Grand Unification, Cosmic Strings, and Gravity Waves

Rinku Maji

Department of Physics, IIT Kanpur

@ What Comes Beyond the SM?

Outline

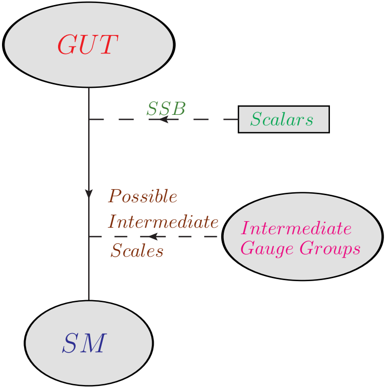
1. Grand Unification Beyond SM, Predictions, and Constraints
2. Evolution of Strings and Gravity Waves
3. GUT Inflation, Phase Transitions, and Strings
4. Possible NANOGrav-PPTA Tension, Inflation, and Resolution
5. Intermediate-mass Monopoles and Strings
6. Summary

1. *Grand Unification Beyond SM, Predictions, and Constraints*

Grand Unification beyond the SM

- The basic idea in a Grand Unified Theory (GUT) is that the SM, $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$, is embedded in a larger simple group, $\mathcal{G}$.

Schematic view



Predictions of GUTs

- Mediation of lepto-quark gauge bosons$\Rightarrow$ proton decay into meson plus antilepton :

$$\boxed{p \to M + \bar{l}}$$

$M \in \{\pi^+, \pi^0, K^+, K^0, \eta\}$ and $l \in \{e, \mu, \nu_{e,\mu,\tau}\}$.

- Topological defects may appear during the SSB of a group $\mathcal{G}$ down to its subgroup $\mathcal{H}$.
- Non-trivial homotopy group $\Pi_k(\mathcal{M})$ of the vacuum manifold $(\mathcal{M} = \mathcal{G}/\mathcal{H})$ implies formation of topological defects.
- Various types of topological defects which can be formed are : domain walls (k = 0), cosmic strings (k = 1), monopoles (k = 2) etc

Observational Constraints on GUTs

- Super-Kamiokande experiment puts a stringent constraint on the partial lifetime for the Golden Channel ($p \to \pi^0 e^+$): $\boxed{\tau_p > 1.6 \times 10^{34} \text{ years.}}$

  Super-K Collaboration, K. Abe et al., PRL 113 (2014), PRD 95 (2017)

- Stable domain walls contradict standard cosmology.

  Y. B. Zeldovich, I. Y. Kobzarev, L. B. Okun, Zh. Eksp. Teor. Fiz. **67**, 3-11 (1974)

- Upper bound on monopole number per comoving volume from MACRO: $\boxed{Y_M = n_M/s < 10^{-27}}$.

  M. Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

- The Parkes Pulsar Timing Arrays (PPTA) put a contraint on the tension of the “undiluted” cosmic strings : $\boxed{G\mu < 1.5 \times 10^{-11}}$.

  J.J. Blanco-Pillado, K.D. Olum, X. Siemens, PLB 778, 392 (2018)

Way Out...

- Proton lifetime constraint ⇒
  1. Threshold Corrections,
  2. Dimension-5 operator.

     J. Chakrabortty, **RM**, S. K. Patra, T. Srivastava, S. Mohanty, PRD **97** (2018) 095010

     J. Chakrabortty, **RM**, S. F. King, PRD **99** (2019) 095008

- Stable topological defects ⇒ Inflation.

2 *Evolution of Strings and Gravity Waves*

Evolution of Cosmic Strings

- The mean inter-string distance at cosmic time t (temp $= T$):

$$\boxed{d_{\text{str}} = p\,\xi(\phi_I)\ \exp(N_{\text{str}})\ \left(\frac{t_r}{\tau}\right)^{\frac{2}{3}}\ \frac{T_r}{T}}$$

Inter-string seperation at production $\xi = \min\left(H^{-1}, m_{\text{eff}}^{-1}\right)$

Expansion during Inflation

Expansion during Inflaton oscillation

Expansion after reheating

- The string network re-enters the post-inflationary horizon at cosmic time t_F if

$$\boxed{d_{\text{str}}(t_F) = d_{\text{hor}}(t_F)}$$

$$\text{with}\ \ d_{\text{hor}}(t_F) = \begin{cases} 2t_F & \text{(radiation dominance)} \\ 3t_F & \text{(matter domination).} \end{cases}$$

Chakrabortty, Lazarides, Maji, Shafi JHEP 02 (2021) 114

Lazarides, Maji, Shafi, arXiv:2104.02016

String Loops and Gravity Waves

- After horizon re-entry, the strings inter-commute and form loops at any subsequent time t_i.
- Loops of initial length $l_i = \alpha t_i$ decay via emission of gravity waves.
- The redshifted frequency of a normal mode k, emitted at time $\tilde{t}$, as observed today, is given by

$$f = \frac{a(\tilde{t})}{a(t_0)} \frac{2k}{\alpha t_i - \Gamma G\mu(\tilde{t} - t_i)}, \quad \text{with} \quad k = 1, 2, 3, \ldots$$

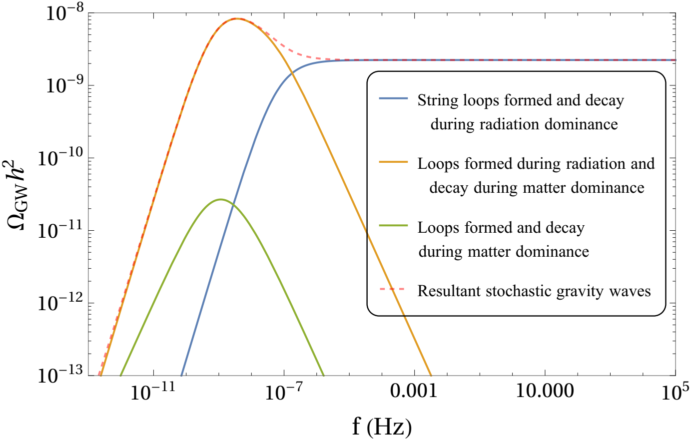
- The total gravity wave background: $\Omega_{\text{GW}}(f) = \sum_k \Omega_{\text{GW}}^{(k)}(f)$, where

$$\Omega_{\text{GW}}^{(k)}(f) = \frac{1}{\rho_c} \frac{2k}{f} \frac{(0.1)\Gamma k^{-4/3} G\mu^2}{\zeta(4/3)\alpha(\alpha+\Gamma G\mu)} \int_{t_F}^{t_0} d\tilde{t}\, \frac{C_{eff}(t_i)}{{t_i}^4} \left(\frac{a(\tilde{t})}{a(t_0)}\right)^5 \left(\frac{a(t_i)}{a(\tilde{t})}\right)^3 \theta(t_i - t_F).$$

$\Gamma \sim 50$, $\alpha \simeq 0.1$, and $C_{eff}(t_i) = 0.5\ (5.7)$ for radiation (matter) dominated universe.

Blanco-Pillado, Olum, Shlaer, PRD 89, 023512 (2014)

Generation of Stochastic Gravity Waves: Analytic Approximation



Sousa, Avelino, Guedes, Phys. Rev. D **101**, 103508 (2020)

3 *GUT Inflation, Phase Transitions, and Strings*

Inflation with GUT-singlet ϕ

- We employ a GUT-inflation driven by CW potential of real GUT-singlet scalar ϕ:

$$V(\phi) = A\phi^4\left[\log\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + V_0.$$

Here $V_0 = AM^4/4$, M is the vacuum expectation value (VEV) of ϕ, and $A = \beta^4 D/(16\pi^2)$, where D is the dimensionality of the representation to which the GUT gauge symmetry breaking real scalar field χ belongs, and β determines the coupling $-\beta^2\phi^2\chi^2/2$ between ϕ and χ.

Shafi, Vilenkin, PRL 52, 691 (1984)

- The GUT scale is $\boxed{M_X = \sqrt{8\pi/\alpha}(V_0/D)^{1/4}}$, α: quartic coupling of χ.

- Successful inflation occurs for $\boxed{V_0^{\frac{1}{4}}/10^{16}\text{GeV} \in [1.51, 1.82].}$

Chakrabortty, Lazarides, RM, Shafi JHEP 02 (2021) 114

Phase Transitions and Ginzburg criterion

- Ginzburg criterion for phase transition is

$$\boxed{\frac{4\pi}{3}\xi^3\Delta V \gtrsim T_H.}$$

Ginzburg, Soviet Phys. Solid State 2, 1824 (1961)

- $\Delta V = \frac{m_{\text{eff}}^4}{16\alpha_{\text{str}}}$ is the potential difference between the local maximum at $\chi = 0$ and the minima.
- $m_{\text{eff}}^2 = 2\left(\beta_{\text{str}}^2\phi^2 - \sigma_{\chi_{\text{str}}}T_H^2\right)$ be the squared effective mass of χ_{str}.
- $T_H = H/2\pi$ is the Hawking temperature.

Symmetry Breaking Scale and String Tension

- Ginzburg criterion gives the intermediate symmetry breaking scale as

$$\boxed{\begin{array}{c} M_{\rm str} = \sqrt{\left(\frac{72\alpha_{\rm str}^2}{\pi^2} + \sigma_{\chi_{\rm str}}\right)\frac{H_{\rm str}}{2\pi\phi_{\rm str}}\frac{M}{\sqrt{\alpha_{\rm str}}}} \\ \text{for } m_{\rm eff}^{-1} \leq H^{-1} \Rightarrow \alpha_{\rm str} \geq \pi^2/6 \end{array}} \qquad \boxed{\begin{array}{c} M_{\rm str} = \sqrt{\left(2\sqrt{6}\pi\alpha_{\rm str}^{1/2} + \sigma_{\chi_{\rm str}}\right)\frac{H_{\rm str}}{2\pi\phi_{\rm str}}\frac{M}{\sqrt{\alpha_{\rm str}}}} \\ \text{for } m_{\rm eff}^{-1} \geq H^{-1} \Rightarrow \alpha_{\rm str} \leq \pi^2/6 \end{array}}$$

where $\phi_{\rm str}$ is the inflaton field value at the phase transition, $H_{\rm str}$ is the corresponding value of the Hubble parameter, and we set $\sigma_{\chi_{\rm str}} = 1$.

- The dimensionless tension of the cosmic strings

$$\boxed{G\mu \simeq \frac{1}{8}B\Big(\frac{\alpha_{\rm str}}{g^2}\Big)\left(\frac{M_{\rm str}}{m_{\rm Pl}}\right)^2}, \quad B(x) = \begin{cases} 1.04\, x^{0.195} & \text{for } 10^{-2} \lesssim x \lesssim 10^2 \\ 2.4/\ln(2/x) & \text{for } x \lesssim 0.01. \end{cases}$$

Intermediate Scale Strings and Gravity Waves

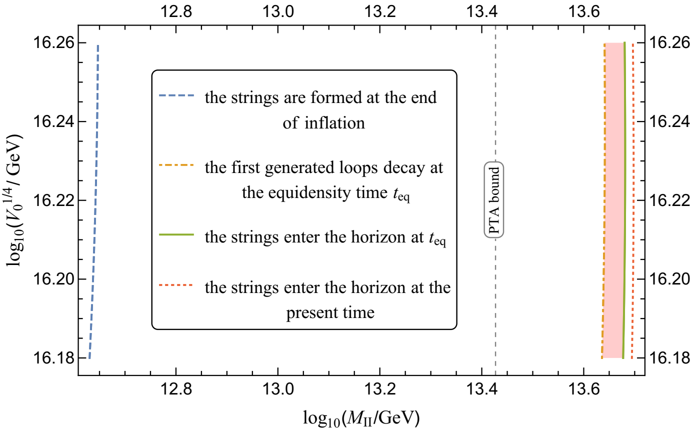


Figure: Intermediate breaking scales M_{II} for the unification scale $\log_{10}(V_0^{1/4}/\text{GeV}) \in [16.18, 16.26]$ for successful inflation with Coleman-Weinberg potential.

Chakrabortty, Lazarides, RM, Shafi JHEP 02 (2021) 114

Lazarides, Maji, Shafi, arXiv:2104.02016

4. *Possible NANOGrav-PPTA Tension, Inflation, and Resolution*

Power-law Approximation for Gravity Waves

- Compute the gravity wave spectrum from string loops within the frequency range $f \in [2.4, 12] \times 10^{-9}$ Hz.
- Fit the results to the power-law expression:

$$\Omega_{\rm GW}(f) = \Omega_0 \left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma}, \quad \Omega_0 = \frac{2\pi^2}{3H_0^2} A^2 f_{\rm yr}^2.$$

- Compare the calculated A and γ values with the NANOGrav 12.5 yr results.

Gravity Waves from Strings: NANOGrav-PPTA Tension

To find the gravitational waves background, the strings are assumed to:

1. be present in the horizon from a very early time (taken to be $t_F = 10^{-25}$ sec without loss of generality),
2. re-enter the horizon at the minimum time t_F to alleviate PPTA bound,
3. re-enter horizon at the maximum t_F allowed by NANOGrav 2σ limit.

Lazarides, Maji, Shafi, arXiv:2104.02016

Ellis, Lewicki, PRL 126, 041304 (2021)

Gravity Waves from Strings: NANOGrav-PPTA Tension

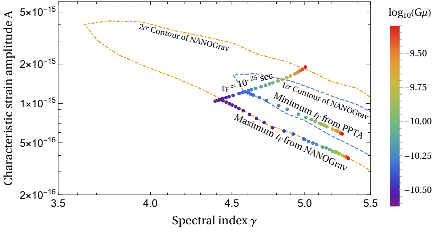


Figure: The amplitude A of the characteristic strain versus the spectral index γ for gravity waves from string loops of different $G\mu$ values is displayed on top of the 1σ and 2σ contours of NANOGrav.

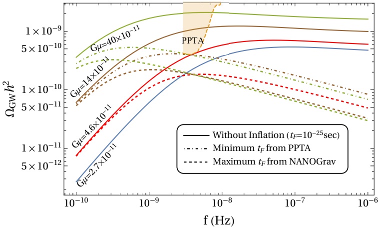


Figure: Gravity wave spectra with and without inflation for $G\mu = (2.7, 4.6, 14, 40) \times 10^{-11}$.

- The ranges of $G\mu$ satisfying 1σ and 2σ of NANOGrav with very low t_F values are $[4.6, 14] \times 10^{-11}$ and $[2.7, 40] \times 10^{-11}$, respectively.
- Apparent tension between NANOGrav and PPTA for $G\mu > 4.6 \times 10^{-11}$!

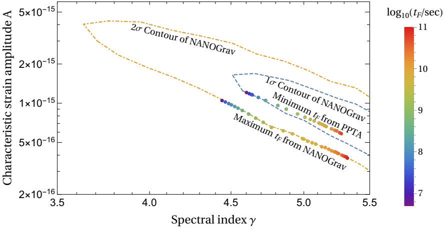
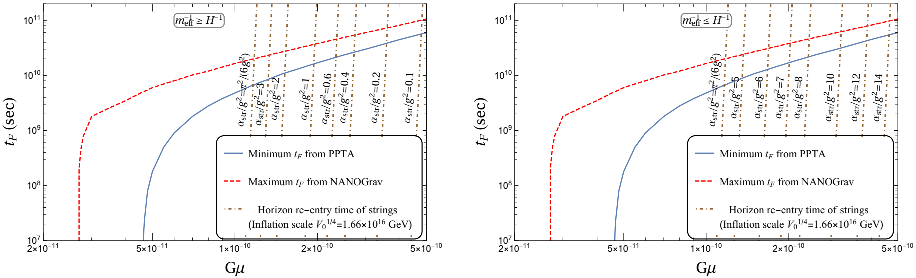


Figure:

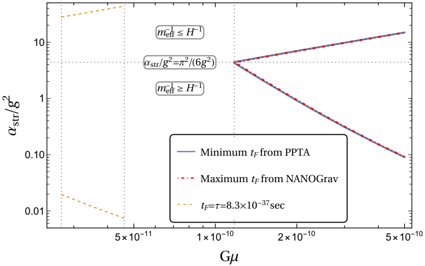
Inflation and Resolution of NANOGrav-PPTA tension

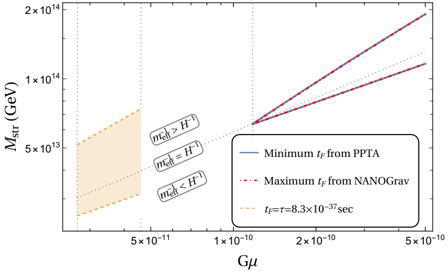


- We employ GUT-inflation with the Coleman-Weinberg potential.
- Horizon re-entry time of the strings as a function of $G\mu$ (brown dash-dotted curves) for different values of the ratio $\alpha_{\rm str}/g^2$ as indicated.

Lazarides, Maji, Shafi, arXiv:2104.02016

Inflation and Resolution of NANOGrav-PPTA tension

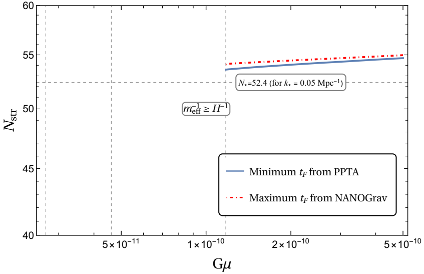


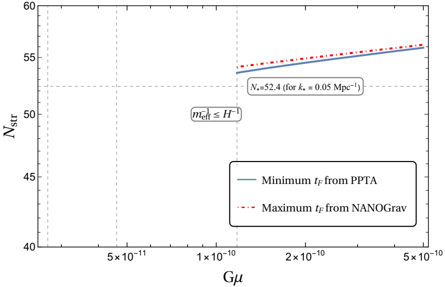


- For $2.7 \times 10^{-11} \leq G\mu \leq 4.6 \times 10^{-11}$, there is no tension ⇒ inflation not needed.
- For $G\mu > 1.1 \times 10^{-10}$, there exist two narrow allowed strips for $m_{\rm eff}^{-1} \geq H^{-1}$ ($\alpha_{\rm str} \leq \pi^2/6$) and $m_{\rm eff}^{-1} \leq H^{-1}$ ($\alpha_{\rm str} \geq \pi^2/6$) as indicated.

Lazarides, Maji, Shafi, arXiv:2104.02016

Inflation and Resolution of NANOGrav-PPTA tension





- In the range $2.7 \times 10^{-11} \leq G\mu \leq 4.6 \times 10^{-11}$, $N_{\rm str}$ can be arbitrary small.

Lazarides, Maji, Shafi, arXiv:2104.02016

5 *Intermediate-mass Monopoles and Strings*

Intermediate-mass Monopoles with Strings

Number density at production

Dilution during Inflation

Dilution from Inflaton oscillation

Monopole yield after reheating :

$$\boxed{Y_M \simeq \frac{\frac{\xi^{-3}}{10}\ \exp(-3N_{\text{str}})\ \left(\frac{\tau}{t_r}\right)^2}{\frac{2\pi^2}{45} g_* T_r^3}}$$

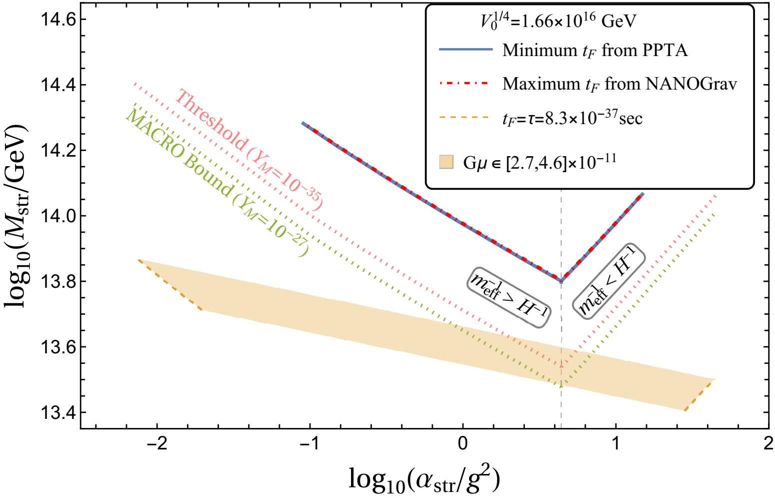
Entropy density after reheating

- MACRO bound: $Y_M \lesssim 10^{-27}$.

Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

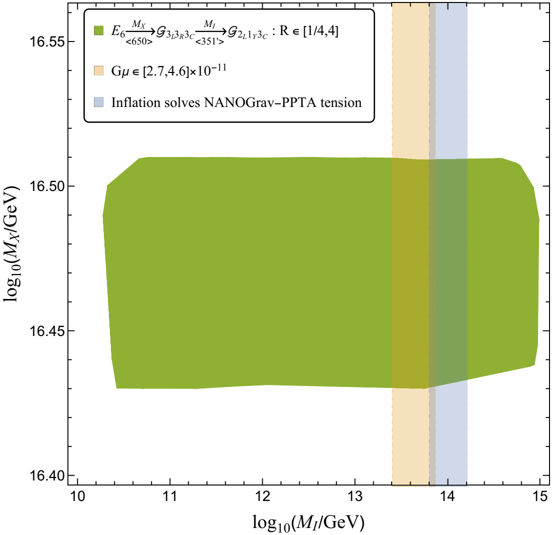
- Adopted threshold for observability: $Y_M \gtrsim 10^{-35}$.

Intermediate-mass Monopoles with Strings



Lazarides, Maji, Shafi, arXiv:2104.02016

Example



Lazarides, Maji, Shafi, arXiv:2104.02016

6 *Summary*

Summary

- Many Non-SUSY GUTs are incompatible with the Super-K proton lifetime bound unless we take the effect of threshold correction into account.
- Effect of dimension-5 operators can improve the proton lifetime for specific breaking paths.
- Topological defects e.g., domain walls, monopoles conflict cosmological observations unless they are inflated away.

Summary

- Inflation can alleviate PPTA bound on the gravitational wave radiation from cosmic string loops.
- We present a concrete example of realistic GUT models supplemented by inflation with a Coleman-Weinberg potential.
- The NANOGrav results can be made compatible with the earlier pulsar timing array bound provided that the strings re-enter the horizon at adequately late times.

Thank You

Backup Slides

Pulsar Timing Arrays

- Pulsars are rapidly spinning neutron stars with a strong magnetic field ⇒ Radiate beam of radio waves.
- Repeating pulses are observed as the radio beam intersects the observers periodically.
- Millisecond pulsar (MSP) produces exceedingly stable and regular pulse profile ⇒ “Perfect Clock”.

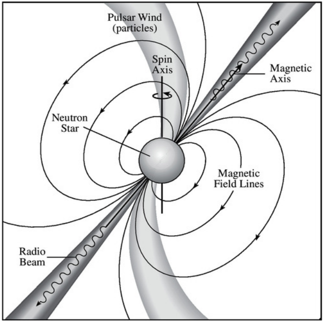


Image source: K.R. Lang, NASA’s Cosmos

- Measurement of the time of arrival (ToA) of pulses can reveal tiny distortion of spacetime fabric due to gravity waves (GWs) ⇒ Pulsar timing!

Pulsar Timing Arrays

- Difference between observed ToA and the expected ToA from timing model gives time residual.
- Time residual contains information about other signals like GWs.
- Impossible to distinguish between GWs signal and other source of signal in the timing residual of a single pulsar.
- Need correlations between the timing residuals of different pulsars ⇒ Pulsar Timing Array (PTA).

Pulsar Timing Arrays

- PTA experiments: EPTA taking data from 42, NANOGrav observing 25, and PPTA taking observations from 45 MSPs.

- Gravity waves generate unique quadrupolar correlations between timing residuals of pulsar pairs.

- Correlations depend on the angular separations between the pulsar pairs and follow the Hellings and Downs correlation curve. **APJ. 265, L39 (1983)**

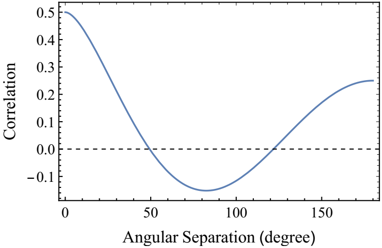


Figure: Hellings and Downs curve.

Generation of Stochastic Gravity Waves: Analytic Approximation

Three different regimes for decaying cosmic string loops:

- Loops that are produced and decay during radiation dominance. They produce the plateau in the spectrum.
- Loops that are produced during radiation dominance but decay during matter dominance. They generate a sharply peaked spectrum at lower frequencies, which is responsible for the overall peak of the spectrum.
- Loops that are produced and decay during matter domination. They also generate a sharply peaked spectrum which is though overshadowed by the previous case.

Sousa, Avelino, Guedes, arXiv:2002.01079

GWs spectrum and observational prospects