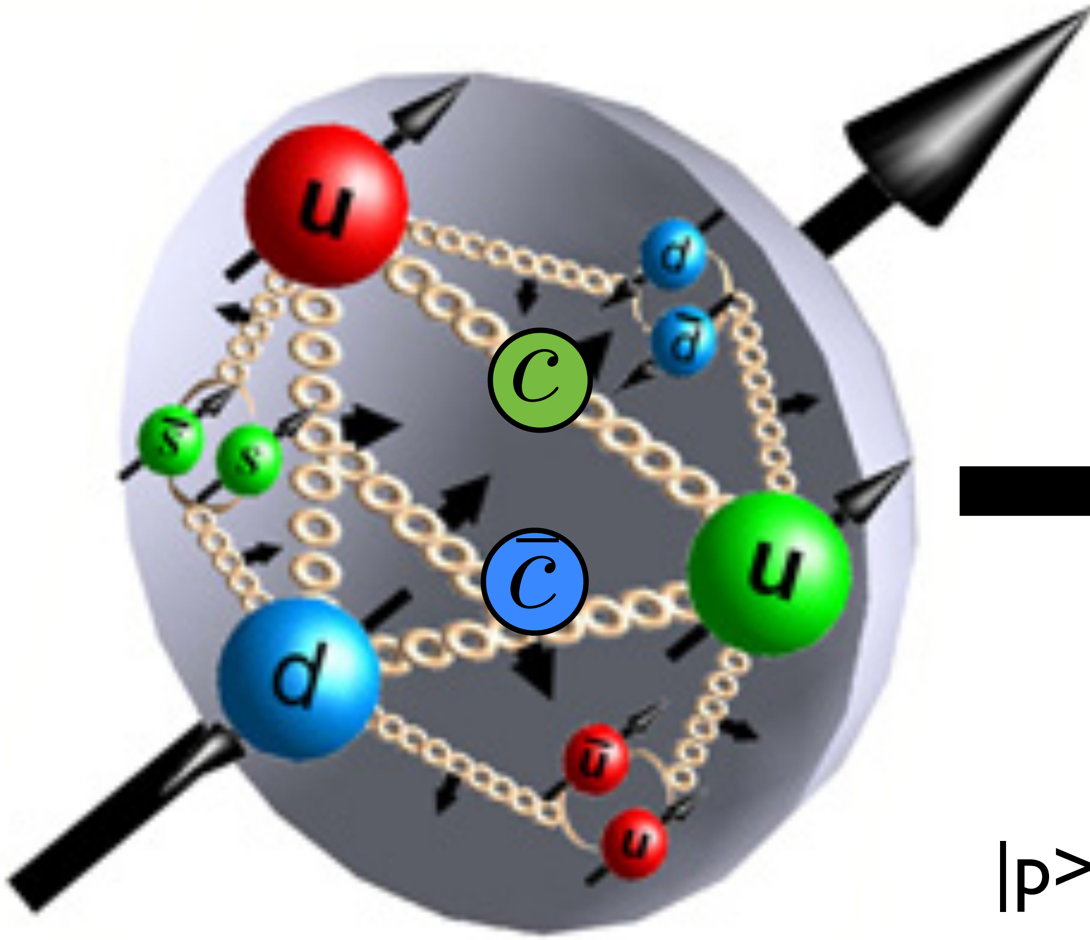
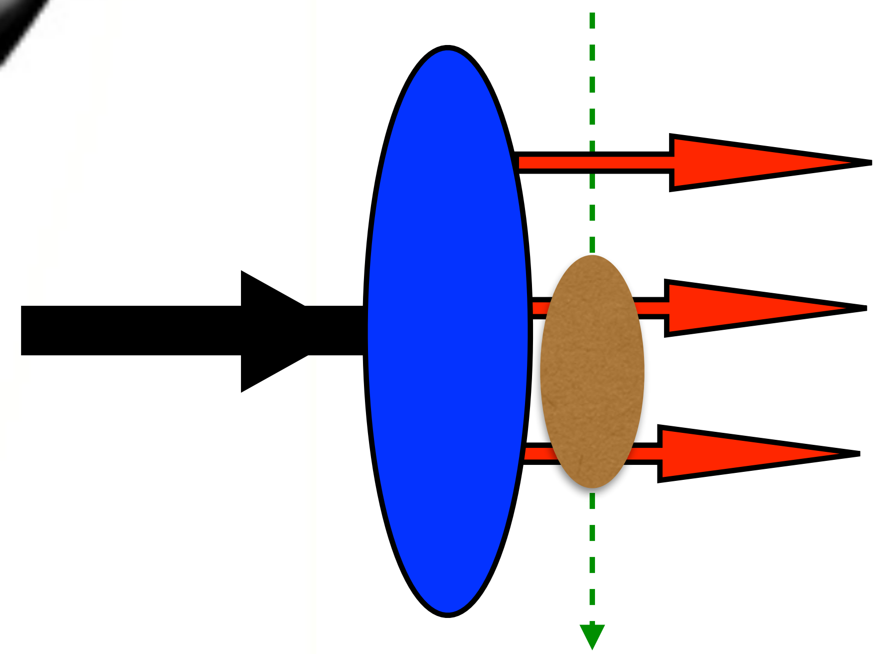
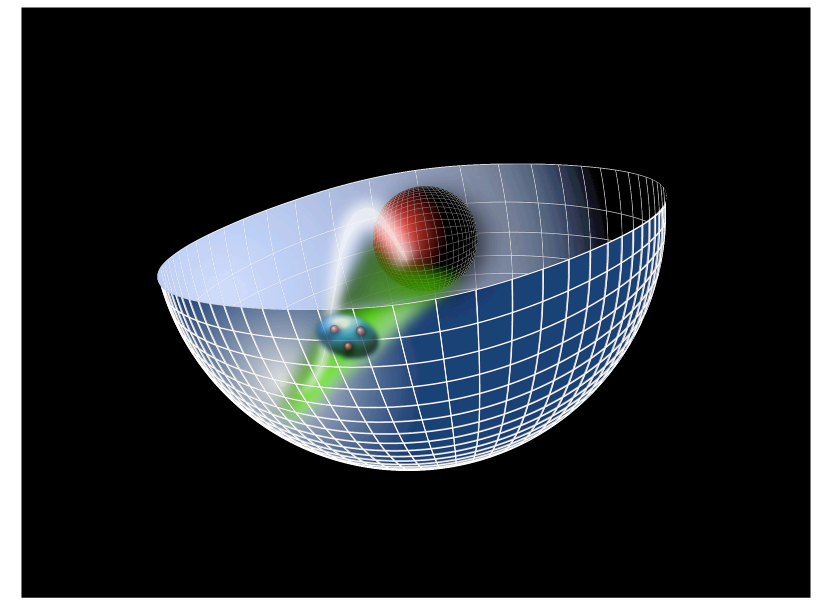
Color Confinement and Supersymmetric Features of Hadron Physics from Light-Front Holography



Fixed $\tau = t + z/c$





|p>=|u[ud]> quark-diquark cluster

with Guy de Tèramond, Hans Günter Dosch, Alexandre Deur, Marina Nielsen, Ivan Schmidt, F. Navarra, Jennifer Rittenhouse West, G. Miller, Keh-Fei Liu, Tianbo LIu, Liping Zou, S. Groote, Joshua Erlich, S. Koshkarev, Xing-Gang Wu, Sheng-Quan Wang, Cedric Lorcè, R. S. Sufian, R. Vogt, G. Lykasov, S. Gardner, S. Liuti

Bled Workshop

What Comes Beyond the Standard Models?

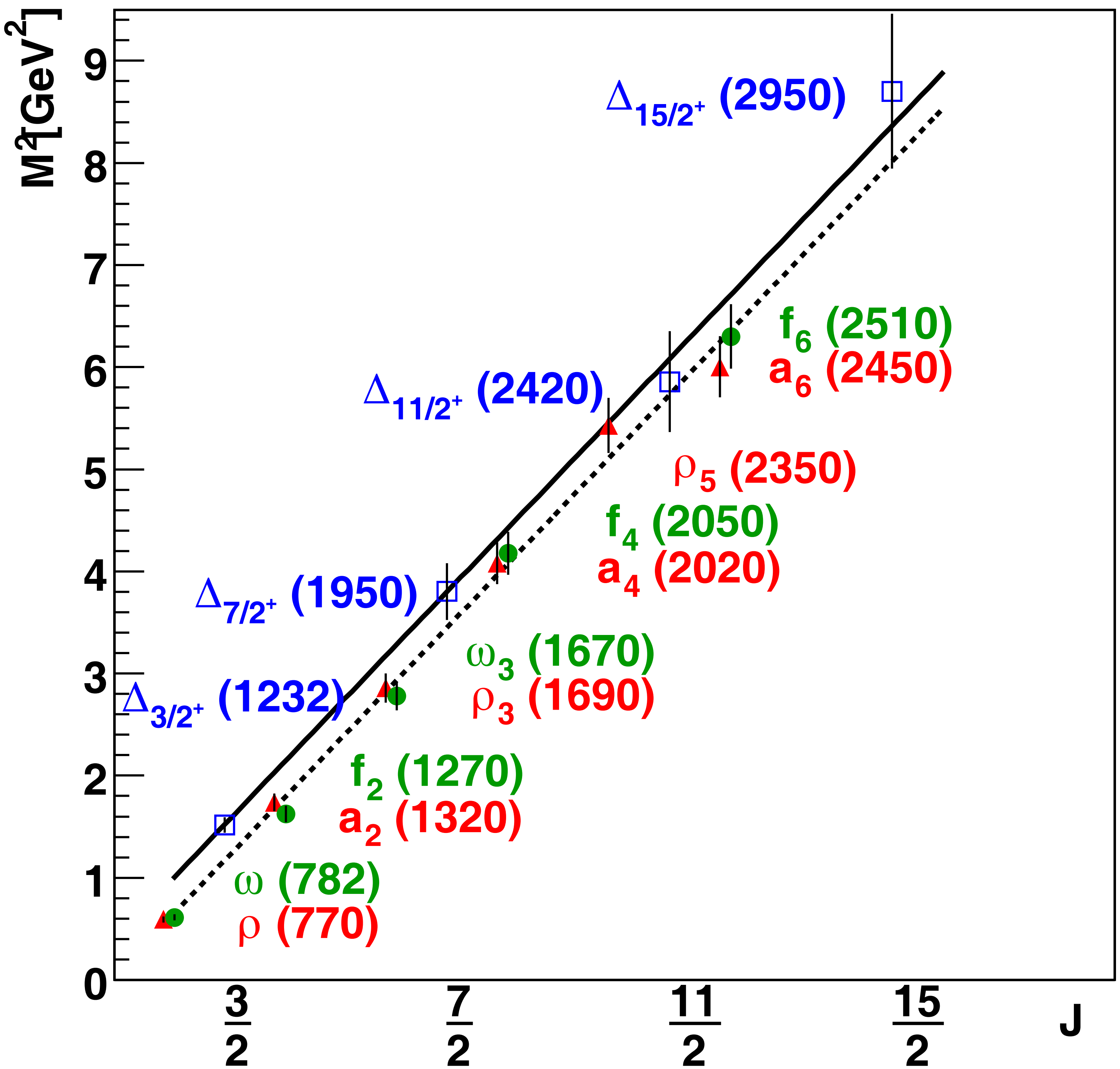
Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

July 6, 2021

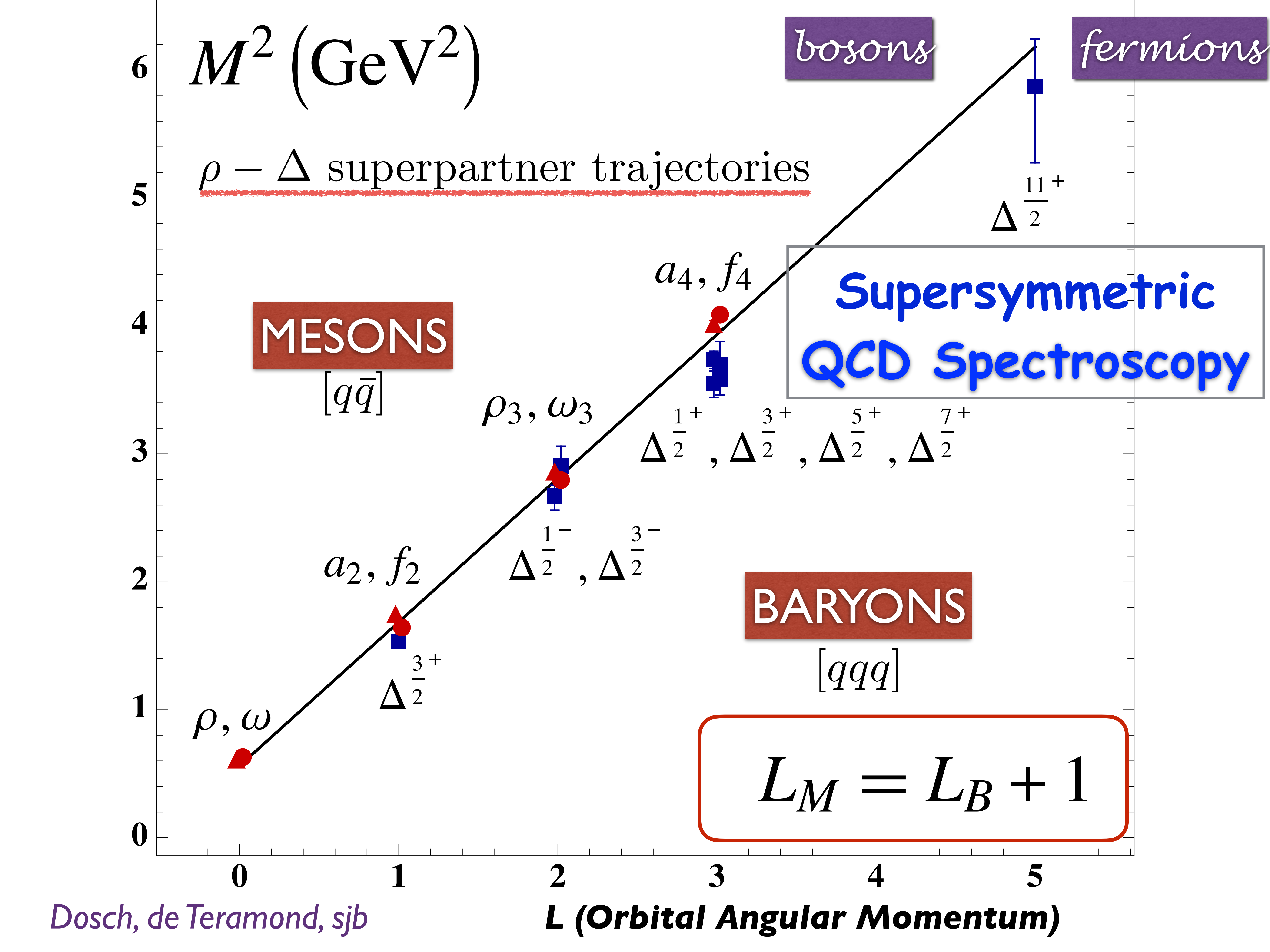
Mesons and Baryons: Same Regge Slope $M^2 \propto J$!

M²[GeV²]



The leading Regge trajectory: Δ resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with J = L+S.

E. Klempt and B. Ch. Metsch



$M^2\,(\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
Supersymmetric
QCD Spectroscopy
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta \frac{3}{2}^+$
$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$
$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$
$\Delta \frac{11}{2}^+$
$L_M = L_B + 1$
L (Orbital Angular Momentum)
0
1
2
3
4
5
6
Dosch, de Teramond, sjb

Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- *Color Confinement*
- *Origin of the QCD Mass Scale*
- *Meson and Baryon Spectroscopy*
- *Exotic States: Tetraquarks, Pentaquarks, Gluonium,*
- *Universal Regge Slopes: n, L, Mesons and Baryons*
- *Almost Massless Pion: GMOR Chiral Symmetry Breaking*

  $M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}\big((m_u + m_d)^2\big)$

- *QCD Coupling at all Scales* $\alpha_s(Q^2)$
- *Eliminate Scale Uncertainties and Scheme Dependence*

$\mathscr{L}_{QCD} \to \psi_n^H(x_i, \overrightarrow{k}_{\perp i}, \lambda_i)$ *Valence and Higher Fock States*

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

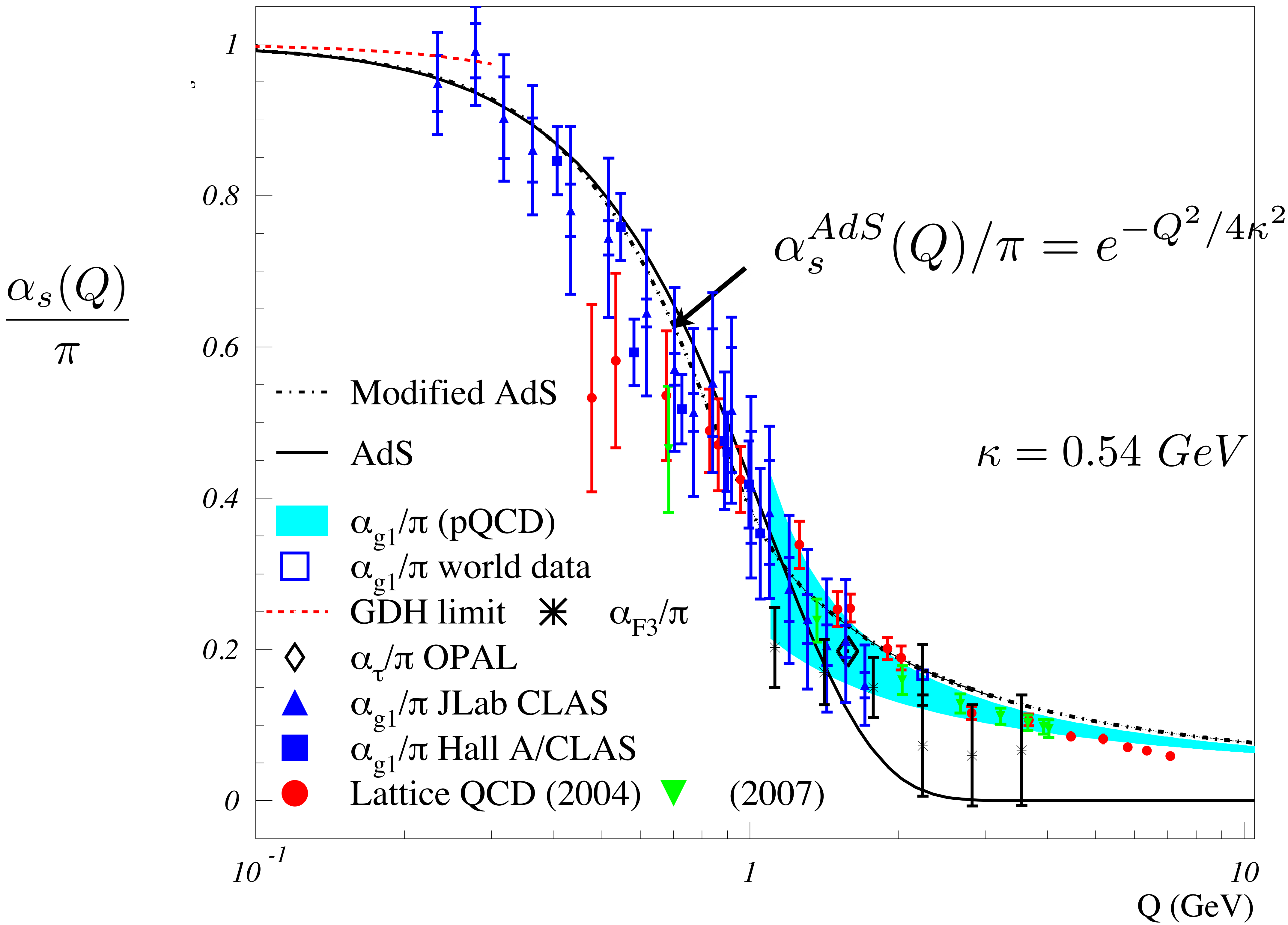
Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale

AdS/QCD
Light-Front Holography
Superconformal Algebra

No parameters except for quark masses!

Analytic, defined at all scales, IR Fixed Point

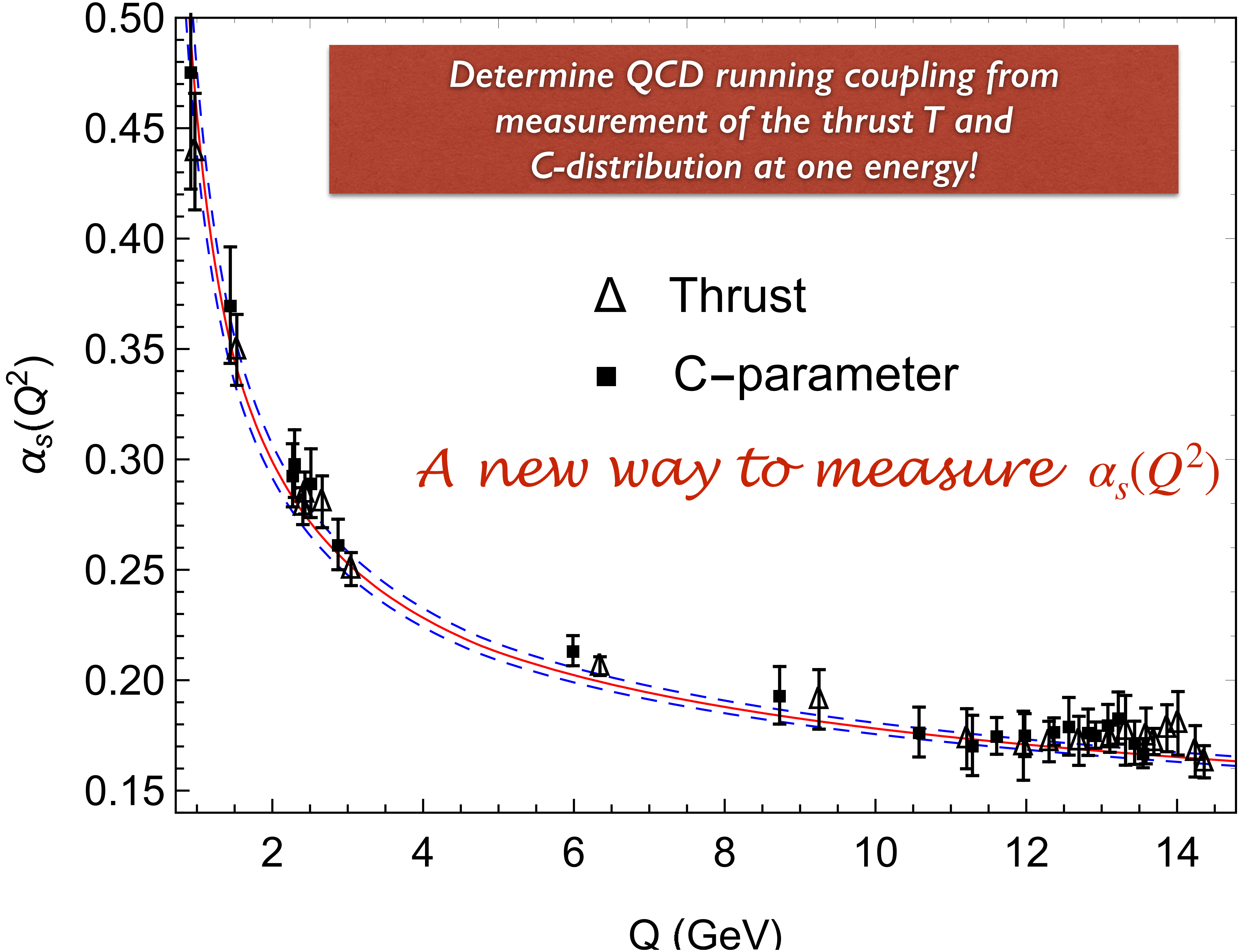


AdS/QCD dilaton predicts the nonperturbative corrections to the QCD running coupling

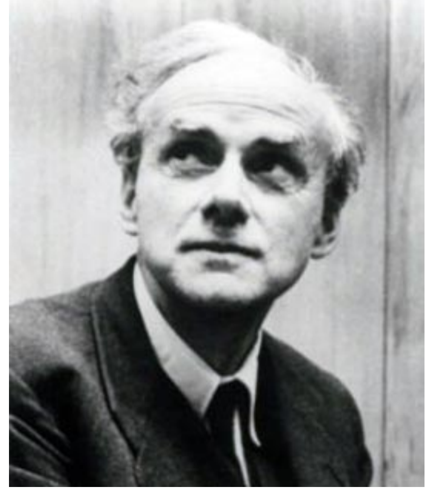
$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

$e^+e^- \to Z^0 \to q\bar{q}g + \cdots \quad \alpha_s(Q^2)$ in $\overline{MS}$ scheme



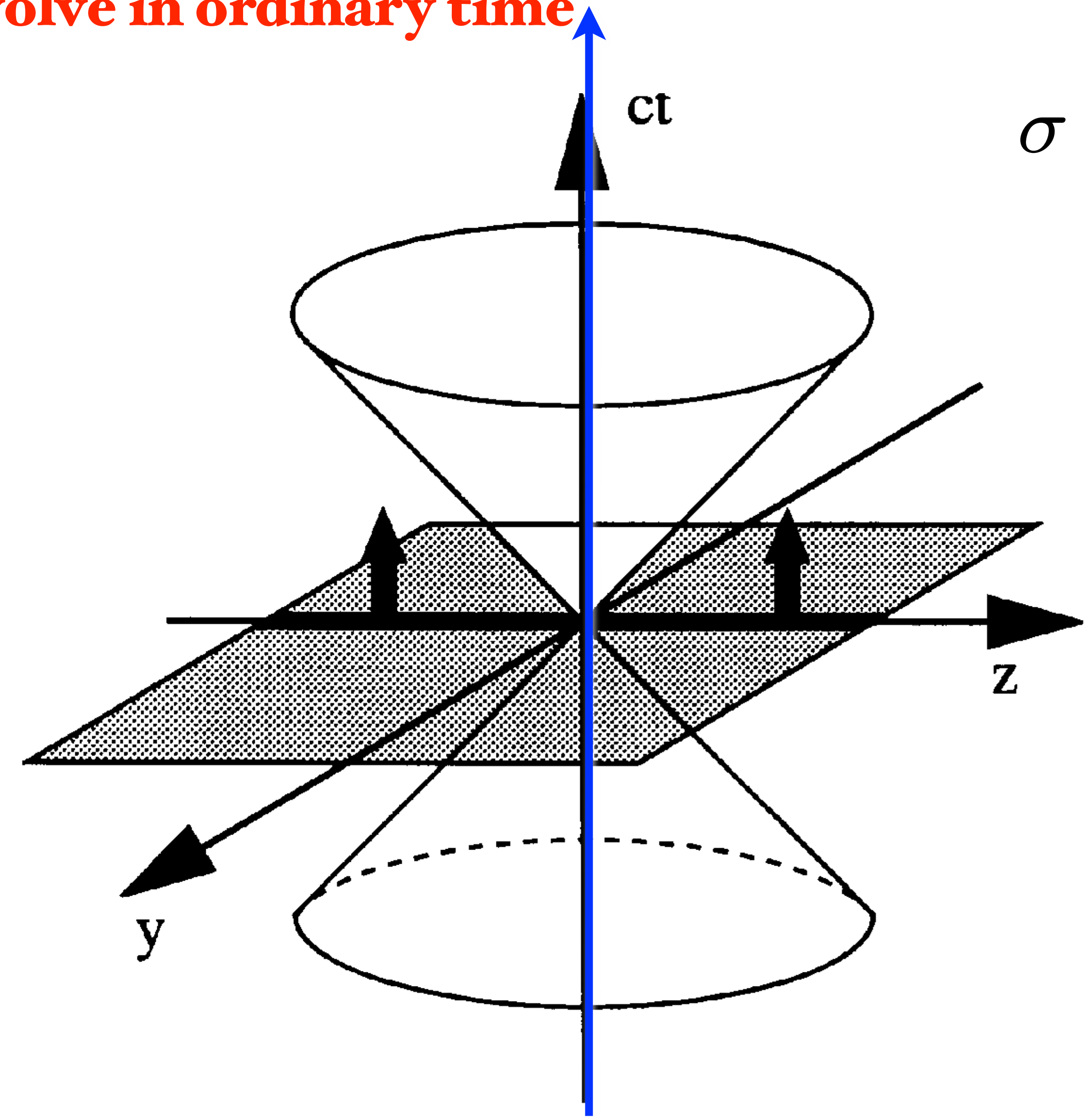
S.-Q. Wang, L. Di Giustino, X.-G. Wu, SJB



P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The "Front Form"

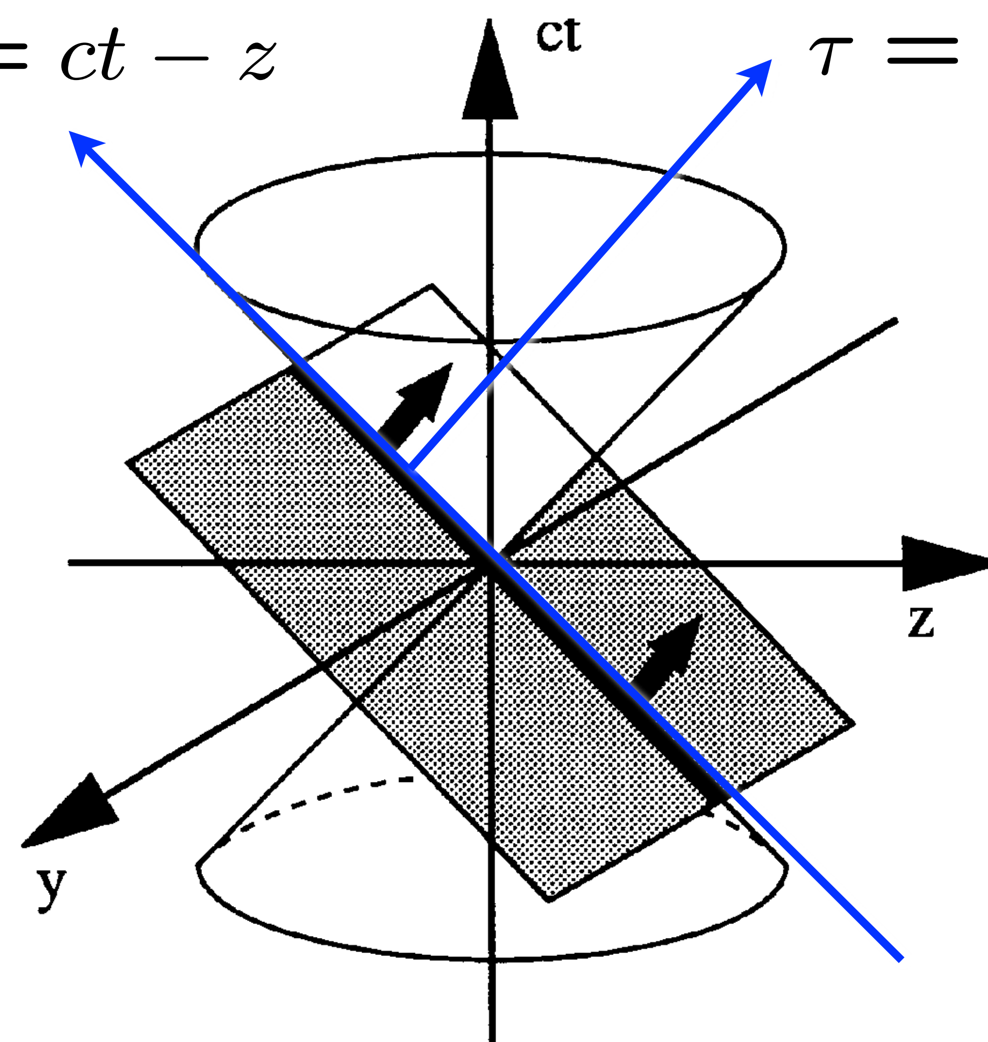
Evolve in ordinary time



Instant Form

Evolve in light-front time!

$\sigma = ct - z$ $\qquad$ $\tau = t + z/c$



Front Form

Casual, Boost Invariant!

- ***Trivial LF Vacuum (up to zero modes)***

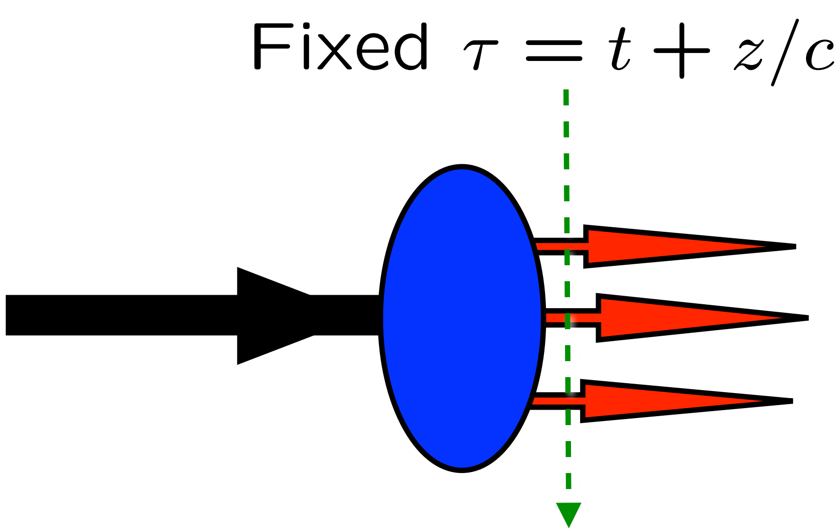
Comparing light-front quantization with instant-time quantization
Philip D. Mannheim(Connecticut U.),
Peter Lowdon(Ecole Polytechnique, CPHT),
Stanley J. Brodsky(SLAC)

- e-Print: 2005.00109 [hep-ph]

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

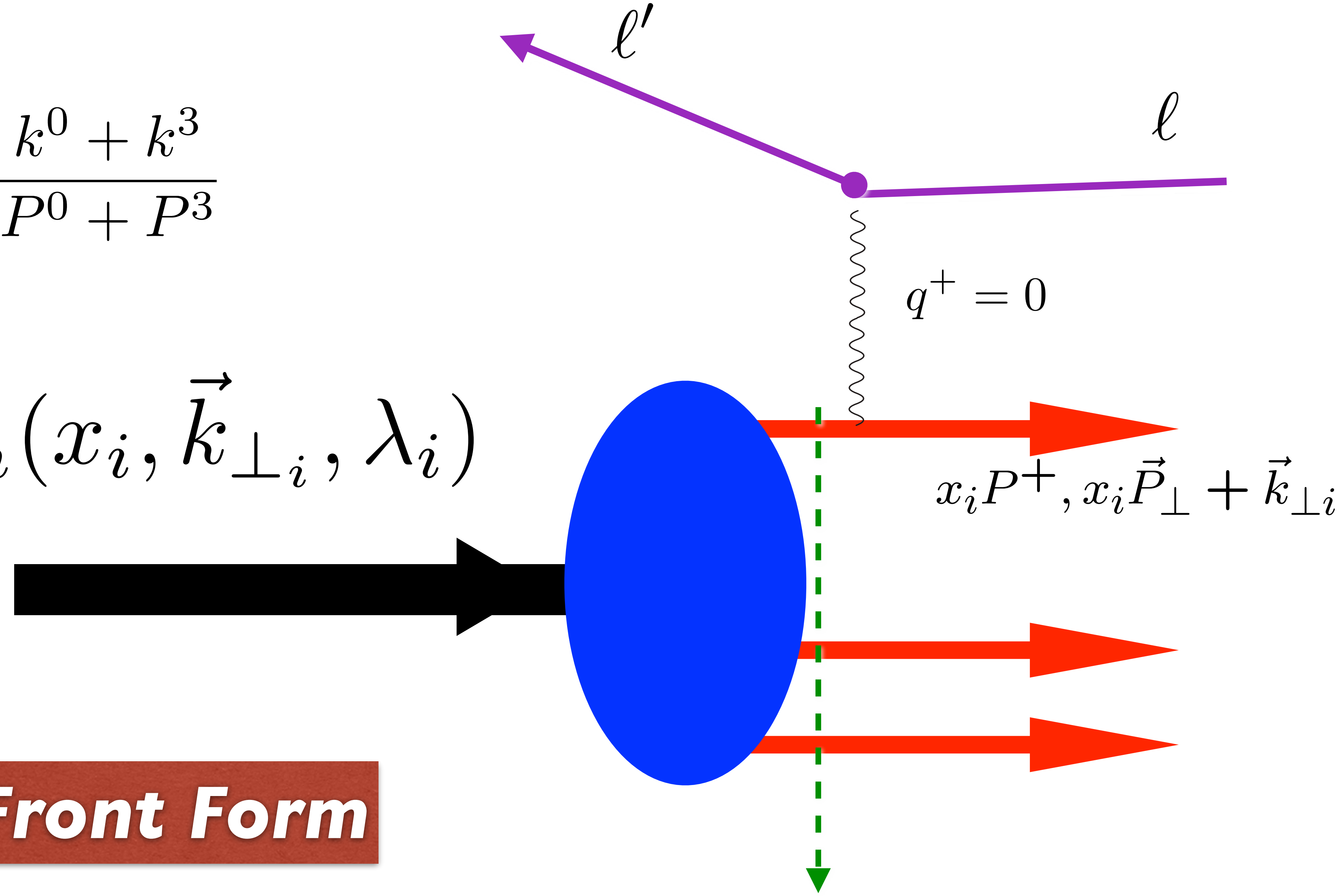
LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$P^+, \vec{P}_\perp$



Dirac: Front Form

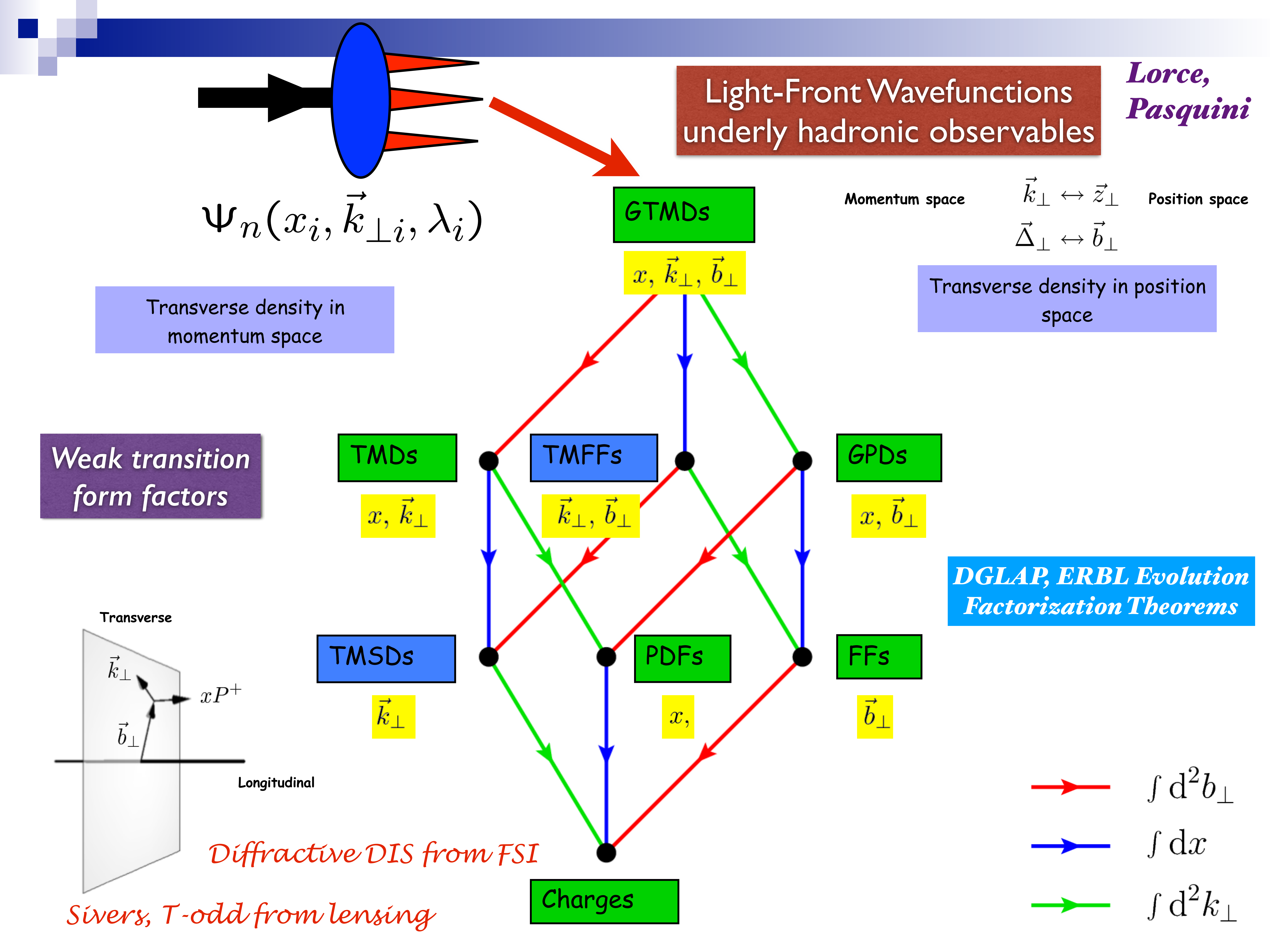
Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

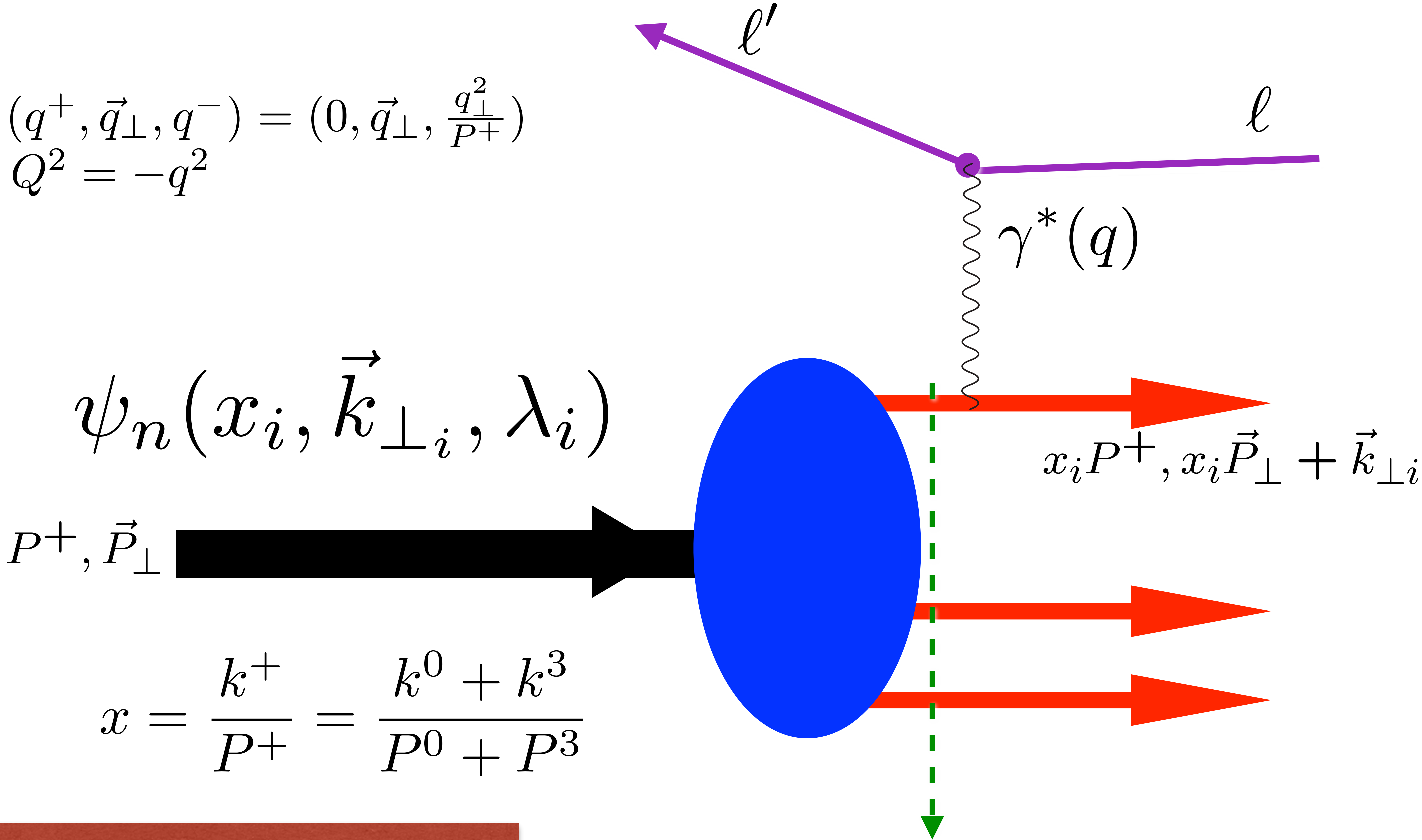
Invariant under boosts! Independent of P^μ



$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
Light-Front Wavefunctions underly hadronic observables
Lorce, Pasquini
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$
GTMDs
$x, \vec{k}_\perp, \vec{b}_\perp$
Transverse density in momentum space
Transverse density in position space
Weak transition form factors
TMDs
$x, \vec{k}_\perp$
TMFFs
$\vec{k}_\perp, \vec{b}_\perp$
GPDs
$x, \vec{b}_\perp$
DGLAP, ERBL Evolution Factorization Theorems
Transverse
$\vec{k}_\perp$
xP^+
$\vec{b}_\perp$
Longitudinal
TMSDs
$\vec{k}_\perp$
PDFs
$x,$
FFs
$\vec{b}_\perp$
$\int d^2 b_\perp$
$\int dx$
$\int d^2 k_\perp$
Diffractive DIS from FSI
Sivers, T-odd from lensing
Charges

$$q^\mu = (q^+, \vec{q}_\perp, q^-) = (0, \vec{q}_\perp, \frac{q_\perp^2}{P^+})$$
$$q_\perp^2 = Q^2 = -q^2$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$



$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Dirac: Front Form

Fixed $\tau = t + z/c$

1912.08911 [hep-ph]

$$x_{bj} = x = \frac{k^+}{P^+}$$

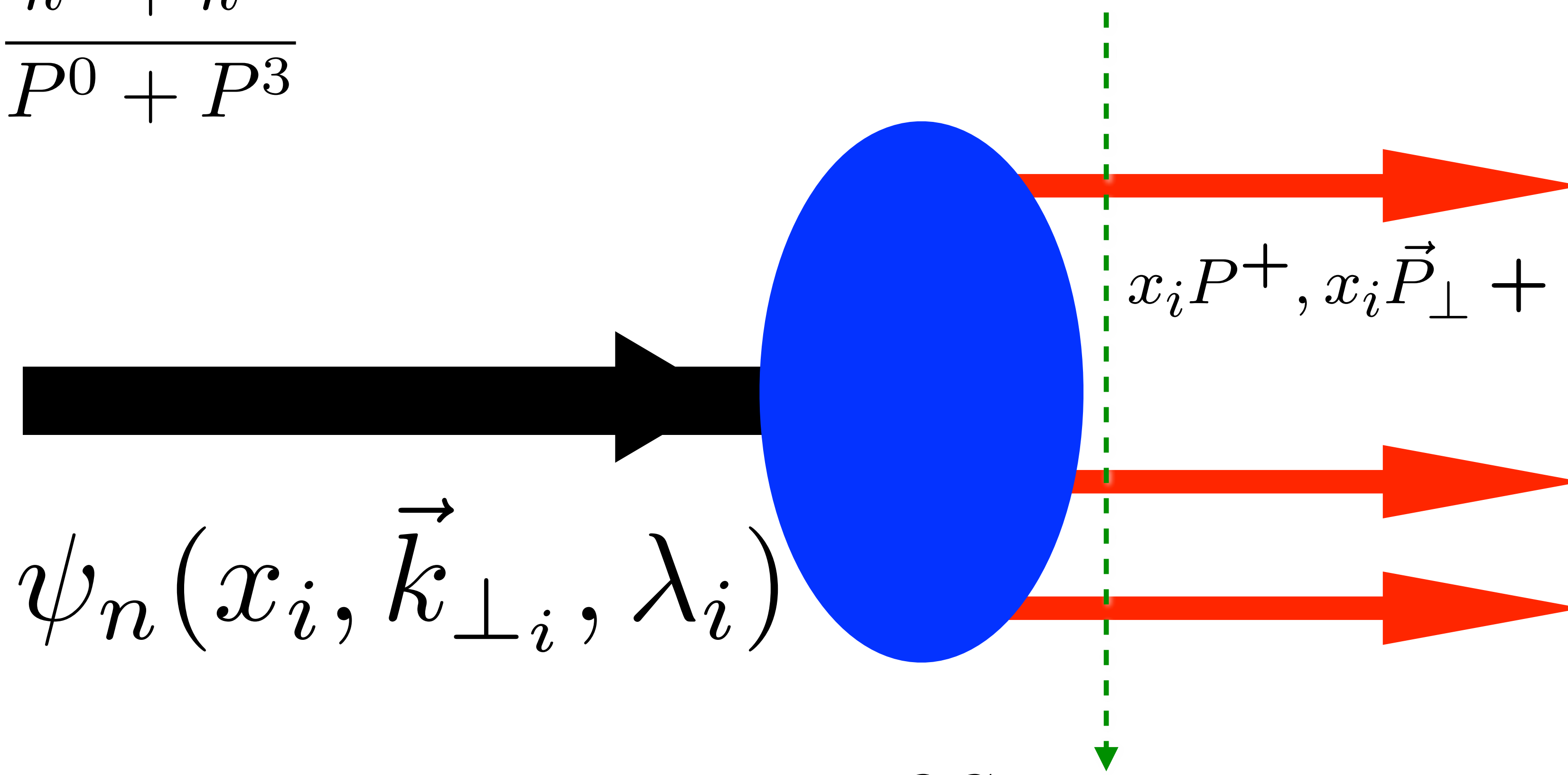
G. A. Miller, sjb:

Ioffe Time: $\tilde{z}$ Third spatial LF coordinate.

Fourier Transform of x in LFWFs

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

Eigenstate of LF Hamiltonian

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Invariant under boosts! Independent of P^μ

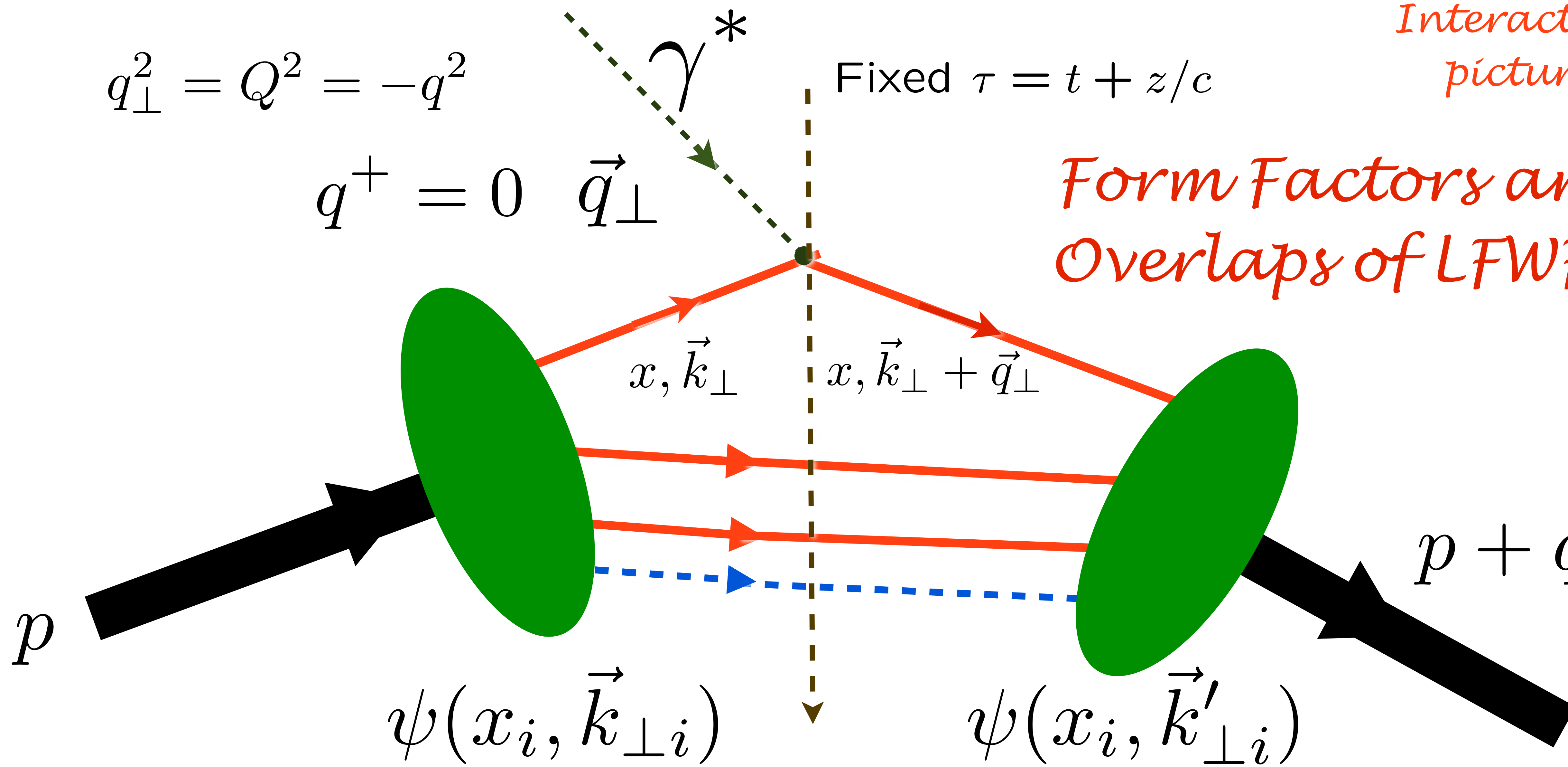
Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Front Form

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Exact LF Formula for Pauli Form Factor

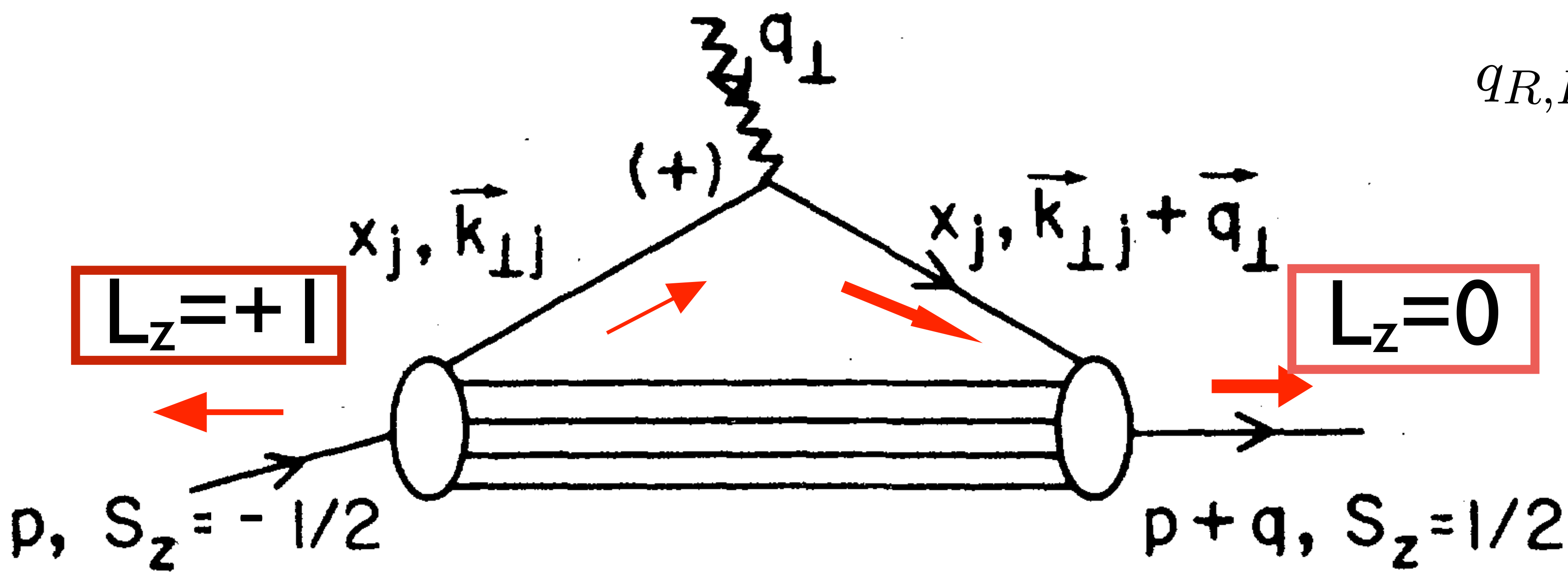
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Dae Sung Hwang, Bo-Qiang Ma, Ivan Schmidt, sjb

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \overline{u}(P')\Big[A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P) \ ,$$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\uparrow\right\rangle = A(q^2) \ ,$$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\downarrow\right\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M} \ .$$

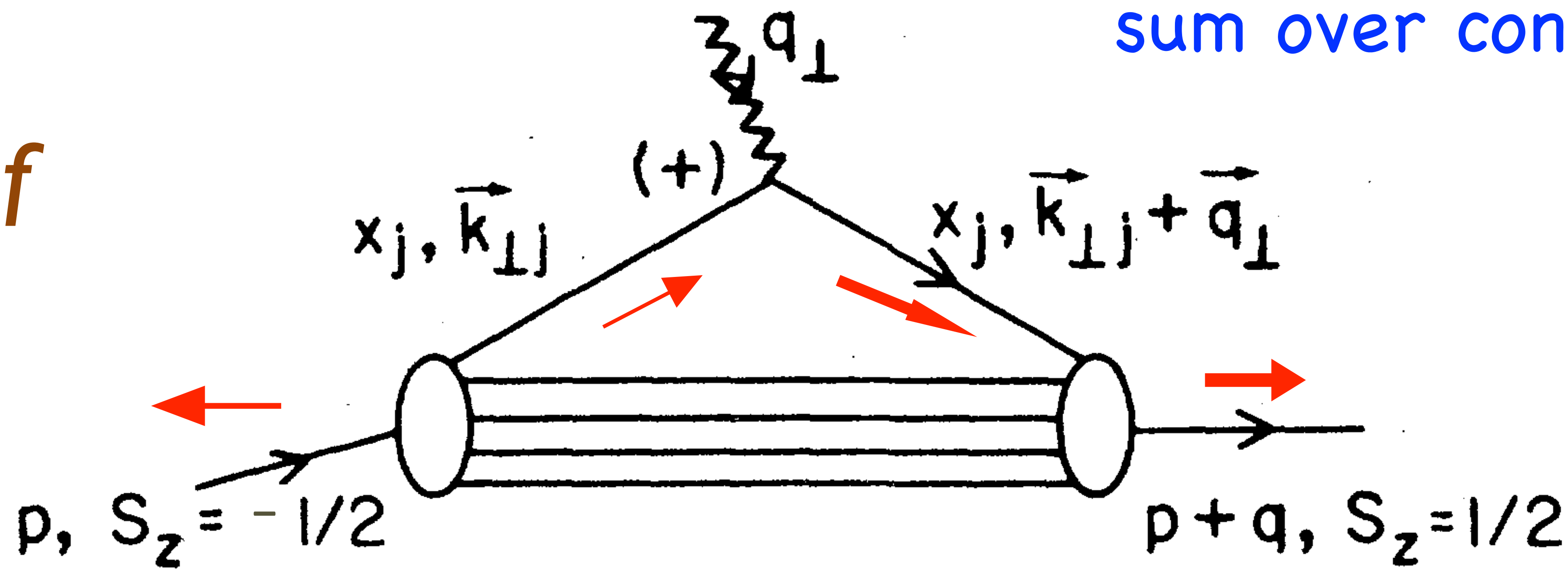
Dae Sung Hwang, Bo-Qiang Ma, Ivan Schmidt, sjb

Terayev, Okun: *B(0) Must vanish because of Equivalence Theorem*

graviton

sum over constituents

LF Proof

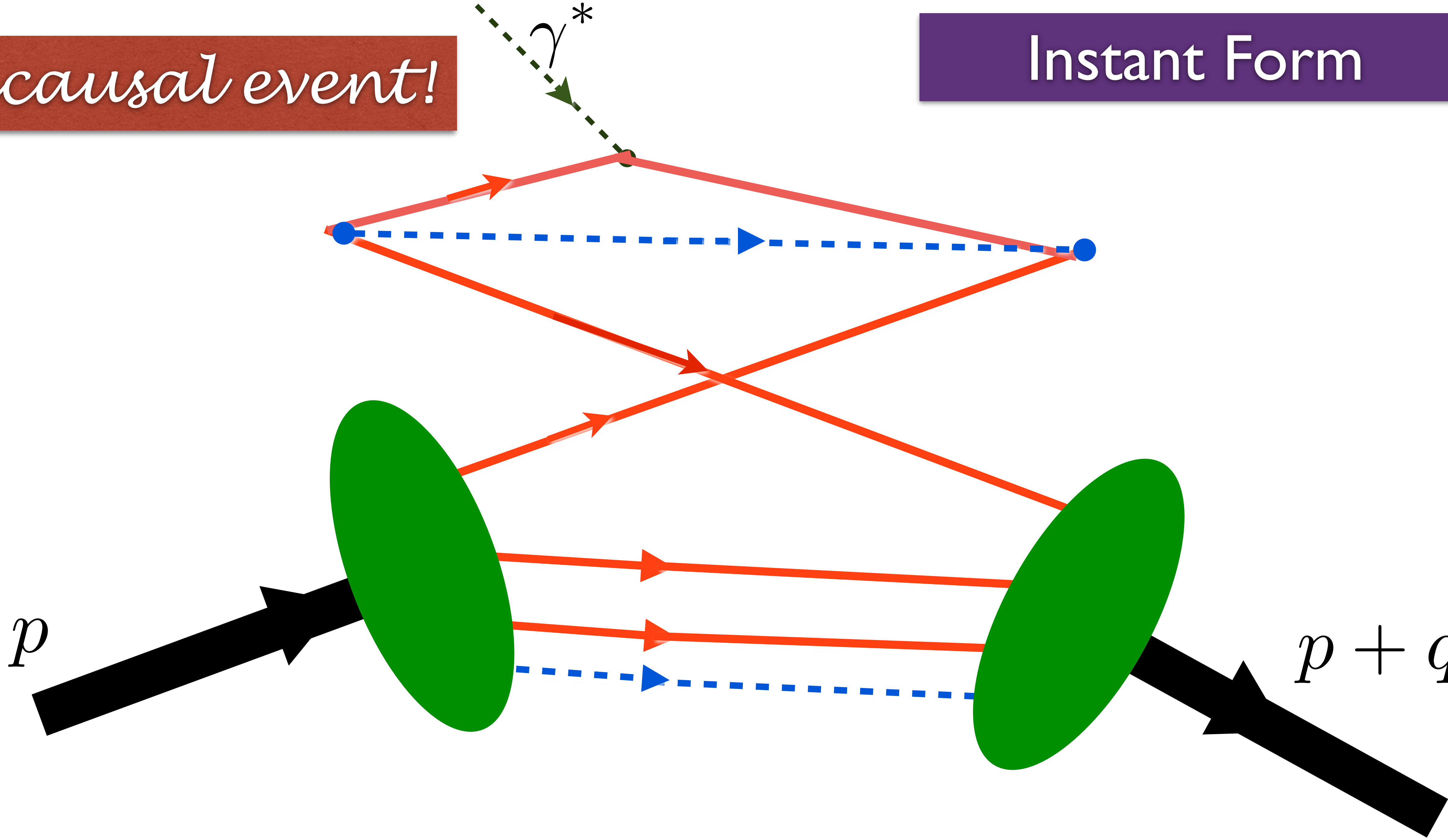


B(0) = 0

Each Fock State

Vanishing Anomalous gravitomagnetic moment B(0) *acausal event!*

Instant Form



Must include vacuum-induced currents to compute form factors and other current matrix elements in instant form

Boost are dynamical in instant form

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i \left[\frac{m^2 + k^2_\perp}{x}\right]_i + H^{int}_{LF}$$

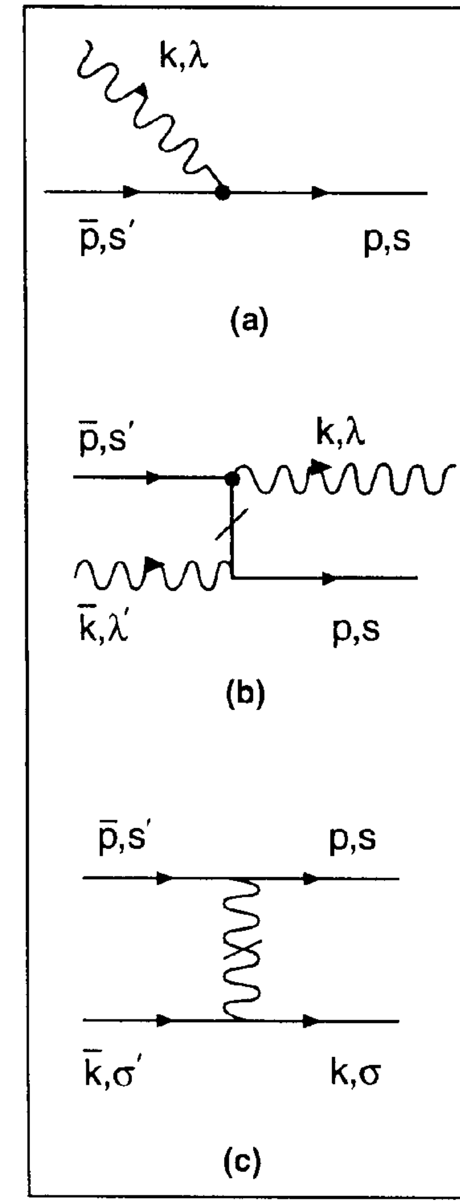
H^{int}_{LF}: Matrix in Fock Space

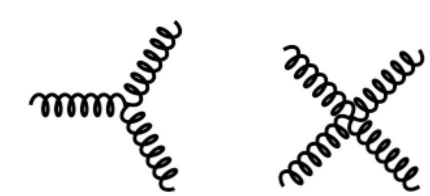
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass





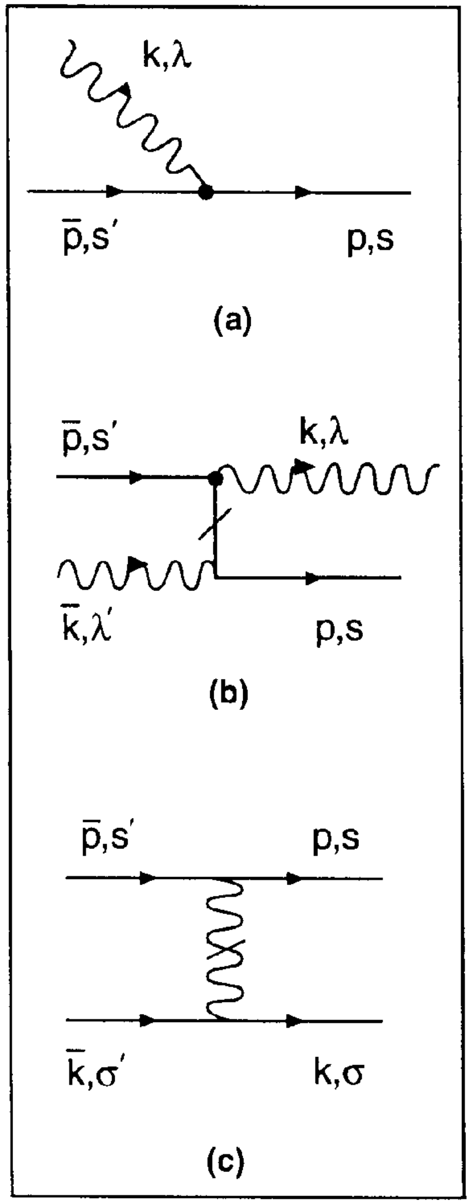
H^{int}_{LF}

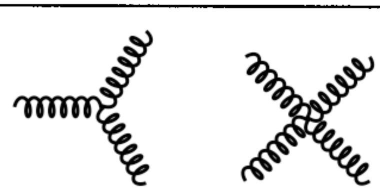
Light-Front QCD Heisenberg Equation

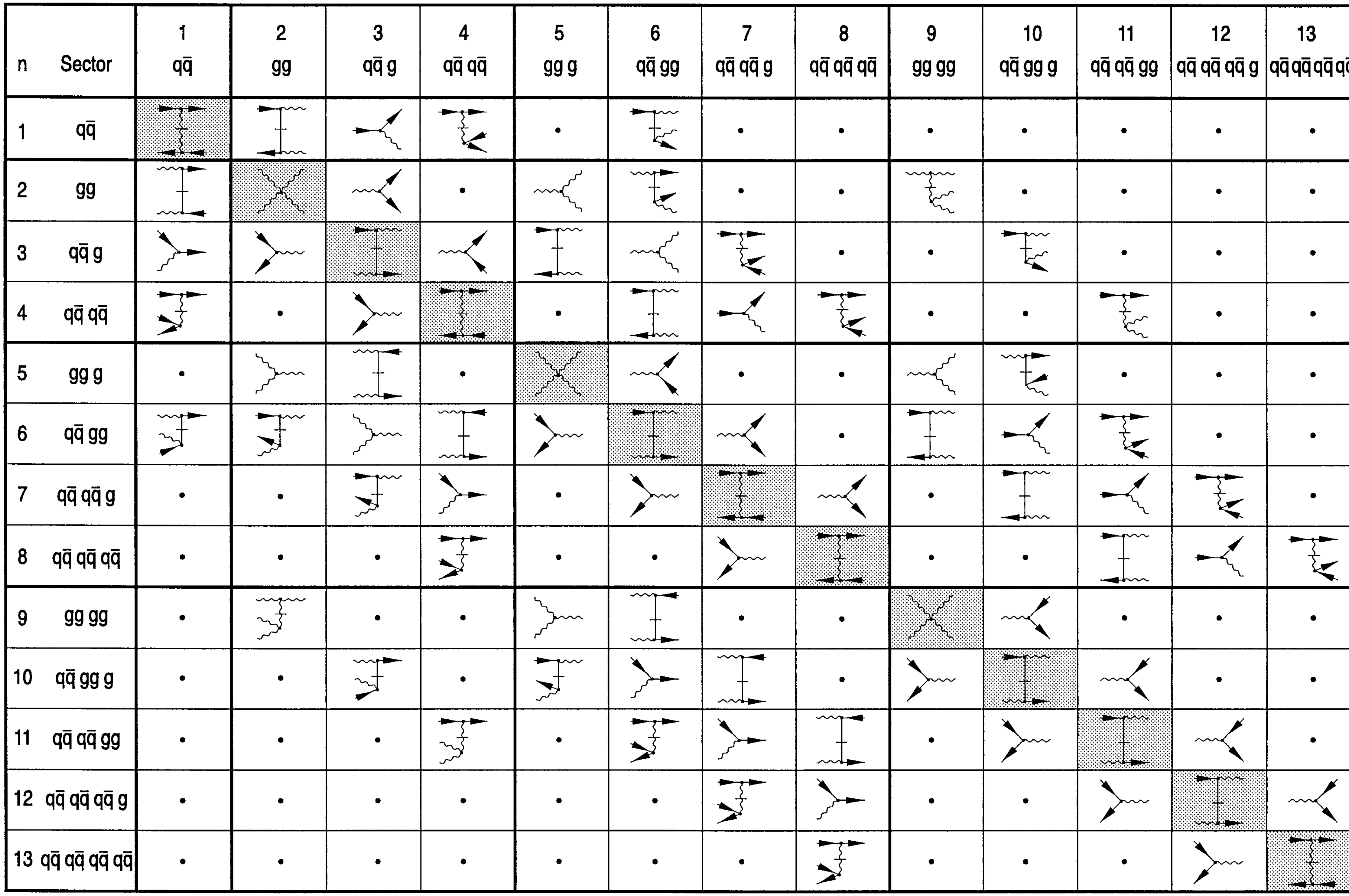
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\,|\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb







Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + \cdots$$

- "History" : Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes! Cluster Decomposition

- Wick Theorem applies, but few amplitudes since all $k^+ > 0$.

- J_z Conservation at every vertex $\left| \sum_{initial} S^z - \sum_{final} S_z \right| \leq n$ at order g^n

- Unitarity is explicit

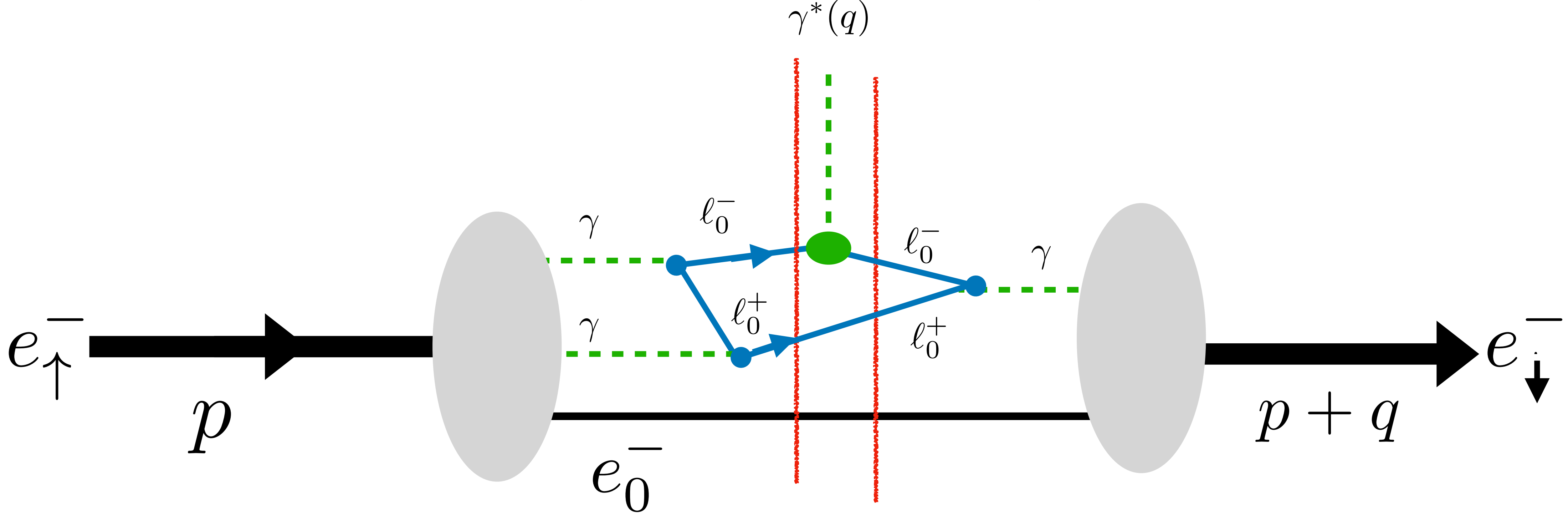
K. Chiu, Lorcé, sjb

- Loop Integrals are 3-dimensional $\int_0^1 dx \int d^2 k_\perp$

- hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Light-by-Light Contribution
to the Pauli form factor and anomalous magnetic moment a_e
of electron eigenstate of H^{LF}_{QED}

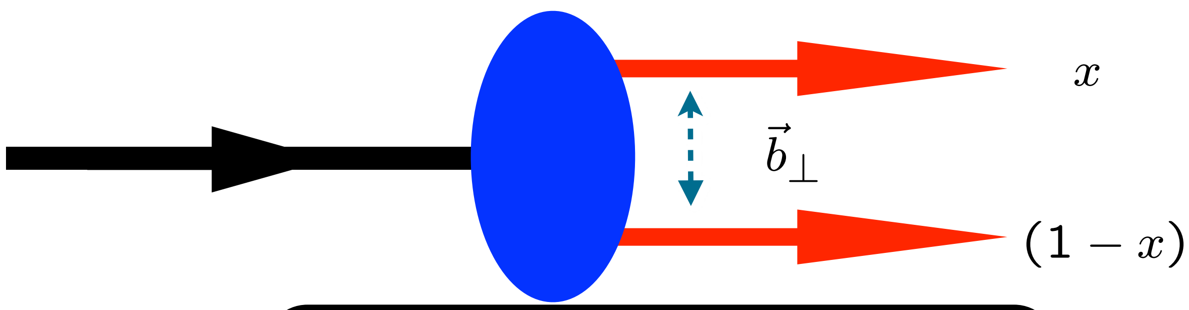
$$F_2(q^2) \propto < \Psi_{e^-_\uparrow}(p+q)|j^+|\Psi_{e_\downarrow}(p) >$$



Overlap of $|e_0^-(\ell_0^+\ell_0^-)_{C=+}>$ and $|e_0^-(\ell_0^+\ell_0^-)_{C=-}>$
Fock state LFWFs of the electron eigenstate $H^{LF}_{QCD}|\Psi_{e^-}>= m_e^2|\Psi_{e^-}>$

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi>= M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x,\vec{k}_\perp) = M^2\ \psi_{LF}(x,\vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis ζ, ϕ

Single variable Equation

$m_q = 0$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

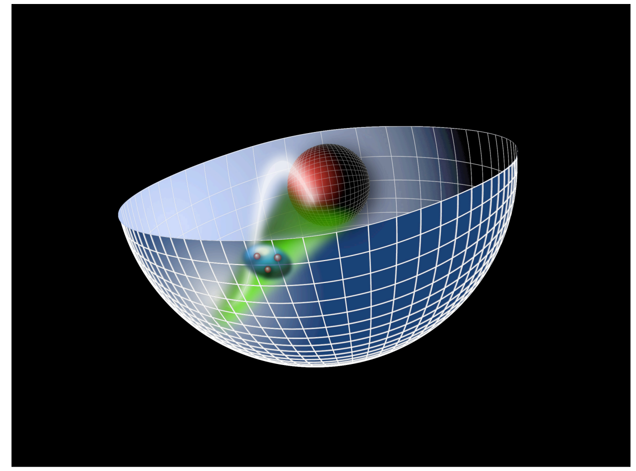
Semiclassical first approximation to QCD

Sums an infinite # diagrams

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique
Confinement Potential!

Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

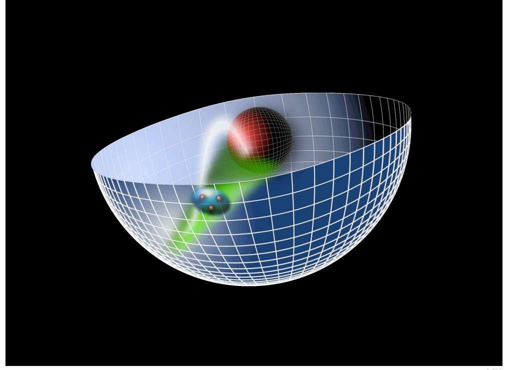
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!

GeV units external to QCD: Only Ratios of Masses Determined

Maldacena

AdS5



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

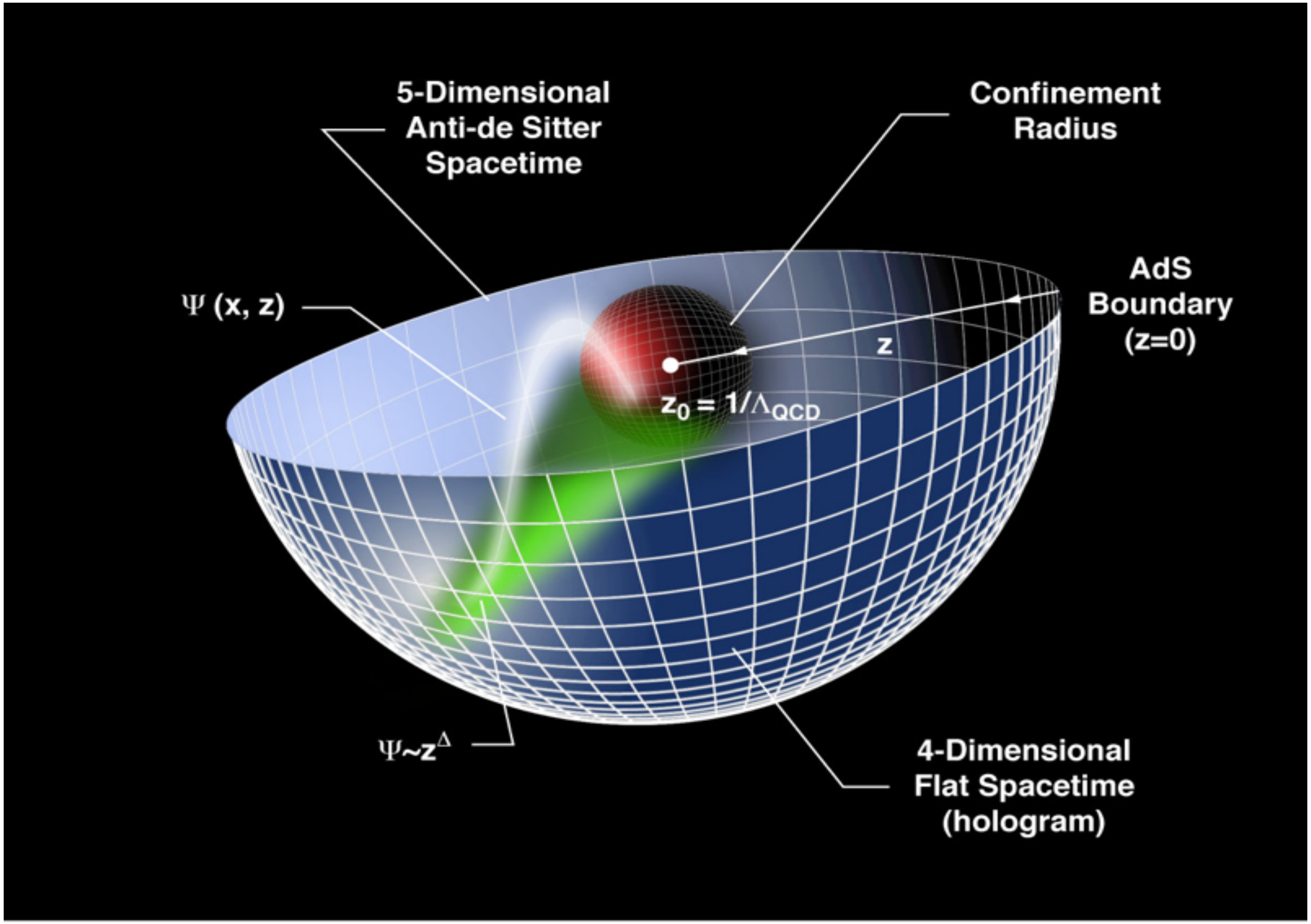
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Applications of AdS/CFT to QCD

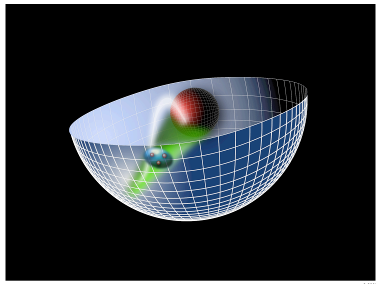


Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond and H. Guenter Dosch

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

SLAC–PUB–15972

Light-Front Holographic QCD and Emerging Confinement

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Stan Brodsky
Bled Workshop

Supersymmetric Features of Hadron Physics from Superconformal Algebra and Light-Front Holography

NATIONAL ACCELERATOR LABORATORY

7 July 2021

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Light-Front Holography

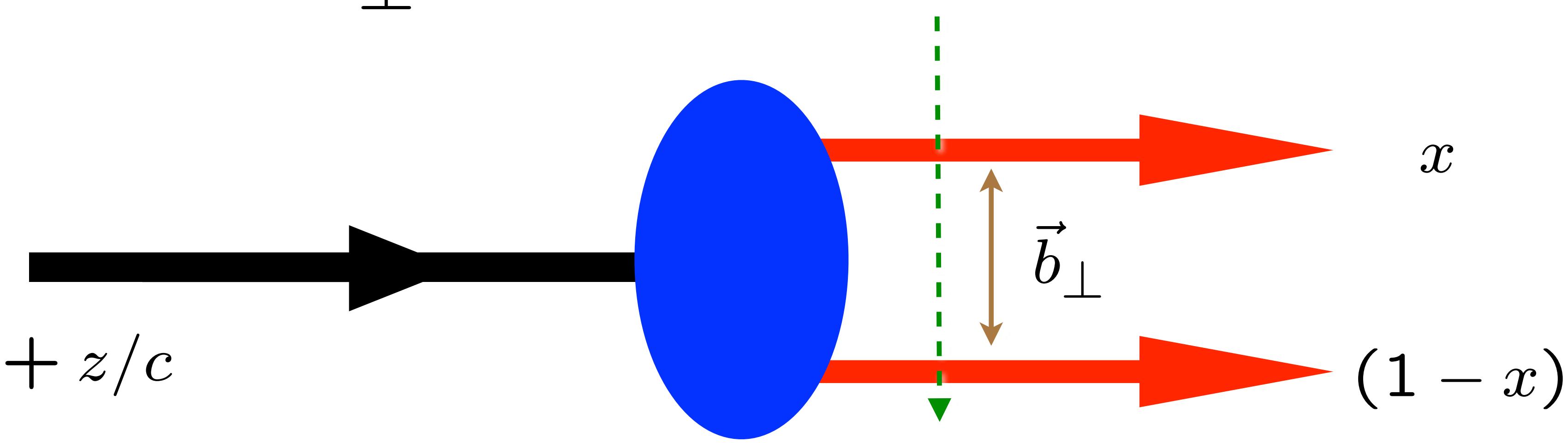
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$$\psi(x,\vec b_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec b_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

$$\vec\zeta^2 = \vec b^2_\perp x(1-x)$$

G. de Teramond, H. G. Dosch, sjb

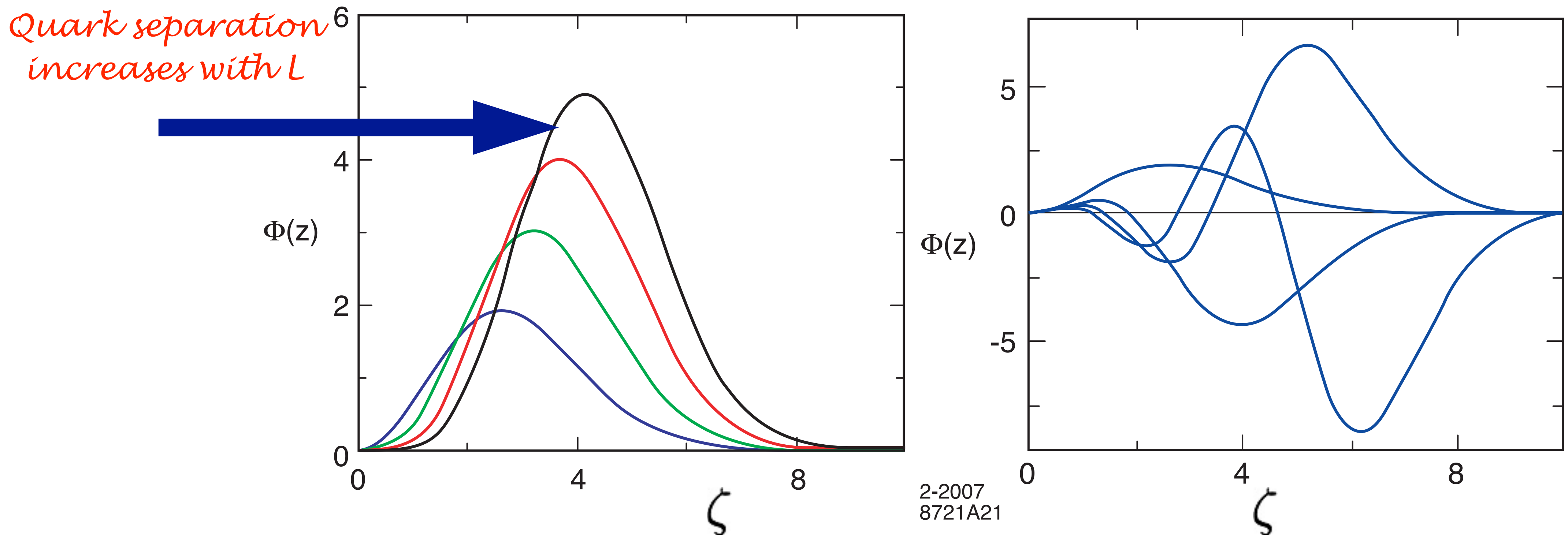
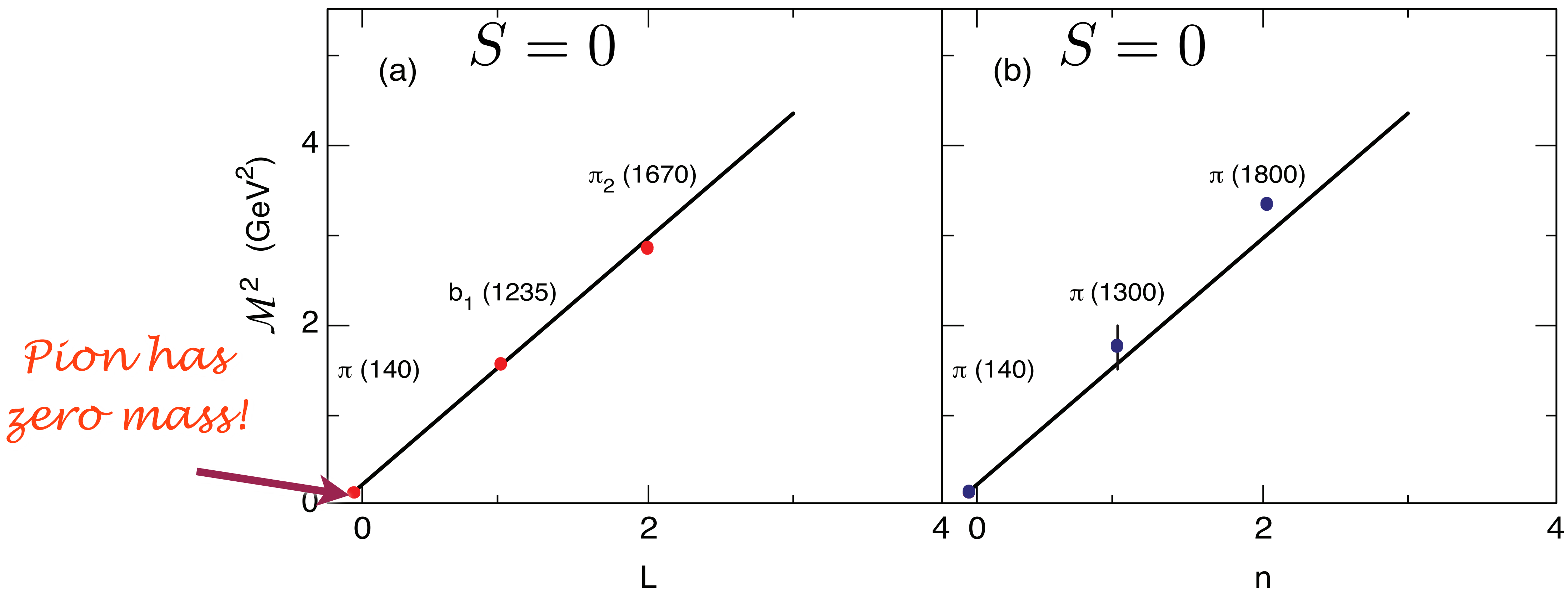


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

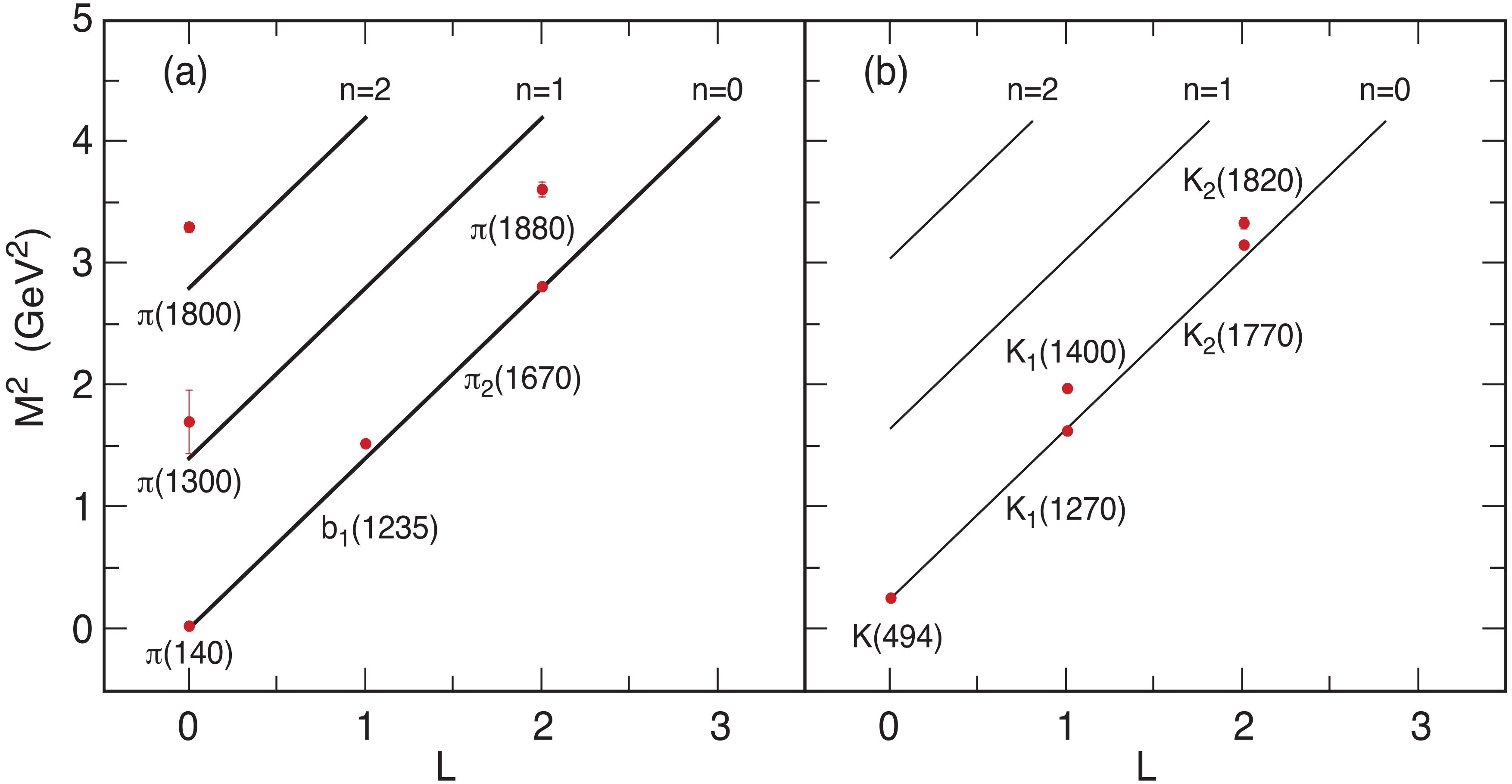
Soft Wall Model

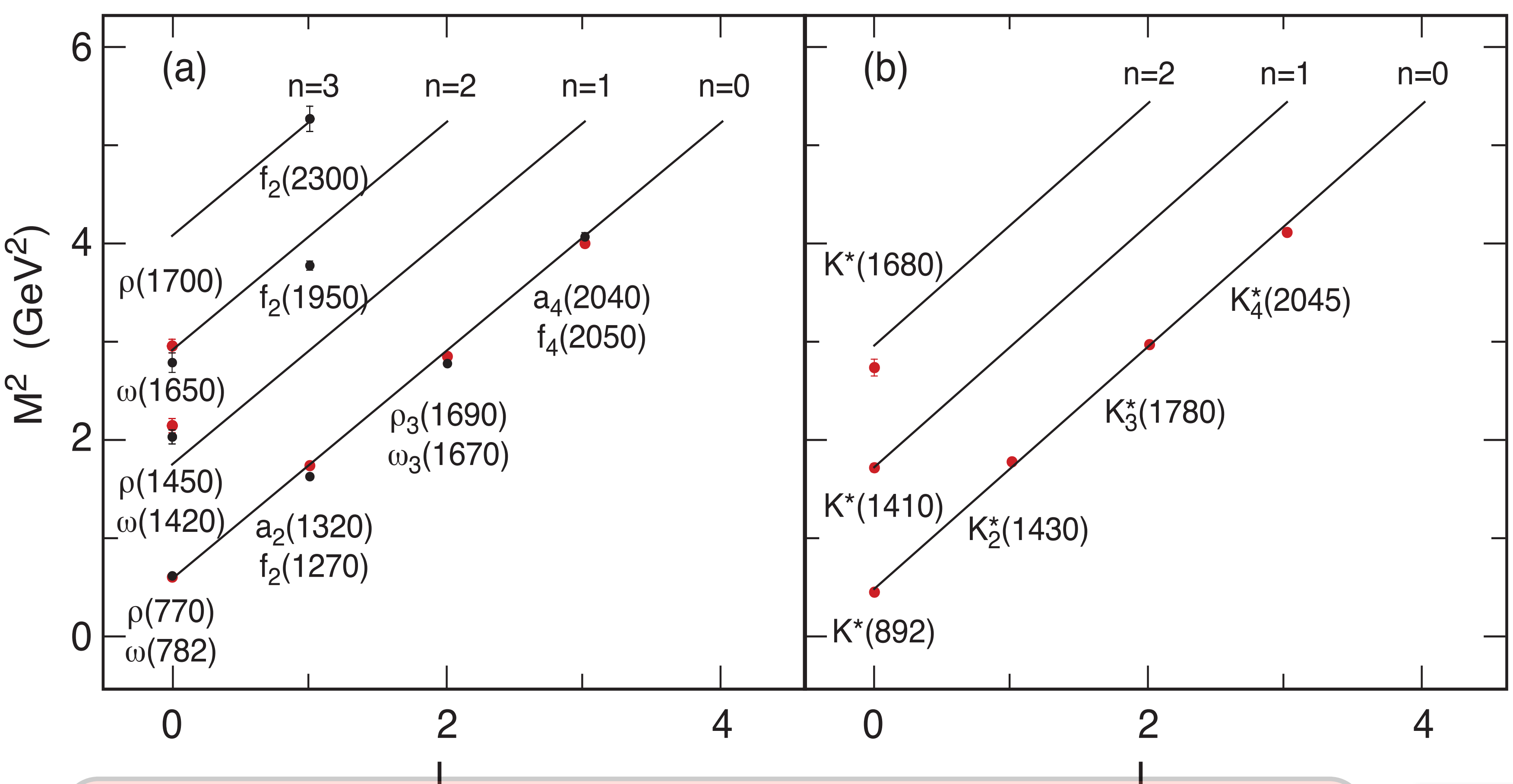
Same slope in n and L!

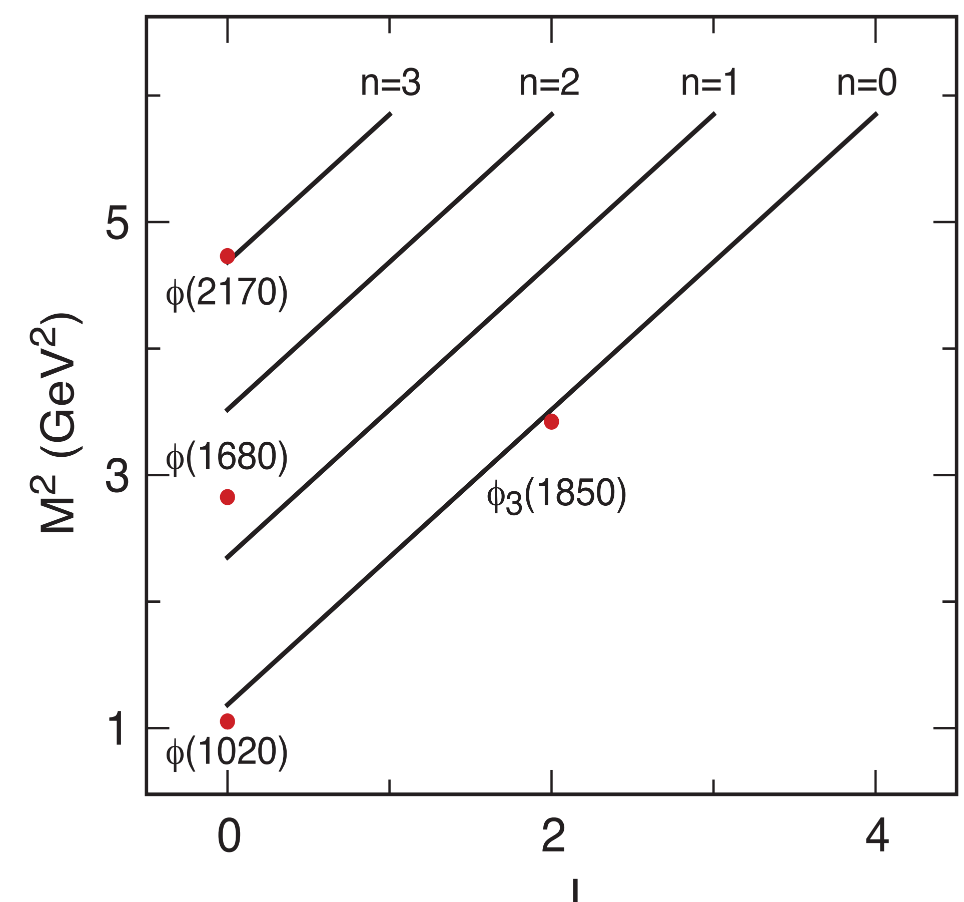


$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.







$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

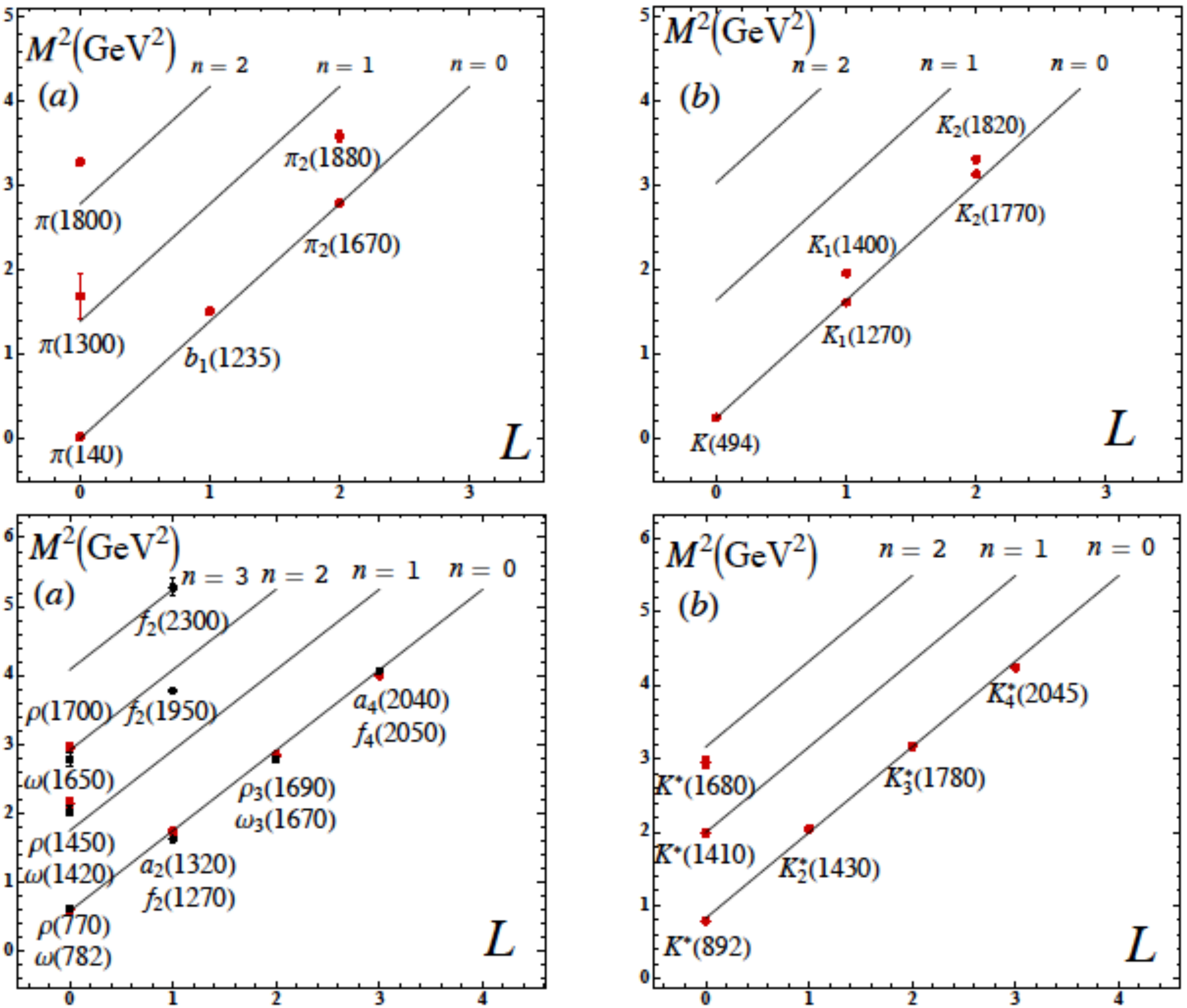
Equal Slope in n and L

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism



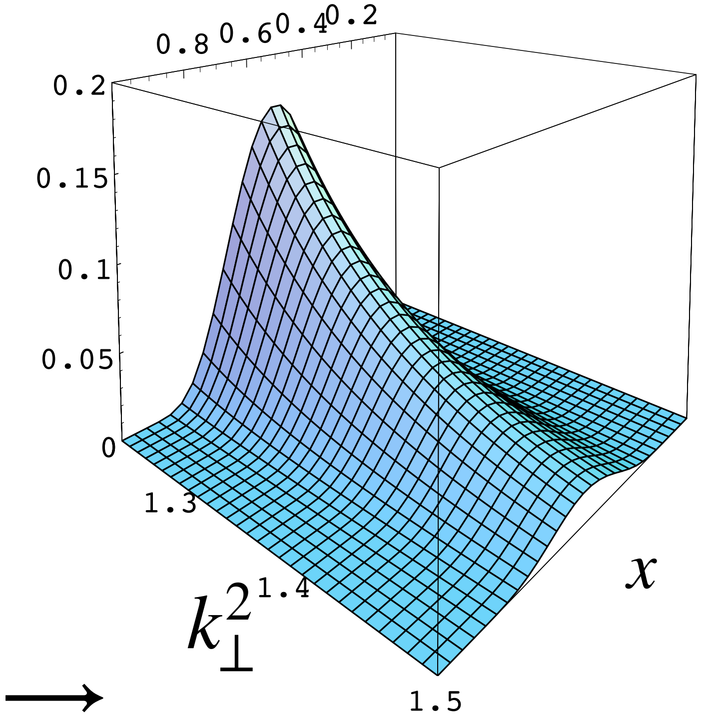
Effective mass from $m(p^2)$

Roberts, et al.

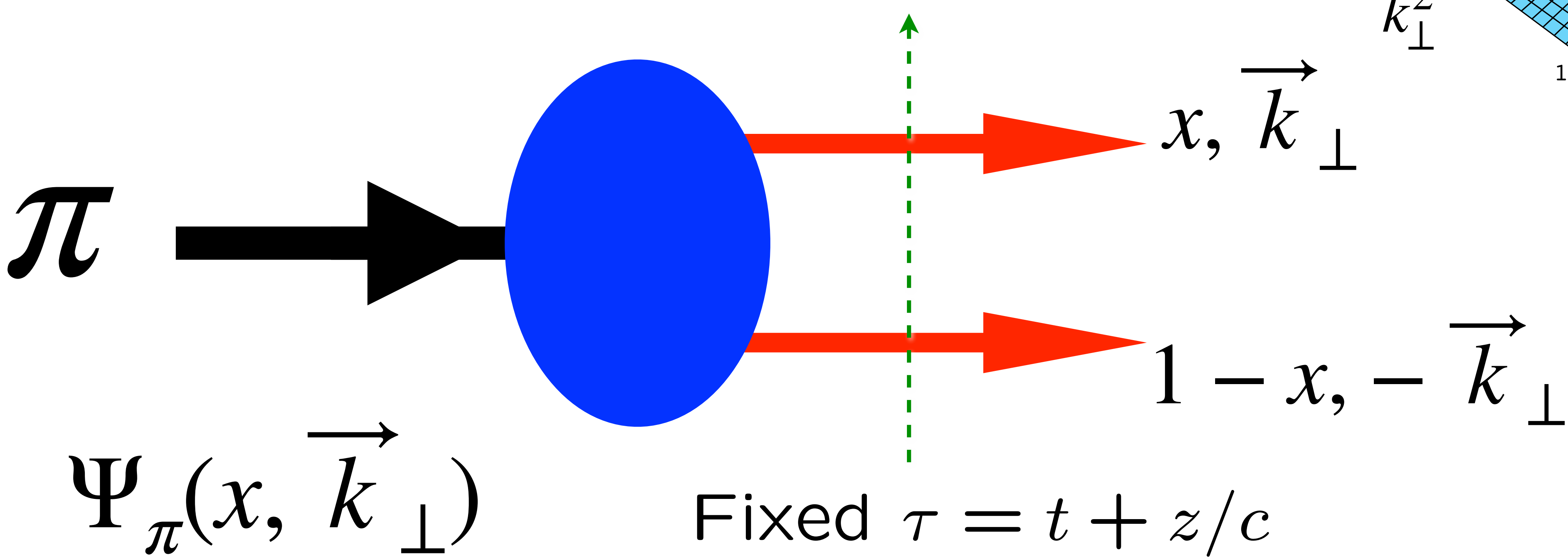
The Pion's Valence Light-Front Wavefunction

- Relativistic Quantum-Mechanical Wavefunction of the pion eigenstate $H_{LF}^{QCD} | \pi > = m_\pi^2 | \pi >$

$$\Psi_\pi(x, \overrightarrow{k}_\perp) = < q(x, \overrightarrow{k}_\perp) \bar{q}(1-x, -\overrightarrow{k}_\perp) | \, \pi >$$



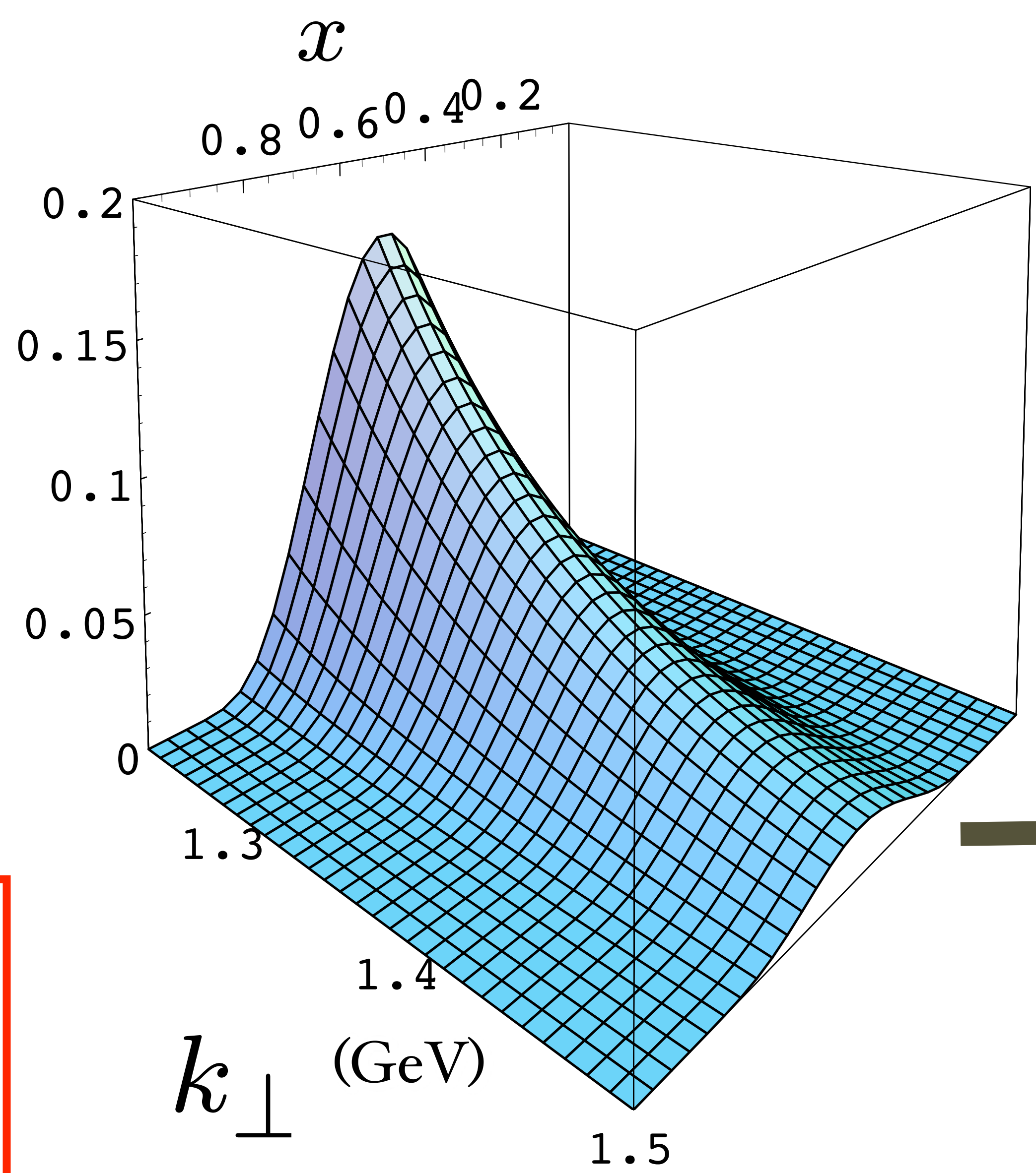
- Independent of the observer's or pion's motion
- No Lorentz contraction; causal
- ***Confined*** quark-antiquark bound state



Prediction from AdS/QCD: Meson LFWF

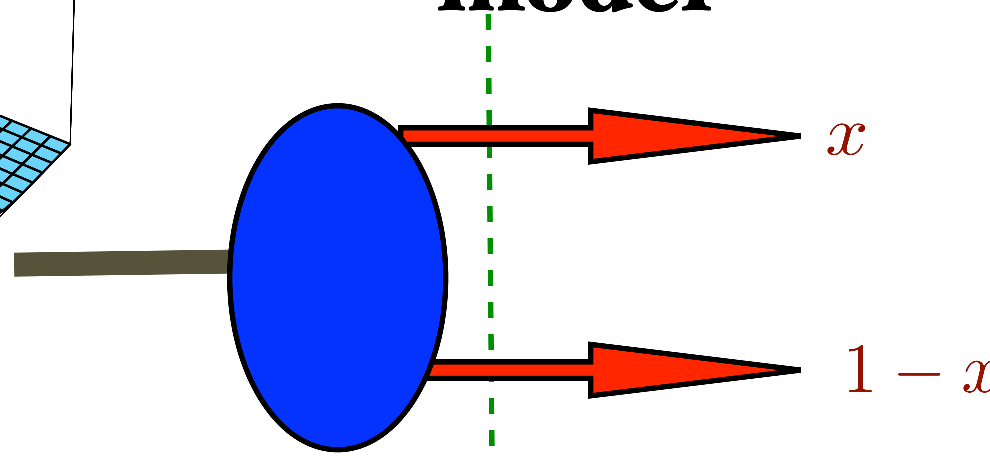
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

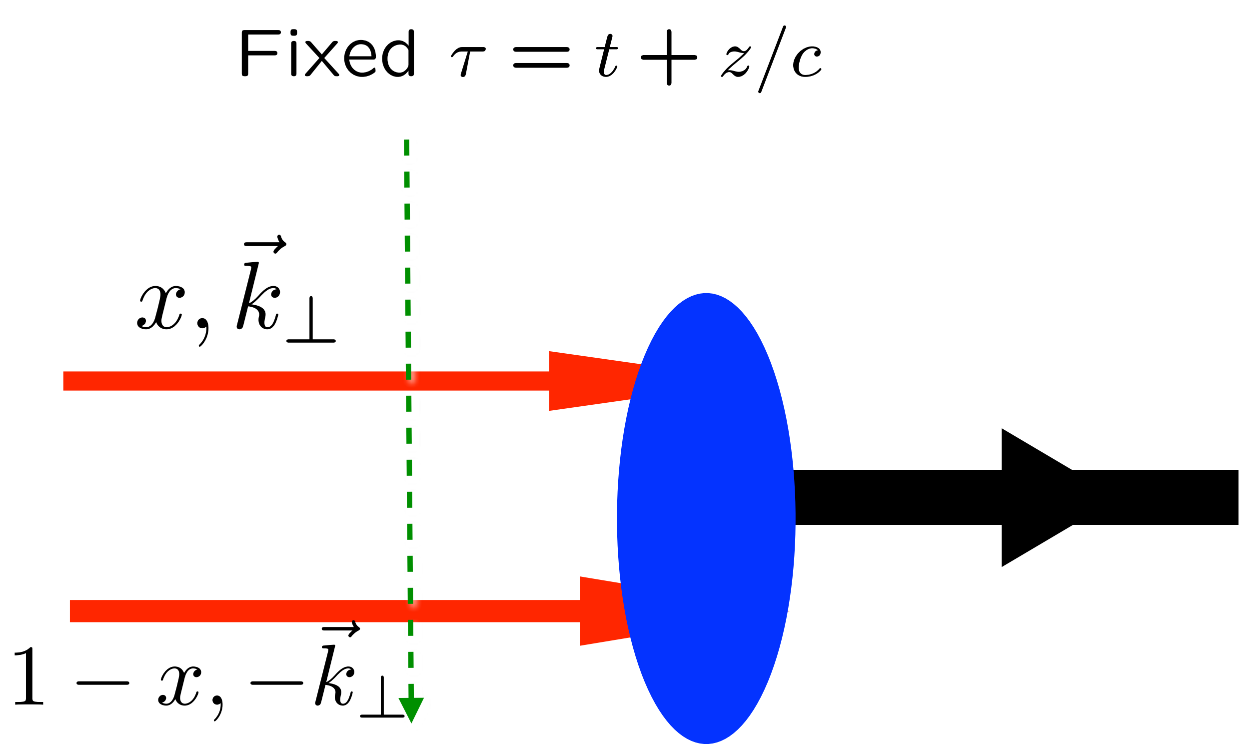
$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

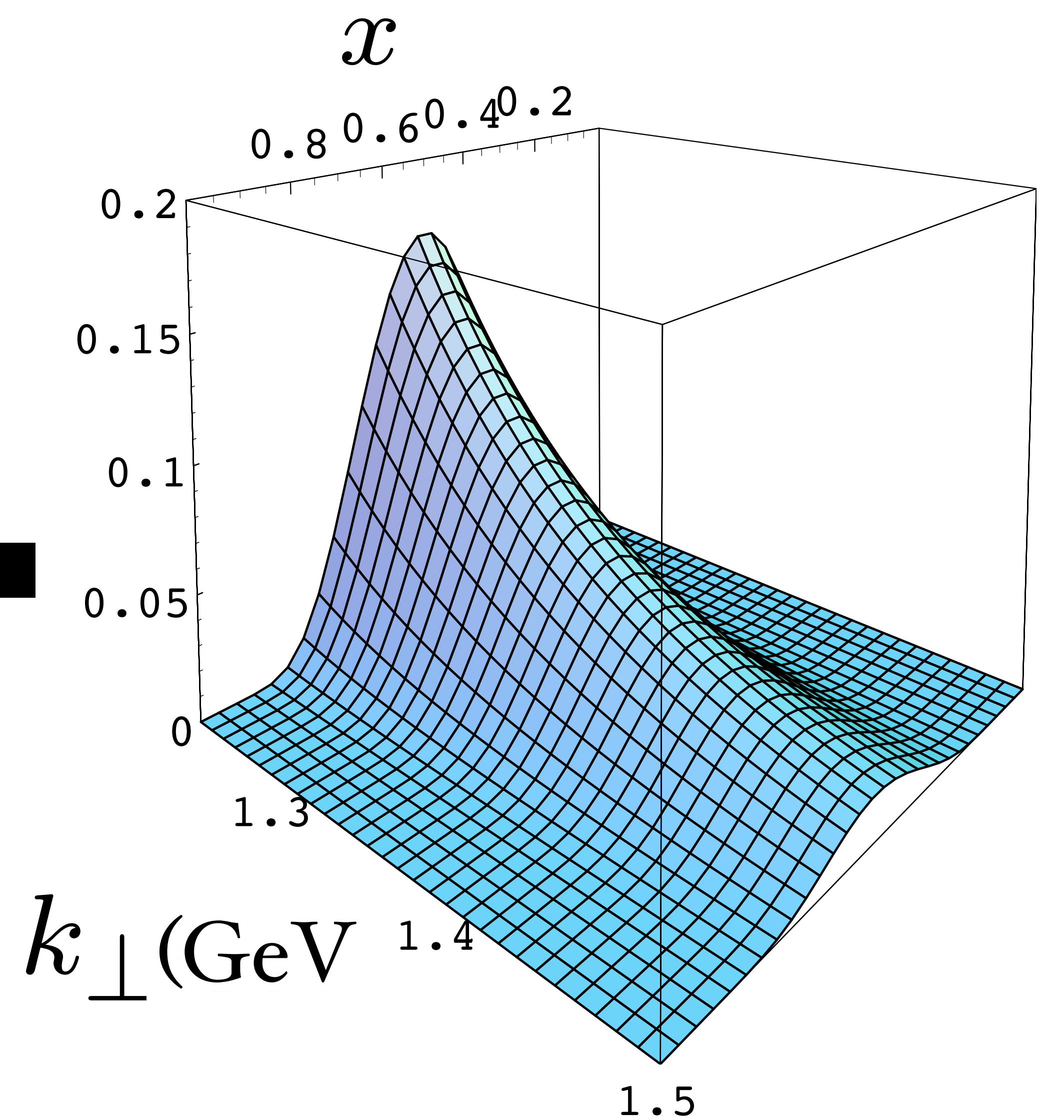
Provides Connection of Confinement to Hadron Structure

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$



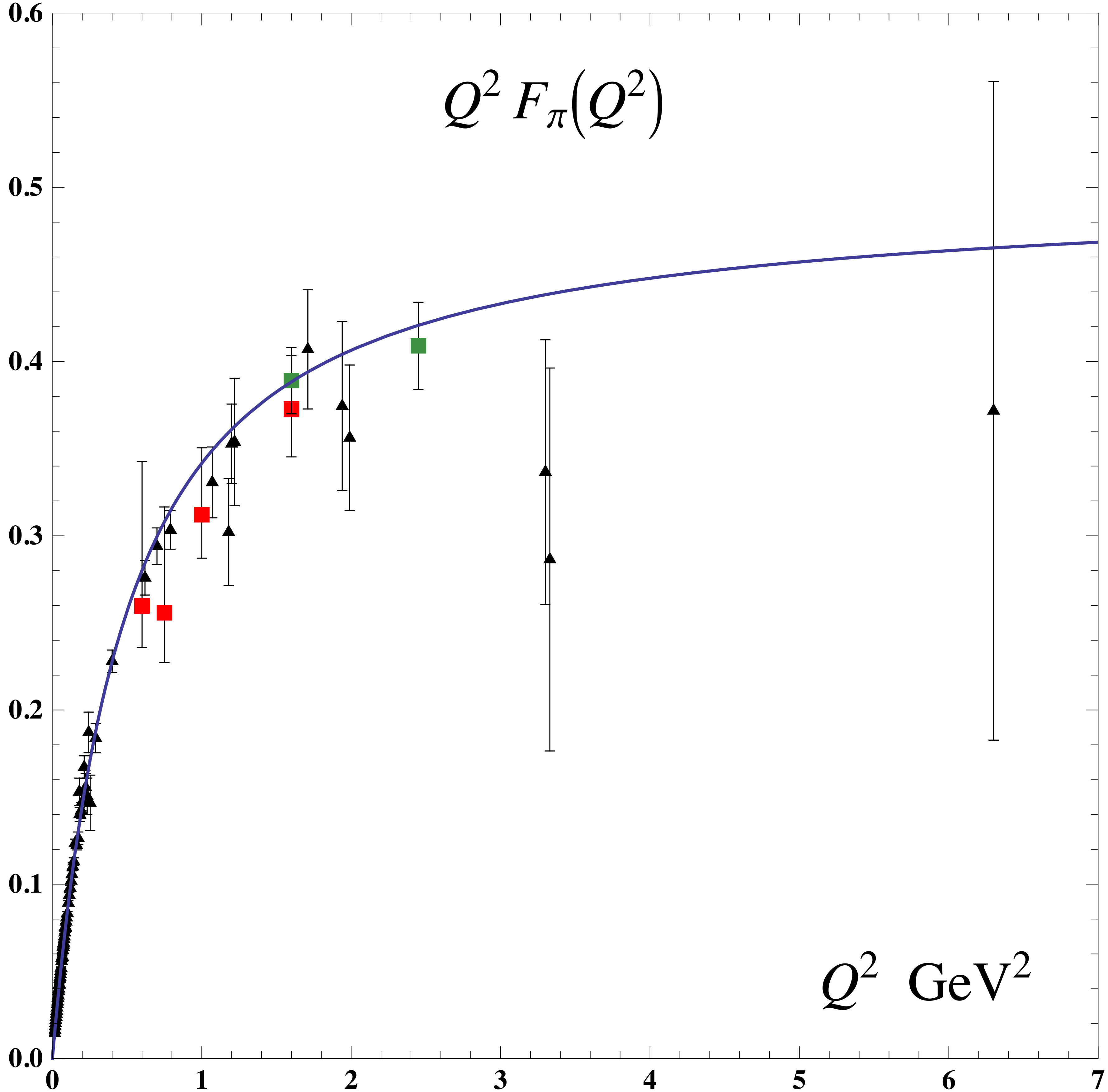
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

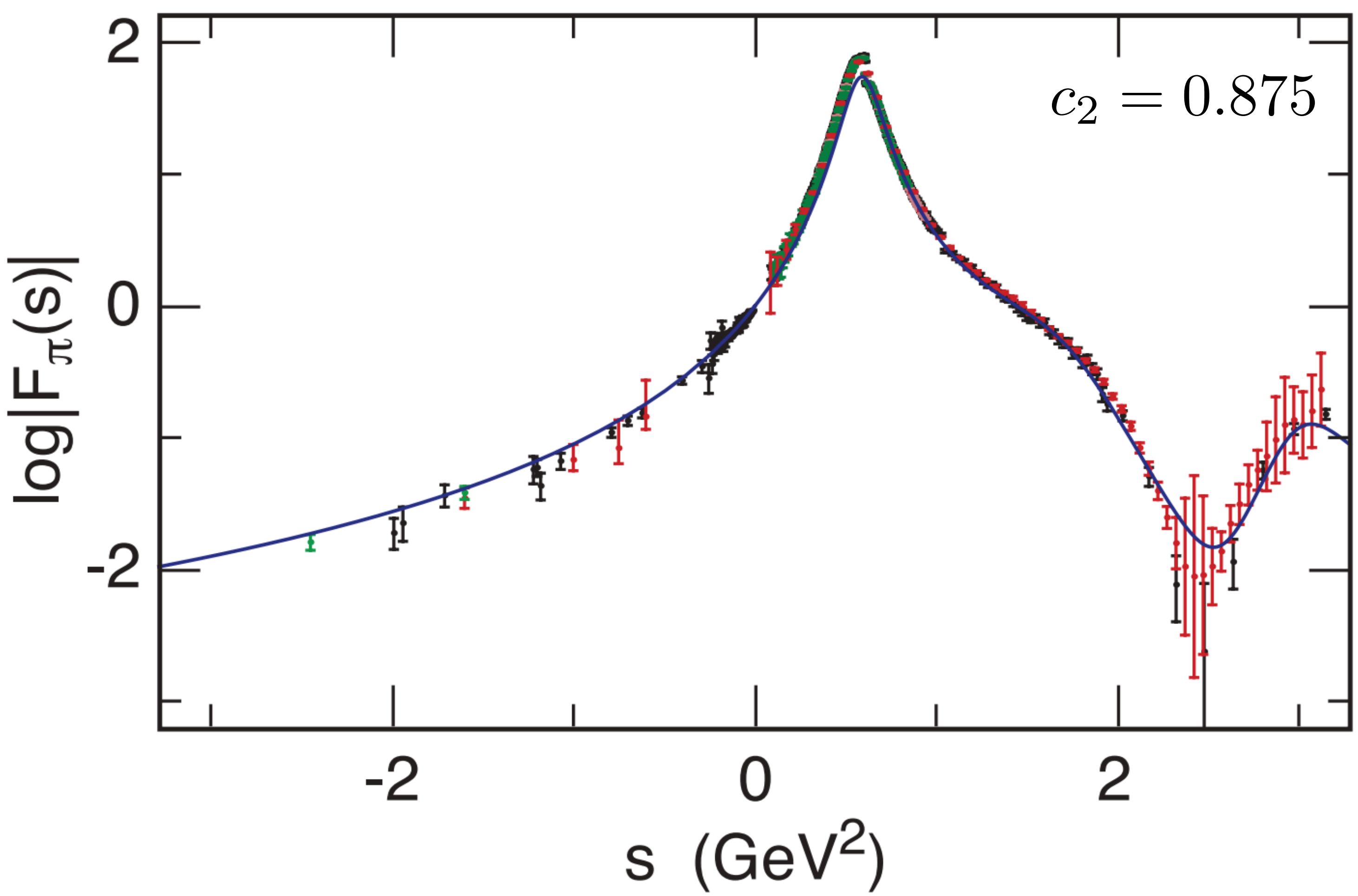
Proceeds in LF time τ within casual horizon
Instant time violates causality



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Pion EM Form Factor

Pion form factor compared with data



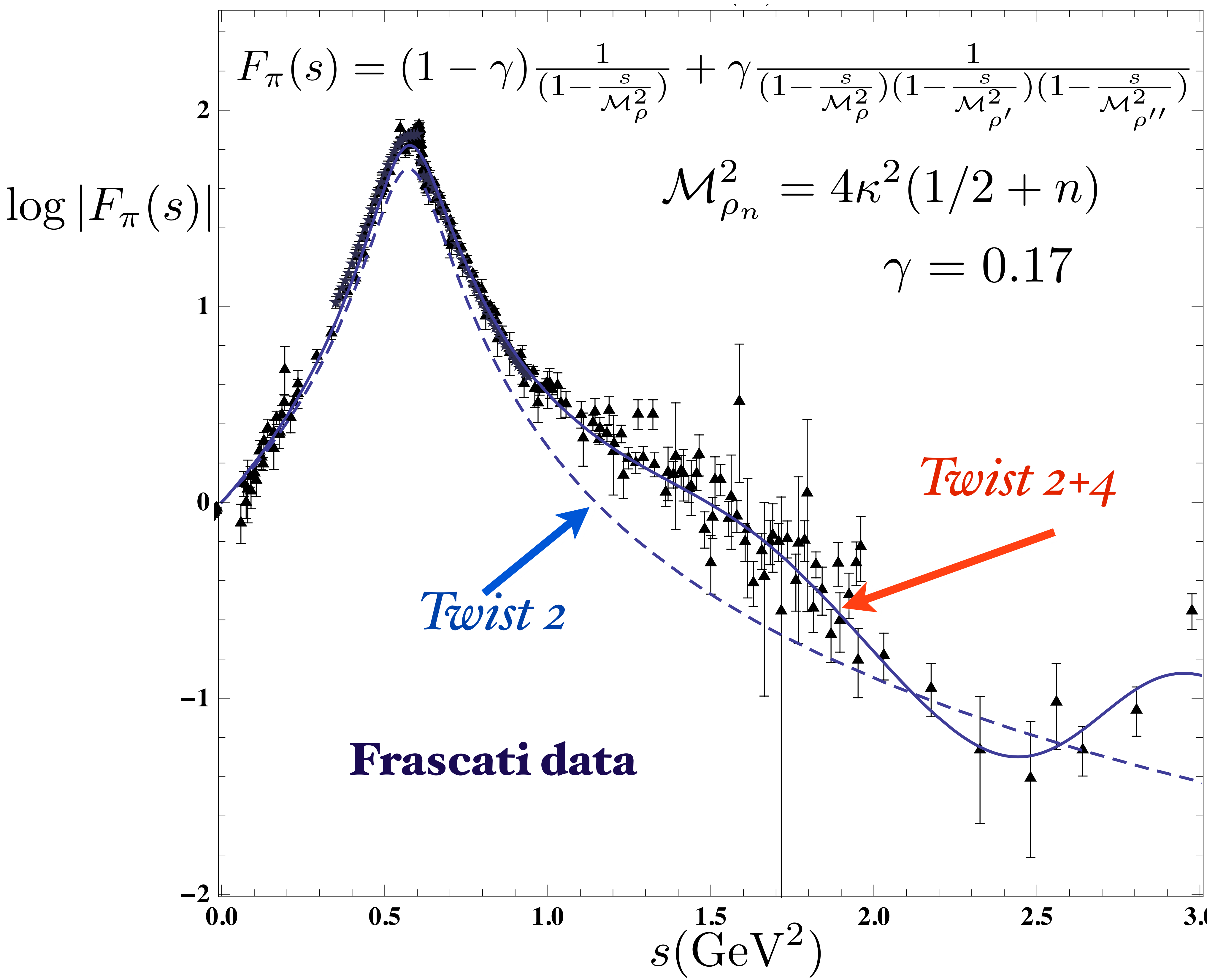
$$F_\pi(t) = \sum_\tau P_\tau F_\tau(t) \qquad \sum_\tau P_\tau = 1$$

Truncated at twist-τ = 4

$$F_\pi(t) = c_2 F_{\tau=2}(t) + (1 - c_2) F_{\tau=4}(t)$$

G.F. de Téramond and S.J. Brodsky, Proc. Sci. LC2010 (2010) 029.
S.J. Brodsky, G.F. de Téramond, H.G. Dosch, J. Erlich, Phys. Rep. 584, 1 (2015). [Sec. 6.1.5]

Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



Prescription for Timelike poles :

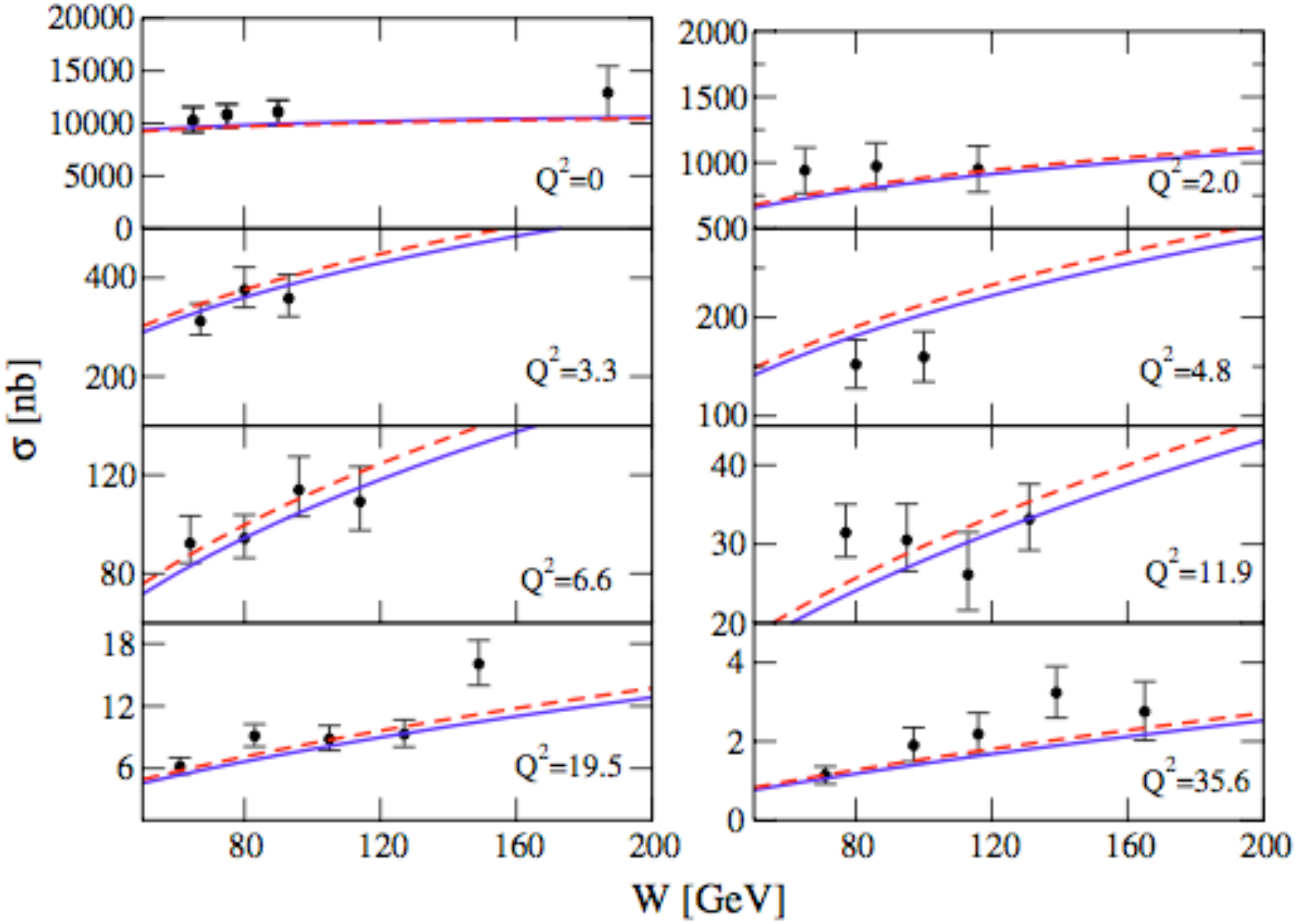
$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

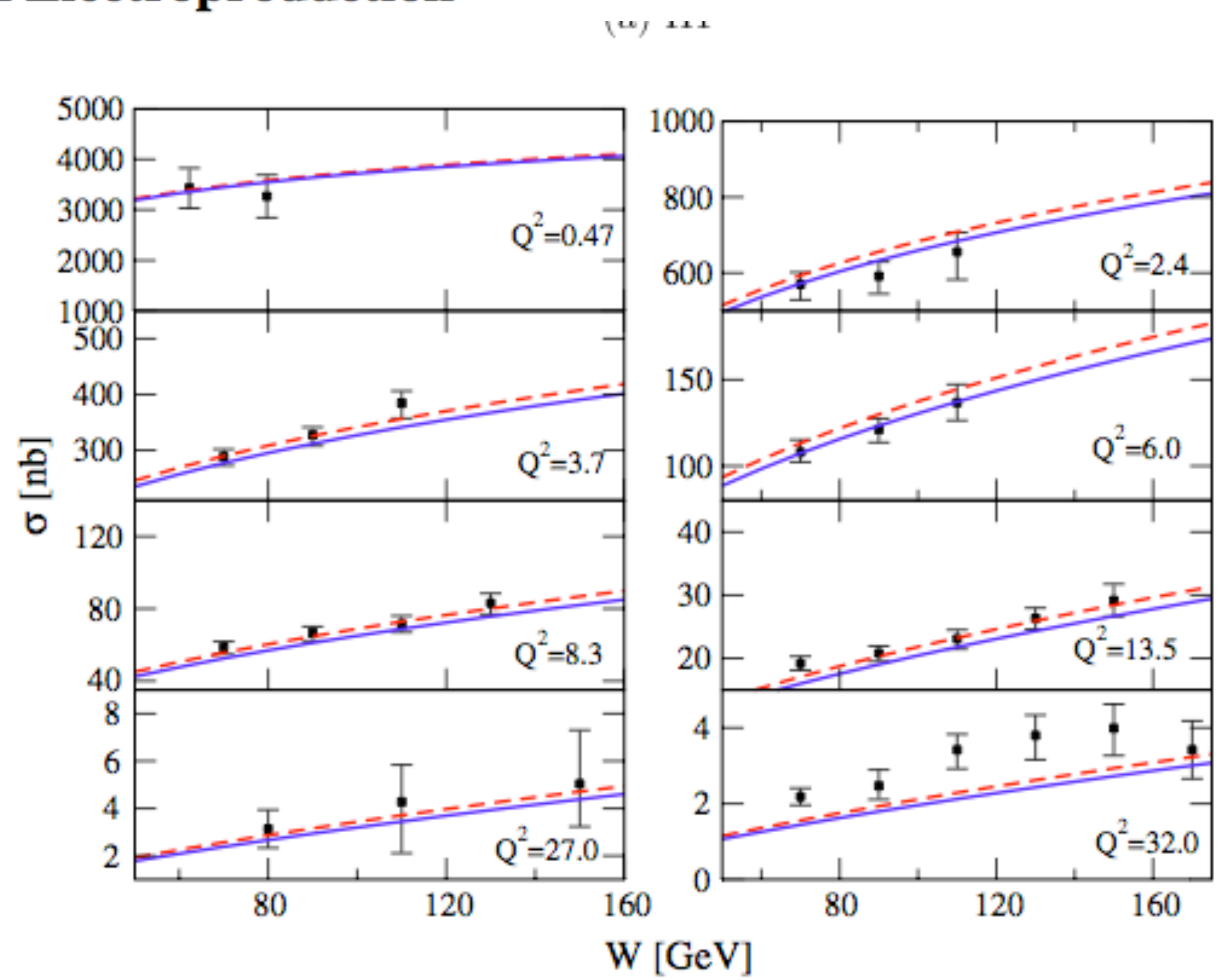
G. de Teramond & sjb

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



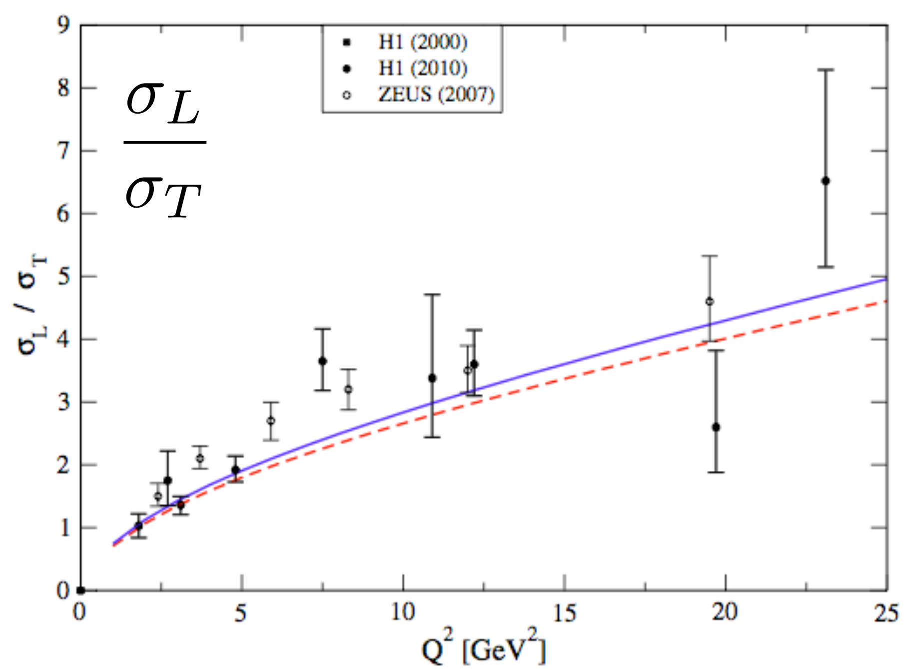
(a) H1



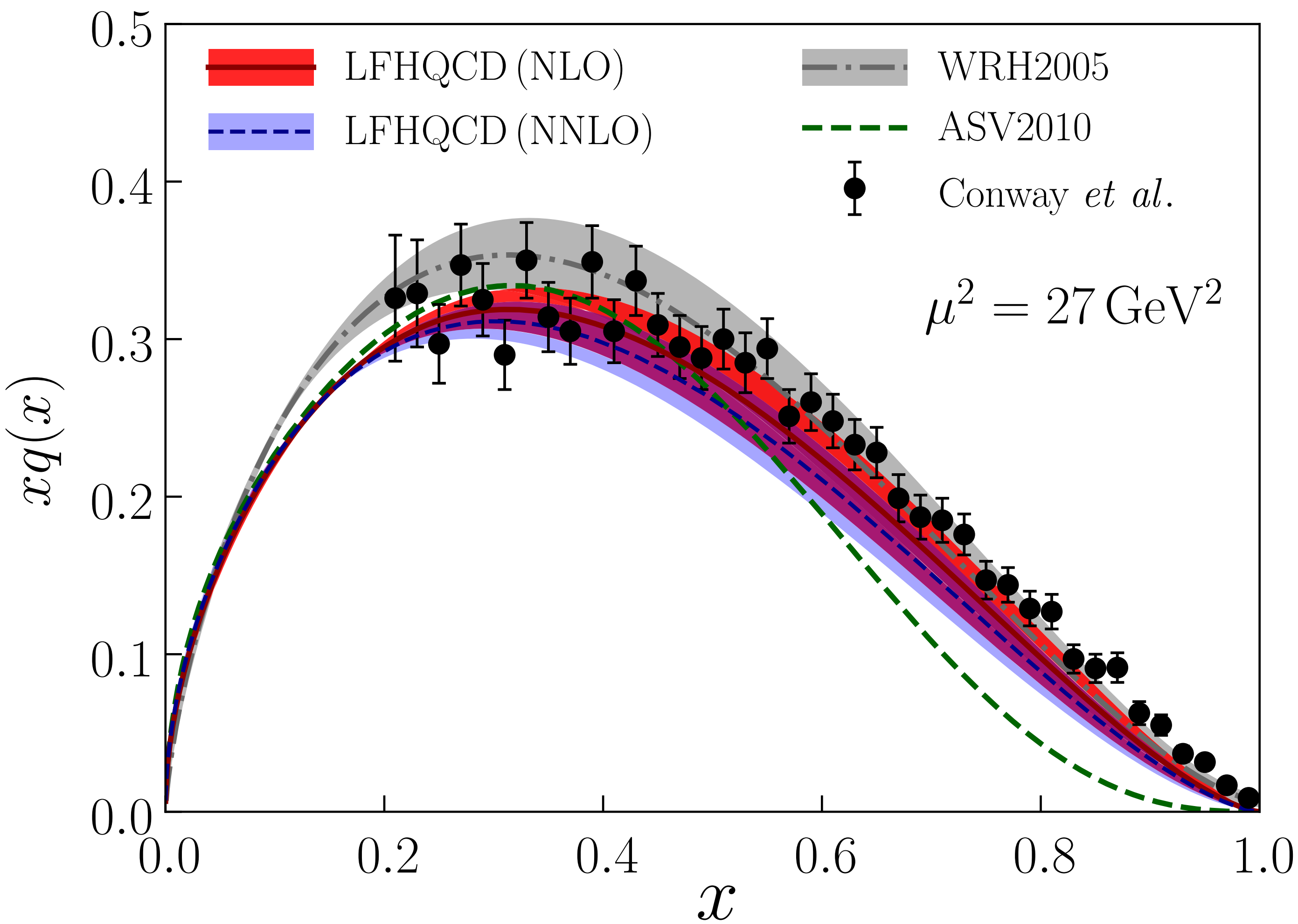
(b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

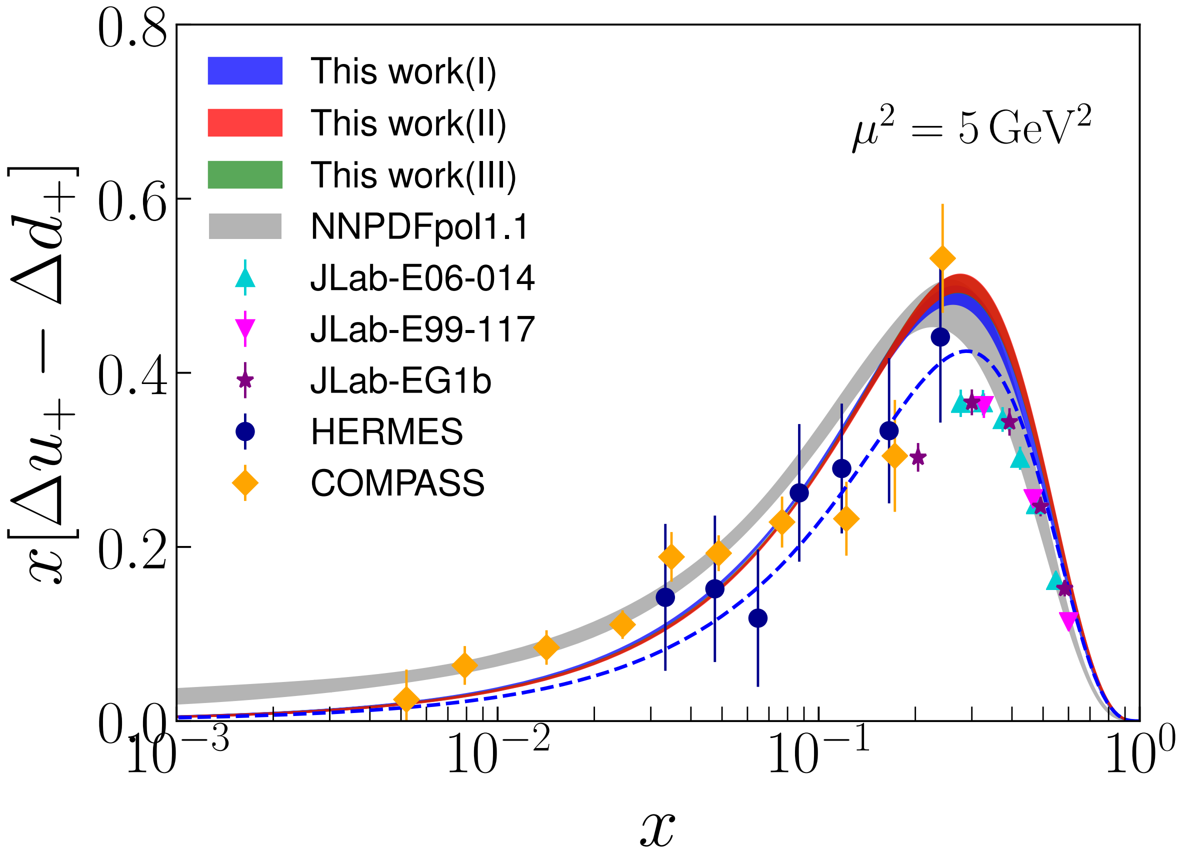


Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1\pm0.2$ GeV at NLO and the initial scale $\mu_0 = 1.06\pm0.15$ GeV at NNLO.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur *PHYSICAL REVIEW LETTERS 120, 182001 (2018)*

Tianbo Liu, Raza Sabbir Sufian, Guy F. de T´eramond, Hans Gunter Dösch, Alexandre Deur, sjb

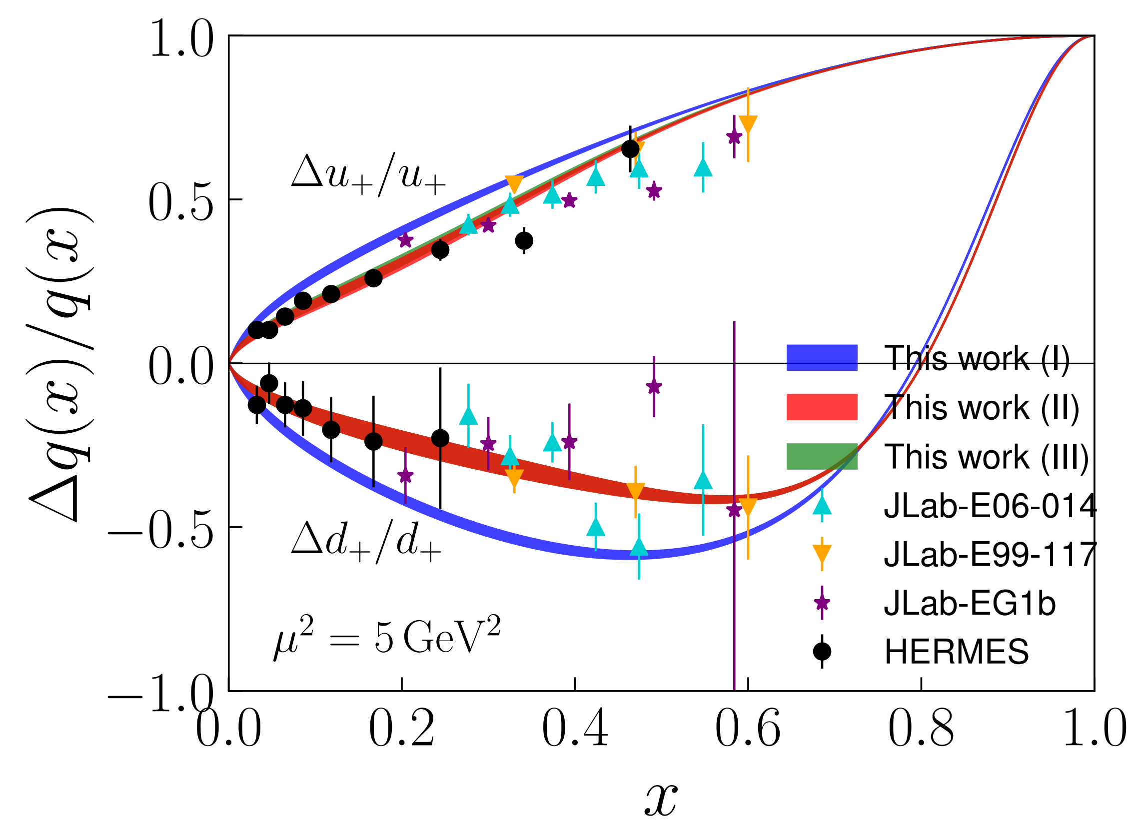


Polarized distributions for the

isovector combination $x[\Delta u_+(x) - \Delta d_+(x)]$

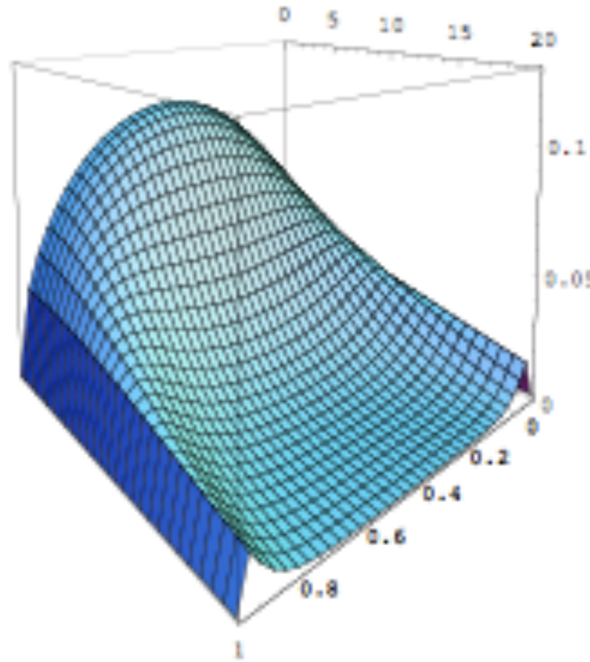
$d_+(x) = d(x) + \bar{d}(x) \qquad u_+(x) = u(x) + \bar{u}(x)$

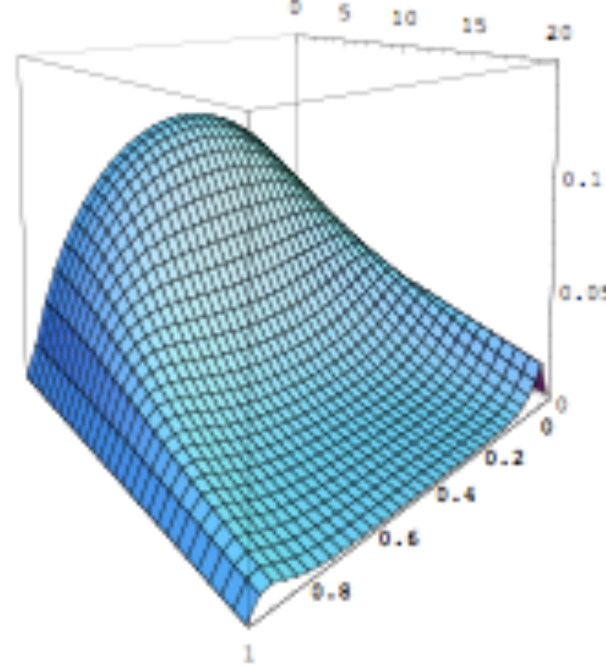
$$\Delta q(x) = q_\uparrow(x) - q_\downarrow(x)$$



$|\pi^+ >= |u\bar{d} >$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



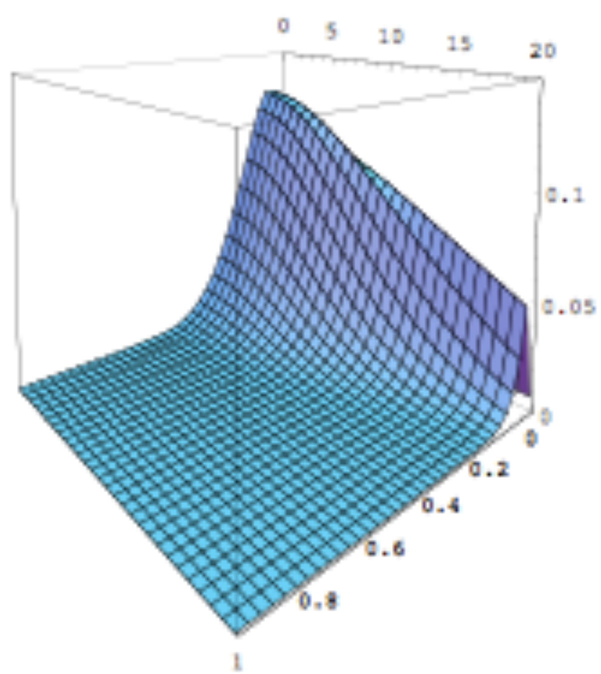


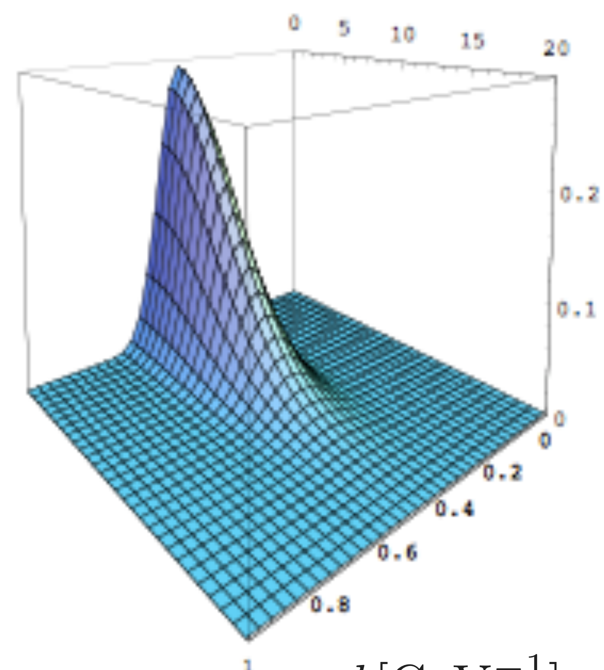
$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

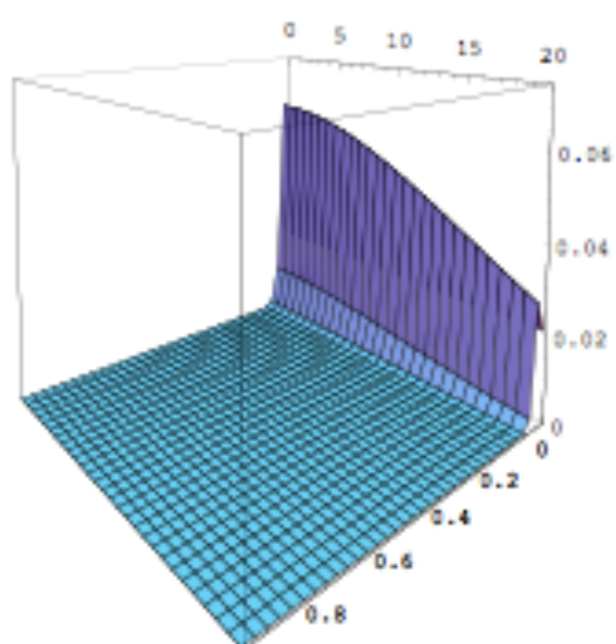


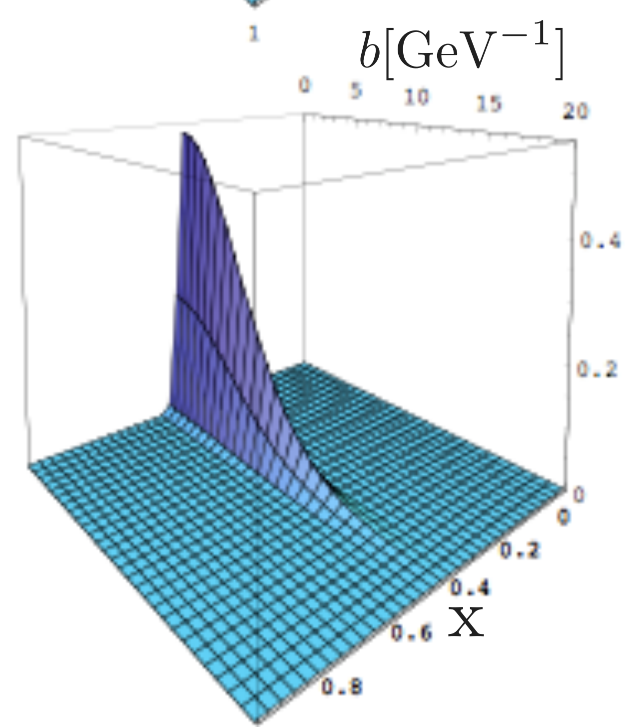


$|\eta_c >= |c\bar{c} >$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$





$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

↕

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau) >= i\frac{\partial}{\partial\tau}|\psi(\tau) >$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(- \frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4} x^2 \right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[- \frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

- **Dosch, de Teramond, sjb**

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

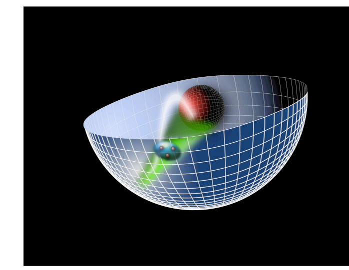
- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

LFHQCD: Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time* τ
- *Causality: Information within causal horizon: Light-Front*
- *Light-Front Holography:* AdS_5 *= LF (3+1)*

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$



- *Introduce Mass Scale* κ *while retaining the Conformal Invariance of the Action (dAFF)*
- *Unique Dilaton in* AdS_5: $e^{+\kappa^2 z^2}$
- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

Consider $R_w = Q + wS;$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B \;, \quad w = \kappa^2$ $\qquad \lambda = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

de Téramond, Dosch, Lorcé, sjb

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\Big(-\partial_\zeta^2+\kappa^4\zeta^2+2\kappa^2(L_B+1)+\frac{4L_B^2-1}{4\zeta^2}\Big)\psi_J^+=M^2\psi_J^+$$

$$\Big(-\partial_\zeta^2+\kappa^4\zeta^2+2\kappa^2L_B+\frac{4(L_B+1)^2-1}{4\zeta^2}\Big)\psi_J^-=M^2\psi_J^-$$

$$M^2(n,L_B)=4\kappa^2(n+L_B+1)$$

S=1/2, P=+

Meson Equation

$$\lambda=\kappa^2$$

$$\Big(-\partial_\zeta^2+\kappa^4\zeta^2+2\kappa^2(J-1)+\frac{4L_M^2-1}{4\zeta^2}\Big)\phi_J=M^2\phi_J$$

S=0, P=+

$$M^2(n,L_M)=4\kappa^2(n+L_M)$$

Same κ*!*

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$

LF Holography

Baryon LFWFs

Superconformal Quantum Mechanics

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

- Eigenvalues

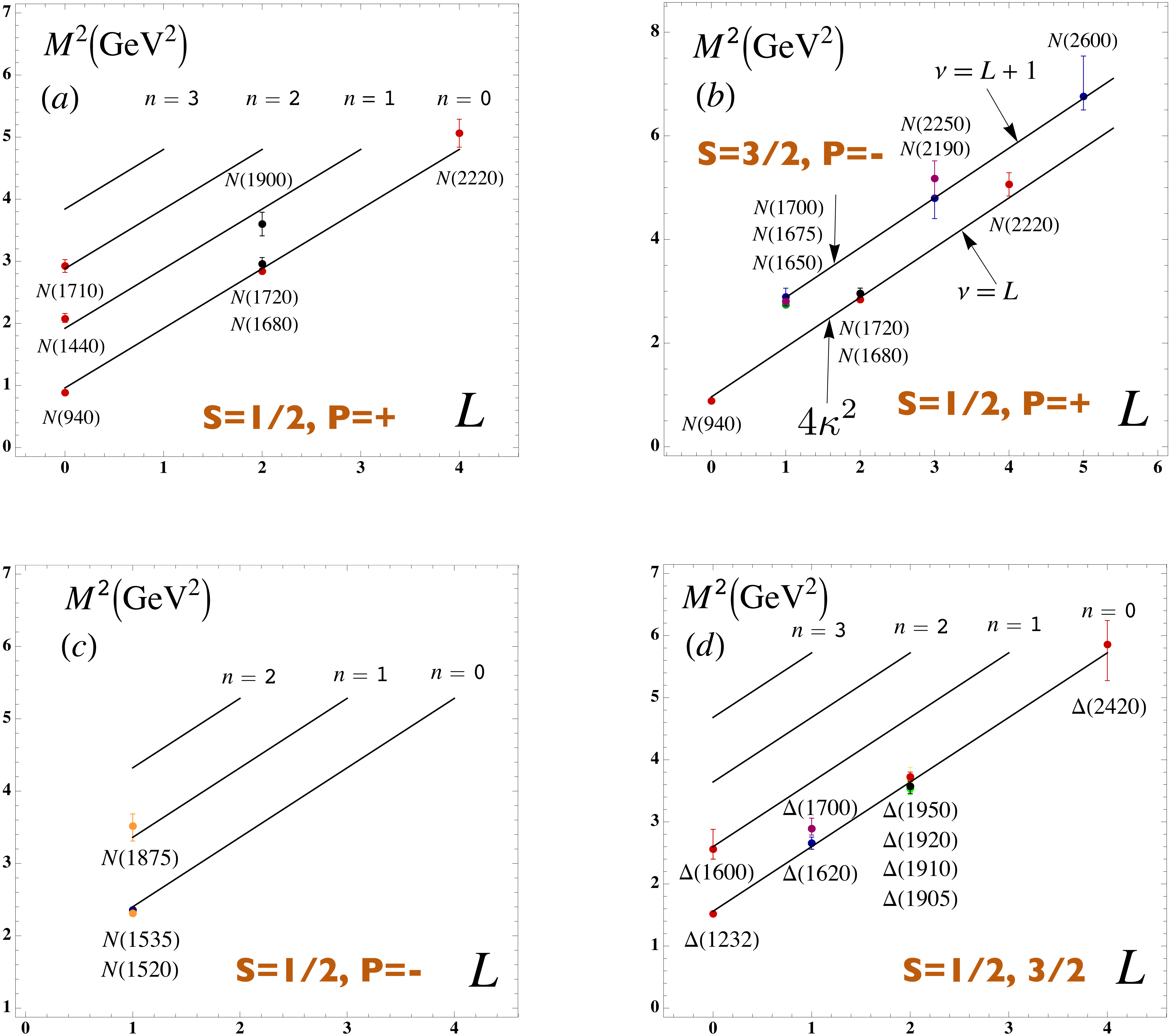
$$\int_0^\infty d\zeta \int_0^1 dx \psi_+^2(\zeta^2, x) = \int_0^\infty d\zeta \int_0^1 dx \psi_-^2(\zeta^2, x) = \frac{1}{2}$$

Quark Chiral Symmetry of Eigenstate!

Nucleon: Equal Probability for L=0,1

$$J^z = +1/2: \quad \frac{1}{\sqrt{2}}[\,|S_q^z = +1/2, L^z = 0 > + |S_q^z = -1/2, L^z = +1 >\,]$$

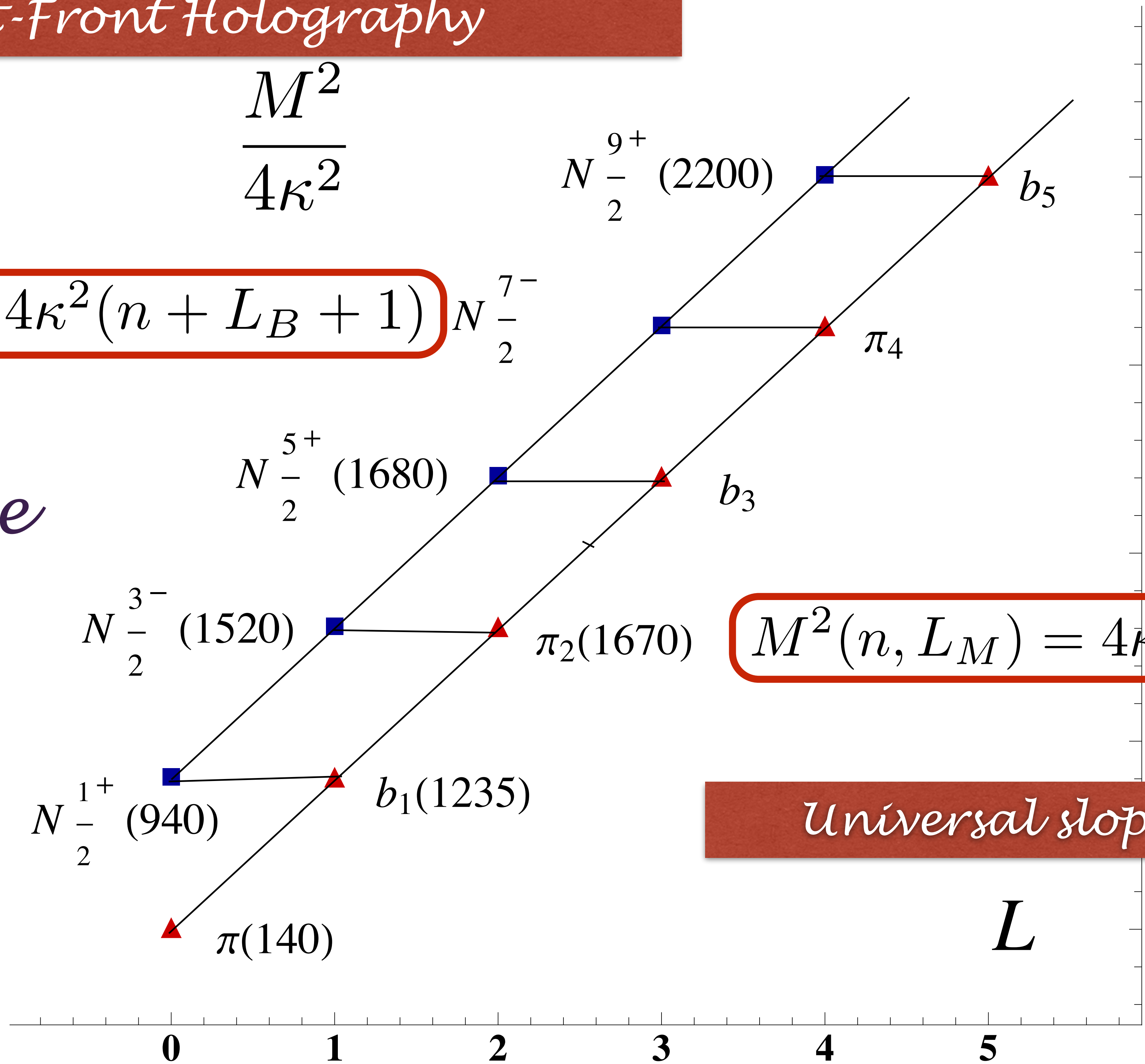
Nucleon spin carried by quark orbital angular momentum



$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
$\nu = L + 1$
S=3/2, P=-
N(2250)
N(2190)
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

Superconformal Quantum Mechanics
Light-Front Holography

de Téramond, Dosch, Lorcé, sjb



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

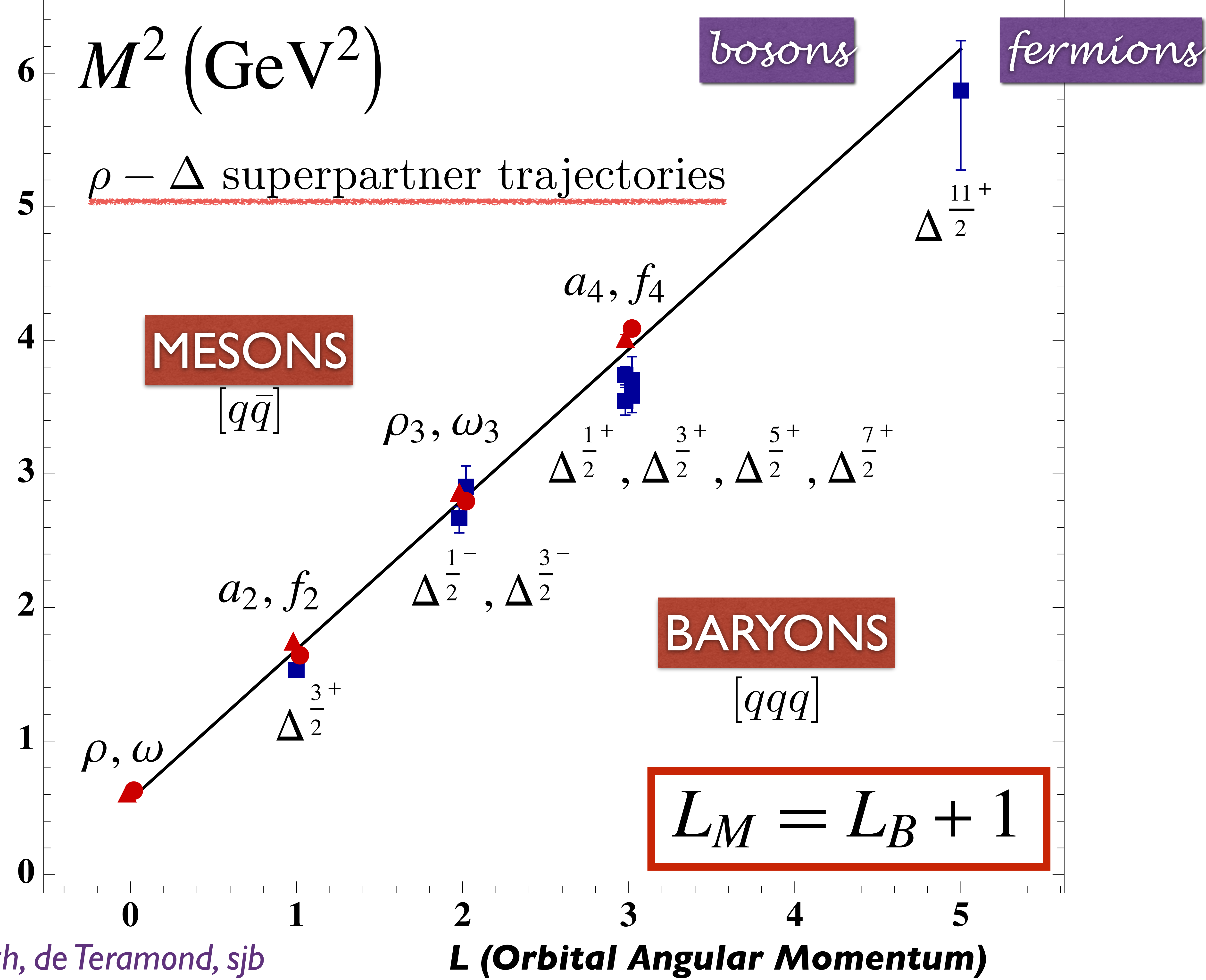
Same slope

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



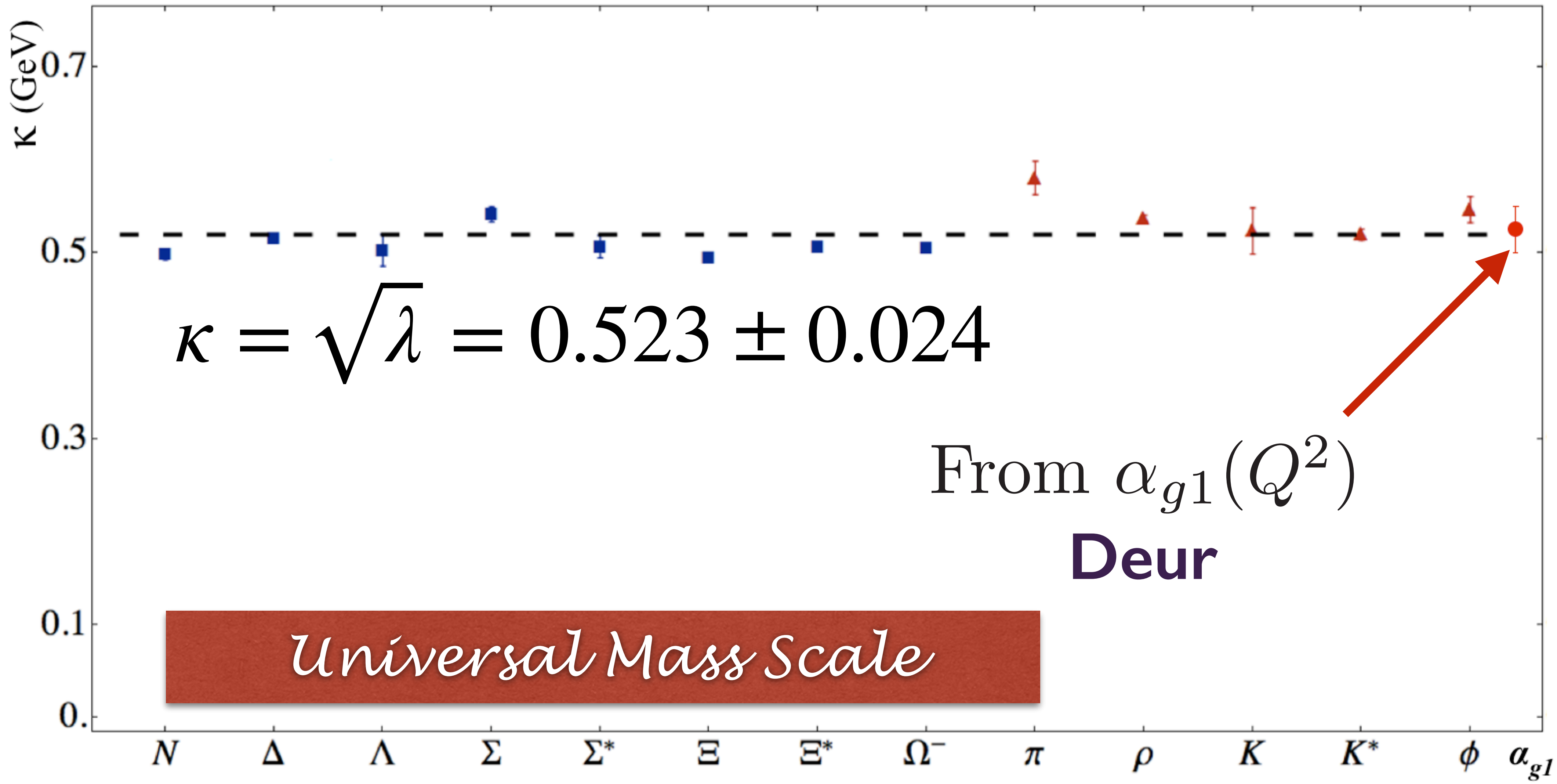
Dosch, de Teramond, sjb

$\lambda = \kappa^2$

de Téramond, Dosch, Lorcé, sjb

$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$

κ



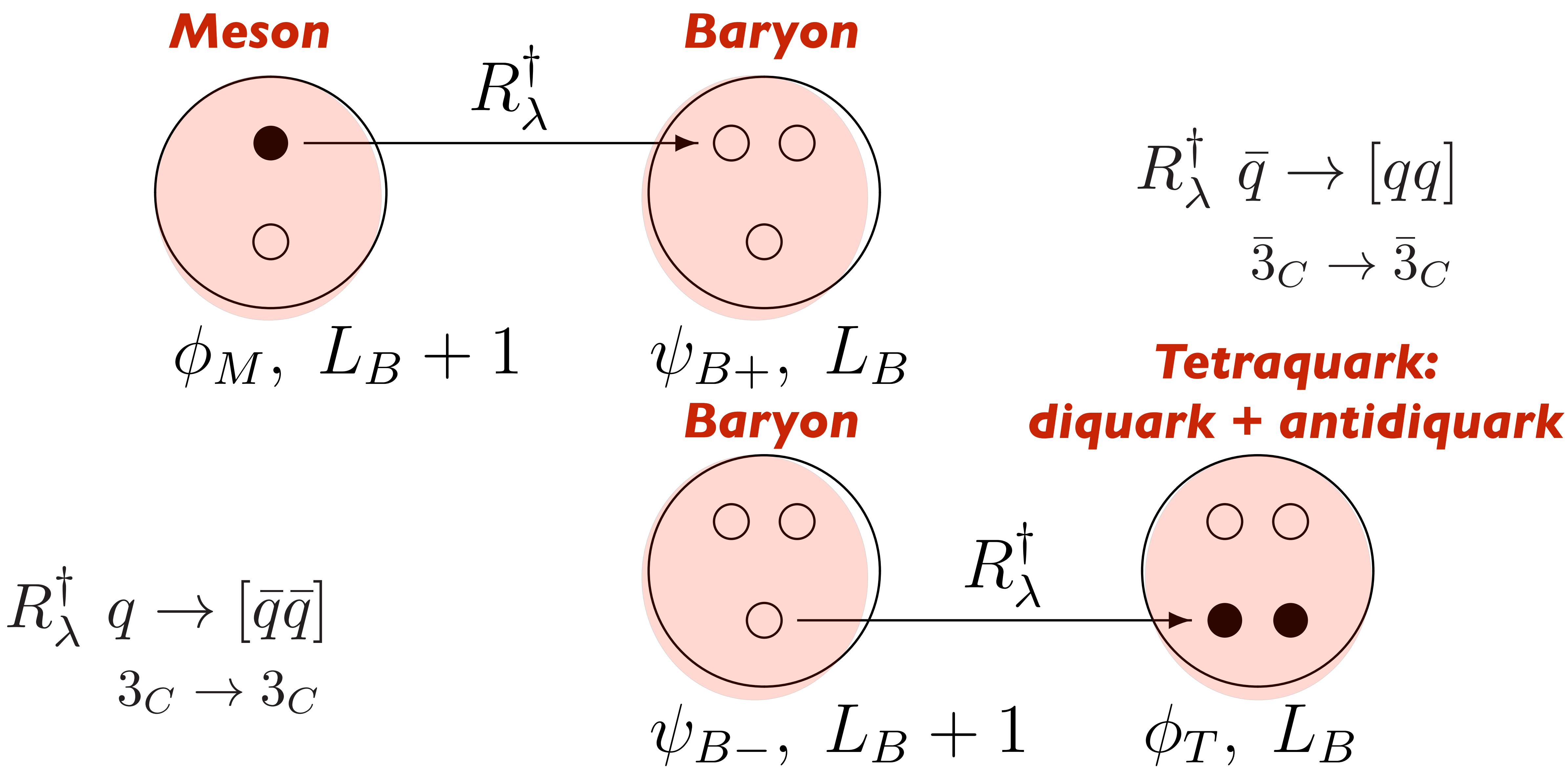
Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: |u[ud]> Quark + Scalar Diquark

Equal Weight: L=0, L=1

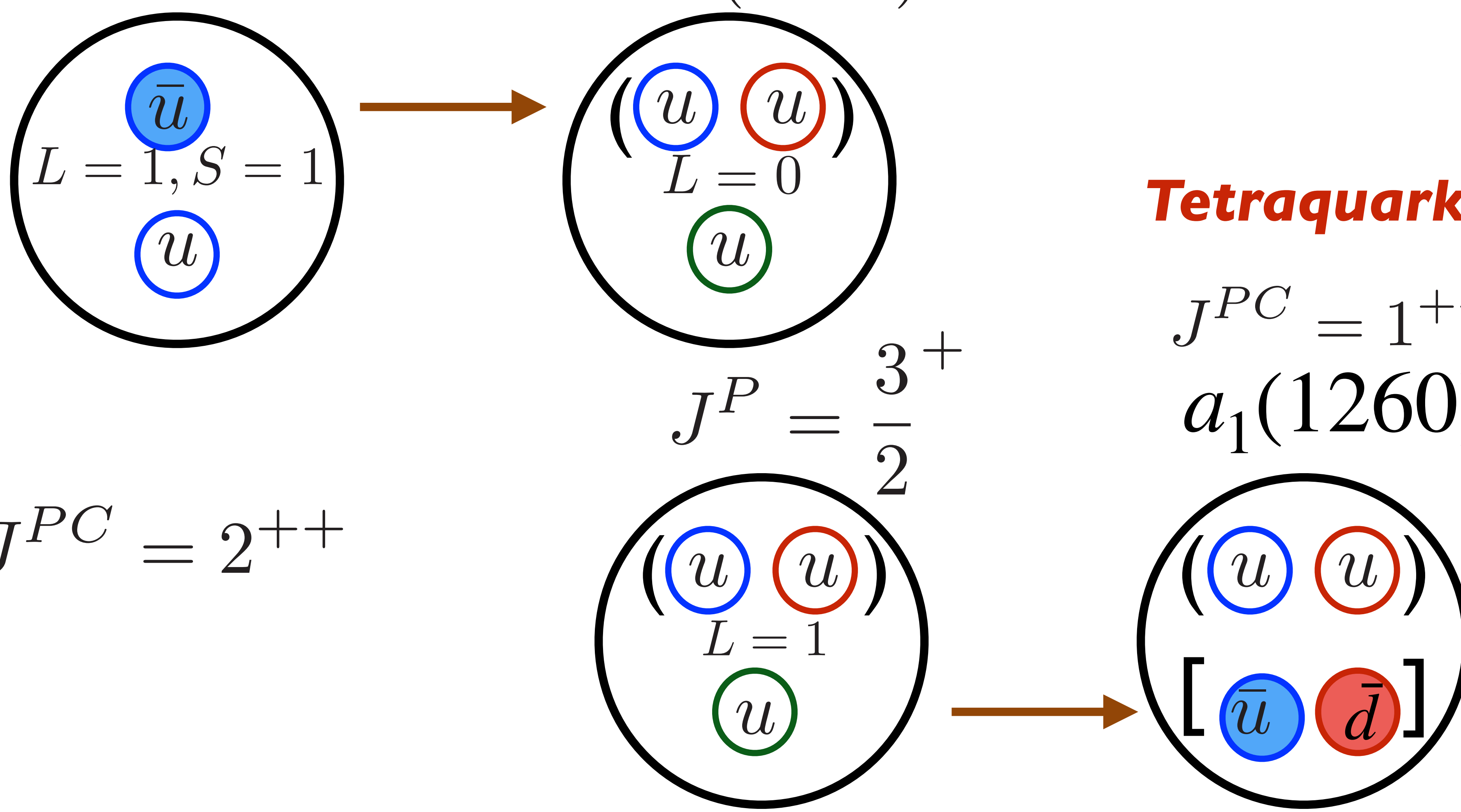
Superconformal Algebra 4-Plet

$R^\dagger_\lambda \quad \bar{q} \to (qq) \quad S = 1$

$\bar{3}_C \to \bar{3}_C$

Vector ()+ Scalar [] Diquarks

$f_2(1270)$ $\quad \Delta^+(1232)$



Tetraquark

$J^{PC} = 1^{++}$

$a_1(1260)$

$J^P = \frac{3}{2}^+$

$J^{PC} = 2^{++}$

$S = 0$

$L = 0$

$R^\dagger_\lambda \quad q \to [\bar{q}\bar{q}]$

$3_C \to 3_C$

Meson **Baryon**

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770),\ \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320),\ f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690),\ \omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040),\ f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)?$
$\bar{q}s$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$K_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)?$
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)?$
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)?$
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)?$
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)?$

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

Complete Regge spectrum in n, L

$$3_C \times 3_C = \bar{3}_C + 6_C$$

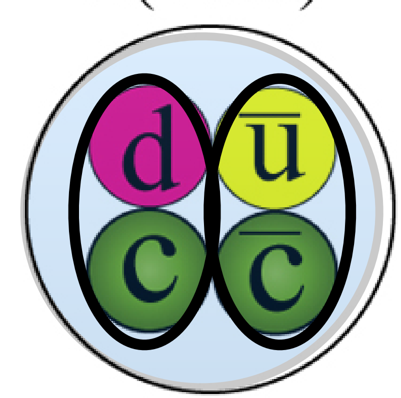
Bound!

- Diquark Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states
- Confinement Force Similar to quark-antiquark mesons $\bar{3}_C \times 3_C = 1_C$
- Isospin $I = 0, \pm 1, \pm 2$ Charge $Q = 0, \pm 1, \pm 2$

$uu\bar{d}\bar{d}$ $uu\bar{s}\bar{d}$ $uu\bar{s}\bar{s}$

Q= +2

Z(4430)



Q= - 1

Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- **Universal quark light-front kinetic energy**

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$

Equal: Virial Theorem

- **Universal quark light-front potential energy**

$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- **Universal Constant Contribution from AdS and Superconformal Quantum Mechanics**

$$\Delta\mathcal{M}^2_{spin} = 2\kappa^2(L + 2S + B - 1)$$

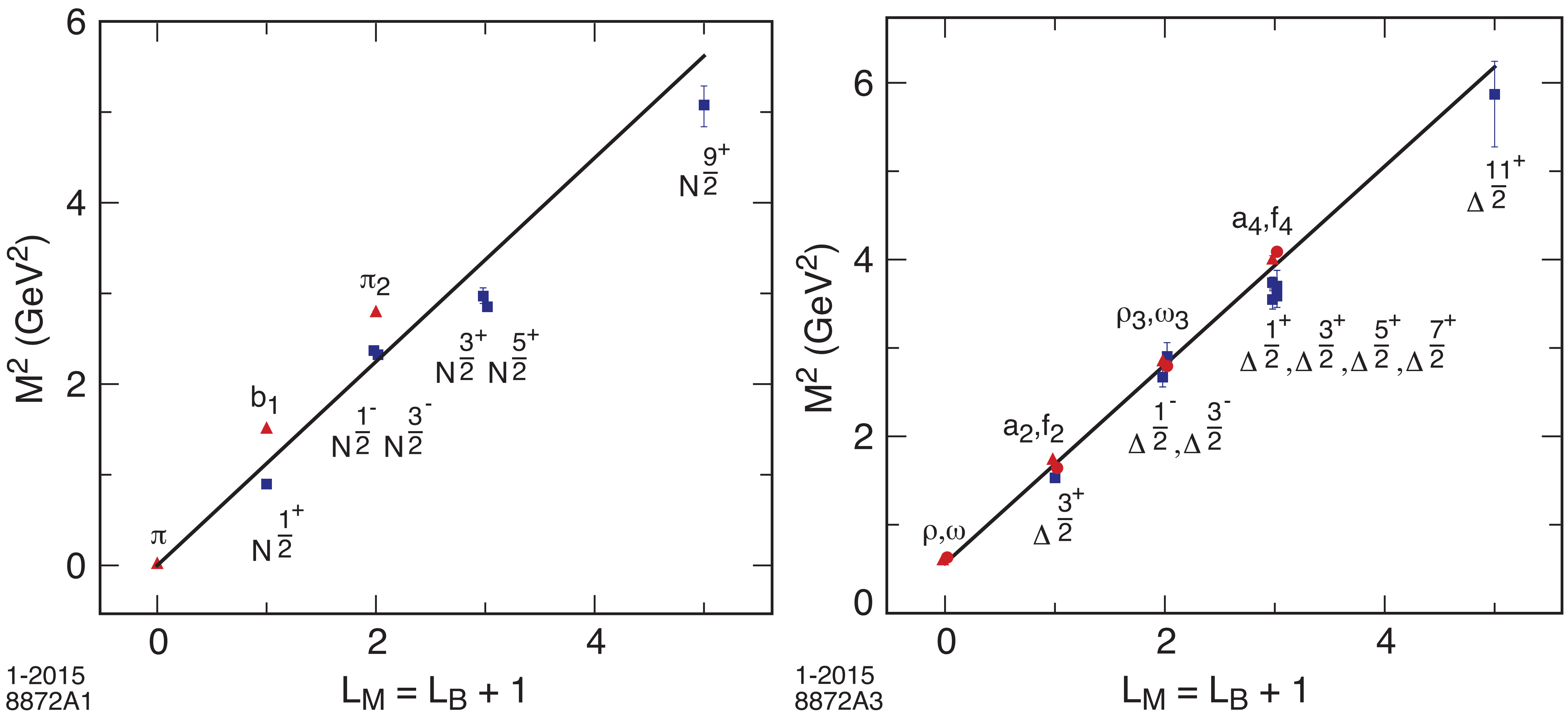
hyperfine spin-spin

- Superconformal spin-dependent Hamiltonian to describe mesons and baryons (chiral limit)

  [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]

$$G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\, S \qquad S = 0, 1$$

$$\text{Mesons}: M^2 = 4\lambda\,(n + L_M) + 2\lambda S, \quad \text{Baryons}: M^2 = 4\lambda\,(n + L_B + 1) + 2\lambda S$$



Superconformal meson-nucleon partners: solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z)\Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2}\sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

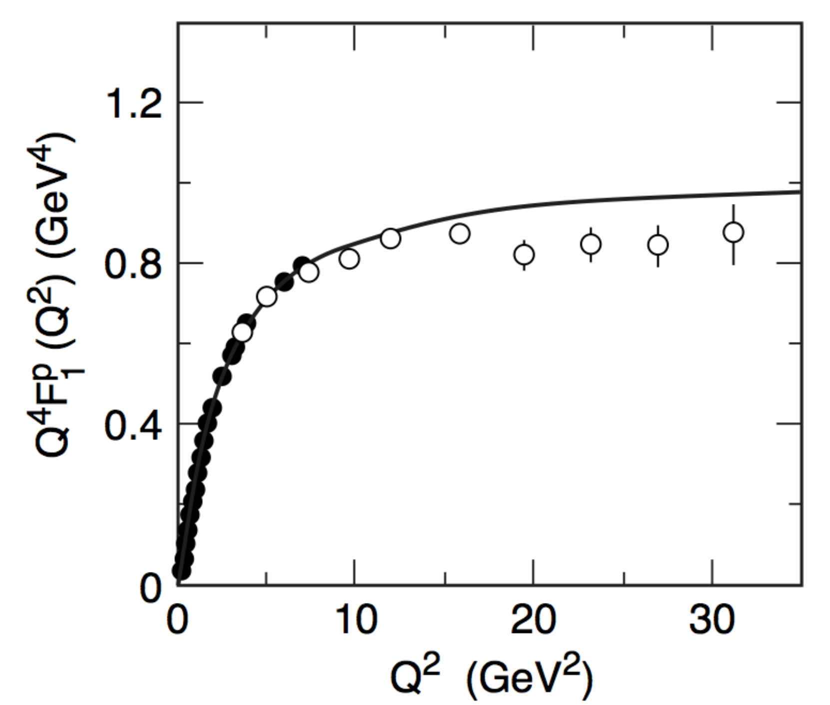
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$

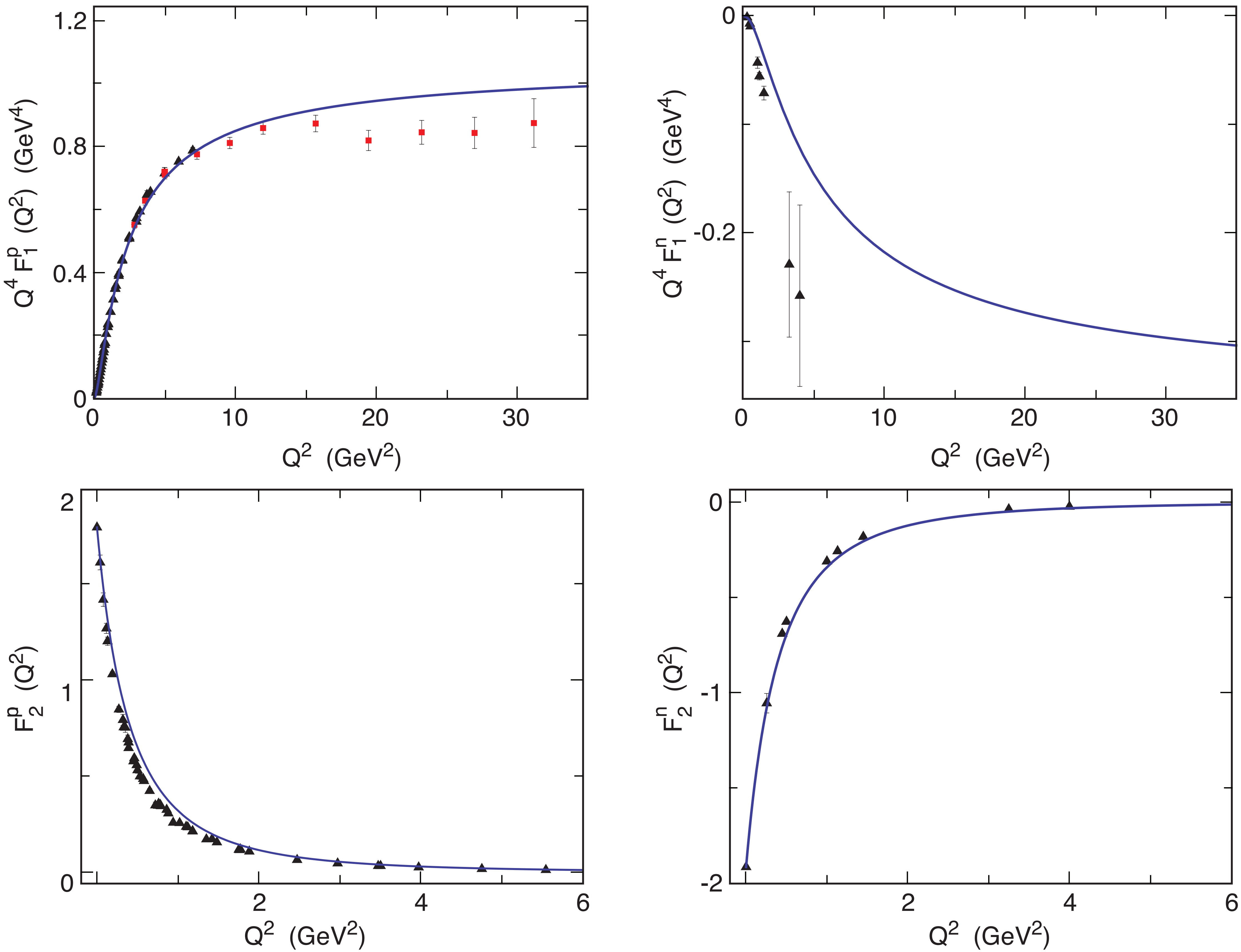
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

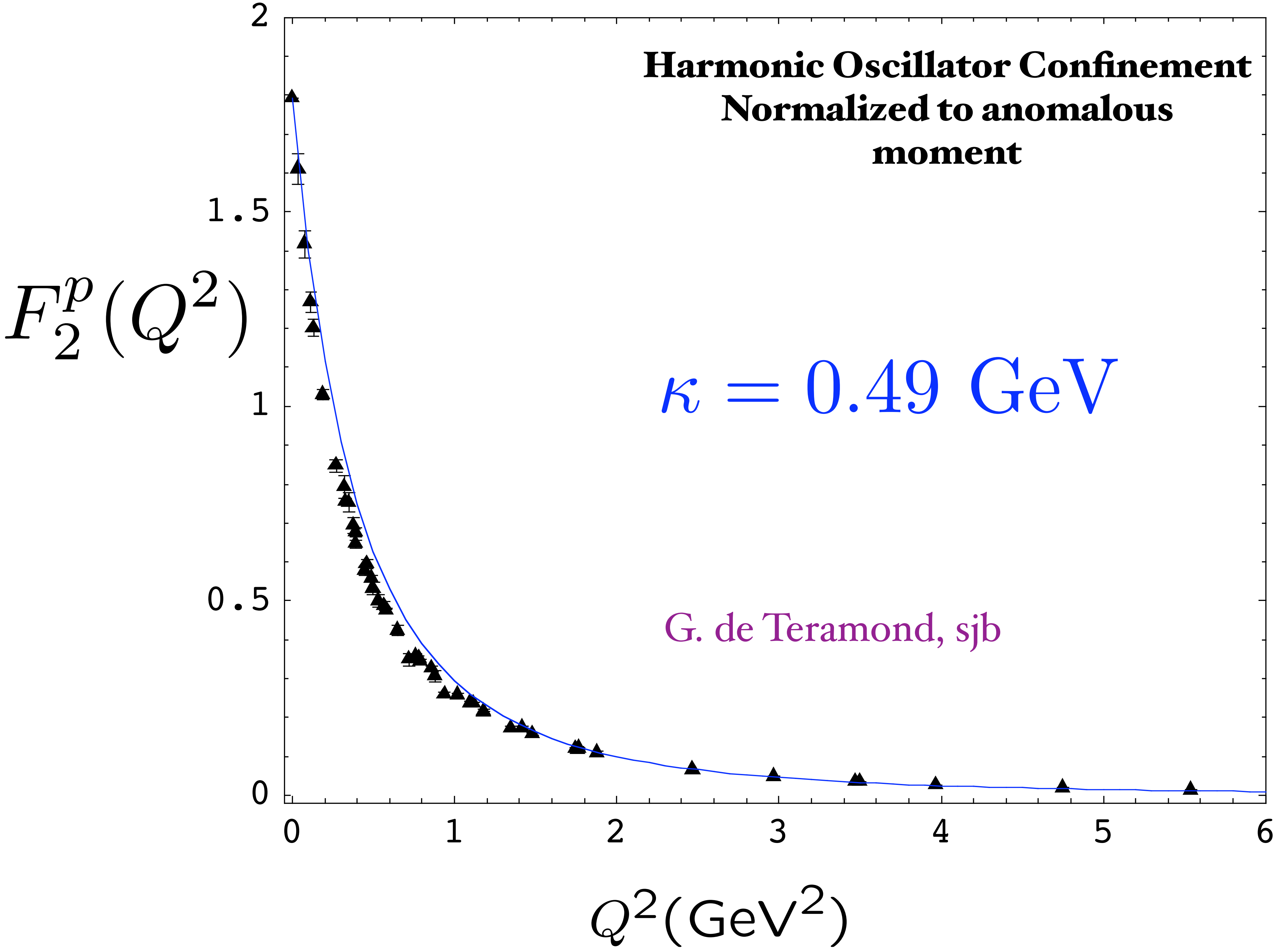


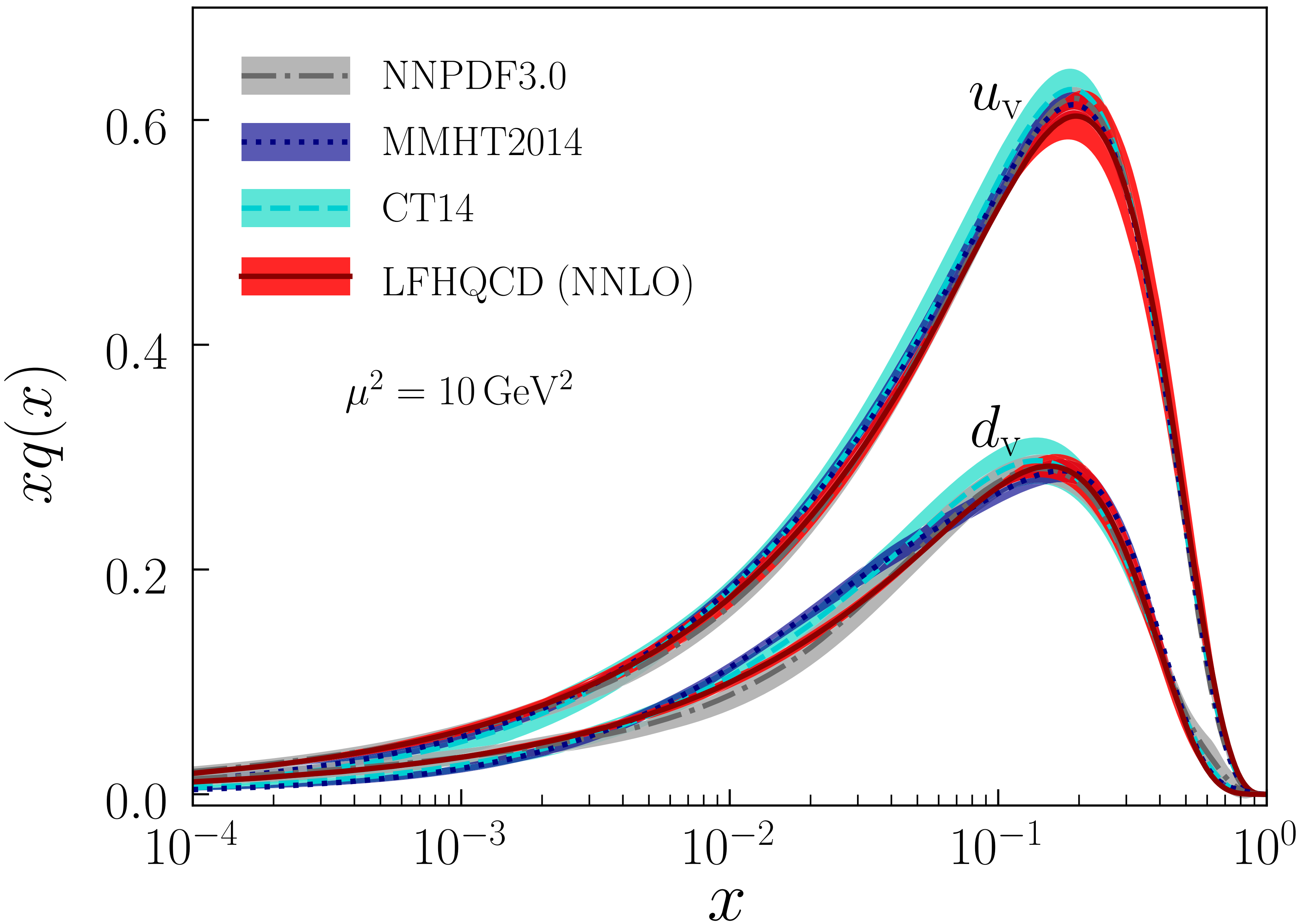
Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs





Comparison for $xq(x)$ in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

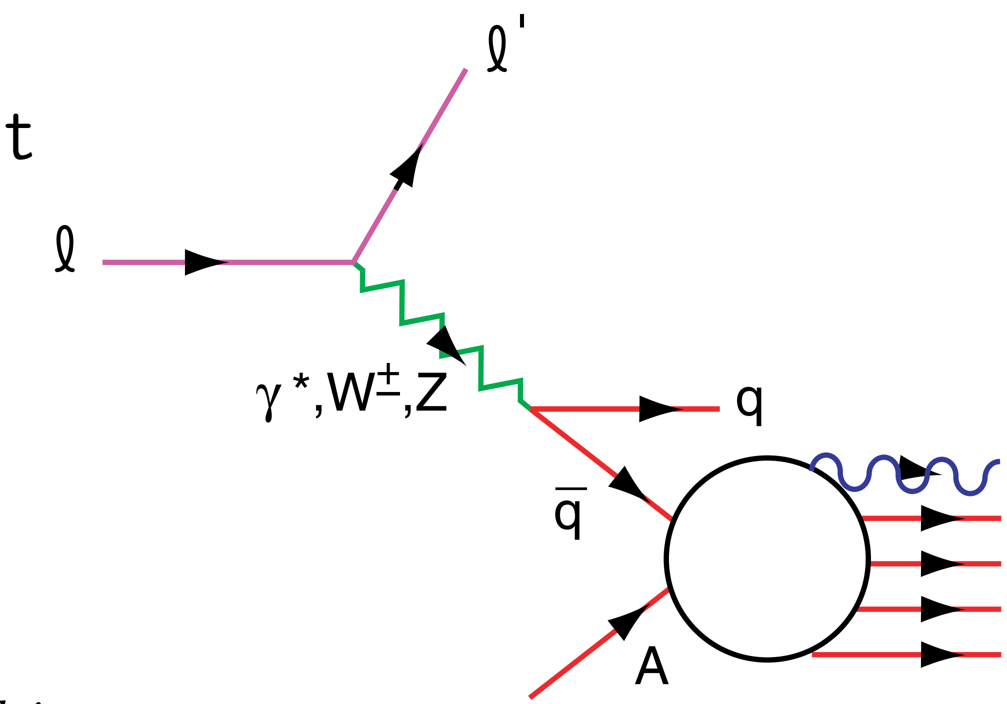
Guy F. de Te´ramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur

PHYSICAL REVIEW LETTERS 120, 182001 (2018)

Origin of Regge Behavior of Deep Inelastic Structure Functions

$$F_{2p}(x) - F_{2n}(x) \propto x^{1/2}$$

Antiquark interacts with target nucleus at energy $\widehat{s} \propto \frac{1}{x_{bj}}$



Regge contribution: $\sigma_{\bar{q}N} \sim \widehat{s}^{\alpha_R - 1}$

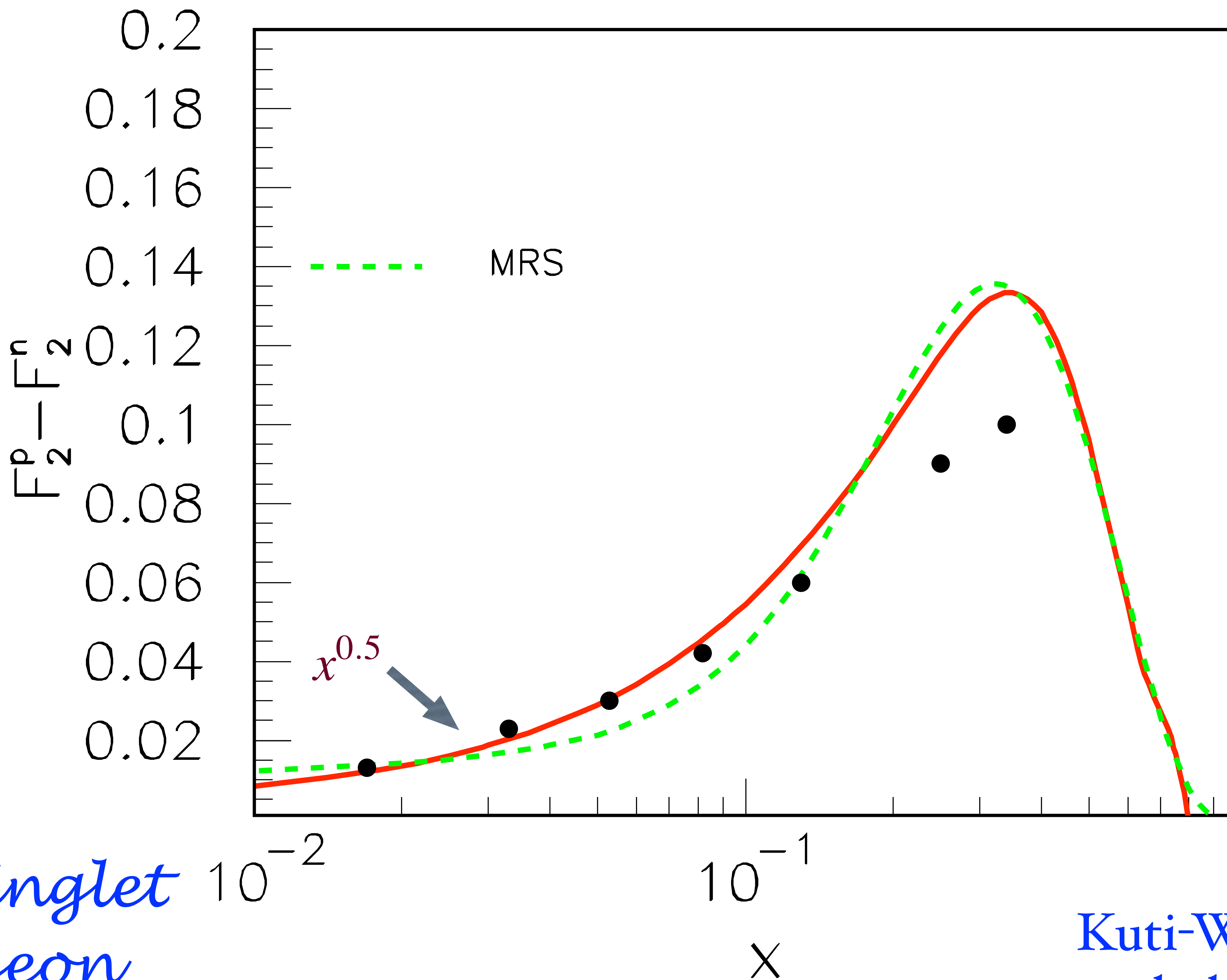
Nonsinglet Kuti-Weisskoff $F_{2p} - F_{2n} \propto \sqrt{x}_{bj}$ at small x_{bj}.

Shadowing of $\sigma_{\bar{q}M}$ produces shadowing of nuclear structure function.

Landshoff, Polkinghorne, Short

Close, Gunion, sjb

Schmidt, Yang, Lu, sjb



Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

Stan Brodsky

UNIVERSITY OF MARYLAND

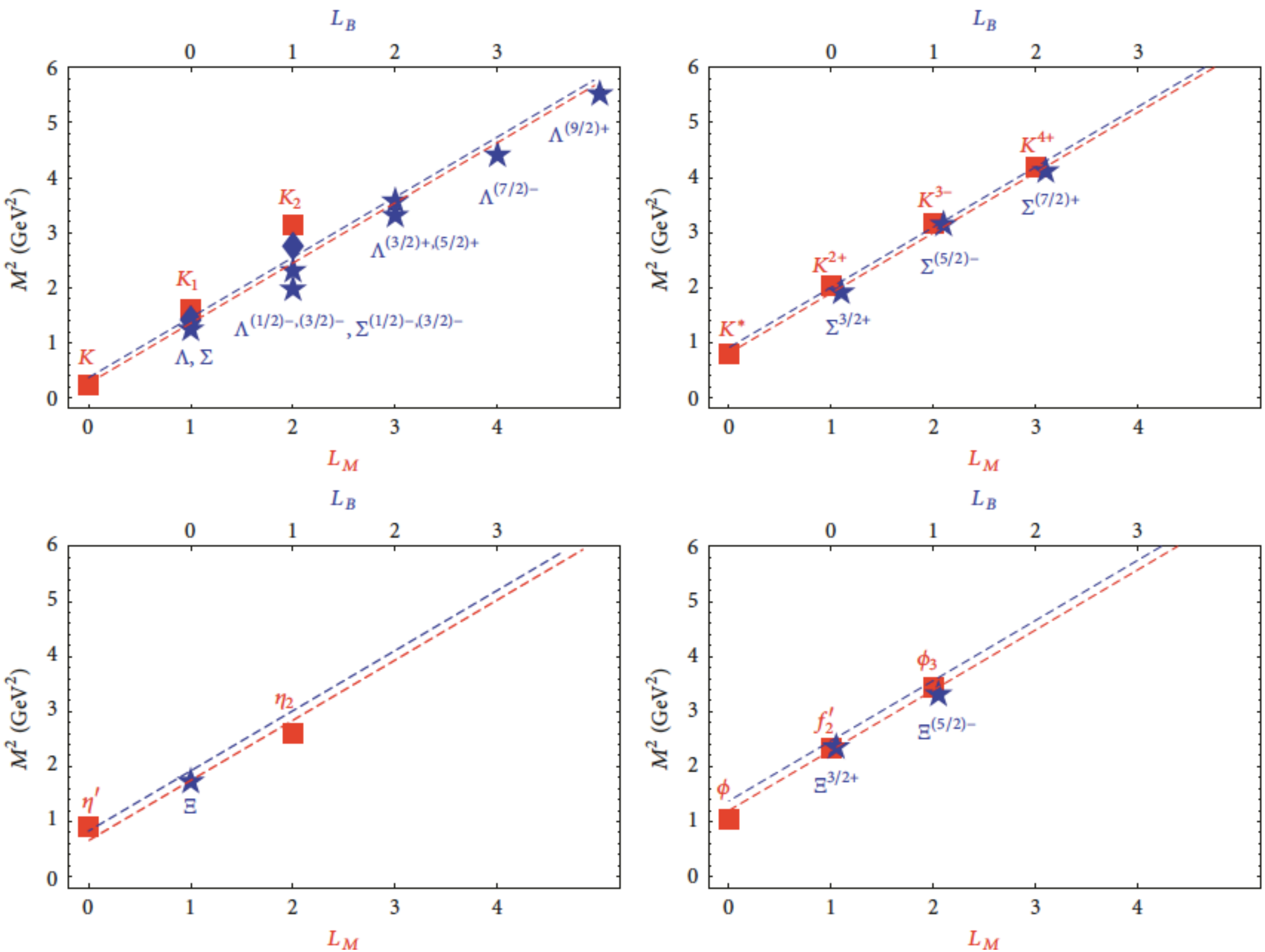
SLAC NATIONAL ACCELERATOR LABORATORY

Supersymmetric Features of Hadron Physics from Superconformal Algebra and Light-Front Holography

28 May 2020

de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum

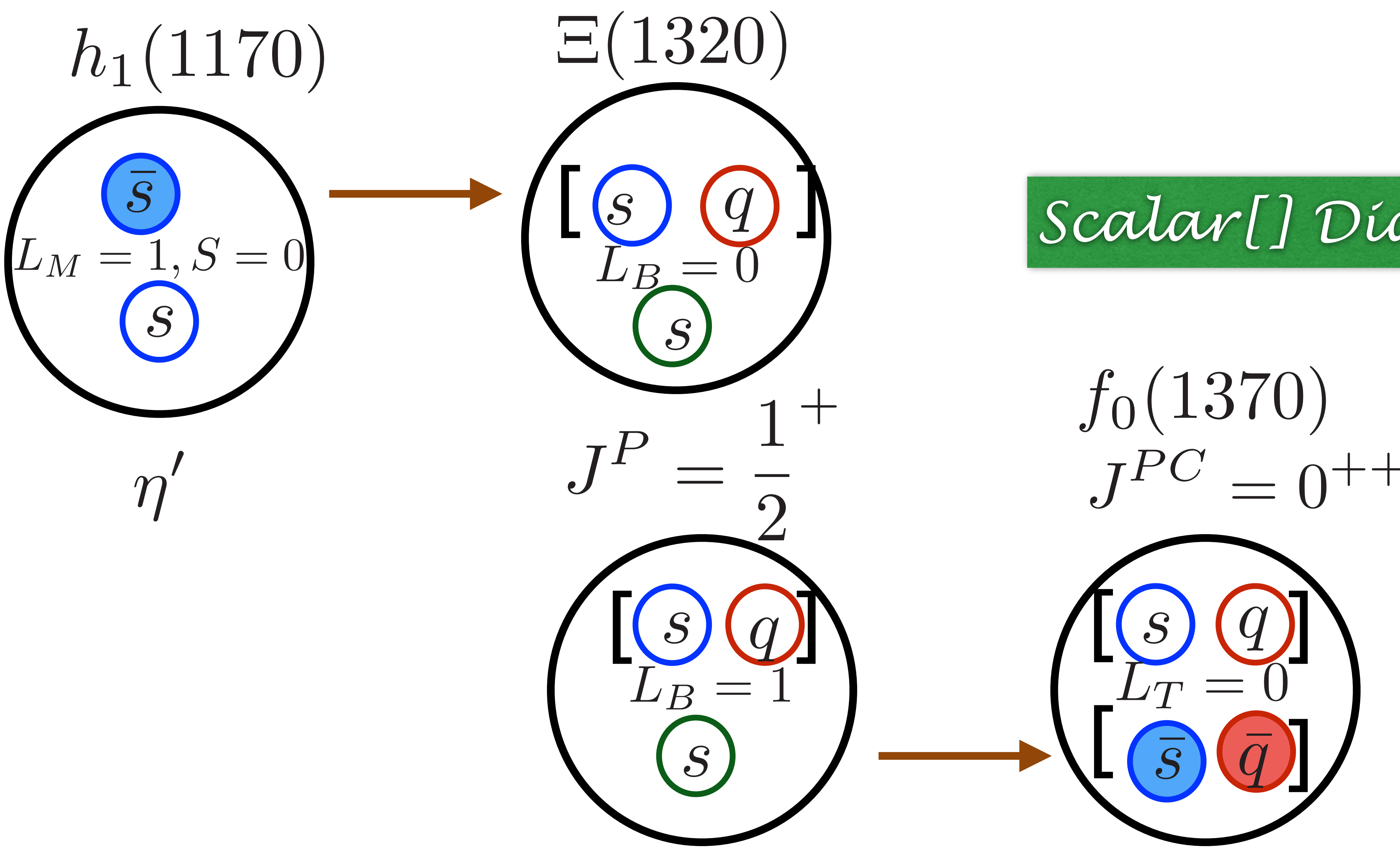


Double-Strange Baryon

$$R_\lambda^\dagger \; \bar{q} \to [qq]$$

$$\bar{3}_C \to \bar{3}_C$$

Scalar[] Diquarks

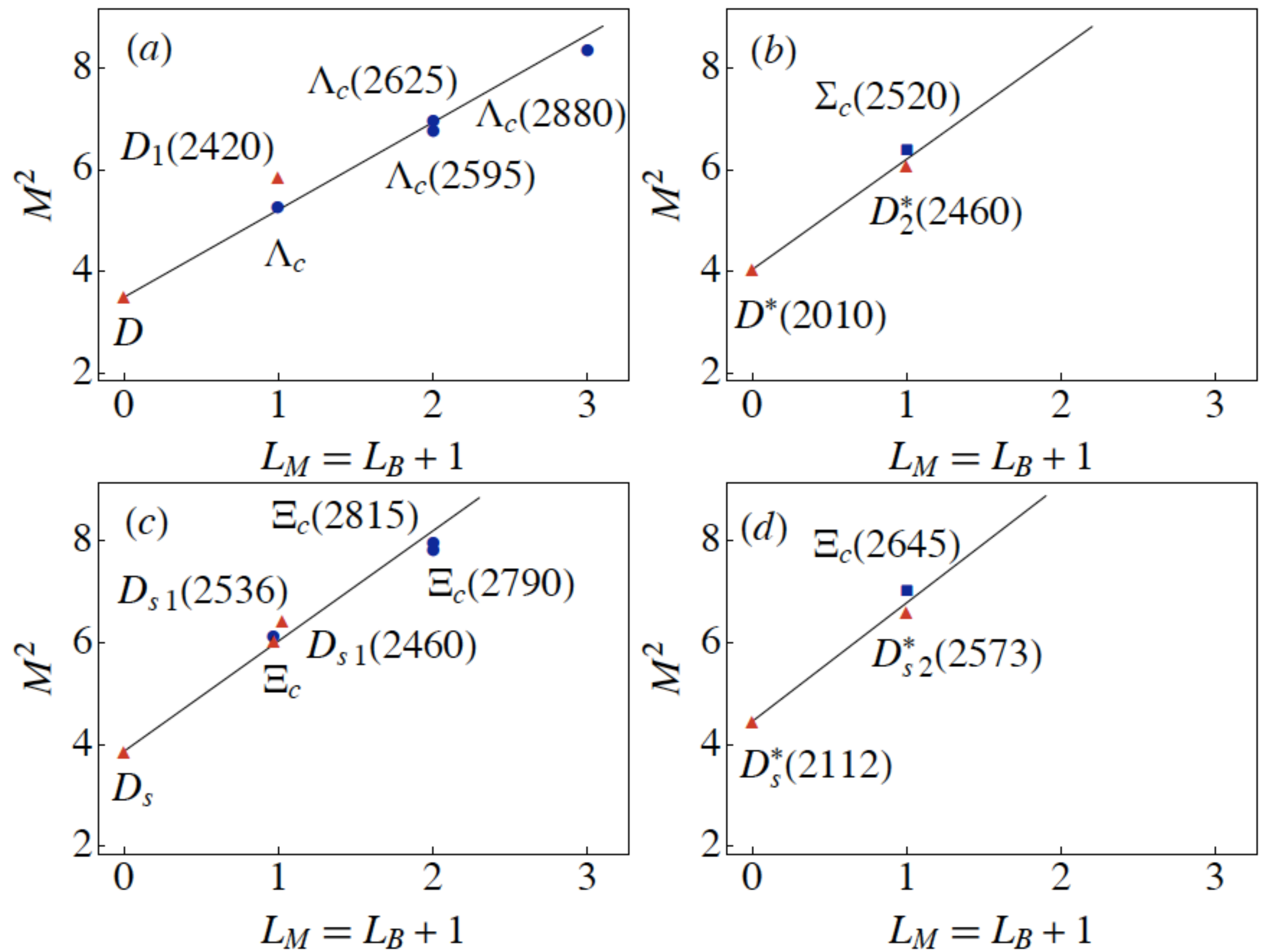


$$R_\lambda^\dagger \; q \to [\bar{q}\bar{q}] \qquad S = 0$$

$$3_C \to 3_C$$

de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

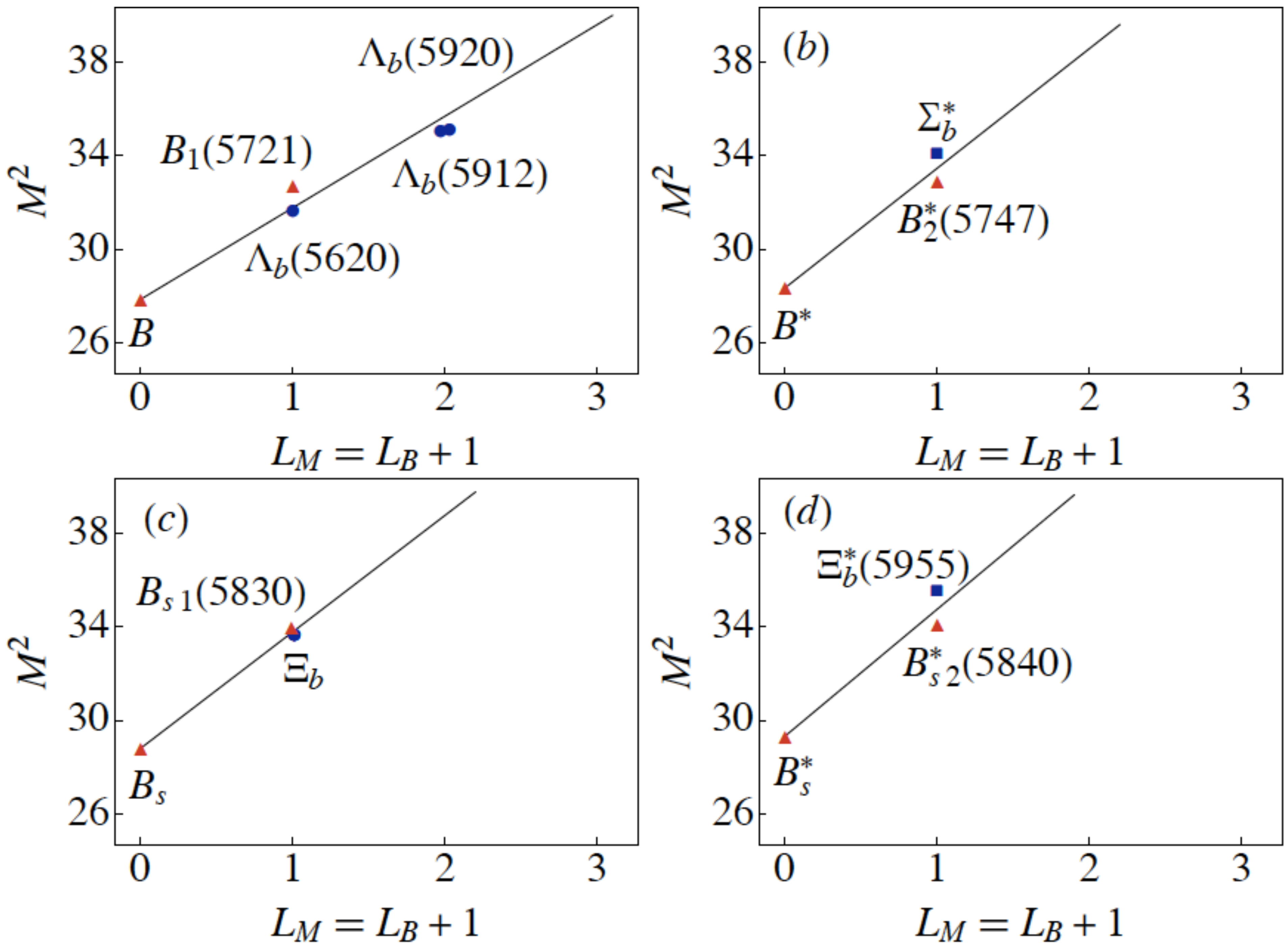
predictions

beautiful agreement!

M. Nielsen, sjb

de Téramond, Dosch, Lorcé, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

Heavy-light and heavy-heavy hadronic sectors

- Extension to the heavy-light hadronic sector

  [H. G. Dosch, GdT, S. J. Brodsky, PRD **92**, 074010 (2015), PRD **95**, 034016 (2017)]

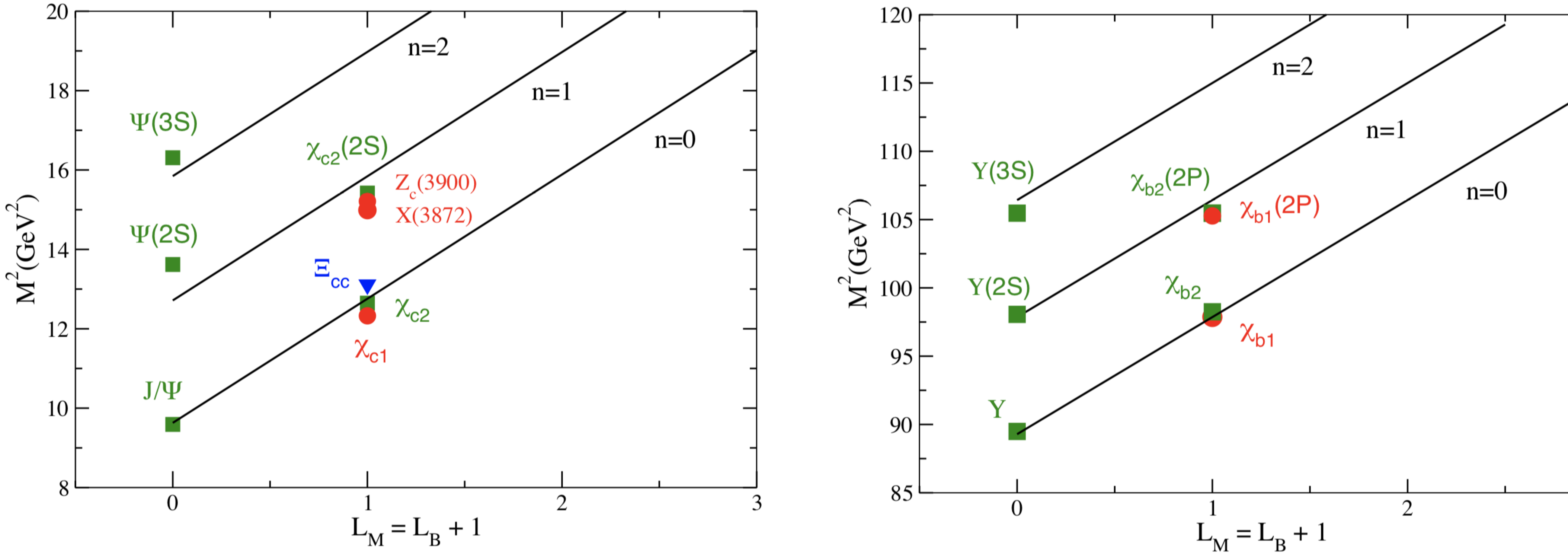
- Extension to the double-heavy hadronic sector

  [M. Nielsen and S. J. Brodsky, PRD, 114001 (2018)]

  [M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch, F. S. Navarra, L. Zou, PRD **98**, 034002 (2018)]

- Extension to the isoscalar hadronic sector

  [L. Zou, H. G. Dosch, GdT,S. J. Brodsky, arXiv:1901.11205 [hep-ph]]

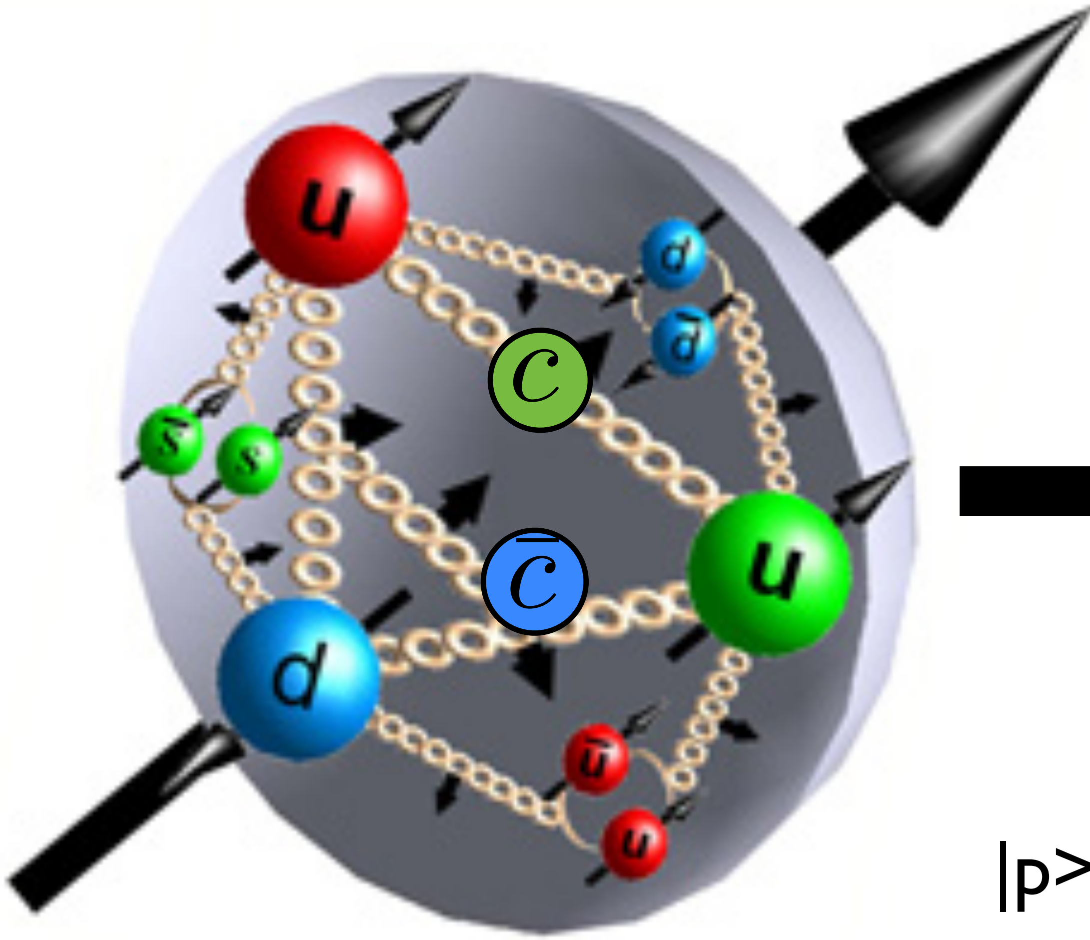


Supersymmetry in QCD

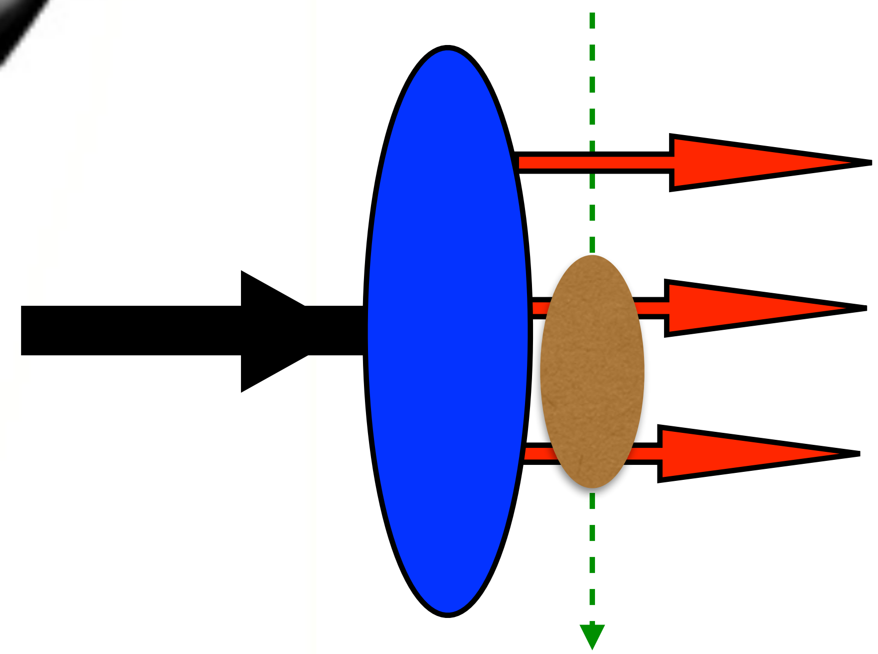
- A hidden symmetry of Color $SU(3)_C$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

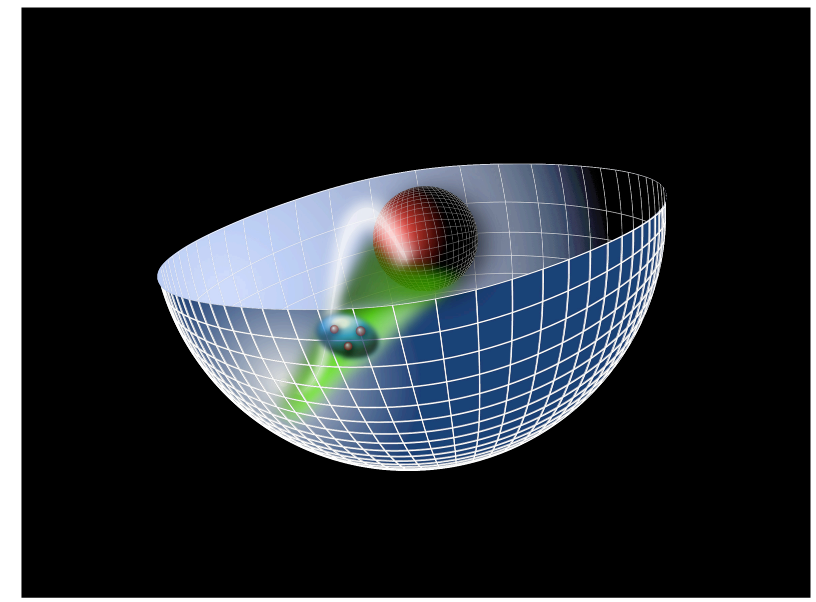
de Téramond, Dosch, Lorcé, sjb

Color Confinement and Supersymmetric Features of Hadron Physics from Light-Front Holography



Fixed $\tau = t + z/c$





|p>=|u[ud]> quark-diquark cluster

with Guy de Tèramond, Hans Günter Dosch, Alexandre Deur, Marina Nielsen, Ivan Schmidt, F. Navarra, Jennifer Rittenhouse West, G. Miller, Keh-Fei Liu, Tianbo LIu, Liping Zou, S. Groote, Joshua Erlich, S. Koshkarev, Xing-Gang Wu, Sheng-Quan Wang, Cedric Lorcè, R. S. Sufian, R. Vogt, G. Lykasov, S. Gardner, S. Liuti

Bled Workshop

What Comes Beyond the Standard Models?

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

July 6, 2021