Dynamics and mergers of primordial black holes in a cluster

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Introduction

The idea of the PBHs formation was proposed by Zel'dovich and Novikov[1].

Some astrophysical effects can be attributed to PBHs:

- PBHs form a part of the Dark Matter[2].
- Early structure formation[3].
- Supermassive black holes at large redshift[4] $z > 7$.
- Gravitational waves[5].

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[1] Sov. Astron. 10 (1967) 602
[2] Annu. Rev. Nucl. Part. Sci. 70 (2020) 355-394
[3] Phys. Rev. D. 100 (2019) 083528
[4] Nature 553 (2018) 473-476
[5] Phys. Rev. Lett. 116 (2016) 201301

Introduction

This work studies the dynamics of PBHs cluster[6] and estimates the merger rate of black hole in the cluster $\rightarrow$ restriction on the PBHs clusters.

Formation redshift $z_f = 10^4$.
Condition of detachment from expansion of the Universe $\delta\rho/\rho \sim 1$ $\rightarrow \langle \rho_{cl} \rangle \sim \rho_{DM}(z_f) \sim 10^4\, M_\odot$ pc$^{-3} \rightarrow R_{cl} = 1$ pc, $M_{cl} = 10^5\, M_\odot$.

Fraction of the cluster[7] $\Omega_{cl}/\Omega_{DM} \lesssim 0.05 \rightarrow$ formation of the dark matter halo around PBHs cluster[8] $\rho_H \propto r^{-9/4}$.

[6] Astropart. Phys. 23 (2005) 265-277
[7] Phys. Rev. Lett. 123 (2019) 071102
[8] Astrophys. J. Suppl. 58 (1985) 39-65

Equations

The Fokker-Planck equation[9]:

$$\frac{\partial N}{\partial t} = -\frac{\partial}{\partial E}\left(N\langle\Delta E\rangle\right) + \frac{1}{2}\frac{\partial^2}{\partial E^2}\left(N\left\langle(\Delta E)^2\right\rangle\right) - \nu_{lc}N, \qquad (1)$$

$N = 4\pi^2 p(E)f(E)$ и $\nu_{lc}N$ — the lose cone term[10], which describes the capture of PBHs by the central massive BH.

The gravitational potential:

$$\phi(r) = -4\pi G\left[\frac{1}{r}\int_0^r dr'\, r'^2\rho(r') + \int_r^\infty dr'\, r'\rho(r')\right], \qquad (2)$$

the density profile:

$$\rho(r) = 4\pi\sum_i m_i\int_{\phi(r)}^0 dE\sqrt{2(E-\phi(r))}\, f_i(E). \qquad (3)$$

[9] Astrophys. J. 848 (2017) 10

[10] Class. Quantum Grav. 30 (2013) 244005

The merger rate

The cross-section of binary black hole formation (direct channel)[11]:

$$\sigma = 2\pi \left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7} \frac{G^2(m_i + m_j)^{10/7} m^{2/7} m'^{2/7}}{c^{10/7} v_{rel}^{18/7}}, \qquad (4)$$

m_i и m_j — masses of merging black holes, v_{rel} — relative velocity of these BHs.

The merger rate of PBHs with masses m_i and m_j

$$\Gamma_{i,j} = \int dV\, n_i n_j \langle \sigma v_{rel} \rangle, \qquad (5)$$

where n_i is the number density of PBHs with mass m_i.

[11] Astrophys J. Lett. 566 (2002) L17

Initial data

The density profile of PBHs:

$$\rho_i(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-1} \left[1 + \left(\frac{r}{r_0}\right)^2\right]^{-2}, \tag{6}$$

the density profile of dark matter:

$$\rho_{DM} = \rho_{0,DM} \left(1 + \frac{r}{r_0}\right)^{-9/4}, \tag{7}$$

$\rho_{0,DM} = \rho_{DM}(z = 10^4) = 10^4\, M_\odot\ \mathrm{pc}^{-3}$, $r_0 = 1$ pc, $M_{cl} = 10^5\, M_\odot$
The mass spectrum of PBHs[12]:

$$\frac{dN}{dm} \propto \frac{1}{M_\odot} \left(\frac{m}{M_\odot}\right)^{-2} \Bigg|_{0.01\, M_\odot}^{10\, M_\odot}, \quad M_\bullet = 100\, M_\odot. \tag{8}$$

[12] Eur. Phys. J. C 79 (2019) 246

The evolution of mass distribution

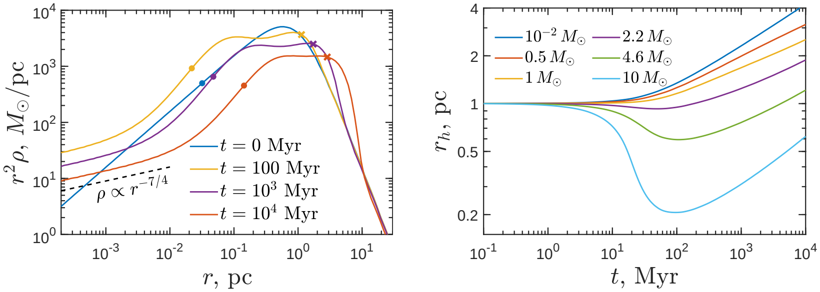

Рис. 1 — Left: the total mass distribution of PBHs for different times. Dots are radii containing $M_\bullet$ mass, crosses are half mass radii r_h. Right: the evolution of half mass radii for different type of PBHs

$r^2\rho(r) \propto \frac{dM(r)}{dr}$, $\rho \propto r^{-7/4}$ is the Bahcall-Wolf cusp[13].

[13] Astrophys. J. 209 (1976) 214

The evolution of mass distribution

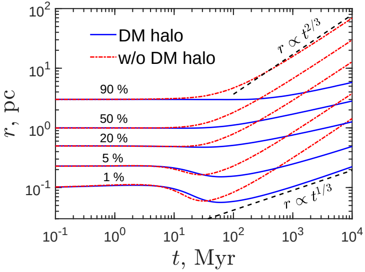

Рис. 2 — The evolution of mass shells. Left: calculations both with and without a halo. Right: calculations with mass spectrum(8) and monochromatic mass spectrum $m = 10\,M_\odot$

The grow of the central BH mass

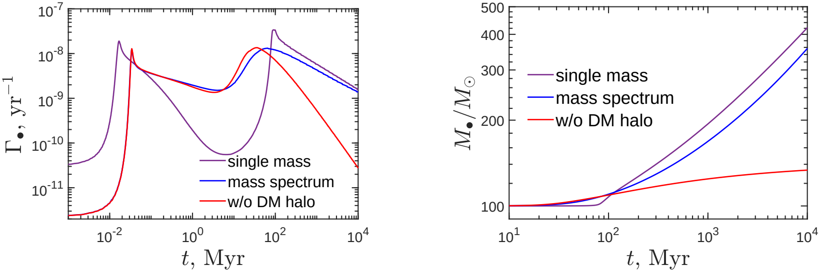

Рис. 3 — Left: The merger rate of black holes with 10 $M_\odot$ mass and central BH. Right: time dependence of the central BH mass

The merger rate evolution

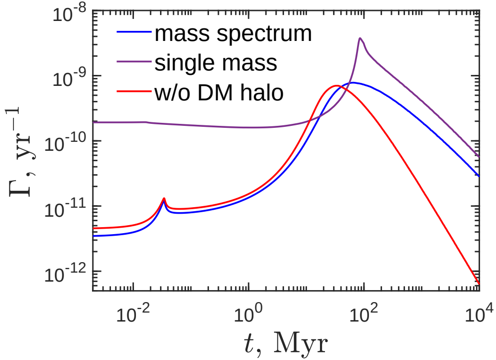

Рис. 4 — The merger rate of PBHs with mass $10\,M_\odot$ depending on the time

LIGO/Virgo[14]: $\Gamma_V \sim 100$ yr^{-1} Gpc^{-3} $\to \Omega_{cl}/\Omega_{DM} \lesssim 0.01$.

[14] Phys. Rev. Lett. 118 (2017) 221101

Conclusion

The dynamic of PBHs cluster is studied.

- Taking into account the dark matter halo slows down the cluster expansion rate.
- The modern cluster size is $r_h \sim 3$ pc.
- The mass of the central BH increases to $\sim 400\, M_\odot$.
- The merger rate of PBHs imposes restriction on the abundance of the clusters $\Omega_{cl}/\Omega_{DM} \lesssim 0.01$.