Novel String Field Theory and Bound State, Projective Line

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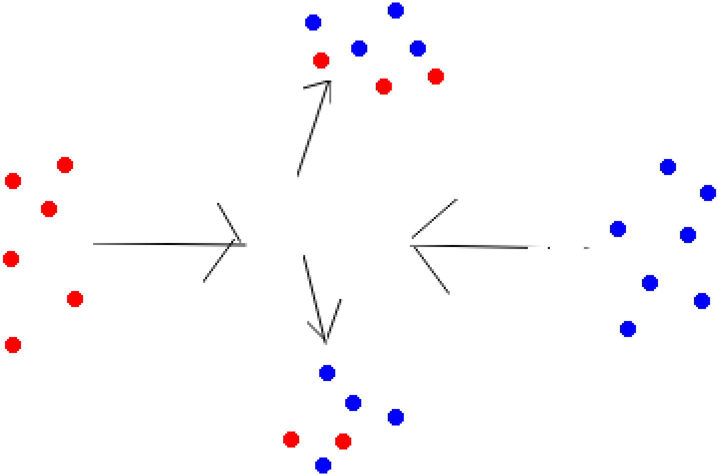
Bound State with Infinitely Many Constituents

For a bound state of infinitely many constituents you would at first expect that the momentum of such a bound state would be shared evenly, so that each constituent would have a negligible part of the total momentum of the bound state. This is of course not safe, since a small part of the constituents might carry the bulk of the momentum, but then the majority of constituents would carry even less. One usually talks about a Bjorken-x defined for each constituent and denoting the (average) fraction of the bound state momentum carried by that constituent.

Scattering of Constituent on Constiuent Not Important for Many Constituents

If the single constituents carry only infinitesially small fraction of the momentum of the bound state, the scattering of one constituent in one bound state with one in another bound state would not be much connected to the scattering of the two bound states.
Rather **scattering of bound states on each other would be dominated by one bound state exchnging a bunch of consituents with the other bound state.**

Scattering by Excange of Constituents



Scattering by Excange of Constutents

This figure illustrates the scattering of two bound states with (inifinitely) many constituents marked in the one as blue and in the other bound state as red. After the scattering there appears again two bound states, but now notice that both of them have partly blue-marked and red-marked constituents. This we may call an "exchange of parts of the constituents"-scattering.

A Motivation for Poeple Interested in making Higher Dimensional Theories:

Wellknown: You do not have genuine renormalizable quantum field theories in higher than 3+1 dimensions.
Except: A very bad scalar theory with ϕ^3-interaction in up to 5+1, and **completely free theories**.
According to the Novel String Field Theory of ours, whicch I shall go a bit inot later, (super)**string theory can be considered a completely free theory**!

"exchange of parts of constituents"-scattering known from our Novel String Field Theory

In our novel string field theory, on which we worked much earlier, the strings are - somewhat similarly, but differently, to/from C. Thorns string bit resolution of the string into "bits" (consituents) - described by means of "objects". After scattering of a couple of strings (almost, except for a nul-set) all the objects from the initial strings are either refound in the final state strings or recognized as having been annihilated. Although we can consider the "objects" constituents they do not scatter on each other, but rather do not interact at all.

Motivation and Plan

- Interpret th geat feature of string theory to be that it is indeed - in our Novel String field theory - a basically free and therefore solvable theory, so that even no divergence problems appear.
- Ask if we can generalize such a string theory, still clinging the idea that the "constituents" (identified with our "objects") do not interact under the scattering of the strings (identified as "bound states").
- Thereby getting e.g. meaningfull (renormalizable) theories in higher than 3+1 dimesnions.

This work especialy: Generalize Möbius Transformations

Our series of objects making up so to say an open string in our novel string field theory are organized in what we call a cyclically ordered chain, which is topologically a circle. It has in fact a "natural" symmetry under a Möbius group, as we shall explain, and can also be considered a projective line (meaning a line as in projective geometry, in which one adds to the lines an extra "point at inifinity", so that the line tomopolgically becomes a circle rather than a usual line).
As a major part of presentation we like to seek to go back from a very general group being the analogon to the Möbius group to see to what extend we can reconstruct projective line for some field (in the sense of the algabraic structure with unit element and invertibility for both a multiplication and an addition).

Reminder of: Möbius transformations in Veneziano model and string theory

From very early times in string theory and Veneziano model theory the Möbius transformations has shown up. In fact physicists were so kind as to call the variable in the formulation of the Veneziano model with some extra variables so that the formulation became precisely invariant under Möbius transformations Koba-Nielsen variables.

What is Möbius transformations ?

A priori the Möbius transformations are defined as transformations of the extended complex number set $\mathrm{C} \cup \{\infty\}$ of the complex numbers with a number ∞ added, a set equivalent to the complex projective line CP^1 given by the transformation function

$$z \rightarrow f(z) = \frac{az+b}{cz+d}, \tag{1}$$

but shall in the present article be more interested just in the real number version transforming only $\mathrm{R} \cup \{\infty\}$, and with the constants a, b, c, d being real numbers.

How we thought in our Novel String Field Theory Articles

We used the splitting of the position variable field on the string into left and right-moving arts

$$X^{\mu}(\sigma,\tau) \;=\; X^{\mu}_{R}(\tau-\sigma)+X^{\mu}_{L}(\tau+\sigma), \qquad (2)$$

where σ is the "spatial" coordinate enumerating the points along the string and τ a "time" for the single string, both arranged in the conformal gauge, meaning they have been partially gauge chosen so that the Lagrangian simplified to a usual 1+1 dimensional massless scalar for each value of the external index μ enumerating the imbedding space dimensions 25+1.

Crucial Featrue of Our Novel String Field Theory, Use X_R and X_L.

Our approach was to discretize into small pieces - analogous to the string bits by Charles Thorn, who used the full X - in the varibles on which these X_R and X_L only depends, namely $\tau - \sigma$ and $\tau + \sigma$ respectively. This means that **we** contrary to C. Thorn discretize into pieces variables which are not a priori physically enumerating the material of which the string consists, but a priori could be just formal parameters enumerating some degrees of freedom of the system(=the string). Therefore a priori we could not be sure if the "objects" corresponding to the small pieces in variables $\tau - \sigma$ or $\tau + \sigma$ can be considered "constituents".

By Changing Physical Interpretation a bit the "Objects" may be Constituents

A priori the "òbjects" associated only with half the degrees of freedom of a string bit - namely only the right or the left moving d.o.f. - are **not** genuine constituents. If you speculate that **the string is just a smart way of looking at it, but not neccessarily the only way,** then we may speculate physically to split up a string bit (as by C. Thorn) into two physically seperate objects, a right and a left. Since the two when interpreted as the "objects" do not interact, are not really needed to be considered the same constituent, we can then make the physical speculation or interpretation rather that the two "objects" for same string bit are two quite independent **constituents**.

So we are allowed to **take it that the "objects" are constituents.**

How strings are seen as crossing places of the objects

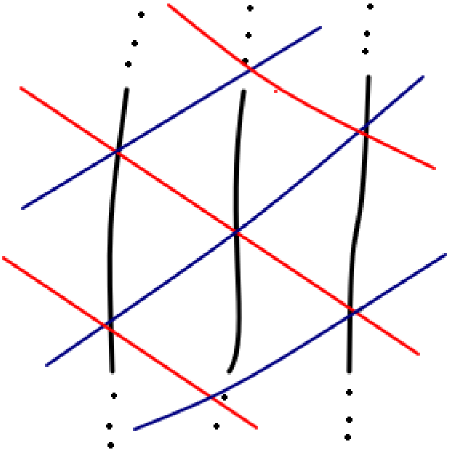


Figure of Objects being Constituents Representing Strings Locally

The figure shall illustrate how two kinds of "objects" denoted by red and blue colored lines telling their path through space (respectively R and L), when flowing through space in long series - infinitesimally close to the neighbors -can represent/look like a string moving with lower velocity.

The "objects" move with velocity of light - actually they are free so they never change even direction -, but the string seemingly there move typically slower. The string at one moment is just where the objects meet at that moment.

The strings are just some way of seeing the objects.

Philosophy of Looking at String Theory in this Talk:

String Theory is a successful theory in higher dimensions because it is actually - according to our Novel String Field Theory - **a free theory**, so that it is one can say renomalizable even in higher dimensions. The " objects" are namely free massless particles.

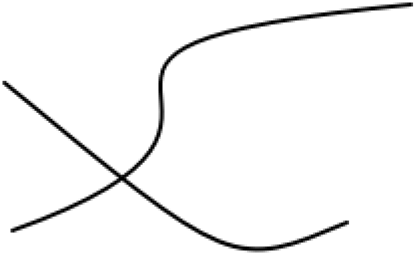
String Versus Novel Object Chains?, Is there a truth?

We claim that there seemingly are **two different ways** of imagining the strings in string theory:

- 1, The **strings** are the true physical objects.
- 2. The **chains of "objects"** are the true physical objects.

You may of course claim, that if we are right that the two ways of looking at it are equivalent, then both are right!
But you could also begin to find argument, that one viewpoint is better or more true than the other one:
In a moment we shall give a couple of weak arguments, that the **chains of objects are more true!**

Mysterious in String Theory: Cross sections for End and String crossing are same order of magnitude?

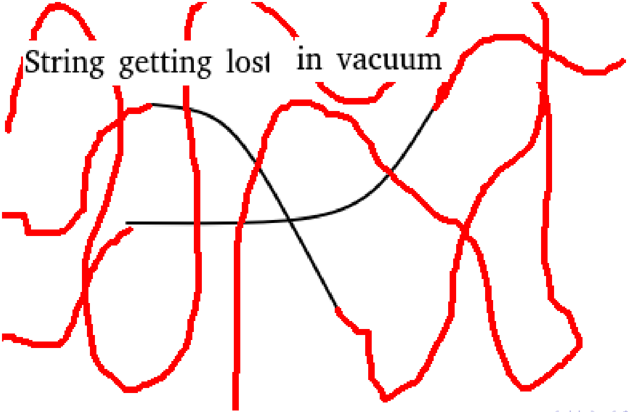


You expect End hitting much more unlikely than hitting of proper string bulk

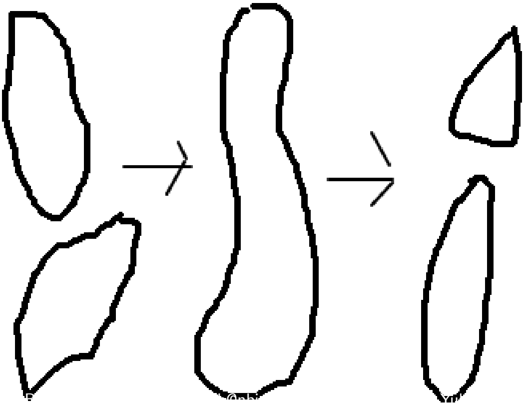
- For two sticks or strings you expect the cross section for that they hit to be of the order of the product of their length.
- But two genuine point particles will have in principle zero crossection for hitting each other.

Conclusion: Something wrong with string interpretation!

Can Vacuum Extensions of String Tails Solve Mystery by one string having an end common with another without knowing



Scattering of two circlar chains (of "objects") allways goes with two local interactions of the chains



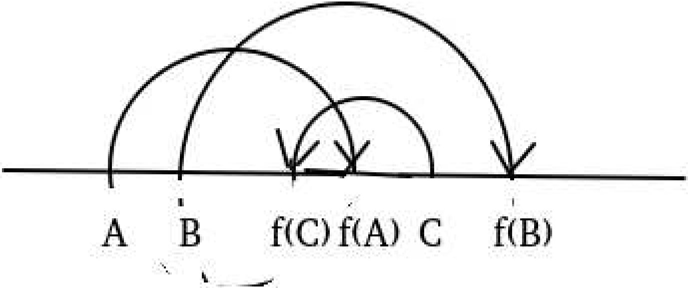
Would be Wonderful to Generalize String Theory, Now we say it is Free

Basically as soon as you calculate scatterings by approximating that all constituents continue without interacting, you are in our present sense generalizing string theory.
So bound states of very many constituents so as each of them having very little momentum share are scattering as a generalization of the strings seen as composed from objects.

Symmetry

If a bound state or an almost bound state consists of infinitely many constituents, then one will unless there are infinitely many types of particles, expect that most of the constituents are in many ways very similar in their way of sitting in the bound stae.
One thus expects a large amount of symmetry between the constituents.
An idea to implement this expectation is to postulate a group of transformmations of the constituents into each other under which the "structure" of the bound state is invariant.

Much Transfromations / Much Symmetry High n Transitivity



3 transitive group

3-transitive group action

We say the group G acts on the space S n-transitively, when you for every n points A ,...,K can find a group element $f \in G$ transforming these n into n prescribed image points. We call it sharply, when the group element acheiving that is unique.

$$
\begin{aligned}
\text{Any } f &\in G && (3)\\
\text{acts } f &: S \to S && (4)\\
\text{For any } n \text{ points } A, \ldots, K & && (5)\\
\text{and another set of } n \text{ points } A', \ldots, K' & \qquad \text{there exists } f && (6)\\
\text{so that } f(A) &= A' && (7)\\
&\vdots && (8)\\
f(K) &= K' && (9)
\end{aligned}
$$

Zassenhaus, a not quite true theorem on 3-transitive transformations

A mathematical article by Katrin Tent, Advances in Mathematics Volume 286, 2 January 2016, Pages 722-728, begins:
" The finite sharply 2- and 3-transitive groups were classified by Zassenhaus in [8] and [9] in the 1930's and were shown to arise from so-called near-fields. They essentially look like the groups of affine linear transformations $x \to ax + b$ or Moebius transformations $x \to \frac{ax+b}{cx+d}$, respectively."

Our own Zassenhaus-like dreamt about theorem:

Thinking instead Zassenhaus finite groups on infinite ones:

A set on which transforms a group in a sharply 3-transitive way will be a projective line corresponding to some field F and the transformations under the group will be Möbius transformations $x \rightarrow \frac{ax+b}{cx+d}$ with the variable enumerating the points on the "projrctive line" x as well as the constants a, b, c, d of the transformation(group element) belong to the field F.

Planning to “‘derive” this doubtfull theorem, like Zassenhaus

We should at least reconstruct the field of real numbers R in the case we consider the Möbius transformations of the real projective line $R \cup \infty$ as the sharply 3-transitive group of transformations.

First step in Reconstructiing the Field F from the Group of sharply 3-transitive transformations

Choose a point in the set S being transformed sharply 3-transitive under the group G and call it ∞. Then look for the subgroup G_1 of the group G consisting of the elements in G with only one fixed point in S, being ∞, (and the unit element in G)
The idea is to identify the subgroup G_1 having ∞ as the only fix point with the additive group of the field F to be found.
The group multiplicaion in G_1 inherited from G of course shall be written with $+$.
(Say $y, z \in (G_1, *)$, then $y * x = y + z$).
(Here $*$ is the group multiplication in G.)

Second Step in Derivation, Identify Scalngs from Two Fix-point Transformations

Next we notice that by requiring just one more fixed point than the ∞ we get (at least in the true Möbius case) a group of scalings of the "numbers" (the elements in G_1) around a certain number. We might call the second fix-point 0 and a similarity transformation of G_1 (the subgroup with one fix-point) by one in the group leaving 0 and ∞ say G_2,say

$$y \in G_1 \quad \rightarrow \quad m * y * m^{-1} \in G_1 \tag{10}$$

$$\text{would be called } y \quad \rightarrow \quad m \cdot y. \tag{11}$$

This would first be a multiplication with an $m \in G_2$.

Third step, Get Identification of G_2 with G_1 by Selecting Point in S to call 1

A priori the subgroup G_1 leaving ∞ and no other points in S invariant, is of course different from the subgroup G_2 of elents in the 3-transitive transformation group of S, which we called G having only two invariant points ∞ and $0 \in S$.
We may, however, choose a third point $1 \in S$ different from the two points ∞ and $0 \in S$ and define a correspondance:

$$y \in G_1 \quad \sim \quad m_y = m \text{ so that } y = m \cdot 1 = m * 1 * m^{-1} \tag{12}$$

(here we needed $1 \in G_1$ but we can make a corresponding 1 in S as $1_{in\ S} = 1_{in\ G_1}(0)$.)

Conclusion

We have proposed the of an approximation applicable hopefully to some bound states: that they have so many constituents with so equally divided momenta - or better Bjorken x's - that we can ignore the scattering of the constituents, when the bound states scatter.

(This means the constituents are in the approximation free, and thus the bound state not truly bound)

Conclusion Details

- Requiring High Symmetry in form of 3-transitive symmetry operation we expected - like Zassenhaus - the constituents to form a structure like a projective line $F \cup \{\infty\}$ for a field F. The srting is the case $F = \mathrm{R}$ i.e. the field is the real number field.(Topologically the projective line is a circle.)
- We suggest that such string theory might be used when the approximation of many constituents with little momentum each becomes good. (of course string theory historically started as attempt to describe hadron physics)
- The p-adic theory of Veneziano model is suggestively incorporated.