \partial^a A^b \partial_\sigma A_b c_{a\mu}{}^{\mu\sigma} \\ & +\partial^a b^{b\mu}{}_{\mu}\, \partial_\sigma A_a \chi_b^\sigma - \partial^a b^{b\mu}{}_{\mu}\, \partial_\sigma A_b \chi_a^\sigma + 2\partial^a \chi_\nu^b (\partial_\sigma A_a\, b_b{}^{\sigma\nu} - \partial_\sigma A_b\, b_a{}^{\sigma\nu} \\ & +\partial_\sigma \chi_a^\nu \chi_b^\sigma - \partial_\sigma \chi_b^\nu \chi_a^\sigma) + \dots \Big) \Big\} \end{aligned} \qquad (1)$$

The propagator

The quadratic part of the action takes the form

$$\mathcal{S}_2 = \int d^d x \sum_{\{\mu\},\{\nu\}} h^a_{\{\mu\}}(x) K^{\{\mu\}\{\nu\}}_{ab}(x,\mathbf{m}) h^b_{\{\nu\}}(x)$$

where

$$K^{\{\mu\}\{\nu\}}_{ab}(x,\mathbf{m}) = \left(\eta_{ab}\Box_x - \frac{\alpha-1}{\alpha}\partial^x_a\partial^x_b\right) N^{\{\mu\}\{\nu\}}(\mathbf{m}) \equiv \mathcal{K}^x_{ab} N^{\{\mu\}\{\nu\}}(\mathbf{m})$$

and

$$N^{\{\mu\}\{\nu\}}(\mathbf{m}) = \begin{pmatrix} 1 & 0 & \eta^{\nu_1\nu_2}\frac{\mathbf{m}^2}{2d} & 0 & \Pi^{\nu_1\nu_2\nu_3\nu_4}\frac{\mathbf{m}^4}{4!d(d+2)} & 0 \\ 0 & \eta^{\mu_1\nu_1}\frac{\mathbf{m}^2}{d} & 0 & \Pi^{\mu_1\nu_1\nu_2\nu_3}\frac{\mathbf{m}^4}{3!d(d+2)} & 0 & \dots \\ \eta^{\mu_1\mu_2}\frac{\mathbf{m}^2}{2d} & 0 & \Pi^{\mu_1\mu_2\nu_1\nu_2}\frac{\mathbf{m}^4}{4d(d+2)} & 0 & \dots & \dots \\ 0 & \Pi^{\mu_1\mu_2\mu_3\nu_1}\frac{\mathbf{m}^4}{3!d(d+2)} & 0 & \dots & \dots & \dots \\ \Pi^{\mu_1\mu_2\mu_3\mu_4}\frac{\mathbf{m}^4}{4!d(d+2)} & 0 & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

This matrix is not invertible. We must impose tracelessness on the indices $\mu_1,\dots,\mu_n$. This is a further gauge condition one has to impose for a propagator to exist.

The propagator (cnt.)

If we define $M_{\{\nu\}\{\lambda\}}$ to be the inverse of $N_{\{\nu\}\{\lambda\}}$:

$$N^{\{\mu\}\{\nu\}}(\mathfrak{m})M_{\{\nu\}\{\lambda\}}(\mathfrak{m}) = \delta^{\{\mu\}}_{\{\lambda\}}$$

then we find

$$M^{\{\mu\}}_{\{\nu\}}(u) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \dots \\ 0 & \delta^{\mu_1}_{\nu_1}\frac{1}{\mathfrak{m}^2} & 0 & 0 & 0 & \dots \\ 0 & 0 & \Delta^{\mu_1\mu_2}_{\nu_1\nu_2}\frac{1}{\mathfrak{m}^4} & 0 & 0 & \dots \\ 0 & 0 & 0 & \Delta^{\mu_1\mu_2\mu_3}_{\nu_1\nu_2\nu_3}\frac{1}{\mathfrak{m}^6} & 0 & \dots \\ 0 & 0 & 0 & 0 & \Delta^{\mu_1\mu_2\mu_3\mu_4}_{\nu_1\nu_2\nu_3\nu_4}\frac{1}{\mathfrak{m}^8} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

where

$$\Delta^{\mu_1\dots\mu_n}{}_{\nu_1\dots\nu_n} = \frac{1}{n!}\left(\delta^{\mu_1}_{\nu_1}\dots\delta^{\mu_n}_{\nu_n} + \mathrm{perm}(\mu_1,\dots,\mu_n)\right)$$

Finally the propagator is

$$P^{\{\mu\}\{\nu\}}_{ab}(k,u) = \left(\frac{\eta_{ab}}{k^2} + (\alpha-1)\frac{k_a k_b}{k^4}\right) M^{\{\mu\}\{\nu\}}(u)$$

Recall: Physical modes are transverse and traceless (with respect to $\mu_1, \dots \mu_n$).

Physical modes

The issue is to decompose

$$\Delta^{\mu_1\ldots\mu_n}{}_{\nu_1\ldots\nu_n} = \frac{1}{n!}\left(\delta^{\mu_1}_{\nu_1}\ldots\delta^{\mu_n}_{\nu_n} + \text{perm}(\mu_1,\ldots,\mu_n)\right),$$

which is the identity operator in the space of simmetric tensors, according to irreducible representations of the Lorentz group. This can be done as follows Let us introduce the elementary projectors

$$\pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \qquad \omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}$$

with the properties: $\pi_{\mu\nu}\,\pi^\nu{}_\lambda = \pi_{\mu\lambda}$, $\quad \omega_{\mu\nu}\,\omega^\nu{}_\lambda = \omega_{\mu\lambda}$, $\quad \pi_{\mu\nu}\,\omega^\nu{}_\lambda = 0.$

Then, for $n = 2$ we have

$$\Delta_{\mu_1\mu_2,\nu_1\nu_2} = P^{(2)}_{\mu_1\mu_2,\nu_1\nu_2} + P^{(1)}_{\mu_1\mu_2,\nu_1\nu_2} + P^{(0)}_{\mu_1\mu_2,\nu_1\nu_2} + \overline{P}^{(0)}_{\mu_1\mu_2,\nu_1\nu_2}$$

where

$$\begin{aligned}
P^{(2)}_{\mu_1\mu_2,\nu_1\nu_2} &= \frac{1}{2}\left(\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2} + \pi_{\mu_1\nu_2}\pi_{\mu_2\nu_1}\right) - \frac{1}{d-1}\pi_{\mu_1\mu_2}\pi_{\nu_1\nu_2}\\
P^{(1)}_{\mu_1\mu_2,\nu_1\nu_2} &= \frac{1}{2}\left(\pi_{\mu_1\nu_1}\omega_{\mu_2\nu_2} + \pi_{\mu_1\nu_2}\omega_{\mu_2\nu_1} + \omega_{\mu_1\nu_1}\pi_{\mu_2\nu_2} + \omega_{\mu_1\nu_2}\pi_{\mu_2\nu_1}\right)\\
P^{(0)}_{\mu_1\mu_2,\nu_1\nu_2} &= \frac{1}{d-1}\pi_{\mu_1\mu_2}\pi_{\nu_1\nu_2}, \qquad \overline{P}^{(0)}_{\mu_1\mu_2,\nu_1\nu_2} = \omega_{\mu_1\mu_2}\omega_{\nu_1\nu_2}
\end{aligned}$$

These are projectors

$$P^{(2)}_{\mu_1\mu_2,\nu_1\nu_2}P^{(2)\nu_1\nu_2,}{}_{\lambda_1\lambda_2} = P^{(2)}_{\mu_1\mu_2,\lambda_1\lambda_2}, \qquad \ldots\ldots$$

These projectors are orthogonal to one another, $P^{(2)}$ is transverse and traceless, $P^{(1)}$ is traceless but not transverse. $P^{(0)}$ is transverse but not traceless, $\overline{P}^{(0)}$ is neither transverse nor traceless.

Physical modes

For $n = 3$

$$\Delta_{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} = P^{(3)}_{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} + P^{(2)}_{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} + P^{(0)}_{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} + P^{(0a)}_{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} + P^{(0b)}_{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} + \overline{P}^{(0)}_{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3}$$

where

$$\begin{aligned} P^{(3)}_{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} = {} & \frac{1}{6}\Big(\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3} + \pi_{\mu_1\nu_1}\pi_{\mu_2\nu_3}\pi_{\mu_3\nu_2} + \pi_{\mu_1\nu_2}\pi_{\mu_2\nu_3}\pi_{\mu_3\nu_1} \\ & + \pi_{\mu_1\nu_2}\pi_{\mu_2\nu_1}\pi_{\mu_3\nu_3} + \pi_{\mu_1\nu_3}\pi_{\mu_2\nu_1}\pi_{\mu_3\nu_2} + \pi_{\mu_1\nu_3}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_1}\Big) \\ & - \frac{1}{3(d+1)}\Big(\pi_{\mu_1\mu_2}\pi_{\nu_1\nu_2}\pi_{\mu_3\nu_3} + \pi_{\mu_1\mu_3}\pi_{\nu_1\nu_3}\pi_{\mu_2\nu_2} + \pi_{\mu_2\mu_3}\pi_{\nu_2\nu_3}\pi_{\mu_1\nu_1} \\ & + \pi_{\mu_1\mu_2}\pi_{\nu_1\nu_3}\pi_{\mu_3\nu_2} + \pi_{\mu_1\mu_2}\pi_{\nu_2\nu_3}\pi_{\mu_3\nu_1} + \pi_{\mu_1\mu_3}\pi_{\nu_1\nu_2}\pi_{\mu_2\nu_3} \\ & + \pi_{\mu_1\mu_3}\pi_{\nu_2\nu_3}\pi_{\mu_2\nu_1} + \pi_{\mu_2\mu_3}\pi_{\nu_1\nu_3}\pi_{\mu_1\nu_2} + \pi_{\mu_2\mu_3}\pi_{\nu_1\nu_2}\pi_{\mu_1\nu_3}\Big) \end{aligned}$$

and so on.

Only $P^{(3)}$ is tranverse and traceless. Only transverse and traceless models are physical. Therefore

Replace Δ in the propagator with the unique transverse and traceless projector.

$$\Delta_{\mu_1\ldots\mu_n,\nu_1\ldots\nu_n} \longrightarrow \mathcal{P}^{(n)}_{\mu_1\ldots\mu_n,\nu_1\ldots\nu_n}$$

This guarantees that only physical modes propagate and no physical modes are lost.

Physical modes

For instance, the two-point function for traceless transverse modes is

$$\langle \tilde{h}_a^{\mu_1\ldots\mu_m}(q_1)\tilde{h}_b^{\nu_1\ldots\nu_n}(q_2)\rangle_0 = g^2\, q_1^b\, q_2^a\, \delta(q_1+q_2)\int \frac{d^dp}{(2\pi)^d}\,\frac{1}{p^2(p-q_1)^2}\,\frac{\mathrm{m}^{n+m}}{n!m!}$$

$$\times \int_{S^{d-1}} dn_1 \int_{S^{d-1}} dn_2\; n_{1\mu_1}\ldots n_{1\mu_m} n_{2\nu_1}\ldots n_{2\nu_n}$$

$$\sum_{l,r=0}^{\infty} \frac{1}{(l!)^2(r!)^2}\,\frac{1}{\mathrm{m}^{2l}}\,\frac{1}{\mathrm{m}^{2r}}\,\mathcal{P}^{(l)\lambda_1\ldots\lambda_l\sigma_1\ldots\sigma_l}(p-q_1)\,\mathcal{P}^{(r)\rho_1\ldots\rho_r\tau_1\ldots\tau_r}(p)$$

$$\times \left[\left(n_1+\frac{p-q_1}{2\mathrm{m}}\right)_{\lambda_1}\ldots\left(n_1+\frac{p-q_1}{2\mathrm{m}}\right)_{\lambda_l}\left(n_1+\frac{p}{2\mathrm{m}}\right)_{\rho_1}\ldots\left(n_1+\frac{p}{2\mathrm{m}}\right)_{\rho_r}\right.$$

$$\left.-\left(n_1-\frac{p-q_1}{2\mathrm{m}}\right)_{\lambda_1}\ldots\left(n_1-\frac{p-q_1}{2\mathrm{m}}\right)_{\lambda_l}\left(n_1-\frac{p}{2\mathrm{m}}\right)_{\rho_1}\ldots\left(n_1-\frac{p}{2\mathrm{m}}\right)_{\rho_r}\right]$$

$$\times \left[\left(n_2+\frac{p-q_1}{2\mathrm{m}}\right)_{\sigma_1}\ldots\left(n_2+\frac{p-q_1}{2\mathrm{m}}\right)_{\sigma_s}\left(n_2+\frac{p}{2\mathrm{m}}\right)_{\tau_1}\ldots\left(n_2+\frac{p}{2\mathrm{m}}\right)_{\tau_j}\right.$$

$$\left.-\left(n_2-\frac{p-q_1}{2\mathrm{m}}\right)_{\sigma_1}\ldots\left(n_2-\frac{p-q_1}{2\mathrm{m}}\right)_{\sigma_s}\left(n_2-\frac{p}{2\mathrm{m}}\right)_{\tau_1}\ldots\left(n_2-\frac{p}{2\mathrm{m}}\right)_{\tau_j}\right]$$

But what are the physical modes?

Consider a free state $\zeta_{\mu\nu} e^{ik\cdot x}$, where ζ is symmetric. Projecting it with the symmetric transverse traceless projector $\mathcal{P}^{(2)} \equiv P^{(2)}$ we obtain a symmetric transverse traceless polarization

$$\bar{\zeta}_{\mu_1\mu_2} = P^{(2)}_{\mu_1\mu_2}{}^{\nu_1\nu_2}\zeta_{\nu_1\nu_2}, \qquad \bar{\zeta}_{\mu_1\mu_2}k^{\mu_1} = \zeta_{\mu_1\mu_2}k^{\mu_2} = 0$$

Setting $k_0 = k_1 = k$ implies $\bar{\zeta}_{00} = \bar{\zeta}_{01} = \bar{\zeta}_{10} = \bar{\zeta}_{11}$ and $\bar{\zeta}_{0i} = \bar{\zeta}_{1i}$. Thus

$$\bar{\zeta}_{\mu_1\mu_2}\bar{\zeta}^{\mu_1\mu_2} = \sum_{i,j=2}^{d-1} \bar{\zeta}^2_{ij} \geq 0$$

The state with such a polarization is physical. But in order to be physical and preserve Lorentz covariance it must include also zero norm states.

For instance in 10 dimensions the symmetric representation of SO(1,9) is the sum of the traceless representation **54** and the identity **1** (the trace). The corresponding little group is SO(8), whose symmetric representation splits as a traceless **35** and **1**. The strictly physical degrees of freedom are 35. But in order to preserve Lorentz covariance we have to add a vector representation **8** plus an identity one, **1**. The sum of all these degrees of freedom $35 + 1 + 8 + 1 = 45$ corresponds to the dimension of the (traceless) adjoint representation of SO(1,9).

Physical modes (cont.)

In d dimensions the symmetric tensor n representation of the Lorentz group and the corresponding irreducible traceless representation have dimension

$$\binom{d+n-1}{n} \qquad \text{and} \qquad \frac{(d+n-3)(d+n-2)\dots(d-2)(d-3)}{n!}(d+2s-2),$$

The symmetric tensor n representation of the little group and the corresponding irreducible traceless representation have dimension

$$\binom{d+n-3}{n} \qquad \text{and} \qquad \frac{(d+n-5)(d+n-4)\dots(d-2)(d-3)}{n!}(d+2s-4),$$

To the last one we have to add zero norm representations of the little group in order to preserve Lorentz covariance. These states form a (in general reducible) symmetric traceless representation of the Lorentz group of dimension

$$\binom{d+n-2}{n}$$

The total number of dofs to be eliminated (in order to preserve physicality and Lorentz covariance) is

$$\binom{d+n-2}{n-1} \tag{1}$$

Hidden symmetry

Returning to the master field $h_a(x,u)$, to compensate for the unphysical degrees of freedom we need a gauge symmetry parametrized as follows

$$\delta h_a^{\mu_1\dots\mu_n}(x) = n\,\partial^{(\mu_1}\Lambda_a^{\mu_2\dots\mu_n)}(x) + \dots, \qquad n \geq 2.$$

This symmetry is hidden, i.e. not manifest in the initial action.

Unfolding the hidden gauge symmetry

Replace everywhere in the action any component field $h_a^{\mu_1\cdots\mu_n}$(with $n \geq 2$) with its projection by the appropriate $\mathcal{P}^{(n)}$ in configuration space, i.e. replacing $\pi_{\mu\nu}$ with

$$\breve{\pi}_{\mu\nu}(\partial) = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}$$

For instance

$$b_a^{\mu_1\mu_2} \longrightarrow \breve{b}_a^{\mu_1\mu_2} = \left(\frac{1}{2} \left(\breve{\pi}^{\mu_1}{}_{\nu_1} \breve{\pi}^{\mu_2}{}_{\nu_2} + \breve{\pi}^{\mu_1}{}_{\nu_2} \breve{\pi}^{\mu_2}{}_{\nu_1} \right) - \frac{1}{d-1} \breve{\pi}^{\mu_1\mu_2} \breve{\pi}_{\nu_1\nu_2} \right) b_a^{\nu_1\nu_2}$$

the transformation becomes

$$\delta_\Lambda h_a = \partial \Lambda_a$$

for any component field. For instance, the new b_a field takes the form

$$\breve{b}_a = b_a - \frac{\partial}{\Box} \partial \cdot b_a + \frac{\partial^2}{\Box^2} \partial \cdot \partial \cdot b_a - \frac{1}{d-1} \left(\eta - \frac{\partial^2}{\Box} \right) \left(b_a' - \frac{1}{\Box} \partial \cdot \partial \cdot b_a \right)$$

Similarly the component field $c_a^{\mu_1\mu_2\mu_3}$ is replaced by

$$\begin{aligned} \breve{c}_a \;\; = \;\; & c_a - \frac{\partial}{\Box} \partial \cdot c_a + \frac{\partial^2}{\Box^2} \partial \cdot \partial \cdot c_a - \frac{\partial^3}{\Box^3} \partial \cdot \partial \cdot \partial \cdot c_a \\ & - \frac{1}{d+1} \left(\eta - \frac{\partial^2}{\Box} \right) \left(c_a' - \frac{\partial}{\Box} \partial \cdot c_a' - \frac{1}{\Box} \partial \cdot \partial \cdot c_a + \frac{\partial}{\Box^2} \partial \cdot \partial \cdot \partial \cdot c_a \right) \end{aligned}$$

and so on. In these formulas a dot, $\cdot$, denotes index contraction, a prime $'$ denotes a trace, free ∂ a gradient, and symmetrization of the free indices is understood.

Gauge symmetry and non-locality

After the above replacements the action

$$\mathcal{YM}(\check{\mathbf{h}}) = -\frac{1}{4g^2}\langle\langle \check{G}^{ab} * \check{G}_{ab} \rangle\rangle$$

becomes non-local. It is however invariant under

$$\delta h_a^{\mu_1 \ldots \mu_n}(x) = n\,\partial^{(\mu_1}\Lambda_a^{\mu_2 \ldots \mu_n)}(x), \qquad n \geq 2.$$

(this follows from the fact that all $\mathcal{P}^{(n)}$ are transverse to the momentum and from the form of the transformation where a derivative always factors out). And it is also invariant under the new HS gauge transformation

$$\delta_{\check{\varepsilon}} \check{h}_a(x,u) = \partial_a \check{\varepsilon}(x,u) - i[\check{h}_a(x,u) \stackrel{*}{,} \check{\varepsilon}(x,u)]$$

A summary about (quantum) HS-YM theories

- They contain Maxwell (or non Abelian YM) and gravity
- There are precise recipes to compute any physical amplitude
- No ghosts
- They are gauge theories with two types of gauge symmetries
- They are non-local with a ‘controlled’ non-locality (non-locality disappears when fixing the gauge symmetry)
- They are supersymmetrizable

In summary: we can construct a theory for massless spin fields that are covariant under HS gauge transformation, which is ghost-free, in any dimension Minkowski spacetime!

This seems to contradict well-known no-go theorems concerning HS: an interacting massless HS theory in Minkowski spacetime cannot exist!

The no-go theorems:

- some no-go theorems are S-matrix statements
- some concern theories with a finite number of fields
- most concern theories with minimal coupling to gravity
- HS gauge transformations are different
- others are based on the Fronsdal equations
- they assume (non-perturbative) locality

Weinberg, Coleman, Mandula, Witten, Benincasa, Cachazo, Porrati, Bekaert, Boulanger, Leclercq, Conde, Young, Mkrtchyan, Ponomarev, Skvortsov, Metsaev,….

HS YM-like theories do not match some of these features (in particular locality)!

It is however necessary to show that they are consistent theories.

The first requirement is absence of ghosts: ok!

Other problems:

- they are non-local
- have high IR singularities
- can they be made local (with addition of auxiliary fields)?

In (temporary) conclusion:

There are serious problems to be tackled with HS-YM models….

….but they are worth studying

What could all this be related to?

String field theory may be the favorite playground to answer this question.

The first analogy which comes to mind is tensionless strings.

Tensionless strings

In the tensionless limit ($\alpha' \to \infty$) of the free string spectrum becomes massless. This limit is well defined. Recalling that $\alpha_0^\mu = \sqrt{2\alpha'}p^\mu$ and $p_\mu = -i\frac{\partial}{\partial x^\mu}$ and redefining the Virasoro generators as follows:

$$L_0 \to \frac{1}{\alpha'}L_0, \qquad L_k \to \frac{1}{\sqrt{2\alpha'}}L_k, \qquad k \neq 0$$

one obtains the reduced generators

$$L_0 \to l_0 = p^2, \qquad L_k \to l_k = p \cdot \alpha_k, \qquad k \neq 0$$

which satisfy the reduced Virasoro algebra

$$[l_k, l_n] = k\, l_0\, \delta_{k+n,0}$$

Tensionless strings (cont.)

Rescaling also the ghost oscillators C_k, B_k as follows

$$C_k \to c_k = \sqrt{2\alpha'} C_k, \qquad B_k \to b_k = \frac{1}{2\alpha'} B_k, \qquad k \neq 0$$

$$C_0 \to c_0 = \alpha' C_0, \qquad B_0 \to b_0 = \frac{1}{\alpha'} B_0$$

in the $\alpha' \to \infty$ one finds a reduced BRST charge

$$Q = \sum_{k=-\infty}^{\infty} \left(c_k l_k - \frac{k}{2} b_0 c_{-k} c_k \right)$$

There is no central charge. $Q^2 = 0$ in any dimension and $Q^\dagger = Q$.
The SFT action reduces to the free action (Bonelli, 2003)

$$S = \langle \Phi | Q | \Phi \rangle$$

It is invariant under the BRST transformation $\delta |\Phi\rangle = Q|\Lambda\rangle$.

Tensionless string (cont.)

The states of this theory have been analyzed by Francia-Sagnotti-Tsulaia. The simplest states are the so-called triplets:

$$|\Phi\rangle = |\phi_0\rangle + c_0|\phi_1\rangle, \qquad |\Lambda\rangle = |\Lambda_0\rangle + c_0|\Lambda_1\rangle,$$

where

$$\begin{aligned} |\phi_0\rangle &= \sum_{s=0}^{\infty} \frac{1}{s!}\varphi_{\mu_1\ldots\mu_s}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_s}c_1|0\rangle \\ &+\sum_{s=0}^{\infty} \frac{1}{(s-2)!}D_{\mu_1\ldots\mu_{s-2}}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_{s-2}}c_{-1}|0\rangle \end{aligned}$$

and

$$|\phi_1\rangle = \sum_{s=0}^{\infty} \frac{1}{(s-1)!}C_{\mu_1\ldots\mu_{s-1}}(x)\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_{s-1}}|0\rangle$$

where $|k\rangle = |0\rangle e^{ikx}$. Satisfying the eom $Q|\Phi\rangle = 0$ implies

Tensionless strings (cont.)

$$\Box\varphi = \partial C,$$
$$\partial\cdot\varphi - \partial D = C$$
$$\Box D = \partial\cdot C$$

Now take the gradient of the second and replace into the first. Then take the trace of the first, and insert the result into the previous equation. The end result is (in unnormalized notation)

$$\Box\varphi - \partial\partial\cdot\varphi + \partial\partial\varphi' = 3\frac{\partial^3}{\Box}C' \qquad (1)$$

This is the Fronsdal equation written in terms of a 'compensator' α field

$$\alpha = \frac{C'}{\Box}$$

This is actually general for all the tensionless strings. This reduces tensionless string theory to a particular HS theory.

- Tensionless strings are non-interacting
- They reproduce Fronsdal-type equations
- They do not seem to be related to HS YM-like theories

A suggestion

In open string theory let us rescale $\alpha_n^\mu \to \frac{1}{\ell^n}\alpha_n^\mu = \tilde{\alpha}_n^\mu$. This induces the rescaling $L_n \to \frac{1}{\ell^n} L_n = \tilde{L}_n$, and an automorphism of the Virasoro algebra.

In a similar way we can rescale the ghosts oscillators $c_n^\mu \to \frac{1}{\ell^n} c_n^\mu = \tilde{c}_n^\mu$ and $b_n^\mu \to \frac{1}{\ell^n} b_n^\mu = \tilde{b}_n^\mu$. The BRST operator remains the same $Q = \tilde{Q}$.

Let us construct the states

$$
\begin{aligned}
|\phi\rangle &= \phi_{\mu_1\ldots\mu_n}(x)\, a_{-1}^{\mu_1} \ldots a_{-1}^{\mu_n}\, c_1|k\rangle \\
|H_a\rangle &= h_{a\mu_1\ldots\mu_n}(x)\, a_{-2}^{a}\, a_{-1}^{\mu_1} \ldots a_{-1}^{\mu_n}\, c_1|k\rangle \\
|H_{ab}\rangle &= h_{ab\mu_1\ldots\mu_n}(x)\, a_{-2}^{a} a_{-2}^{b}\, a_{-1}^{\mu_1} \ldots a_{-1}^{\mu_n}\, c_1|k\rangle \\
&\ldots\ldots
\end{aligned}
$$

They scale like

$$
\begin{aligned}
\phi_{\mu_1\ldots\mu_n}(x) &\to \ell^{n-1}\, \phi_{\mu_1\ldots\mu_n}(x) \\
h_{a\mu_1\ldots\mu_n}(x) &\to \ell^{n+1}\, h_{a\mu_1\ldots\mu_n}(x) \\
h_{ab\mu_1\ldots\mu_n}(x) &\to \ell^{n+3}\, h_{ab\mu_1\ldots\mu_n}(x) \\
&\ldots\ldots
\end{aligned}
$$

A suggestion (cnt.)

Now construct the master fields

$$
\begin{aligned}
\Phi(x,u) &= \sum_{n=0}^{\infty} \phi^{\mu_1\dots\mu_n}(x)u_{\mu_1}\dots u_{\mu_n} \\
h_a(x,u) &= \sum_{n=0}^{\infty} h_a^{\mu_1\dots\mu_n}(x)u_{\mu_1}\dots u_{\mu_n} \\
h_{ab}(x,u) &= \sum_{n=0}^{\infty} h_{ab}^{\mu_1\dots\mu_n}(x)u_{\mu_1}\dots u_{\mu_n} \\
&\dots\dots
\end{aligned}
$$

If we rescale u_μ as $u_\mu \to \frac{1}{\ell}u_\mu$, they scale like

$$
\begin{aligned}
\Phi(x,u) &\to \ell^{-1}\Phi(x,u) \\
h_a(x,u) &\to \ell\, h_a(x,u) \\
h_{ab}(x,u) &\to \ell^3\, h_{ab}(x,u) \\
&\dots\dots
\end{aligned}
$$

In this way one slices the states of string theory according to the powers of ℓ, reorganizing them in terms master fields. The question one may pose is:

For which of these master fiields can we formulate a local field theory?

The answer seems to be : only for Φ and h_a.

THANKS