

XXIV Workshop
“What Comes Beyond the Standard Models?”
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Galactic model with a phase transition
from dark matter to dark energy

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Thanks for hosting: Virtual Institute of Astroparticle physics

Content

Previous work

- Planck stars and Planck cores
- RDM-stars: fast radio bursts, galactic rotation curves
- 5 hypotheses on composition of astrophysical dark matter

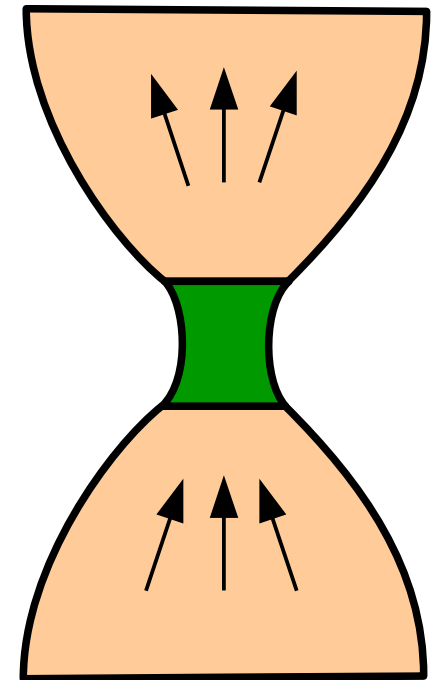
Review in Proc. of 2020 Bled Workshop "What Comes Beyond Standard Models", arXiv: 2102.07769; Lecture COSMOVIA July 5, 2019

New results

- Joining RDM galactic model with the cosmological background
- 4 rejected scenarios, demonstrating various problems for this joining
- 4 accepted scenarios, including phase transitions from DM to DE

Planck stars and Planck cores

- calculations in quantum gravity (QG) for a cosmological model with a scalar field (Ashtekar et al. 2006) give a correction to the mass density: $\rho_x = \rho (1 - \rho/\rho_c)$, $\rho_c \sim \rho_P$
- $\rho = \rho_c \Rightarrow \rho_x = 0$ at critical density the gravity is switched off
- $\rho > \rho_c \Rightarrow \rho_x < 0$ in excess of critical density the effective negative mass appears (**exotic matter**), with gravitational repulsion (a quantum bounce phenomenon)
- Planck star model: Rovelli, Vidotto (2014), Barceló et al. (2015): collapse of a star replaced by extension, black hole turns white
- in this talk we consider a stationary version of Planck star, stabilized under the pressure of the external matter (a Planck core)



Planck star

General remark on negative masses

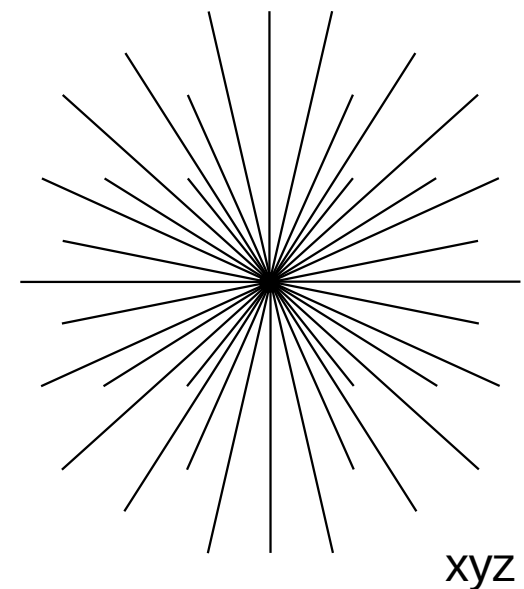
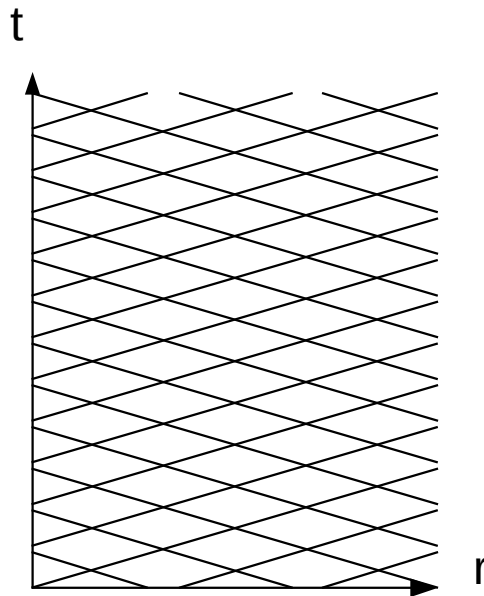
Evolution of viewpoint:

- Energy conditions (Einstein, Hawking): there are **NO** negative masses
- 't Hooft (1985): "... negative mass solutions unattractive to work with but ***perhaps they cannot be completely excluded.***"
- negative masses ***are needed*** to create the wormholes (Morris-Thorne 1988), warp drives (Alcubierre 1994), time machines (Visser 1996)
- Barceló, Visser (2002), ***Twilight for the energy conditions?***
- Rovelli, Vidotto (2014), Barceló et al. (2015): negative masses ***can be obtained*** effectively by an excess of Planck density
- alternative way: Tippett, Tsang (2017), many interesting solutions can be obtained with non-exotic matter in ***f(R)-gravity*** (examples are wormholes and accelerating cosmology)
- ...

Dark Stars

- also known as ***quasi black holes***, boson stars, gravastars, fuzzballs ...
- solutions of general theory of relativity, which first follow Schwarzschild profile and then *are modified*
- outside are similar to black holes, inside are constructed differently (depending on the model of matter used)
- review of the models: Visser et al., ***Small, dark, and heavy: But is it a black hole?***, arXiv: 0902.0346
- our contribution to this family: *RDM stars* (quasi black holes coupled to Radial Dark Matter)

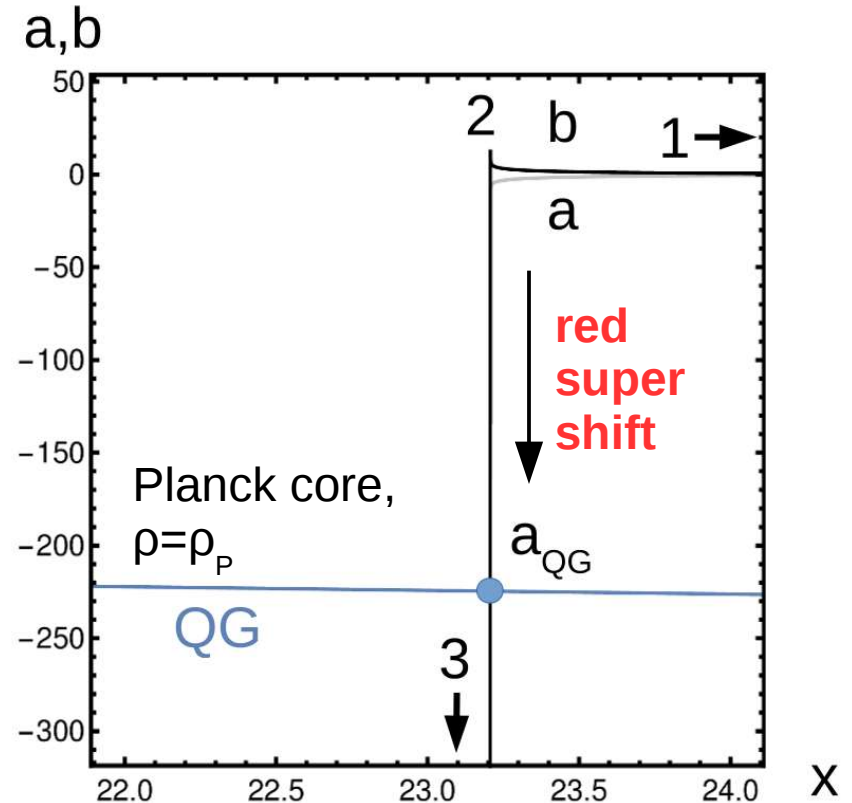
Stationary solution,
including T-symmetric
superposition of ingoing
and outgoing radially
directed flows of dark matter



RDM-stars, interior

computation in strong fields shows:

- event horizon is erased, replaced by a deep gravitational well (red supershift)
- density is rapidly increasing towards the center (mass inflation phenomenon)
- Planck density is reached, the central region is replaced with Planck core
- large negative mass of the core is compensated by large positive mass of surrounding matter
- from outside, the object looks like a moderate mass (quasi)BH



calculation for the Milky Way galaxy

$$x = \log r, \quad a = \log A, \quad b = \log B$$

(logarithms of metric profiles)

RDM-stars, interior

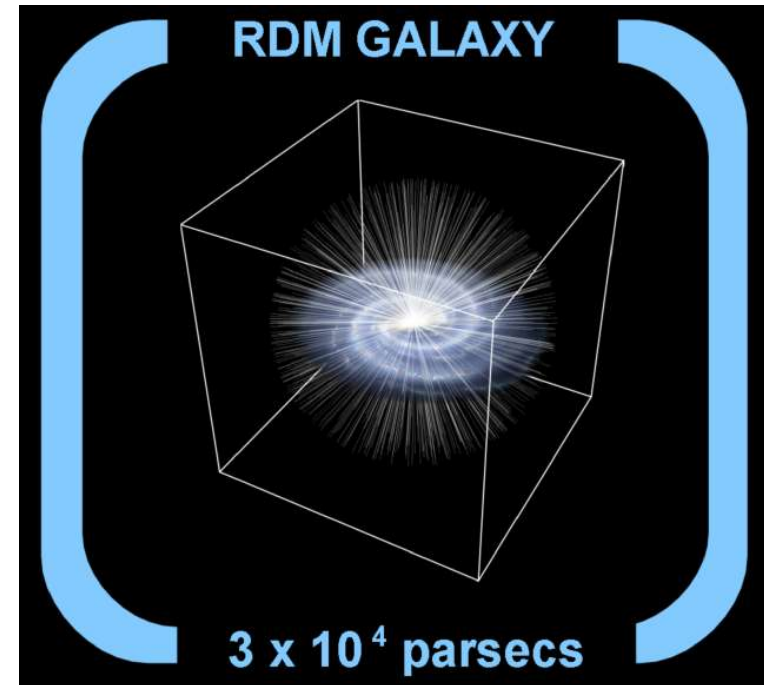
observational consequences:

- the depth of the well: redshift factor $z \sim 10^{49}$ is reached in scenario with central BH in MW galaxy
- on the core surface, Planck regime is assumed: one Planck energy particle emitted from Planck area in Planck time
- Planck energy particles from the core are redshifted to $\lambda \sim 10^{14} \text{m}$ outside
- although low-energetic, the particles come in densities sufficient to explain the hidden mass and rotation curves of the galaxies (see below)
- particles that can escape the gravitational well in this scenario:
massive ultralight (Compton $\lambda > 10^{14} \text{m}$, mass $< 10^{-20} \text{eV}$, axion-alike), massless (Standard Model photons, gravitons, or extensions), tachyons (theoretically)
- an external object (asteroid) falling onto an RDM-star produces a photonic flash redshifted to $\lambda \sim 1 \text{m}$ that can be identified with FRB (note: the formula for wavelength depends on RDM-star mass, mass of nuclei composing the asteroid and the local density of dark matter, the latter fixed by rotation curves of the galaxy)

RDM-stars, exterior

computation in weak fields shows:

- gravitating density following geometric profile
 $\rho_{\text{grav}} \sim 1/r^2$
- composed of mass density and radial pressure:
 $\rho_{\text{grav}} = \rho + p_r$, both following such profile
- important: transverse pressure vanishes $p_t = 0$
- galactic model in a single center approximation has flat rotation curve: $M \sim r$, $v^2 = GM/r = \text{const}$
- this simplified model assumes that the whole dark matter in the galaxy is coupled to the central black hole, considered as RDM-star
- alternatively, it is coupled to a large number of black holes in the galactic nuclei
- alternatively, it is coupled to all black holes in the galaxy, but considered at large distance from the center, so that the whole galaxy can be treated as a single unresolved point

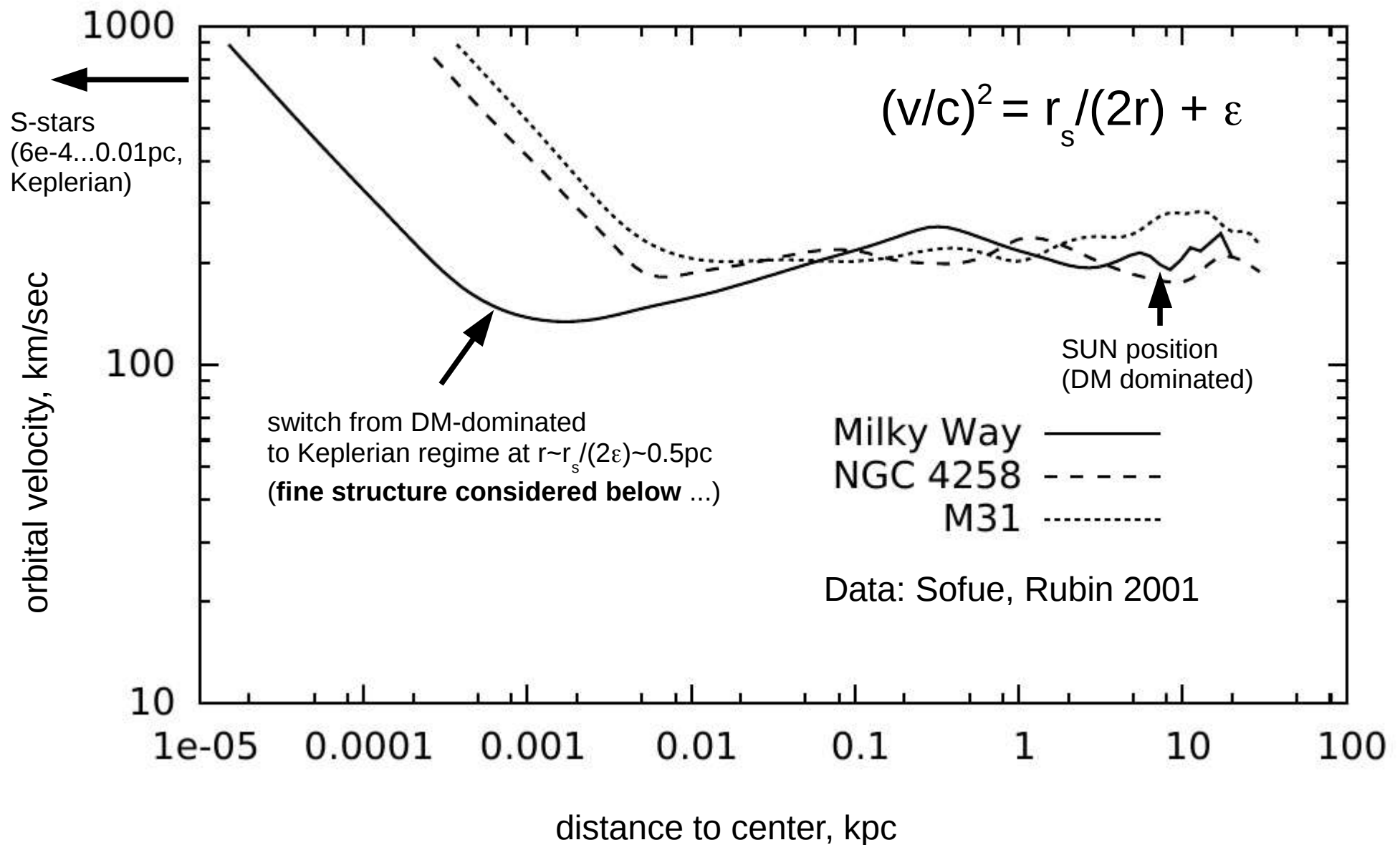


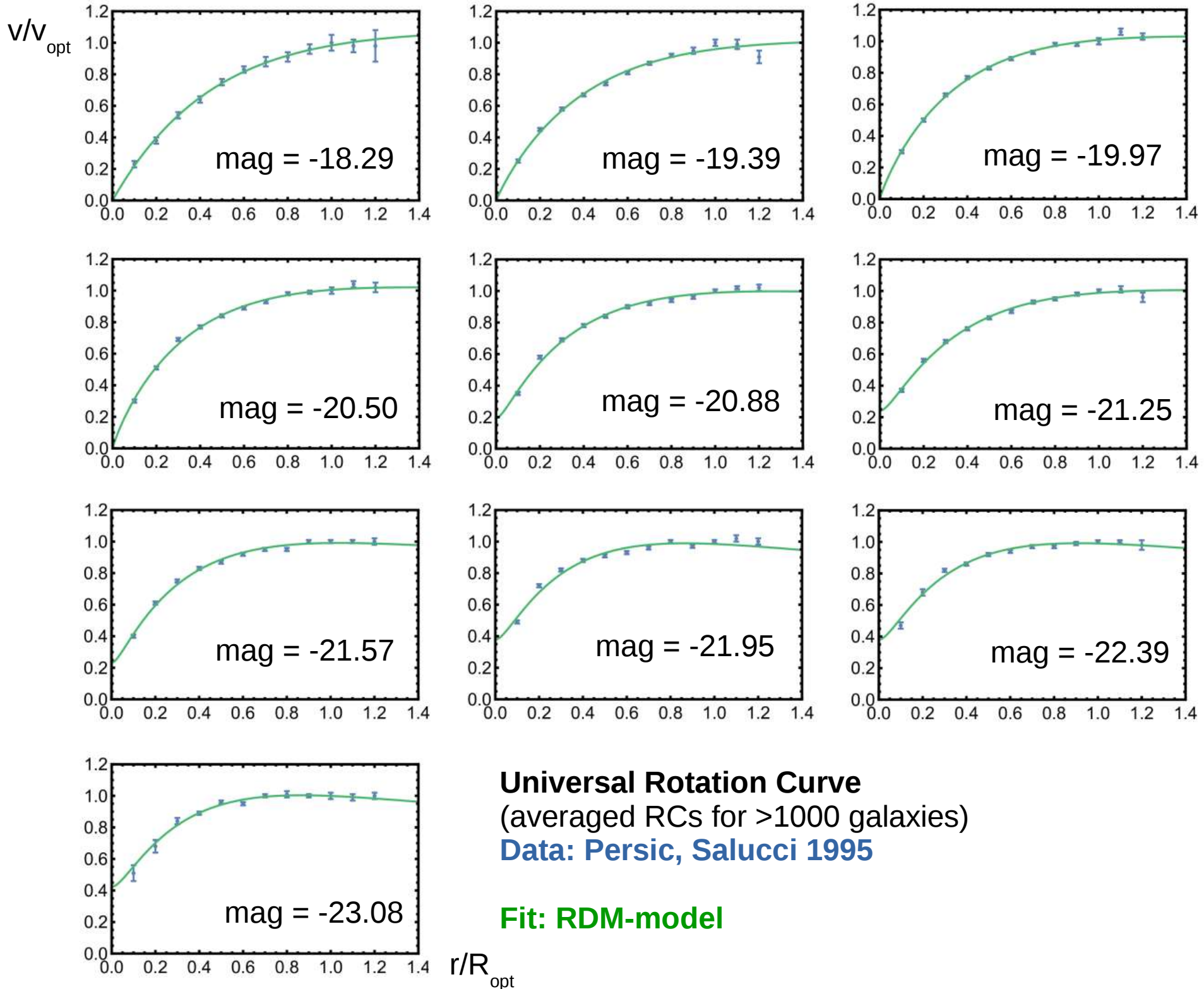
RDM-stars, exterior

difference from a standard halo model (isothermal sphere):

- in RDM-model the rotation curves (the shape, the amplitude) do not depend on DM type (M/N/T - massive, null, tachyonic << considered for completeness)
- isothermal sphere model has a different EOS with isotropic pressure $p_t = p_r$, computation shows that relativistic DM particles produce also relativistic rotation curves, excluded by observation => only cold dark matter remains possible
- in RDM-model all M/N/T cases are possible

Typical behaviour of galactic rotation curves (RDM in single center approximation, Kepler + constant)

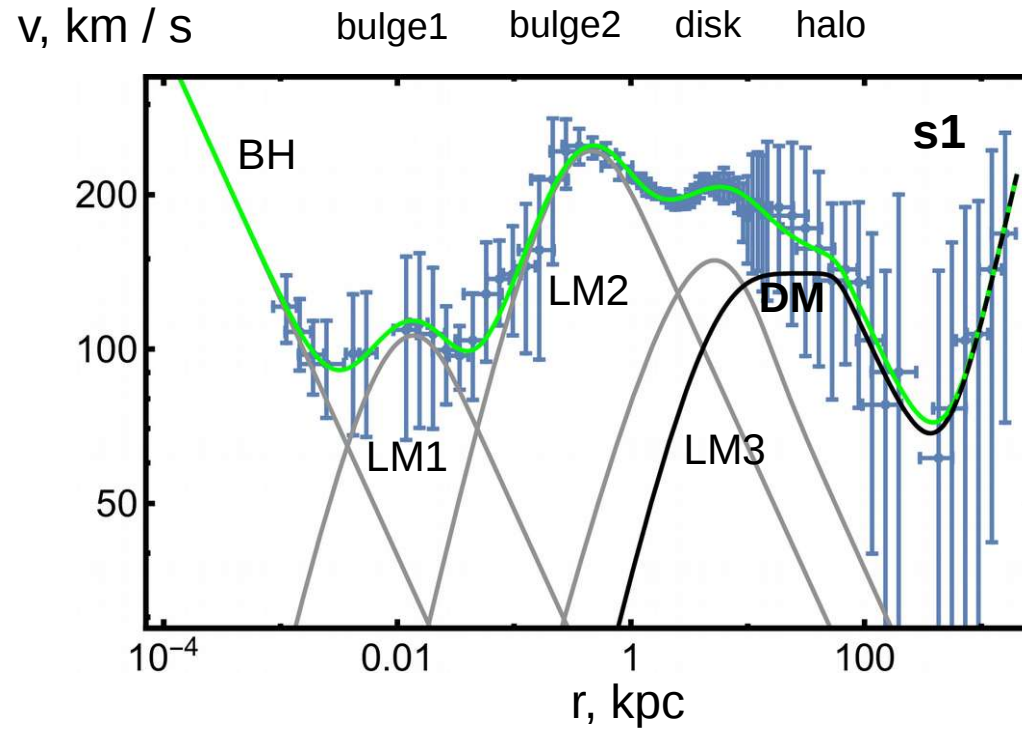




Universal Rotation Curve
 (averaged RCs for >1000 galaxies)
Data: Persic, Salucci 1995

Fit: RDM-model

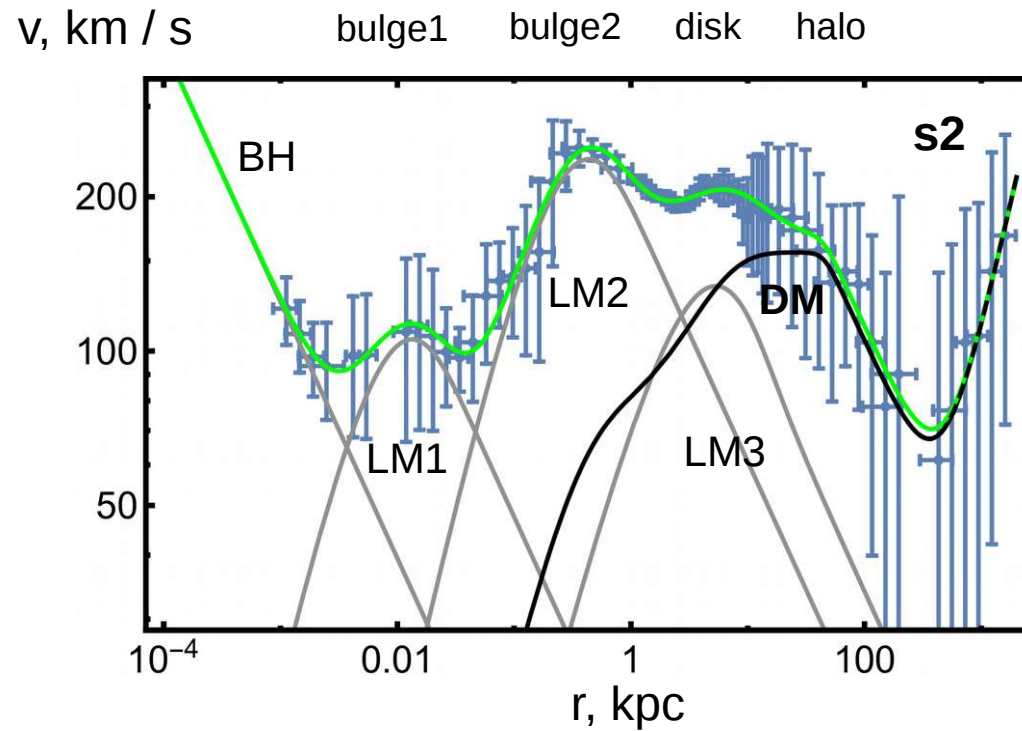
Rotation curve of Milky Way



Data: Sofue et al. 2009-2013

Fit: RDM-model
(different coupling constants)

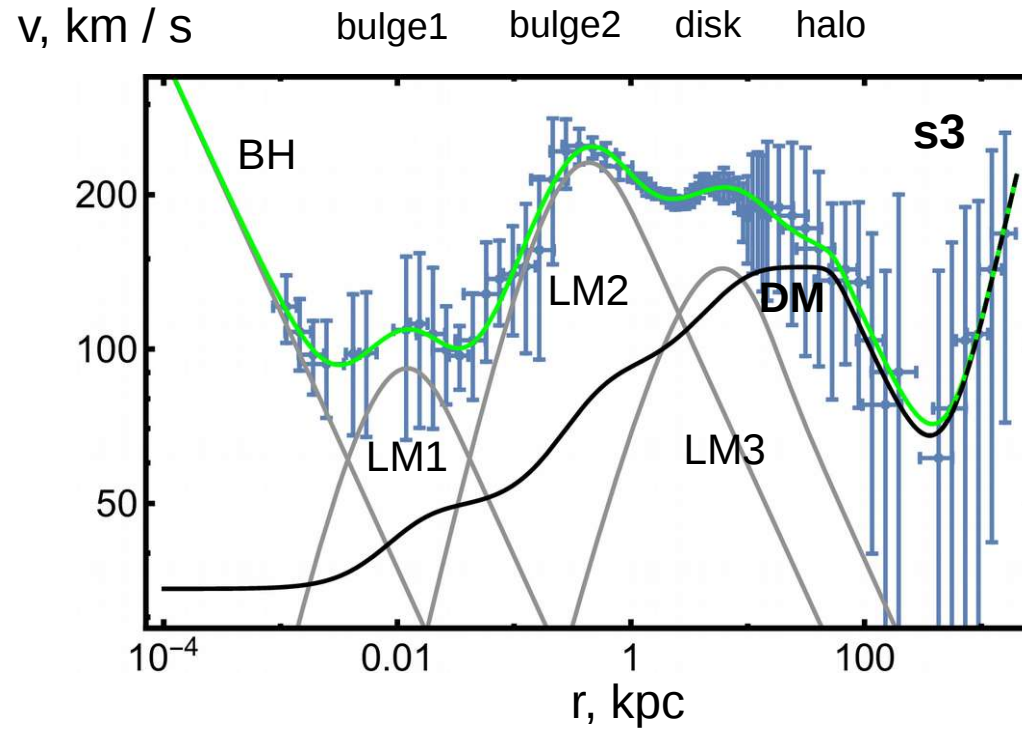
Rotation curve of Milky Way



Data: Sofue et al. 2009-2013

Fit: RDM-model
(different coupling constants)

Rotation curve of Milky Way



Data: Sofue et al. 2009-2013

Fit: RDM-model
(different coupling constants)

Rotation curve of Milky Way

GRC: fitting results, central values of parameters*

coupling constants,
regulate DM coupling
to various structures

λ_{KT}	s1	s2	s3
λ_{bh}	0	1	10^3
λ_1	0	1	10^2
λ_2	0	1	2
λ_{disk}	1	1	1

<i>par</i>	s1	s2	s3
M_{bh}	3.6×10^6	3.6×10^6	3.2×10^6
M_1	5.5×10^7	5.2×10^7	3.6×10^7
a_1	0.0041	0.0039	0.0036
M_2	9.7×10^9	8.6×10^9	8.2×10^9
a_2	0.13	0.13	0.13
M_{disk}	3.2×10^{10}	2.7×10^{10}	3.5×10^{10}
R_D	2.4	2.5	2.8
L_{KT}	7.0	6.3	12.0
r_{cut}	58	45	53
$M_{dm}(r_{cut})$	2.7×10^{11}	2.5×10^{11}	2.6×10^{11}
ρ_0	646	653	649

approx
equal
for all
scenarios

* masses in M_\odot , lengths in kpc , density in M_\odot/kpc^3

$$\varepsilon = (v/c)^2 |dm = G M_{dm}(r_{cut}) / (r_{cut} c^2) \sim 2.5 \cdot 10^{-7}$$

5 DM Hypotheses

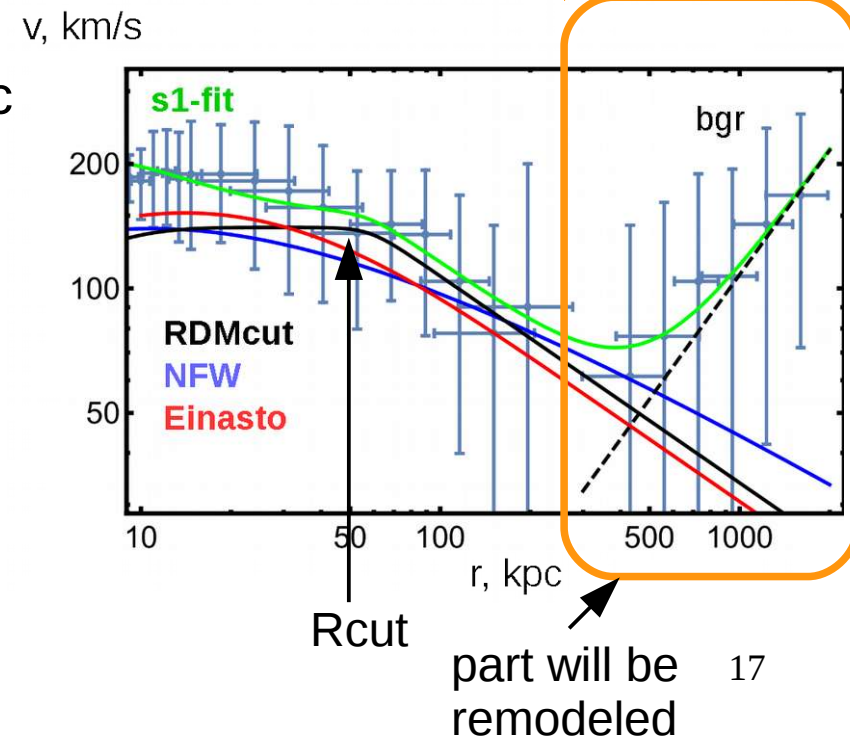
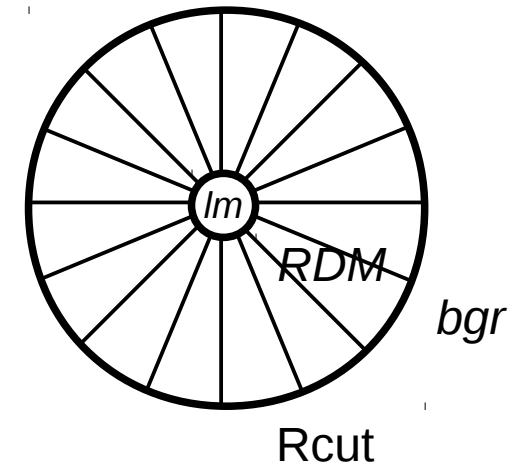
Hypothesis1: galactic DM can be cold, hot, or tachyonic, producing *the same rotation curves*

- cold = massive non-relativistic, standard case, in the considered scenario only possible if the initial energy \gg Planck one, to escape gravitational well; the energy should be fine-tuned to provide non-relativistic behavior outside; will not be considered here
- tachyonic case is yet too exotic, also will not be considered here
- hot = massive ultralight or massless cases are close to each other, will be considered here as null case (NRDM)

5 DM Hypotheses

Hypothesis2: (cut&paste approach) galactic DM of any nature stitched at R_{cut} to cold cosmological DM or other background

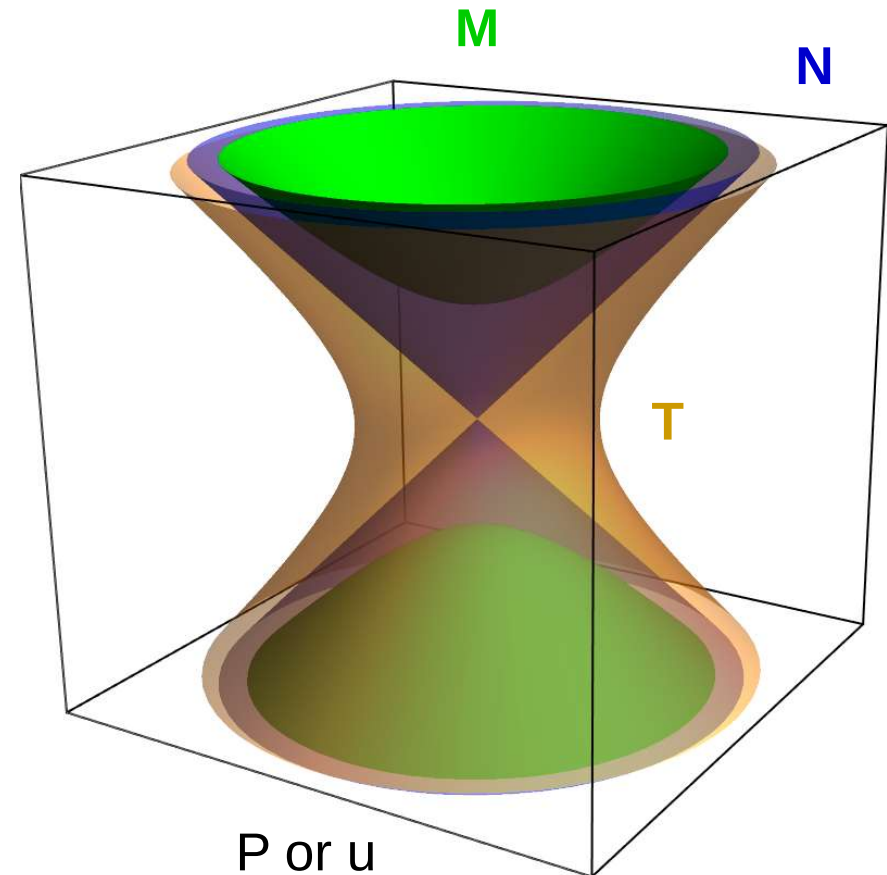
- caused by a self-interaction of DM at R_{cut} limit
- similar to a termination shock on the border of the solar system, where the radially directed solar wind meets the uniform interstellar medium
- MW RC fit with RDMcut model gives $R_{\text{cut}} \sim 50 \text{ kpc}$ (although does not distinguish between different models due to high scatter in the outer region)
- this case will be considered here in various scenarios



5 DM Hypotheses

Hypothesis3: emission of galactic DM from Planck core can be *acausal*

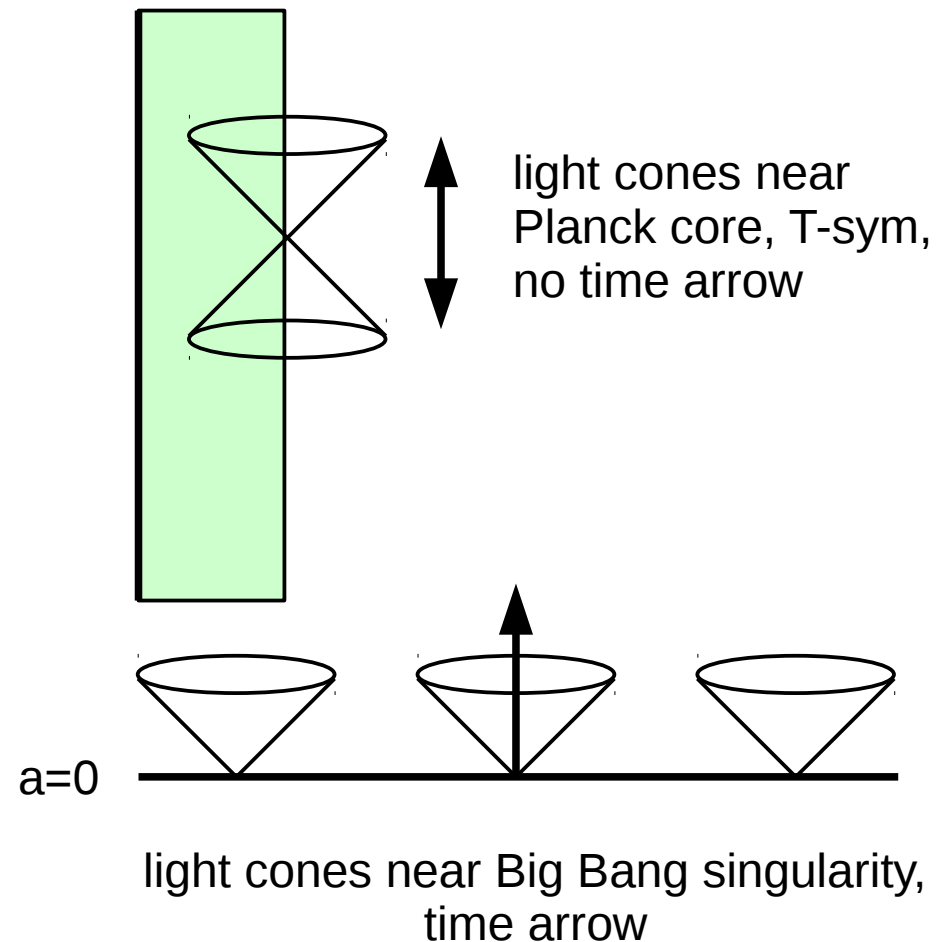
- RDM star contains two Tsym flows, ingoing and outgoing
- sterile DM, no interaction with the rest of the world (except of non-local gravitational and local high temperature at Planck core)
- can have decoupled thermodynamics, with other time arrow or absence of it (T-sym thermodyn., max entropy, equilibr. state)
- mass shells:
 - one-sheet tachyonic, contains both ingoing and outgoing directions
 - two-sheet massive/null, T-sym occupied



Remark: $P_0 < 0$ corresponds to T-conj flow of the same particles as $P_0 > 0$
 $A = m \int dt |x'_\mu x'^{\mu}|^{1/2}$ and $T^{\mu\nu} = \rho u^\mu u^\nu$ are invariant under T-reflection

5 DM Hypotheses

- Planck core temperature conditions are similar to Big Bang
- with a difference that RDM singularity and Planck core are timelike, while Big Bang singularity is spacelike
- different orientation of light cones can lead to the absence of time arrow (recovered T-sym) near Planck core and its presence near/after Big Bang



5 DM Hypotheses

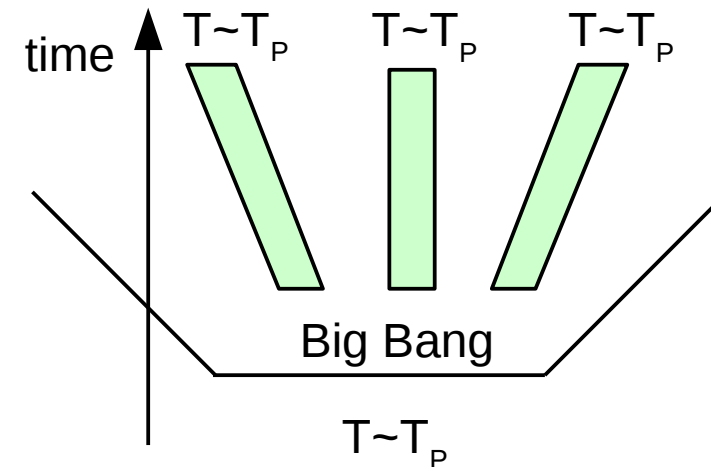
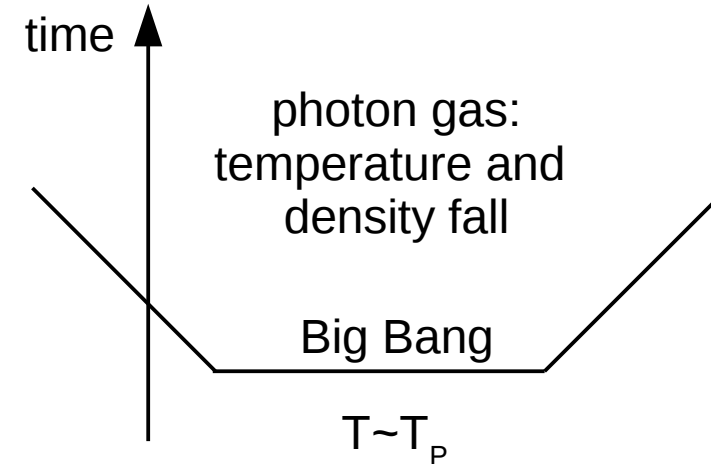
Hypothesis4: sterile DM vs normal matter in *unusual condition*

- sterile DM: new type of particles not interacting with the known ones (except of gravitational and Planck temperature interaction)
- alternative are known massless particles (photons, gravitons...)
with extremely large wavelength, $\lambda_{\text{out}} \sim 10^{14} \text{m} \sim 4 \text{ light days} \sim 16 \times |\text{Sun-Pluto}|$
- such longwave particles are not registered by usual means
- they come in density corresponding to the measured halo mass
- here we will concentrate on generic case of sterile massless particles
- the question whether DM particles can be real longwave photons, will be considered separately

5 DM Hypotheses

Hypothesis 5: cosmological DM *mimics* CDM

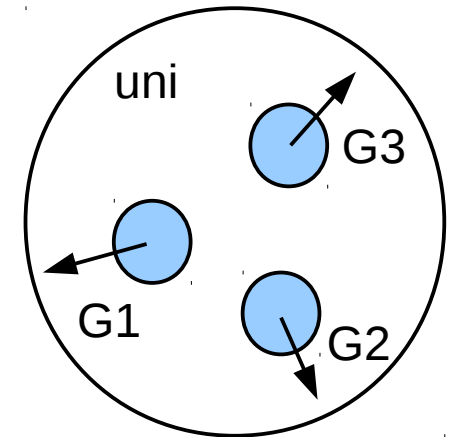
- consider FLRW-cosmology, evolution of uniform photon gas: initial flash, then temperature and density fall in expanding universe
- for CDM only density falls
- differences: RDM model is non-uniform
- thermalization: RDM-stars continuously absorb&inject energy, possess constant $T \sim T_p$
- if DM has a constant T , then in long-range evolution it will behave like CDM



DM: Planck cores support constant local temperature, density falls

5 DM Hypotheses

- other mechanism: effective EOS of cosmological DM should not be identical to the galactic one
- Swiss cheese model: galaxies and their halos do not change their size and structure under cosmological expansion, move as a whole
- cosmological expansion acts only on the level where the matter distribution can be considered as uniform
- clustering: galaxies coated in massive halos can behave like macro-particles of CDM
- compare with balloons (Dyson spheres) filled with radiation, externally act like cold massive particles
- the described mechanisms will be considered further, in various scenarios



galactic vs cosmological DM:
hot inside, cold outside

New results

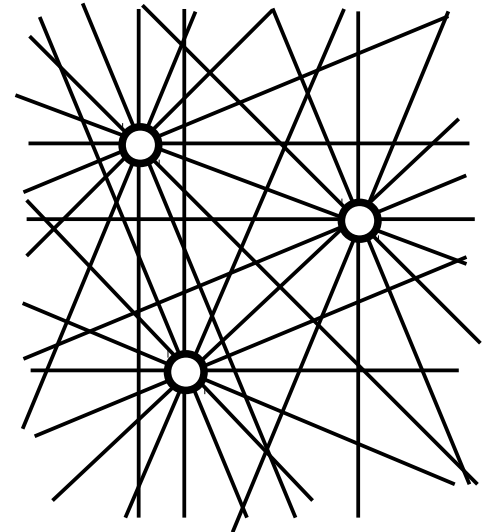
- at first, we present 4 rejected scenarios, to demonstrate certain non-trivial problems appearing in their construction
- then, we present 4 accepted scenarios
- the most successful ones involve dark energy as a background and perform **a phase transition** between interior DM and exterior DE
- for similar ideas see arXiv: 2012.01407, 1907.06353, 0912.1609, ...

- Aside note: many recent works 2010.10823, 2002.06127, 1804.08558, 1812.03540, 1907.12551, 2001.05103, 1908.04281, ... consider interaction between DM and DE as a source of **cosmological tensions**. The reason is that non-interacting DM and DE possess cosmological evolution with separate conservation laws, while interaction produces energy exchange leading to observable deviations from the standard cosmological model.
- In this talk, however, we will build scenarios completely equivalent to the standard model. Our purpose is to find mechanisms by which NRDM model can become equivalent to Λ CDM cosmology, while possible deviations from it can be considered in the next step.

Rejected scenarios

S0.1: superposition of galactic halos without cutting

- halos are extended till $R_{uni} \sim 14 \text{ Gpc}$ and superimposed additively
- scenario is equivalent to CDM: in cosmological expansion, the number density of galaxies decreases as a^{-3} , energy of photons on exit from every RDM-star is constant, energetic density is a^{-3} , like CDM
- scenario is **rejected** due to the following computation
- estimated number of galaxies in the universe $N_{gal} \sim 2 \cdot 10^{12}$
- assume now, for simplicity, that all galaxies have parameters of MW (corrections for true distribution of galaxies will be applied below)
- in RDM model: $M_{dm}(r) \sim r$, linearly increasing profile
- $R_{cut} \sim 50 \text{ kpc}$, $M_{dm}(R_{cut}) \sim 2.6 \cdot 10^{11} \text{ Msun} \ll$ from MW fit
- $M_{dm}(R_{uni}) \sim 7.3 \cdot 10^{16} \text{ Msun} \ll$ continued to R_{uni}
- $M_{dm} \sim 1.5 \cdot 10^{29} \text{ Msun} \ll$ multiplied to N_{gal}
- $M_{dm,uni} \sim 4.5 \cdot 10^{23} \text{ Msun} \ll$ cosmological estimation from $\Omega_{dm} \rho_{crit} = 2.7 \cdot 10^{-27} \text{ kg/m}^3$, **mismatch factor $\sim 3.2 \cdot 10^5$**



Rejected scenarios

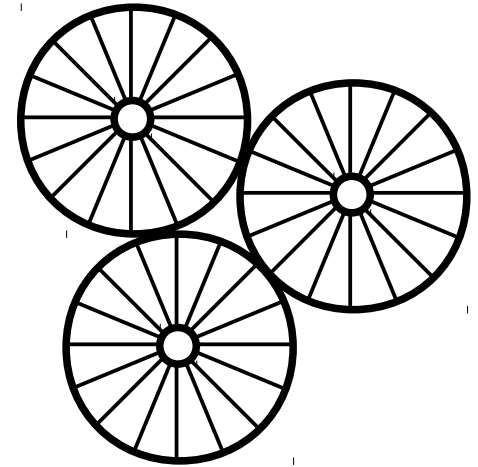
S0.1: superposition of galactic halos without cutting (cont'd)

- corrections to galaxies distribution and cosmological redshift introduce smaller factors and do not help
- if the cutting will be applied not at R_{uni} , but at some R_{gal} , then the exact match to cosmological value will be at $R_{gal} = 44\text{kpc} \sim R_{cut, MW}$
- in other words, if the universe would consist of $N_{gal} \sim 2 \cdot 10^{12}$ copies of MW, with DM halo cut at $R_{cut} \sim 50\text{kpc}$ and DM absent inbetween the galaxies, it will approximately satisfy the cosmological DM mass estimation
- the same computation with $R_{gal} = 1\text{Mpc}$ gives exact match for corrected $N_{gal}' = 8.7 \cdot 10^{10}$
- however, such "simple cut" solutions in NRDM model are not possible, since the radial pressure component $p_r \sim \rho$ on R_{cut} radius remains unbalanced

Rejected scenarios

S0.2: adjacent halos in dynamic equilibrium

- assume that the galaxies can exchange dark matter: null DM leaking from one galaxy is absorbed by neighboring galaxies, and vice versa
- world lines of DM form a network connecting galaxies, the concept of spherical halos is only an approximation
- equivalently, halos touch each other in the outer region, and the radial pressure p_r is balanced between the galaxies
- scenario is **rejected** due to the following reason
- in cosmological expansion, pressure forces develop a negative work $-p_r 4\pi r^2 dr = -\epsilon/2 \cdot dr$, where $p_r = \rho = \epsilon/(8\pi r^2)$, $M_{gal} = \epsilon/2 \cdot r \ll$ NRDM model
- by conservation of energy, the total mass-energy of the galaxy is reduced by this value $dM_{gal} = -\epsilon/2 \cdot dr \Rightarrow \epsilon \sim a^{-2}$, $r \sim a$, $M_{gal} \sim a^{-1} \ll$ just like a single photon!
- multiplied to num.density of galaxies $\sim a^{-3}$, this gives cosmological mass density $\sim a^{-4}$ (radiation epoch), for expansion rate „today“ \ll **contradiction**



Rejected scenarios

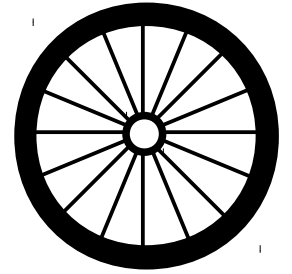
S0.2: adjacent halos in dynamic equilibrium (cont'd)

- equivalent consideration: DM particles from neighboring galaxies receive a small cosmological redshift, decreasing their energy and flux density by the corresponding factor
- we consider RDM stars in a stationary T-symmetric scenario, so the energy and flux density coincide for the incoming and outgoing flows
- therefore, the outgoing photons also have reduced energy and flux density
- with multiple reflections between galaxies, the redshift of photons accumulates, just as it would in a homogeneous environment
- RDM stars act as spherical mirrors that change the direction of the photons, but not their energy characteristics
- such an environment is equivalent to hot DM, its evolution coincides with the radiation epoch, different from the observed evolution of the universe today.

Rejected scenarios

S0.3: halo surrounded by a massive thin shell

- scenario of termination shock type
- this phenomenon occurs at the edge of the solar system when the radially directed solar wind meets the isotropic interstellar medium
- similar phenomena can occur with dark matter at the edge of the galaxy when the radial flow of dark matter meets the intergalactic background
- in this particular scenario, NRDM galaxy at radius R_{cut} is surrounded by a thin CDM layer, with a vacuum outside. The CDM layer is held in equilibrium by NRDM pressure and gravity. If such a scenario is possible, the galaxies would be isolated massive balls floating in a vacuum. On a cosmological level, such matter is equivalent to CDM.



Rejected scenarios

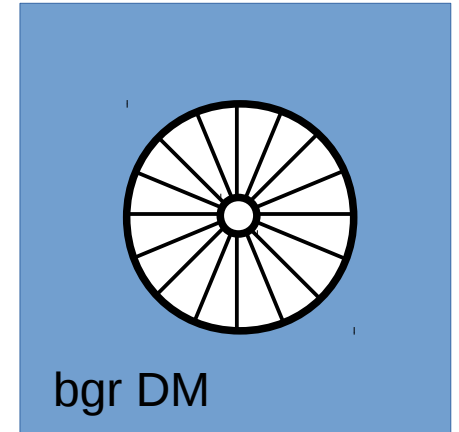
S0.3: halo surrounded by a massive thin shell (cont'd)

- scenario is **rejected** due to the following reason
- The condition for the balance of forces can be written as
$$\varepsilon/(8\pi r^2) \cdot 4\pi r^2 = \varepsilon/r \cdot m, \quad r=R_{\text{cut}} \Rightarrow m=R_{\text{cut}}/2 \gg M_{\text{dm}}(R_{\text{cut}}) = \varepsilon R_{\text{cut}}$$
where $\varepsilon \ll 1$, for MW $\varepsilon = 2.5 \cdot 10^{-7}$
- The shell has a huge mass, strongly exceeding the mass of the galaxy. Formally, with such a mass, the galaxy will be covered by its event horizon, becoming **a black hole**. More precisely, the calculation uses Newtonian equations and only shows that there is no solution in weak fields. The interpretation of this result is that the relativistic pressure at the boundary of the NRDM galaxy can be compensated only by relativistic gravitational forces.
- (detailed computation of hydrostatic equilibrium in CDM shell gives the same result)

Rejected scenarios

S0.4: halo surrounded by uniform DM

- a variation of the previous scenario, where, instead of vacuum, there is uniform dark matter with isotropic EOS:
 $p_{bgr} = w \rho_{bgr}$
- we will consider two options: CDM $0 < w \ll 1$, HDM $w = 1/3$
- for HDM case the hope is that the galactic DM will dominate in mass estimation, resulting in effective cosmological CDM type
- pressure equilibrium at the halo boundary: $\epsilon / (8\pi R_{cut}^2) = w \rho_{bgr}$
- gravitating masses: $M_{dm,gal} = N_{gal} \epsilon R_{cut}$,
 $M_{dm,bgr} = (1 + 3w)\rho_{bgr} \cdot (4\pi/3)(R_{uni}^3 - N_{gal} R_{cut}^3)$
- an estimate of the total mass of dark matter in the universe:
 $M_{dm,uni} = M_{dm,gal} + M_{dm,bgr}$



Rejected scenarios

S0.4: halo surrounded by uniform DM (cont'd)

- $M_{\text{dm,uni}} = N_{\text{gal}} \varepsilon R_{\text{cut}} + \varepsilon(1 + 3w)/(6wR_{\text{cut}}^2)(R_{\text{uni}}^3 - N_{\text{gal}} R_{\text{cut}}^3)$
- according to earlier calculations, the first term already corresponds to the cosmological DM mass estimate, while the formula with the 2nd term gives
 $\varepsilon = 2.5 \cdot 10^{-7}$, $R_{\text{uni}} = 14\text{Gpc}$, $R_{\text{cut}} = 50\text{kpc}$, $N_{\text{gal}} = 2 \cdot 10^{12}$
 $M_{\text{dm,uni}} / M_{\text{sun}} = 5.3 \cdot 10^{23} + 5.7 \cdot 10^{27} (1 + 3w)/(6w) \ll$ 2nd term prevails
..., $R_{\text{cut}} = 1\text{Mpc}$, $N_{\text{gal}} = 8.7 \cdot 10^{10}$
 $M_{\text{dm,uni}} / M_{\text{sun}} = 4.5 \cdot 10^{23} + 1.4 \cdot 10^{25} (1 + 3w)/(6w) \ll$ 2nd term prevails
- **no match**, already for $w=1/3$ and even more for $0 < w < 1$
- scenario does not allow CDM / HDM as background matter when continuously stitching with NRDM pressure at halo boundaries, is **rejected**

Accepted scenarios

- next, we look at scenarios involving dark energy (DE)
- we will represent DE as a kind of matter, perhaps a kind of dark matter (DM) or its other phase state
- standard isotropic EOS $p_{de} = -\rho_{de}$, that is, $w = -1$, with positive ρ_{de} , constant within each phase
- the gravitating mass density for such matter is negative and is equal to $\rho_{de,grav} = \rho_{de} + 3p_{de} = -2\rho_{de}$
- the negativity of this density, provided that it prevails over other components, is the driving mechanism for the accelerated expansion of the universe

Accepted scenarios

S1.1: a jump of DE density on the border of galactic halo

- let there be two different densities of dark energy, outside the halo $\rho_{de,bgr}$, inside the halo $\rho_{de,gal}$, with a jump at R_{cut}

- equilibrium condition of pressures:

$$\varepsilon/(8\pi R_{cut}^2) = p_{de,bgr} - p_{de,gal} = \rho_{de,gal} - \rho_{de,bgr}$$

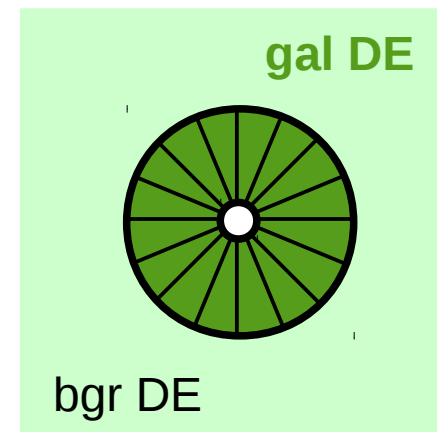
- gravitating masses:

$$M_{dm,gal} = N_{gal} \varepsilon R_{cut} , M_{de,gal} = -2\rho_{de,gal} N_{gal} \cdot (4\pi/3)R_{cut}^3,$$

$$M_{de,bgr} = -2\rho_{de,bgr} \cdot (4\pi/3)(R_{uni}^3 - N_{gal} R_{cut}^3)$$

- estimate of the total mass of dark matter and dark energy in the universe:

$$M_{dm+de,uni} = M_{dm,gal} + M_{de,gal} + M_{de,bgr}$$



Accepted scenarios

S1.1: a jump of DE density on the border of galactic halo (cont'd)

- $M_{dm+de,uni} = (2/3) N_{gal} \varepsilon R_{cut} - (8\pi/3) \rho_{de,bgr} R_{uni}^3$
- the second term here describes the total gravitating mass of dark energy, as if it was filling homogeneously the entire universe, including galactic halos
- the first term - reduced by a factor (2/3) the gravitating mass of the galactic halo
- in general, the model behaves like a mixture of uniform CDM and uniform DE, equivalent to Λ CDM
- in order of magnitude, for $R_{cut} = 50\text{kpc}$, CDM mass corresponds to cosmological estimates
- in exact match, the factor (2/3) can be compensated at $N_{gal}' = 2.6 \cdot 10^{12}$

Accepted scenarios

S1.1: a jump of DE density on the border of galactic halo (cont'd)

- for MW parameters with $R_{\text{cut}} = 50\text{kpc}$, $p_r(R_{\text{cut}}) = \rho(R_{\text{cut}}) = \varepsilon / (8\pi R_{\text{cut}}^2)$

internal DE density: $\rho_{\text{de,gal}} = \rho(R_{\text{cut}}) + \rho_{\text{de,bgr}} = 5.6 \cdot 10^{-24} \text{ kg/m}^3$

external DE density: $\rho_{\text{de,bgr}} = \Omega_{\text{de}} \rho_{\text{crit}} = 6.8 \cdot 10^{-27} \text{ kg/m}^3$

density jump factor $\sim 10^3$

- gravitating density is C^0 -continuous: $2\rho(R_{\text{cut}}) - 2\rho_{\text{de,gal}} = -2\rho_{\text{de,bgr}}$
- gravitating mass function is C^1 -continuous:

$$M(r < R_{\text{cut}}) = \varepsilon r - (8\pi/3) \rho_{\text{de,gal}} r^3$$

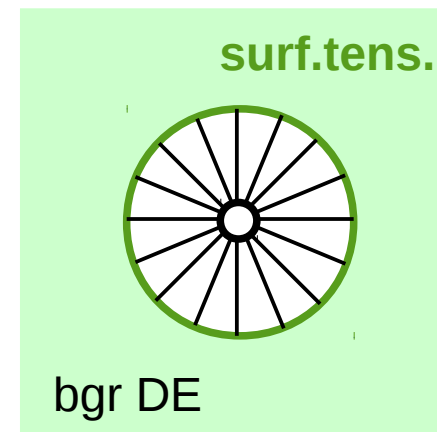
$$M(r > R_{\text{cut}}) = (2/3)\varepsilon R_{\text{cut}} - (8\pi/3) \rho_{\text{de,bgr}} r^3$$

- in $M(r < R_{\text{cut}})$ the first term dominates at $r \sim 8\text{kpc}$ (Sun location), the second term active at $r \sim 50\text{kpc}$; in $M(r > R_{\text{cut}})$ the second term active at $r > 0.6\text{Mpc}$
- resume: the first **accepted** scenario connecting NRDM and DE bgr, cosmologically equivalent to ΛCDM

Accepted scenarios

S1.2: a surface tension on the border of DM halo and DE background

- let there be NRDM inside R_{cut} , DE with density $\rho_{\text{de,bgr}}$ outside R_{cut} , and surface tension with coefficient σ on the boundary
- equilibrium condition for pressures:
$$\varepsilon/(8\pi R_{\text{cut}}^2) = 2\sigma/R_{\text{cut}} + \rho_{\text{de,bgr}} = 2\sigma/R_{\text{cut}} - \rho_{\text{de,bgr}}$$
- gravitating masses:
$$M_{\text{dm,gal}} = N_{\text{gal}} \varepsilon R_{\text{cut}}, \quad M_{\text{de,surf}} = -N_{\text{gal}} \sigma \cdot 4\pi R_{\text{cut}}^2,$$
$$M_{\text{de,bgr}} = -2\rho_{\text{de,bgr}} \cdot (4\pi/3)(R_{\text{uni}}^3 - N_{\text{gal}} R_{\text{cut}}^3)$$
- estimate of the total mass of DM and DE in the universe:
$$M_{\text{dm+de,uni}} = M_{\text{dm,gal}} + M_{\text{de,surf}} + M_{\text{de,bgr}}$$



Accepted scenarios

S1.2: a surface tension on the border of DM halo and DE background (cont'd)

- $= (3/4) N_{gal} \varepsilon R_{cut} + (2\pi/3) N_{gal} R_{cut}^3 \rho_{de,bgr} - (8\pi/3) \rho_{de,bgr} R_{uni}^3$
- Here the third term corresponds to the cosmological contribution of DE, it grows in negative in proportion to the volume of the expanding universe.
- The first and second terms are preserved during the expansion and represent the CDM. At $R_{cut} = 50\text{kpc}$, the first term dominates, and, as in the previous scenario, allows fine tuning to the cosmological CDM density value.
- The calculation of the gravitating mass of the boundary layer. Surface tension is related to negative transverse pressure and positive energy density as $-p_t = \rho = \sigma/dr$, where dr is the layer thickness. The grav.mass of the spherical layer is $M = (\rho + 2p_t)S_{dr} = -\sigma \cdot 4\pi R_{cut}^2$. There is also a radial pressure p_r inside the layer, which continuously interpolates the boundary values, remains bounded, and makes a vanishing contribution at $dr \rightarrow 0$.

Accepted scenarios

S1.2: a surface tension on the border of DM halo and DE background (cont'd)

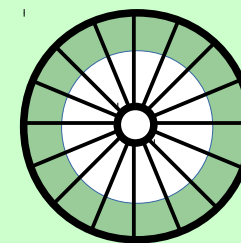
- With $R_{\text{cut}} = 50\text{kpc}$, the density jump between DE and NRDM is still $\sim 10^3$ times, but here it is compensated by surface tension.
- The mass function initially coincides with the NRDM dependence
 $M(r < R_{\text{cut}}) = \epsilon r \ll$ the inner rotation curve does not change.
- When passing R_{cut} , the mass function undergoes a jump :
 $M(R_{\text{cut}}+0) = (3/4)\epsilon R_{\text{cut}} - 2\pi R_{\text{cut}}^3 \rho_{\text{de,bgr}}$, \ll the first term dominates
- Further, the mass receives the cosmological term dominating at $r > 0.6\text{Mpc}$:
 $M(r > R_{\text{cut}}) = (3/4)\epsilon R_{\text{cut}} + (2\pi/3)R_{\text{cut}}^3 \rho_{\text{de,bgr}} - (8\pi/3)\rho_{\text{de,bgr}} r^3$.
- Resume: this **accepted** scenario is very close to the previous one, only a different mechanism to compensate for the pressure jump at the galaxy's boundary is used. Phenomenologically, if we consider DM&DE as media consisting of interacting particles, the presence of a boundary can lead to the appearance of a surface term in the equations, as for classical media.

Accepted scenarios

S1.3: a phase transition from DM to DE

- assume that DE is a form of DM, and there is a continuous transition between the corresponding EOS:
- $p_r = w_r \rho$, $p_t = w_t \rho$, where (w_r, w_t) is changed from $(1, 0)$ at R_{cut1} to $(-1, -1)$ at $R_{\text{cut2}} > R_{\text{cut1}}$
- the result depends on the transition path, fixed from physical considerations as follows (alternatives will be also tried)
- Initially, from R_{cut1} to the intermediate point R_{cut1b} , only w_t changes, from 0 to -1. The transverse attraction between DM flows leads to Joule-Thomson effect known in gas dynamics, the cooling of flows, which in our case manifests itself in a rapid decrease of the mass density ρ .
- Further, from R_{cut1b} to R_{cut2} only w_r changes, from 1 to -1. In this region, DM contributions from different sources are mixed, the matter becomes isotropic. Further, the matter obeys the isotropic EOS for DE, and its density and pressure become constant.

DM-DE transition



bgr DE

Accepted scenarios

S1.3: a phase transition from DM to DE (cont'd)

- logarithmic variables: $x = \log r$, $\xi = \log \rho$, $\rho > 0$
- interpolation of w_t, w_r linear in x , in intervals
- $\{x_1, x_{1b}, x_2\} = \log\{R_{\text{cut}1}, R_{\text{cut}1b}, R_{\text{cut}2}\}$
- hydrostatic eqn for anisotropic medium:
- $r(\rho r + \rho)A'_r + 2A(r(\rho r)'_r + 2\rho r - 2p_t) = 0$
- \ll the first term describes gravitational self-interaction, in our problems is quadratically small, neglected, from the second term:
- $w_r \xi'_x + (w_r)'_x + 2(w_r - w_t) = 0$, solution: $\xi = - \int dx ((w_r)'_x + 2(w_r - w_t))/w_r$
- regularity condition: $w_r = 0$, $(w_r)'_x = 2w_t$, in one point
- in our scenario with lin.interpolation: $(w_r)'_x = 2w_t = -2$ on the interval $[x_{1b}, x_2]$

Accepted scenarios

S1.3: a phase transition from DM to DE (cont'd)

- integrals can be evaluated analytically (details omitted)
- input data: $R_{\text{cut1}}=R_{\text{cut}}=50\text{kpc}$, $\varepsilon=2.5 \cdot 10^{-7}$, $\rho_2=\rho_{\text{de,bgr}}=6.8 \cdot 10^{-27} \text{ kg/m}^3$
- output data: $\rho_1/\rho_2=824$, $R_{\text{cut1b}}=0.24\text{Mpc}$, $R_{\text{cut2}}=0.65\text{Mpc}$
- \ll physically reasonable configuration
- $\rho_{\text{grav}} = (1+w_r + 2w_t) \rho$, $\Delta M_{\text{grav}} = 4\pi \int \rho_{\text{grav}} r^2 dr$
- $\{M_1, \Delta M_1, \Delta M_2, M_{\text{vac}}\} = \{2.60, 2.67, -2.60, 2.35\} \cdot 10^{11} \text{ Msun}$
- $M_1 = \varepsilon R_{\text{cut}}$ – mass of NRDM halo
- $\Delta M_{1,2}$ – masses of spherical layers for two interpolation intervals
- $M_{\text{vac}} = (8\pi/3) \rho_{\text{de,bgr}} R_{\text{cut2}}^3$ – compensation mass of vacuole

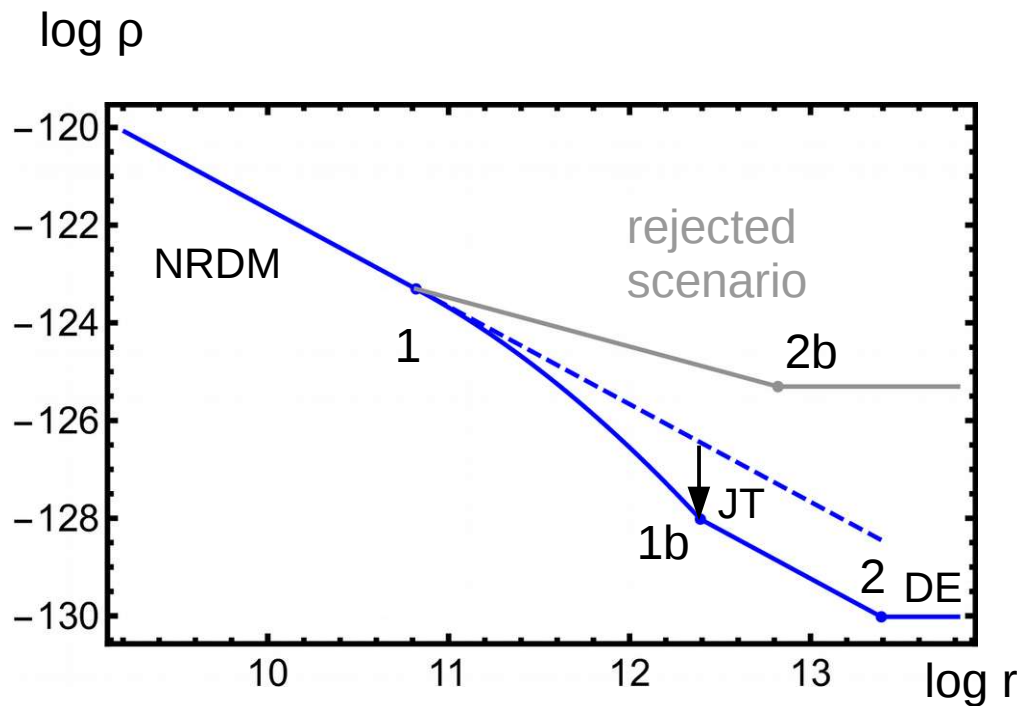
Accepted scenarios

S1.3: a phase transition from DM to DE (cont'd)

- M_{vac} appears due to the terms regrouping
- $M_{dm+de,uni} = N_{gal} M_{dm+de,gal} - (8\pi/3) \rho_{de,bgr} (R_{uni}^3 - N_{gal} R_{cut}^2)^3$
- $= N_{gal} (M_{dm+de,gal} + M_{vac}) - (8\pi/3) \rho_{de,bgr} R_{uni}^3$
- M_{vac} is formally attributed to CDM in cosmological computations, should be excluded from rotation curves
- $M_{dm+de,gal} + M_{vac} = 5 \cdot 10^{11} M_{sun}$, exact match with cosmological CDM mass at $N_{gal}' = 9 \cdot 10^{11}$, factor 2.2 less than the nominal value
- the constructed scenario contains a wide arbitrariness in a choice of interpolating functions and is rather **a proof of the existence** for a solution that satisfies cosmological estimates
- for comparison, the alternative scenarios with the other order of interpolation (first w_r , than w_t) does not satisfy conditions of regularity, while simultaneous interpolation of (w_t, w_r) misses the exp. estimate $\rho_1/\rho_2 \sim 824$.

Accepted scenarios

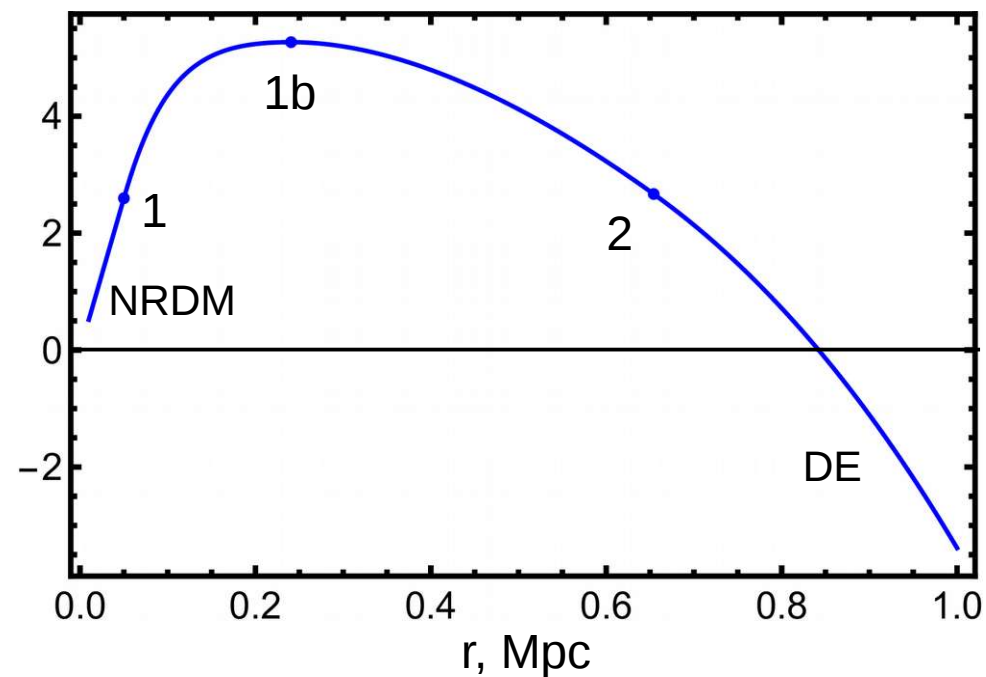
S1.3: a phase transition from DM to DE (cont'd)



accepted scenario

with NRDM phase,
Joule-Thompson effect (JT),
mixing of flows, towards
the constant DE phase

$M_{\text{grav}}, 10^{11} M_{\text{sun}}$

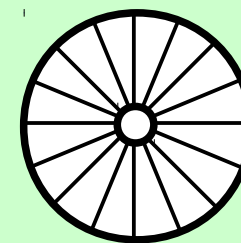


the same phase transition for
gravitating mass function, from the
initial linear to the final negative cubic

Accepted scenarios

S1.4: Bose-Einstein condensation

- let there be two phases: the internal NRDM phase, described by the model of classical particles, and the external DE phase, described by a complex scalar field
- such field theory is used in phenomenological models of Bose-Einstein condensation (Ginzburg-Landau theory of superconductivity), as well as in cosmological models of quintessence and its variants (k-essence, quartessence, Chaplygin gas), see 0912.1609 and references therein
- thus, this scenario assumes that DM particles are emitted by RDM stars in the galaxy and undergo Bose-Einstein condensation at large distances
- alternatively, these can be particles of different types that are in contact equilibrium at the edge of the galactic halo.



bgr DE = scal.field

Accepted scenarios

S1.4: Bose-Einstein condensation (cont'd)

$$L = -(\partial_\mu \phi^* \partial^\mu \phi)/2 - V(|\phi|^2), \quad \ll \text{Lagrangian}$$
$$T_{\mu\nu} = (\partial_\mu \phi^* \partial_\nu \phi + \partial_\nu \phi^* \partial_\mu \phi)/2 + g_{\mu\nu} L, \quad \ll \text{energy-momentum tensor}$$
$$(-\partial^2/\partial t^2 + \Delta)\phi = 2V'(|\phi|^2)\phi. \quad \ll \text{equations of motion}$$

- the equations of motion are written in a flat background, and the rest of the expressions are valid for an arbitrary metric
- the influence of gravity on the scalar field is neglected, assuming that the gravitational fields are weak and the corresponding solutions are relativistic
- the field equations belong to **nonlinear Klein-Gordon** type with the specifics in choosing a potential for a complex scalar field

$$V(|\phi|^2) = \text{Const} + m^2 |\phi|^2/2$$

special case, when the equations become linear and describe the behavior of a free massive scalar field

Accepted scenarios

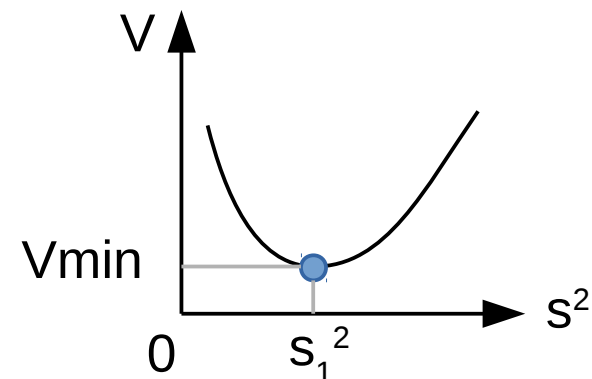
S1.4: Bose-Einstein condensation (cont'd)

- let us use a smooth potential $V(s^2)$, possessing a minimum for a nonzero value of the argument $V(s_1^2) = V_{min}$, $s_1^2 > 0$; for this minimum, the constant function $\varphi = s_1$ is the exact solution
- for such a function, using a spherical coordinate system and a metric of signature (- +++):

$$T_{\mu}^{\nu} = \text{diag}(-\rho, p_r, p_t, p_t) = -V_{min} \cdot \text{diag}(1, 1, 1, 1),$$

$$\rho = V_{min}, \quad p_r = p_t = -V_{min},$$

$$\rho_{grav} = \rho + p_r + 2p_t = -2V_{min}.$$



- the result coincides with the standard EOS of dark energy, which determines the interest in this model in the cosmological context
- here we will fix $V_{min} > 0$, and for simplicity assume $V > 0$ everywhere

Accepted scenarios

S1.4: Bose-Einstein condensation (cont'd)

- we consider stationary spherically symmetric problems for complex scalar field with exact particular solutions of the form $\varphi = e^{iEt} s(r)$
- this substitution reduces the dimension: $(E^2 + \Delta)s = 2V'(s^2)s$
- next, we look at static solutions: $E = 0$, $\varphi = s(r)$
- the uniqueness of solutions with static boundary conditions can be proven
- thus, all solutions that can be attached to the constant $\varphi = s_1$ are globally static and have the form above
- theory of real scalar field has the same static solutions, but dynamical ones, with non-zero E , in the form $\varphi = \cos(Et) s(r)$, are not exact solutions, since the time-dependence does not disappear in $V(|\varphi|)$

Accepted scenarios

S1.4: Bose-Einstein condensation (cont'd)

- calculating EOS for static solutions

$$T_{\mu}^{\nu} = \text{diag}(0, s'^2, 0, 0) - \text{diag}(1, 1, 1, 1) \cdot (s'^2/2 + V(s^2)),$$

$$\rho = -p_t = s'^2/2 + V(s^2) > 0, \quad p_r = s'^2/2 - V(s^2),$$

$$\rho_{grav} = \rho + p_r + 2p_t = -2V(s^2).$$

- if the potential is shallow, then $\rho_{grav} \sim -2V_{min}$, as for DE, everywhere
- this result is quite remarkable: the scenario can be configured in such a way that the gravitating density profile immediately after NRDM phase drops sharply to DE phase
- it reproduces RDMcut scenario with a sharp cutoff of the density to almost zero at R_{cut} radius
- DE contribution is small at R_{cut} and begins to be felt at much larger distances, where it reproduces the accelerated cosmological expansion

Accepted scenarios

S1.4: Bose-Einstein condensation (cont'd)

- technically, the equilibrium condition for the radial pressure component at the interface between the phases must be met
- this condition is satisfied if the model has enough degrees of freedom to ensure that in $pr=s'^2/2-V(s^2)$ the first term dominates over the second
- in this case, it is possible to keep the connection with the positive pr from the NRDM phase, no matter how large this value may be
- the physical manifestations are defined only by p_{grav} and do not depend on the details of this connection
- we will make such a connection for a particular choice of the potential. First, write the r.h.s. of the field equation in the form $2V'(s^2)s = V(s^2)'_s$. Next, use the reparametrization of the argument $V(s^2)=V_1(s)$, we choose the potential as follows...

Accepted scenarios

S1.4: Bose-Einstein condensation (cont'd)

$$V_1(s) = V_{min} + a/2 (s - s_1)^2, \quad a > 0, \quad s_1 > 0,$$

$$s'' + 2s'/r = a(s - s_1),$$

$$s = s_1 + (e^{-\sqrt{ar}} C_1)/r + (e^{\sqrt{ar}} C_2)/(2\sqrt{ar}).$$

- the remarkable properties of such a potential are the linearity of the field equation, the existence of an analytical solution, and also the fact that any potential in the vicinity of the minimum can be written in such a way
- selecting a branch with finite $s \rightarrow s_1$ at $r \rightarrow \infty$, obtain $C_2 = 0$
- also impose $C_1 > 0$ in order to ensure $s > s_1$ on the solutions. For $s > s_1$, the ascending branch of $V_1(s)$ corresponds to the positive square of the mass, normal particles. For $s < s_1$, the descending branch of $V_1(s)$ formally corresponds to the negative square of the mass, the tachyon case, but this branch is not activated in the solutions we consider.

Accepted scenarios

S1.4: Bose-Einstein condensation (cont'd)

- evaluating relevant components in EOS:

$$p_r = e^{-2\sqrt{ar}} C_1^2 (1 + 2\sqrt{ar}) / (2r^4) - V_{min},$$

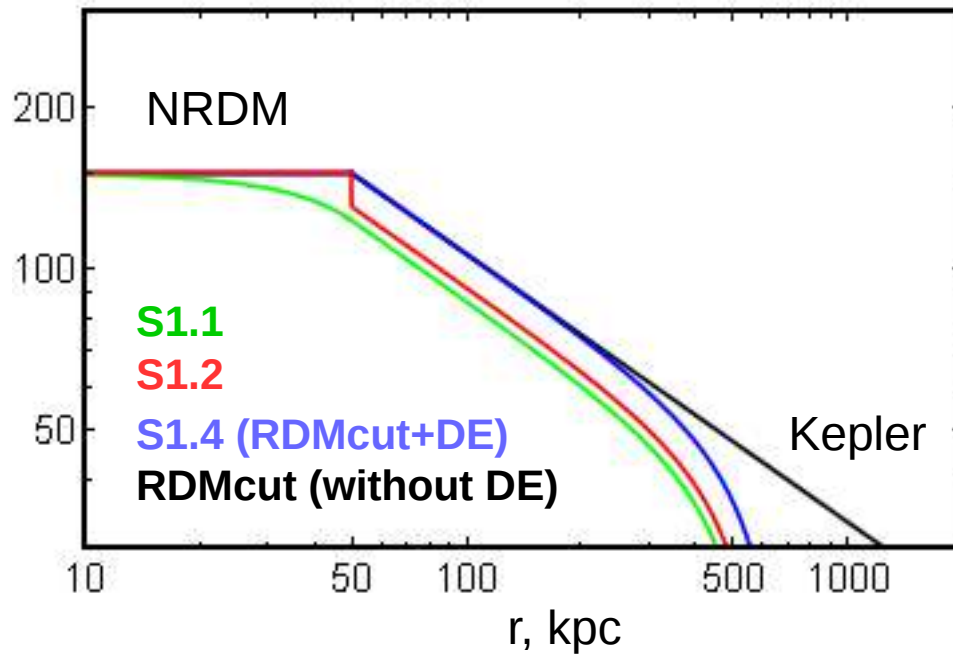
$$\rho_{grav} = -a C_1^2 e^{-2\sqrt{ar}} / r^2 - 2V_{min},$$

- we see that by choosing C_1 it is always possible to achieve stitching with positive p_r from NRDM phase
- then, choosing small a , achieve $\rho_{grav} \sim -2V_{min}$
- with such a choice of parameters, the solution comes arbitrarily close to **RDMcut + DE profile**: NRDM sharply cut at R_{cut} , constant DE follows
- thereby the considered scenario provides a deeper physical foundation for this phenomenological profile
- **exact match** with cosmological estimations at $R_{cut}=50\text{kpc}$, $N_{gal}'=1.7 \cdot 10^{12}$ or $R_{cut}=44\text{kpc}$, $N_{gal}=2 \cdot 10^{12}$ or $R_{cut}=0.6\text{Mpc}$, $N_{gal}'=1.4 \cdot 10^{11} \dots$

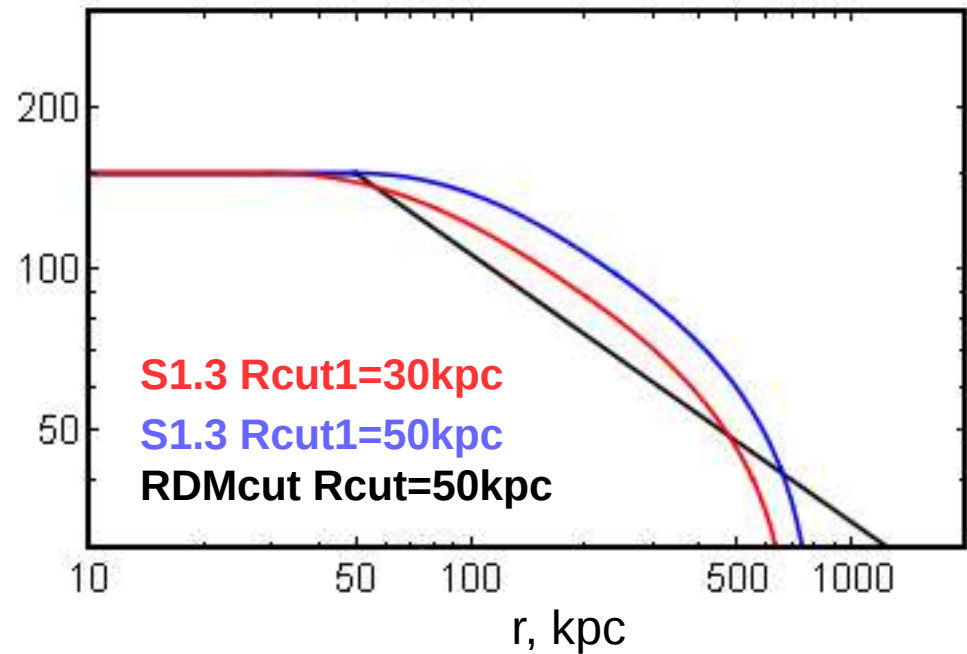
Further details

All accepted scenarios: the outer part of MW rotation curve

v, km/s



v, km/s

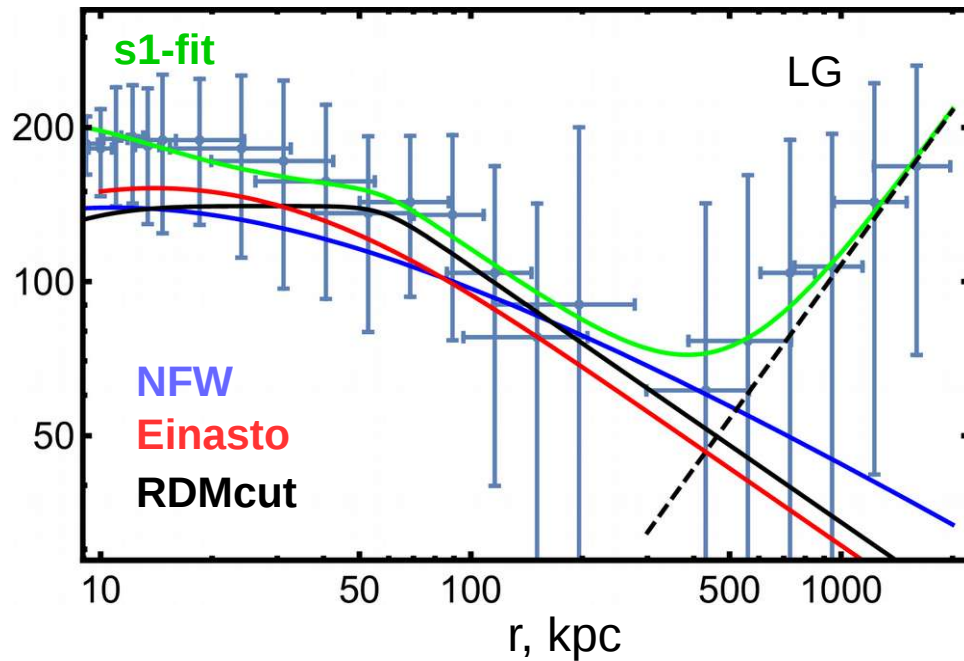


- the profiles go close to each other
- can be additionally adjusted to become even closer

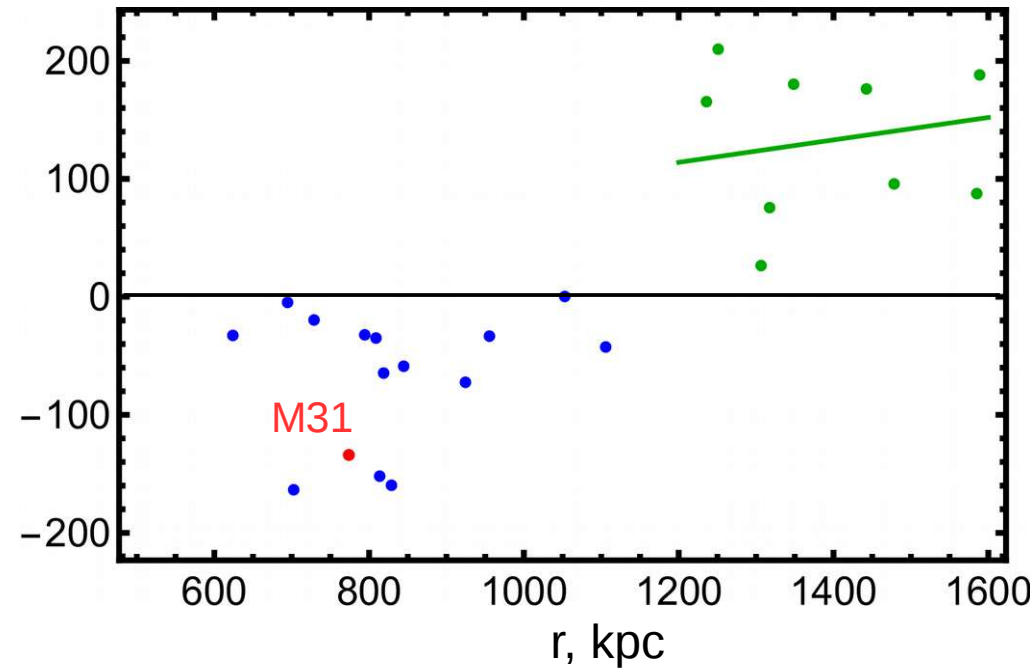
Further details

All accepted scenarios: the outer part of MW rotation curve

v , km/s



v_r , km/s

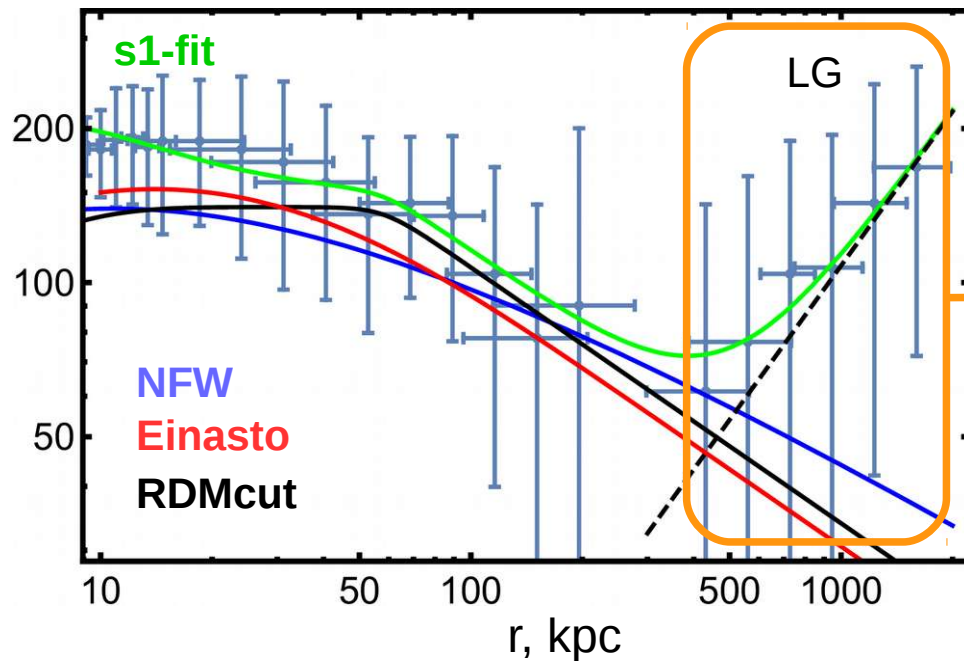


- large experimental scatter (data from 1307.8241)
- cannot distinguish between the profiles (including standard ones)
- **the main result:** 4 scenarios of connection NRDM to DE bgr, all equivalent to Λ CDM, not contradicting to outer MW RC

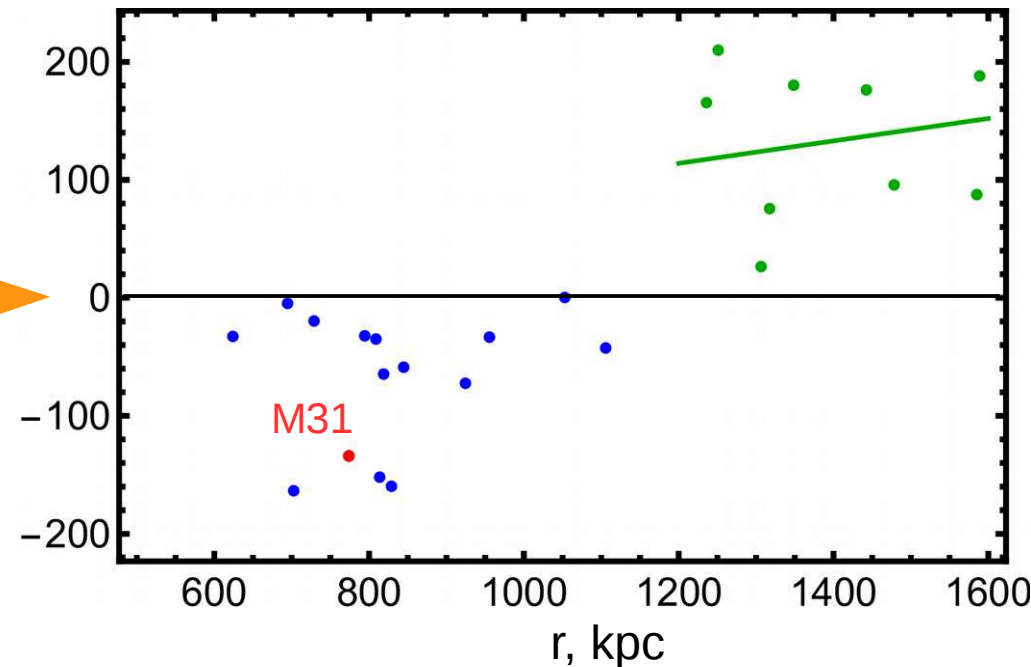
Further details

All accepted scenarios: the outer part of MW rotation curve

v , km/s



v_r , km/s



- the outer segment corresponds to Local Group, should be remodeled
- only radial velocity component is measured, clear separation to negative (grav.attraction of LG) and positive (Hubble flow) parts
- such a picture is confirmed by detailed modeling in 1405.0306
- we concentrate on isolated galaxies and exclude LG tail from the modeling

Further details

On model-independent reconstruction of EOS from RC

- let RC $v(r)$ for an average-mass galaxy, approximated by some empirical profile, be given \Rightarrow $M_{\text{grav}}(r)$ and $\rho_{\text{grav}}(r)$ are known
- match with cosmological estimates:
 $M_{\text{grav}}(R_{\text{max}}) + M_{\text{vac}}(R_{\text{max}}) = M_{\text{dm,uni}} / N_{\text{gal}},$
 $M_{\text{vac}}(r) = (8\pi/3) \rho_{\text{de,bgr}} r^3, \quad \rho_{\text{grav}}(R_{\text{max}}) = -2 \rho_{\text{de,bgr}}$
 \ll these conditions are imposed directly on the experimental curves and not on the EOS components
- use: $\rho_{\text{grav}} = \rho + p_r + 2p_t, \quad r(p_r)'_r + 2p_r - 2p_t = 0,$
2 relations on 3 profiles (ρ, p_r, p_t) , 1 functional dof remains
- e.g., set an arbitrary p_r , then (ρ, p_r, p_t) will be reconstructed by linear formulas, even without solving differential equations

Further details

On model-independent reconstruction of EOS from RC (cont'd)

- boundary conditions on R_{\max} : $\rho = \rho_{\text{de,bgr}}$, $p_r = p_t = -\rho_{\text{de,bgr}}$, $p_r' = 0$,
restrict $(p_r, p_r')(R_{\max})$
- \Rightarrow EOS in the parametric form $(\rho, p_r, p_t)(r)$ is reconstructed
- this algorithm can be supplemented with boundary conditions for NRDM $\rho = \rho_r$, $p_t = 0$ at the inner radius R_{\min} , by introducing the gravitational term into the hydrostatic equation and other model corrections
- similar reconstruction done in 1301.6785, where EOS was assumed to be isotropic $p_r = p_t$, the solution did not contain functional ambiguities, but the anisotropic NRDM-type solution was missed
- the main obstacle to the implementation of such algorithms is the large scatter in the outer region of the rotation curves, leading to inaccurate reconstruction of EOS in this region

Further details

Taking the mass distribution of galaxies into account

- we used the **estimated number of galaxies** $N_{gal}=2 \cdot 10^{12}$ from 1607.03909
- this value takes into account the evolution of the universe and estimates the number of observable galaxies up to redshift values $z < 8$
- to compare with dark matter density today, we need the number of galaxies in a simultaneous slice, in a ball of radius $R_{uni} \sim 14 \text{Gpc}$
- this radius is nominal, the final formulas include the ratio $M_{dm,uni} / N_{gal}$, from which this radius drops out
- in fact, we need an estimate of the density of galaxies dN_{gal} / dV near our position, for small z
- the mentioned ratio is expressed through this density:
$$M_{dm,uni} / N_{gal} = \rho_{dm} / (dN_{gal} / dV)$$

Further details

Taking the mass distribution of galaxies into account (cont'd)

- in 1607.03909, density of galaxies is modeled using the **Schechter function**:

$$dN_{gal}/dV/dM = \phi^* \log(10) 10^{(M-M^*)(1+\alpha)} \exp(-10^{(M-M^*)}),$$

where $M = \log_{10} (M_{lm,gal} / M_{sun})$, $M_{lm,gal}$ is the stellar mass of the galaxy;

parameters selected from the most accurate fit for the closest galaxies:

$$\alpha = -1.29, M^* = 11.44, \phi^* = 12.2 \cdot 10^{-4} \text{ Mpc}^{-3} \ll \text{2nd row Tab1 1607.03909}$$

- integrating this expression over $6 < M < 12$, obtain $dN_{gal}/dV = 0.154 \text{ Mpc}^{-3}$, multiplying by $(4\pi/3)R_{uni}^3$, get $N_{gal} = 1.766 \cdot 10^{12}$, close to $2 \cdot 10^{12}$, found in 1607.03909 for the same mass range, taking into account the evolution of the universe

Further details

Taking the mass distribution of galaxies into account (cont'd)

- next, we need the mean $\langle v^2 \rangle$ for the square of the outer orbital velocity, for the same distribution
- use **Tully-Fisher relation** $v \sim M_{\text{lm}}^p$ with $p = 1/4$
- normalizing to MW value and denoting $\eta_p = \langle (M_{\text{lm}}/M_{\text{lm},\text{MW}})^p \rangle$, have
 $\langle v^2 \rangle / v_{\text{MW}}^2 = \eta_{1/2}$. With $M_{\text{lm},\text{MW}} = 6.08 \cdot 10^{10} \text{ Msun}$ from 1407.1078, compute $\eta_{1/2} = 0.0455$, Ngal $\eta_{1/2} = 1.196 \cdot 10^{11}$.
- This estimate is based only on experimental data in the form of Schechter and Tully-Fisher relations. It needs to be compared with the corrected Ngal' parameter in our scenarios.

Further details

Taking the mass distribution of galaxies into account (cont'd)

- Before the comparison: integration used the lower limit $M_{\min} = 6$, as in 1607.03909. This limit is slightly below the limit of exp data $M_{\min} = 8$, that is, extrapolation is used in the calculations. The number of galaxies depends on this limit, for $M_{\min} = 8$ get $N_{\text{gal}} = 4.189 \cdot 10^{11}$. At the same time, $\eta_{1/2}$ will increase approx by the same factor and $N_{\text{gal}} \eta_{1/2}$ will not change. The same effect is observed for all $p > 0.3$. The reason for this is that the cumulative value $N_{\text{gal}} \eta_p$ is expressed by an integral dominated by large masses.
- Also note that the modeling for Schechter function has scatter 0.4-1dex and Tully-Fisher relation 0609076 for v^2 has scatter 0.8dex. Therefore, deviations in comparison of model and experiment of **<1.8dex can be tolerated**.

Further details

Taking the mass distribution of galaxies into account (cont'd)

- most of our scenarios have clear algebraic structure:

$$M_{dm,uni} = N_{gal} (M_{dm,gal} + M_{vac}),$$

$$M_{dm,gal} = k_1 \varepsilon R_{cut} c^2/G, \quad M_{vac} = k_2 (8\pi/3) \rho_{de,bgr} R_{cut}^3,$$

- the constants for scenarios {S1.1, S1.2, S1.4} are

$$k_1 = \{2/3, 3/4, 1\}, \quad k_2 = \{0, 1/4, 1\}$$

- in calculations by order of magnitude, $k_1 \sim 1$, while M_{vac} can be neglected for

$$N_{gal} = 2 \cdot 10^{12}, \quad M_{dm,uni} = 4.5 \cdot 10^{23} M_{sun}, \quad R_{cut} < 0.6 \text{Mpc}$$

- => there is a single relation for these 3 scenarios that should be checked

$$\text{with experiment: } M_{dm,uni} \sim N_{gal} \varepsilon R_{cut} c^2/G$$

- At first, for these 3 scenarios, assume R_{cut} fixed, and ε distributed over the galaxies. In this case, $M_{dm,uni} \sim N_{gal} \langle \varepsilon \rangle R_{cut} c^2/G$. Also, if R_{cut} is distributed but uncorrelated with ε , then $M_{dm,uni} \sim N_{gal} \langle \varepsilon \rangle \langle R_{cut} \rangle c^2/G$.

Further details

Taking the mass distribution of galaxies into account (cont'd)

- further, $\varepsilon = (v/c)^2$ and using $\eta_{1/2}$ introduced above, get
$$M_{\text{dm,uni}} \sim N_{\text{gal}} \eta_{1/2} M_{\text{dm,MW}}, \text{ with } M_{\text{dm,MW}} = \varepsilon_{\text{MW}} R_{\text{cut}} c^2/G$$
- rewrite: $N_{\text{gal}}' \sim N_{\text{gal}} \eta_{1/2}$, where $N_{\text{gal}}' = M_{\text{dm,uni}} / M_{\text{dm,MW}}$ is corrected number of galaxies introduced above in scenarios with MW copies
- for $\varepsilon_{\text{MW}} = 2.5 \cdot 10^{-7}$ and R_{cut} varying within 50kpc-0.6Mpc, get
$$N_{\text{gal}}' = 1.7 \cdot 10^{12} - 1.4 \cdot 10^{11}, \text{ in agreement with the exp estimate}$$

$$N_{\text{gal}} \eta_{1/2} = 1.196 \cdot 10^{11} \text{ within } \mathbf{1.2-0.1dex}, \text{ with the preference for larger values of } R_{\text{cut}}.$$

Further details

Taking the mass distribution of galaxies into account (cont'd)

- For an assessment of S1.3 scenario, **scaling** of various galactic parameters should be known. As a working hypothesis, suppose that mass density is scaled as $\rho(r) \rightarrow \rho(r/a)$, its consequences: $M_{\text{grav}}(R) \rightarrow a^3 M_{\text{grav}}(R/a)$, $M_{\text{vac}}(R) \rightarrow a^3 M_{\text{vac}}(R/a)$, $M_{\text{dm,gal}} \rightarrow M_{\text{dm,gal}} a^3$, $v^2 = GM/R \rightarrow v^2 a^2$, $v \rightarrow va$. From Tully-Fisher relation, $M_{\text{lm}} \rightarrow M_{\text{lm}} a^4$. Thus, $M_{\text{dm}} \sim M_{\text{lm}}^{3/4}$, the required correction factor is $N_{\text{gal}} \eta_{3/4} = 8.302 \cdot 10^{10}$.
- The function $N_{\text{gal}} \eta_p$ has a minimum at $p \sim 0.9$ and almost const in the range $p = 0.3 \dots 2$, so all dependencies $M_{\text{dm}} \sim M_{\text{lm}}^p$ with such p lead to a similar result. In other works, other p -values were obtained, Schaeffer_1993 $p=0.3$, Girardi_2002 $p=1.34$, 0703115 Eq. (7) $p=0.3-1.1$ for spiral galaxies, 1609.06903 Eq. (21) $p=1.05-1.24$ for dwarf disc galaxies. The result depends on the choice of the mass profile and the halo cutoff radius. In our scenario S1.3, the cutoff occurs at the outer radius R_{cut2} , where the phase transition of DM into DE is completed, outside of which the density of DM vanishes. In other works, other definitions of R_{cut} were used.

Further details

Taking the mass distribution of galaxies into account (cont'd)

- Compared with the value obtained in S1.3 for joining the relations $N_{gal} \eta_{3/4} = 8 \cdot 10^{10}$ and $N_{gal}' = 9 \cdot 10^{11}$, there is a discrepancy of **1.1dex**
- => our assumption about the scale invariance of scenario S1.3 fits into the existing scatter of experimental data
- The discrepancy is not related to the details of our modeling, it is the result of direct comparison of different experimental estimates. Using p-values from the experimental works cited above, a similar result will be obtained.
- A similar result will also be obtained in our other scenarios if we accept the same scaling assumptions: $N_{gal}' = 1.7 \cdot 10^{12} - 1.4 \cdot 10^{11}$ for $R_{cut} = 50\text{kpc} - 0.6\text{Mpc}$. Deviation from $N_{gal} \eta_{3/4} = 8 \cdot 10^{10}$ is **1.3dex-0.2dex**, with a preference for larger values of R_{cut} .

Open Questions

- 4 scenarios with sterile DM particles are constructed, for massive ultralight ($\lambda_{\text{Compt}} > 10^{14} \text{m}$, $\text{mass} < 10^{-20} \text{eV}$, axion-alike) or massless ($\lambda_{\text{out}} \sim 10^{14} \text{m}$) particles
- bosons for scenario S1.4 involving Bose-Einstein condensation (BEC), considered in frames of (generalized) Ginzburg-Landau theory
- **Q1: can DM particles be real photons?**
- **Q1a:** photons are not sterile, at $E > 1 \text{MeV}$ e^+e^- pairs are created. NRDM core is replaced with ultrarelativistic plasma, $w=1/3$ TOV core. Recomputation is needed combining Planck | TOV | NRDM cores. λ_{out} can be changed.
- **Q1b:** massless photons cannot undergo BEC. The other opinion: 1305.1210 and references therein. Due to the interaction with ISM/IGM, the photons receive dispersion relation with a small mass, then can go BEC. The mass depends on the wavelength. Recomputation for λ_{out} and the corresponding remodeling of BEC is necessary.
- **Q2: can they be gravitons?** are they sterile? what is about BEC for gravitons?

Conclusion

- 8 scenarios connecting NRDM galactic model with constant density background are considered
- 4 scenarios survive, only those containing DE as a background
- scenarios are cosmologically equivalent to Λ CDM, although DM particles are massless or ultralight
- a phase transition of DM to DE is the major component of these scenarios
- in particular, Bose-Einstein condensation can be a mechanism for this transition
- general type (sterile, massless or ultralight, presumably bosonic) DM particles are considered
- the possibility that DM particles are Standard Model's photons and/or gravitons with extraordinary large wavelength should be investigated

Thank you!