Gravitational wave and modified gravity

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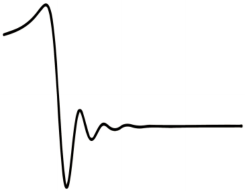
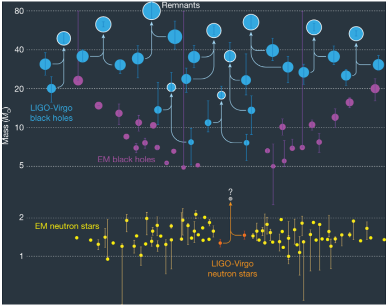
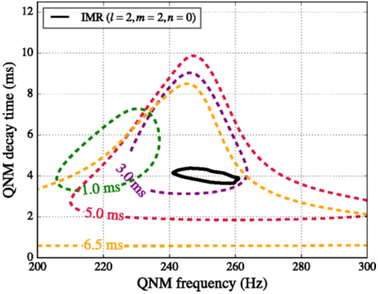


Black hole (BH)



- Black holes form in nature as binary systems.
- Binary black holes coalesce within the Hubble time.

✓ First attempt to detect BH Quasi-Normal-Modes.

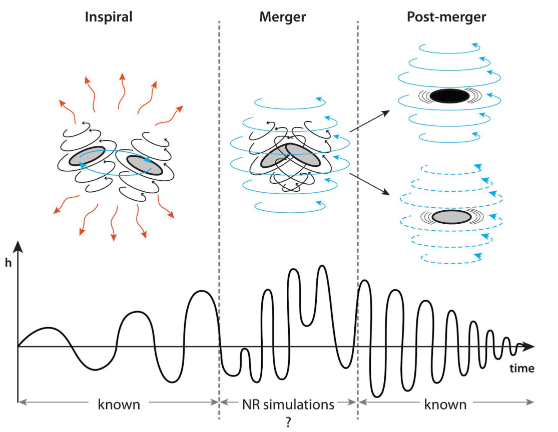


[Abbott et al. LVC 2016-2018]

Why modified gravity can be a plausible alternative?

- The exterior of BH is free of pathology, but the interior is not. Singularities, Cauchy horizons, signalling predictability breakdown.
- Observations limit us from testing strong field regime around BHs.
- Other Compact Objects may be the output of stellar collapse, which can form without an event horizon.
- The gravitational wave detected by LIGO project, from black hole merger, can give us a better understanding of the interior of the black hole nd the nature of the gravitational wave.

Binary Coalescence



Neutron stars significantly distinguished from black holes in the merger and postmerger signal instead of the inspiral phase of the binaries observed by gravitational-wave detectors.

Modified Gravity

- The $f(R, T)$ gravity, which was recently developed by Harko et al., has been one of the promising alternatives.
- The nature of the matter source affects the gravitational field equations in general.
- The equations of motion for test particles that follow from the covariant divergence of a stress-energy tensor.
- The gravitational field equations developed from the metric formalism, modified action principle.
- GW can have up to six possible polarization states in alternative metric theories, four more than GR allows.

[T. Harko et al., Phys. Rev. D 84, 024020 (2011)]

Basic Formalism

- The total action with the modification, and a self interacting vacuum field Lagrangian and the matter lagrangian,
$$S = \int d^4x\sqrt{-g}\big[f(R, T^{\phi}) + \mathcal{L}(\phi, \partial_{\mu}\phi) + K\mathcal{L}_m\big], \qquad (1)$$
- Considered the system in the absence of matter, therefore $\mathcal{L}_m = 0$.
- We considered Lagrangian density for a real scalar field(ϕ) as
$$\mathcal{L}_{\phi} = \frac{1}{2}\nabla_{\alpha}\phi\nabla^{\alpha}\phi - V(\phi). \qquad (2)$$
  Here $V(\phi)$ is a self-interacting potential.
- We assume that the modified gravity function $f(R, T^{\phi})$ is given by $f(R, T^{\phi}) = \alpha R + f(T^{\phi})$.
- The field equation immediately takes the following form,
$$G_{\mu\nu} = \frac{1}{2\alpha}[T^{\phi}_{\mu\nu} + g_{\mu\nu}f(T^{\phi}) - 2f_T(T^{\phi})\nabla_{\mu}\phi\nabla_{\nu}\phi]. \qquad (3)$$

[P. H. R. S. Moraes and J. R. L. Santos, Eur. Phys. J. C 76, 60 (2016)].

- We consider the four-dimensional, spatially statistically homogeneous and isotropic as well as curved spacetime with the jordan frame FLRW metric.

- The Friedmann eqn. for the field as follow,

$$3\left[\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}\right]=-\frac{1}{2\alpha}\left[\frac{1}{2}(1+4f_T)\left\{\dot{\phi}^2-\frac{1-kr^2}{a^2(t)}\left(\frac{\partial\phi}{\partial r}\right)^2\right.\right.$$
$$\left.\left.-\frac{1}{r^2a^2(t)}\left(\frac{\partial\phi}{\partial\theta}\right)^2\right\}-V(\phi)+f(T^{\phi})\right] \tag{4}$$

Here, $a(t)$ stands for the scale factor (in the unit of [length]).

Vacuum Scalar Field

- The Ricci tensor can be written from the Eq. (3) as,

$$R_{\mu\nu} = \frac{1}{2\alpha}[\alpha R g_{\mu\nu} + g_{\mu\nu} f(T^{\phi}) + T^{\phi}_{\mu\nu} - 2f_T \nabla_{\mu}\phi\nabla^{\mu}\phi] \tag{5}$$

- On contraction and simplification the Ricci scalar of can be obtained as follows,

$$R = -\frac{1}{2\alpha}[4f(T^{\phi}) + T^{\phi} - 2f_T \nabla_{\mu}\phi\nabla^{\mu}\phi] \tag{6}$$

- The equation of motion for the scalar field can be found from the covariant divergence of the field Eq. (3) as follows,

$$(1 + 2f_T)\Box\phi + (1 + 4f_T)\left(\frac{\partial V}{\partial \phi}\right) + 2f_{TT}\nabla^{\mu}\phi\nabla_{\mu}T^{\phi} = 0. \tag{7}$$

- With the help of following mathematical identity,

$$\nabla_{\lambda}T^{\phi} = 2(\nabla_{\lambda}\nabla_{\mu}\phi)(\nabla^{\mu}\phi) - 4\left(\frac{\partial V}{\partial \phi}\right)(\nabla_{\lambda}\phi).$$

• We assumed $f(T^{\phi})$ is a power function of the trace of the energy-momentum tensor of the scalar field, i.e., $f(T^{\phi}) = \beta T^{n}$.
• The potential is associated with the Higgs field.

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4, \tag{8}$$

where, μ and λ are constants.
• With such assumption and first order restrictions, following the same methodology as before, we find,

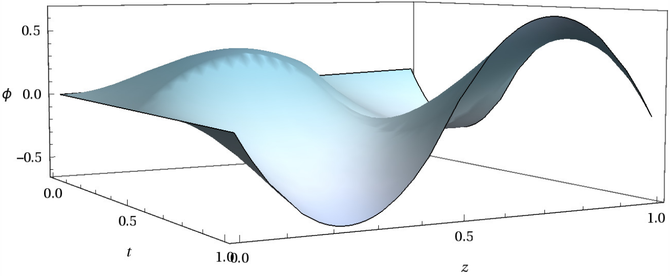
$$\Box\phi + \left(\frac{1 + 4(-1)^{n-1}\beta n(V_0)^{n-1}}{1 + 2(-1)^{n-1}\beta n(V_0)^{n-1}}\right)\left(\frac{\partial V}{\partial \phi}\right) = 0. \tag{9}$$

• The field equations in the linear region has the solution in the following form,

$$\phi(x) = \phi_0 + \phi_1 \exp\left(iq_{\rho}x^{\rho}\right), \tag{10}$$

Effects of perturbation

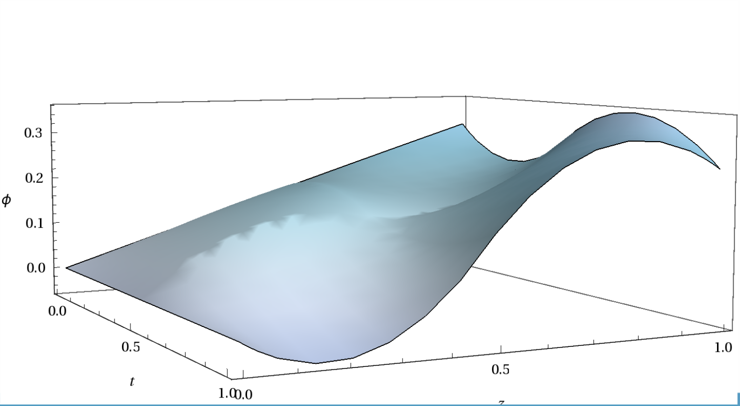
- For the $\mu^2 > 0$ the steady minimum value of scalar field is zero, and this leads to the effective cosmological constant (Λ) to zero.



GW wave propagation for the perturbation of vacuum scalar field

- The minimum scalar field is non zero for $\mu^2 < 0$, effective cosmological constant is this consideration is leads to non-zero. The effective cosmological constant is

$$\Lambda = \frac{1}{2\alpha}\left[(-1)^{2n-1}\beta\left(\frac{\mu^4}{\lambda}\right)^n - \frac{\mu^4}{4\lambda}\right]. \qquad (11)$$



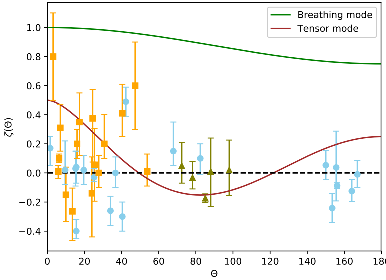
Detection of GWs' polarization modes

- The irreducible parts of the Riemann tensor $R_{\lambda\mu\kappa\nu}$ are represented by ten components of Wely tensor (Ψ's), nine component of traceless Ricci tensor (Φ's), and a curvature scalar (Λ) in the NP formalism, which are algebraically independent.

- The components of the Riemann tensor in the null tetrad basis are related to these NP quantities as follows

$$\begin{aligned}
\Psi_2 &= -\frac{1}{6}R_{lklk} \sim \text{longitudinal scalar mode},\\
\Psi_3 &= -\frac{1}{2}R_{lkl\bar{m}} \sim \text{vector-x \& vector-y modes},\\
\Psi_4 &= -R_{l\bar{m}l\bar{m}} \sim +\text{,x tensorial mode},\\
\Phi_{22} &= -R_{lml\bar{m}} \sim \text{breathing scalar mode}.
\end{aligned}$$

Detection from polarization modes



- For tensor modes, the correlation function is as follows

$$C^{+,\times}(\theta) = \xi^{GR}(\theta) \int_0^\infty \frac{|h_c^{+,\times}|^2}{24\pi^2 f^3} df, \tag{12}$$

- The correlation function for the corresponding scalar modes is

$$C^{b}(\theta) = \xi^{b}(\theta) \int_0^\infty \frac{|h_c^{b}|^2}{12\pi^2 f^3} df, \tag{13}$$

✓ The normalized correlation function is defined as

$$\zeta(\theta) = \frac{C(\theta)}{C(0)}. \tag{14}$$

Conclusion

- Based on the **nature of potential** the structural behaviour of the scalar field varies and we have considered the spontaneous symmetry breaking potential for our system. The behaviour of the scalar field varies with the **sign of order parameter** (μ^2).
- $\mu^2 > 0$ leads that the system is **independent** of the order of degree.
- Conjuncture with the scalar field Lagrangian leads to a **new set of Friedmann equations**.
- It's worth noting that if GWs have nontensorial polarization modes, as discussed, an evaluated signal, such as a stochastic cosmological background of GWs, would be a **combination of all of those modes**.

Thank you..!!