How far has so far the Spin-Charge-Family theory succeeded to offer the explanation for the observed phenomena in elementary particle physics and cosmology :

i. The new way of second quantization of fermion and of boson fields, explaining the postulates

ii. Short overview of the spin-charge-family theory and its achievements

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Some publications:

- *Phys. Lett.* **B 292**, 25-29 (**1992**), *J. Math. Phys.* **34**, 3731-3745 (**1993**), *Mod. Phys. Lett.* **A 10**, 587-595 (**1995**), *Int. J. Theor. Phys.* **40**, 315-337 (**2001**),
- *Phys. Rev.* **D 62** (04010-14) (**2000**), *Phys. Lett.* **B 633** (**2006**) 771-775, **B 644** (**2007**) 198-202, **B** (**2008**) 110.1016, *JHEP* **04** (**2014**) 165, *Fortschritte Der Physik-Progress in Physics*, **(2017) with** H.B.Nielsen,
- *Phys. Rev.* **D 74** 073013-16 (**2006**), **with** A.Borštnik Bračič,
- *New J. of Phys.* **10** (**2008**) 093002, arxiv:1412.5866, **with** G.Bregar, M.Breskvar, D.Lukman,
- *Phys. Rev.* **D** (**2009**) 80.083534, **with** G. Bregar,
- *New J. of Phys.* (**2011**) 103027, *J. Phys. A: Math. Theor.* **45** (**2012**) 465401, *J. Phys. A: Math. Theor.* **45** (**2012**) 465401, *J. of Mod. Phys.* **4** (**2013**) 823-847, arxiv:1409.4981, **6** (**2015**) 2244-2247, *Phys. Rev.* **D 91** (**2015**) 6, 065004, . *J. Phys.: Conf. Ser. 845 01* **IARD 2017**, *Eur. Phys. J.C.* **77** (2017) 231, Rev. Artile in **Progress in Particle and Nuclear Physics**, **http://doi.org/10.1016.j.ppnp.2021.103890**

More than **50 years ago** the **electroweak (and colour) standard model** offered an **elegant new step** in **understanding the origin of fermions and bosons** by **postulating**:

A.

- **The existence** of **massless family members** with the **charges** in the **fundamental representation of the groups** -
  o the **coloured triplet quarks and colourless leptons**,
  o the **left handed members as the weak charged doublets**,
  o the **right handed weak chargeless members**,
  o the **left handed quarks distinguishing in the hyper charge from the left handed leptons**,
  o each right handed member having a different hyper charge.
- **The existence** of **massless families to each of a family member.**

α name	hand-edness $-4iS^{03}S^{12}$	weak charge τ^{13}	hyper charge Y	colour charge	elm charge Q
u^i_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	colour triplet	$\frac{2}{3}$
d^i_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	colour triplet	$-\frac{1}{3}$
ν^i_L	-1	$\frac{1}{2}$	$-\frac{1}{2}$	colourless	0
e^i_L	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	colourless	-1
u^i_R	1	weakless	$\frac{2}{3}$	colour triplet	$\frac{2}{3}$
d^i_R	1	weakless	$-\frac{1}{3}$	colour triplet	$-\frac{1}{3}$
ν^i_R	1	weakless	0	colourless	0
e^i_R	1	weakless	-1	colourless	-1

Members of each of the $i = 1, 2, 3$ families, $i = 1, 2, 3$ massless before the electroweak break. Each family contains the left handed weak charged quarks and the right handed weak chargeless quarks, belonging to the colour triplet $(1/2, 1/(2\sqrt{3}))$, $(-1/2, 1/(2\sqrt{3}))$, $(0, -1/(\sqrt{3}))$.

And the anti-fermions to each family and family member.

B.

- **The existence** of **massless vector gauge fields** to the observed **charges** of the **family members**, **carrying charges** in the **adjoint representation of the charge groups**.

Masslessness needed for gauge invariance.

Gauge fields before the electroweak break

- **Three massless vector fields, the gauge fields of the three charges.**

name	hand-edness	weak charge	hyper charge	colour charge	elm charge
hyper photon	0	0	0	colourless	0
weak bosons	0	triplet	0	colourless	triplet
gluons	0	0	0	colour octet	0

They all are vectors in $d = (3 + 1)$, in the adjoint representations with respect to the weak, colour and hyper charges.

C.

- The **existence of a massive scalar field - the higgs**,

  o carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$ (as it would be in the fundamental representation of the groups.)

  o gaining at some step the **imaginary mass** and consequently the **constant value** , breaking the weak and the hyper charge and correspondingly breaking the **mass protection**.

- The **existence** of the **Yukawa couplings**, taking care of

  o the properties of **fermions** and

  o the masses of the **heavy bosons**.

- The **Higgs's field**, **the scalar in** $d = (3+1)$, **a doublet with respect to the weak charge**.

name	hand-edness	weak charge	hyper charge	colour charge	elm charge
$0\cdot$ **Higgs**$_u$	0	$\frac{1}{2}$	$\frac{1}{2}$	**colourless**	**1**
$< Higgs_d >$	0	$-\frac{1}{2}$	$\frac{1}{2}$	**colourless**	**0**

name	hand-edness	weak charge	hyper charge	colour charge	elm charge
$< Higgs_u >$	0	$\frac{1}{2}$	$-\frac{1}{2}$	**colourless**	**0**
$0\cdot$ **Higgs**$_d$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	**colourless**	-1

D.

- There is the **gravitational field** in $\mathbf{d=(3+1)}$.

- **The *standard model* assumptions have been confirmed without offering surprises.**
- **The last unobserved field as a field, the Higgs's scalar, detected in June 2012, was confirmed in March 2013.**
- **The waves of the gravitational field were detected in February 2016 and again 2017.**

There remain not understood phenomena:

- The *Standard model* **assumptions need explanation.**

- **There are several cosmological observations which do not look to be explainable within the** *standard model*, **like**

  o The **existence of the** *dark matter*

  o The *matter/antimatter asymmetry* **in the universe**

  o The **need for the** *dark energy*
- the **observed dimension of space time**,
- the **quantization of the gravitational field**,
- ...

- **The Standard model assumptions have in the literature several explanations, but with many new not explained assumptions.**

- **I am proposing the Spin-Charge-Family theory,** which **offers the explanation for**

  i. **all the assumptions of the** *standard model*,

  ii. **for many observed phenomena**:

  ii.a. the **dark matter**,

  ii.b. the **matter-antimatter asymmetry**,

  ii.c. **others observed phenomena**,

  iii. **explaining the Dirac's postulates for the second quantized fermion and second quantized boson fields**,

  iv. **making several predictions.**

  Is the Spin-Charge-Family theory the right next step beyond both standard models?

- **Work done so far on the spin-charge-family theory is promising.**

** We try to understand:

- What are elementary constituents and interactions among constituents in our Universe, in any universe?
- Can the elementary constituent be of only one kind? Are the four observed interactions — gravitational, elektromagnetic, weak and colour — of the common origin?
- Can the postulated second quantized fermions and second quantized bosons be understood trough the algebra, like it is the quantization of the coordinates? Can fermions and bosons be second quantized in an equivalent way?
- Is the space-time $(3 + 1)$? If yes why $(3+1)$?
- If not $(3 + 1)$ may it be that the space-time is infinite?
- How has the space-time of our universe started?
- What is the matter and what the anti-matter?

Obviously it is the time to make the next step beyond both standard models.

What questions should one ask to be able to find next steps beyond the *standard models* and to understand not yet understood phenomena?

- **o Where do family members originate?**
  o Where do charges of family members originate?
  o Why are the charges of family members so different?
  o Why have the left handed family members so different charges from the right handed ones?
- **o Where do families of family members originate?**
  o How many different families exist?
  o Why do family members – quarks and leptons – manifest so different properties if they all start as massless?

- **o How is the origin of the scalar field (the Higgs's scalar) and the Yukawa couplings connected with the origin of families?**

  o How many scalar fields determine properties of the so far (and others possibly be) **observed fermions and masses of weak bosons?** (The Yukawa couplings certainly speak for the existence of several scalar fields with the properties of Higgs's scalar.)
- **Why is the Higgs's scalar**, or are all **scalar fields**, if there are several, **doublets with respect to the weak and the hyper charge?**
- **Do exist also scalar fields with the colour charge in the fundamental representation and where, if they are, do they manifest?**

- Where do the **charges and correspondingly the so far** (and others possibly be) **observed vector gauge fields originate?**
- **Where** does the **dark matter originate?**
- **Where** does the **"ordinary" matter-antimatter asymmetry originate?**
- **Where** does the **dark energy originate?**
- What is the dimension of space? $(3+1)$?, $((d-1)+1)$?, ∞?
- **What** is the role of the **symmetries**– discrete, continuous, global and gauge – in our **universe, in Nature?**
- And many others.

My statement:

- An elegant trustworthy next step must offer answers to several open questions, explaining:
  o The origin of the family members and the charges.
  o The origin of the families and their properties.
  o The origin of the scalar fields and their properties.
  o The origin of the vector fields and their properties.
  o The origin of the internal space of fermions and bosons and of their properties.
  o The origin of the dark matter.
  o The origin of the "ordinary" matter-antimatter asymmetry.

My statement continues:

- **There exist not yet observed families, gauge vector and gauge scalar fields.**
- **Dimension of space is larger than** 4 (very probably infinite).
- **Inventing a next step which covers one of the open questions, might be of a help but can hardly show the right next step in understanding nature.**

In the literature NO explanation for the existence of the families can be found, which would not just assume the family groups.
Several extensions of the standard model are, however, proposed, like:

- The $SU(3)$ group is assumed to describe – not explain – the existence of three families.
  Like the Higgs's scalar charges are in the fundamental representations of the groups, also the Yukawas are assumed to emerge from the scalar fields, in the fundamental representation of the $SU(3)$ group.

- SU(5) and SO(10) **grand unified theories are proposed, unifying all the charges.** But the **spin** (the handedness) **is** obviously connected with the (weak and the hyper) charges, what these theories do "by hand" as it does the *standard model*, and the appearance of families is not explained.
- **Supersymmetric theories, assuming the existence of bosons with the charges of quarks and leptons and fermions with the charges of the gauge vector fields,** although having several nice properties **but not explaining the appearance of families (except again by assuming larger groups), are not, to my understanding, the right next step beyond the** *standard model*.

o The **Spin-Charge-Family theory does offer the explanation for all the assumptions of the standard model**, **answering up to now several of the above cited open questions!**

o The **more effort is put into this theory, the more answers to the open questions in elementary particle physics and cosmology is the theory offering**.

o I shall first make a short introduction into the **Spin-Charge-Family theory.**

o I shall report on **how does the odd Clifford algebra explain the second quantization postulates of Dirac.**
Rev. article in JPPNP –2021 Progress in Particle and Nuclear Physics http://doi.org/10.1016.j.ppnp.2021.103890

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o I shall report on **how does the even Clifford algebra explain the second quantization of boson fields.** [arXiv:2108.05718]

o I shall make an overview of achievements so far of the **Spin-Charge-Family theory.**

- A brief introduction into the **spin-charge-family theory**.

- **There are two kinds of the Clifford algebra objects in any d . I recognized that in Grassmann space.**

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θ^a**'s and** p^{θ}_a**'s**, $p^{\theta}_a = \frac{\partial}{\partial \theta_a}$
with the property
$(\theta^a)^\dagger = \eta^{aa} \frac{\partial}{\partial \theta_a}$.

i. The **Dirac** γ^a (recognized 90 years ago in $d = (3+1)$).
ii. The **second one:** $\tilde{\gamma}^a$,

$$\gamma^a = (\theta^a - i\, p^{\theta a})\,, \quad \tilde{\gamma}^a = i\,(\theta^a + i\, p^{\theta a})\,,$$

References can be found in
Progress in Particle and Nuclear Physics,
http://doi.org/**10.1016.j.ppnp.2021.103890** .

- The two kinds of the **Clifford algebra objects** **anticommute**

$$\{\gamma^{a},\gamma^{b}\}_{+} = 2\eta^{ab} = \{\tilde{\gamma}^{a},\tilde{\gamma}^{b}\}_{+},$$
$$\{\gamma^{a},\tilde{\gamma}^{b}\}_{+} = 0,$$

- the **postulate**

$$(\tilde{\gamma}^{a}\mathbf{B} = \mathbf{i}(-)^{n_B}\mathbf{B}\gamma^{a}\,)\,|\psi_0>,$$
$$(\mathbf{B} = a_0 + a_a\gamma^a + a_{ab}\gamma^a\gamma^b + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\ldots\gamma^{a_d})|\psi_o>$$

with $(-)^{n_B} = +1, -1$, if B has a Clifford even or odd character, respectively, $|\psi_o>$ is a vacuum state on which the operators γ^a **apply**, **reduces the Clifford space for fermions for the factor of two**, while the operators $\tilde{\gamma}^a\tilde{\gamma}^b = -2i\tilde{S}^{ab}$ **define the family quantum numbers**.

- It is convenient to write all the **"basis vectors" describing the internal space of either fermion fields or boson fields as products of nilpotents and projectors, which are eigenvectors of the chosen Cartan subalgebra**

$$S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\,d},$$
$$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\,d},$$
$$\mathbf{S}^{ab} = S^{ab} + \tilde{S}^{ab}.$$

nilpotents

$$S^{ab}\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b) = \frac{k}{2}\,\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b), \quad \overset{ab}{(\mathbf{k})} := \frac{\mathbf{1}}{\mathbf{2}}(\gamma^{\mathbf{a}} + \frac{\eta^{\mathbf{aa}}}{\mathbf{ik}}\gamma^{\mathbf{b}}),$$

projectors

$$S^{ab}\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b) = \frac{k}{2}\,\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b), \quad \overset{ab}{[\mathbf{k}]} := \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{1} + \frac{\mathbf{i}}{\mathbf{k}}\gamma^{\mathbf{a}}\gamma^{\mathbf{b}}),$$

$$(\overset{ab}{(\mathbf{k})})^2 = \mathbf{0}, \qquad (\overset{ab}{[\mathbf{k}]})^2 = \overset{ab}{[\mathbf{k}]},$$

$$\overset{ab}{(\mathbf{k})}^{\dagger} = \eta^{\mathbf{aa}}\,\overset{ab}{(-\mathbf{k})}, \quad \overset{ab}{[\mathbf{k}]}^{\dagger} = \overset{ab}{[\mathbf{k}]}.$$

$$
\begin{aligned}
\mathbf{S}^{ab}\stackrel{ab}{(\mathbf{k})} &= \frac{k}{2}\stackrel{ab}{(\mathbf{k})}, \quad \mathbf{S}^{ab}\stackrel{ab}{[\mathbf{k}]} = \frac{k}{2}\stackrel{ab}{[\mathbf{k}]},\\
\tilde{\mathbf{S}}^{ab}\stackrel{ab}{(\mathbf{k})} &= \frac{k}{2}\stackrel{ab}{(\mathbf{k})}, \quad \tilde{\mathbf{S}}^{ab}\stackrel{ab}{[\mathbf{k}]} = -frack2\stackrel{ab}{[\mathbf{k}]}\,.
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{a}\stackrel{ab}{(\mathbf{k})} &= \eta^{aa}\stackrel{ab}{[-\mathbf{k}]}\,,\; \gamma^{b}\stackrel{ab}{(\mathbf{k})} = -ik\stackrel{ab}{[-\mathbf{k}]}\,,\; \gamma^{a}\stackrel{ab}{[\mathbf{k}]} = \stackrel{ab}{(-\mathbf{k})}\,,\; \gamma^{b}\stackrel{ab}{[\mathbf{k}]} = -ik\eta^{aa}\stackrel{ab}{(-\mathbf{k})}\,,\\
\tilde{\gamma}^{a}\stackrel{ab}{(\mathbf{k})} &= -i\eta^{aa}\stackrel{ab}{[\mathbf{k}]}\,,\; \tilde{\gamma}^{b}\stackrel{ab}{(\mathbf{k})} = -k\stackrel{ab}{[\mathbf{k}]}\,,\; \tilde{\gamma}^{a}\stackrel{ab}{[\mathbf{k}]} = \; i\stackrel{ab}{(\mathbf{k})}\,,\; \tilde{\gamma}^{b}\stackrel{ab}{[\mathbf{k}]} = -k\eta^{aa}\stackrel{ab}{(\mathbf{k})}\,,\\
\stackrel{ab}{(\mathbf{k})}\stackrel{ab}{(-\mathbf{k})} &= \eta^{aa}\stackrel{ab}{[\mathbf{k}]}\,,\; \stackrel{ab}{[\mathbf{k}]}\stackrel{ab}{(\mathbf{k})} = \stackrel{ab}{(\mathbf{k})}\,,\; \stackrel{ab}{(\mathbf{k})}\stackrel{ab}{[-\mathbf{k}]} = \stackrel{ab}{(\mathbf{k})}\,,\\
\stackrel{ab}{(\mathbf{k})}\stackrel{ab}{[\mathbf{k}]} &= \mathbf{0}\,,\; \stackrel{ab}{[\mathbf{k}]}\stackrel{ab}{(-\mathbf{k})} = \mathbf{0}\,,\; \stackrel{ab}{[\mathbf{k}]}\stackrel{ab}{[-\mathbf{k}]} = \mathbf{0}\,,\\
\stackrel{ab}{\widetilde{(-\mathbf{k})}}\stackrel{ab}{(\mathbf{k})} &= -i\eta^{aa}\stackrel{ab}{[\mathbf{k}]}\,,\; \stackrel{ab}{\widetilde{[\mathbf{k}]}}\stackrel{ab}{(\mathbf{k})} = \stackrel{ab}{(\mathbf{k})}\,,\; \stackrel{ab}{\widetilde{(\mathbf{k})}}\stackrel{ab}{[\mathbf{k}]} = i\stackrel{ab}{(\mathbf{k})}\,,\; \stackrel{ab}{\widetilde{[-\mathbf{k}]}}\stackrel{ab}{[\mathbf{k}]} = \stackrel{ab}{[\mathbf{k}]}\,,\\
\stackrel{ab}{\widetilde{(\mathbf{k})}}\stackrel{ab}{(\mathbf{k})} &= \mathbf{0}\,,\; \stackrel{ab}{\widetilde{[-\mathbf{k}]}}\stackrel{ab}{(\mathbf{k})} = \mathbf{0}\,,\; \stackrel{ab}{\widetilde{(\mathbf{k})}}\stackrel{ab}{[-\mathbf{k}]} = \mathbf{0}\,,\; \stackrel{ab}{\widetilde{[\mathbf{k}]}}\stackrel{ab}{[\mathbf{k}]} = \mathbf{0}\,.
\end{aligned}
$$

- γ^a **transforms** $\overset{ab}{(k)}$ **into** $\overset{ab}{[-k]}$**, never to** $\overset{ab}{[k]}$**.**

- $\tilde{\gamma}^a$ **transforms** $\overset{ab}{(k)}$ **into** $\overset{ab}{[k]}$, **never to** $\overset{ab}{[-k]}$**.**

- **There are the Clifford odd "basis vector", that is the "basis vector" with an odd number of nilpotents, at least one, the rest are projectors, such "basis vectors" anti commute among themselves.**

- **There are the Clifford even "basis vector", that is the "basis vector" with an even number of nilpotents, the rest are projectors, such "basis vectors" commute among themselves.**

- **Let us see how does one family of the Clifford odd "basis vector" in $d = (13+1)$ look like, if spins in $d = (13+1)$ are analysed with respect to the Standard Model groups.**

- **One irreducible representation** of one **family contains $2^{\frac{(13+1)}{2}-1} = 64$ members which include all the family members, quarks and leptons with the right handed neutrinos included, as well as all the antimembers, antiquarks and antileptons, reachable by either S^{ab} (or by $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ on a family member)**.

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Int. J. of Modern Phys. **A 9**, 1731 (**1994**),
J. of Math. Phys. **44** 4817 (**2003**), hep-th/030322 .

S^{ab} generate **all the members of one family**. The **eightplet** (represent. of $SO(7,1)$) **of quarks of a particular colour charge. All are Clifford odd "basis vectors"** .

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Octet, $\Gamma^{(7,1)}=1,\ \Gamma^{(6)}=-1$, of quarks							
1	$\mathbf{u_R^{c1}}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
4	$\mathbf{d_R^{c1}}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	$\mathbf{d_L^{c1}}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	$\mathbf{u_L^{c1}}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\mathbf{u_R}$ of the 1^{st} row into $\mathbf{u_L}$ of the 7^{th} row, and $\mathbf{d_R}$ of the 4^{rd} row into $\mathbf{d_L}$ of the 6^{th} row, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} generate **all the members of one family with leptons included.** Here is The **eightplet** (represent. of $SO(7,1)$) **of leptons colour chargeless. the $SO(7,1)$ part is identical with the one of quarks.**

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(7,1)} = 1$, $\Gamma^{(6)} = -1$, of leptons							
1	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
2	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
3	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
4	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
5	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
6	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
7	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
8	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform ν_R of the 1^{st} line into ν_L of the 7^{th} line, and e_R of the 4^{rd} line into e_L of the 6^{th} line, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} **generate also all the anti-eightplet** (repres. of $SO(7,1)$) of **anti-quarks of the anti-colour charge bellonging to the same family of the Clifford odd basis vectors** .

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Antioctet, $\Gamma^{(7,1)}=-1$, $\Gamma^{(6)}=1$, of antiquarks							
33	$\bar{\mathbf{d}}^{\bar{c1}}_{\mathbf{L}}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
34	$\bar{d}^{\bar{c1}}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
35	$\bar{u}^{\bar{c1}}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
36	$\bar{\mathbf{u}}^{\bar{c1}}_{\mathbf{L}}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	- 1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
37	$\bar{\mathbf{d}}^{\bar{c1}}_{\mathbf{R}}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
38	$\bar{d}^{\bar{c1}}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{(+)}\stackrel{78}{[-]}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
39	$\bar{u}^{\bar{c1}}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
40	$\bar{\mathbf{u}}^{\bar{c1}}_{\mathbf{R}}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\,\vert\,\stackrel{56}{[-]}\stackrel{78}{(+)}\,\vert\vert\,\stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\bar{\mathbf{d}}_{\mathbf{L}}$ of the 1^{st} row into $\bar{\mathbf{d}}_{\mathbf{R}}$ of the 5^{th} row, and $\bar{\mathbf{u}}_{\mathbf{L}}$ of the 4^{rd} row into $\bar{\mathbf{u}}_{\mathbf{R}}$ of the 8^{th} row.

- **We discuss so far the internal space of fermions describing their internal space with Clifford odd "basis vectors".**
- **Before we start to discuss Clifford even "basis vectors" describing the internal space of bosons let us write down the action.**
- **Fermions and bosons can exist even if they do not interact**, at least mathematically.
- **Describing their internal space we do not pay attention on their interactions. We treat them as free fields.**
- **Describing the properties of fermions and bosons as we observe, the interaction should be included: A simple and elegant one (this is how I "see nature") demonstrating at low energies all the observed phenomena.**

I use in the spin-charge-family theory a simple action. Fermions carry in $d=(13+1)$ only spins, two kinds of spins (no charges) and interact with the gauge gravitational fields.

$$\begin{aligned} \mathbf{S} &= \int d^d x \, E \, \mathcal{L}_f + \\ & \int d^d x \, E \, (\alpha \, R + \tilde{\alpha} \, \tilde{R}) \end{aligned}$$

- $$\begin{aligned} \mathcal{L}_f &= \frac{1}{2}(\bar{\psi} \, \gamma^a p_{0a} \psi) + h.c. \\ p_{0a} &= f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^{\alpha}{}_a\}_- \\ \mathbf{p_{0\alpha}} &= \mathbf{p}_\alpha - \frac{\mathbf{1}}{\mathbf{2}} \mathbf{S^{ab}} \omega_{\mathbf{ab}\alpha} - \frac{\mathbf{1}}{\mathbf{2}} \mathbf{\tilde{S}^{ab}} \tilde{\omega}_{\mathbf{ab}\alpha} \end{aligned}$$

- The Einstein action for a free gravitational field is assumed to be linear in the curvature

$$\mathcal{L}_{\mathbf{g}} = \mathbf{E}\,(\alpha\,\mathbf{R} + \tilde{\alpha}\tilde{\mathbf{R}}),$$
$$\mathbf{R} = \mathbf{f}^{\alpha[\mathbf{a}}\mathbf{f}^{\beta\mathbf{b}]}\,(\omega_{\mathbf{ab}\alpha,\beta} - \omega_{\mathbf{ca}\alpha}\omega^{\mathbf{c}}{}_{\mathbf{b}\beta}),$$
$$\tilde{\mathbf{R}} = \mathbf{f}^{\alpha[\mathbf{a}}\mathbf{f}^{\beta\mathbf{b}]}\,(\tilde{\omega}_{\mathbf{ab}\alpha,\beta} - \tilde{\omega}_{\mathbf{ca}\alpha}\tilde{\omega}^{\mathbf{c}}{}_{\mathbf{b}\beta}),$$

  with $E = \det(e^{a}{}_{\alpha})$
  and $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$.

- **I shall first treat "basis vectors" and correspondingly the creation operators for either the Clifford odd fermion fields or for the Clifford even boson fields in the limit of free fields.**

- **Let me discuss the Clifford even "basis vectors", offering the description of the internal space of bosons within a toy model in $d = (5+1)$, pointing out the difference between the "basis vectors" of odd and "basis vectors" of even Clifford algebra elements.**

- **One can learn in Progress in Particle and Nuclear Physics**, http://doi.org/**10.1016.j.ppnp.2021.103890** , Eq. (14, 16, 28), **that there are** 2^d **Grassmann polynomials of** θ^a**'s and** 2^d **their Hermitian conjugated partners** $\frac{\partial}{\partial \theta_a}$, $(\theta^a)^\dagger = \eta^{aa} \frac{\partial}{\partial \theta_a}$.
- **One can also learn that there are** 2^d **Clifford objects, which are products** of γ^a**'s**

$$\gamma^a = (\theta^a + \frac{\partial}{\partial \theta_a}),$$

half of them form Clifford odd "basis vectors" , half of them form Clifford even "basis vectors" .

- **There are** $2^{\frac{d}{2}-1}$ **Clifford odd family members, appearing** $2^{\frac{d}{2}-1}$ **irreducible representations, carrying family quantum numbers.**

  And there are $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ **their Hermitian conjugated partners. Together there are** 2^{d-1} **Clifford odd "basis vectors".**
- **And there are** 2^{d-1} **Clifford even "basis vectors".**

- Let us start now to learn about properties of "basis vectors" constituting the creation operators of boson fields on the case of $d = (5 + 1)$.
- In $d = (5 + 1)$ there are $2^{\frac{6}{2}-1}$ members in each of $2^{\frac{6}{2}-1}$ families.
- Clifford odd "basis vectors", $\hat{b}^{m\dagger}_{f}$, have their Hermitian conjugated partners, $\hat{b}^{m}_{f}$, in the separate group not reachable either by S^{ab} or by $\tilde{S}^{ab}$. Due to

$$\overset{ab}{(\mathbf{k})}^{\dagger} = \eta^{\mathbf{aa}} \overset{ab}{(-\mathbf{k})}, \overset{ab}{[\mathbf{k}]}^{\dagger} = \overset{ab}{[\mathbf{k}]}.$$

- Clifford even "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$, have their Hermitian conjugated partners, ${}^{I}\hat{\mathcal{A}}^{m}_{f}$, within the same group reachable by S^{ab} or by $\tilde{S}^{ab}$.

basis vect. $\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}$	m $\rightarrow$	$f=1$ $\frac{i}{2}, -\frac{1}{2}, -\frac{1}{2}$	$f=2$ $-\frac{i}{2}, -\frac{1}{2}, \frac{1}{2}$	$f=3$ $-\frac{i}{2}, \frac{1}{2}, -\frac{1}{2}$	$f=4$ $\frac{i}{2}, \frac{1}{2}, \frac{1}{2}$	S^{03}	S^{12}	S^{56}
odd I $\hat{b}^{m\dagger}_{f}$	1 2 3 4	03 12 56 $(+i)[+][+]$ $[-i](-)[+]$ $[-i][+](-)$ $(+i)(-)(-)$	03 12 56 $[+i][+](+)$ $(-i)(-)(+)$ $(-i)[+][-]$ $[+i](-)[-]$	03 12 56 $[+i](+)[+]$ $(-i)[-][+]$ $(-i)(+)(-)$ $[+i][-](-)$	03 12 56 $(+i)(+)(+)$ $[-i][-](+)$ $[-i](+)[-]$ $(+i)[-][-]$	$\frac{i}{2}$ $-\frac{i}{2}$ $-\frac{i}{2}$ $\frac{i}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$
S^{03}, S^{12}, S^{56}	$\rightarrow$	$-\frac{i}{2}, \frac{1}{2}, \frac{1}{2}$ 03 12 56	$\frac{i}{2}, \frac{1}{2}, -\frac{1}{2}$ 03 12 56	$\frac{i}{2}, -\frac{1}{2}, \frac{1}{2}$ 03 12 56	$-\frac{i}{2}, -\frac{1}{2}, -\frac{1}{2}$ 03 12 56	$\tilde{S}^{03}$	$\tilde{S}^{12}$	$\tilde{S}^{56}$
odd II $\hat{b}^{m}_{f}$	1 2 3 4	$(-i)[+][+]$ $[-i](+)[+]$ $[-i][+](+)$ $(-i)(+)(+)$	$[+i][+](-)$ $(+i)(+)(-)$ $(+i)[+][-]$ $[+i](+)[-]$	$[+i](-)[+]$ $(+i)[-][+]$ $(+i)(-)(+)$ $[+i][-](+)$	$(-i)(-)(-)$ $[-i][-](-)$ $[-i](-)[-]$ $(-i)[-][-]$	$-\frac{i}{2}$ $\frac{i}{2}$ $\frac{i}{2}$ $-\frac{i}{2}$	$-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$	$-\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}$	$\rightarrow$	$-\frac{i}{2}, \frac{1}{2}, \frac{1}{2}$ 03 12 56	$\frac{i}{2}, -\frac{1}{2}, \frac{1}{2}$ 03 12 56	$-\frac{i}{2}, -\frac{1}{2}, -\frac{1}{2}$ 03 12 56	$\frac{i}{2}, \frac{1}{2}, -\frac{1}{2}$ 03 12 56	S^{03}	S^{12}	S^{56}
even I ${}^{I}\mathcal{A}^{m}_{f}$	1 2 3 4	$[+i](+)(+)$ $(-i)[-](+)$ $(-i)(+)[-]$ $[+i][-][-]$	$(+i)[+](+)$ $[-i](-)(+)$ $[-i][+][-]$ $(+i)(-)[-]$	$[+i][+][+]$ $(-i)(-)[+]$ $(-i)[+](-)$ $[+i](-)(-)$	$(+i)(+)[+]$ $[-i][-][+]$ $[-i](+)(-)$ $(+i)[-](-)$	$\frac{i}{2}$ $-\frac{i}{2}$ $-\frac{i}{2}$ $\frac{i}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$
$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}$	$\rightarrow$	$\frac{i}{2}, \frac{1}{2}, \frac{1}{2}$ 03 12 56	$-\frac{i}{2}, -\frac{1}{2}, \frac{1}{2}$ 03 12 56	$\frac{i}{2}, -\frac{1}{2}, -\frac{1}{2}$ 03 12 56	$-\frac{i}{2}, \frac{1}{2}, -\frac{1}{2}$ 03 12 56	S^{03}	S^{12}	S^{56}
even II ${}^{II}\mathcal{A}^{m}_{f}$	1 2 3 4	$[-i](+)(+)$ $(+i)[-](+)$ $(+i)(+)[-]$ $[-i][-][-]$	$(-i)[+](+)$ $[+i](-)(+)$ $[+i][+][-]$ $(-i)(-)[-]$	$[-i][+][+]$ $(+i)(-)[+]$ $(+i)[+](-)$ $[-i](-)(-)$	$(-i)(+)[+]$ $[+i][-][+]$ $[+i](+)(-)$ $(-i)[-](-)$	$-\frac{i}{2}$ $\frac{i}{2}$ $\frac{i}{2}$ $-\frac{i}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$

- **Clifford odd "basis vectors"** describing the internal space of **fermions in the case of** $d = (5+1)$ are presented in the table as *odd I* $\hat{b}^{m\dagger}_{f}$, **having odd numbers of nilpotents**
- $\hat{b}^{m}_{f}$ **is presented in the same table as** *odd II* $\hat{b}^{m}_{f}$**.** The two groups are not reachable by either S^{ab} or by $\tilde{S}^{ab}$.
- **Clifford even "basis vectors"** describing the internal space of **bosons in the case of** $d = (5+1)$ are presented in the table as *even I, II* ${}^{I,II}\hat{\mathcal{A}}^{m\dagger}_{f}$, **having an even numbers of nilpotents.**
- **Their Hermitian conjugated partner appear within the same group of "basis vectors", either I or II, demonstrating correspondingly the properties of the internal space of the gauge fields to the fermion "basis vectors".**

- **Clifford odd "basis vector" describing the internal space of quark** $u^{c1\dagger}_{\uparrow R}$, $\Leftrightarrow$ $b^{1\dagger}_{1} := \overset{03}{(+i)}\overset{12}{[+]}\,|\,\overset{56}{[+]}\overset{78}{(+)}\,||\,\overset{9\,10}{(+)}\,\overset{11\,12}{[-]}\,\overset{13\,14}{[-]}$,
  has the Hermitian conjugated partner equal to
  $u^{c1}_{\uparrow R} \Leftrightarrow (\mathbf{b}^{1\dagger}_{1})^{\dagger} = \overset{13\,14}{[-]}\,\overset{11\,12}{[-]}\,\overset{9\,10}{(-)}\,||\,\overset{78}{(-)}\overset{56}{[+]}\,|\,\overset{12}{[+]}\overset{03}{(-\mathbf{i})}$, **both with an odd number of nilpotents,**
  both are the Clifford odd objects, belonging to two group.
- **Quarks "basis vectors" contain** $b^{1\dagger}_{1} = \overset{03}{(+i)}\overset{12}{[+]}\,|\,\overset{56}{[+]}$ **from d=(5+1).**
- **Clifford even "basis vectors", having an even number of nilpotents, describe the internal space of the corresponding boson field**
  ${}^{I}\mathcal{A}^{m}_{f} = \overset{03}{(+i)}\overset{12}{(+)}\,|\,\overset{56}{[+]}\overset{78}{(+)}\,||\,\overset{9\,10}{(+)}\,\overset{11\,12}{[-]}\,\overset{13\,14}{[-]}$,
- **it contains** ${}^{I}\mathcal{A}^{m}_{f} = \overset{03}{(+i)}\overset{12}{(+)}\,|\,\overset{56}{[+]}$ **from d=(5+1).**

Anti-commutation relations for Clifford odd "basis vectors", representing the internal space of fermion fields of quarks and leptons ($i = (u_{R,L}^{c,f,\uparrow,\downarrow}, d_{R,L}^{c,f,\uparrow,\downarrow}, \nu_{R,L}^{f,\uparrow,\downarrow}, e_{R,L}^{f,\uparrow,\downarrow})$), and anti-quarks and anti-leptons, with the family quantum number f.

- $\{\mathbf{b}_{\mathbf{f}}^{\mathbf{m}}, \mathbf{b}_{\mathbf{f'}}^{\mathbf{k}\dagger}\}_{*_{\mathbf{A}}+} |\psi_{\mathbf{o}}> = \delta_{\mathbf{f}\,\mathbf{f'}}\, \delta^{\mathbf{mk}}\, |\psi_{\mathbf{o}}>$,
- $\{\mathbf{b}_{\mathbf{f}}^{\mathbf{m}}, \mathbf{b}_{\mathbf{f'}}^{\mathbf{k}}\}_{*_{\mathbf{A}}+} |\psi_{\mathbf{o}}> = 0 \cdot |\psi_{\mathbf{o}}>$,
- $\{\mathbf{b}_{\mathbf{f}}^{\mathbf{m}\dagger}, \mathbf{b}_{\mathbf{f'}}^{\mathbf{k}\dagger}\}_{*_{\mathbf{A}}+} |\psi_{\mathbf{o}}> = 0 \cdot |\psi_{\mathbf{o}}>$,
- $\mathbf{b}_{\mathbf{f}}^{\mathbf{m}}\, |\psi_{\mathbf{o}}> = 0 \cdot |\psi_{\mathbf{o}}>$,
- $\mathbf{b}_{\mathbf{f}}^{\mathbf{m}\dagger}\, |\psi_{\mathbf{o}}> = |\psi_{\mathbf{f}}^{\mathbf{m}}>$,

  $$|\psi_{\mathbf{o}}> = \stackrel{03}{[-\mathbf{i}]}\stackrel{12}{[-]}\stackrel{56}{[-]} \cdots \stackrel{13\,14}{[-]}\ |\mathbf{1}>$$

  define the vacuum state for quarks and leptons and antiquarks and antileptons of the family f .

[arXiv:1802.05554v1], [arXiv:1802.05554v4], [arXiv:1902.10628]

Commutation relations for Clifford even "basis vectors", representing the internal space of boson fields of two kinds, ${}^{i}\hat{\mathcal{A}}^{m\dagger}_{f}$, $i = (I, II)$, which are the gauge fields of the fermion fields

- $$\mathbf{{}^{i}\hat{\mathcal{A}}^{m\dagger}_{f} *_{A} {}^{i}\hat{\mathcal{A}}^{m'\dagger}_{f`} \rightarrow \left\{ \begin{array}{r} {}^{i}\hat{\mathcal{A}}^{m\dagger}_{f`}\,, \\ \text{or } 0\,, i = (I, II)\,. \end{array} \right.}$$

- $$\mathbf{{}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} *_{A} {}^{II}\hat{\mathcal{A}}^{m\dagger}_{f} \;=\; 0 = {}^{II}\hat{\mathcal{A}}^{m\dagger}_{f} *_{A} {}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}\,.}$$

I shall demonstrate the properties of ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ as the gauge fields of the corresponding $\hat{b}^{m\dagger}_{f}$ in what follows.

Let us come back to d=(5+1) case and to the properties of the Clifford odd and the Clifford even "basiss vectors"

Let us first treat the properties **of the "basis vectors" for fermion fields in** $d=(5+1)$, then we shall treat properties **of the "basis vectors" for boson fields in** $d=(5+1)$, as well as their mutual interaction.

The "basis vectors" for fermion fields in $d=(5+1)$, **appear in four families, each family is identical with respect to** $S^{ab}=\frac{i}{4}(\gamma^a\gamma^b-\gamma^b\gamma^a)$, **distinguishing only in** $\tilde{S}^{ab}=\frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b-\tilde{\gamma}^b\tilde{\gamma}^a)$.

The nilpotents and projectors are chosen to be eigenstates of the Cartan subalgebra of the Lorentz algebra

$$\begin{aligned}
\mathbf{S}^{ab}\overset{ab}{(\mathbf{k})} &= \frac{k}{2}\overset{ab}{(\mathbf{k})}, \quad \mathbf{S}^{ab}\overset{ab}{[\mathbf{k}]}=\frac{k}{2}\overset{ab}{[\mathbf{k}]},\\
\tilde{\mathbf{S}}^{ab}\overset{ab}{(\mathbf{k})} &= \frac{k}{2}\overset{ab}{(\mathbf{k})}, \quad \tilde{\mathbf{S}}^{ab}\overset{ab}{[\mathbf{k}]}=-\frac{k}{2}\overset{ab}{[\mathbf{k}]}.
\end{aligned}$$

$\tilde{S}^{01}\overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}=-\frac{i}{2}\overset{03}{[+i]}\overset{12}{(+)}\overset{56}{[+]}$,

and the $\hat{b}^{m\dagger}_f$ are eigenvectors of all the Cartan subalgebra members.

"Basis vectors" for fermions

f	m	$\hat{b}_f^{m\dagger}$	S^{03}	S^{12}	S^{56}	Γ^{3+1}	N_L^3	N_R^3	τ^3	τ^8	τ	$\tilde{S}^{03}$	$\tilde{S}$
I	1	$\overset{03}{(+i)}\,\overset{12}{[+]} \mid \overset{56}{[+]}$	$\frac{i}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{i}{2}$	−
	2	$\overset{03}{[-i]}\,\overset{12}{(-)} \mid \overset{56}{[+]}$	$-\frac{i}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	−
	3	$\overset{03}{[-i]}\,\overset{12}{[+]} \mid \overset{56}{(-)}$	$-\frac{i}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	−
	4	$\overset{03}{(+i)}\,\overset{12}{(-)} \mid \overset{56}{(-)}$	$\frac{i}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	−
II	1	$\overset{03}{[+i]}\,\overset{12}{(+)} \mid \overset{56}{[+]}$	$\frac{i}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{i}{2}$	
	2	$\overset{03}{(-i)}\,\overset{12}{[-]} \mid \overset{56}{[+]}$	$-\frac{i}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	
	3	$\overset{03}{(-i)}\,\overset{12}{(+)} \mid \overset{56}{(-)}$	$-\frac{i}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	
	4	$\overset{03}{[+i]}\overset{12}{[-]} \mid \overset{56}{(-)}$	$\frac{i}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	
III	1	$\overset{03}{[+i]}\,\overset{12}{[+]} \mid \overset{56}{(+)}$	$\frac{i}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{i}{2}$	−
	2	$\overset{03}{(-i)}\,\overset{12}{(-)} \mid \overset{56}{(+)}$	$-\frac{i}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	−
	3	$\overset{03}{(-i)}\,\overset{12}{[+]} \mid \overset{56}{[-]}$	$-\frac{i}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	−
	4	$\overset{03}{[+i]}\,\overset{12}{(-)} \mid \overset{56}{[-]}$	$\frac{i}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{i}{2}$	−
IV	1	$\overset{03}{(+i)}\,\overset{12}{(+)} \mid \overset{56}{(+)}$	$\frac{i}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{i}{2}$	
	2	$\overset{03}{[-i]}\,\overset{12}{[-]} \mid \overset{56}{(+)}$	$-\frac{i}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	
	3	$\overset{03}{[-i]}\,\overset{12}{(+)} \mid \overset{56}{[-]}$	$-\frac{i}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	
	4	$\overset{03}{(+i)}\,\overset{12}{[-]} \mid \overset{56}{[-]}$	$\frac{i}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{i}{2}$	

To demonstrate properties of the internal space of fermions using the odd Clifford subalgebra let us use the superposition of members of Cartan subalgebra for the subgroup $SO(3,1) \times U(1)$: $(N^3_\pm, \tau)$

$$N^3_\pm (= N^3_{(L,R)}) := \frac{1}{2}(S^{12} \pm i S^{03})\,, \quad \tau = S^{56}\,,$$

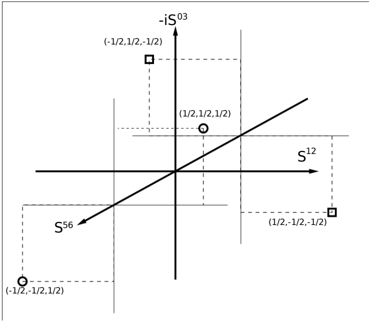
what is meaningful if we understand S^{03} and S^{12} as spins of fermions and S^{56} as their charge,

and for the subgroup $SU(3) \times U(1)$: (τ', τ^3, τ^8)

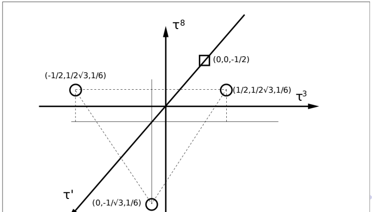
$$\tau^3 := \quad \frac{1}{2}(-S^{12} - iS^{03})\,, \qquad \tau^8 = \frac{1}{2\sqrt{3}}(-iS^{03} + S^{12} - 2S^{56})\,,$$
$$\tau' = \quad -\frac{1}{3}(-iS^{03} + S^{12} + S^{56})\,,$$

if we treat the colour properties for fermions to learn from this toy model as much as we can. The number of commuting operators is three in both cases.

We recognize twice 2 "basis vectors" with charge $\pm\frac{1}{2}$, and with spins up and down.



We recognize one colour triplet of "basis vectors" with $\tau' = \frac{1}{6}$ and one colour singlet with $\tau' = -\frac{1}{2}$.



- Let us see the algebraic application, $*_A$, of the Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f=3}$, $m = (1,2,3,4)$, presented in the first table in the third column of *even* I, on $\hat{b}^{m=1\dagger}_{f=1}$, presented as the first Clifford *odd* I "basis vector" on the first and the second table.

- The algebraic application, $*_A$, can easily be evaluated by taking into account

$$
\begin{aligned}
\overset{ab}{(\mathbf{k})}\overset{ab}{(-\mathbf{k})} &= \eta^{aa}\overset{ab}{[\mathbf{k}]}\,,\;\; \overset{ab}{[\mathbf{k}]}\overset{ab}{(\mathbf{k})} = \overset{ab}{(\mathbf{k})}\,,\;\; \overset{ab}{(\mathbf{k})}\overset{ab}{[-\mathbf{k}]} = \overset{ab}{(\mathbf{k})}\,,\\
\overset{ab}{(\mathbf{k})}\overset{ab}{[\mathbf{k}]} &= \mathbf{0}\,,\overset{ab}{[\mathbf{k}]}\overset{ab}{(-\mathbf{k})} = \mathbf{0}\,,\;\; \overset{ab}{[\mathbf{k}]}\overset{ab}{[-\mathbf{k}]} = \mathbf{0}\,,
\end{aligned}
$$

for any m and f.

$$
\begin{aligned}
&{}^{I}\hat{\mathcal{A}}^{1\dagger}_{3}\,(\equiv \overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]}) *_{A} \hat{\mathbf{b}}^{1\dagger}_{1}\,(\equiv \overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}) \rightarrow \hat{\mathbf{b}}^{1\dagger}_{1}\,,\text{selfadjoint}\\
&{}^{I}\hat{\mathcal{A}}^{2\dagger}_{3}\,(\equiv \overset{03}{(-i)}\overset{12}{(-)}\overset{56}{[+]}) *_{A} \hat{\mathbf{b}}^{1\dagger}_{1} \rightarrow \hat{\mathbf{b}}^{2\dagger}_{1}\,(\equiv \overset{03}{[-i]}\overset{12}{(-)}\overset{56}{[+]})\,,\\
&{}^{I}\hat{\mathcal{A}}^{3\dagger}_{3}\,(\equiv \overset{03}{(-i)}\overset{12}{[+]}\overset{56}{(-)}) *_{A} \hat{\mathbf{b}}^{1\dagger}_{1} \rightarrow \hat{\mathbf{b}}^{3\dagger}_{1}\,(\equiv \overset{03}{[-i]}\overset{12}{[+]}\overset{56}{(-)})\,,\\
&{}^{I}\hat{\mathcal{A}}^{4\dagger}_{3}\,(\equiv \overset{03}{[+i]}\overset{12}{(-)}\overset{56}{(-)}) *_{A} \hat{\mathbf{b}}^{1\dagger}_{1} \rightarrow \hat{\mathbf{b}}^{4\dagger}_{1}\,(\equiv \overset{03}{(+i)}\overset{12}{(-)}\overset{56}{(-)})\,.
\end{aligned}
$$

Looking at the eigenvalues of the $\hat{b}^{m\dagger}_{1}$ we see that ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{3}$ obviously carry the integer eigenvalues of $\mathcal{S}^{03}, \mathcal{S}^{12}, \mathcal{S}^{56}$.

Let us look at the eigenvalues of (τ^3, τ^8, τ') of $\hat{b}_1^{m\dagger}$.

$\hat{b}_1^{1\dagger}$ has $(\tau^3, \tau^8, \tau') = (0, 0, -\frac{1}{2})$,
$\hat{b}_1^{2\dagger}$ has $(\tau^3, \tau^8, \tau') = (0, -\frac{1}{\sqrt{3}}, \frac{1}{6})$,
$\hat{b}_1^{3\dagger}$ has $(\tau^3, \tau^8, \tau') = (-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{6})$,
$\hat{b}_1^{4\dagger}$ has $(\tau^3, \tau^8, \tau') = (\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{6})$.

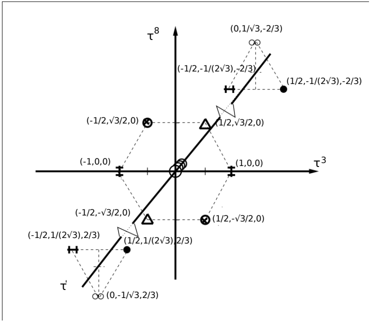
The eigenvalues of (τ^3, τ^8, τ') of ${}^{I}\hat{\mathcal{A}}_3^{1\dagger}$ are obviously
${}^{I}\hat{\mathcal{A}}_3^{1\dagger}$ has $(\tau^3, \tau^8, \tau') = (0, 0, 0)$,
${}^{I}\hat{\mathcal{A}}_3^{2\dagger}$ has $(\tau^3, \tau^8, \tau') = (0, -\frac{1}{\sqrt{3}}, \frac{2}{3})$,
${}^{I}\hat{\mathcal{A}}_3^{3\dagger}$ has $(\tau^3, \tau^8, \tau') = (-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{2}{3})$,
${}^{I}\hat{\mathcal{A}}_3^{4\dagger}$ has $(\tau^3, \tau^8, \tau') = (\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{2}{3})$,

It can be concluded: $\mathcal{S}^{ab} = S^{ab} + \tilde{S}^{ab}$. Using this recognition we find the properties of the Clifford even "basis vectors":

f	m	$*$	${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$	$\mathcal{S}^{03}$	$\mathcal{S}^{12}$	$\mathcal{S}^{56}$	$\mathcal{N}^3_L$	$\mathcal{N}^3_R$	τ^3	τ^8	τ'
I	1	$\star\star$	$\stackrel{03}{[+i]}\stackrel{12}{(+)}\stackrel{56}{(+)}$	0	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$
	2	$\triangle$	$\stackrel{03}{(-i)}\stackrel{12}{[-]}\stackrel{56}{(+)}$	$-i$	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2\sqrt{3}}$	0
	3	$\ddagger$	$\stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{[-]}$	$-i$	1	0	1	0	-1	0	0
	4	$\bigcirc$	$\stackrel{03}{[+i]}\stackrel{12}{[-]}\stackrel{56}{[-]}$	0	0	0	0	0	0	0	0
II	1	$\bullet$	$\stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{(+)}$	i	0	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$
	2	$\otimes$	$\stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{(+)}$	0	-1	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2\sqrt{3}}$	0
	3	$\bigcirc$	$\stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{[-]}$	0	0	0	0	0	0	0	0
	4	$\ddagger$	$\stackrel{03}{(+i)}\stackrel{12}{(-)}\stackrel{56}{[-]}$	i	-1	0	-1	0	1	0	0
III	1	$\bigcirc$	$\stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}$	0	0	0	0	0	0	0	0
	2	$\odot\odot$	$\stackrel{03}{(-i)}\stackrel{12}{(-)}\stackrel{56}{[+]}$	$-i$	-1	0	0	-1	0	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$
	3	$\bullet$	$\stackrel{03}{(-i)}\stackrel{12}{[+]}\stackrel{56}{(-)}$	$-i$	0	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$
	4	$\star\star$	$\stackrel{03}{[+i]}\stackrel{12}{(-)}\stackrel{56}{(-)}$	0	-1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{2}{3}$
IV	1	$\odot\odot$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{[+]}$	i	1	0	0	1	0	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$
	2	$\bigcirc$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\stackrel{56}{[+]}$	0	0	0	0	0	0	0	0
	3	$\otimes$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{(-)}$	0	1	-1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2\sqrt{3}}$	0
	4	$\triangle$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\stackrel{56}{(-)}$	i	0	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2\sqrt{3}}$	0

Selfadjoint members are denoted by $\bigcirc$, Hermitian conjugated partners are denoted by the same symbol.

Fig. analyses ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ with respect to Cartan subalgebra members (τ^3, τ^8, τ'). There are
one sextet with $\tau' = 0$,
four singlets with $(\tau^3 = 0, \tau^8 = 0, \tau' = 0)$,
one triplet with $\tau' = \frac{2}{3}$ and one triplet with $\tau' = -\frac{2}{3}$.
Families play NO role!



We now know how to describe the internal space of bosons with "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and fermions with "basis vectors" $\hat{b}^{m'\dagger}_{f'}$.

And we know the action

$$\mathbf{A} = \int \; d^d x \; E \; \mathcal{L}_f + \int \; d^d x \; E \; (\alpha \, R + \tilde{\alpha} \, \tilde{R}) \, ,$$

defining the interaction between fermions and bosons

$$\mathcal{L}_f - \frac{1}{2}(\bar{\psi} \, \gamma^a p_{0a} \psi) + h.c. p_{0a} = f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E} \, \{p_\alpha, E f^{\alpha}{}_a\}_-$$

$$\mathbf{p}_{0\alpha} = \mathbf{p}_\alpha - \frac{1}{2} \mathbf{S}^{\mathbf{ab}} \omega_{\mathbf{ab}\alpha} - \frac{1}{2} \tilde{\mathbf{S}}^{\mathbf{ab}} \tilde{\omega}_{\mathbf{ab}\alpha}$$

- It is time now to relate the **boson fields** with the **fermion fields**.

- It is time to relate the **boson fields** with the **boson fields**.

- Let us point out that ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ concern only the internal space of **bosons**, while in the action it appears beside S^{ab}, which apply on the **fermion field**, also $\omega_{ab\alpha}$ which have the **vector index** in addition.

  o o To relate ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ with $\omega_{ab\alpha}$ we must multiply ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ by a vector ${}^{I}\mathcal{C}^{m}_{f\alpha}$.

- **We treat fermion and bosons as free fields, that is as plane waves. We can now relate the application of** $^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ $^{I}\mathcal{C}^{m}_{f\alpha}$ and $\omega_{ab\alpha}$ **by applying both on** $\sum_{m'} \hat{b}^{m'\dagger}_{f'}$ $\beta^{m'}$

$$\{\sum_{\mathbf{m,f}} {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} \mathcal{C}^{\mathbf{mf}}_{\alpha}\} *_{\mathbf{A}} \{\sum_{\mathbf{m}'} \hat{\mathbf{b}}^{\mathbf{m}'\dagger}_{\mathbf{f}'} \beta^{\mathbf{m}'}\} = \{\sum_{\mathbf{ab}} \mathbf{S}^{\mathbf{ab}} \omega_{\mathbf{ab}\alpha}\} \{\sum_{\mathbf{m}''} \hat{\mathbf{b}}^{\mathbf{m}''\dagger}_{\mathbf{f}'} \beta^{\mathbf{m}''}\}$$

  for a chosen family f', the same in in $\{\sum_{\mathbf{m}'} \hat{\mathbf{b}}^{\mathbf{m}'\dagger}_{\mathbf{f}'} \beta^{\mathbf{m}'}\}$ **and in** $\{\sum_{\mathbf{m}''} \hat{\mathbf{b}}^{\mathbf{m}''\dagger}_{\mathbf{f}'} \beta^{\mathbf{m}''}\}$.

- **We relate** $(2^{\frac{d}{2}-1})^2$ **of** ${}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}}$ **with** $\frac{\mathbf{d(d-1)}}{\mathbf{2}}$ **of** $\omega_{\mathbf{ab}\alpha}$ **for a particular** α.

Let us check how it works for $d=(3+1)$ with four $\{{}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}}\,{}^{\mathbf{I}}\mathcal{C}^{\mathbf{m}}_{\mathbf{f}\alpha}\}$ and with six $\{\mathbf{S}^{\mathbf{ab}}\,\omega_{\mathbf{ab}\alpha}\}$.
For ${}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}}\,{}^{\mathbf{I}}\mathcal{C}^{\mathbf{mf}}_{\alpha}$ we get from

$$\{{}^{\mathbf{I}}\hat{\mathcal{A}}^{1\dagger}_{1}(\overset{03}{[+\mathbf{i}]}\overset{12}{[+]})\,{}^{\mathbf{I}}\mathcal{C}^{1}_{1\alpha}+{}^{\mathbf{I}}\hat{\mathcal{A}}^{2\dagger}_{1}(\overset{03}{(-\mathbf{i})}\overset{12}{(-)})\,{}^{\mathbf{I}}\mathcal{C}^{2}_{1\alpha}+{}^{\mathbf{I}}\hat{\mathcal{A}}^{1\dagger}_{2}(\overset{03}{(+\mathbf{i})}\overset{12}{(+)})\,{}^{\mathbf{I}}\mathcal{C}^{1}_{2\alpha}+{}^{\mathbf{I}}\hat{\mathcal{A}}^{2\dagger}_{2}(\overset{03}{[-\mathbf{i}]}\overset{12}{[-]})\,{}^{\mathbf{I}}\mathcal{C}^{2}_{2\alpha}\}$$

$$\{\hat{\mathbf{b}}^{1\dagger}_{1}\beta^{1}_{1}+\hat{\mathbf{b}}^{2\dagger}_{1}\beta^{2}_{1}+\hat{\mathbf{b}}^{3\dagger}_{1}\beta^{3}_{1}+\hat{\mathbf{b}}^{4\dagger}_{1}\beta^{4}_{1}\}$$

$$=\frac{1}{2}\sum_{\mathbf{ab}}\mathbf{S}^{\mathbf{ab}}\omega_{\mathbf{ab}\alpha}\{\hat{\mathbf{b}}^{1\dagger}_{1}\beta^{1}_{1}+\hat{\mathbf{b}}^{2\dagger}_{1}\beta^{2}_{1}+\hat{\mathbf{b}}^{3\dagger}_{1}\beta^{3}_{1}+\hat{\mathbf{b}}^{4\dagger}_{1}\beta^{4}_{1}\}\,.$$

the expressions for four ${}^{\mathbf{I}}\mathcal{C}^{\mathbf{mf}}_{\alpha}$ in terms of six $\omega_{ab\alpha}$.

$${}^{\mathbf{I}}\mathcal{C}^{1}_{1\alpha}=\frac{1}{2}(\mathbf{i}\,\omega_{03\alpha}+\omega_{12\alpha})\,,\quad {}^{\mathbf{I}}\mathcal{C}^{2}_{2\alpha}=-\frac{1}{2}(\mathbf{i}\,\omega_{03\alpha}+\omega_{12\alpha})$$

$${}^{\mathbf{I}}\mathcal{C}^{1}_{2\alpha}=\mathbf{i}\,\frac{1}{2}(\omega_{01\alpha}-\mathbf{i}\,\omega_{02\alpha}-\omega_{31\alpha}+\mathbf{i}\,\omega_{32\alpha})$$

$${}^{\mathbf{I}}\mathcal{C}^{2}_{1\alpha}=\mathbf{i}\,\frac{1}{2}(\omega_{01\alpha}+\mathbf{i}\,\omega_{02\alpha}+\omega_{31\alpha}+\mathbf{i}\,\omega_{32\alpha})$$

. **For $d>(5+1)$ we get more ${}^{I}\mathcal{C}^{m}_{f\alpha}$, $(2^{\frac{d}{2}-1})^2$, than $\omega_{ab\alpha}$, $\frac{d}{2}(d-1)$.**
But they are related.

Let us repeat some general properties of the Clifford even "basis vector" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ when they apply on each other.

- Let us denote the self adjoint member in each group of "basis vectors" of particular f as ${}^{I}\hat{\mathcal{A}}^{m_0\dagger}_{f}$. We easily see that

$$
\begin{aligned}
\{{}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}, {}^{I}\hat{\mathcal{A}}^{m'\dagger}_{f},\}_{-} &= 0\,, \quad \text{if}\,(m,m') \neq m_0 \,\text{or}\, m = m_0 = m'\,, \forall f\,,\\
{}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} *_{A} {}^{I}\hat{\mathcal{A}}^{m_0\dagger}_{f} &\rightarrow {}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}\,, \quad \forall\, m\,, \forall\, f\,.
\end{aligned}
$$

- Two "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{I}\hat{\mathcal{A}}^{m'\dagger}_{f}$ of the same f and of $(m,m') \neq m_0$ are orthogonal.

-

$$
{}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} *_{A} {}^{I}\hat{\mathcal{A}}^{m'\dagger}_{f'} \rightarrow \begin{cases} {}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}\,, \\ \text{or zero}\,. \end{cases}\,.
$$

Looking at the properties of free gravitational fields we can relate also the interaction among ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}\,{}^{I}\mathcal{C}^{m}_{f\alpha}$ **and the interaction among gravitational fields.**

We can proceed in equivalent way also when looking for relations between

$\sum_{ab} \tilde{S}^{ab}\ \tilde{\omega}_{ab\alpha}$ and $\sum_{mf}\ {}^{I}\hat{\tilde{\mathcal{A}}}^{m\dagger}_{f}\ \ {}^{I}\tilde{\mathcal{C}}^{m}_{f\alpha}$

We are then able to replace

$\sum_{ab} S^{ab}\ \omega_{ab\alpha}$ by $\sum_{mf}\ {}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}\ \ {}^{I}\mathcal{C}^{m}_{f\alpha}$ and

$\sum_{ab} \tilde{S}^{ab}\ \tilde{\omega}_{ab\alpha}$ by $\sum_{mf}\ {}^{I}\hat{\tilde{\mathcal{A}}}^{m\dagger}_{f}\ \ {}^{I}\tilde{\mathcal{C}}^{m}_{f\alpha}$

in a covariant derivative

$$\mathcal{L}_f - \frac{1}{2}(\bar{\psi}\,\gamma^a p_{0a}\psi) + h.c. \quad \text{with} \mathbf{p_{0a}} = \mathbf{f}^{\alpha}{}_{\mathbf{a}}\mathbf{p}_{\mathbf{0}\alpha} + \frac{\mathbf{1}}{\mathbf{2E}}\{p_\alpha, Ef^{\alpha}{}_a\}_-$$

$$\mathbf{p}_{\mathbf{0}\alpha} = \mathbf{p}_\alpha - \frac{1}{2}\sum_{\mathbf{ab}} \mathbf{S}^{\mathbf{ab}}\omega_{\mathbf{ab}\alpha} - \frac{1}{2}\sum_{\mathbf{ab}}\tilde{\mathbf{S}}^{\mathbf{ab}}\tilde{\omega}_{\mathbf{ab}\alpha}$$

$$\mathbf{p}_{\mathbf{0}\alpha} = \mathbf{p}_\alpha - \sum_{\mathbf{mf}} {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}}\,{}^{\mathbf{I}}\mathcal{C}^{\mathbf{m}}_{\mathbf{f}\alpha} - \sum_{\mathbf{mf}} {}^{\mathbf{I}}\hat{\tilde{\mathcal{A}}}^{\mathbf{m}\dagger}_{\mathbf{f}}\,{}^{\mathbf{I}}\tilde{\mathcal{C}}^{\mathbf{m}}_{\mathbf{f}\alpha}\,,$$

provided that ${}^{I}\mathcal{C}^{m}_{f\alpha}$ and ${}^{I}\tilde{\mathcal{C}}^{m}_{f\alpha}$ fulfil also the application of both operators on the fermion fields $\sum_{mf}\beta^m\ \hat{b}^{m\dagger}_{f}$ for any β^m and any f.

Although I almost "see" (almost prove) the general relations among

$${}^{I,II}\hat{\mathcal{A}}^{m\dagger}_{f}\ {}^{I,II}\mathcal{C}^{m}_{f\alpha}\ ,\quad {}^{I,II}\hat{\widetilde{\mathcal{A}}}^{m\dagger}_{f}\ {}^{I,II}\widetilde{\mathcal{C}}^{m}_{f\alpha}$$

and

$$S^{ab}\omega_{ab\alpha}\ ,\quad \tilde{S}^{ab}\widetilde{\omega}_{ab\alpha},$$

for any even d

it still remains to see what new, if any, this new way of second quantization of fermions and bosons brings.

I hope I have convinced you that the Clifford algebra objects, if used to describe the internal space — "basis vectors" — of fermion and boson fields, offer the explanation for the postulates of the usual second quantization procedure.

- **The internal space offers a finite number of degrees of freedom for either fermion or boson fields.**

  It is the ordinary momentum or coordinate basis which offers the continuously infinite basis.

  Progress in Particle and Nuclear Physics, http://doi.org/10.1016.j.ppnp.2021.103890

  The second quantization of bosons is newer, partly presented in **Proceedings of the Bled workshop 2021, [arXiv:2112.04378].**

- **Let me introduce the basis in momentum representation** $\{\hat{p}^i\,,\hat{p}^j\}_- = 0$, $\{\hat{x}^k\,,\hat{x}^l\}_- = 0$, $\{\hat{p}^i\,,\hat{x}^j\}_- = i\eta^{ij}$.

$$
\begin{aligned}
|\vec{p}> &= \hat{b}^{\dagger}_{\vec{p}}\,|\,0_p\;>\,,\quad <\vec{p}\,| = <\,0_p\,|\,\hat{b}_{\vec{p}}\,,\\
<\vec{p}\,|\,\vec{p}'> &= \delta(\vec{p}-\vec{p}') = <\,0_p\,|\hat{b}_{\vec{p}}\,\hat{b}^{\dagger}_{\vec{p}'}|\,0_p\;>\,, <\,0_p\,|\,0_p\;> = 1\,,\\
&\text{leading to}\\
\hat{b}_{\vec{p'}}\,\hat{b}^{\dagger}_{\vec{p}} &= \delta(\vec{p'}-\vec{p})\,,
\end{aligned}
$$

It follows

$$
\begin{aligned}
<\vec{p}\,|\,\vec{x}> &= <0_{\vec{p}}\,|\,\hat{b}_{\vec{p}}\,\hat{b}^{\dagger}_{\vec{x}}|0_{\vec{x}}\;> = (<0_{\vec{x}}\,|\,\hat{b}_{\vec{x}}\,\hat{b}^{\dagger}_{\vec{p}}\,|0_{\vec{p}}\;>)^{\dagger}\\
\{\hat{b}^{\dagger}_{\vec{p}},\,\hat{b}^{\dagger}_{\vec{p}\,'}\}_- &= 0\,,\quad \{\hat{b}_{\vec{p}},\,\hat{b}_{\vec{p}\,'}\}_- = 0\,,\quad \{\hat{b}_{\vec{p}},\,\hat{b}^{\dagger}_{\vec{p}\,'}\}_- = 0\,,\\
\{\hat{b}^{\dagger}_{\vec{x}},\,\hat{b}^{\dagger}_{\vec{x}\,'}\}_- &= 0\,,\quad \{\hat{b}_{\vec{x}},\,\hat{b}_{\vec{x}\,'}\}_- = 0\,,\quad \{\hat{b}_{\vec{x}},\,\hat{b}^{\dagger}_{\vec{x}\,'}\}_- = 0\,,\\
&\text{while}\\
\{\hat{b}_{\vec{p}},\,\hat{b}^{\dagger}_{\vec{x}}\}_- &= e^{i\vec{p}\cdot\vec{x}}\frac{1}{\sqrt{(2\pi)^{d-1}}}\,,\quad \{\hat{b}_{\vec{x}},\,\hat{b}^{\dagger}_{\vec{p}}\}_- = e^{-i\vec{p}\cdot\vec{x}}\frac{1}{\sqrt{(2\pi)^{d-1}}}\,,
\end{aligned}
$$

$\vec{p}$ determines momentum in ordinary space, $|\psi_o > *_T |0_{\vec{p}} >$ is the vacuum state for fermions ($|\psi_o >= |\psi_{oc} >$) or for bosons ($|\psi_o >= |\psi_{ob} >$) with the zero momentum, $\hat{b}^{\dagger}_{\vec{p}}$ pushes the momentum by $\vec{p}$.

- For fermions we can write

$$\{\hat{\mathbf{b}}^{s\dagger}_{f}(\vec{p}) \;=\; \sum_{m} c^{sm}{}_{f}\,(\vec{p})\,\hat{b}^{\dagger}_{\vec{p}} *_{T}\, \hat{b}^{m\dagger}_{f}\}\,|\psi_{oc}> *_{T} |0_{\vec{p}}>\,,$$

- For bosons we can write

$$\{{}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{s}\dagger}_{\mathbf{f}\alpha}(\tilde{\mathbf{p}}) \;=\; \sum_{mf} \mathcal{C}^{\mathbf{sm}}{}_{\mathbf{f}\alpha}\,(\tilde{\mathbf{p}})\,\hat{\mathbf{b}}^{\dagger}_{\tilde{\mathbf{p}}} *_{\mathbf{T}}\, {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{s}\dagger}_{\mathbf{f}}\}\,|\phi_{ob}> *_{T} |0_{\vec{p}}> \;.$$

boson fields need additional space index α, as we have seen.

While the internal space of fermions if describable by the finite number of the Clifford odd "basis vectors" and the internal space of bosons if describable by the finite of the Clifford even "basis vectors", (for bosons and fermions it is the ordinary space which brings the infinite number of degrees of freedom) the usual second quantization postulates the creation and annihilation operators, anticommuting for fermions on the whole Hilbert space

$$
\begin{aligned}
\{\hat{\mathbf{b}}^{\mathbf{s}\dagger}_{\mathbf{f}}(\tilde{\mathbf{p}}), \hat{\mathbf{b}}^{\mathbf{s}'\dagger}_{\mathbf{f}'}(\tilde{\mathbf{p}}\,')\}_{+}\mathcal{H} &= \mathbf{0}\,,\\
\{\hat{\mathbf{b}}^{\mathbf{s}\dagger}_{\mathbf{f}}(\tilde{\mathbf{p}}), \hat{\mathbf{b}}^{\mathbf{s}'\dagger}_{\mathbf{f}'}(\tilde{\mathbf{p}}\,')\}_{+}\mathcal{H} &= \mathbf{0}\,,\\
\{\hat{\mathbf{b}}^{\mathbf{s}\dagger}_{\mathbf{f}}(\tilde{\mathbf{p}}), \hat{\mathbf{b}}^{\mathbf{s}'\dagger}_{\mathbf{f}'}(\tilde{\mathbf{p}}\,')\}_{+}\mathcal{H} &= \delta^{\mathbf{ss}'}\delta_{\mathbf{ff}'}\delta(\vec{p} - \vec{p'})\mathcal{H}\,,
\end{aligned}
$$

and commuting for bosons.

The Clifford algebra used in the spin-charge-family theory explains the second postulates of fields.

We have treated so far free fermion fields and boson fields in any even dimensional space. We describe the internal space of fermion fields and boson fields with the odd and even Clifford algebra elements, respectively.

- We learn that all the family members of fermions, they are reachable by S^{ab}, are equivalent, and all the families, they are reachable by $\tilde{S}^{ab}$, are equivalent. We learn that the Hermitian conjugated partners of fermion fields form their own group.
- We learn that the boson fields have their Hermitian conjugated partners within the same group of Clifford even members, and that families play no role for bosons. Boson fields carry in addition the space index.

- The spin-charge-family theory assumes a simple starting action for fermions and bosons in $d \geq (13+1)$, with the gravity as the only gauge fields.
- It is the break of the starting symmetry which causes that fermion fields and gravitational fields manifest in $d = (3+1)$ as all the observed quarks and leptons and the corresponding vector and scalar gauge fields.
- Is the spin-charge-family theory following what nature does while breaking starting symmetries?

- **Spinors carry** in $d \geq (13+1)$ **two kinds of spin, no charges**, *Phys. Rev.* **D 91** 065004 (2015), *J.of Mod. Physics* **6** (2015) 2244, **Rev. article** in **JPPNPhttp://doi.org/10.1016.j.ppnp.2021.103890** .

  o The **Dirac spin** (γ^a) in $d=(13+1)$ **describes** in $d=(3+1)$ **spin** and **ALL the charges of quarks and leptons and anti-quarks and anti-leptons**, **left and right handed, explaining all the assumptions about the charges and the handedness of the Standard Model**, *J. of Math. Phys.* **34** (1993), 3731, *J. of Math. Phys.* **43**, 5782 (2002) [hep-th/0111257].

  o The **second kind** of **spin** $(\tilde{\gamma}^a)$ **describes FAMILIES, explaining the origin and number of families**, *J. of Math. Phys.* **44** 4817 (2003) [hep-th/0303224].

  o There is **NO third kind of spin.**

- **C,P,T symmetries in** $d=(3+1)$ **follow from the C,P,T symmetry in** $d \geq (13+1)$. (*JHEP* **04** (**2014**) 165)

- **All** vector and **scalar gauge fields origin in gravity, explaining the origin of the vector and scalar gauge fields, which in the Standard model are assumed**, *Eur. Phys. J. C* **77** (2017) 231:

  o Vector and **scalar gauge fields origin in two spin connection fields**, the **gauge fields** of $\gamma^a \gamma^b$ and $\tilde{\gamma}^a \tilde{\gamma}^b$, and in

  o vielbeins, the gauge fields of momenta
  Eur. Phys. J. C **77** (2017) 231, [arXiv:1604.00675]

- **If there are no spinor sources present, then either vector ($\vec{A}^A_m$, $m = 0, 1, 2, 3$) or scalar ($\vec{A}^A_s$, $s = 5, 6, .., d$) gauge fields are determined by vielbeins uniquely**.

- **Spinors (fermions) interact correspondingly with**
  o the vielbeins and
  o the **two kinds of the** spin connection fields, *Eur. Phys. J. C* **77** (2017) 231.
- **In** $d = (3 + 1)$ the **spin-connection fields**,
  together with the vielbeins,
  manifest either as
  o **vector gauge fields** with all the **charges** in the **adjoint representations** or as
  o **scalar gauge fields** with the **charges with respect to the space index** in the **"fundamental" representations** and all the other **charges** in the **adjoint representations** or as
  o **tensor gravitational field**.
- **I shall discuss the internal space of fermions and bosons using the Clifford algebra objects, the Clifford odd algebra to describe internal space of fermions and Clifford even algebra to describe the internal space of bosons, what explains the second quantization postulates for fermions and for bosons.**

There are two kinds of scalar fields with respect to the space index s — this is with respect to $d = (3+1)$:

- **Those** with **($s = 5, 6, 7, 8$)** (they carry zero **"spinor charge"**) **are doublets** with respect to the $SU(2)_I$ **(the weak) charge** and the **second $SU(2)_{II}$ charge (determining the hyper charge)**. They are in the **adjoint representations** with respect to the **family** and the **family members charges**.

  o These scalars explain the Higgs's scalar and the Yukawa couplings.

  Phys. Rev. **D 91 (2015)** 6, 065004

- **Those** with the "spinor charge" of a quark and **(**$s = 9, 10, ..d$**)** are **colour triplets. Also they are in the adjoint representations** with respect to the **family** and the **family members charges**.

  o These scalars transform antileptons into quarks, and antiquarks into quarks and back and correspondingly **contribute to matter-antimatter asymmetry of our universe and to proton decay**.

- **There are no additional scalar fields in the spin-charge-family theory**, if $d = (13 + 1)$.

  Phys. Rev. **D 91** (**2015**) 6, 065004
  J. of Mod. Phys. **6** (**2015**) 2244

Breaking symmetry from M^{13+1} into $M^{7+1} \times M^6$

- **We start with the massless solutions of the Weyl equation in $d = (13+1)$ with the "basis vectors", described by the odd Clifford algebra objects, determining the internal space of fermions.**
- **With the spin (or the total angular momentum) in extra dimensions, $d > (7+1)$, determining the charge in $d = (7+1)$.**
- **Also all the boson fields are in $d = (13+1)$ massless free fields with the "basis vectors", described by the even Clifford algebra objects, determining the internal space of bosons.**

- We then let the $\mathcal{M}^{13+1}$ manifold to break into $\mathcal{M}^{7+1}$ $\times$ an almost S^6 sphere.
- The Weyl equation, $m = (0, 1, 2, 3, 5, 6, 7, 8)$ and $s = 9, 10, \ldots 13, 14$ is

$$(\gamma^m p_m + \gamma^s p_{0s})\psi = 0\,,$$
$$p_{0s} = f^{\sigma}_{s}(p_\sigma - \frac{1}{2} S^{ab}\omega_{ab\sigma} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\sigma}) + \frac{1}{2E}\{p_\sigma, f^{\sigma}_{s} E\}_- \,.$$

- With the choice of the vielbein fields and the spin spinconnection fields of both kinds one can achieve that the infinite surface $d = (9, 10, 11, \ldots, 13, 14)$ curls into an almost S^6 (with one hole with the substructure of $SU(3) \times U(1)$) with massless fermions in $d = (7 + 1)$.
- This is the project, not yet done. The simpler problem with breaking $\mathcal{M}^{5+1}$ manifold into $\mathcal{M}^{3+1}$ $\times$ an almost S^2 sphere with one hole is done, without taking into account families and with families included.

New J. Phys. 13:103027, 2011.
J. Phys. A: Math. Theor. 45:465401, 2012.

Condensate

- **The (assumed so far, waiting to be derived how does this spontaneously appear) scalar condensate of two right handed neutrinos with the family quantum numbers of the upper four families (there are two four family groups in the theory)**, appearing $\approx 10^{16}$ GeV or higher,

  o breaks the CP symmetry, causing the matter-antimatter asymmetry and the proton decay,

  o couples to all the scalar fields, making them massive,

  o couples to all the phenomenologically unobserved vector gauge fields, making them massive.

  o Before the electroweak break all the so far observed vector gauge fields are massless.

  Phys. Rev. **D 91** (**2015**) 6, 065004,
  J. of Mod. Phys. **6** (**2015**) 2244,
  J. Phys.: Conf.Ser. 845 01, **IARD 2017**

- The **vector fields, which do not couple to the condensate and remain massless, are:**

  o the hyper charge vector field.

  o the weak vector fields,

  o the colour vector fields,

  o the gravity fields.

  The $SU(2)_{II}$ symmetry breaks due to the **condensate**, leaving the **hyper charge unbroken**.

Nonzero vacuum expectation values of scalars

— waiting to be shown how does such an event, making the masses of the scalar gauge fields imaginary, appear in the *spin-charge-family* **spontaneously**.

- **The scalar fields with the space index $(7, 8)$, gaining nonzero vacuum expectation values**, a constant values, **cause the electroweak break**,

  o **breaking the weak and the hyper charge,**

  o **changing their own masses**,

  o **bringing masses to the weak bosons**,

  o **bringing masses to the families of quarks and leptons**.

Phys. Rev. **D 91** (**2015**) 6, 065004,
J. Phys.: Conf.Ser. 845 01 **IARD 2017**,
Eur. Phys. J.C. **77** (2017) 231 [arXiv:1604.00675],
J. of Mod. Phys. **6** (**2015**) 2244, [arXiv:1502.06786, arXiv:1409.4981]

- **The only gauge fields which do not couple to these scalars and remain massless are**

  o electromagnetic,

  o colour vector gauge fields,

  o gravity.

- **There are two times four decoupled massive families of quarks and leptons after the electroweak break**:

  o There are the observed three families among the **lower four, the fourth to be observed**.

  o The stable among the upper four families form the **dark matter**.

  Phys. Rev. **D 80**, 083534 (**2009**),
  Phys. Rev. **D 91** (**2015**) 6, 065004,
  J. Phys.: Conf.Ser. 845 01, **IARD 2017**

- **It is extremely encouraging for the spin-charge-family theory, that a simple starting action contains all the degrees of freedom observed at low energies, directly or indirectly, and that only**

  o the **break of manifold** $M^{(13,1)}$ **to** $M^{(7,1)} \times M^{(6)}$ **is needed so that the manifold** $M^{(6)}$ **makes an almost** S^n **sphere.**

  o the **condensate** and

  o **constant values of all the scalar fields with** $s = (7, 8)$

  are needed that the theory explains

  o **all the assumptions of the standard model, with the gauge fields, scalar fields, families of fermions, masses of fermions and of bosons included**,

  o **explaining also the dark matter**,

  o **the matter/antimatter asymmetry**,

  o **the triangle anomalies cancellation in the standard model** (Forts. der Physik, Prog.of Phys.) (2017) 1700046)

  and...

**

Variation of the action brings for $\omega_{ab\alpha}$

$$\begin{aligned}
\omega_{ab\alpha} &= -\frac{1}{2E}\Big\{ e_{e\alpha} e_{b\gamma}\, \partial_\beta (E f^{\gamma[e} f^{\beta}{}_{a]}) + e_{e\alpha} e_{a\gamma}\, \partial_\beta (E f^{\gamma}{}_{[b} f^{\beta e]}) \\
&\qquad - e_{e\alpha} e^{e}{}_{\gamma}\, \partial_\beta \big(E f^{\gamma}{}_{[a} f^{\beta}{}_{b]}\big) \Big\} \\
&- \frac{e_{e\alpha}}{4} \Big\{ \bar{\Psi} \Big(\gamma_e S_{ab} + \frac{3i}{2} (\delta^e_b \gamma_a - \delta^e_a \gamma_b) \Big) \Psi \Big\} \\
&- \frac{1}{d-2} \Big\{ e_{a\alpha} \Big[\frac{1}{E} e^{d}{}_{\gamma} \partial_\beta \Big(E f^{\gamma}{}_{[d} f^{\beta}{}_{b]} \Big) + \frac{1}{2} \bar{\Psi} \gamma^{d} S_{db} \Psi \Big] \\
&\qquad - e_{b\alpha} \Big[\frac{1}{E} e^{d}{}_{\gamma} \partial_\beta \Big(E f^{\gamma}{}_{[d} f^{\beta}{}_{a]} \Big) + \frac{1}{2} \bar{\Psi} \gamma^{d} S_{da} \Psi \Big\} \Big]
\end{aligned}$$

IARD, J. Phys.: Conf. Ser. 845 01**2017** and the refs. therein

**

and for $\tilde{\omega}_{ab\alpha}$,

$$\begin{aligned}
\tilde{\omega}_{ab\alpha} &= -\frac{1}{2E}\Big\{ e_{e\alpha} e_{b\gamma}\, \partial_\beta (E f^{\gamma[e} f^{\beta}{}_{a]}) + e_{e\alpha} e_{a\gamma}\, \partial_\beta (E f^{\gamma}{}_{[b} f^{\beta e]}) \\
&\qquad - e_{e\alpha} e^{e}{}_{\gamma}\, \partial_\beta \left(E f^{\gamma}{}_{[a} f^{\beta}{}_{b]} \right) \Big\} \\
&- \frac{e_{e\alpha}}{4} \left\{ \bar{\Psi} \left(\gamma_e \tilde{S}_{ab} + \frac{3i}{2} (\delta^e_b \gamma_a - \delta^e_a \gamma_b) \right) \Psi \right\} \\
&- \frac{1}{d-2} \Big\{ e_{a\alpha} \left[\frac{1}{E} e^{d}{}_{\gamma} \partial_\beta \left(E f^{\gamma}{}_{[d} f^{\beta}{}_{b]} \right) + \frac{1}{2} \bar{\Psi} \gamma^d \tilde{S}_{db} \Psi \right] \\
&\qquad - e_{b\alpha} \left[\frac{1}{E} e^{d}{}_{\gamma} \partial_\beta \left(E f^{\gamma}{}_{[d} f^{\beta}{}_{a]} \right) + \frac{1}{2} \bar{\Psi} \gamma^d \tilde{S}_{da} \Psi \Big\} \right]
\end{aligned}$$

Eur. Phys. J. C, **77** (2017) 231 and the refs. therein.
If there are no spinors present, the two spin connections are uniquely described by vielbeins $f^{\alpha}{}_{a}$.

Fermions

- The action for spinors "seen" from $d=(3+1)$ and analyzed with respect to the standard model groups as subgroups of $SO(13+1)$:

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_m)\psi + \\
&\{\sum_{s=[7],[8]} \bar{\psi}\gamma^s p_{0s}\,\psi\} + \\
&\{\sum_{s=[5],[6]} \bar{\psi}\gamma^s p_{0s}\,\psi + \\
&\sum_{t=[9],\dots[14]} \bar{\psi}\gamma^t p_{0t}\,\psi\}\,. \\
&+\text{the rest}\;,\;,
\end{aligned}
$$

J. of Mod. Phys. **4** (**2013**) 823

Covariant momenta

$$
\begin{aligned}
p_{0m} &= \{p_m - \sum_A g^A\, \vec{\tau}^A\, \vec{A}^A_m\} \\
\mathbf{m} &\quad n \quad (\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3})\,, \\
p_{0s} &= f^{\sigma}_{s}\,[p_\sigma - \sum_A g^A\, \vec{\tau}^A\, \vec{A}^A_\sigma - \sum_A \tilde{g}^A\, \vec{\tilde{\tau}}^A\, \vec{\tilde{A}}^A_\sigma]\,, \\
\mathbf{s} &\in (\mathbf{7}, \mathbf{8})\,, \\
p_{0s} &= f^{\sigma}_{s}\,[p_\sigma - \sum_A g^A\, \vec{\tau}^A\, \vec{A}^A_\sigma - \sum_A \tilde{g}^A\, \vec{\tilde{\tau}}^A\, \vec{\tilde{A}}^A_\sigma]\,, \\
\mathbf{s} &\in (\mathbf{5}, \mathbf{6})\,, \\
p_{0t} &= f^{\sigma'}_{t}\,(p_{\sigma'} - \sum_A g^A\, \vec{\tau}^A\, \vec{A}^A_{\sigma'} - \sum_A \tilde{g}^A\, \vec{\tilde{\tau}}^A\, \vec{\tilde{A}}^A_{\sigma'})\,, \\
\mathbf{t} &\in (\mathbf{9}, \mathbf{10}, \mathbf{11}, \ldots, \mathbf{14})\,,
\end{aligned}
$$

$$
\begin{aligned}
A_s^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}\, \omega_{abs} \,, \\
A_t^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}\, \omega_{abt} \,, \\
\tilde{A}_s^{Ai} &= \sum_{a,b} \tilde{c}^{Ai}{}_{ab}\, \tilde{\omega}_{abs} \,, \\
\tilde{A}_t^{Ai} &= \sum_{a,b} \tilde{c}^{Ai}{}_{ab}\, \tilde{\omega}_{abt} \,.
\end{aligned}
$$

$$
\begin{aligned}
\tau^{\mathbf{Ai}} &= \sum_{a,b} c^{Ai}{}_{ab}\, \mathbf{S}^{\mathbf{ab}}\,, \\
\tilde{\tau}^{\mathbf{Ai}} &= \sum_{a,b} \tilde{c}^{Ai}{}_{ab}\, \tilde{\mathbf{S}}^{\mathbf{ab}}\,, \\
\{\tau^{\mathbf{Ai}}, \tau^{\mathbf{Bj}}\}_- &= i\delta^{AB} f^{Aijk} \tau^{\mathbf{Ak}}\,, \\
\{\tilde{\tau}^{\mathbf{Ai}}, \tilde{\tau}^{\mathbf{Bj}}\}_- &= i\delta^{AB} f^{Aijk} \tilde{\tau}^{\mathbf{Ak}}\,, \\
\{\tau^{\mathbf{Ai}}, \tilde{\tau}^{\mathbf{Bj}}\}_- &= 0\,.
\end{aligned}
$$

- **o** τ^{Ai} represent the *standard model* charge groups — $SU(3)_c, SU(2)_w$ — the second $SU(2)_{II}$, the "spinor" charge $U(1)$, taking care of the hyper charge Y,
- **o** $\tilde{\tau}^{Ai}$ denote the family quantum numbers.

$$\mathbf{N}^{\mathbf{i}}_{(\mathbf{L},\mathbf{R})} : = \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03}),$$

$$\tau^{\mathbf{i}}_{(\mathbf{1},\mathbf{2})} : = \frac{1}{2}(S^{58} \mp S^{67}, S^{57} \pm S^{68}, S^{56} \mp S^{78}),$$

$$\tau^{\mathbf{i}}_{\mathbf{3}} := \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\},$$

$$\tau^{\mathbf{4}} := -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14}),$$

$$\mathbf{Y} := \tau^4 + \tau^{23},$$

$$\mathbf{Y}' := -\tau^4 \tan^2\vartheta_2 + \tau^{23},$$

$$\mathbf{Q} := \tau^{13} + Y,$$

$$\mathbf{Q}' := -Y \tan^2\vartheta_1 + \tau^{13},$$

and equivalently for family groups $\tilde{S}^{ab}$.

Breaks of symmetries
after starting with

o massless spinors (fermions) ,

o massles vielbeins and two kinds of the spin connection fields

We prove for a toy model that breaking symmetry in Kaluza-Klein theories can lead to massless fermions.

New J. Phys. 13, 103027, 2011.
J. Phys. A. Math. Theor. 45, 465401, 2012.
[arXiv:1205.1714], [arXiv:1312.541], [arXiv:hep-ph/0412208 p.64-84].
[arXiv:1302.4305], p. 157-166.

$$\mathsf{SO}(1,13) \times \widetilde{\mathsf{SO}(1,13)}$$

BREAK I

at E $\geq 10^{16}$GeV

$\downarrow$

$\mathsf{SO}(1,7) \times \widetilde{\mathsf{SO}(1,7)} \times \quad \mathsf{U}(1) \times \quad \mathsf{SU}(3)$

$\swarrow$ eight massless families $\searrow$

$\mathsf{SO}(1,3) \times \mathsf{SO}(4) \times \mathsf{U}(1) \times \quad (\widetilde{\mathsf{SU}}(2)_{\mathsf{I}_{\widetilde{\mathsf{SO}}(1,3)}} \times \widetilde{\mathsf{SU}}(2)_{\mathsf{I}_{\widetilde{\mathsf{SO}}(4)}}) \times \quad (\widetilde{\mathsf{SU}}(2)_{\mathsf{II}_{\widetilde{\mathsf{SO}}(1,3)}} \times \widetilde{\mathsf{SU}}(2)_{\mathsf{II}_{\widetilde{\mathsf{SO}}(4)}}) \times \quad \mathsf{SU}(3)$

(devided into two groups)

BREAK II

$\downarrow$

The Standard Model like way of breaking

$\downarrow$

$\mathsf{SO}(1,3) \times \mathsf{U}(1) \times \mathsf{SU}(3)$

$\times$(two groups of four massive families)

- **Both breaks, the one from $SO(13,1)$ to $SO(7,1) \times SO(6)$ and the appearance of the condensate, leave eight families ($2^{8/2-1} = 8$, determined by the symmetry of $\widetilde{SO}(1,7)$) massless. All the families are $\widetilde{SU}(3)$ chargeless.** Phys. Rev. **D**, 80.083534 (**2009**)

- **The appearance of the condensate of the two right handed neutrinos, coupled to spin 0, makes the boson gauge fields, with which the condensate interacts, massive.** These gauge fields are:

  o **All the scalar gauge fields with the space index** $s \geq 5$.

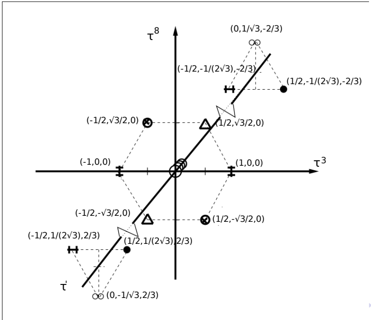
  o **The vector ($m \leq 3$) gauge fields with the** Y' **charges — the superposition of** $SU(2)_{II}$ and $U(1)_{II}$ **charges.**
  J. Phys.: Conf. Ser. 845 (**2017**) 012017

The **condensate has spin** $S^{12} = 0$, $S^{03} = 0$,
weak charge $\vec{\tau}^{1} = 0$, **and**
$\vec{\tilde{\tau}}^{1} = 0, \tilde{Y} = 0, \tilde{Q} = 0, \vec{\tilde{N}}_L = 0.$

state	τ^{23}	τ^{4}	Y	Q	$\tilde{\tau}^{23}$	$\tilde{N}^{3}_{R}$	$\tilde{\tau}^{4}$
$\|\nu^{VIII}_{1R}>_1\ \|\nu^{VIII}_{2R}>_2$	1	−1	0	0	1	1	−1
$\|\nu^{VIII}_{1R}>_1\ \|e^{VIII}_{2R}>_2$	0	−1	−1	−1	1	1	−1
$\|e^{VIII}_{1R}>_1\ \|e^{VIII}_{2R}>_2$	−1	−1	−2	−2	1	1	−1

Let us look at boson **"basis vectors"** as presented in already shown figure, which analyses ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ with respect to Cartan subalgebra members (τ^3, τ^8, τ').

There are
one sextet with $\tau' = 0$,
four singlets with $(\tau^3 = 0, \tau^8 = 0, \tau' = 0)$,
one triplet with $\tau' = \frac{2}{3}$ and one triplet with $\tau' = -\frac{2}{3}$.
The only ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ which couple to condensate are the two triplets with non zero $\tau' = \pm\frac{2}{3}$, which **transform leptons** into **quarks**. **They become massive**.



- The **colour, elm, weak and hyper** **vector gauge fields do not interact with the condensate and remain massless**. J. of Mod. Physics **6** (**2015**) 2244

- **At the electroweak break** from $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$ to $SO(1,3) \times U(1) \times SU(3)$
  o **scalar fields with the space index $s = (7,8)$ obtain constant values and imaginary masses (nonzero vacuum expectation values)**,
  o **break correspondingly the weak and the hyper charge and change their own masses**.
  o **They leave massless** only the **colour, elm** and **gravity gauge fields.**
- **All the eight massless families gain masses**.

Also these is so far just assumed, waiting to be proven that scalar fields, together with boundary conditions, are spontaneously causing also this last breaks.

However, all the needed vector and scalar gauge fields, the fermion fields with all the observed properties, are already in the simple starting action, making the *spin-charge-family* **theory (at least so far) very promising**.

- **To the electroweak break several scalar fields, the gauge fields of twice $\widetilde{SU}(2) \times \widetilde{SU}(2)$ and three $\times U(1)$, contribute, all with the weak and the hyper charge of the *standard model* Higgs.**
- **They carry besides the weak and the hyper charge either o the family members quantum numbers originating in (Q,Q',Y') or o the family quantum numbers originating in twice $\widetilde{SU}(2) \times \widetilde{SU}(2)$.**

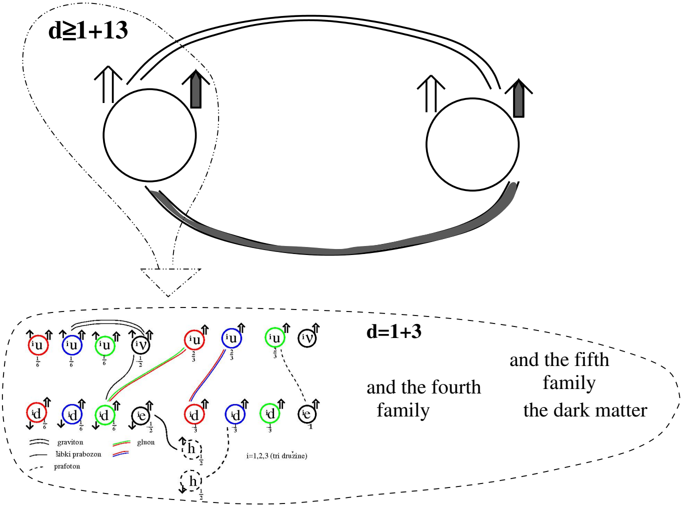
  J. of Mod. Physics **6** (**2015**) 2244.

- **The mass matrices of each family member manifest the $\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$ symmetry, which remains unchanged in all loop corrections**.

[arXiv:1902.02691, arXiv:1902.10628]

- We studied on a toy model of $d = (1+5)$ conditions which lead after breaking symmetries to massless spinors chirally coupled to the Kaluza-Klein-like gauge field.,

  New J. Phys. **13** (**2011**) 103027, 1-25,
  Int. J Mod. Phys. **A 29**, 1450124 (**2014**), 21 pages.


d≧1+13
d=1+3
and the fourth family
and the fifth family
the dark matter
graviton
šibki prabozon
prafoton
gluon
i=1,2,3 (tri družine)

**

The theory explains:

- The appearance of the finite number of the internal space "basis vectors" of fermions, $\hat{b}^{m\dagger}_{f}$.
  The appearance of the finite number of the internal space "basis vectors" of bosons, ${}^{I}\mathcal{A}^{m\dagger}_{f}\,{}^{I}\mathcal{C}^{m}_{f\alpha}$.
- The anticommutation relations among the creation and annihilation operators, creating the anticommuting single fermion states.
  The commutation relations among the creation and annihilation operators, creating the commuting single boson states.
- The continuously infinite number of creation operators due to infinite dimensional ordinary space for fermions and bosons.
- The tensor products of the Clifford odd creation operators explain the Hilbert space of the second quantized fermions.

- **It is worthwhile to notice that ”nature could make a choice” of Grassmann rather than Clifford space:**
  o Also in Grassmann space, namely, one finds the anticommutation relations needed for a fermion field.
  o But in this case spinors would have spins and charges in adjoint representations with respect to particular subgroups.
  o And no families would appear.

Vector gauge fields origin in gravity,

in vielbeins f^{a}_{α} and two kinds of the spin connection fields,

$\omega_{ab\alpha}, \tilde{\omega}_{ab\alpha}$,

the gauge fields of S^{ab} and $\tilde{S}^{ab}$. I showed above that both are expressible by ${}^{I}\mathcal{A}^{m\dagger}_{f}\,{}^{I}\mathcal{C}^{m}_{f\alpha}$ and ${}^{I}\tilde{\mathcal{A}}^{m\dagger}_{f}\,{}^{I}\tilde{\mathcal{C}}^{m}_{f\alpha}$.

**

- **All the vector gauge fields**, A^{Ai}_{m}, $(m,n) = (0,1,2,3)$ **of the observed charges** $\tau^{Ai} = \sum_{s,t} c^{Ai}{}_{st}\, S^{st}$, **manifesting at the observable energies**, **have all the properties as assumed by the** *standard model*.
- **They carry with respect to the space index** $m \in (0,1,2,3)$ **the vector degrees of freedom, while they have additional internal degrees of freedom** (τ^{Ai}) in **the adjoint representations**.
- **They origin as spin conection gauge fields** of S^{ab}: $A^{Ai}_{m} = \sum_{s,t} c^{Aist}\, \omega_{stm}$.
- $\mathcal{S}^{ab}$ **applies on indexes** (s,t,m) **as follows**

$$\mathcal{S}^{ab}\, \omega_{stm\ldots g} \quad = \quad i\,(\delta^{a}_{s}\, \omega^{b}{}_{tm\ldots g} - \delta^{b}_{s}\, \omega^{a}{}_{tm\ldots g})\,.$$

**

The action for vectors with respect to the space index $m=(0,1,2,3)$ origin in gravity

$$\int \mathbf{E}\, d^4x\, d^{(d-4)}x\, \alpha\, R^{(d)} = \int d^4x \{-\frac{1}{4} F^{Ai}{}_{mn} F^{Aimn}\},$$
$$A^{Ai}{}_{m} = \sum_{s,t} c^{Aist}\, \omega_{stm}\,.$$

Eur. Phys. J. C. **77** (**2017**) 231,

*

Also scalar fields
(there are doublets and triplets)
origin in gravity fields — they are spin connections and vielbeins —
with the space index $s \geq 5$, I showed above that also scalar fields are expressible by ${}^{I}\mathcal{A}^{m\dagger}_{f}\,{}^{I}\mathcal{C}^{m}_{f\alpha}$and ${}^{I}\tilde{\mathcal{A}}^{m\dagger}_{f}\,{}^{I}\tilde{\mathcal{C}}^{m}_{f\alpha}$.

Eur. Phys. J. C. **77** (**2017**) 231 ,
Phys. Rev. **D 91** (**2015**) 6, 065004,
J. of Mod. Physics **6** (**2015**) 2244.

- **There are several scalar gauge fields with the space index (s,t,s') = (7,8), all origin in the spin connection fields, either $\tilde{\omega}_{abs}$ or $\omega_{s'ts}$:**
  o Twice two triplets, the scalar gauge fields with the family quantum numbers ($\tilde{\tau}^{Ai} = \sum_{a,b} \tilde{c}^{Ai}{}_{ab}\, \tilde{S}^{ab}$) and
  o three singlets with the family members quantum numbers (Q,Q',Y'), the gauge fields of S^{st}.
- **They are all doublets with respect to the space index (5,6,7,8).**
- **They have all the rest quantum numbers determined by the adjoint representations.**
- **They explain at the so far observable energies the Higgs's scalar** and **the Yukawa couplings.**

The two doublets, determining the properties of the Higgs's scalar and the Yukawa couplings, are:

	state	τ^{13}	$\tau^{23} = Y$	spin	τ^4	Q
$A^{Ai}_{\underset{(-)}{78}}$	$A^{Ai}_7 + iA^{Ai}_8$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
$A^{Ai}_{\underset{(-)}{56}}$	$A^{Ai}_5 + iA^{Ai}_6$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1
$A^{Ai}_{\underset{(+)}{78}}$	$A^{Ai}_7 - iA^{Ai}_8$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0
$A^{Ai}_{\underset{(+)}{56}}$	$A^{Ai}_5 - iA^{Ai}_6$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	$+1$

There are $A^{Ai}_{\underset{(-)}{78}}$ and $A^{Ai}_{\underset{(+)}{78}}$ which gain nonzero vacuum expectation values at the electroweak break.

Index Ai determines the family ($\tilde{\tau}^{Ai}$) quantum numbers and the family members (Q,Q',Y') quantum numbers, both are in adjoint representations.

- **There are besides doublets, with the space index** $s = (5, 6, 7, 8)$, as well **triplets and anti-triplets, with respect to the space index** $s = (9, \ldots, 14)$.
- **There are no additional scalars in the theory for d=(13+1)**.
- **All are massless.**
- All the scalars have the family and the family members quantum numbers in the **adjoint** representations.
- The properties of scalars are to be analyzed with respect to the generators of the corresponding subgroups, expressible with $\mathcal{S}^{ab}$, as it is the case of the vector gauge fields.
- It is the (**so far assumed**) **condensate, which makes those gauge fields, with which it interacts, massive**.
  o The condensate breaks the CP symmetry.

- **The scalar condensate of two right handed neutrinos couple to**
  o all the scalar and vector gauge fields, making them massive,
  o It does not interact with the weak charge $SU(2)_I$, **the hyper charge** $U(1)$, and **the colour** $SU(3)$ **charge gauge fields**, as well as the **gravity,**
  leaving them massless.

J. of Mod.Phys.**4** (**2013**) 823-847,
J. of Mod.Phys. **6** (**2015**) 2244-2247,
Phys Rev.**D 91**(**2015**)6,065004.

Scalars with s=(7,8), which gain nonzero vacuum expectation values, break the weak and the hyper symmetry, while conserving the electromagnetic and colour charge:

$$
\begin{aligned}
A^{Ai}_s &\supset (A^{Q}_s, A^{Q'}_s, A^{Y'}_s, \tilde{\tilde{A}}^{\tilde{1}}_s, \tilde{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_s, \tilde{\tilde{A}}^{\tilde{2}}_s, \tilde{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_s),\\
\tau^{Ai} &\supset (Q, \quad Q', \quad Y', \quad \tilde{\tilde{\tau}}^{1}, \quad \tilde{\tilde{N}}_L, \quad \tilde{\tilde{\tau}}^{2}, \quad \tilde{\tilde{N}}_R),\\
s &= (7, 8).
\end{aligned}
$$

Ai denotes:

o family quantum numbers

($\tilde{\tilde{\tau}}^1$, $\tilde{\tilde{N}}_L$) quantum numbers of the first group of four families and

($\tilde{\tilde{\tau}}^2$, $\tilde{\tilde{N}}_R$)) quantum numbers of the second group of four families.

o And family members quantum numbers (Q, Q', Y')

A_s^{Ai} **are expressible with either** $\omega_{sts'}$ **or** $\tilde{\omega}_{abs'}$**.**

$$
\begin{aligned}
\vec{\tilde{A}}^1_s &= (\tilde{\omega}_{58s} - \tilde{\omega}_{67s}, \tilde{\omega}_{57s} + \tilde{\omega}_{68s}, \tilde{\omega}_{56s} - \tilde{\omega}_{78s})\,,\\
\vec{\tilde{A}}^2_s &= (\tilde{\omega}_{58s} + \tilde{\omega}_{67s}, \tilde{\omega}_{57s} - \tilde{\omega}_{68s}, \tilde{\omega}_{56s} + \tilde{\omega}_{78s})\,,\\
\vec{\tilde{A}}^N_{Ls} &= (\tilde{\omega}_{23s} + i\tilde{\omega}_{01s}, \tilde{\omega}_{31s} + i\tilde{\omega}_{02s}, \tilde{\omega}_{12}s + \tilde{\omega}_{03s})\,,\\
\vec{\tilde{A}}^N_{Rs} &= (\tilde{\omega}_{23s} - i\tilde{\omega}_{01s}, \tilde{\omega}_{31s} - i\tilde{\omega}_{02s}, \tilde{\omega}_{12}s - i\tilde{\omega}_{03s})\,,\\
A^Q_s &= \omega_{56s} - (\omega_{9\,10s} + \omega_{11\,12s} + \omega_{13\,14s})\,,\\
A^Y_s &= (\omega_{56s} + \omega_{78s}) - (\omega_{9\,10s} + \omega_{11\,12s} + \omega_{13\,14s})\\
A^4_s &= -(\omega_{9\,10s} + \omega_{11\,12s} + \omega_{13\,14s})\,.
\end{aligned}
$$

The **mass term**, appearing in the **starting action**,
is (p_s, when treating the lowest energy solutions, is left out)

$$\begin{aligned}
\mathcal{L}_M &= \sum_{s=(7,8),Ai} \bar{\psi}\,\gamma^s\,(-\tau^{Ai}\,A^{Ai}_s\,)\psi = \\
& -\bar{\psi}\,\{\,\stackrel{78}{(+)}\,\tau^{Ai}\,(A^{Ai}_7 - i\,A^{Ai}_8) + \stackrel{78}{(-)}\,\tau^{Ai}\,(A^{Ai}_7 + i\,A^{Ai}_8)\,\}\psi\;,
\end{aligned}$$

$$\stackrel{78}{(\pm)} = \frac{1}{2}\,(\gamma^7 \pm\, i\,\gamma^8\,)\,,\quad A^{Ai}_{\stackrel{78}{(\pm)}} := (A^{Ai}_7 \mp i\,A^{Ai}_8)\,.$$

Operators Y, Q and τ^{13}, applied on $(A^{Ai}_7 \mp i\, A^{Ai}_8)$

$$\begin{aligned}
\tau^{\mathbf{13}}\,(A^{Ai}_7 \mp i\, A^{Ai}_8) &= \pm\frac{1}{2}\,(A^{Ai}_7 \mp i\, A^{Ai}_8)\,,\\
\mathbf{Y}\,(A^{Ai}_7 \mp i\, A^{Ai}_8) &= \mp\frac{1}{2}\,(A^{Ai}_7 \mp i\, A^{Ai}_8)\,,\\
\mathbf{Q}\,(A^{Ai}_7 \mp i\, A^{Ai}_8) &= 0\,,
\end{aligned}$$

manifest that **all** $(A^{Ai}_7 \mp i\, A^{Ai}_8)$ have quantum numbers of the **Higgs's scalar of the standard model**, "dressing", after **gaining nonzero expectation values**, the right handed members of a family with appropriate charges, so that they gain charges of the left handed partners:
$(A^{Ai}_7 + iA^{Ai}_8)$ **"dresses"** u_R, ν_R and $(A^{Ai}_7 - iA^{Ai}_8)$ **"dresses"** d_R, e_R, with quantum numbers of their left handed partners, just as required by the "standard model".

Ai determines:

either

o the **Q,Q',Y'** **charges** of the **family members**

or

o family charges ($\vec{\tilde{\tau}}^{\tilde{1}}, \vec{\tilde{N}}_L$), **transforming a family member of one family into the same family member of another family**,

manifesting in each group of four families the
$\widetilde{SU}(2) \times \widetilde{SU}(2) \times U(1)$
symmetry.

** **Eight families** of u_R (spin 1/2, colour $(\frac{1}{2}, \frac{1}{2\sqrt{3}})$) and of colourless ν_R (spin 1/2). All have "tilde spinor charge" $\tilde{\tau}^4 = -\frac{1}{2}$, the weak charge $\tau^{13} = 0$, $\tau^{23} = \frac{1}{2}$. Quarks have "spinor" q.no. $\tau^4 = \frac{1}{6}$ and leptons $\tau^4 = -\frac{1}{2}$. The first four families have $\tilde{\tau}^{23} = 0$, $\tilde{N}^3_R = 0$, the second four families have $\tilde{\tau}^{13} = 0$, $\tilde{N}^3_L = 0$.

	$\tilde{N}^3_R = 0, \quad \tilde{\tau}^{23} = 0$		$\tilde{N}^3_R = 0, \quad \tilde{\tau}^{23} = 0$	$\tilde{\tau}^{13}$	$\tilde{N}^3_L$
$u^{c1}_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$u^{c1}_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	$\frac{1}{2}$
$u^{c1}_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	$-\frac{1}{2}$
$u^{c1}_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	$\frac{1}{2}$
	$\tilde{N}^3_L = 0, \quad \tilde{\tau}^{13} = 0$		$\tilde{N}^3_L = 0, \quad \tilde{\tau}^{13} = 0$	$\tilde{\tau}^{23}$	$\tilde{N}^3_R$
$u^{c1}_{R\,5}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,5}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$u^{c1}_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{[+]}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{[+]}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	$\frac{1}{2}$
$u^{c1}_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	$-\frac{1}{2}$
$u^{c1}_{R\,8}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \mid \stackrel{56}{[+]}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$	$\nu_{R\,8}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \mid \stackrel{56}{[+]}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(+)}\;\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	$\frac{1}{2}$

Before the **electroweak break** all the **families** are **mass protected** and correspondingly **massless**.

- Scalars with the weak and the hyper charge $(\mp\frac{1}{2}, \pm\frac{1}{2})$ determine masses of **all** the **family members** α of the **lower four families**, ν_R **of the lower four families have nonzero** $Y' := -\tau^4 + \tau^{23}$ and interact **with the scalar field** $(A^{Y'}_{(\pm)}, \vec{\tilde{A}}^{\tilde{1}}_{(\pm)}, \vec{\tilde{A}}^{\tilde{N}_L}_{(\pm)})$.
- **The group of the lower four families** manifest the $\widetilde{SU}(2)_{\widetilde{SO}(1,3)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)} \times U(1)$ **symmetry** (also after all loop corrections).

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e^* & -a_2 - a & b & d \\ d^* & b^* & a_2 - a & e \\ b^* & d^* & e^* & a_1 - a \end{pmatrix}^{\alpha} .$$

[arXiv:1412.5866], [arXiv:1902.02691], [arXiv:1902.10628]

We made calculations, treating quarks and leptons in equivalent way, as required by the "spin-charge-family" theory. Although

- any **(n-1)x (n-1)** submatrix of an unitary **n x n** matrix determines the **nxn** matrix for **n** $\geq$ **4** uniquely,
- the **measured mixing matrix elements** of the **3 x 3** submatrix **are not yet accurate enough even for quarks to predict the masses** m_4 **of the fourth family members**.
  o We can say, taking into account the data for the mixing matrices and masses, that m_4 **quark masses might be any in the interval (300** $< m_4 <$ **1000) GeV or even above**. Other experiments require that m_4 are above 1000 GeV.
- **Assuming masses** m_4 **we can predict mixing matrices**.

Results are presented for two choices of $m_{u_4} = m_{d_4}$, [arxiv:1412.5866]:

- 1. $m_{u_4} = 700$ GeV, $m_{d_4} =$ **700** GeV.....new_1
- 2. $m_{u_4} = 1\,200$ GeV, $m_{d_4} =$ **1 200** GeV.....new_2

$|V_{(ud)}| =$

exp_n	0.97425 ± 0.00022	0.2253 ± 0.0008	0.00413 ± 0.00049	
new_1	0.97423(4)	0.22539(7)	0.00299	**0.00776(1)**
new_2	0.97423[5]	0.22538[42]	0.00299	**0.00793[466]**
exp_n	0.225 ± 0.008	0.986 ± 0.016	0.0411 ± 0.0013	
new_1	0.22534(3)	0.97335	0.04245(6)	**0.00349(60)**
new_2	0.22531[5]	0.97336[5]	0.04248	**0.00002[216]**
exp_n	0.0084 ± 0.0006	0.0400 ± 0.0027	1.021 ± 0.032	
new_1	0.00667(6)	0.04203(4)	0.99909	**0.00038**
new_2	0.00667	0.04206[5]	0.99909	**0.00024[21]**
new_1	**0.00677(60)**	**0.00517(26)**	**0.00020**	**0.99996**
new_2	**0.00773**	**0.00178**	**0.00022**	**0.99997[9]**

One can see what
B. Belfatto, R. Beradze, Z. Berezhiani, required in [arXiv:1906.02714v1], that
$V_{u_1 d_4} > V_{u_1 d_3}$, $V_{u_2 d_4} < V_{u_1 d_4}$, and $V_{u_3 d_4} < V_{u_1 d_4}$, ,
what is just happening in my theory.
The newest experimental data, PDG, (P A Zyla at al, Prog. Theor. and Exp. Phys., Vol. 2020, Issue 8, Aug. 2020, 083C01) have not yet been used to fit mass matrix of Eq. (1).

- **o** The **matrix elements V_{CKM} depend strongly on the accuracy** **of the experimental 3 x 3 submatrix**.
  o Calculated 3 x 3 submatrix of 4 x 4 $\mathbf{V_{CKM}}$ depends on the m_{4th} family masses, but not much.
  o $V_{u_i d_4}$, $V_{d_i u_4}$ do not depend strongly on the m_{4th} **family masses** and **are obviously very small**.
- **The higher are the fourth family members masses, the closer are the mass matrices to the democratic matrices for either quarks or leptons, as expected.**
- **The higher are the fourth family members masses, the better are conditions**
  $V_{u_1 d_4} > V_{u_1 d_3}$ **,**
  $V_{u_2 d_4} < V_{u_1 d_4}$ **, and**
  $V_{u_3 d_4} < V_{u_1 d_4}$
  fulfilled.

- The **stable family** of the **upper four families** group is the candidate to form the **Dark Matter**.
- **Masses of the upper four families are influenced** :
  o by the $\widetilde{SU}(2)_{II\,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II\,\widetilde{SO}(4)}$ **scalar fields with the corresponding family quantum numbers**,
  o by the **scalars** ($A^{Q}_{\substack{78\\(\mp)}}$, $A^{Q'}_{\substack{78\\(\mp)}}$, $A^{Y'}_{\substack{78\\(\mp)}}$), and
  o by the **condensate of the two** ν_R **of the upper four families**.

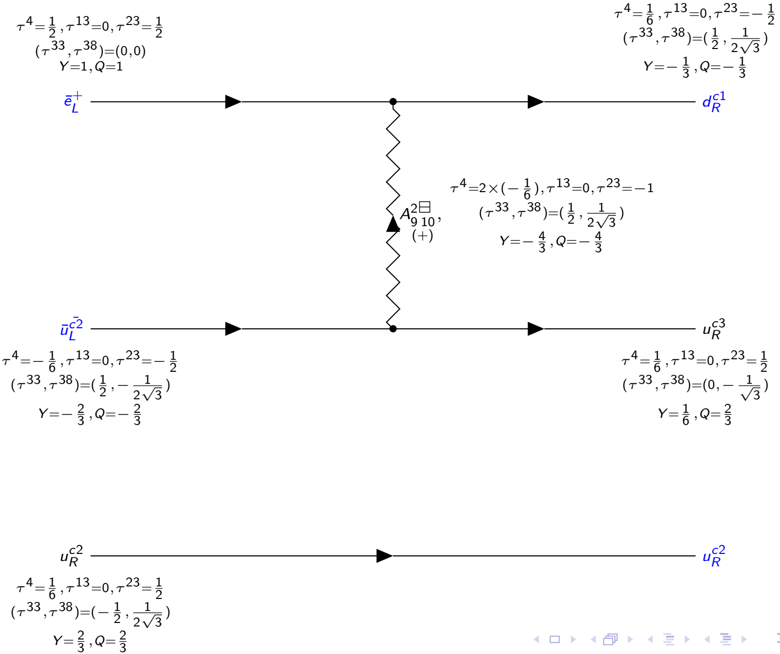
Matter-antimatter asymmetry

There are also triplet and anti-triplet scalars, $s = (9, .., d)$:,

	state	τ^{33}	τ^{38}	spin	τ^{4}	Q
$A^{Ai}_{9\,10\,(+)}$	$A^{Ai}_{9} - iA^{Ai}_{10}$	$+\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{11\,12\,(+)}$	$A^{Ai}_{11} - iA^{Ai}_{12}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{13\,14\,(+)}$	$A^{Ai}_{13} - iA^{Ai}_{14}$	0	$-\frac{1}{\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{9\,10\,(-)}$	$A^{Ai}_{9} + iA^{Ai}_{10}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A^{Ai}_{11\,12\,(-)}$	$A^{Ai}_{11} + iA^{Ai}_{12}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A^{Ai}_{13\,14\,(-)}$	$A^{Ai}_{13} + iA^{Ai}_{14}$	0	$\frac{1}{\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$

They cause transitions from **anti-leptons into quarks and anti-quarks into quarks and back**, **transforming matter into antimatter and back**. **The condensate breaks CP symmetry**, **offering the explanation for the matter-antimatter asymmetry in the universe.**

Let us look at scalar triplets, causing the **birth of a proton** from the left handed **positron**, **antiquark** and **quark**:



These two quarks, d_R^{c1} and u_R^{c3} can bind (at low enough energy) together with u_R^{c2} into the colour **chargeless baryon - a proton**.

After the appearance of the **condensate** the **CP is broken**.

In the expanding universe, fulfilling the Sakharov request for appropriate non-thermal equilibrium, **these triplet scalars have a chance to explain the matter-antimatter asymmetry**.

The opposite transition makes the proton decay.
These processes seems to explain the lepton number non conservation.

Dark matter

$d \to (d-4)+(3+1)$ **before (or at least at) the electroweak break.**

- We follow the evolution of the universe, in particular the abundance of the fifth family members - the candidates for the dark matter in the universe.
- We estimate the behaviour of our stable heavy family quarks and anti-quarks in the expanding universe by solving the system of Boltzmann equations.
- We follow the clustering of the fifth family quarks and antiquarks into the fifth family baryons through the colour phase transition.
- The mass of the fifth family members is determined from the today dark matter density.

Phys. Rev. **D** (**2009**) 80.083534

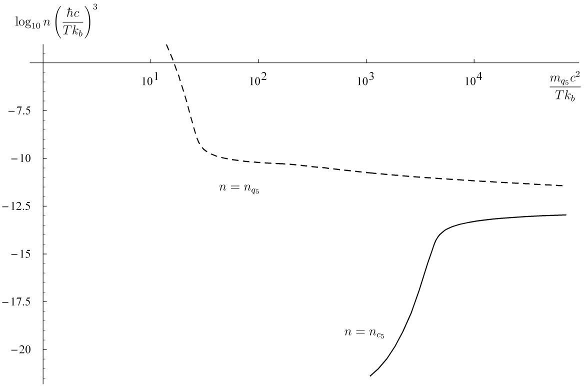


Figure: The dependence of the two number densities n_{q_5} (of the fifth family quarks) and n_{c_5} (of the fifth family clusters) as the function of $\frac{m_{q_5} c^2}{T k_b}$ is presented for the values $m_{q_5} c^2 = 71\ \mathrm{TeV}$, $\eta_{c_5} = \frac{1}{50}$ and $\eta_{(q\bar{q})_b} = 1$. We take $g^* = 91.5$.

We estimated from following the fifth family members in the expanding universe:

- $$\mathbf{10}\ \mathrm{TeV} < \mathbf{m_{q_5}\, c^2} < \mathbf{4} \cdot \mathbf{10^2} \mathrm{TeV}\,.$$

- $$\mathbf{10^{-8}} \mathrm{fm}^{\mathbf{2}} < \sigma_{\mathbf{c_5}} < \mathbf{10^{-6}} \mathrm{fm}^{\mathbf{2}}\,.$$

(It is at least $10^{-6}\times$ smaller than the cross section for the first family neutrons.)

We estimate from the scattering of the fifth family members on the ordinary matter on our Earth, on the direct measurements - DAMA, CDMS,..- ...

- $$200\,\mathrm{TeV} < \mathbf{m_{q_5} c^2} < \mathbf{10^5}\,\mathrm{TeV}\,.$$

- **In the** *standard model* the **family members with all their properties**, the **families**, the **gauge vector fields**, the **scalar Higgs**, the **Yukawa couplings**, exist by the **assumption**.

- ** In the **spin-charge-family theory the appearance and all the properties of all these fields follow from the simple starting action** with **two kinds of spins** and with the **gravity only** .

  ** The theory offers the explanation for the **dark matter**.

  ** The theory offers the explanation for the **matter-antimatter asymmetry**.

  ** All the **scalar** and all the **vector** gauge fields are **directly or indirectly observable**.

- ** The **spin-charge-family theory even offers the creation and annihilation operators without postulation.**

The *spin-charge-family* theory explains also many other properties, which are not explainable in the *standard model*, like "miraculous" non-anomalous triangle Feynman diagrams.

The more work is put into the *spin-charge-family* theory the more explanations for the phenomena follow.

Concrete predictions:

- There are several scalar fields;
  o two triplets , o three singlets ,
  explaining higgss and Yukawa couplings,
  some of them will be observed at the LHC, JMP 6 (**2015**) 2244,
  Phys. Rev. D 91 (**2015**) 6, 065004.
- There is the fourth family, (weakly) coupled to the observed three, which will be observed at the LHC,
  New J. of Phys. 10 (**2008**) 093002.
- There is the dark matter with the predicted properties,
  Phys. Rev. D (**2009**) 80.083534.
- There is the ordinary matter/antimatter asymmetry explained and the proton decay predicted and explained,
  Phys. Rev. D 91 (**2015**) 6, 065004.

We recognize that:

- The **last data for mixing matrix of quarks are in better agreement with our prediction for the** 3×3 **submatrix elements of the** 4×4 **mixing matrix than the previous ones**.
- Our **fit to the last data predicts how will** the 3×3 **submatrix elements change** in the **next more accurate measurements**.
- **Masses** of the **fourth family lie much above the known three, masses of quarks are close to each other.**
- The1**larger are masses of the fourth family the larger are** $V_{u_1 d_4}$ **in comparison with** $V_{u_1 d_3}$ **and the more is valid that** $V_{u_2 d_4} < V_{u_1 d_4}$, $V_{u_3 d_4} < V_{u_1 d_4}$.
  The flavour changing neutral currents are correspondingly weaker.

- **Masses** of the **fifth family lie much above the known three and the predicted fourth family masses**.
- **Although the upper four families carry the weak (of two kinds) and the colour charge, these group of four families are completely decoupled from the lower four families up to the** $< 10^{16}$ **GeV, unless the breaks of symmetries recover**.
- **Baryons of the fifth family are heavy, forming small enough clusters with small enough scattering amplitude among themselves and with the ordinary matter to be the candidate for the dark matter**.
- **The "nuclear" force among them is different from the force among ordinary nucleons.**

- The **spin-charge-family theory is offering an explanation for** the **hierarchy problem**:
  The mass matrices of the two four families groups are almost democratic, causing spreading of the fermion masses from 10^{16} **GeV to** 10^{-8} **MeV**.

- **Using odd and even Clifford algebra objects** the **spin-charge-family theory is offering an explanation for** the **second quantization postulates for fermions and bosons, while describing the internal space of fermions with the Clifford odd anti-commuting "basis vectors" and the internal space of bosons with the Clifford even commuting "basis vectors" .**

- **When all the properties of** $\hat{b}^{m\dagger}_{f}$**, and their Hermitian conjugated partners,** $\hat{b}^{m}_{f}$**, as well as of** $^{\mathrm{I}}\hat{\mathcal{A}}^{m\dagger}_{f}\,^{\mathrm{I}}\mathcal{C}^{m}_{f\alpha}$ **will be understood we very probably will understood nature in** $d=(3+1)$ **much better.**

To summarize:

- I hope that I managed to convince you that I can answer many open questions of particle physics and cosmology. The more work is put into this theory the more observed phenomenas I can explain and the predictions offer.
- The collaborators are very welcome!
- There are namely a lot of properties to derive.

- **Might it be that I could make consistent theory without fermions?**
  That is: Could one relate ${}^{I}\mathcal{A}^{m\dagger}_{f}\,{}^{I}\mathcal{C}^{m}_{f\alpha}$ and ${\omega^{ab}}_{\alpha}$ without **fermions?**.
  Yes, I can relate ${}^{I}\mathcal{A}^{m\dagger}_{f}\,{}^{I}\mathcal{C}^{m}_{f\alpha}$, if I apply $\mathcal{S}^{ab}$ on ${}^{I}\mathcal{A}^{m\dagger}_{f}\,{}^{I}\mathcal{C}^{m}_{f\alpha}$ and on $(\alpha^{cd}{\omega^{cd}}_{\alpha} + \alpha^{ef}{\omega^{ef}}_{\alpha} + \alpha^{gh}{\omega^{gh}}_{\alpha} + \cdots)$
- But **there are in Clifford algebra** Clifford **odd** and Clifford **even** "basis vectors". Then I should explain why nature uses only Clifford **even** "basis vectors". Why not Clifford **odd "basis vectors?**
- **Nature uses obviously both**, **odd** and **even**.

**29.06.2022 at 15:00

Let us compare the above second quantization procedure, describing internal space of fermions with the odd Clifford algebra objects, with the second quantization procedure proposed by Dirac, where he creation operators and correspondingly their Hermitian conjugate operators are assumed.

$$\psi_{\mathbf{i}}(\mathbf{t}, \tilde{\mathbf{x}}) = \sum_{\mathbf{p},\mathbf{i}} \hat{\mathbf{a}}^{\dagger}(\mathbf{p}, \mathbf{i})\, \mathbf{v}(\tilde{\mathbf{p}}, \mathbf{i})\, \mathbf{e}^{-\mathbf{p_a x^a}}).$$

$v(\vec{p}, i)$ determine solutions of equations of motion for a particular $\mathbf{e}^{-\mathbf{p_a x^a}}$.

$\hat{a}^{\dagger}(p, i)$ **is just assumed, together with the (assumed) Hermitian conjugate operator, to fill the anticommutation relation.**

In the spin-charge-family theory the creation operators appear from the odd Clifford objects, representing fermion states, applying on the vacuum state, **in internal space**.

- The **anticommutation relations for creation operators and their Hermitian conjugated partners in the Dirac case** in $d = (3+1)$ **for spin** $(\uparrow, \downarrow)$ and **right and left handedness** $(\pm\mathbf{1}$, respectively)

$$\begin{aligned}\{\hat{a}^{\dagger}_i(\vec{p}),\, \hat{a}^{\dagger}_j(\vec{p}\,')\}_+ &= 0 = \{\hat{a}_i(\vec{p}),\, \hat{a}_j(\vec{p'})\}_+ \,,\\ \{\hat{a}_i(\vec{p}),\, \hat{a}^{\dagger}_j(\vec{p'})\}_+ &= \delta_{ij}\, \delta(\vec{p} - \vec{p'})\,,\end{aligned}$$

  in the case of massless fermions.

- To be able to compare the spin-charge-family theory creation operators for this particular case of $d = (3+1)$, we make a choice of the creation operators representing
  spin $\uparrow$ and right handedness,
  $\hat{b}^{\dagger}_1 := \overset{03}{[+i]}\overset{12}{(+)}$,
  and spin $\downarrow$ and right handedness,
  $\hat{b}^{\dagger}_2 := \overset{03}{(-i)}\overset{12}{[-]}$,
  and shall not pay attention on charges (which in spin-charge-family theory originate in $d \geq 5$) and families.

- In the spin-charge-family case the creation operators $\hat{b}^{\dagger}_{i}, i = (1,2)$ originate in "basis vectors" determining the internal space of fermions.
- Solutions of the Weyl equation – the Dirac equation for massless fermions – are superposition of both "basis vectors" for particular $\vec{p}$, $p^0 = |\vec{p}|$ $\hat{\mathbf{b}}^{s\dagger}(\vec{p}) = \sum_i c^{is}\,(\vec{p})\,\hat{b}^{\dagger}_{\vec{p}} *_T \ \hat{b}^{i\dagger} \ \ i = (1,2)\,.$
- Let us write down both kinds of creation operators, the Dirac one and ours, both for the right handed case, leaving out therefore the index describing handedness h in the Dirac case and f, describing family, in our case

$$\hat{\mathbf{a}}^{s\dagger}(\vec{p}) \stackrel{def}{=} \sum_i \hat{\mathbf{a}}^{s\dagger}_i(\vec{p})\, u^s_i(\vec{p})\,, \quad \hat{\mathbf{b}}^{s\dagger}(\vec{p}) = \sum_i c^{is}(\vec{p})\,\hat{b}^{i\dagger}\,\hat{b}_{\vec{p}}\,,$$

what is my redefinition of Dirac's operators.

- The creation operator of Dirac
  $\hat{\mathbf{a}}^{s\dagger}(\vec{p}) = \sum_i u_i^s(\vec{p})\, \hat{a}_i^{s\dagger}(\vec{p})\,, \qquad v^{si}(\vec{p},\vec{x}) = u^{si}(\vec{p})\, e^{i\vec{p}\cdot\vec{x}},$
  has to be related to
  $\hat{\mathbf{b}}^{s\dagger} = \sum_i c^{is}(\vec{p})\, \hat{b}^i\, \hat{b}_{\vec{p}}.$

  $$\hat{\mathbf{a}}^{s\dagger}(\vec{p}) = \sum_i \hat{\mathbf{a}}_i^{s\dagger}(\vec{p})\, u_i^s(\vec{p}) \text{ to be related to } \hat{\mathbf{b}}^{s\dagger}(\vec{p}) = \sum_i c^{is}(\vec{p})\, \hat{b}^i\, \hat{b}_{\vec{p}}\,.$$

  Both creation operators, $\hat{\mathbf{a}}^{s\dagger}(\vec{p})$ and $\hat{\mathbf{b}}^{s\dagger}(\vec{p})$, fulfill the same anticommutation relations,
  $\hat{\mathbf{a}}^{s\dagger}(\vec{p})$ fulfill also the anticommutation relations of Dirac.

- Dirac equipped the creation operators (and correspondingly also the annihilation operators) with the quantum numbers (s, i) and with $\vec{p}$. He postulated for such creation and annihilation operators anticommutation relations.
- Our creation and annihilation operators, $\hat{\mathbf{b}}^{s\dagger}(\vec{p})$ and $\hat{\mathbf{b}}^{s}(\vec{p})$, have anticommuting properties due to the anticommutativity of $\hat{\mathbf{b}}^{i\dagger}$ and $\hat{\mathbf{b}}^{i}$, which are Clifford odd objects.

The odd Clifford algebra offers the explanation for the Dirac's postulates for the second quantized fermions.

- $\psi^s(\vec{x}) = \int_{-\infty}^{+\infty} d^3p \; \hat{\mathbf{b}}^{\mathbf{s}\dagger}(\tilde{\mathbf{p}}) \, e^{-i(p^0 x^0 - \vec{p}_k \cdot \vec{x})}$

  to be related to

  $\psi^s(\vec{x}) = \int_{-\infty}^{+\infty} d^3p \; \hat{\mathbf{a}}^{\mathbf{s}\dagger}(\tilde{\mathbf{p}}) \, e^{-i(p^0 x^0 - \vec{p}_k \cdot \vec{x})}.$