Do we understand the internal spaces of second quantized fermion and boson fields, in odd dimensional spaces? :

N.S. Mankoč Borštnik, University of Ljubljana
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norma.mankoc@fmf.uni-lj.si

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Although it looks like that we know almost everything about our Universe, due to the fact that the theories and models are to high extend supported by the experiments and the cosmological observations,
it is also true that we do not know why and how the universe has started,
what caused the exponential grow of the size of the universe,
what is happening in the black holes,
we namely do not know how to treat the second quantized gravity.

More than **50 years ago** the **electroweak (and colour) standard model** offered an **elegant new step** in **understanding the origin of fermions and bosons** by **postulating**, **supported by theories, models, experiments and cosmological observations**:

- **The existence** of **massless family members** with the **charges** in the **fundamental representation of the groups** -
  o the **coloured triplet quarks and colourless leptons**,
  o the **left handed members as the weak charged doublets**,
  o the **right handed weak chargeless members**,
  o the **left handed quarks distinguishing in the hyper charge from the left handed leptons**,
  o each right handed member having a different hyper charge.
- **The existence** of **massless families to each of a family member.**

- **The existence** of **massless vector gauge fields** to the observed **charges** (with the space i ndex in (3+1)) of the **family members**,
  carrying charges in the **adjoint representation of the charge groups**.
  (Masslessness needed for gauge invariance.)
  o **Three** **massless vector fields**, **the gauge fields of the three charges**.
  They all are vectors **in** $d = (3+1)$, **in the adjoint representations with respect to the weak SU(2), colour SU(3) and hyper U(1) charges**.
- The **existence of a massive** **scalar field - the higgs**,
  o carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$.
  o gaining at some step the **imaginary mass** and consequently the **constant value** , breaking the weak and the hyper charge and correspondingly breaking the **mass protection**.
- The **existence** of the **Yukawa couplings**, taking care of
  o the properties of **fermions** and
  o the masses of the **heavy bosons**.

- There is the **gravitational field** in **d=(3+1)**, **determined by the vielbeins and spin connections, with the space index in (d=3+1)**.
- **There are the Dirac prescriptions for the second quantized fermion** and **boson fields**
- **There are several trials to explain the appearance of families of quarks and leptons.**
- **There are several trials to explain the appearance of the inflation of the universe.**
- **There are several trials to try to treat the fermions and bosons in a unifying way.**
- **There are several trials to make a next step beyond the both standard models, electroweak and cosmological.**
- **There are several trials to make the theories renormalizable and without anomalies.**

- **The electroweak and colour *standard model* assumptions have been confirmed without offering surprises.**
- **The last unobserved field as a field, the Higgs's scalar, detected in June 2012, was confirmed in March 2013.**
- **The waves of the gravitational field were detected in February 2016 and again 2017.**

The assumptions of the *standard model* remain unexplained.

- **There are several cosmological observations which do not look to be explainable within the** *standard model*,
- the **second quantization of fermion and boson fields are postulated**,
- the **second quantization of the gravitational field is not yet even postulated**,
- the **used groups used in the standard model are postulated**,
- ...

- **The** *standard model* **assumptions have in the literature several explanations**.
- The **string theorists promise the theory offering the understanding the nature.**
- **What is the most promising next step beyond the** *standard model***?**.
- **Physicists suggest theories and look for predictions confirmed by experiments.**
- **We might all agree that the elementary constituents are two kinds of fields: Anti-commuting fermion and commuting boson fields, both assumed to be second quantized fields.**

- **The Spin-Charge-Family theory,**

  assuming the description of the internal spaces of fermions and bosons with the "basis vectors", which are superposition of products of

  odd number of γ^a for fermions and

  even number of γ^a for bosons,

  offers an unique description of boson and fermion second quantized fields.

  There are namely the same number of fermion and boson second quantized fields, manifesting a kind of supersymmetry.

  If the internal space involved in creating our universe has $d \geq (13+1)$ and the ordinary space is active in $d = (3+1)$ and no symmetry is broken, then both "basis vectors" have the same number of elements.

- Making a choice that all "basis vectors" are eigenvectors of the chosen Cartan subalgebra members, and arrange the "basis vectors" to be products of nilpotents and projectors then "basis vectors" of fermions have an odd number of nilpotents , and "basis vectors" of bosons have an even number of nilpotents .
- These description is elegant and simple to use.
- Analysing the "basis vectors" with respect to the symmetry they manifest in $d = (3+1)$ all the second quantized boson fields observed in $d = (3+1)$, and all the second quantized fermion fields observed in $d = (3+1)$ can be described, with the second quantized graviton fields included.
- Choosing the simplest action for fermions and bosons, we can describe all the properties of the observed fields.

- **The Spin-Charge-Family theory offers the explanation for**
  i. all the assumptions of the *standard model*,
  ii. for many observed phenomena:
  ii.a. the **dark matter**,
  ii.b. the **matter-antimatter asymmetry**,
  ii.c. others observed phenomena,
  iii. explaining the Dirac's postulates for the second quantized fermion and second quantized boson fields,
  iv. offering explanation for the appearance of the graviton,
  v. explaining the offer of the Fadeev-Popov ghosts,
  vi. making several predictions.

- Is the Spin-Charge-Family theory the right next step beyond both standard models?
- Work done so far on the spin-charge-family theory is promising.
- Let me comment whether the low energy limit of strings can be presented by the spin-charge-family theory.

- **There are two kinds of the Clifford algebra objects in any d . I recognized that in Grassmann space.**

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$$\theta^a\text{'s and } p^{\theta}_{a}\text{'s}, \; p^{\theta}_{a} = \frac{\partial}{\partial\theta_a}$$

with the property

$$(\theta^a)^{\dagger} = \eta^{aa}\frac{\partial}{\partial\theta_a}.$$

i. The **Dirac** γ^a (recognized 90 years ago in $d = (3+1)$).
ii. The **second one:** $\tilde{\gamma}^a$,

$$\gamma^a = (\theta^a - i\, p^{\theta a})\,, \quad \tilde{\gamma}^a = i\,(\theta^a + i\, p^{\theta a})\,,$$

References can be found in
Progress in Particle and Nuclear Physics,
http://doi.org/**10.1016.j.ppnp.2021.103890** .

- **The two kinds of the Clifford algebra objects anticommute as follows**

$$\{\gamma^{\mathbf{a}}, \gamma^{\mathbf{b}}\}_{+} = \mathbf{2}\eta^{\mathbf{ab}} = \{\tilde{\gamma}^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_{+},$$
$$\{\gamma^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_{+} = 0,$$

- **the postulate**

$$(\tilde{\gamma}^{\mathbf{a}}\mathbf{B} = \mathbf{i}(-)^{\mathbf{n_B}}\mathbf{B}\gamma^{\mathbf{a}}\,)\,|\psi_0\,>,$$
$$(\mathbf{B} = a_0 + a_a\gamma^a + a_{ab}\gamma^a\gamma^b + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\ldots\gamma^{a_d})|\psi_o>$$

with $(-)^{n_B} = +1, -1$, if B has a Clifford even or odd character, respectively, $|\psi_o>$ is a vacuum state on which the operators γ^a **apply**, **reduces the Clifford space for fermions for the factor of two, from** 2×2^d **to** 2^d, while the operators $\tilde{\gamma}^a\tilde{\gamma}^b = -2i\tilde{S}^{ab}$ **define the family quantum numbers**.

- It is convenient to write all the **"basis vectors" describing the internal space of either fermion fields or boson fields as products of nilpotents and projectors, which are eigenvectors of the chosen Cartan subalgebra**

$$
\begin{aligned}
&S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\, d}\,,\\
&\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\, d}\,,\\
&\mathbf{S}^{ab} = S^{ab} + \tilde{S}^{ab}\,.
\end{aligned}
$$

nilpotents

$$
S^{ab}\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b) = \frac{k}{2}\,\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \quad \overset{ab}{(\mathbf{k})} := \mathbf{\frac{1}{2}}(\gamma^{\mathbf{a}} + \frac{\eta^{\mathbf{aa}}}{\mathbf{ik}}\gamma^{\mathbf{b}})\,,
$$

projectors

$$
S^{ab}\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b) = \frac{k}{2}\,\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,, \quad \overset{ab}{[\mathbf{k}]} := \mathbf{\frac{1}{2}}(\mathbf{1} + \frac{\mathbf{i}}{\mathbf{k}}\gamma^{\mathbf{a}}\gamma^{\mathbf{b}})\,,
$$

$$
(\overset{ab}{(\mathbf{k})})^2 = \mathbf{0}\,, \qquad (\overset{ab}{[\mathbf{k}]})^2 = \overset{ab}{[\mathbf{k}]}\,,
$$

$$
\overset{ab}{(\mathbf{k})}^{\dagger} = \eta^{\mathbf{aa}}\,\overset{ab}{(-\mathbf{k})}\,, \quad \overset{ab}{[\mathbf{k}]}^{\dagger} = \overset{ab}{[\mathbf{k}]}\,.
$$

$$
\begin{aligned}
S^{ab}\overset{ab}{(\mathbf{k})} &= \frac{k}{2}\overset{ab}{(\mathbf{k})}, \quad S^{ab}\overset{ab}{[\mathbf{k}]} = \frac{k}{2}\overset{ab}{[\mathbf{k}]},\\
\tilde{S}^{ab}\overset{ab}{(\mathbf{k})} &= \frac{k}{2}\overset{ab}{(\mathbf{k})}, \quad \tilde{S}^{ab}\overset{ab}{[\mathbf{k}]} = -\frac{k}{2}\overset{ab}{[\mathbf{k}]}\,.
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{a}\overset{ab}{(\mathbf{k})} &= \eta^{aa}\overset{ab}{[-\mathbf{k}]}\,, \;\gamma^{b}\overset{ab}{(\mathbf{k})} = -ik\overset{ab}{[-\mathbf{k}]}\,, \;\gamma^{a}\overset{ab}{[\mathbf{k}]} = \overset{ab}{(-\mathbf{k})}\,, \;\gamma^{b}\overset{ab}{[\mathbf{k}]} = -ik\eta^{aa}\overset{ab}{(-\mathbf{k})}\,,\\
\tilde{\gamma}^{a}\overset{ab}{(\mathbf{k})} &= -i\eta^{aa}\overset{ab}{[\mathbf{k}]}\,, \;\tilde{\gamma}^{b}\overset{ab}{(\mathbf{k})} = -k\overset{ab}{[\mathbf{k}]}\,, \;\tilde{\gamma}^{a}\overset{ab}{[\mathbf{k}]} = i\overset{ab}{(\mathbf{k})}\,, \;\tilde{\gamma}^{b}\overset{ab}{[\mathbf{k}]} = -k\eta^{aa}\overset{ab}{(\mathbf{k})}\,,\\
\overset{ab}{(\mathbf{k})}\overset{ab}{(-\mathbf{k})} &= \eta^{aa}\overset{ab}{[\mathbf{k}]}\,, \;\overset{ab}{[\mathbf{k}]}\overset{ab}{(\mathbf{k})} = \overset{ab}{(\mathbf{k})}\,, \;\overset{ab}{(\mathbf{k})}\overset{ab}{[-\mathbf{k}]} = \overset{ab}{(\mathbf{k})}\,, **\\
\overset{ab}{(\mathbf{k})}\overset{ab}{[\mathbf{k}]} &= \mathbf{0}\,, \;\overset{ab}{[\mathbf{k}]}\overset{ab}{(-\mathbf{k})} = \mathbf{0}\,, \;\overset{ab}{[\mathbf{k}]}\overset{ab}{[-\mathbf{k}]} = \mathbf{0}\,, **\\
\overset{ab}{\widetilde{(-\mathbf{k})}}\overset{ab}{(\mathbf{k})} &= -i\eta^{aa}\overset{ab}{[\mathbf{k}]}\,, \;\overset{ab}{\widetilde{[\mathbf{k}]}}\overset{ab}{(\mathbf{k})} = \overset{ab}{(\mathbf{k})}\,, \;\overset{ab}{\widetilde{(\mathbf{k})}}\overset{ab}{[\mathbf{k}]} = i\overset{ab}{(\mathbf{k})}\,, \;\overset{ab}{\widetilde{[-\mathbf{k}]}}\overset{ab}{[\mathbf{k}]} = \overset{ab}{[\mathbf{k}]}\,,\\
\overset{ab}{\widetilde{(\mathbf{k})}}\overset{ab}{(\mathbf{k})} &= \mathbf{0}\,, \;\overset{ab}{\widetilde{[-\mathbf{k}]}}\overset{ab}{(\mathbf{k})} = \mathbf{0}\,, \;\overset{ab}{\widetilde{(\mathbf{k})}}\overset{ab}{[-\mathbf{k}]} = \mathbf{0}\,, \;\overset{ab}{\widetilde{[\mathbf{k}]}}\overset{ab}{[\mathbf{k}]} = \mathbf{0}\,.
\end{aligned}
$$

- γ^a **transforms** $\stackrel{ab}{(k)}$ **into** $\stackrel{ab}{[-k]}$, **never to** $\stackrel{ab}{[k]}$.

- $\tilde{\gamma}^a$ **transforms** $\stackrel{ab}{(k)}$ **into** $\stackrel{ab}{[k]}$, **never to** $\stackrel{ab}{[-k]}$.

- **There are the Clifford odd "basis vectors", that is the "basis vectors" with an odd number of nilpotents, at least one, the rest are projectors, such "basis vectors" anti commute among themselves.**

- **There are the Clifford even "basis vectors", that is the "basis vectors" with an even number of nilpotents, the rest are projectors, such "basis vectors" commute among themselves.**

- **There are the** $2^{\frac{d}{2}-1}$ **Clifford odd "basis vectors" appearing in** $2^{\frac{d}{2}-1}$ **families and the same number of their Hermitian conjugated partners;** $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$.

- **There are** $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ **Clifford even "basis vectors" appearing in two orthogonal groups.**

- **Let us see how does one family of the Clifford odd "basis vector" in $d=(13+1)$ look like, if spins in $d=(13+1)$ are analysed with respect to the Standard Model groups: $SO(3,1)\times$ $SU(2)\times SU(2)\times$ $SU(3)\times U(1)$.**
- **One irreducible representation** of one **family contains $2^{\frac{(13+1)}{2}-1}$ = 64 members which include all the family members, quarks and leptons with the right handed neutrinos included**, **as well as all the antimembers, antiquarks and antileptons, reachable by either S^{ab} (or by $\mathbb{C}_{\mathcal{N}}$ $\mathcal{P}_{\mathcal{N}}$ on a family member)**.

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Int. J. of Modern Phys. **A 9**, 1731 (**1994**),
J. of Math. Phys. **44** 4817 (**2003**), hep-th/030322 .

S^{ab} generate **all the members of one family**. The **eightplet** (represent. of $SO(7,1)$) **of quarks of a particular colour charge. All are Clifford odd "basis vectors"** , with $SU(3) \times U(1)$ **part** ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, **and** $\tau^{41} = 1/6$)

i		$\vert ^a\psi_i >$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Octet, $\Gamma^{(7,1)} = 1$, $\Gamma^{(6)} = -1$, of quarks							
1	$\mathbf{u_R^{c1}}$	$\stackrel{03}{(+\mathbf{i})}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
4	$\mathbf{d_R^{c1}}$	$\stackrel{03}{[-\mathbf{i}]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	$\mathbf{d_L^{c1}}$	$\stackrel{03}{(+\mathbf{i})}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	$\mathbf{u_L^{c1}}$	$\stackrel{03}{[-\mathbf{i}]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\mathbf{u_R}$ of the 1^{st} row into $\mathbf{u_L}$ of the 7^{th} row, and $\mathbf{d_R}$ of the 4^{rd} row into $\mathbf{d_L}$ of the 6^{th} row, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} generate **all the members** of **one family of quarks, leptonsantiquarks, antileptons**. **Here is the eightplet (represent. of** $SO(7,1)$**) of the colour chargeless leptons. The** $SO(7,1)$ **part is identical with the one of quarks, while the** $SU(3)\times U(1)$ **part is:** $\tau^{33}=0$, $\tau^{38}=0$, $\tau^{41}=-\frac{1}{2}$.

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(7,1)}=1$, $\Gamma^{(6)}=-1$, of leptons							
1	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	**1**	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
2	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	**1**	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
3	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	**1**	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
4	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	**1**	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
5	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	**-1**	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
6	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	**-1**	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
7	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	**-1**	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
8	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \vert\vert \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	**-1**	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform ν_R of the 1^{st} line into ν_L of the 7^{th} line, and e_R of the 4^{rd} line into e_L of the 6^{th} line, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} **generate also all the anti-eightplet** (repres. of $SO(7,1)$) of **anti-quarks of the anti-colour charge belonging to the same family of the Clifford odd basis vectors . (**$\tau^{33} = -1/2$**,** $\tau^{38} = -1/(2\sqrt{3})$**,** $\tau^{41} = -1/6$**).**

i		$\|^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Antioctet, $\Gamma^{(7,1)} = -1$, $\Gamma^{(6)} = 1$, of antiquarks							
33	$\bar{d}^{\bar{c}1}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
34	$\bar{d}^{\bar{c}1}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
35	$\bar{u}^{\bar{c}1}_L$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \mid \stackrel{56}{[-]}\stackrel{78}{[-]} \mid\mid \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
36	$\bar{u}^{\bar{c}1}_L$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \mid \stackrel{56}{[-]}\stackrel{78}{[-]} \mid\mid \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	- 1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
37	$\bar{d}^{\bar{c}1}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[-]} \mid\mid \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
38	$\bar{d}^{\bar{c}1}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \mid \stackrel{56}{(+)}\stackrel{78}{[-]} \mid\mid \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
39	$\bar{u}^{\bar{c}1}_R$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{[-]}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
40	$\bar{u}^{\bar{c}1}_R$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \mid \stackrel{56}{[-]}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\bar{d}_L$ of the 1^{st} line into $\bar{d}_R$ of the 5^{th} line, and $\bar{u}_L$ of the 4^{rd} line into $\bar{u}_R$ of the 8^{th} line.

- The **Clifford odd "basis vector"** describing the internal space of **quark** $u^{c1\dagger}_{\uparrow R}$, $\Leftrightarrow b^{1\dagger}_{1} :=$
$\stackrel{03}{(+i)}\stackrel{12}{[+]} \mid \stackrel{56}{[+]}\stackrel{78}{(+)} \parallel \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{[-]}\;\stackrel{13\,14}{[-]}$,
has the Hermitian conjugated partner equal to
$u^{c1}_{\uparrow R} \Leftrightarrow (\mathbf{b}^{1\dagger}_{1})^{\dagger} = \stackrel{13\,14}{[-]}\;\stackrel{11\,12}{[-]}\;\stackrel{9\,10}{(-)} \parallel \stackrel{78}{(-)}\stackrel{56}{[+]} \mid \stackrel{12}{[+]}\stackrel{03}{(-\mathbf{i})}$, **both with an odd number of nilpotents,
both are the Clifford odd objects — forming two separate groups.**

Anti-commutation relations for Clifford odd "basis vectors", representing the internal space of fermion fields of quarks and leptons ($i = (u^{c,f,\uparrow,\downarrow}_{R,L}, d^{c,f,\uparrow,\downarrow}_{R,L}, \nu^{f,\uparrow,\downarrow}_{R,L}, e^{f,\uparrow,\downarrow}_{R,L})$), and anti-quarks and anti-leptons, with the family quantum number f.

- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*\mathbf{A}+}|\psi_{\mathbf{o}}> = \delta_{\mathbf{f}\,\mathbf{f}'}\,\delta^{\mathbf{mk}}\,|\psi_{\mathbf{o}}>$,
- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}}_{\mathbf{f}'}\}_{*\mathbf{A}+}|\psi_{\mathbf{o}}> = 0\cdot|\psi_{\mathbf{o}}>$,
- $\{\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*\mathbf{A}+}|\psi_{\mathbf{o}}> = 0\cdot|\psi_{\mathbf{o}}>$,
- $\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}\,|\psi_{\mathbf{o}}> = 0\cdot|\psi_{\mathbf{o}}>$,
- $\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}\,|\psi_{\mathbf{o}}> = |\psi^{\mathbf{m}}_{\mathbf{f}}>$,

  $$|\psi_{\mathbf{o}}> = \overset{03}{[-\mathbf{i}]}\overset{12}{[-]}\overset{56}{[-]}\cdots\overset{13\,14}{[-]}\;|\mathbf{1}>$$

  define the vacuum state for quarks and leptons and antiquarks and antileptons of the family $\mathbf{f}$.

- **Clifford even "basis vectors"**, having an even number of nilpotents, describe the internal space of the corresponding **boson** field. The **gluon** field, for example, ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}$, which transforms the u_R^{c1} into u_R^{c2} looks like: ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}$ $(\equiv \overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{[+]}\overset{9\,10}{(-)}\overset{11\,12}{(+)}\overset{13\,14}{[-]})$.

  If it algebraically multiplies on u_R^{c1} $(\equiv \overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]})$ it follows

$$
\begin{aligned}
&{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}(\equiv \overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{[+]}\overset{9\,10}{(-)}\overset{11\,12}{(+)}\overset{13\,14}{[-]}) *_A \\
&u_R^{c1\dagger}, (\equiv \overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{[-]}\overset{13\,14}{[-]}) \rightarrow \\
&u_R^{c2\dagger}, (\equiv \overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{(+)}\overset{9\,10}{[-]}\overset{11\,12}{(+)}\overset{13\,14}{[-]}),\\
&\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}} = \mathbf{u_R^{c2\dagger}} *_{\mathbf{A}} (\mathbf{u_R^{c1\dagger}})^{\dagger},\\
&{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c2}\to u_R^{c1}}(\equiv \overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{[+]}\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{[-]}) *_A u_R^{c2\dagger} \rightarrow u_R^{c1\dagger},\\
&\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c2}\to u_R^{c1}}} = \mathbf{u_R^{c1\dagger}} *_{\mathbf{A}} (\mathbf{u_R^{c2\dagger}})^{\dagger}.
\end{aligned}
$$

- These gluons ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u^{ci}_{R} \to u^{cj}_{R}} = u^{cj\dagger}_{R} *_{A} (u^{ci\dagger}_{R})^{\dagger}$ transform quarks of a particular colour charge to quarks of all the rest colour charges.
  Let us notice that they all are expressed as the algebraic product of a family member and one of the Hermitian conjugated partner.
- We can in an equivalent way express the weak boson ${}^{I}\hat{\mathcal{A}}^{\dagger}_{weak\; u^{ci}_{R} \to d^{ci}_{R}} = d^{ci\dagger}_{R} *_{A} (u^{ci\dagger}_{R})^{\dagger}$ transforming quarks of a particular weak charge to quarks of another weak charge (keeping the colour charges unchanged).

There is the second kind of the Clifford even "basis vectors", having as well an even number of nilpotents, and consequently commute, describing the internal space of boson fields; they are orthogonal to all ${}^{I}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$.

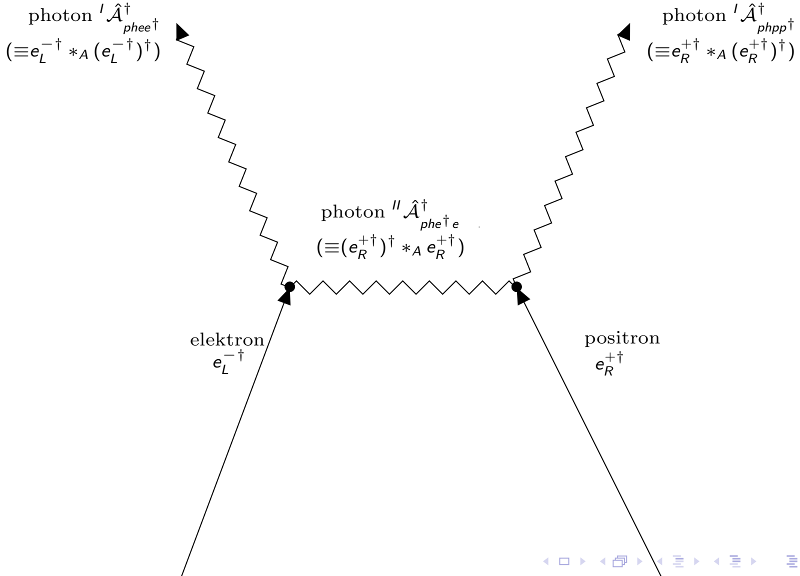
- We call them ${}^{II}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$.
  ${}^{II}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$ transform a family member of a particular family of fermions to the same family member of all the rest families of quarks and leptons and antiquarks and antileptons.
  Let $e^{-\dagger}_{L\uparrow f=1}$ be $(\equiv \overset{03}{[-i]}\overset{12}{[+]}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})$, and $e^{-\dagger}_{L\uparrow f=2}$ be $(\equiv \overset{03}{(-)}\overset{12}{(+)}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})$.
  It follows that ${}^{II}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$ apply fermions from the right hand side
  $e^{-\dagger}_{L\uparrow f=2}\ (\equiv \overset{03}{(-)}\overset{12}{(+)}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})\ *_A\ {}^{II}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$
  $(\equiv \overset{\mathbf{03}}{(+)}\overset{\mathbf{12}}{(-)}\overset{\mathbf{56}}{[+]}\ \overset{\mathbf{78}}{[-]}\overset{\mathbf{9\,10}}{[-]}\overset{\mathbf{11\,12}}{[-]}\ \overset{\mathbf{13\,14}}{[-]}) \rightarrow e^{-\dagger}_{L\uparrow f=1}$
  $(\equiv \overset{03}{[-i]}\overset{12}{[+]}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})$.
  $\gamma^{\mathbf{a}}\overset{\mathbf{ab}}{(\mathbf{k})} = \eta^{aa}\overset{\mathbf{ab}}{[-\mathbf{k}]}$, $\gamma^{\mathbf{a}}\overset{\mathbf{ab}}{[\mathbf{k}]} = \overset{\mathbf{ab}}{(-\mathbf{k})}$, $\tilde{\gamma}^{\mathbf{a}}\overset{\mathbf{ab}}{(\mathbf{k})} = -i\eta^{aa}\overset{\mathbf{ab}}{[\mathbf{k}]}$, $\tilde{\gamma}^{\mathbf{a}}\overset{\mathbf{ab}}{[\mathbf{k}]} = i\overset{\mathbf{ab}}{(\mathbf{k})}$.
  $\overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{(-\mathbf{k})} = \eta^{aa}\overset{\mathbf{ab}}{[\mathbf{k}]}$, $\overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{(\mathbf{k})} = \overset{\mathbf{ab}}{(\mathbf{k})}$, $\overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{[-\mathbf{k}]} = \overset{\mathbf{ab}}{(\mathbf{k})}$, $\overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{[\mathbf{k}]} =$
  $\mathbf{0}$, $\overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{(-\mathbf{k})} = \mathbf{0}$, $\overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{[-\mathbf{k}]} = \mathbf{0}$.

Also $^{II}\hat{\mathcal{A}}^{\dagger m}_{f}$ can be expressed as algebraic products of the Hermitian conjugate fermion fields and **one of the Clifford odd "basis vector'.**

- **photon** $^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{-\dagger}e^{-}} = (e_L^{-\dagger})^{\dagger} *_A e_L^{-\dagger} =$
  photon $^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{+\dagger}e^{+}} = (e_R^{+\dagger})^{\dagger} *_A e_R^{+\dagger}$

- All bosons "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ (describing internal spaces of boson fields) are expressible as algebraic products of "basis vectors" and their Hermitian conjugated partners as $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`'})^\dagger$ or as $(\hat{b}^{m'\dagger}_{f`})^\dagger *_A \hat{b}^{m''\dagger}_{f``}$.
- Knowing "basis vectors" of fermions appearing in families we know all the boson fields as well.

Let us see how does the annihilation of electron and positron look like.



- Let us present **graviton** ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1}_{R\uparrow} \to u^{c1}_{R\downarrow}}$ **, which must leave all the charges of fermions, except the spin** (S^{03}, S^{12}) **in** $d=(3+1)$**, unchanged.**

$$
{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1\dagger}_{R\uparrow} \to u^{c1\dagger}_{R\downarrow}} (\equiv \stackrel{03}{(-i)}\stackrel{12}{(-)}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{[+]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) *_A
$$

$$
u^{c1\dagger}_{R\uparrow}, (\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) \to
$$

$$
u^{c1\dagger}_{R\downarrow} (\equiv \stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) ,
$$

$$
\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1\dagger}_{R\uparrow} \to u^{c1\dagger}_{R\downarrow}}} = \mathbf{u^{c1\dagger}_{R\downarrow}} \; \mathbf{*_A} \; (\mathbf{u^{c1\dagger}_{R\uparrow}})^{\dagger} ,
$$

$$
{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1\dagger}_{R\downarrow} \to u^{c1\dagger}_{R\uparrow}} (\equiv \stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{[+]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) *_A \; u^{c1\dagger}_{R\downarrow} \to u^{c1\dagger}_{R\uparrow} ,
$$

$$
\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\; u^{c1\dagger}_{R\downarrow} \to u^{c1\dagger}_{R\uparrow}}} = \mathbf{u^{c1\dagger}_{R\uparrow}} \; \mathbf{*_A} \; (\mathbf{u^{c1\dagger}_{R\downarrow}})^{\dagger} .
$$

Let be recognized again

- All bosons "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ (describing internal spaces of boson fields) are expressible as algebraic products of "basis vectors" and their Hermitian conjugated partners as $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f``})^\dagger$ or as $(\hat{b}^{m'\dagger}_{f`})^\dagger *_A \hat{b}^{m''\dagger}_{f``}$.
- Knowing "basis vectors" of fermions appearing in families we know all the boson fields as well.

- The Clifford odd and the Clifford even “basis vectors” differ essentially in their properties:

  The Clifford odd "basis vectors" in even dimensional spaces appear in $2^{\frac{d}{2}-1}$ families, each family having $2^{\frac{d}{2}-1}$ members, and have their Hermitian conjugated partners in a separate group, with $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ contributions.

  The Clifford even "basis vectors" in even dimensional spaces appear in two groups, each with $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ members, having the Hermitian conjugated partners within the same group. They have no families.
- The Clifford odd "basis vectors" in even dimensional spaces carry the eigenvalues of the Cartan subalgebra members $\pm\frac{i}{2}$ or $\pm\frac{i}{2}$.

  The Clifford even "basis vectors" in even dimensional spaces carry the eigenvalues of the Cartan subalgebra members $(\pm i, 0)$ or $(\pm 1, 0)$.

- There are in $d = 2(2n+1)$ dimensional spaces $2^{\frac{d}{2}-1}$ Clifford odd families, each family having $2^{\frac{d}{2}-1}$ members. The Clifford odd "basis vectors" have their Hermitian conjugated partners in a separate group of $2^{\frac{d}{2}-1}$ families with $2^{\frac{d}{2}-1}$ members.
  In a tensor product with the basis in ordinary space the Clifford odd "basis vectors" together with their Hermitian conjugated partners form anti commuting creation and annihilation operators, fulfilling on the vacuum state the postulates of the second quantized fermion fields.
- There are in even dimensional spaces two times $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ Clifford even "basis vectors", with their Hermitian conjugated partners within the same group.

  In a tensor product with the basis in ordinary space the Clifford even "basis vectors" form commuting creations and annihilation operators, fulfilling the postulates of the second quantized boson fields.

Properties of the Clifford odd and Clifford even "basis vectors" in odd dimensional spaces $d=(2n+1)$ differ essentially from the corresponding properties in even dimensional spaces.

While in even dimensional spaces the Clifford odd "basis vectors" fulfil the postulates for the second quantized fermion fields, and Clifford even "basis vectors" fulfil the postulates for the second quantized boson fields,

have the Clifford odd and even "basis vectors" in odd dimensional spaces unusual properties resembling properties of the internal spaces of the Faddeev-Popov ghosts.

- In $d=(2n+1)$-dimensional cases, $n=1,2,\dots$, we find for the Clifford odd and Clifford even "basis vectors" the "basis vectors" from the $2n$-dimensional part of space having the properties of the even-dimensional space; they form half of the "basis vectors".
- The rest of the "basis vectors" in odd dimensional spaces follow if applying $S^{0\,2n+1}$ on the obtained half of the Clifford odd and the Clifford even "basis vectors". Since $S^{0\,2n+1}$ are Clifford even operators; they do not change oddness or evenness of the "basis vectors". But the rest half of the "basis vectors" do change their character:
  The anti-commuting part appears in two orthogonal groups, resembling properties of ${}^{I}\hat{\mathcal{A}}^{m}_{f}$ ${}^{II}\hat{\mathcal{A}}^{m}_{f}$.
  The commuting part appears as $2^{\frac{d-1}{2}-1}$ members in $2^{\frac{d-1}{2}-1}$ families and the same number of their Hermitian conjugated partners.

Here are the Clifford odd "basis vectors";
On the right hand side is the first part paying attention of $d = 2n$**.**
On the left hand side is the second part, following from the first by application of $S^{0\,2n+1}$ **on the first part.**

$$
\begin{array}{rlrl}
 & \mathbf{d} = & & \mathbf{2(2n+1)+1} \\
\hat{\mathbf{b}}^{1\dagger}_{1} = & \overset{03}{(+\mathbf{i})}\overset{12}{(+)}\overset{56}{(+)}\cdots\overset{d-2\,d-1}{(+)}\,, & \hat{\mathbf{b}}^{1\dagger}_{2^{\frac{d-1}{2}-1}+1} = & \overset{03}{[-\mathbf{i}]}\overset{12}{(+)}\overset{56}{(+)}\cdots\overset{d-2\,d-1}{(+)}\;\gamma^{d}\,, \\
\hat{\mathbf{b}}^{2\dagger}_{1} = & \overset{03}{[-\mathbf{i}]}\overset{12}{[-]}\overset{56}{(+)}\cdots\overset{d-2\,d-1}{(+)}\,, & \hat{\mathbf{b}}^{2\dagger}_{2^{\frac{d-1}{2}-1}+1} = & \overset{03}{(+\mathbf{i})}\overset{12}{[-]}\overset{56}{(+)}\cdots\overset{d-2\,d-1}{(+)}\;\gamma^{d}\,, \\
 & \cdots & & \cdots \\
\hat{\mathbf{b}}^{2^{\frac{d-1}{2}-1}\dagger}_{1} = & \overset{03}{[-\mathbf{i}]}\overset{12}{[-]}\overset{56}{(+)}\ldots\overset{d-2\,d-1}{[-]}\,, & \hat{\mathbf{b}}^{2^{\frac{d-1}{2}-1}\dagger}_{2^{\frac{d-1}{2}-1}+1} = & \overset{03}{(+\mathbf{i})}\overset{12}{[-]}\overset{56}{(+)}\ldots\overset{d-2\,d-1}{[-]}\;\gamma^{d}\,, \\
 & \cdots & & \cdots\,.
\end{array}
$$

Here are the Clifford even “basis vectors”;
On the right hand side is the first part paying attention of $d = 2n$.
On the left hand side is the second part, following from the first by application of $S^{0\,2n+1}$ on the first part.

$$
\begin{array}{rcl}
 & \mathbf{d} = & \mathbf{2(2n+1)+1} \\
{}^{\mathrm{I}}\mathcal{A}^{1\dagger}_{1} = \stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{(+)}\cdots\stackrel{d-2\,d-1}{[+]}, & & {}^{\mathrm{I}}\mathcal{A}^{1\dagger}_{2^{\frac{d-1}{2}-1}+1} = \stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{(+)}\cdots\stackrel{d-2\,d-1}{[+]}\,\gamma^{d}, \\
{}^{\mathrm{I}}\mathcal{A}^{2\dagger}_{1} = \stackrel{03}{[-i]}\stackrel{12}{[-]}\stackrel{56}{(+)}\cdots\stackrel{d-2\,d-1}{[+]}, & & {}^{\mathrm{I}}\mathcal{A}^{2\dagger}_{2^{\frac{d-1}{2}-1}+1} = \stackrel{03}{(+i)}\stackrel{12}{[-]}\stackrel{56}{(+)}\cdots\stackrel{d-2\,d-1}{[+]}\,\gamma^{d}, \\
\cdots & & \cdots \\
{}^{\mathrm{I}}\mathcal{A}^{2^{\frac{d-1}{2}-1}\dagger}_{1} = \stackrel{03}{[-i]}\stackrel{12}{[-]}\stackrel{56}{[-]}\dots\stackrel{d-2\,d-1}{[+]}, & & {}^{\mathrm{I}}\mathcal{A}^{2^{\frac{d-1}{2}-1}\dagger}_{2^{\frac{d-1}{2}-1}+1} = \stackrel{03}{(+i)}\stackrel{12}{[-]}\stackrel{56}{[-]}\dots\stackrel{d-2\,d-1}{[+]}\,\gamma^{d}, \\
\cdots & & \cdots\,.
\end{array}
$$

Let us show on chosen $d = (2n+1)$ how do the right hand sides get — although anti commuting — properties of the left hand sides of commuting Clifford even “basis vectors” and
let us show on chosen cases $d = (2n+1)$ how do the right hand sides get — although commuting — manifest properties of the left hand sides of anticommuting Clifford odd “basis vectors”.

Let us point out that the Lorentz transformations in internal spaces of fermion and boson fields transform the left hand sides of equations into the corresponding right hand sides and opposite.

$$\mathbf{d} = \qquad 4+1$$

Clifford odd

$$\hat{b}^{1\dagger}_1 = \stackrel{03}{(+i)}\stackrel{12}{[+]}\,, \quad \hat{b}^{1\dagger}_2 = \stackrel{03}{[+i]}\stackrel{12}{(+)}\,, \qquad \hat{b}^{1\dagger}_3 = \stackrel{03}{[-i]}\stackrel{12}{[+]}\,\gamma^5\,, \quad \hat{b}^{1\dagger}_4 = \stackrel{03}{(-i)}\stackrel{12}{(+)}\,\gamma^5\,,$$

$$\hat{b}^{2\dagger}_1 = \stackrel{03}{[-i]}\stackrel{12}{(-)}\,, \quad \hat{b}^{2\dagger}_2 = \stackrel{03}{(-i)}\stackrel{12}{[-]}\,, \qquad \hat{b}^{2\dagger}_3 = \stackrel{03}{(+i)}\stackrel{12}{(-)}\,\gamma^5\,, \quad \hat{b}^{2\dagger}_4 = \stackrel{03}{[+i]}\stackrel{12}{[-]}\,\gamma^5\,,$$

$$\hat{b}^{1}_1 = \stackrel{03}{(-i)}\stackrel{12}{[+]}\,, \quad \hat{b}^{1}_2 = \stackrel{03}{[+i]}\stackrel{12}{(-)}\,, \qquad \hat{b}^{1}_3 = \stackrel{03}{[+i]}\stackrel{12}{[+]}\,\gamma^5\,, \quad \hat{b}^{1}_4 = \stackrel{03}{(-i)}\stackrel{12}{(-)}\,\gamma^5\,,$$

$$\hat{b}^{2}_1 = \stackrel{03}{[-i]}\stackrel{12}{(+)}\,, \quad \hat{b}^{2}_2 = \stackrel{03}{(+i)}\stackrel{12}{[-]}\,, \qquad \hat{b}^{2}_3 = \stackrel{03}{(+i)}\stackrel{12}{(+)}\,\gamma^5\,, \quad \hat{b}^{2}_4 = \stackrel{03}{[-i]}\stackrel{12}{[-]}\,\gamma^5\,,$$

Clifford even

$${}^{I}\mathcal{A}^{1\dagger}_1 = \stackrel{03}{[+i]}\stackrel{12}{[+]}\,, \quad {}^{I}\mathcal{A}^{1\dagger}_2 = \stackrel{03}{(+i)}\stackrel{12}{(+)}\,, \qquad {}^{I}\mathcal{A}^{1}_3 = \stackrel{03}{(-i)}\stackrel{12}{[+]}\,\gamma^5\,, \quad {}^{I}\mathcal{A}^{1}_4 = \stackrel{03}{[-i]}\stackrel{12}{(+)}\,\gamma^5\,,$$

$${}^{I}\mathcal{A}^{2\dagger}_1 = \stackrel{03}{(-i)}\stackrel{12}{(-i)}\,, \quad {}^{I}\mathcal{A}^{2\dagger}_2 = \stackrel{03}{[-i]}\stackrel{12}{[-]}\,, \qquad {}^{I}\mathcal{A}^{2}_3 = \stackrel{03}{[+i]}\stackrel{12}{(-)}\,\gamma^5\,, \quad {}^{I}\mathcal{A}^{2}_4 = \stackrel{03}{(+i)}\stackrel{12}{[-]}\,\gamma^5\,,$$

$${}^{II}\mathcal{A}^{1\dagger}_1 = \stackrel{03}{[-i]}\stackrel{12}{[+]}\,, \quad {}^{II}\mathcal{A}^{1\dagger}_2 = \stackrel{03}{(-i)}\stackrel{12}{(+)}\,, \qquad {}^{II}\mathcal{A}^{1\dagger}_3 = \stackrel{03}{(+i)}\stackrel{12}{[+]}\,\gamma^5\,, \quad {}^{II}\mathcal{A}^{1\dagger}_4 = \stackrel{03}{[+i]}\stackrel{12}{(+)}\,\gamma^5\,,$$

$${}^{II}\mathcal{A}^{2\dagger}_1 = \stackrel{03}{(+i)}\stackrel{12}{(-)}\,, \quad {}^{II}\mathcal{A}^{2\dagger}_2 = \stackrel{03}{[+i]}\stackrel{12}{[-]}\,, \qquad {}^{II}\mathcal{A}^{2\dagger}_3 = \stackrel{03}{[-i]}\stackrel{12}{(-)}\,\gamma^5\,, \quad {}^{II}\mathcal{A}^{2\dagger}_4 = \stackrel{03}{(-i)}\stackrel{12}{[-]}\,\gamma^5\,.$$

Let us repeat the properties of "basis vectors" of the left hand side:

- **On the left hand sides we have**
  $\hat{b}^{m\dagger}_{f} *_A \hat{b}^{m'\dagger}_{f`}$ **are orthogonal** $(=0)$,
  $\hat{b}^{m}_{f} *_A \hat{b}^{m'}_{f`}$ **are orthogonal** $(=0)$,
  $\hat{b}^{m\dagger}_{f} *_A \hat{b}^{m'}_{f`}$ **are not orthogonal** $(\neq 0)$,
  $\hat{b}^{m}_{f} *_A \hat{b}^{m'\dagger}_{f`}$ **are not orthogonal** $(\neq 0)$,

  ${}^{\mathbf{I}}\mathcal{A}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{II}}\mathcal{A}^{\mathbf{m}'\dagger}_{\mathbf{f`}}$ **are orthogonal** $(=0)$
  ${}^{\mathbf{II}}\mathcal{A}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{I}}\mathcal{A}^{\mathbf{m}'\dagger}_{\mathbf{f`}}$ **are orthogonal** $(=0)$
  ${}^{\mathbf{I}}\mathcal{A}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{I}}\mathcal{A}^{\mathbf{m}'\dagger}_{\mathbf{f`}}$ **are not orthogonal** $(\neq 0)$
  ${}^{\mathbf{II}}\mathcal{A}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{II}}\mathcal{A}^{\mathbf{m}'\dagger}_{\mathbf{f`}}$ **are not orthogonal** $(\neq 0)$,

paying attention on

$$\begin{aligned}
\overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{(-\mathbf{k})} &= \eta^{aa}\overset{\mathbf{ab}}{[\mathbf{k}]}\,,\; \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{(\mathbf{k})} = \overset{\mathbf{ab}}{(\mathbf{k})}\,,\; \overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{[-\mathbf{k}]} = \overset{\mathbf{ab}}{(\mathbf{k})}\,, **\\
\overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{[\mathbf{k}]} &= \mathbf{0}\,, \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{(-\mathbf{k})} = \mathbf{0}\,,\; \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{[-\mathbf{k}]} = \mathbf{0}\,, **
\end{aligned}$$

Let us see properties of the "basis vectors" for the right hand side

- $\hat{b}^{m\dagger}_{f} *_A \hat{b}^{m'}_{f`}$ are orthogonal $(=0)$,
  $\hat{b}^{m\dagger}_{f} *_A \hat{b}^{m'}_{f`}$ are orthogonal $(=0)$,
  $\hat{b}^{m\dagger}_{f} *_A \hat{b}^{m'\dagger}_{f`}$ are not orthogonal $(\neq 0)$,
  $\hat{b}^{m}_{f} *_A \hat{b}^{m'}_{f`}$ are not orthogonal $(\neq 0)$,

  ${}^{\mathbf{I}}\mathcal{A}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{I}}\mathcal{A}^{\mathbf{m}'\dagger}_{\mathbf{f`}}$ are orthogonal $(=0)$
  ${}^{\mathbf{II}}\mathcal{A}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{II}}\mathcal{A}^{\mathbf{m}'\dagger}_{\mathbf{f`}}$ are orthogonal $(=0)$
  ${}^{\mathbf{I}}\mathcal{A}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{II}}\mathcal{A}^{\mathbf{m}'\dagger}_{\mathbf{f`}}$ are not orthogonal $(\neq 0)$
  ${}^{\mathbf{II}}\mathcal{A}^{\mathbf{m}\dagger}_{\mathbf{f}} *_{\mathbf{A}} {}^{\mathbf{I}}\mathcal{A}^{\mathbf{m}'\dagger}_{\mathbf{f`}}$ are not orthogonal $(\neq 0)$
- The anticommuting right hand side have the properties of the commuting left hand side.
- The commuting right hand side have the properties of the anti commuting left hand side.

We can conclude that neither Clifford odd nor the Clifford even "basis vectors", have in odd dimensional spaces the properties which they do demonstrate in even dimensional spaces: Only half of the "basis vectors" have in $d = 2n + 1$ the properties which they demonstrate in $d = 2(2n' + 1)$.

The other half of the Clifford odd "basis vectors" demonstrate properties of the Clifford even "basis vectors" and the other half of the Clifford even "basis vectors" demonstrate properties of the Clifford odd "basis vectors", keeping their anti commutativity or commutativity unchanged.
(Arbitrary Lorentz transformations transform the "basis vectors" of the left hand sides into the "basis vectors" of the right sides, and vice versa.)

These are properties of the internal spaces of the ghost scalar fields, used in the quantum field theory to make contributions of the Feynman diagrams finite.

General points of my way for cosmology.