Can the “basis vectors”, describing the internal spaces of fermion and boson fields with the Clifford odd (for fermion) and Clifford even (for boson) objects, explain interactions among fields, with gravitons included? :

A short overview of the spin-charge-family theory and its achievements so far

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Some publications:

- *Phys. Lett.* **B 292**, 25-29 (**1992**), *J. Math. Phys.* **34**, 3731-3745 (**1993**), *Mod. Phys. Lett.* **A 10**, 587-595 (**1995**), *Int. J. Theor. Phys.* **40**, 315-337 (**2001**),
- *Phys. Rev.* **D 62** (04010-14) (**2000**), *Phys. Lett.* **B 633** (**2006**) 771-775, **B 644** (**2007**) 198-202, **B** (**2008**) 110.1016, *JHEP* **04** (**2014**) 165, *Fortschritte Der Physik-Progress in Physics*, **(2017)**,
- *Phys. Rev.* **D 74** 073013-16 (**2006**),
- *New J. of Phys.* **10** (**2008**) 093002, arxiv:1412.5866,
- *Phys. Rev.* **D** (**2009**) 80.083534,
- *New J. of Phys.* (**2011**) 103027, *J. Phys. A: Math. Theor.* **45** (**2012**) 465401, *J. Phys. A: Math. Theor.* **45** (**2012**) 465401, *J. of Mod. Phys.* **4** (**2013**) 823-847, arxiv:1409.4981, **6** (**2015**) 2244-2247, *Phys. Rev.* **D 91** (**2015**) 6, 065004, *Eur. Phys. J.C.* **77** (2017) 231, Rev. Artile in **Progress in Particle and Nuclear Physics**, **http://doi.org/10.1016.j.ppnp.2021.103890** , Nucl. Phys. B NUPHB 994 (2023) 116326 , [arXiv: 2210.06256], Symmetry 2023,15,818-12-V2 94818, [arXiv:2301.04466]

More than **50 years ago** the **electroweak (and colour) standard model** offered an **elegant new step** in **understanding the origin of fermions and bosons** by **postulating**:

A.

- **The existence** of **massless family members** with the **charges** in the **fundamental representation of the groups** -
  o the **coloured triplet quarks and colourless leptons**,
  o the **left handed members as the weak charged doublets**,
  o the **right handed weak chargeless members**,
  o the **left handed quarks distinguishing in the hyper charge from the left handed leptons**,
  o each right handed member having a different hyper charge.
- **The existence** of **massless families to each of a family member.**

α name	hand-edness $-4iS^{03}S^{12}$	weak charge τ^{13}	hyper charge Y	colour charge	elm charge Q
u^i_L	-1	$\frac{1}{2}$	$\frac{1}{6}$	colour triplet	$\frac{2}{3}$
d^i_L	-1	$-\frac{1}{2}$	$\frac{1}{6}$	colour triplet	$-\frac{1}{3}$
ν^i_L	-1	$\frac{1}{2}$	$-\frac{1}{2}$	colourless	0
e^i_L	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	colourless	-1
u^i_R	1	weakless	$\frac{2}{3}$	colour triplet	$\frac{2}{3}$
d^i_R	1	weakless	$-\frac{1}{3}$	colour triplet	$-\frac{1}{3}$
ν^i_R	1	weakless	0	colourless	0
e^i_R	1	weakless	-1	colourless	-1

Members of each of the $i = 1, 2, 3$ families, $i = 1, 2, 3$ massless before the electroweak break. Each family contains the left handed weak charged quarks and the right handed weak chargeless quarks, belonging to the colour triplet $(1/2, 1/(2\sqrt{3}))$, $(-1/2, 1/(2\sqrt{3}))$, $(0, -1/(\sqrt{3}))$.

And the anti-fermions to each family and family member.

B.

- **The existence** of **massless vector gauge fields** to the observed **charges** of the **family members**, **carrying charges** in the **adjoint representation of the charge groups**.

Masslessness needed for gauge invariance.

Gauge fields before the electroweak break

- Three **massless vector fields**, **the gauge fields of the three charges**.

name	hand-edness	weak charge	hyper charge	colour charge	elm charge
hyper photon	0	0	0	colourless	0
weak bosons	0	triplet	0	colourless	triplet
gluons	0	0	0	colour octet	0

They all are vectors in $d = (3 + 1)$, in the adjoint representations with respect to the weak, colour and hyper charges.

C.

- The **existence of a massive scalar field - the higgs**,

  o carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$.

  o gaining at some step the **imaginary mass** and consequently the **constant value** , breaking the weak and the hyper charge and correspondingly breaking the **mass protection**.

- The **existence** of the **Yukawa couplings**, taking care of

  o the properties of **fermions** and

  o the masses of the **heavy bosons**.

- The **Higgs's field**, **the scalar** **in** $d = (3 + 1)$, **a doublet with respect to the weak charge**.

name	hand-edness	weak charge	hyper charge	colour charge	elm charge
$0\cdot$ **Higgs**$_u$	0	$\frac{1}{2}$	$\frac{1}{2}$	**colourless**	**1**
$< Higgs_d >$	0	$-\frac{1}{2}$	$\frac{1}{2}$	**colourless**	**0**

name	hand-edness	weak charge	hyper charge	colour charge	elm charge
$< Higgs_u >$	0	$\frac{1}{2}$	$-\frac{1}{2}$	**colourless**	**0**
$0\cdot$ **Higgs**$_d$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	**colourless**	-1

D.

- There is the **gravitational field** in $\mathbf{d=(3+1)}$.

- **The *standard model* assumptions have been confirmed without offering surprises.**
- **The last unobserved field as a field, the Higgs's scalar, detected in June 2012, was confirmed in March 2013.**
- **The waves of the gravitational field were detected in February 2016 and again 2017.**

The assumptions of the *standard model* remain unexplained.

- **There are several cosmological observations which do not look to be explainable within the** *standard model*,
- the **quantization of fermion and boson fields are postulated**,
- the **quantization of the gravitational field is not yet postulated**,
- the **used groups are postulated**,
- ...

- **It is obviously the time to make the step beyond the *standard model*.**

- **The Spin-Charge-Family theory offers the explanation for**
  i. **all the assumptions of the** *standard model*,
  ii. **for many observed phenomena**:
  ii.a. the **dark matter**,
  ii.b. the **matter-antimatter asymmetry**,
  ii.c. **others observed phenomena**,
  iii. **explaining the Dirac's postulates for the second quantized fermion and second quantized boson fields**,
  iv. **offering explanation for the appearance of the graviton**,
  v. **explaining the offer of the Fadeev-Popov ghosts**,
  vi. **making several predictions.**

- **Is the Spin-Charge-Family theory the right next step beyond both standard models?**
- **Work done so far on the spin-charge-family theory is promising.**

Trying to understand what the elementary constituents of our universe are and what are the laws of nature; physicists suggest theories and look for predictions which need confirmation of experiments.

What seems to be trustworthy is that the elementary constituents are two kinds of fields: Anti-commuting fermion and commuting boson fields, both assumed to be second quantized fields.

We try to understand:

- Are the elementary constituent of only one kind? Are the observed interactions — gravitational, electromagnetic, weak and colour — of the common origin?
- Can the postulates for the second quantized fermions and for the second quantized bosons be understood in equivalent way?

I found that it is the Clifford algebra offering the equivalent procedure for both kinds of the second quantized fields. The Clifford odd algebra offers the description of the internal space of fermion second quantized fields . The Clifford even algebra offers the description of the internal space of boson second quantized fields .

I found in 1990 that it is the Clifford algebra — the algebra of the superposition of products of γ^a's — offering the equivalent procedure for both kinds of the second quantized fields.

The Clifford odd algebra — the superposition of odd products of γ^a's — offers the description of the internal space of fermion second quantized fields .

The Clifford even algebra — the superposition of even products of γ^a's — offers the description of the internal space of boson second quantized fields .

- The Clifford odd algebra, arranged in the Clifford odd "basis vectors", which are eigenvectors of the (chosen) Cartan subalgebra members of $S^{ab} = \frac{i}{2}\gamma^a\gamma^b, a \neq b$, describe the internal space of fermions.

  o Appearing in $2^{\frac{d}{2}-1}$ irreducible representations — families — each irreducible representation with $2^{\frac{d}{2}-1}$ members (which include particles and antiparticles) the "basis vectors" describe the internal space of fermions.

  o Their Hermitian conjugated partners, with again $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ members, appear in a separate group.

- The Clifford even algebra, arranged in the Clifford even "basis vectors", which are eigenvectors of the (chosen) Cartan subalgebra members of $\mathcal{S}^{ab} = \frac{i}{2}\gamma^a\gamma^b, a \neq b$, describe the internal space of bosons.

  o Appearing in two groups, with $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ members each, having their Hermitian conjugated partners within the same group, the "basis vectors" describe the internal space of bosons.

- Clifford odd “basis vector demonstrate families of family members,

  o which manifest quarks and leptons and antiquarks and anti-leptons as observed so far in d=$(3+1)$,

  o the quarks distinguishing from leptons (and the antiquarks distinguishing from antiptons) only in the part determined by the eigenvalues of $S^{9\,10}, S^{11\,12}, S^{13\,14}$) predicting the fourth family to the observed three,

  o predicting the additional group of four families, the lowest of which determine properties of the dark matter.

  o explaining also why do family members – quarks and leptons – manifest so different properties.

- Clifford even “basis vector demonstrate all the vector (with the space index $\alpha = (0, 1, 2, 3)$) and scalar (with the space index $\alpha = (5, 6, ..., d)$) gauge fields:

o The vector gauge fields – gluons, photons, weak bosons, (two kinds of weak bosons), gravitons .

o The scalar gauge fields (the Higgs's scalar) and the Yukawa couplings (The Yukawa couplings certainly speak for the existence of several scalar fields with the properties of Higgs's scalar, which do appear in this theory, explaining the quantum numbers of scalars.)

o Predicting the additional scalar gauge fields, which explain matter/antimatter asymmetry in our universe.

Many a question remains unanswered like:

- **Is the space-time $(3+1)$? If yes why $(3{+}1)$?**
- **If not $(3+1)$ may it be that the space-time is infinite?**
- **How has the space-time of our universe started?**
- **What caused the start of our universe?**
- **What caused the inflation of our universe**

o The Spin-Charge-Family theory, assuming that the elementary fermion fields are quarks, leptons, antiquarks, and antileptons, the internal space of which is described by the Clifford odd "basis vectors",

o and the elementary boson fields are $SO(3,1)$ graviton fields, $SU(2)\times SU(2)$ weak boson fields, $SU(3)$ gluon fields and (photon) $U(1)$ fields, the internal space of which is described by the Clifford even "basis vectors",

o while recognizing that $SO(3,1)\times$ $SU(2)\times SU(2)\times$ $SU(3)\times U(1)$ are subgroups of the group $SO(13,1)$,

o assuming as well that the dynamics in ordinary space are non-zero only in $d=(3+1)$ space (that is, the momentum is non-zero only if space concerns $x_\mu=(x_0,x_1,x_2,x_3)$), the vector gauge fields (photons, weak bosons, gluons, gravitons) carry the (additional) space index $\alpha=\mu=(0,1,2,3)$, while scalars have the space index $\alpha\geq 5$,

o do offer the explanation for all the assumptions of the standard model.

**o The more effort is put into this theory,
the more answers to the open questions in elementary particle physics and cosmology is the theory offering.**

o Let us make a short introduction into the **Spin-Charge-Family theory.**

o I shall report on **how does the odd Clifford algebra explain the second quantization postulates of Dirac.**
Rev. article in JPPNP –2021 Progress in Particle and Nuclear Physics http://doi.org/10.1016.j.ppnp.2021.103890 , Symmetry 2023,15,818-12-V2 94818, https:doi.org/10.3390/sym15040818, [arXiv:2301.04466] .

o I shall report on **how does the even Clifford algebra explain the second quantization of boson fields.** Nucl. Phys. B, NUPHB 994 (2023) 116326 , [arXiv: 2210.06256, V2].

o I shall make a very short overview of achievements so far of the **Spin-Charge-Family theory.**

- **There are two kinds of the Clifford algebra objects in any d . I recognized that in Grassmann space.**

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  θ^a**'s and** p^{θ}_a**'s**, $p^{\theta}_a = \frac{\partial}{\partial \theta_a}$
  with the property
  $(\theta^a)^\dagger = \eta^{aa} \frac{\partial}{\partial \theta_a}$.

  i. The **Dirac** γ^a (recognized 90 years ago in $d = (3+1)$).
  ii. The **second one:** $\tilde{\gamma}^a$,

  $$\gamma^a = (\theta^a - i\, p^{\theta a})\,, \quad \tilde{\gamma}^a = i\,(\theta^a + i\, p^{\theta a})\,,$$

  References can be found in
  Progress in Particle and Nuclear Physics,
  http://doi.org/**10.1016.j.ppnp.2021.103890** .

- The two kinds of the **Clifford algebra objects** anticommute as follows

$$\{\gamma^{\mathbf{a}}, \gamma^{\mathbf{b}}\}_{+} = 2\eta^{\mathbf{ab}} = \{\tilde{\gamma}^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_{+},$$
$$\{\gamma^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_{+} = 0,$$

- the **postulate**

$$(\tilde{\gamma}^{\mathbf{a}}\mathbf{B} = \mathbf{i}(-)^{\mathbf{n_B}}\mathbf{B}\gamma^{\mathbf{a}})\,|\psi_0>,$$
$$(\mathbf{B} = a_0 + a_a\gamma^a + a_{ab}\gamma^a\gamma^b + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\ldots\gamma^{a_d})|\psi_o>$$

with $(-)^{n_B} = +1, -1$, if B has a Clifford even or odd character, respectively, $|\psi_o>$ is a vacuum state on which the operators γ^a **apply**, **reduces the Clifford space for fermions for the factor of two, from** 2×2^d **to** 2^d, while the operators $\tilde{\gamma}^a\tilde{\gamma}^b = -2i\tilde{S}^{ab}$ **define the family quantum numbers**.

- It is convenient to write all the **"basis vectors" describing the internal space of either fermion fields or boson fields as products of nilpotents and projectors, which are eigenvectors of the chosen Cartan subalgebra**

$$S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\, d},$$
$$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\, d},$$
$$\mathbf{S}^{ab} = S^{ab} + \tilde{S}^{ab}.$$

nilpotents

$$S^{ab}\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b) = \frac{k}{2}\,\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b), \quad \overset{\mathbf{ab}}{(\mathbf{k})} := \frac{\mathbf{1}}{\mathbf{2}}(\gamma^{\mathbf{a}} + \frac{\eta^{\mathbf{aa}}}{\mathbf{ik}}\gamma^{\mathbf{b}}),$$

projectors

$$S^{ab}\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b) = \frac{k}{2}\,\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b), \quad \overset{\mathbf{ab}}{[\mathbf{k}]} := \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{1} + \frac{\mathbf{i}}{\mathbf{k}}\gamma^{\mathbf{a}}\gamma^{\mathbf{b}}),$$

$$(\overset{\mathbf{ab}}{(\mathbf{k})})^2 = \mathbf{0}, \qquad (\overset{\mathbf{ab}}{[\mathbf{k}]})^2 = \overset{\mathbf{ab}}{[\mathbf{k}]},$$

$$\overset{\mathbf{ab}}{(\mathbf{k})}^{\dagger} = \eta^{\mathbf{aa}}\, \overset{\mathbf{ab}}{(-\mathbf{k})}, \quad \overset{\mathbf{ab}}{[\mathbf{k}]}^{\dagger} = \overset{\mathbf{ab}}{[\mathbf{k}]}.$$

$$
\begin{aligned}
\mathbf{S}^{ab}\stackrel{ab}{(\mathbf{k})} &= \frac{k}{2}\stackrel{ab}{(\mathbf{k})}, \quad \mathbf{S}^{ab}\stackrel{ab}{[\mathbf{k}]} = \frac{k}{2}\stackrel{ab}{[\mathbf{k}]},\\
\tilde{\mathbf{S}}^{ab}\stackrel{ab}{(\mathbf{k})} &= \frac{k}{2}\stackrel{ab}{(\mathbf{k})}, \quad \tilde{\mathbf{S}}^{ab}\stackrel{ab}{[\mathbf{k}]} = -\frac{k}{2}\stackrel{ab}{[\mathbf{k}]}\,.
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{a}\stackrel{ab}{(\mathbf{k})} &= \eta^{aa}\stackrel{ab}{[-\mathbf{k}]}\,,\ \gamma^{b}\stackrel{ab}{(\mathbf{k})} = -ik\stackrel{ab}{[-\mathbf{k}]}\,,\ \gamma^{a}\stackrel{ab}{[\mathbf{k}]} = \stackrel{ab}{(-\mathbf{k})}\,,\ \gamma^{b}\stackrel{ab}{[\mathbf{k}]} = -ik\eta^{aa}\stackrel{ab}{(-\mathbf{k})}\,,\\
\tilde{\gamma}^{a}\stackrel{ab}{(\mathbf{k})} &= -i\eta^{aa}\stackrel{ab}{[\mathbf{k}]}\,,\ \tilde{\gamma}^{b}\stackrel{ab}{(\mathbf{k})} = -k\stackrel{ab}{[\mathbf{k}]}\,,\ \tilde{\gamma}^{a}\stackrel{ab}{[\mathbf{k}]} = i\stackrel{ab}{(\mathbf{k})}\,,\ \tilde{\gamma}^{b}\stackrel{ab}{[\mathbf{k}]} = -k\eta^{aa}\stackrel{ab}{(\mathbf{k})}\,,\\
\stackrel{ab}{(\mathbf{k})}\stackrel{ab}{(-\mathbf{k})} &= \eta^{aa}\stackrel{ab}{[\mathbf{k}]}\,,\ \stackrel{ab}{[\mathbf{k}]}\stackrel{ab}{(\mathbf{k})} = \stackrel{ab}{(\mathbf{k})}\,,\ \stackrel{ab}{(\mathbf{k})}\stackrel{ab}{[-\mathbf{k}]} = \stackrel{ab}{(\mathbf{k})}\,, **\\
\stackrel{ab}{(\mathbf{k})}\stackrel{ab}{[\mathbf{k}]} &= \mathbf{0}\,,\ \stackrel{ab}{[\mathbf{k}]}\stackrel{ab}{(-\mathbf{k})} = \mathbf{0}\,,\ \stackrel{ab}{[\mathbf{k}]}\stackrel{ab}{[-\mathbf{k}]} = \mathbf{0}\,, **\\
\overset{ab}{\widetilde{(-\mathbf{k})}}\stackrel{ab}{(\mathbf{k})} &= -i\eta^{aa}\stackrel{ab}{[\mathbf{k}]}\,,\ \overset{ab}{\widetilde{[\mathbf{k}]}}\stackrel{ab}{(\mathbf{k})} = \stackrel{ab}{(\mathbf{k})}\,,\ \overset{ab}{\widetilde{(\mathbf{k})}}\stackrel{ab}{[\mathbf{k}]} = i\stackrel{ab}{(\mathbf{k})}\,,\ \overset{ab}{\widetilde{[-\mathbf{k}]}}\stackrel{ab}{[\mathbf{k}]} = \stackrel{ab}{[\mathbf{k}]}\,,\\
\overset{ab}{\widetilde{(\mathbf{k})}}\stackrel{ab}{(\mathbf{k})} &= \mathbf{0}\,,\ \overset{ab}{\widetilde{[-\mathbf{k}]}}\stackrel{ab}{(\mathbf{k})} = \mathbf{0}\,,\ \overset{ab}{\widetilde{(\mathbf{k})}}\stackrel{ab}{[-\mathbf{k}]} = \mathbf{0}\,,\ \overset{ab}{\widetilde{[\mathbf{k}]}}\stackrel{ab}{[\mathbf{k}]} = \mathbf{0}\,.
\end{aligned}
$$

- γ^a transforms $\overset{ab}{(k)}$ into $\overset{ab}{[-k]}$, never to $\overset{ab}{[k]}$.
- $\tilde{\gamma}^a$ transforms $\overset{ab}{(k)}$ into $\overset{ab}{[k]}$, never to $\overset{ab}{[-k]}$.
- There are the Clifford odd "basis vector", that is the "basis vector" with an odd number of nilpotents, at least one, the rest are projectors, such "basis vectors" anti commute among themselves.
- There are the Clifford even "basis vectors", that is the "basis vectors" with an even number of nilpotents, the rest are projectors, such "basis vectors" commute among themselves.

- **Let us see how does one family of the Clifford odd "basis vector" in $d=(13+1)$ look like, if spins in $d=(13+1)$ are analysed with respect to the Standard Model groups: $SO(3,1)\times$ $SU(2)\times SU(2)\times$ $SU(3)\times U(1)$.**

- **One irreducible representation** of one **family contains $2^{\frac{(13+1)}{2}-1}=64$ members which include all the family members, quarks and leptons with the right handed neutrinos included**, **as well as all the antimembers, antiquarks and antileptons, reachable by either S^{ab} (or by $\mathbb{C}_{\mathcal{N}}$ $\mathcal{P}_{\mathcal{N}}$ on a family member)**.

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S^{ab} generate **all the members of one family**. The **eightplet** (represent. of $SO(7,1)$) **of quarks of a particular colour charge. All are Clifford odd "basis vectors"** , with $SU(3)\times U(1)$ **part** **($\tau^{33}=1/2$, $\tau^{38}=1/(2\sqrt{3})$, and $\tau^{41}=1/6$)**

i		$\|^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Octet, $\Gamma^{(7,1)}=1,\ \Gamma^{(6)}=-1$, of quarks							
1	$\mathbf{u_R^{c1}}$	$\overset{03}{(+\mathbf{i})}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{(+)}\mid\mid\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
2	u_R^{c1}	$\overset{03}{[-i]}\overset{12}{[-]}\mid\overset{56}{(+)}\overset{78}{(+)}\mid\mid\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
3	d_R^{c1}	$\overset{03}{(+i)}\overset{12}{(+)}\mid\overset{56}{[-]}\overset{78}{[-]}\mid\mid\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
4	$\mathbf{d_R^{c1}}$	$\overset{03}{[-\mathbf{i}]}\overset{12}{[-]}\mid\overset{56}{[-]}\overset{78}{[-]}\mid\mid\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
5	d_L^{c1}	$\overset{03}{[-i]}\overset{12}{(+)}\mid\overset{56}{[-]}\overset{78}{(+)}\mid\mid\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	$\mathbf{d_L^{c1}}$	$\overset{03}{(+\mathbf{i})}\overset{12}{[-]}\mid\overset{56}{[-]}\overset{78}{(+)}\mid\mid\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	$\mathbf{u_L^{c1}}$	$\overset{03}{[-\mathbf{i}]}\overset{12}{(+)}\mid\overset{56}{(+)}\overset{78}{[-]}\mid\mid\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^{c1}	$\overset{03}{(+i)}\overset{12}{[-]}\mid\overset{56}{(+)}\overset{78}{[-]}\mid\mid\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

$\gamma^{\mathbf{0}}\gamma^{\mathbf{7}}$ and $\gamma^{\mathbf{0}}\gamma^{\mathbf{8}}$ transform $\mathbf{u_R}$ of the 1^{st} row into $\mathbf{u_L}$ of the 7^{th} row, and $\mathbf{d_R}$ of the 4^{rd} row into $\mathbf{d_L}$ of the 6^{th} row, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} generate **all the members** of **one family of quarks, leptonsantiquarks, antileptons**. Here is the **eightplet** (represent. of $SO(7,1)$) of the **colour chargeless leptons.** The $SO(7,1)$ **part is identical** with the one of **quarks**, while the $SU(3) \times U(1)$ **part is:** $\tau^{33} = 0$, $\tau^{38} = 0$, $\tau^{41} = -\frac{1}{2}$.

i		$\vert^{a}\psi_i >$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(7,1)} = 1$, $\Gamma^{(6)} = -1$, of leptons							
1	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
2	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
3	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
4	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
5	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
6	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
7	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
8	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform ν_R of the 1^{st} line into ν_L of the 7^{th} line, and e_R of the 4^{rd} line into e_L of the 6^{th} line, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} **generate also all the anti-eightplet** (repres. of $SO(7,1)$) of **anti-quarks of the anti-colour charge belonging to the same family of the Clifford odd basis vectors . ($\tau^{33} = -1/2$, $\tau^{38} = -1/(2\sqrt{3})$, $\tau^{41} = -1/6$).**

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Antioctet, $\Gamma^{(7,1)} = -1$, $\Gamma^{(6)} = 1$, of antiquarks							
33	$\bar{\mathbf{d}}^{\bar{\mathbf{c}}1}_{\mathbf{L}}$	$\stackrel{03}{[-\mathbf{i}]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
34	$\bar{d}^{\bar{c}1}_{L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
35	$\bar{u}^{\bar{c}1}_{L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
36	$\bar{\mathbf{u}}^{\bar{\mathbf{c}}1}_{\mathbf{L}}$	$\stackrel{03}{(+\mathbf{i})}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	- 1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
37	$\bar{\mathbf{d}}^{\bar{\mathbf{c}}1}_{\mathbf{R}}$	$\stackrel{03}{(+\mathbf{i})}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
38	$\bar{d}^{\bar{c}1}_{R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
39	$\bar{u}^{\bar{c}1}_{R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
40	$\bar{\mathbf{u}}^{\bar{\mathbf{c}}1}_{\mathbf{R}}$	$\stackrel{03}{[-\mathbf{i}]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$

$\gamma^{\mathbf{0}}\gamma^{\mathbf{7}}$ and $\gamma^{\mathbf{0}}\gamma^{\mathbf{8}}$ transform $\bar{\mathbf{d}}_{\mathbf{L}}$ of the 1^{st} line into $\bar{\mathbf{d}}_{\mathbf{R}}$ of the 5^{th} line, and $\bar{\mathbf{u}}_{\mathbf{L}}$ of the 4^{rd} line into $\bar{\mathbf{u}}_{\mathbf{R}}$ of the 8^{th} line.

- **Clifford odd "basis vector"** **describing the internal space of** **quark** $u^{c1\dagger}_{\uparrow R}$, $\Leftrightarrow$ $b^{1\dagger}_1 := \stackrel{03}{(+i)}\stackrel{12}{[+]} | \stackrel{56}{[+]}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$,
has the Hermitian conjugated partner equal to
$u^{c1}_{\uparrow R} \Leftrightarrow (\mathbf{b}^{1\dagger}_{\mathbf{1}})^{\dagger} = \stackrel{\mathbf{13\,14}}{[-]}\stackrel{\mathbf{11\,12}}{[-]}\stackrel{\mathbf{9\,10}}{(-)} || \stackrel{\mathbf{78}}{(-)}\stackrel{\mathbf{56}}{[+]} | \stackrel{\mathbf{12}}{[+]}\stackrel{\mathbf{03}}{(-\mathbf{i})}$, **both with an odd number of nilpotents,**
both are the Clifford odd objects — forming two separate groups.

Anti-commutation relations for Clifford odd "basis vectors", representing the internal space of fermion fields of quarks and leptons ($i = (u^{c,f,\uparrow,\downarrow}_{R,L}, d^{c,f,\uparrow,\downarrow}_{R,L}, \nu^{f,\uparrow,\downarrow}_{R,L}, e^{f,\uparrow,\downarrow}_{R,L})$), and anti-quarks and anti-leptons, with the family quantum number f.

- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = \delta_{\mathbf{f}\,\mathbf{f}'}\,\delta^{\mathbf{mk}}\,|\psi_\mathbf{o}>$,
- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}}_{\mathbf{f}'}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\{\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}\,|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}\,|\psi_\mathbf{o}> = |\psi^{\mathbf{m}}_{\mathbf{f}}>$,

  $$|\psi_\mathbf{o}> = \overset{\mathbf{03}}{[-\mathbf{i}]}\overset{\mathbf{12}}{[-]}\overset{\mathbf{56}}{[-]}\cdots\overset{\mathbf{13\;14}}{[-]}\;|\mathbf{1}>$$

  define the vacuum state for quarks and leptons and antiquarks and antileptons of the family f .

- **Clifford even "basis vectors", having an even number of nilpotents, describe the internal space of the corresponding boson field. The gluon field, for example, ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}$, which transforms the u_R^{c1} into u_R^{c2} looks like: ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}$ $(\equiv \overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{[+]}\overset{9\,10}{(-)}\overset{11\,12}{(+)}\overset{13\,14}{[-]})$.**

  If it algebraically multiplies on u_R^{c1} $(\equiv \overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]})$ it follows

$$
\begin{aligned}
&{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}(\equiv \overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{[+]}\overset{9\,10}{(-)}\overset{11\,12}{(+)}\overset{13\,14}{[-]}) *_A \\
&u_R^{c1\dagger},(\equiv \overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}) \to \\
&u_R^{c2\dagger},(\equiv \overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{(+)}\overset{9\,10}{[-]}\overset{11\,12}{(+)}\overset{13\,14}{[-]}), \\
&\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}} = u_R^{c2\dagger} *_A (u_R^{c1\dagger})^{\dagger}}, \\
&{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c2}\to u_R^{c1}}(\equiv \overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{[+]}\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{[-]}) *_A u_R^{c2\dagger} \to u_R^{c1\dagger}, \\
&\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c2}\to u_R^{c1}} = u_R^{c1\dagger} *_A (u_R^{c2\dagger})^{\dagger}}.
\end{aligned}
$$

There are two kinds of the Clifford even "basis vectors", having an even number of nilpotents, describing the internal space of boson field:

- Two gluon fields, ${}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$, which transform family members of a particular family of fermions among themselves; the same ${}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$ make transformations for any family of quarks and leptons and antiquarks and antileptons.

  ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\, u_R^{c1} \to u_R^{c2}}$ and ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\, u_R^{c2} \to u_R^{c2}}$ were presented above.
  Let us point out that both have all the $\mathcal{S}^{ab}$ of the Cartan subalgebra members equal zero, except two of the group $SU(3) \times U(1)$ ($\mathcal{S}^{9\,10}$, $\mathcal{S}^{11\,12}$, $\mathcal{S}^{13\,14}$).
  They can correspondingly change only the colour charge of fermions.

- **Let us present graviton ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1}_{R\uparrow}\to u^{c1}_{R\downarrow}}$, which must leave all the charges of fermions, except the spin (S^{03}, S^{12}) in $d=(3+1)$, unchanged.**

$$
{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1\dagger}_{R\uparrow}\to u^{c1\dagger}_{R\downarrow}} (\equiv \stackrel{03}{(-i)}\stackrel{12}{(-)}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{[+]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) *_A
$$

$$
u^{c1\dagger}_{R\uparrow}, (\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) \to
$$

$$
u^{c1\dagger}_{R\downarrow} (\equiv \stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}),
$$

$$
\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1\dagger}_{R\uparrow}\to u^{c1\dagger}_{R\downarrow}}} = \mathbf{u^{c1\dagger}_{R\downarrow}} *_{\mathbf{A}} (\mathbf{u^{c1\dagger}_{R\uparrow}})^{\dagger},
$$

$$
{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1\dagger}_{R\downarrow}\to u^{c1\dagger}_{R\uparrow}} (\equiv \stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{[+]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) *_A u^{c1\dagger}_{R\downarrow} \to u^{c1\dagger}_{R\uparrow},
$$

$$
\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1\dagger}_{R\downarrow}\to u^{c1\dagger}_{R\uparrow}}} = \mathbf{u^{c1\dagger}_{R\uparrow}} *_{\mathbf{A}} (\mathbf{u^{c1\dagger}_{R\downarrow}})^{\dagger}.
$$

There is the second kind of the Clifford even "basis vectors", having an even number of nilpotents, and consequently commute, describing the internal space of boson fields; they are orthogonal to all ${}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$.

- We call them ${}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$.
  ${}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$ transform a family member of a particular family of fermions to the same family member of all the rest families of quarks and leptons and antiquarks and antileptons.

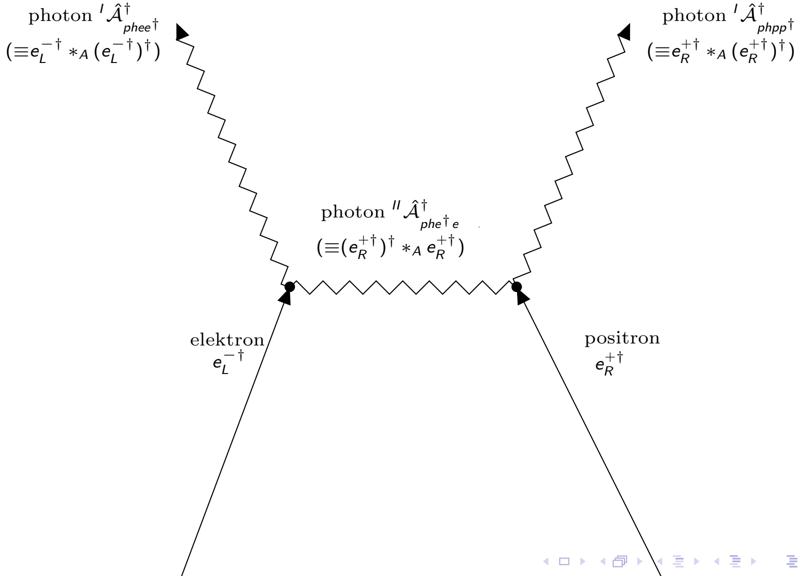
  Let $e^{-\dagger}_{L\uparrow f=1}$ be $(\equiv\overset{03}{[-i]}\overset{12}{[+]}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})$, and $e^{-\dagger}_{L\uparrow f=2}$ be $(\equiv\overset{03}{(-)}\overset{12}{(+)}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})$.

  It follows

  $$e^{-\dagger}_{L\uparrow f=2}\,(\equiv\overset{03}{(-)}\overset{12}{(+)}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})\; *_A\; {}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$$
  $$(\equiv\overset{\mathbf{03}}{(+)}\overset{\mathbf{12}}{(-)}\overset{\mathbf{56}}{[+]}\,\overset{\mathbf{78}}{[-]}\overset{\mathbf{9\,10}}{[-]}\,\overset{\mathbf{11\,12}}{[-]}\;\overset{\mathbf{13\,14}}{[-]}\,) \rightarrow e^{-\dagger}_{L\uparrow f=1}$$
  $$(\equiv\overset{03}{[-i]}\overset{12}{[+]}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}).$$

  $$\gamma^{\mathbf{a}}\overset{\mathbf{ab}}{(\mathbf{k})}=\eta^{aa}\overset{\mathbf{ab}}{[-\mathbf{k}]},\; \gamma^{\mathbf{a}}\overset{\mathbf{ab}}{[\mathbf{k}]}=\overset{\mathbf{ab}}{(-\mathbf{k})},\; \tilde{\gamma}^{\mathbf{a}}\overset{\mathbf{ab}}{(\mathbf{k})}=-i\eta^{aa}\overset{\mathbf{ab}}{[\mathbf{k}]},\; \tilde{\gamma}^{\mathbf{a}}\overset{\mathbf{ab}}{[\mathbf{k}]}=i\overset{\mathbf{ab}}{(\mathbf{k})}.$$
  $$\overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{(-\mathbf{k})}=\eta^{aa}\overset{\mathbf{ab}}{[\mathbf{k}]},\; \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{(\mathbf{k})}=\overset{\mathbf{ab}}{(\mathbf{k})},\; \overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{[-\mathbf{k}]}=\overset{\mathbf{ab}}{(\mathbf{k})},\; \overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{[\mathbf{k}]}=$$
  $$\mathbf{0},\; \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{(-\mathbf{k})}=\mathbf{0},\; \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{[-\mathbf{k}]}=\mathbf{0},$$

Let us see how does the annihilation of electron and positron look like.



Let be recognized again

- photon ${}^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{-\dagger} e^{-}} = (e_{L}^{-\dagger})^{\dagger} *_{A} e_{L}^{-\dagger} =$
  photon ${}^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{+\dagger} e^{+}} = (e_{R}^{+\dagger})^{\dagger} *_{A} e_{R}^{+\dagger}$
- All bosons "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ (describing internal spaces of boson fields) are expressible as algebraic products of "basis vectors" and their Hermitian conjugated partners as $\hat{b}^{m'\dagger}_{f`} *_{A} (\hat{b}^{m''\dagger}_{f`'})^{\dagger}$ or as $(\hat{b}^{m'\dagger}_{f`})^{\dagger} *_{A} \hat{b}^{m''\dagger}_{f``}$.
- Knowing "basis vectors" of fermions appearing in families we know all the boson fields as well.

- **We discuss so far the internal space of fermions describing their internal space with Clifford odd "basis vectors"**;

  And the internal space of bosons described with the Clifford even "basis vectors".

- **Let us write down the action**.

  Fermions and bosons can exist even if they do not interact, but have non zero momenta in ordinary space, at least mathematically.

- **Describing the properties of fermions and bosons as we observe, the interaction should be included.**

  Let us assume a simple and elegant one (this is how I "see nature") demonstrating at low energies all the observed phenomena.

Let us take into account what we have learned up to now

- If "nature uses" the Clifford algebra to describe internal degrees of freedom of fermions and bosons
then most of the action is determined:

  There are fermions appearing in families and there are their Hermitian conjugated partners .
  Families and family members demonstrate symmetries.

  There are bosons, the "basis vectors" of which are expressible as algebraic products of "basis vectors" and their Hermitian conjugated partners as $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`'})^\dagger$ or as $(\hat{b}^{m'\dagger}_{f`})^\dagger *_A \hat{b}^{m''\dagger}_{f``}$.

  There are two kinds of bosons again demonstrating symmetries determined by their internal spaces.

I use in the spin-charge-family theory a simple action.

- Internal spaces of fermions , of their "basis vectors", are determined in $d=(1+13)$, demonstrating $2^{\frac{d}{2}-1}$ members (which include particles and antiparticles) appearing in $2^{\frac{d}{2}-1}$ families and the same number of their Hermitian conjugated partners. Making a choice of the subgroups of the group SO(13,1) determines symmetries of fermions.
- Internal spaces of two kinds of bosons have also twice $2^{\frac{d}{2}-1}\times\, 2^{\frac{d}{2}-1}$ members. Making a choice of the subgroups of the group SO(13,1) determines symmetries also of bosons.
- The action must have an even number of γ^a's and two kinds of boson fields: $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$.

$$\mathbf{S} = \int d^dx\, E\, \mathcal{L}_f + \int d^dx\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})$$

- $$\mathcal{L}_f = \frac{1}{2}(\psi^{\dagger}\, \gamma^0 \gamma^a p_{0a} \psi) + h.c.$$
$$p_{0a} = f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\{p_{\alpha}, Ef^{\alpha}{}_a\}_-$$
$$\mathbf{p_{0\alpha}} = \mathbf{p}_{\alpha} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{S^{ab}}\omega_{\mathbf{ab}\alpha} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{\tilde{S}^{ab}}\tilde{\omega}_{\mathbf{ab}\alpha}$$

We have two kinds of $\omega_{ab\alpha}$, $\tilde{\omega}_{ab\alpha}$, in the the spin-charge-family theory already. They must be related by ${}^{I}\hat{\mathcal{A}}^{m\dagger}_f$ **and** ${}^{II}\hat{\mathcal{A}}^{m\dagger}_f$. It is not difficult to relate them.

We relate the application of **bosons,** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ ${}^{I}\mathcal{C}^{m}_{f\alpha}$, and $S^{ab}\omega_{ab\alpha}$ by applying both on **fermions** $\sum_{m'} \hat{b}^{m'\dagger}_{f'}$ $\beta^{m'}$

$$\{\sum_{\mathbf{m,f}} {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} \mathcal{C}^{\mathbf{mf}}_{\alpha}\} *_{\mathbf{A}} \{\sum_{\mathbf{m}'} \hat{\mathbf{b}}^{\mathbf{m}'\dagger}_{\mathbf{f}'} \beta^{\mathbf{m}'}\} = \{\sum_{\mathbf{ab}} \mathbf{S}^{\mathbf{ab}} \mathbf{c}_{\mathbf{ab}} \omega_{\mathbf{ab}\alpha}\} \{\sum_{\mathbf{m}''} \hat{\mathbf{b}}^{\mathbf{m}''\dagger}_{\mathbf{f}'} \beta^{\mathbf{m}''}\}$$

for a chosen **family** f', the same one in $\{\sum_{\mathbf{m}'} \hat{\mathbf{b}}^{\mathbf{m}'\dagger}_{\mathbf{f}'} \beta^{\mathbf{m}'}\}$ and in $\{\sum_{\mathbf{m}''} \hat{\mathbf{b}}^{\mathbf{m}''\dagger}_{\mathbf{f}'} \beta^{\mathbf{m}''}\}$.

Let us try to relate the case of **graviton**.
Having no charges the **gravitons** must be of the kind:

$${}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\,\mu}\ (\equiv \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\stackrel{56}{[\pm]} \dots \stackrel{11\,12}{[\pm]}\ \stackrel{13\,14}{[\pm]}\ {}^{I}\mathcal{C}_{gr\,\mu}.$$

$${}^{II}\hat{\mathcal{A}}^{\dagger}_{gr\,\mu}\ (\equiv \stackrel{03}{(\pm i)}\stackrel{12}{(\pm)}\stackrel{56}{[\mp]} \dots \stackrel{11\,12}{[\pm]}\ \stackrel{13\,14}{[\pm]}\ {}^{II}\mathcal{C}_{gr\,\mu}.$$

Requiring that the superpositions of $\sum_{ab}$ $c_{ab}\, \omega_{ab\alpha}$ and ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} \mathcal{C}^{mf}_{\alpha}$ have the same values of the Cartan subalgebra members, that is of $\mathcal{S}^{ab}$ we find

One finds for "gravitons" with $\mathcal{S}^{03} = i$ and $\mathcal{S}^{12} = 1$

$$
{}^{I}\hat{\mathcal{A}}^{1\dagger}_{4\alpha}(\equiv \overset{03}{(+i)}\overset{12}{(+)}\overset{56}{[+]}\overset{78}{[+]} \dots \overset{13\,14}{[\pm]}\; {}^{I}\mathcal{C}_{gr\,\mu} \Leftrightarrow
$$
$$
c_1\,(S^{01}\omega_{01\alpha} + i\, S^{02}\omega_{02\alpha} + S^{13}\omega_{13\alpha} + i\, S^{23}\omega_{23\alpha})\,,
$$

and similarly

$$
{}^{II}\hat{\mathcal{A}}^{1\dagger}_{4\alpha}(\equiv \overset{03}{(+i)}\overset{12}{(+)}\overset{56}{[-]}\overset{56}{[+]} \dots \overset{13\,14}{[\pm]}\; {}^{I}\mathcal{C}_{gr\,\mu} \Leftrightarrow
$$
$$
c_1\,(\tilde{S}^{01}\tilde{\omega}_{01\alpha} + i\, \tilde{S}^{02}\tilde{\omega}_{02\alpha} + \tilde{S}^{13}\tilde{\omega}_{13\alpha} + i\, \tilde{S}^{23}\tilde{\omega}_{23\alpha})\,,
$$

This allows that

$p_{0\alpha} \;= p_{\alpha}$ - $\sum_{ab} \frac{1}{2}\; S^{ab}\; \omega_{ab\alpha}$ - $\sum_{ab} \frac{1}{2}\; \tilde{S}^{ab}\; \tilde{\omega}_{ab\alpha}$

is replaced by

$p_{0\alpha} = p_{\alpha}$ - $\sum_{mf}\; {}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}\;\; {}^{I}\mathcal{C}^{m}_{f\alpha}$ - $\sum_{mf}\; {}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}\;\; {}^{II}\mathcal{C}^{m}_{f\alpha}$,

- The Einstein action for a free gravitational field is assumed to be linear in the curvature

$$
\begin{aligned}
\mathcal{L}_{\mathbf{g}} &= \mathbf{E}\,(\alpha\,\mathbf{R} + \tilde{\alpha}\tilde{\mathbf{R}}),\\
\mathbf{R} &= \mathbf{f}^{\alpha[\mathbf{a}}\mathbf{f}^{\beta\mathbf{b}]}\,(\omega_{\mathbf{ab}\alpha,\beta} - \omega_{\mathbf{ca}\alpha}\omega^{\mathbf{c}}{}_{\mathbf{b}\beta}),\\
\tilde{\mathbf{R}} &= \mathbf{f}^{\alpha[\mathbf{a}}\mathbf{f}^{\beta\mathbf{b}]}\,(\tilde{\omega}_{\mathbf{ab}\alpha,\beta} - \tilde{\omega}_{\mathbf{ca}\alpha}\tilde{\omega}^{\mathbf{c}}{}_{\mathbf{b}\beta}),
\end{aligned}
$$

with $E = \det(e^a{}_\alpha)$
and $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$.

Let us repeat the anti-commutation and commutation relations of the Clifford odd and the Clifford even "basis vectors".

Anti-commutation relations for Clifford odd "basis vectors", representing the internal space of fermion fields of quarks and leptons ($i = (u_{R,L}^{c,f,\uparrow,\downarrow}, d_{R,L}^{c,f,\uparrow,\downarrow}, \nu_{R,L}^{f,\uparrow,\downarrow}, e_{R,L}^{f,\uparrow,\downarrow})$), and anti-quarks and anti-leptons, with the family quantum number f.

- $\{\mathbf{b_f^m}, \mathbf{b_{f'}^{k\dagger}}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = \delta_{\mathbf{f}\,\mathbf{f}'}\,\delta^{\mathbf{mk}}\,|\psi_\mathbf{o}>$,
- $\{\mathbf{b_f^m}, \mathbf{b_{f'}^{k}}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\{\mathbf{b_f^{m\dagger}}, \mathbf{b_{f'}^{k\dagger}}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\mathbf{b_f^m}\,|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\mathbf{b_f^{m\dagger}}\,|\psi_\mathbf{o}> = |\psi_\mathbf{f}^\mathbf{m}>$,

  $$|\psi_\mathbf{o}> = \stackrel{03}{[-\mathbf{i}]}\stackrel{12}{[-]}\stackrel{56}{[-]}\cdots\stackrel{13\,14}{[-]}\;|\mathbf{1}>$$

  define the vacuum state for quarks and leptons and antiquarks and antileptons of the family f .

[arXiv:1802.05554v1], [arXiv:1802.05554v4], [arXiv:1902.10628]

Commutation relations for Clifford even "basis vectors", representing the internal space of boson fields of two kinds, ${}^{i}\hat{\mathcal{A}}^{m\dagger}_{f}\,, i=(I,II)$, which are the gauge fields of the fermion fields

- $$ {}^{\mathbf{i}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{i}}\hat{\mathcal{A}}^{\mathbf{m}'\dagger}_{\mathbf{f}`} \rightarrow \left\{ \begin{array}{r} {}^{\mathbf{i}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}`}\,, \\ \text{or}\,\mathbf{0}\,, \mathbf{i}=(\mathbf{I},\mathbf{II})\,. \end{array} \right. $$

- $$ {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{II}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} = \mathbf{0} = {}^{\mathbf{II}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{I}}\hat{\mathcal{A}}^{\mathbf{m}\dagger}_{\mathbf{f}}\,. $$

${}^{i}\hat{\mathcal{A}}^{m\dagger}_{f}$, i=(I,II) must carry the space index α:
${}^{i}\hat{\mathcal{A}}^{m\dagger}_{f}\,{}^{i}\mathcal{C}^{m\dagger}_{f}$ i=(I,II)
(in order to represent the gauge fields of the corresponding fermion fields).

- **One finds** (Prog. in Part. and Nucl. Phys., http://doi.org/**10.1016.j.ppnp.2021.103890**, Eqs. (14,16,28), and refs.therein.)
  that there are 2^d Grassmann polynomials of θ^a's and 2^d their Hermitian conjugated partners $\frac{\partial}{\partial\theta_a}$, $(\theta^a)^\dagger = \eta^{aa}\frac{\partial}{\partial\theta_a}$.
- **We have demonstrated that there are 2^d Clifford objects, which are products** of γ^a**'s**
  $$\gamma^a = (\theta^a + \frac{\partial}{\partial\theta_a})\,,$$
  half of them form Clifford odd "basis vectors", **half of them form Clifford even "basis vectors"**.
- **There are $2^{\frac{d}{2}-1}$ Clifford odd family members, appearing $2^{\frac{d}{2}-1}$ irreducible representations, carrying family quantum numbers, determined by** $\tilde{\gamma}^a$
  $$\tilde{\gamma}^a = i\,(\theta^a - \frac{\partial}{\partial\theta_a})\,,$$
  and there are $2^{\frac{d}{2}-1}\times\, 2^{\frac{d}{2}-1}$ **their Hermitian conjugated partners**. **Together there are 2^{d-1} Clifford odd "basis vectors".**
- **And there are 2^{d-1} Clifford even "basis vectors".**

- **The 2^{d-1} Clifford even "basis vectors" are of two kinds:**
  $^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ **and** $^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$.

  Both are expressible as algebraic products of the Clifford odd "basis vectors" and their Hermitian conjugated partners as

  $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f``})^{\dagger} \, (^{I}\hat{\mathcal{A}}^{m\dagger}_{f})$

  or as

  $(\hat{b}^{m'\dagger}_{f`})^{\dagger} *_A \hat{b}^{m''\dagger}_{f``} \, (^{II}\hat{\mathcal{A}}^{m\dagger}_{f})$.

- **Fermion** and **boson** second quantized fields manifest all the properties assumed by the *standard model* before the electroweak break, with the **Higgs scalars** included and the **gravitational field** included.
- The **break of symmetry**, caused by the **two right-handed neutrinos**, makes the **boson gauge fields** , which are not observed at low energies, **massive**.

The **condensate** has spin $S^{12} = 0$, $S^{03} = 0$,
weak charge $\vec{\tau}^{1} = 0$, **and**
$\vec{\tilde{\tau}}^{1} = 0, \tilde{Y} = 0, \tilde{Q} = 0, \vec{\tilde{N}}_L = 0$.

state	τ^{23}	τ^{4}	Y	Q	$\tilde{\tau}^{23}$	$\tilde{N}^3_R$	$\tilde{\tau}^{4}$
$\vert\nu^{\mathbf{VIII}}_{\mathbf{1R}}>_1 \vert\nu^{\mathbf{VIII}}_{\mathbf{2R}}>_2$	1	-1	0	0	1	1	-1
$\vert\nu^{VIII}_{1R}>_1 \vert e^{VIII}_{2R}>_2$	0	-1	-1	-1	1	1	-1
$\vert e^{VIII}_{1R}>_1 \vert e^{VIII}_{2R}>_2$	-1	-1	-2	-2	1	1	-1

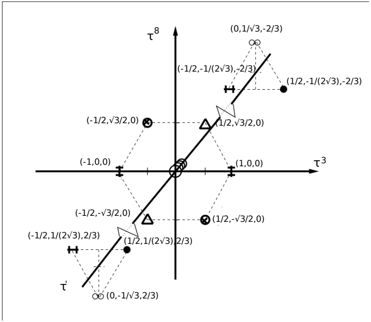
The gluon ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c2} \rightarrow u_R^{c1}}$ **has**, for example, **with respect to the Cartan subalgebra members** (τ^3, τ^8, τ') **the properties:**

one sextet with $\tau' = 0$**,**

four singlets with $(\tau^3 = 0, \tau^8 = 0, \tau' = 0)$**,**

one triplet with $\tau' = \frac{2}{3}$ **and one triplet with** $\tau' = -\frac{2}{3}$**.**

The only ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ **which couple to the condensate are the two triplets with non zero** $\tau' = \pm\frac{2}{3}$**, which transform leptons into quarks**. **They become massive**.



The only boson fields which remain massless after the appearance of the condensate of the two right handed neutrinos are

- gravitons,
- U(1) photon fields,
- SU(2) weak fields,
- SU(3) gluon fields.

The scalar fields, gaining masses as well in interaction with the condensate–if carrying space index $(7,8)$ — bring masses to quarks and leptons and antiquarks and antileptons and to weak bosons at the electroweak break.

Due to the recognitions that all the boson fields' "basis vectors" are expressible with the algebraic products of fermion's "basis vectors" and their Hermition conjugated partners appearing in families, we can get all the properties of all the boson fields' "basis vectors" knowing the symmetries of the "basis vectors" of fermions:

- **The fermion's "basis vectors" appear in twice four families of quarks and leptons and antiquarks and antileptons demonstrating** $SU(2) \times SU(2) \times U(1)$ **symmetry.**
- **The observed three families of quarks and lepton belong to the lower group of four families.**
  New J. of Phys. **10** (2008) 093002,
  Phys. Rev. D **80**, 083534 (2009), 1-16,
  J. of Modern Phys. **4** (2013) 823 [arXiv:1312.1542],
  Progr. in Part. and Nucl.r Phys., and references therein,
  http://doi.org/10.1016.j.ppnp.2021.103890,

- **There exists (at low energies decoupled from the lower group) another group of four families** (the masses of which are determined by another group of scalar fields)**offering the explanation for the dark matter**.
  Phys. Rev. D **80**, 083534 (2009),1-16,
  J. of Mod. Phys. **4** (2013) 823 [arXiv:1312.1542],
- **There exist scalar triplet and antitriplet fields, offering an explanation for the matter/antimatter asymmetry in our universe**.
  Phys. Rev. **D 91** (2015) 065004 [arXiv:1409.7791].
- **The description the internal spaces of fermions by the anticommuting Clifford odd "basis vectors" and bosons by the commuting Clifford even "basis vectors offers the explanation for the second quantization of fermion and boson fields**.
  Nucl. Phys. B **NUPHB 994** (2023) 116326 , [arXiv: 2210.06256],
  Symmetry 2023,15,818-12-V2 94818,
  https:doi.org/10.3390/sym15040818,

There remain questions to be answered:

- **Do two kinds of boson fields, ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$,appearing in this new recognition in my** *spin-charge-family* **theory (offering the interpretation of the Feynman diagrams, and elegantly confirming the requirement of the two kinds of fields, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, used so far in the** *spin-charge-family*) **offer the correct (true) description of boson fields?**
- **Does this way of describing the internal spaces of fermion and boson fields offer easier explanation for breaking symmetries from $SO(13,1)$ to $SO(3,1)\times U(1) \times SU(3)$?**
- **Can in this theory appear the gravitino?**
- **How has our universe gotten non-zero momenta only in $d = (3+1)$?**
- **Does this way of describing the internal spaces of fermion and boson fields with the "basic vectors" "open a new door" in understanding nature)?**
- And many other questions to be answered.

Let us present some of the achievements so far.
Let us repeat: All the boson gauge fields with the gravity included have the common origin.

The action for vectors with respect to the space index $m = (0, 1, 2, 3)$ can be written as

$$\int \mathbf{E}\, d^4x\, \alpha\, \mathbf{R}^{(d)} = \int d^4x \{-\frac{1}{4} \mathbf{F}^{Ai}{}_{mn}\, \mathbf{F}^{Aimn}\},$$
$$\mathbf{A}^{Ai}{}_{m} = \sum_{s,t} \mathbf{c}^{Aist}\, \omega_{stm}\,.$$

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Also scalar fields
(there are doublets and triplets)
origin in spin connections and vielbeins — expressible with ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f\alpha}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f\alpha}$
with the space index $\alpha \geq 5$.

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- Scalars with the weak and the hyper charge $(\mp\frac{1}{2}, \pm\frac{1}{2})$ determine masses of **all** the **family members** α of the **lower four families**, ν_R **of the lower four families have nonzero** $Y' := -\tau^4 + \tau^{23}$ and interact **with the scalar field** $(A^{Y'}_{(\pm)}, \vec{\tilde{A}}^{\tilde{1}}_{(\pm)}, \vec{\tilde{A}}^{\tilde{N}_L}_{(\pm)})$.
- **The group of the lower four families** manifest the $\widetilde{SU}(2)_{\widetilde{SO}(1,3)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)} \times U(1)$ **symmetry** (also after all loop corrections).

$$\mathcal{M}^\alpha = \begin{pmatrix} -a_1 - a & e & d & b \\ e^* & -a_2 - a & b & d \\ d^* & b^* & a_2 - a & e \\ b^* & d^* & e^* & a_1 - a \end{pmatrix}^\alpha .$$

[arXiv:1412.5866], [arXiv:1902.02691], [arXiv:1902.10628]

We made calculations, treating quarks and leptons in equivalent way, as required by the "spin-charge-family" theory. Although

- any **(n-1)x (n-1)** submatrix of an unitary **n x n** matrix determines the **nxn** matrix for **n** $\geq$ **4** uniquely,
- the **measured mixing matrix elements** of the **3 x 3** submatrix **are not yet accurate enough even for quarks to predict the masses** m_4 **of the fourth family members**.
  o We can say, taking into account the data for the mixing matrices and masses, that m_4 **quark masses might be any in the interval (**$300 < m_4 < 1000$**) GeV or even above**. Other experiments require that m_4 are above 1000 GeV.
- **Assuming masses** m_4 **we can predict mixing matrices**.

Results are presented for two choices of $m_{u_4} = m_{d_4}$, [arxiv:1412.5866]:

- 1. $m_{u_4} = 700$ GeV, $m_{d_4} =$ **700** GeV.....new_1
- 2. $m_{u_4} = 1\,200$ GeV, $m_{d_4} =$ **1 200** GeV.....new_2

$|V_{(ud)}| =$

exp_n	0.97425 ± 0.00022	0.2253 ± 0.0008	0.00413 ± 0.00049	
new_1	0.97423(4)	0.22539(7)	0.00299	**0.00776(1)**
new_2	0.97423[5]	0.22538[42]	0.00299	**0.00793[466]**
exp_n	0.225 ± 0.008	0.986 ± 0.016	0.0411 ± 0.0013	
new_1	0.22534(3)	0.97335	0.04245(6)	**0.00349(60)**
new_2	0.22531[5]	0.97336[5]	0.04248	**0.00002[216]**
exp_n	0.0084 ± 0.0006	0.0400 ± 0.0027	1.021 ± 0.032	
new_1	0.00667(6)	0.04203(4)	0.99909	**0.00038**
new_2	0.00667	0.04206[5]	0.99909	**0.00024[21]**
new_1	**0.00677(60)**	**0.00517(26)**	**0.00020**	**0.99996**
new_2	**0.00773**	**0.00178**	**0.00022**	**0.99997[9]**

.

We found:

$V_{u_1 d_4} > V_{u_1 d_3}$, $V_{u_2 d_4} < V_{u_1 d_4}$, and $V_{u_3 d_4} < V_{u_1 d_4}$, .

The newer are experimental data, the better agreement with our calculations offer.

The newest experimental data, have not yet been used to fit mass matrix.

- The **stable family** of the **upper four families** group is the candidate to form the **Dark Matter**.
- **Masses of the upper four families are influenced** :
  o **by the** $\widetilde{SU}(2)_{II\,\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{II\,\widetilde{SO}(4)}$ **scalar fields with the corresponding family quantum numbers**,
  o **by the scalars** $(A^{Q}_{\substack{78\\(\mp)}}, A^{Q'}_{\substack{78\\(\mp)}}, A^{Y'}_{\substack{78\\(\mp)}})$, and
  o **by the condensate of the two** ν_R **of the upper four families**.

Dark matter

$d \to (d-4) + (3+1)$ **before (or at least at) the electroweak break.**

- We follow the evolution of the universe, in particular the abundance of the fifth family members - the candidates for the dark matter in the universe.
- We estimate the behaviour of our stable heavy family quarks and anti-quarks in the expanding universe by solving the system of Boltzmann equations.
- We follow the clustering of the fifth family quarks and antiquarks into the fifth family baryons through the colour phase transition.
- The mass of the fifth family members is determined from the today dark matter density.

Phys. Rev. **D** (**2009**) 80.083534

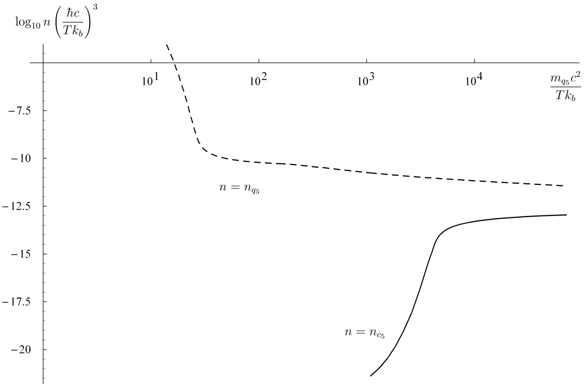


Figure: The dependence of the two number densities n_{q_5} (of the fifth family quarks) and n_{c_5} (of the fifth family clusters) as the function of $\frac{m_{q_5} c^2}{T k_b}$ is presented for the values $m_{q_5} c^2 = 71$ TeV, $\eta_{c_5} = \frac{1}{50}$ and $\eta_{(q\bar{q})_b} = 1$. We take $g^* = 91.5$.

We estimated from following the fifth family members in the expanding universe:

- $$\mathbf{10}\ \mathrm{TeV} < \mathbf{m_{q_5}\, c^2} < \mathbf{4 \cdot 10^2}\mathrm{TeV}\,.$$

- $$\mathbf{10^{-8}}\mathrm{fm}^{\mathbf{2}}\ < \sigma_{\mathbf{c_5}} <\ \mathbf{10^{-6}}\mathrm{fm}^{\mathbf{2}}\,.$$

(It is at least $10^{-6}\times$ smaller than the cross section for the first family neutrons.)

We estimate from the scattering of the fifth family members on the ordinary matter on our Earth, on the direct measurements - DAMA, CDMS,..- ...

- $$\mathbf{200}\,\mathrm{TeV} < \mathbf{m_{q_5} c^2} < \mathbf{10^5}\,\mathrm{TeV}\,.$$

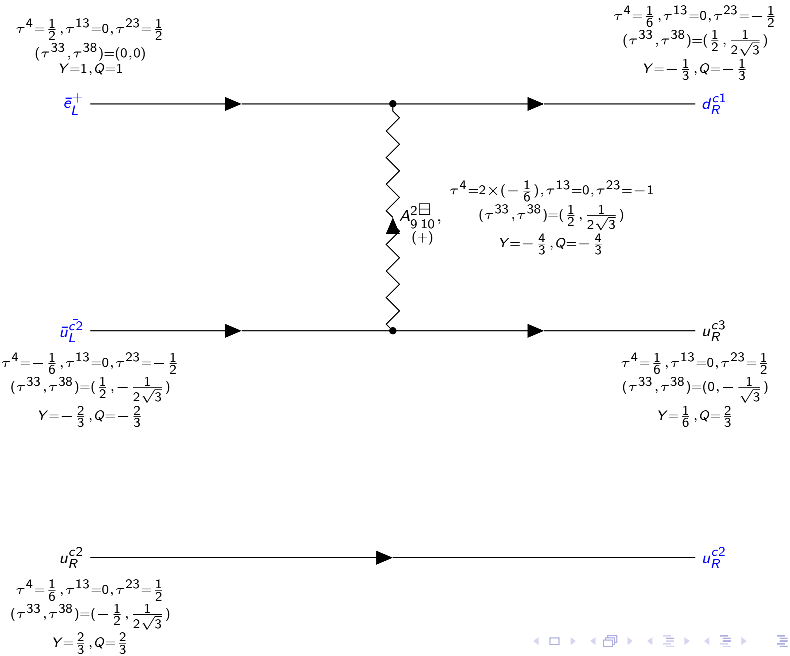
Matter-antimatter asymmetry

There are also triplet and anti-triplet scalars, $s = (9, .., d)$:,

	state	τ^{33}	τ^{38}	spin	τ^{4}	Q
$A^{Ai}_{9\,10\,(+)}$	$A^{Ai}_{9} - iA^{Ai}_{10}$	$+\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{11\,12\,(+)}$	$A^{Ai}_{11} - iA^{Ai}_{12}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{13\,14\,(+)}$	$A^{Ai}_{13} - iA^{Ai}_{14}$	0	$-\frac{1}{\sqrt{3}}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$A^{Ai}_{9\,10\,(-)}$	$A^{Ai}_{9} + iA^{Ai}_{10}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A^{Ai}_{11\,12\,(-)}$	$A^{Ai}_{11} + iA^{Ai}_{12}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
$A^{Ai}_{13\,14\,(-)}$	$A^{Ai}_{13} + iA^{Ai}_{14}$	0	$\frac{1}{\sqrt{3}}$	0	$+\frac{1}{3}$	$+\frac{1}{3}$

They cause transitions from **anti-leptons into quarks and anti-quarks into quarks and back**, **transforming matter into antimatter and back**. **The condensate breaks CP symmetry**, **offering the explanation for the matter-antimatter asymmetry in the universe.**

Let us look at scalar triplets, causing the **birth of a proton** from the left handed **positron**, **antiquark** and **quark**:



These two quarks, d_R^{c1} and u_R^{c3} can bind (at low enough energy) together with u_R^{c2} into the colour **chargeless baryon - a proton**.

After the appearance of the **condensate** the **CP is broken**.

In the expanding universe, fulfilling the Sakharov request for appropriate non-thermal equilibrium, **these triplet scalars have a chance to explain the matter-antimatter asymmetry**.

The opposite transition makes the proton decay.
These processes seems to explain the lepton number non conservation.

Let me conclude:

- Describing internal space of boson fields with the two kinds of the Clifford even "basis vectors", having an even number of nilpotents (each),
- and internal space of fermion fields which appear in families with the Clifford odd "basis vectors" having an odd number of nilpotents
  with the Hermitian conjugated partners in a different group,
  it follows that either fermion or boson (vector and scalar) second quantized gauge fields, with gravity included, can be successfully described.
  Only the boundary conditions and correspondingly break of symmetry are not known.
  However, a lot of work is still needed.

Thank you!