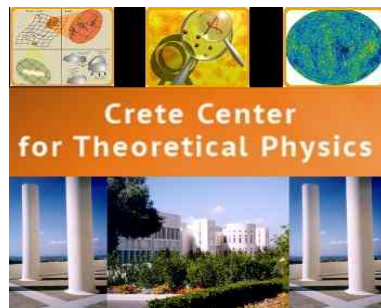


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QFTs on De Sitter, holography and Coleman-de Lucia transitions

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Based on earlier work:

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Introduction

- Quantum Systems can have in general many ground states (minima).
- Transitions to lower minima proceed via tunneling effects.
- As was understood by Coleman, this is mediated by instanton solutions of the Euclidean theory.
- Euclidean instantons provide, eventually, boundary conditions for the subsequent Lorentzian evolution.
- In the absence of gravity, instanton solutions start in the “false vacuum” and end in the “true vacuum”.
- In the presence of gravity this process is richer.
- It was first studied by Coleman and De Luccia in 1980 (the corresponding solution is the Coleman-De Luccia instanton).

- In the presence of gravity, **the absolute value** of the ground state energies are important, as they determine the space-time geometry (**de Sitter, Minkowski or AdS**).
- Decays to de Sitter or Minkowski turn out to be qualitatively similar to that in the absence of gravity.
- Decays in **the AdS case** are different for two reasons:
 - ♠ **Not all decays are allowed**: gravity stabilises some vacua.
 - ♠ The end-point of the decay is **not the true vacuum** but an **FRW universe that undergoes (singular) collapse**.

• There are some “folk beliefs” in the fields concerning CdL decays since the CdL Paper:

♠ The thin-wall approximation is applicable generically once the vacua are near degenerate (compared to the Planck scale).

♠ The instanton solutions generically exist!

♠ Later a condition was derived in the thin wall approximation:

Harlow

We parametrize

$$V_{false} = -\frac{12}{l_f^2} \quad , \quad V_{true} = -\frac{12}{l_t^2}$$

and consider an AdS inside the bubble, and AdS outside, separated by a thin wall with tension σ .

Then σ must satisfy the inequality

$$\frac{\sigma}{M_P^2} \leq 4 \left(\frac{1}{l_f} - \frac{1}{l_t} \right)$$

- It is not clear how to relate σ to the details of the scalar potential.
- AdS CdL decays has received attention in the recent past because:
 - ♠ String theory is full of AdS vacua and therefore the tunneling issue is relevant
 - ♠ The **AdS/CFT correspondence** provides a dual view to such processes.

The logistics of CDL transitions

- To compute the transition rate, we must first find the CdL instanton solution in Euclidean signature: $\bar{g}_{ab}, \bar{\phi}_i$.
- This solution is a Euclidean bubble, with the false vacuum asymptotically and the true vacuum inside.
- Then, the tunneling rate is given by

$$\Gamma = A e^{-S_E(\bar{g}_{ab}, \bar{\phi}_i)}$$

- The subsequent evolution, involves a real time evolution of the “bubble” with initial conditions given by the Euclidean instanton solution.

- The bubble expands with the speed of light until it covers the whole space.

- Two hypotheses are made typically:

- ♠ **The instanton solution has maximal symmetry** : $O(d+1)$ invariance in Euclidean case:

$$ds^2 = d\xi^2 + a^2(\xi)d\Omega_d^2 \quad , \quad \phi_i(\xi) \quad , \quad \xi \in [0, +\infty)$$

The solution must be **regular** at the center and must approach the false vacuum at $\xi \rightarrow +\infty$.

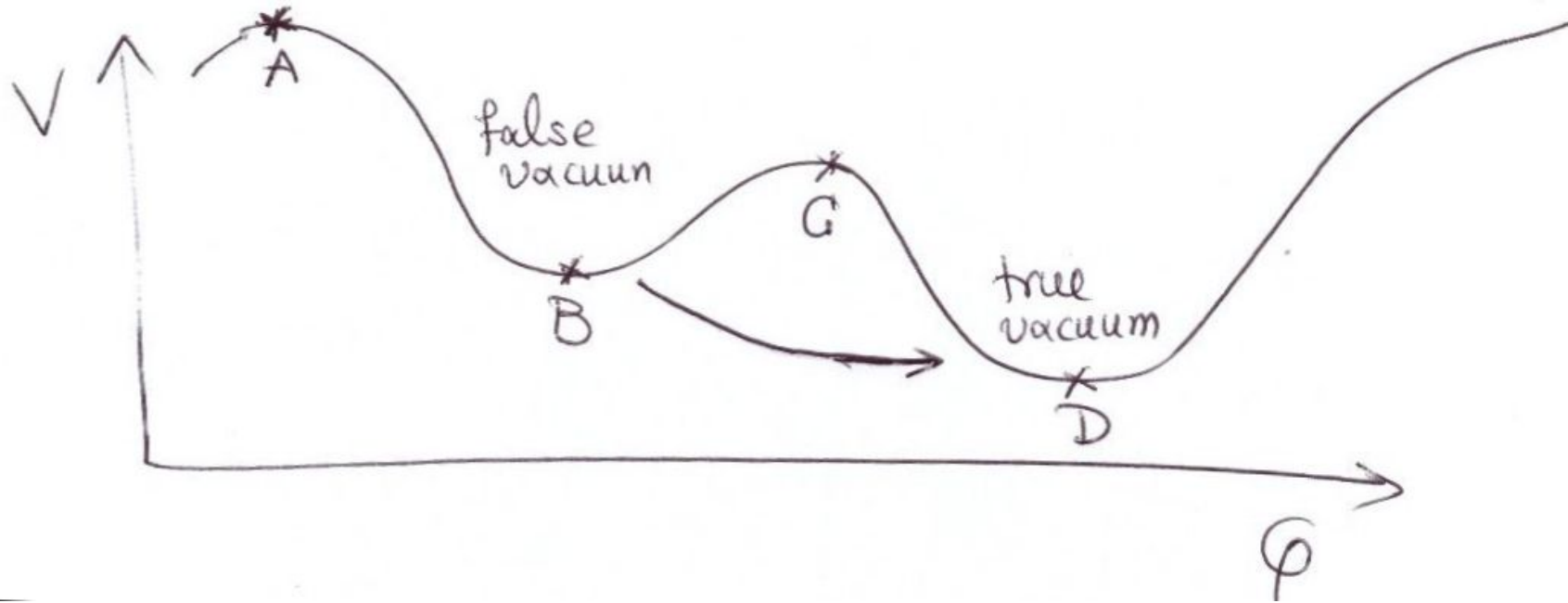
- ♠ It describes a “bubble” of the true vacuum inside a space-time that approaches the false vacuum at infinity.

- ♠ The second assumption is **the thin-wall approximation**: the solution is approximated with the true vacuum inside, the false vacuum outside and an infinitesimally thin wall separating the two.

- It was shown by Coleman that in the absence of gravity, this approximation is good if the difference in energy of the true and false vacuum is small compared to the height of the barrier.
- In the presence of gravity in AdS it was not clear when this is applicable.
- I would be considering for simplicity a gravitational theory with a single scalar.

$$S_{grav} = M^{d-1} \int d^{d+1}x \sqrt{g} \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right]$$

- The potential landscape in the gravity theory looks as follows.



- The $O(d+1)$ -invariant instanton ansatz

$$ds^2 = d\xi^2 + a^2(\xi)d\Omega_d^2 \quad , \quad \phi(\xi) \quad , \quad \xi \in [0, +\infty)$$

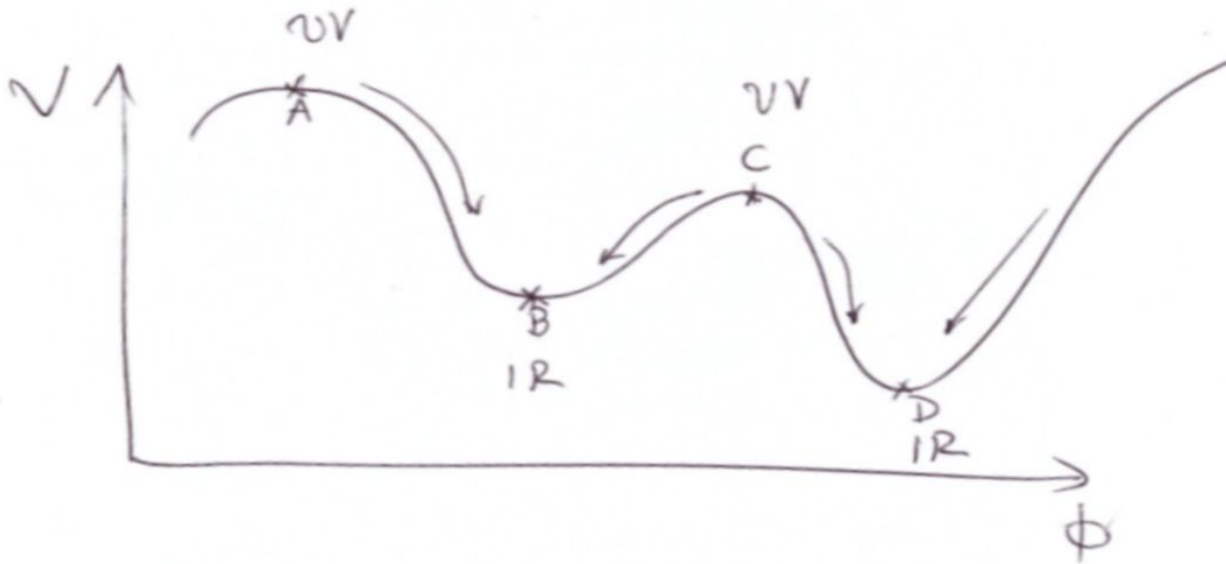
amounts to

$$\phi(0) = \phi_D \quad , \quad \phi(\infty) = \phi_B$$

while the metric is **near-AdS** both near B and D for an $\text{AdS} \rightarrow \text{AdS}$ transition.

Holography

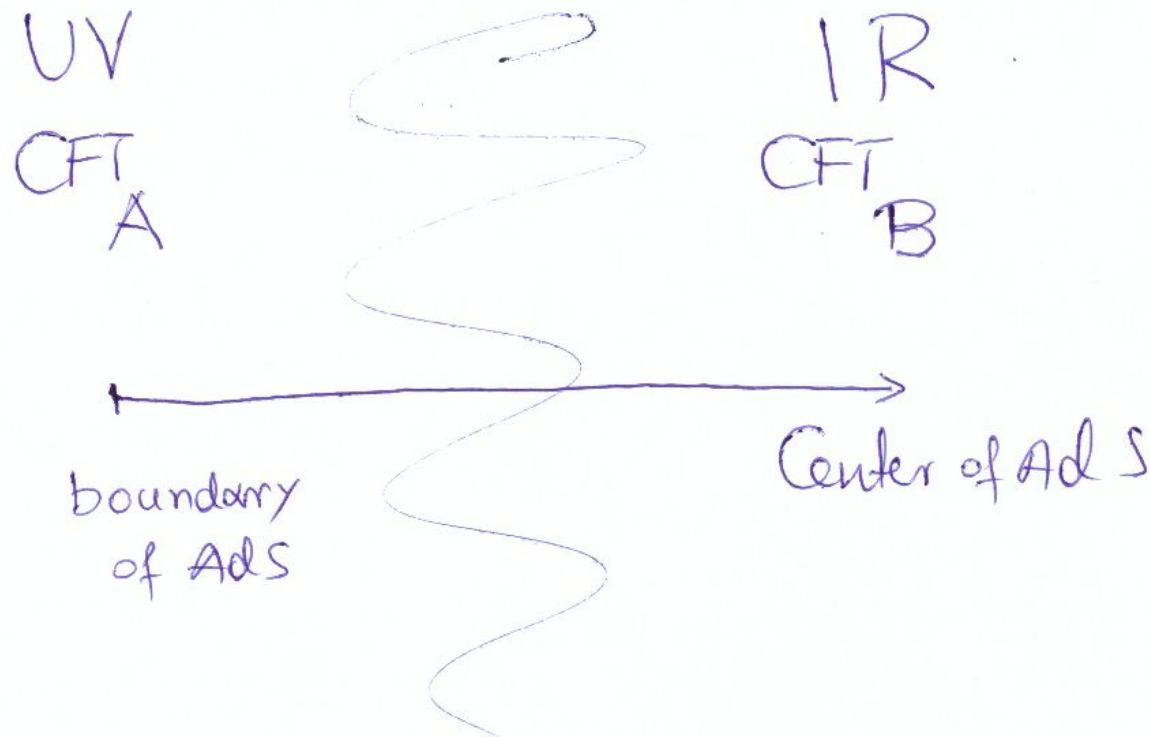
- Holography interprets the same potential landscape of the gravitational theory in terms of **the dual QFT**.



- ϕ is dual to a scalar operator in the dual QFT.
- At the extrema of the potential, $V'(\phi_A) = 0$, we have the solution:

$\phi = \text{constant}$ and the metric is AdS \rightarrow CFT in d dimensions.

- According to Wilson a QFT is an RG Flow between a UV CFT to an IR CFT.
- The maxima are UV fixed points (CFTs). The minima are IR fixed points* (modulo an exception).
- A (regular) solution where ϕ varies from A to B is dual to an RG flow from the UV CFT_A to the IR CFT_B.



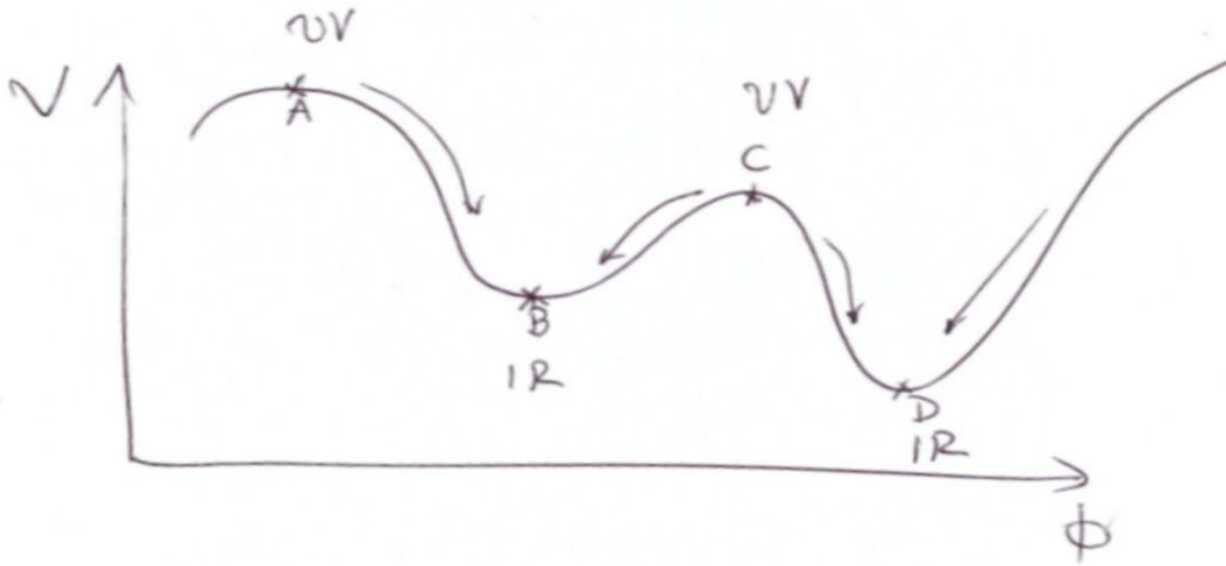
- The metric varies from AdS (the boundary=UV in QFT) to AdS (the center=IR in QFT).

- The solution ansatz is

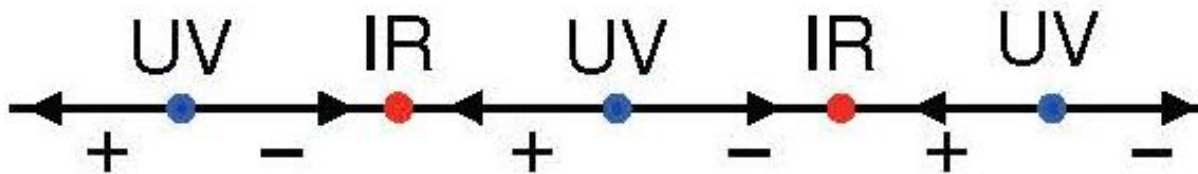
$$ds^2 = du^2 + a(u)^2 ds_d^2 \quad , \quad ds_d^2 = g_{ij} dx^i dx^j \quad , \quad \phi(u)$$

- The metric g_{ij} is also the metric at the AdS boundary ($u \rightarrow -\infty$)
- According to the duality, this solution is dual to the ground state of the QFT defined on the metric g_{ij} .
- If g_{ij} is Minkowski_d \rightarrow QFT on a flat metric
- If g_{ij} is a S^d \rightarrow QFT on a d-sphere.

The analytic continuation in this case gives a QFT on a dS_d metric.



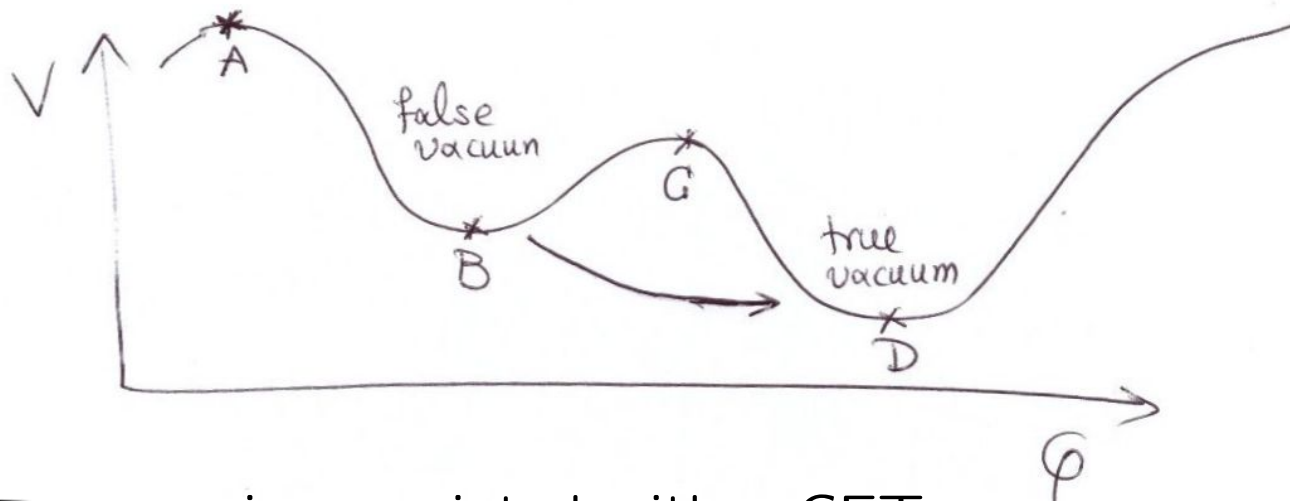
- Therefore, the “vanilla” holographic RG flows look like in field theory



with UV and IR fixed points interspersed.

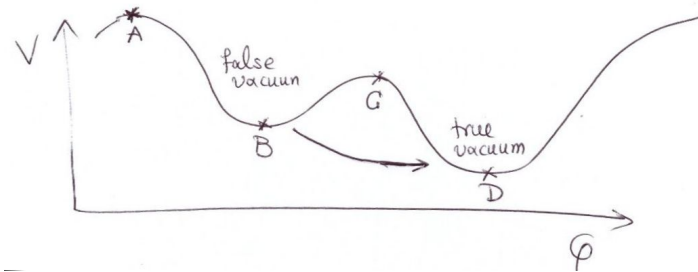
Holographic interpretation of CdL

We may now **interpret**, the CdL instanton solution (and the associated transition) in holography:



- The false vacuum is associated with a CFT_B .
- The true vacuum is associated with a CFT_D .
- ♠ This is why, it has been claimed that **the existence of a CdL instanton implies that the CFT_B is non-perturbatively unstable to decay to the CFT_D .**

- But the story is subtler!



- The CdL instanton solution, is a flow that starts from a minimum of the potential.
- Such (regular) flows do NOT exist in general. They exist only if the potential is special.

Ghosh+Kiritsis+Nitti+Witkowski

- The $O(d+1)$ -invariant instanton ansatz

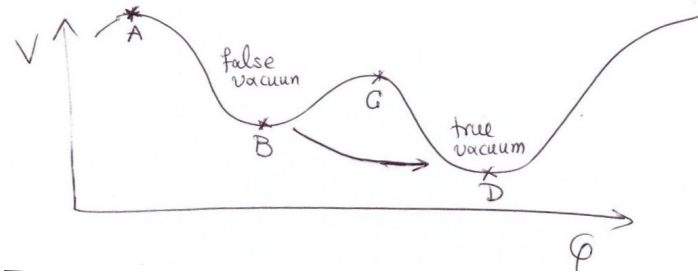
$$ds^2 = du^2 + a^2(u)d\Omega_d^2 \quad , \quad \phi(u) \quad , \quad u \in (-\infty, +\infty)$$

amounts to a solution with

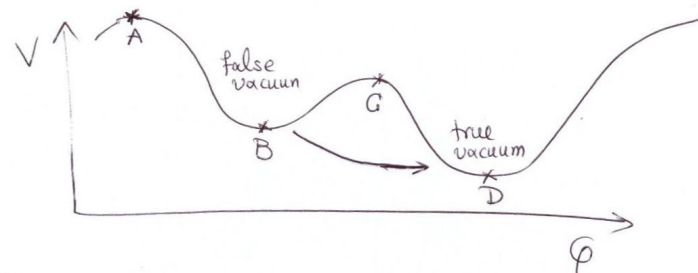
$$\phi(-\infty) = \phi_B \quad , \quad \phi(u_0) = \text{near } \phi_D$$

while the metric is **near-AdS** both near B and D for an $\text{AdS} \rightarrow \text{AdS}$ transition.

- This solution is interpreted as follows in the CFT_B associated to point D (false vacuum):



- ♠ It is a ground state of this CFT, in which **scale symmetry is broken** (because the scalar runs, and the metric is not exactly AdS).
- ♠ The operator **dual to ϕ** (which is an irrelevant operator here, as B is a minimum) has a non-zero vev which **breaks the scale symmetry spontaneously**.
- ♠ This non-zero vev triggers **an RG Flow towards the true vacuum** (at point D).
- ♠ The CFT_B is defined on the sphere S^d .



♠ All the above imply that the CFT_B on S^d has **two competing ground states**:

(a) **A conformally-invariant ground state** dual to the $\phi = \phi_B$ solution with exact AdS metric.

(b) **A conformally non-invariant ground state** dual to the flowing solutions towards point D , where the $O(d+1,1)$ conf. symmetry is broken to $O(d+1)$.

- **This second solution is the CdL instanton.**

CONCLUSION: A necessary and sufficient condition for the **existence of a CdL instanton** that drives tunneling from $B \rightarrow D$ is that **CFT_B on a sphere has a symmetry breaking ground-state.**

The strategy

We have two, a priori distinct, problems:

PROBLEM A: a gravitational theory in $d+1$ dimensions, with several ground states, and **CdL instantons** driving transitions between them.

PROBLEM B: a **(holographic) QFT_d** in d dimensions, that is dual to a gravitational theory in $d+1$ dimensions.

In problem B :

(a) Holography maps what happens in the QFT to what happens in the gravitational theory and vice versa.

(b) The QFT fixes “ambiguities” in the gravitational theory (like boundary conditions).

- We shall “define” the gravitational theory via holography.
- We shall find under what conditions instanton solutions exist.
- We shall examine whether the thin-wall approximation is valid in the string-theory/holographic regime.
- We shall show that CdL transitions have implications for CFTs on de Sitter space only.
- We shall analyze the physical implications of the subsequent Lorentzian evolution and discuss the observability of the bulk singularity from the boundary.

Thin vs thick walls

- We define the radius at the center of the wall \bar{r} as the radius at the locus where φ has interpolated half way between its boundary values φ_f (false vacuum) and φ_0 (true vacuum), i.e.

$$\bar{r} \equiv \alpha e^{A(\bar{u})}, \quad \text{with} \quad \varphi(\bar{u}) = \frac{\varphi_f + \varphi_0}{2}.$$

- The wall itself denotes the region where the interpolation between φ_f and φ_0 effectively occurs.
- There is no universal definition for this and here we make a choice. We define the wall as the interval $[u_{\text{in}}, u_{\text{out}}]$ with

$$\varphi(u_{\text{in}}) = \frac{\varphi_f + \varphi_0}{2} - \gamma \frac{\varphi_f - \varphi_0}{2}, \quad \varphi(u_{\text{out}}) = \frac{\varphi_f + \varphi_0}{2} + \gamma \frac{\varphi_f - \varphi_0}{2},$$

with a parameter $\gamma < 1$ that controls how close $\varphi(u_{\text{in}})$ is to φ_0 and $\varphi(u_{\text{out}})$ to φ_f .

- In all practical examples we choose

$$\varphi(u_{\text{in}}) = \varphi_0 + 0.12(\varphi_f - \varphi_0) \quad , \quad \varphi(u_{\text{out}}) = \varphi_f + 0.12(\varphi_0 - \varphi_f).$$

- We can then define

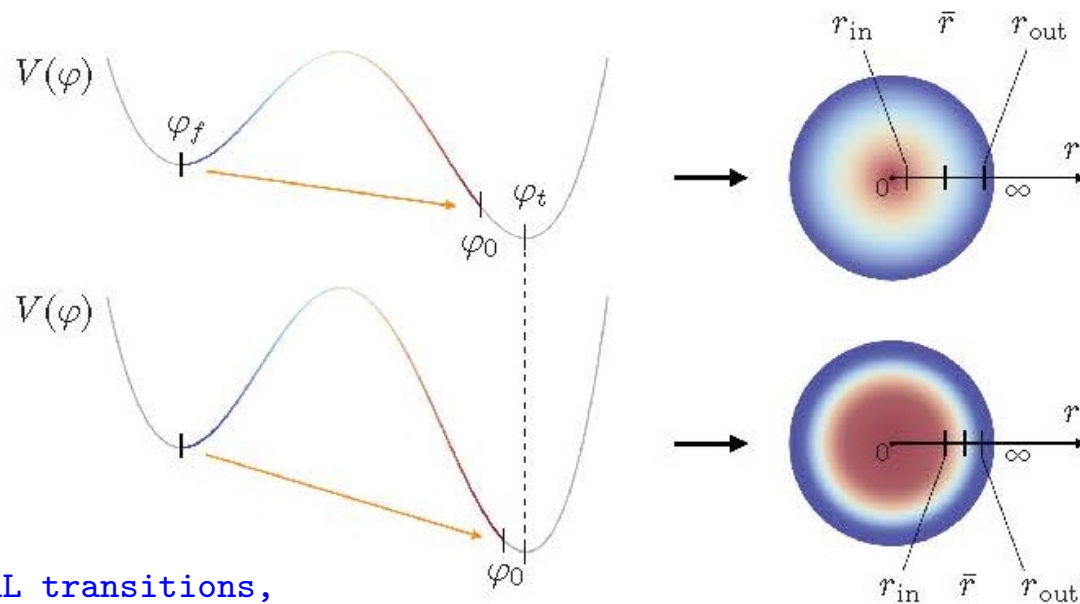
$$r_{\text{out}} = \alpha e^{A(u_{\text{out}})} \quad , \quad r_{\text{in}} = \alpha e^{A(u_{\text{in}})}$$

as the radii corresponding to the outer and inner edge of the wall.

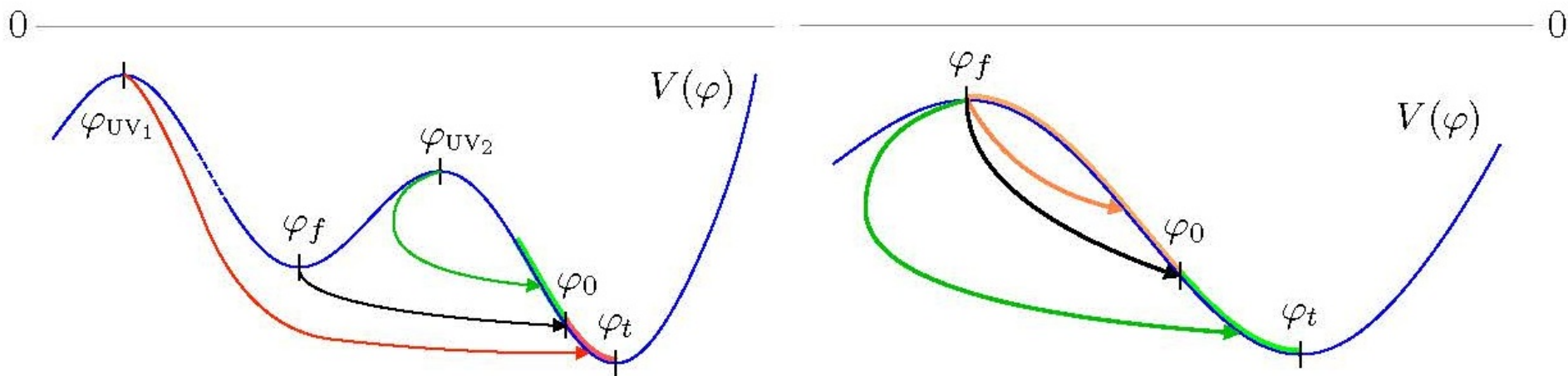
- We define the ‘thin-ness’ parameter η as

$$\eta \equiv \frac{r_{\text{out}} - r_{\text{in}}}{\bar{r}} = \frac{e^{A(u_{\text{out}})} - e^{A(u_{\text{in}})}}{e^{A(\bar{u})}} .$$

- We refer to a wall as ‘thin’, if $\eta \ll 1$ with $\eta \rightarrow 0$ the limit of a vanishingly thin wall.



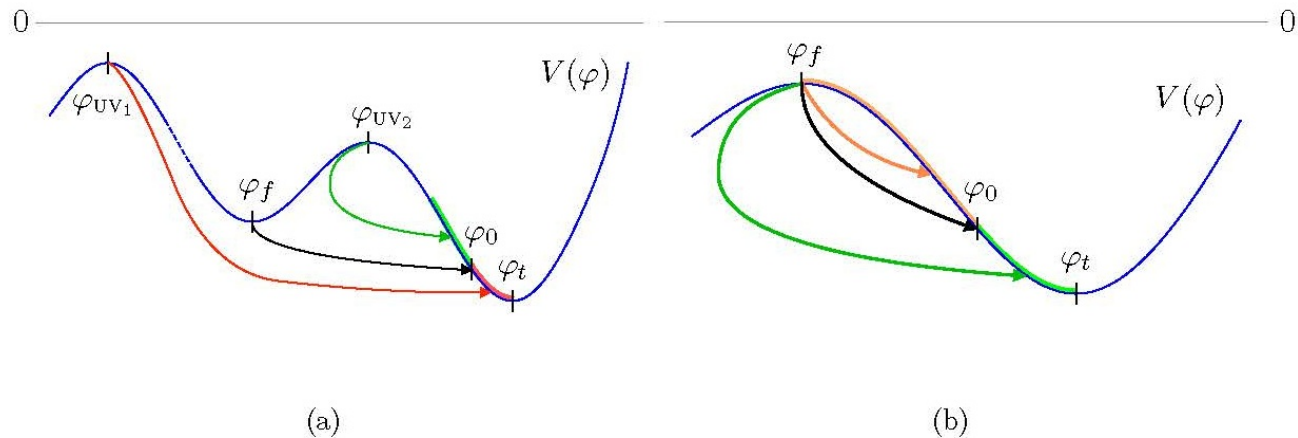
Instantons vs Skipping RG flows



- By studying the ^(a)gravitational equations we can ^(b)prove the following connection.

If there is a regular $O(d+1)$ -invariant instanton solution describing tunnelling from φ_f to φ_0 (black arrow) then also exist:

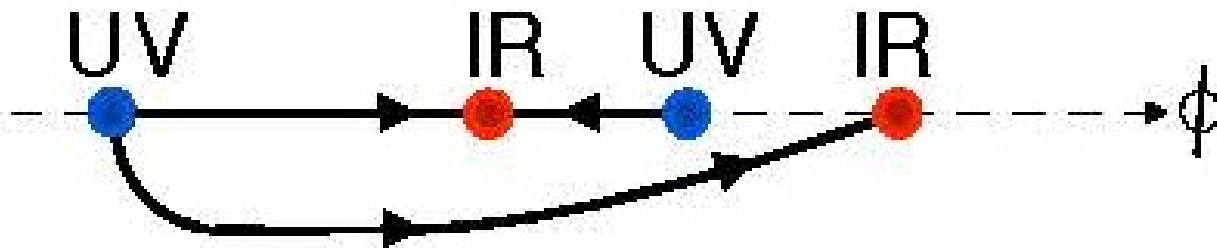
(a) A holographic RG flow solutions from the UV fixed point at φ_{UV1} to an end point in the red region that skips past the other maximum at φ_{UV2} .



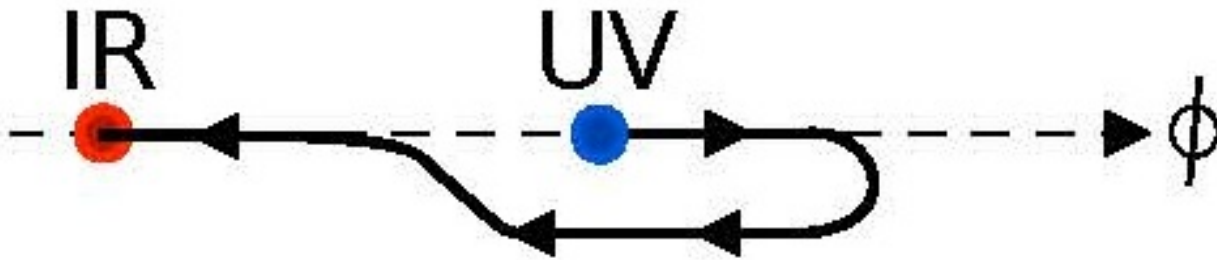
- This is a so-called skipping holographic flow (red arrow).

Kiritsis+Nitti+Silva-Pimenta

- This flow is non-perturbative from the point of view of QFT:



- In addition, the potential will allow for flows leaving φ_{UV_2} to the left before changing direction and flowing to an end point in the green region.
- This is a so-called bouncing flow.

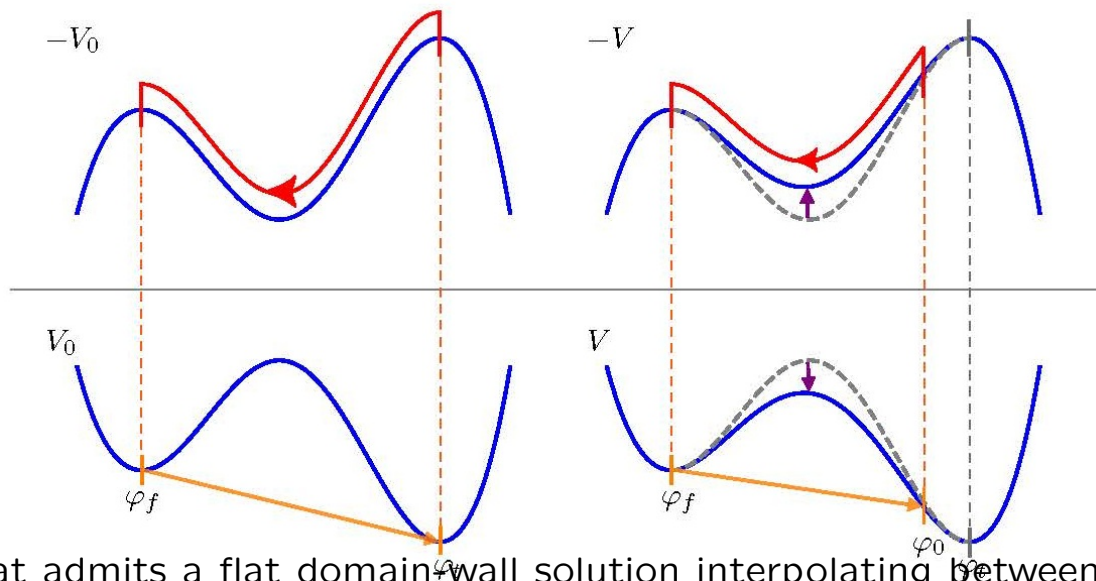


- All such holographic RG flows are for a QFT defined on S^d .

How to construct the relevant potentials

- We start from a potential V_0 that admits flat-sliced skipping RG flows.
- Such potentials can be reversed engineered.
- Then we modify slightly the potential so that it has spherical domain wall solutions

$$V(\varphi) = V_0(\varphi) + v(\varphi)$$



Left: Potential V_0 that admits a flat domain wall solution interpolating between the two minima at φ_f and φ_t , indicated by the orange arrow. In the mechanical picture in the inverted potential $-V_0$ this corresponds to a trajectory for a particle that is released from rest at φ_t and coming to rest again at φ_f (red trajectory).

Right: Potential V that differs from V_0 only by exhibiting a lower barrier separating the two minima at φ_f and φ_t . In turn the inverted potential $-V$ exhibits a shallower valley separating φ_f and φ_t than $-V_0$. A particle released from φ_t in $-V$ will overshoot φ_f as the shallower valley leads to a lower velocity and hence less friction. By reducing the initial potential energy and releasing the particle lower down the slope at some φ_0 , a trajectory can be found so that the particle neither over- nor undershoots, but comes to rest exactly at φ_f (red trajectory). In the Euclidean picture, this corresponds to an $O(D)$ -instanton describing tunnelling from φ_f to φ_0 (orange arrow).

A numerical study

- We parametrize V_0 as

$$V_0(\varphi) = -\frac{6}{l_f^2} - \frac{\Delta(3-\Delta)}{2l_f^2}\varphi^2 + \frac{\Delta(1-\Delta)}{l_f^2\varphi_t}\varphi^3 + \frac{\Delta^2(16-3\varphi_t^2)}{32l_f^2\varphi_t^2}\varphi^4 \\ + \frac{\Delta^2}{8l_f^2\varphi_t}\varphi^5 - \frac{\Delta^2}{24l_f^2\varphi_t^2}\varphi^6$$

This has two AdS minima at $\varphi_f = 0$ and φ_t separated by a barrier.

- The parameters are, φ_t, Δ, l_f .
- $\Delta > 3$ is the dimension of the (irrelevant) scalar operator in the false vacuum.

$$v(\varphi) = -\frac{64v_0}{l_f^2} \frac{\varphi^3(\varphi_t - \varphi)^3}{\varphi_t^6}$$

with an extra parameter v_0 .

- The dimensions at φ_f and φ_t are not affected by $v(\varphi)$.
- We choose $d = 3$.
- The dual CFT has two scales: one is the curvature of the sphere, R_s and the other is the non-trivial vev $\langle O \rangle$ of the operator dual to ϕ .
- All physical quantities depend on a single dimensionless parameter, the **dimensionless curvature**

$$\mathcal{R} \equiv \frac{R_u}{\langle O \rangle^{\frac{2}{\Delta}}}$$

- We derive numerically $O(4)$ -instanton solutions for various potentials (varying their parameters).
- For a given instanton solution, we then record the value of the end-point φ_0 , extract the dimensionless curvature \mathcal{R} , compute the thinness parameter η and calculate **the instanton action B**

$$B \equiv -S_{\text{instanton}} = -(M\ell_f)^2 V_3 \mathcal{B} \quad , \quad (M\ell_f)^2 \sim N_c^2$$

- Reminder: the flow can never end at the true minimum if the slices are curved (as here). It ends at a point $\varphi_0 \neq \varphi_t$.

- By varying \mathcal{R} , φ_0 varies.

$\Delta = 30.1, \quad \varphi_t = 1, \quad \eta_{\text{flat}} = 0.211$				
v_0	φ_0	\mathcal{R}	η	$B/(M\ell_f)^2$
1	0.999999999999999996	0.36	0.216	123
2	0.99999999999999995	0.73	0.222	73
5	0.99999999999996	1.90	0.243	31
10	0.9999999985	3.9	0.287	13
20	0.999965	8.3	0.466	3.5

- We observe that the closer φ_0 to φ_t , the smaller \mathcal{R} and η and the larger $B/(M\ell_f)^2$,

$$\varphi_0 \rightarrow \varphi_t : \quad \mathcal{R} \downarrow, \quad B/(M\ell_f)^2 \uparrow .$$

- We can also prove that $\eta \geq \eta_{flat}$.

The thin-wall limit revisited

- To achieve $\eta \rightarrow 0$ we must first achieve $\eta_{flat} \rightarrow 0$.

- For our potential, we have an analytic expression of η_{flat} :

$$\eta_{flat} = 2 e^{-\frac{l_f/l_t - 1}{4\Delta} [\gamma^2 - 2 \log(1 - \gamma^2)]} \sinh \left(\frac{1 + l_f/l_t}{\Delta} \tanh^{-1} \gamma \right),$$

which depends on Δ , l_f/l_t and (γ), where γ was defined by

$$\varphi(u_{in}) = \frac{\varphi_f + \varphi_0}{2} - \gamma \frac{\varphi_f - \varphi_0}{2}, \quad \varphi(u_{out}) = \frac{\varphi_f + \varphi_0}{2} + \gamma \frac{\varphi_f - \varphi_0}{2},$$

- We obtain $\eta_{flat} \rightarrow 0$ in the following limit

$$\Delta \rightarrow +\infty, \quad \Delta \frac{l_t}{l_f} \rightarrow +\infty$$

- In this limit

(a) $\varphi_f \rightarrow \varphi_t$

(b) The potential barrier goes to $+\infty$.

- We also have

$$\Delta(\varphi_f) = \Delta \rightarrow \infty \quad , \quad \Delta(\varphi_t) = \Delta \frac{\frac{l_t}{l_f}}{1 - \frac{l_t}{l_f}} \rightarrow \infty \quad , \quad \frac{l_t}{l_f} < 1$$

- These arguments and conditions generalize qualitatively to more general potentials

The thin-wall limit vs holography

- We have seen that we need to have large dimensions for the irrelevant operators near a false vacuum in order to be in the thin wall approximation.
- From the bootstrap program it seems that we obtain upper bounds on the dimensions of the least irrelevant operators:

$$\Delta_{irr} \leq \Delta_0$$

where Δ_0 is dimension-dependent only and is a number of order one.

- In the multiscalar case, instanton solutions extend mostly along scalars with the lowest (irrelevant) dimensions.
- Therefore, **the bound above excludes the existence of thin-walled instantons.**

The Lorentzian continuation

- At the end of tunneling, the instanton provides initial conditions for the subsequent Lorentzian evolution.

- We start with the instanton solution for the metric

$$ds^2 = d\xi^2 + r^2(\xi) \left(d\theta^2 + \sin^2(\theta) d\Omega_{d-1}^2 \right) ,$$

- We need two analytic continuations of the above metric. One gives all points that are spacelike separated from the center of the bubble (**outside the bubble**).

$$\theta = \frac{\pi}{2} + i\chi \quad , \quad ds_{\text{out}}^2 = d\xi^2 + r^2(\xi) \left[-d\chi^2 + \cosh^2(\chi) d\Omega_{d-1}^2 \right]$$

- The metric multiplying $r^2(\xi)$ is that of unit radius dS_d .
- For this analytic continuation, the Euclidean equations are unchanged.
- Therefore the instanton solution, can be trivially continued in this region.

- This implies that $\phi(\xi)$ and $r(\xi)$ from the instanton solution, give the (Lorentzian) metric in this regime.

- To access the region ‘inside the bubble’ we instead continue as

$$\xi = i\tau, \quad \theta = i\eta \quad , \quad a(\tau) = -ir(i\tau)$$

- The metric is now

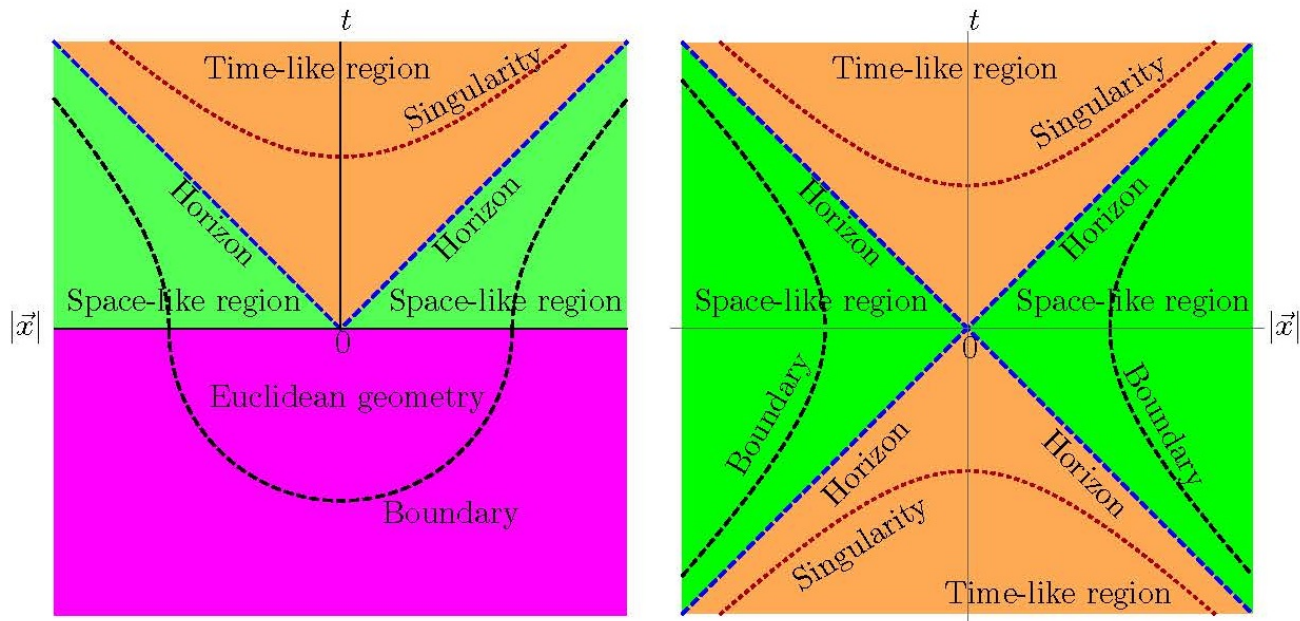
$$ds_{\text{in}}^2 = -d\tau^2 + a^2(\tau) \left(d\eta^2 + \sinh^2 \eta d\Omega_{d-1}^2 \right) \quad ,$$

- It describes a FRW universe with hyperbolic slicing with scale factor $a(\tau)$.
- In Euclidean signature, the dilaton $\varphi = \varphi(\xi)$ was just a function of ξ .
- In this Lorentzian continuation, we instead have $\varphi = \varphi(\tau)$.
- The equations of motion are the Euclidean ones but with the crucial change $V \rightarrow -V$.
- For a continuous space-time and solution for the scalar, the two metrics and the scalar field have to be matched at the center of the bubble.

- The relevant matching conditions are

$$\begin{aligned} \varphi(\xi = 0) &= \varphi(\tau = 0), & \frac{d}{d\xi}\varphi(\xi)|_{\xi=0} &= \frac{d}{d\tau}\varphi(\tau)|_{\tau=0}, \\ r(\xi = 0) &= a(\tau = 0), & \frac{d}{d\xi}r(\xi)|_{\xi=0} &= \frac{d}{d\tau}a(\tau)|_{\tau=0}. \end{aligned} \quad (1)$$

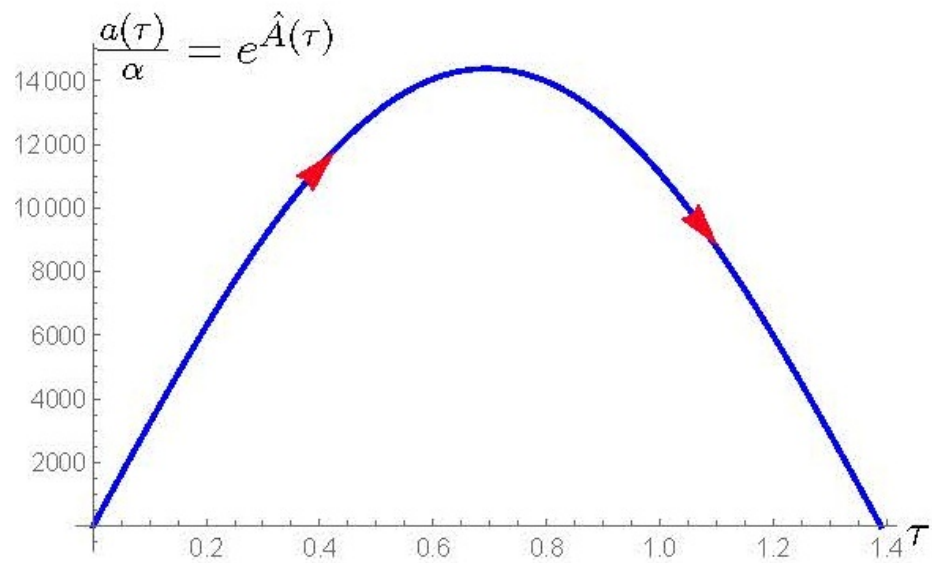
- The conditions above imply that all the geometric quantities are continuous across $r(0) = 0$.
- In the region ‘inside the bubble’ the equations of motion differ from those in the Euclidean setting and hence the Euclidean solution itself cannot be continued into this region.
- Instead, we have to solve afresh for $a(\tau)$ and $\varphi(\tau)$.
- However, the Euclidean instanton will provide the initial conditions for this analysis via the matching conditions above.



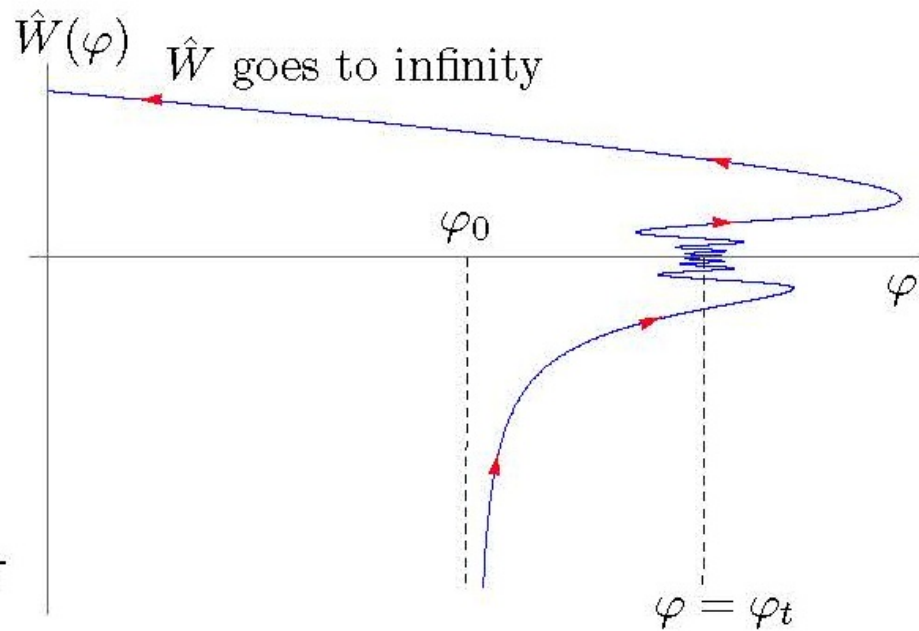
(a): Space-time diagram of the CdL geometry including the Lorentzian continuation. The purple region is the Euclidean geometry. Green and orange are space-like and time-like regions from the origin respectively. The boundary is denoted as the dashed black line whereas the blue dashed line denotes the horizon. Inside the time-like region, there is a singularity which is denoted as the red dashed line. This singularity is hidden behind the horizon. **(b)**: Space-time diagram for a holographic RG flow for a QFT on dS_d , shown for comparison with the CdL case. The causal structure is identical to two copies of the Lorentzian continuation of the CdL geometry, connected at the surface $t = 0$. The solution in the green regions has been previously studied in [?]. The solution in the orange regions can be computed using the same method as in the CdL case. By analogy, we once more expect singularities in the orange regions.

The bulk singularity

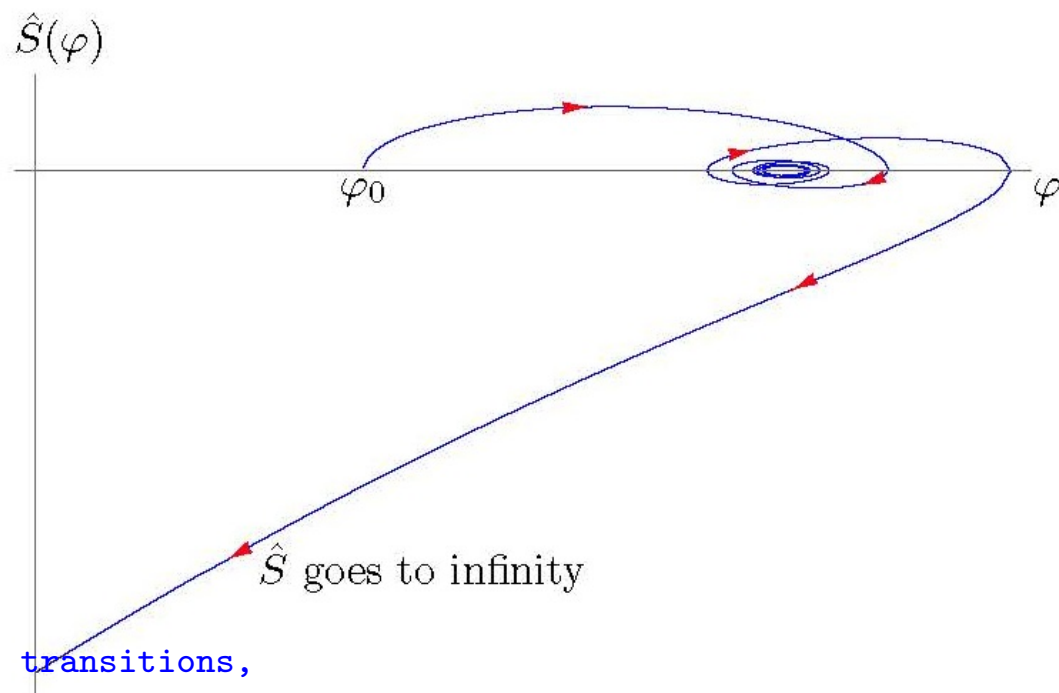
- Coleman has argued the existence of a singularity based on the thin wall approximation.
- Banks has given plausibility arguments about its existence in a more general context.
- We can prove that generically a singularity exists.
- This is based on a proof that the scale factor $a(\tau)$ starts increasing in the beginning, but always turns around and starts decreasing again as $\ddot{a} < 0$.
- Therefore $\dot{a} > 0$ turns to $\dot{a} < 0$ and the friction in the equations becomes negative friction, that send the system to $\phi = \infty$.
- This is a geometric singularity.



(a)

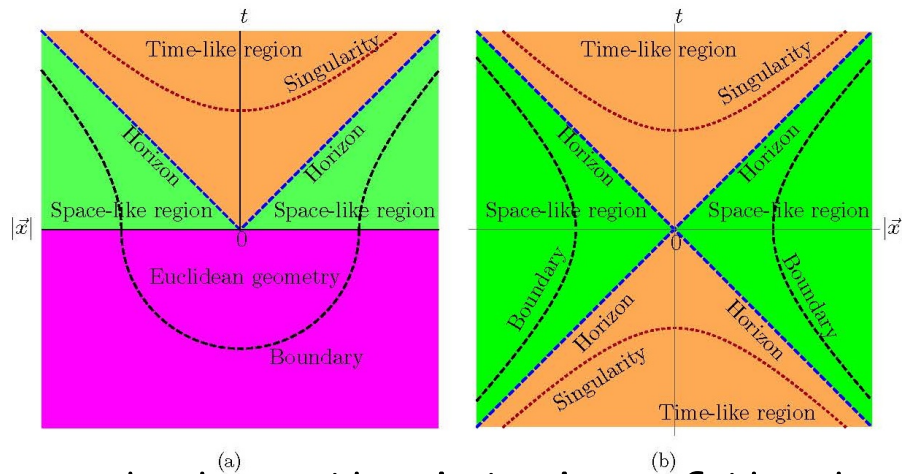


(b)



The meaning from the QFT Point of view

- We have argued that the solution near the boundary is that of the "unstable" CFT_d on dS_d , perturbed by a vev of an irrelevant operator, that drives the RG Flow



- We must therefore ask, how the interior of the bubble can be seen from the boundary?

♠ The singularity and the horizon both reach the boundary only in the asymptotic de Sitter future $\chi \rightarrow +\infty$.

♠ **No information** from the orange region in the future light-cone of the origin can reach the green region, nor the boundary.

♠ This then also applies to the big crunch singularity (shown as the red dotted line) as this is confined to the future light-cone of the origin.

- We conclude that **the dual QFT is unaffected by the crunch.**

- The boundary CFT (on dS_d) is the false-vacuum CFT at all times, and no “vacuum decay” occurs.

- This CFT has two ground states, $|C_1\rangle$ and $|C_2\rangle$.

$$|C_1\rangle \rightarrow \varphi = \varphi_{false}$$

and an AdS metric.

- This ground state preserves the full conformal symmetry of the CFT on dS_d $O(d, 2)$ and

$$\langle C_1|O|C_1\rangle = 0$$

- The other ground state $|C_2\rangle$ corresponds to the vev flow solution with $\varphi(\xi)$ non-trivial, and the metric not being AdS.

- This ground state breaks the full conformal symmetry $O(d, 2) \rightarrow O(d, 1)$ and

$$\langle C_2 | O | C_2 \rangle \neq 0$$

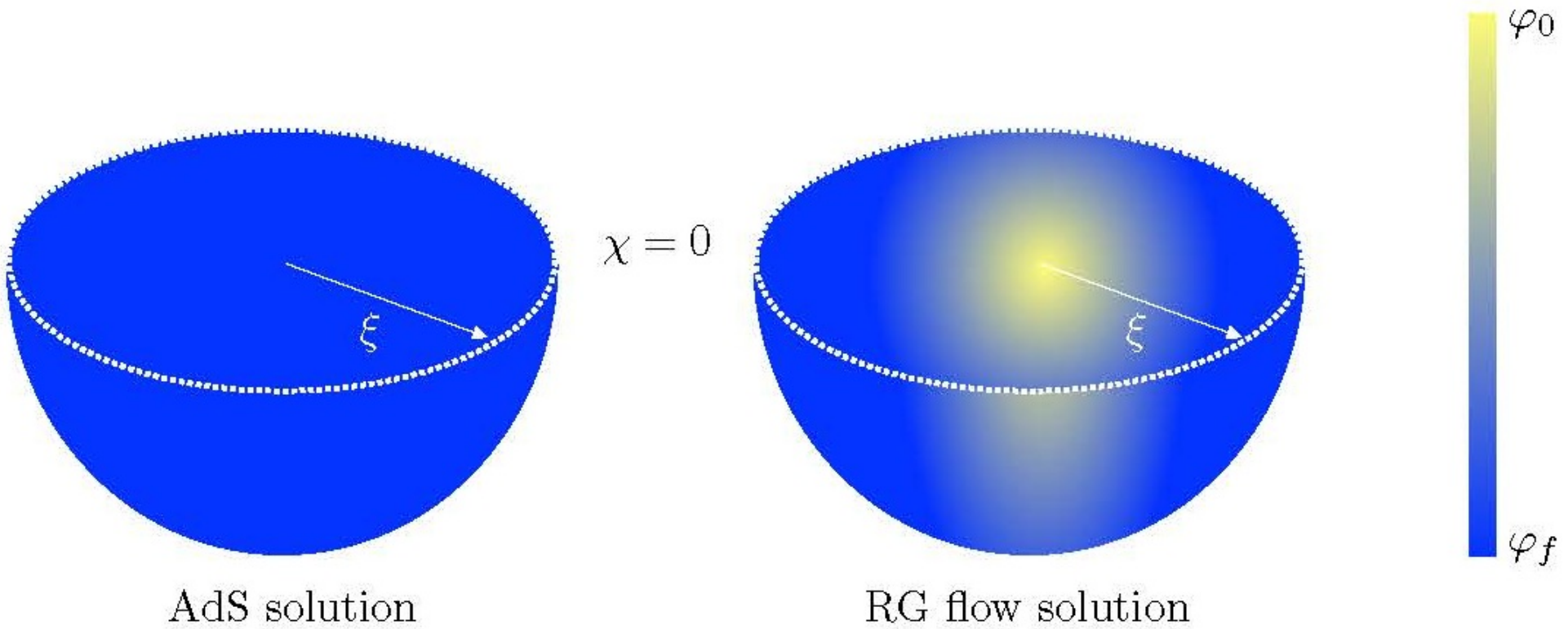
- Both states give the same vev for the stress tensor.

The Hartle-Hawking state

- The false vacuum CFT on dS_d has a special state, known as the Hartle-Hawking state, or the no-boundary state.

Hartle+Hawking, Hartle+Hawking+Hertog

- This is defined by doing the gravitational path integral with AdS_{d+1} boundary conditions.
- The wave function describing the HH state is given by the on-shell gravitational action that is a functional of the boundary data.
- As the theory has two semiclassical ground states, the HH state contain some overlap with these states.
- For perturbative QFTs on de Sitter, **the HH state is the one associated to the Bunch-Davies vacuum.**



No-boundary Euclidean solutions with two different spatial metric and dilaton at the fixed time-slice $\theta = \pi/2$ (Lorentzian time $\chi = 0$). The left figure represents the false-vacuum solution, in which the scalar field is constant, $\varphi = \varphi_f$, and the metric is AdS_D sliced by S^d . On the right, the solution corresponding to the CdL instanton, a.k.a. holographic RG flow on S^d , in which the scalar field flows from φ_f (for $\xi \rightarrow +\infty$) to φ_0 at the center, and the metric deviates from AdS in the interior.

$$\langle HH|C_1\rangle = \Psi[\gamma_1, \phi_1] \sim e^{-S_E(\gamma_1, \phi_1)} \quad , \quad \langle HH|C_2\rangle = \Psi[\gamma_2, \phi_2] \sim e^{-S_E(\gamma_2, \phi_2)}$$

so that

$$\frac{|\langle HH|C_2\rangle|^2}{|\langle HH|C_1\rangle|^2} = e^{S_1 - S_2} = e^{-S_{\text{Instanton}}}$$

- Therefore the instanton probability is controlling how much $|C_2\rangle$ is in $|HH\rangle$.
- This indicates that the symmetry breaking vacuum is exponentially suppressed.
- The true vacuum CFT is “hidden behind the horizon”.

CFTs on $R \times S^{d-1}$

- Can these CdL instantons tell us something about CFTs on the cylinder $R \times S^{d-1}$ or flat space, R^d ?
- There are bulk diffeomorphisms that change the boundary conditions at the boundary.
- Such diffeos provide new solutions, but with different boundary conditions that correspond to different theories.
- Here we want to change the boundary metric from S^d to $R \times S^{d-1}$.
- Such diffeos can be easily found in the AdS case, and can be extended also in the asymptotically AdS case.
- They have the property that they map the infinite future of de Sitter to a finite time in $R \times S^{d-1}$ or R^d .

- As the singularity hits the boundary at that point, this seems to imply that the boundary CFT on $R \times S^{d-1}$ or R^d is hit by the singularity at finite time.
- We shall argue the $O(d+1)$ -symmetric instanton, upon coordinate transformation to a different slicing, does not in fact describe *a spontaneous decay* in a finite time of the dual field theory.
- Rather, it describes *a driven decay*, not unlike what one would obtain by turning on, in the UV CFT, a time-dependent source which becomes singular at a finite time.
- The reason is that the coordinate transformation generates a non-trivial and singular source for the scalar field at the finite end of time in $R \times S^{d-1}$ or R^d .
- This is a different theory, and it is a CFT that is perturbed by a large time-dependent source.
- Therefore such solutions cannot tell anything about the sourceless CFT case.

Conclusions and Outlook

- We have considered Coleman-de Luccia transitions from AdS to AdS.
- We have used holography to constrain such transitions, and interpreted them from the point of view of the dual CFT.
- We have found that generically, there are no instanton solutions.
- We have found that in order for instantons to exist, the dual quantum field theory must have (non-perturbative) RG flows that skip fixed points.
- The dual CFT must be defined on the Euclidean sphere.
- We have given an algorithm to construct potentials that have instantons starting from those that generate skipping RG Flows in the flat case.
- We have clarified the conditions for a thin-wall approximation to exist.

- We have found that for gravitational theories dual to QFTs, **there are no thin-wall instantons.**
- Equivalently, **thin-walled instantons are in the swampland.**
- We have considered the Lorentzian evolution, and shown generally that there is **always a singularity in the future.**
- Such a singularity is protected by a horizon. It cannot affect the dual QFT on de Sitter, except after infinite time.
- The dual CFT is living on de Sitter space.
- If there is an instanton, then it has two ground states, a symmetry preserving one and a symmetry breaking one.
- We have interpreted the instanton amplitude as the probability to find the symmetry breaking state in the Hartle-Hawking vacuum of the CFT on de Sitter.

- It is not known whether in the absence of $O(d)$ -symmetric instantons, there might be ones with less symmetry.
- The implications of this analysis **for inflationary models** remain to be investigated.

THANK YOU!

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 5 minutes
- The logistics of CdL transitions 9 minutes
- Holography 15 minutes
- Holographic Interpretation of CdL 20 minutes
- The strategy 22 minutes
- Thick vs thin domain walls 24 minutes
- Instantons vs skipping flows 27 minutes
- How to construct the relevant potentials 29 minutes
- A numerical study 33 minutes
- The thin-wall limit revisited 35 minutes
- Thin-wall limit vs holography 36 minutes
- The Lorentzian continuation 40 minutes

- The bulk singularity 42 minutes
- The meaning from a QFT point of view 44 minutes
- The HH state 47 minutes
- CFTs on $R \times S^{d-1}$ 49 minutes
- Conclusions 51 minutes

QFT on deSitter and CdL transitions,

Elias Kiritsis