Ultra-relativistic Tachyonic and Tardyonic Wavepackets on Cosmic Scales

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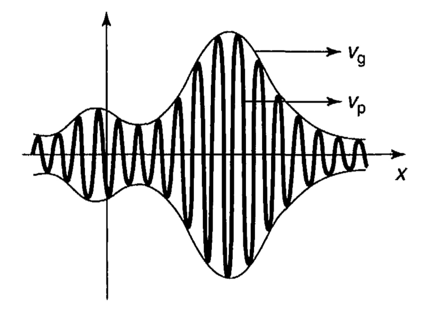
Abstract

A famous "early" arrival of a neutrino burst from the supernova SN1987A (before the light burst) was observed in a detector under Mont Blanc, as reported in [V. L. Dadykin et al.*, JETP Lett. 45, 593 (1987)]. This event still provides some grounds for speculations about a possible tachyonic (faster-than-light) nature of at least some of the known neutrino species. It is well known that quantum mechanical wave packets describing massive particles disperse while propagating on cosmic distance scales, in contrast to the deterministic trajectories of classical particles. This applies both to tachyonic (superluminal) and tardyonic (subluminal) massive relativistic wave packets. This constitutes quite a fundamental question, in fact, which is relevant for tardyons and tachyons alike. Hence, on the basis of the dispersion of quantum-mechanical wave packets, it is interesting to ask to which extent quantum dispersion of the wave packets could contribute to the uncertainties of arrival times of cosmic rays consisting of massive particles on Earth, possibly even "mimicking" superluminal propagation, purely due to dispersion of the wave packet. Furthermore, it is interesting to ask about possible general formulas describing the quantum mechanical spreading of wave packets on cosmic scales, in the ultrarelativistic limit.*

Shining a Laser at the Moon...

Question: Why can we shine a laser at the moon and measure the distance, without having to worry about dispersion?
Answer: Because the phase and group velocities of luxons (massless particles) in intrastellar space are equal to the speed of light, and there is no spatial dispersion of a traveling wavepacket.

$$v_g = v_p = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\omega}{k} = c\,.$$



Nonrelativistic Dispersion...

"Herd of cows": The faster cows will form the tip of the herd, while the slower cows will stay behind. The same can be said about the different momentum components of a nonrelativistic wave packet under free propagation.

Units with $\hbar = c = \epsilon_0 = 1$:
Energy $E = \omega$, wave number $k = p$.

$$\omega = \frac{p^2}{2m}\,, \qquad v_g = \frac{\mathrm{d}\omega}{\mathrm{d}p} = \frac{p}{m} \neq v_p = \frac{\omega}{p} = \frac{p}{2m}\,.$$

Huge dispersion of the wave packet:

$$v_p \propto v_g \propto p\,.$$

The wave packet will disperse considerably under free propagation!

Ultrarelativistic Case

Dispersion relation (+: tardyons, − tachyons):

$$E = \sqrt{p^2 \pm m^2}\,.$$

Group velocity is almost the speed of light:

$$v_g = \frac{\mathrm{d}E}{\mathrm{d}p} = \frac{p}{\sqrt{p^2 \pm m^2}} = \frac{p}{E} \approx 1\,.$$

Phase velocity is almost the speed of light:

$$v_p = \frac{E}{p} = \frac{\sqrt{p^2 \pm m^2}}{p} \approx 1\,.$$

... but there is some dispersion!
Tardyons: "The bigger cows are faster than the smaller ones, the group velocity approaches the speed of light from below as the energy increases."
Tardyons: "The bigger cows are slower than the smaller ones, the group velocity approaches the speed of light from above as the energy increases."
How does this work out in practice?

Anticipating the Main Result

Let us suppose a neutrino is created in a supernova in the Large Magellanic Cloud. Its initial momentum is p_0, and its momentum spread is δp. It travels for a time t. The mass of the particle (tardyonic or tachyonic) is m. The particle travels in the "x direction" toward Earth. It is being detected under the Mont Blanc [Dadykin *et al.*, 1987].

Result: Irrespective of whether the particle is a tachyon or tardyon, the positional uncertainty at the arrival on Earth is given by the approximate formula

$$\sigma_x(t) \approx \frac{m^2\, c^3\, \delta p}{p_0^3}\, t\,.$$

This result describes the uncertainty in the detection position $\sigma_x(t)$, where δp is the initial momentum spread of the wave packet *in statu nascendi*, m is the particle mass, c is the speed of light, p_0 is the central value of the momentum of the wave packet.

This result holds independently for tachyons and tardyons. It is proportional to m^2 and has the correct massless limit.

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Article

Dispersion of Ultrarelativistic Tardyonic and Tachyonic Wave Packets on Cosmic Scales

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Abstract: We investigate the time propagation of tachyonic (superluminal) and tardyonic (subluminal, ordinary) massive wave packets on cosmic scales. A normalizable wave packet cannot be monochromatic in momentum space and thus acquires a positional uncertainty (or packet width) that increases with travel distance. We investigate the question of how this positional uncertainty affects the uncertainty in the detection time for cosmic radiation on Earth. In the ultrarelativistic limit, we find a unified result, $\delta x(t)/c^3 = m^2 \delta p t / p_0^3$, where $\delta x(t)$ is the positional uncertainty, m is the mass parameter, δp is the initial momentum spread of the wave function, and p_0 is the central momentum of the wave packet, which, in the ultrarelativistic limit, is equal to its energy. This result is valid for tachyons and tardyons; its interpretation is being discussed.

Keywords: symmetry and conservation laws; gauge field theories; gauge bosons

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Detour: Looking at Neutrinos

Looking at Neutrinos

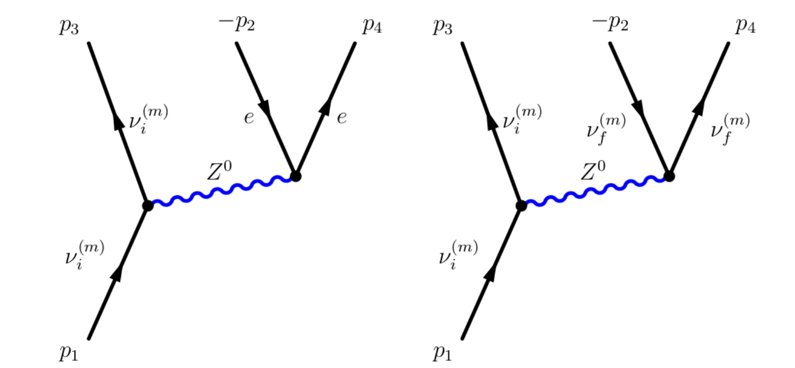
- Neutrinos are very elusive particles.
- Speculation about tachyonic nature
[Chodos, Hauser, Kostelecky, PLB 1985]
- Lorentz–Violating Extension of Standard Model (SME) developed with strong inspiration from neutrinos.
- Anyway, decay among neutrino mass eigenstates kinematically allowed due to their mass differences.
- However, decay rates for “ordinary” neutrinos
(both Dirac as well as Majorana)
exceed lifetime of Universe by orders of magnitude.
- We look only at Lorentz-conserving neutrinos (tachyonic).
- Lorentz-violating neutrinos undergo stronger decay and energy loss mechanims than “ordinary” neutrinos because of their dispersion relation $E \approx p\,v$ with $v > 1$ (at high energy), which makes a number of decays kinematically possible.

Looking at Neutrinos

- Early arrival of the 1987A neutrinos from the supernova.
- Consistent (statistically insignificant) experimental results $\delta_\nu \gtrsim 0$ by various groups. ($v_\nu = \sqrt{1 + \delta_\nu}$.)
- Neutrinos cannot be used to transmit information (at least not easily) because of their small interaction cross sections. Superluminality of neutrinos would not necessarily lead to violation of causality.
- Cutoff in the cosmic spectrum seen by IceCUBE at about 2 PeV.

Lorentz Violation Constrained by Decay

Left: LPCR=Lepton–Pair Cerenkov Radiation
Right: NPCR=Neutrino–Pair Cerenkov Radiation



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Neutrino splitting for Lorentz-violating neutrinos: Detailed analysis

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Lorentz-violating neutrino parameters have been severely constrained on the basis of astrophysical considerations. In the high-energy limit, one generally assumes a superluminal dispersion relation of an incoming neutrino of the form $E \approx |\vec{p}|v$, where E is the energy, $\vec{p}$ is the momentum and $v = \sqrt{1+\delta} > 1$. Lepton-pair creation due to a Cerenkov-radiation-like process ($\nu \to \nu + e^- + e^+$) becomes possible above a certain energy threshold, and bounds on the Lorentz-violating parameter δ can be derived. Here, we investigate a related process, $\nu_i \to \nu_i + \nu_f + \bar{\nu}_f$, where ν_i is an incoming neutrino mass eigenstate, while ν_f is the final neutrino mass eigenstate, with a superluminal velocity that is slightly slower than that of the initial state. This process is kinematically allowed if the Lorentz-violating parameters at high energy differ for the different neutrino mass eigenstates. Neutrino splitting is not subject to any significant energy threshold condition and could yield quite a substantial contribution to decay and energy loss processes at high energy, even if the differential Lorentz violation among neutrino flavors is severely constrained by other experiments. We also discuss the $SU(2)_L$-gauge invariance of the superluminal models and briefly discuss the use of a generalized *vierbein* formalism in the formulation of the Lorentz-violating Dirac equation.

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Strict bound on Lorentz violation: $\delta_\nu < 2.0 \times 10^{-20}$

particles

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Article

Squeezing the Parameter Space for Lorentz Violation in the Neutrino Sector with Additional Decay Channels

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Abstract: The hypothesis of Lorentz violation in the neutrino sector has intrigued scientists for the last two to three decades. A number of theoretical arguments support the emergence of such violations, first and foremost for neutrinos, which constitute the "most elusive" and "least interacting" particles known to mankind. It is of obvious interest to place stringent bounds on the Lorentz-violating parameters in the neutrino sector. In the past, the most stringent bounds have been placed by calculating the probability of neutrino decay into a lepton pair, a process made kinematically feasible by Lorentz violation in the neutrino sector, above a certain threshold. However, even more stringent bounds can be placed on the Lorentz-violating parameters if one takes into account, additionally, the possibility of neutrino splitting, i.e., of neutrino decay into a neutrino of lower energy, accompanied by "neutrino-pair Čerenkov radiation." This process has a negligible threshold and can be used to improve the bounds on Lorentz-violating parameters in the neutrino sector. Finally, we take the opportunity to discuss the relation of Lorentz and gauge symmetry breaking, with a special emphasis on the theoretical models employed in our calculations.

Keywords: lorentz violation; neutrinos; gauge invariance; mass mixing; icecube detector; physics beyond the standard models

End of the Detour

Tachyonic and Tardyonic Dirac Equations

Tardyons:
For spin-1/2, satisfy the Dirac equation

$$(\mathrm{i}\gamma^\mu \partial_\mu - m)\ \psi(x) = 0\,.$$

Dispersion relation:

$$E = \sqrt{p^2 + m^2}\,.$$

Tachyons:
For spin-1/2, satisfy the Dirac equation

$$\left(\mathrm{i}\gamma^\mu \partial_\mu - \gamma^5\, m\right)\ \Psi(x) = 0\,.$$

Dispersion relation:

$$E = \sqrt{p^2 - m^2}\,.$$

Tachyonic Dirac Solution

Helicity basis:

$$a_+(\vec{k}) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) \mathrm{e}^{\mathrm{i}\,\varphi} \end{pmatrix}, \quad a_-(\vec{k}) = \begin{pmatrix} -\sin\left(\frac{\theta}{2}\right) \mathrm{e}^{-\mathrm{i}\,\varphi} \\ \cos\left(\frac{\theta}{2}\right) \end{pmatrix}.$$

Propagating in the x direction:

$$\Psi(t,x) = \int \frac{\mathrm{d}p}{2\pi} \frac{f(p)}{2} \begin{pmatrix} -\sqrt{(p-m)/p} \\ \sqrt{(p-m)/p} \\ \sqrt{(p+m)/p} \\ -\sqrt{(p+m)/p} \end{pmatrix} \exp\left(-\mathrm{i}\sqrt{p^2 - m^2}\,t + \mathrm{i}p\,x\right).$$

Normalization:

$$\int \mathrm{d}x\, |\Psi(t,x)|^2 = \int \mathrm{d}x\, \Psi^+(t,x)\, \Psi(t,x) = \int \frac{\mathrm{d}p}{2\pi} |f(p)|^2 = 1\,.$$

Approximation of prefactor:

$$\Psi(t,x) \approx \int \frac{\mathrm{d}p}{2\pi} \frac{f(p)}{2}\, \underline{u} \exp\left(-\mathrm{i}\sqrt{p^2 - m^2}\,t + \mathrm{i}p\,x\right).$$

Gaussian Envelope

Here, we employ the Gaussian envelope function

$$f(p) = \frac{(2\pi)^{1/4}}{\sqrt{\delta p}} \exp\left(-\frac{(p-p_0)^2}{4\delta p^2}\right) ,$$

which is normalized to unity

$$\int \frac{\mathrm{d}p}{2\pi} |f(p)|^2 = 1 ,$$

and has the property

$$\langle p^2 \rangle - \langle p \rangle^2 = \delta p^2 .$$

The mean-square momentum uncertainty is equal to δp^2.

Tachyonic Standard Wavepacket

Standard wave packet:

$$\Psi(t,x) = \frac{(2\pi)^{1/4}}{\delta p} \int \frac{\mathrm{d}p}{2\pi} \exp\left(-\mathrm{i}\sqrt{p^2 - m^2}\, t + \mathrm{i}p\, x - \frac{(p-p_0)^2}{4\delta p^2} \right)$$

Interesting expectation values: $\langle X(t) \rangle = \int \mathrm{d}x\, x\, |\Psi(t,x)|^2$ and $\langle X(t)^2 \rangle = \int \mathrm{d}x\, x^2\, |\Psi(t,x)|^2$. We are interested in $\langle X(t) \rangle$ and $\langle X(t)^2 \rangle$.
Secret to the integration: Do the x integral first, using

$$\int \mathrm{d}x\, x\, \exp\left(\mathrm{i}(p-p')x\right) = -\mathrm{i}\frac{\partial}{\partial p}\delta(p-p')\,.$$

formulating the bra and ket wavepackets with momentum integration variables p and p'. Then apply the Dirac-δ function, reducing the problem to a one-dimensional p integral with an exponential weight factor. In the last step, one does the remaining p integral under the appropriate ultrarelativistic approximations. No saddle point approximation! Result:

$$\langle [X(t)]^2 \rangle = \frac{1}{4\delta p^2} + t^2 + \frac{m^2\, t^2}{p_0^2} + \frac{m^4 + 3m^2\delta p^2}{p_0^4}\, t^2 + \mathcal{O}(p_0^{-6})\,,$$

for the mean square position, and for the square:

$$[\langle X(t) \rangle]^2 = t^2 + \frac{m^2\, t^2}{p_0^2} + \frac{m^4 + 3m^2\delta p^2}{p_0^4}\, t^2 + \mathcal{O}(p_0^{-6})\,.$$

Tachyonic Uncertainty

${p_0}^{-6}$ terms lead to the result:

$$\delta X(t)^2 = \langle [X(t)^2] \rangle - [\langle X(t) \rangle]^2 = \frac{1}{4\delta p^2} + \frac{m^4\,\delta p^2\,t^2}{p_0^6} + \mathcal{O}({p_0}^{-8})\,.$$

Tardyonic Standard Wavepacket

Standard wave packet:

$$\psi(t,x) = \frac{\sqrt{2\pi}}{\delta p} \int \frac{\mathrm{d}p}{2\pi} \exp\left(-\mathrm{i}\sqrt{p^2+m^2}\,t + \mathrm{i}p\,x - \frac{(p-p_0)^2}{4\delta p^2}\right).$$

Interesting expectation values: $\langle X(t)\rangle = \int \mathrm{d}x\,x\,|\Psi(t,x)|^2$ and $\langle X(t)^2\rangle = \int \mathrm{d}x\,x^2\,|\Psi(t,x)|^2$. We are interested in $\langle X(t)\rangle$ and $\langle X(t)^2\rangle$. Secret to the integration: Do the x integral first, using

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$$\langle [x(t)]^2\rangle = \frac{1}{4\delta p^2} + t^2 - \frac{m^2\,t^2}{p_0^2} + \frac{m^4 - 3m^2\,\delta p^2}{p_0^4}\,t^2 + \mathcal{O}(p_0^{-6}),$$

for the mean square position, and for the square:

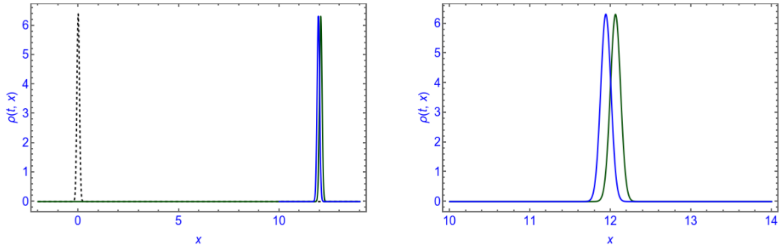
$$[\langle x(t)\rangle]^2 = t^2 - \frac{m^2\,t^2}{p_0^2} + \frac{m^4 - 3m^2\,\delta p^2}{p_0^4}\,t^2 + \mathcal{O}(p_0^{-6}).$$

Tardyonic Uncertainty

p_0^{-6} terms lead to the result:

$$\delta x(t)^2 = \langle [x(t)^2] \rangle - [\langle x(t) \rangle]^2 = \frac{1}{4\delta p^2} + \frac{m^4\,\delta p^2\,t^2}{p_0^6} + \mathcal{O}(p_0^{-8})\,.$$

Result of Propagation



(a) (b)

Figure 1. We illustrate the time-propagated tachyonic and tardyonic wave functions for the example case $m = 10$, $p_0 = 100$, $\delta p = 8$, and $t_0 = 12$, given in Equation (27). The dashed curve in (**a**) displays the initial density $\rho(t = 0, x)$, while the blue curve shows the tardyonic density $\rho(t = t_0, x)$ and the dark green curve shows the tachyonic time-evolved function $R(t = t_0, x)$. As demonstrated more clearly in the close-up in (**b**), the tachyonic wave has propagated a little faster in the positive x direction as compared to the tardyonic wave. The positional uncertainty of the time-evolved tachyonic and tardyonic wave packets is almost the same, as is evident from Equations (28), (29), (39), (40) and (42).

Cosmic Limit

We remember (universally for tachyons and tardyons):

$$\delta X(t)^2 \approx \delta x(t)^2 \approx \frac{1}{4\delta p^2} + \frac{m^4\,\delta p^2\,t^2}{p_0^6} \approx \frac{m^4\,\delta p^2\,t^2}{p_0^6}$$

for large t, but also large p_0, which suppresses the higher-order terms in δp, which are accompanied by inverse powers of p_0.
We choose as the cosmic travel time an interval of 168,000 light years, which is the distance to the Large Magellanic Could, where the supernova 1987A originated. One finds

$$\left.\frac{\delta X(t)}{c}\right|_{t=168{,}000\,\mathrm{yr}} \approx \left.\frac{\delta x(t)}{c}\right|_{t=168{,}000\,\mathrm{yr}} \approx 5.298 \times 10^{-6}\,\frac{\delta\xi}{\xi}\,\left(\frac{\chi}{\xi}\right)^2\,\mathrm{s}\,,$$

where "s" of course is the symbol for the unit "second", $\delta\xi$ is the momentum spread in GeV$/c$, ξ is equal to the central momentum p_0 in GeV$/c$, and χ is the mass of the particle, measured in eV$/c^2$. It means that, if the particle wave function is centered about a well-defined ultrarelativistic mean momentum $p_0 \gg m$ (i.e., $\delta\xi/\xi \ll 1$ and $\chi/\xi \ll 1$), then the detection time uncertainty amounts to less than a microsecond even for cosmic travel over appreciable distances (here, as an example, the distance to the Large Magellanic Could).

Conclusions

- The universal result
$$\sigma_x(t) \approx \frac{m^2\, c^3\, \delta p}{p_0^3}\, t$$
has been found for the dispersion of ultrarelativistic tardyonic and tachyonic wave packets on cosmic scales.
- This applies to all Lorentz-conserving wave packets (tardyons and tachyons).
- The dispersion for the neutrinos from the supernova SN1987A is small and cannot explain the early arrival.
This finding also is reassuring for the timing of other cosmic events.
- However, the dispersion cannot be ignored if the wave packet is created with considerable momentum uncertainty, and the travel time exceeds a few billion years.