Can the “basis vectors”, describing the internal space of point fermion and boson fields with the Clifford odd (for fermions) and Clifford even (for bosons) objects, be meaningfully extended to strings”?

N.S. Mankoč Borštnik, University of Ljubljana
H.B. Nielsen, University of Copenhagen

Bled 8-17 of July, 2024 Bled Slovenia

July 7, 2024

Although it looks like that we know almost everything about our Universe, due to the fact that the theories and models are to high extend supported by the experiments and the cosmological observations,
it is also true that we do not know why and how the universe has started,
what caused the exponential grow of the size of the universe,
what is happening in the black holes,
we namely do not know how to treat the second quantized gravity.

More than **50 years ago** the **electroweak** **(and colour)** **standard model** offered an **elegant new step** in **understanding the origin of fermions and bosons** by **postulating**, **supported by theories, models, experiments and cosmological observations**:

- **The existence** of **massless family members** with the **charges** in the **fundamental representation of the groups** -
  o the **coloured triplet quarks and colourless leptons**,
  o the **left handed members as the weak charged doublets**,
  o the **right handed weak chargeless members**,
  o the **left handed quarks distinguishing in the hyper charge from the left handed leptons**,
  o each right handed member having a different hyper charge.
- **The existence** of **massless families to each of a family member.**

- **The existence** of **massless vector gauge fields** to the observed **charges** (with the space i ndex in (3+1)) of the **family members**,
  carrying charges in the **adjoint representation of the charge groups**.
  (**Masslessness needed for gauge invariance.**)
  o Three massless vector fields, **the gauge fields of the three charges**.
  They all are vectors in $d = (3+1)$, **in the adjoint representations with respect to the weak SU(2), colour SU(3) and hyper U(1) charges**.
- The **existence of a massive scalar field - the higgs**,
  o carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$.
  o gaining at some step the **imaginary mass** and consequently the **constant value** , breaking the weak and the hyper charge and correspondingly breaking the **mass protection**.
- The **existence** of the **Yukawa couplings**, taking care of
  o the properties of **fermions** and
  o the masses of the **heavy bosons**.

- There is the gravitational field in $d=(3+1)$, **determined by the vielbeins and spin connections, with the space index in $(d=3+1)$**.
- **There are the Dirac prescriptions for the second quantized fermion** and **boson fields**
- **There are several trials to explain the appearance of families of quarks and leptons.**
- **There are several trials to explain the appearance of the inflation of the universe.**
- **There are several trials to try to treat the fermions and bosons in a unifying way.**
- **There are several trials to make a next step beyond the both standard models, electroweak and cosmological.**
- **There are several trials to make the theories renormalizable and without anomalies.**

- **The electroweak and colour *standard model* assumptions have been confirmed without offering surprises.**
- **The last unobserved field as a field, the Higgs's scalar, detected in June 2012, was confirmed in March 2013.**
- **The waves of the gravitational field were detected in February 2016 and again 2017.**

The assumptions of the *standard model* remain unexplained.

- **There are several cosmological observations which do not look to be explainable within the** *standard model*,
- the **second quantization of fermion and boson fields are postulated**,
- the **second quantization of the gravitational field is not yet even postulated**,
- the **used groups used in the standard model are postulated**,
- ...

- **The *standard model* assumptions have in the literature several explanations**.
- The **string theorists promise the theory offering the understanding the nature.**
- **What is the most promising next step beyond the** *standard model***?**.
- **Physicists suggest theories and look for predictions confirmed by experiments.**
- **We might all agree that the elementary constituents are two kinds of fields: Anti-commuting fermion and commuting boson fields, both assumed to be second quantized fields.**

- **The Spin-Charge-Family theory,**

  assuming the description of the internal spaces of fermions and bosons with the "basis vectors", which are superposition of products of

  odd number of γ^a for fermions and

  even number of γ^a for bosons,

  offers an unique description of boson and fermion second quantized fields.

  There are namely the same number of fermion and boson second quantized fields, manifesting a kind of supersymmetry.

  If the internal space involved in creating our universe has $d \geq (13+1)$ and the ordinary space is active in $d = (3+1)$ and no symmetry is broken, then both "basis vectors" have the same number of elements.

- **Making a choice that all "basis vectors" are eigenvectors of the chosen Cartan subalgebra members**, and **arrange the "basis vectors" to be products** of **nilpotents and projectors then "basis vectors" of fermions have an odd number of nilpotents** , and **"basis vectors" of bosons have an even number of nilpotents** .
- **These description is elegant and simple to use.**
- **Analysing the "basis vectors" with respect to the symmetry they manifest in** $d = (3+1)$ **all the second quantized boson fields observed in** $d = (3+1)$**, and all the second quantized fermion fields observed in** $d = (3+1)$ **can be described, with the second quantized graviton fields included.**
- **Choosing the simplest action for fermions and bosons, we can describe all the properties of the observed fields**.

- **The Spin-Charge-Family theory offers the explanation for**
  i. **all the assumptions of the** *standard model*,
  ii. **for many observed phenomena**:
  ii.a. the **dark matter**,
  ii.b. the **matter-antimatter asymmetry**,
  ii.c. **others observed phenomena**,
  iii. **explaining the Dirac's postulates for the second quantized fermion and second quantized boson fields**,
  iv. **offering explanation for the appearance of the graviton**,
  v. **explaining the offer of the Fadeev-Popov ghosts**,
  vi. **making several predictions.**

- Is the Spin-Charge-Family theory the right next step beyond both standard models?
- Work done so far on the spin-charge-family theory is promising.
- Let me comment whether the low energy limit of strings can be presented by the spin-charge-family theory.

- **There are two kinds of the Clifford algebra objects in any d . I recognized that in Grassmann space.**

J. of Math. Phys. **34** (1993) 3731

θ^a**'s and** p^{θ}_a**'s**, $p^{\theta}_a = \frac{\partial}{\partial \theta_a}$
with the property
$(\theta^a)^{\dagger} = \eta^{aa} \frac{\partial}{\partial \theta_a}$.

i. The **Dirac** γ^a (recognized 90 years ago in $d = (3+1)$).
ii. The **second one:** $\tilde{\gamma}^a$,

$$\gamma^a = (\theta^a - i\, p^{\theta a})\,, \quad \tilde{\gamma}^a = i\,(\theta^a + i\, p^{\theta a})\,,$$

References can be found in
Progress in Particle and Nuclear Physics,
http://doi.org/**10.1016.j.ppnp.2021.103890** .

- **The two kinds of the Clifford algebra objects anticommute as follows**

$$
\begin{aligned}
\{\gamma^{\mathbf{a}}, \gamma^{\mathbf{b}}\}_+ &= \mathbf{2}\eta^{\mathbf{ab}} = \{\tilde{\gamma}^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_+ ,\\
\{\gamma^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_+ &= 0,
\end{aligned}
$$

- **the postulate**

$$
\begin{aligned}
(\tilde{\gamma}^{\mathbf{a}}\mathbf{B} &= \mathbf{i}(-)^{\mathbf{n_B}}\mathbf{B}\gamma^{\mathbf{a}}\,)\,|\psi_0>,\\
(\mathbf{B} &= a_0 + a_a\gamma^a + a_{ab}\gamma^a\gamma^b + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\ldots\gamma^{a_d})|\psi_o>
\end{aligned}
$$

with $(-)^{n_B} = +1, -1$, if B has a Clifford even or odd character, respectively, $|\psi_o>$ is a vacuum state on which the operators γ^a **apply**, **reduces the Clifford space for fermions for the factor of two, from** 2×2^d **to** 2^d, while the operators $\tilde{\gamma}^a\tilde{\gamma}^b = -2i\tilde{S}^{ab}$ **define the family quantum numbers**.

- It is convenient to write all the **"basis vectors"** **describing the internal space of either fermion fields or boson fields as products of nilpotents and projectors, which are eigenvectors of the chosen Cartan subalgebra**

$$S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\; d}\,,$$
$$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\; d}\,,$$
$$\mathbf{S}^{ab} = S^{ab} + \tilde{S}^{ab}\,.$$

nilpotents

$$S^{ab}\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b) = \frac{k}{2}\,\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \quad \overset{ab}{(\mathbf{k})} := \frac{\mathbf{1}}{\mathbf{2}}(\gamma^{\mathbf{a}} + \frac{\eta^{\mathbf{aa}}}{\mathbf{ik}}\gamma^{\mathbf{b}})\,,$$

projectors

$$S^{ab}\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b) = \frac{k}{2}\,\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,, \quad \overset{ab}{[\mathbf{k}]} := \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{1} + \frac{\mathbf{i}}{\mathbf{k}}\gamma^{\mathbf{a}}\gamma^{\mathbf{b}})\,,$$

$$(\overset{ab}{(\mathbf{k})})^2 = \mathbf{0}\,, \qquad (\overset{ab}{[\mathbf{k}]})^2 = \overset{ab}{[\mathbf{k}]}\,,$$

$$\overset{ab}{(\mathbf{k})}^{\dagger} = \eta^{\mathbf{aa}}\,\overset{ab}{(-\mathbf{k})}\,, \quad \overset{ab}{[\mathbf{k}]}^{\dagger} = \overset{ab}{[\mathbf{k}]}\,.$$

$$
\begin{aligned}
S^{ab}\stackrel{ab}{(k)} &= \frac{k}{2}\stackrel{ab}{(k)}, \quad S^{ab}\stackrel{ab}{[k]} = \frac{k}{2}\stackrel{ab}{[k]},\\
\tilde{S}^{ab}\stackrel{ab}{(k)} &= \frac{k}{2}\stackrel{ab}{(k)}, \quad \tilde{S}^{ab}\stackrel{ab}{[k]} = -\frac{k}{2}\stackrel{ab}{[k]}\,.
\end{aligned}
$$

$$
\begin{aligned}
\gamma^a\stackrel{ab}{(k)} &= \eta^{aa}\stackrel{ab}{[-k]}\,,\ \gamma^b\stackrel{ab}{(k)} = -ik\stackrel{ab}{[-k]}\,,\ \gamma^a\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}\,,\ \gamma^b\stackrel{ab}{[k]} = -ik\eta^{aa}\stackrel{ab}{(-k)}\,,\\
\tilde{\gamma}^a\stackrel{ab}{(k)} &= -i\eta^{aa}\stackrel{ab}{[k]}\,,\ \tilde{\gamma}^b\stackrel{ab}{(k)} = -k\stackrel{ab}{[k]}\,,\ \tilde{\gamma}^a\stackrel{ab}{[k]} = \ i\stackrel{ab}{(k)}\,,\ \tilde{\gamma}^b\stackrel{ab}{[k]} = -k\eta^{aa}\stackrel{ab}{(k)}\,,\\
\stackrel{ab}{(k)}\stackrel{ab}{(-k)} &= \eta^{aa}\stackrel{ab}{[k]}\,,\ \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,,\ \stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}\,, **\\
\stackrel{ab}{(k)}\stackrel{ab}{[k]} &= 0\,,\ \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0\,,\ \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0\,, **\\
\stackrel{ab}{\widetilde{(-k)}}\stackrel{ab}{(k)} &= -i\eta^{aa}\stackrel{ab}{[k]}\,,\ \stackrel{ab}{\widetilde{[k]}}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}\,,\ \stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{[k]} = i\stackrel{ab}{(k)}\,,\ \stackrel{ab}{\widetilde{[-k]}}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}\,,\\
\stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{(k)} &= 0\,,\ \stackrel{ab}{\widetilde{[-k]}}\stackrel{ab}{(k)} = 0\,,\ \stackrel{ab}{\widetilde{(k)}}\stackrel{ab}{[-k]} = 0\,,\ \stackrel{ab}{\widetilde{[k]}}\stackrel{ab}{[k]} = 0\,.
\end{aligned}
$$

- γ^a transforms $\overset{ab}{(k)}$ into $\overset{ab}{[-k]}$, never to $\overset{ab}{[k]}$.
- $\tilde{\gamma}^a$ transforms $\overset{ab}{(k)}$ into $\overset{ab}{[k]}$, never to $\overset{ab}{[-k]}$.
- There are the Clifford odd "basis vectors", that is the "basis vectors" with an odd number of nilpotents, at least one, the rest are projectors, such "basis vectors" anti commute among themselves.
- There are the Clifford even "basis vectors", that is the "basis vectors" with an even number of nilpotents, the rest are projectors, such "basis vectors" commute among themselves.
- There are the $2^{\frac{d}{2}-1}$ Clifford odd "basis vectors" appearing in $2^{\frac{d}{2}-1}$ families and the same number of their Hermitian conjugated partners; $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$.
- There are $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ Clifford even "basis vectors" appearing in two orthogonal groups.

- **Let us see how does one family of the Clifford odd "basis vector" in $d = (13+1)$ look like, if spins in $d = (13+1)$ are analysed with respect to the Standard Model groups: $SO(3,1)\times$ $SU(2) \times SU(2)\times$ $SU(3) \times U(1)$.**

- **One irreducible representation** of one **family contains $2^{\frac{(13+1)}{2}-1}$ = 64 members which include all the family members, quarks and leptons with the right handed neutrinos included**, **as well as all the antimembers, antiquarks and antileptons, reachable by either S^{ab} (or by $\mathbb{C}_{\mathcal{N}}$ $\mathcal{P}_{\mathcal{N}}$ on a family member)**.

Jour. of High Energy Phys. **04** (**2014**) 165
J. of Math. Phys. **34**, 3731 (**1993**),
Int. J. of Modern Phys. **A 9**, 1731 (**1994**),
J. of Math. Phys. **44** 4817 (**2003**), hep-th/030322 .

S^{ab} generate **all the members of one family**. The **eightplet** (represent. of $SO(7,1)$) **of quarks of a particular colour charge. All are Clifford odd "basis vectors"** , with $SU(3)\times U(1)$ **part** $(\tau^{33}=1/2,\ \tau^{38}=1/(2\sqrt{3})$**, and** $\tau^{41}=1/6)$

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Octet, $\Gamma^{(7,1)}=1,\ \Gamma^{(6)}=-1$, of quarks							
1	$\mathbf{u_R^{c1}}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	**1**	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	**1**	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	**1**	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
4	$\mathbf{d_R^{c1}}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	**1**	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	**-1**	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	$\mathbf{d_L^{c1}}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	**-1**	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	$\mathbf{u_L^{c1}}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	**-1**	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\mid\mid\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	**-1**	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\mathbf{u_R}$ of the 1^{st} row into $\mathbf{u_L}$ of the 7^{th} row, and $\mathbf{d_R}$ of the 4^{rd} row into $\mathbf{d_L}$ of the 6^{th} row, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} generate **all the members of one family of quarks, leptonsantiquarks, antileptons. Here is the eightplet (represent. of** $SO(7,1)$**) of the colour chargeless leptons. The** $SO(7,1)$ **part is identical with the one of quarks, while the** $SU(3)\times U(1)$ **part is:** $\tau^{33}=0$, $\tau^{38}=0$, $\tau^{41}=-\frac{1}{2}$.

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(7,1)}=1$, $\Gamma^{(6)}=-1$, of leptons							
1	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
2	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
3	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
4	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
5	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
6	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
7	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
8	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform ν_R of the 1^{st} line into ν_L of the 7^{th} line, and e_R of the 4^{rd} line into e_L of the 6^{th} line, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} **generate also all the anti-eightplet** (repres. of $SO(7,1)$) **of anti-quarks of the anti-colour charge belonging to the same family of the Clifford odd basis vectors .** ($\tau^{33} = -1/2$, $\tau^{38} = -1/(2\sqrt{3})$, $\tau^{41} = -1/6$).

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Antioctet, $\Gamma^{(7,1)} = -1$, $\Gamma^{(6)} = 1$, of antiquarks							
33	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
34	$\bar{d}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
35	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
36	$\bar{u}_L^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	- 1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
37	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
38	$\bar{d}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
39	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
40	$\bar{u}_R^{\bar{c}1}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \vert \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\bar{d}_L$ of the 1^{st} line into $\bar{d}_R$ of the 5^{th} line, and $\bar{u}_L$ of the 4^{rd} line into $\bar{u}_R$ of the 8^{th} line.

- **The Clifford odd "basis vector" describing the internal space of quark $u^{c1\dagger}_{\uparrow R}$, $\Leftrightarrow$ $b^{1\dagger}_{1}$:=**
  $\stackrel{03}{(+i)}\stackrel{12}{[+]}\,|\,\stackrel{56}{[+]}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}$,
  has the Hermitian conjugated partner equal to
  $u^{c1}_{\uparrow R} \Leftrightarrow (\mathbf{b}^{1\dagger}_{\mathbf{1}})^{\dagger} = \stackrel{\mathbf{13\,14}}{[-]}\,\stackrel{\mathbf{11\,12}}{[-]}\,\stackrel{\mathbf{9\,10}}{(-)}\,||\,\stackrel{\mathbf{78}}{(-)}\stackrel{\mathbf{56}}{[+]}\,|\,\stackrel{\mathbf{12}}{[+]}\stackrel{\mathbf{03}}{(-\mathbf{i})}$, **both with an odd number of nilpotents,**
  both are the Clifford odd objects — forming two separate groups.

Anti-commutation relations for Clifford odd "basis vectors", representing the internal space of fermion fields of quarks and leptons ($i = (u^{c,f,\uparrow,\downarrow}_{R,L}, d^{c,f,\uparrow,\downarrow}_{R,L}, \nu^{f,\uparrow,\downarrow}_{R,L}, e^{f,\uparrow,\downarrow}_{R,L})$), and anti-quarks and anti-leptons, with the family quantum number f.

- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*_{\mathbf{A}}+} |\psi_{\mathbf{o}}> = \delta_{\mathbf{f}\,\mathbf{f}'}\, \delta^{\mathbf{mk}}\, |\psi_{\mathbf{o}}>$,
- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}}_{\mathbf{f}'}\}_{*_{\mathbf{A}}+} |\psi_{\mathbf{o}}> = 0 \cdot |\psi_{\mathbf{o}}>$,
- $\{\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*_{\mathbf{A}}+} |\psi_{\mathbf{o}}> = 0 \cdot |\psi_{\mathbf{o}}>$,
- $\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}\, |\psi_{\mathbf{o}}> = 0 \cdot |\psi_{\mathbf{o}}>$,
- $\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}\, |\psi_{\mathbf{o}}> = |\psi^{\mathbf{m}}_{\mathbf{f}}>$,

$$|\psi_{\mathbf{o}}> = \overset{03}{[-\mathbf{i}]}\overset{12}{[-]}\overset{56}{[-]} \cdots \overset{13\,14}{[-]}\ |\mathbf{1}>$$

define the vacuum state for quarks and leptons and antiquarks and antileptons of the family $\mathbf{f}$.

- **Clifford even "basis vectors"**, having an even number of nilpotents, describe the internal space of the corresponding **boson** field. The **gluon** field, for example, ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}$, which transforms the u_R^{c1} into u_R^{c2} looks like: ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}$ $(\equiv \stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{(-)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]})$.
  If it algebraically multiplies on u_R^{c1} $(\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]})$ it follows

$$
\begin{aligned}
&{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}(\equiv \stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{(-)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}) *_A \\
&u_R^{c1\dagger}, (\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) \rightarrow \\
&u_R^{c2\dagger}, (\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{[-]}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}), \\
&\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}} = \mathbf{u_R^{c2\dagger}} \;\mathbf{*_A}\; (\mathbf{u_R^{c1\dagger}})^{\dagger}, \\
&{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c2}\to u_R^{c1}}(\equiv \stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{[-]}) *_A u_R^{c2\dagger} \rightarrow u_R^{c1\dagger}, \\
&\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c2}\to u_R^{c1}}} = \mathbf{u_R^{c1\dagger}} \;\mathbf{*_A}\; (\mathbf{u_R^{c2\dagger}})^{\dagger}.
\end{aligned}
$$

- These **gluons** ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u^{ci}_{R} \rightarrow u^{cj}_{R}} = u^{cj\dagger}_{R} *_{A} (u^{ci\dagger}_{R})^{\dagger}$ **transform quarks of a particular colour charge to quarks of all the rest colour charges.**
  Let us notice that they all are expressed as the algebraic product of a family member and one of the Hermitian conjugated partner.
- **We can in an equivalent way express the weak boson** ${}^{I}\hat{\mathcal{A}}^{\dagger}_{weak\; u^{ci}_{R} \rightarrow d^{ci}_{R}} = d^{ci\dagger}_{R} *_{A} (u^{ci\dagger}_{R})^{\dagger}$ **transforming quarks of a particular weak charge to quarks of another weak charge (keeping the colour charges unchanged).**

There is the second kind of the Clifford even "basis vectors", having as well an even number of nilpotents, and consequently commute, describing the internal space of boson fields; they are orthogonal to all ${}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$.

- We call them ${}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$.

  ${}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$ transform a family member of a particular family of fermions to the same family member of all the rest families of quarks and leptons and antiquarks and antileptons.

  Let $e^{-\dagger}_{L\uparrow f=1}$ be $(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)})$, and $e^{-\dagger}_{L\uparrow f=2}$ be $(\equiv \stackrel{03}{(-)}\stackrel{12}{(+)}\stackrel{56}{(-)}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)})$.

  It follows that ${}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$ apply fermions from the right hand side

  $e^{-\dagger}_{L\uparrow f=2}\ (\equiv \stackrel{03}{(-)}\stackrel{12}{(+)}\stackrel{56}{(-)}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)})\ *_A\ {}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger \mathbf{m}}_{\mathbf{f}}$
  $(\equiv \stackrel{\mathbf{03}}{(+)}\stackrel{\mathbf{12}}{(-)}\stackrel{\mathbf{56}}{[+]}\ \stackrel{\mathbf{78}}{[-]}\stackrel{\mathbf{9\,10}}{[-]}\stackrel{\mathbf{11\,12}}{[-]}\ \stackrel{\mathbf{13\,14}}{[-]}) \to e^{-\dagger}_{L\uparrow f=1}$
  $(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)})$.

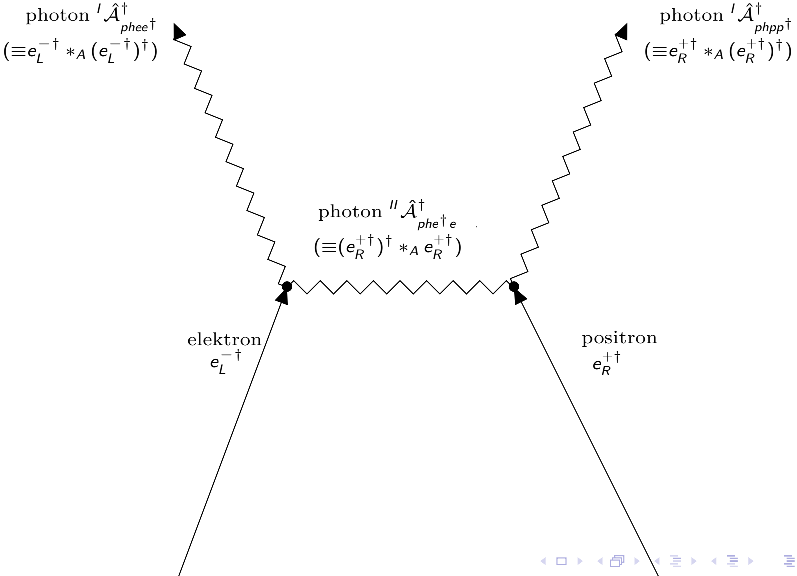
  $\gamma^{\mathbf{a}}\stackrel{\mathbf{ab}}{(\mathbf{k})} = \eta^{aa}\stackrel{\mathbf{ab}}{[-\mathbf{k}]}$, $\gamma^{\mathbf{a}}\stackrel{\mathbf{ab}}{[\mathbf{k}]} = \stackrel{\mathbf{ab}}{(-\mathbf{k})}$, $\tilde{\gamma}^{\mathbf{a}}\stackrel{\mathbf{ab}}{(\mathbf{k})} = -i\eta^{aa}\stackrel{\mathbf{ab}}{[\mathbf{k}]}$, $\tilde{\gamma}^{\mathbf{a}}\stackrel{\mathbf{ab}}{[\mathbf{k}]} = i\stackrel{\mathbf{ab}}{(\mathbf{k})}$.
  $\stackrel{\mathbf{ab}}{(\mathbf{k})}\stackrel{\mathbf{ab}}{(-\mathbf{k})} = \eta^{aa}\stackrel{\mathbf{ab}}{[\mathbf{k}]}$, $\stackrel{\mathbf{ab}}{[\mathbf{k}]}\stackrel{\mathbf{ab}}{(\mathbf{k})} = \stackrel{\mathbf{ab}}{(\mathbf{k})}$, $\stackrel{\mathbf{ab}}{(\mathbf{k})}\stackrel{\mathbf{ab}}{[-\mathbf{k}]} = \stackrel{\mathbf{ab}}{(\mathbf{k})}$, $\stackrel{\mathbf{ab}}{(\mathbf{k})}\stackrel{\mathbf{ab}}{[\mathbf{k}]} = \mathbf{0}$, $\stackrel{\mathbf{ab}}{[\mathbf{k}]}\stackrel{\mathbf{ab}}{(-\mathbf{k})} = \mathbf{0}$, $\stackrel{\mathbf{ab}}{[\mathbf{k}]}\stackrel{\mathbf{ab}}{[-\mathbf{k}]} = \mathbf{0}$.

Also $^{II}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$ **can be expressed as algebraic products of the Hermitian conjugate fermion fields** and **one of the Clifford odd "basis vector'.**

- **photon** $^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{-\dagger}e^{-}} = (e_L^{-\dagger})^{\dagger} *_A e_L^{-\dagger} =$
  photon $^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{+\dagger}e^{+}} = (e_R^{+\dagger})^{\dagger} *_A e_R^{+\dagger}$

- All bosons "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ (describing internal spaces of boson fields) are expressible as algebraic products of "basis vectors" and their Hermitian conjugated partners as $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`'})^\dagger$ or as $(\hat{b}^{m'\dagger}_{f`})^\dagger *_A \hat{b}^{m''\dagger}_{f``}$.
- Knowing "basis vectors" of fermions appearing in families we know all the boson fields as well.

Let us see how does the annihilation of electron and positron look like.



- Let us present graviton ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\, u^{c1}_{R\uparrow} \to u^{c1}_{R\downarrow}}$, which must leave all the charges of fermions, except the spin (S^{03}, S^{12}) in $d = (3+1)$, unchanged.

$$
\begin{aligned}
&{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\, u^{c1\dagger}_{R\uparrow} \to u^{c1\dagger}_{R\downarrow}} (\equiv \stackrel{03}{(-i)}\stackrel{12}{(-)}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{[+]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) *_A \\
&u^{c1\dagger}_{R\uparrow}, (\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) \to \\
&u^{c1\dagger}_{R\downarrow} (\equiv \stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}), \\
&\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\, u^{c1\dagger}_{R\uparrow} \to u^{c1\dagger}_{R\downarrow}}} = \mathbf{u^{c1\dagger}_{R\downarrow}} \; \mathbf{*_A} \; \mathbf{(u^{c1\dagger}_{R\uparrow})^{\dagger}}, \\
&{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\, u^{c1\dagger}_{R\downarrow} \to u^{c1\dagger}_{R\uparrow}} (\equiv \stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{[+]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) *_A \; u^{c1\dagger}_{R\downarrow} \to u^{c1\dagger}_{R\uparrow}, \\
&\mathbf{{}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\, u^{c1\dagger}_{R\downarrow} \to u^{c1\dagger}_{R\uparrow}}} = \mathbf{u^{c1\dagger}_{R\uparrow}} \; \mathbf{*_A} \; \mathbf{(u^{c1\dagger}_{R\downarrow})^{\dagger}}.
\end{aligned}
$$

Let be recognized again

- All bosons "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ (describing internal spaces of boson fields) are expressible as algebraic products of "basis vectors" and their Hermitian conjugated partners as $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f``})^\dagger$ or as $(\hat{b}^{m'\dagger}_{f`})^\dagger *_A \hat{b}^{m''\dagger}_{f``}$.
- Knowing "basis vectors" of fermions appearing in families we know all the boson fields as well.

- The Clifford odd and the Clifford even “basis vectors” differ essentially in their properties:

  The Clifford odd "basis vectors" in even dimensional spaces appear in $2^{\frac{d}{2}-1}$ families, each family having $2^{\frac{d}{2}-1}$ members, and have their Hermitian conjugated partners in a separate group, with $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ contributions.

  The Clifford even "basis vectors" in even dimensional spaces appear in two groups, each with $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ members, having the Hermitian conjugated partners within the same group. They have no families.

- The Clifford odd "basis vectors" in even dimensional spaces carry the eigenvalues of the Cartan subalgebra members $\pm\frac{i}{2}$ or $\pm\frac{i}{2}$.

  The Clifford even "basis vectors" in even dimensional spaces carry the eigenvalues of the Cartan subalgebra members $(\pm i, 0)$ or $(\pm 1, 0)$.

- There are in $d = 2(2n+1)$ dimensional spaces $2^{\frac{d}{2}-1}$ Clifford odd families, each family having $2^{\frac{d}{2}-1}$ members. The Clifford odd "basis vectors" have their Hermitian conjugated partners in a separate group of $2^{\frac{d}{2}-1}$ families with $2^{\frac{d}{2}-1}$ members.
  In a tensor product with the basis in ordinary space the Clifford odd "basis vectors" together with their Hermitian conjugated partners form anti commuting creation and annihilation operators, fulfilling on the vacuum state the postulates of the second quantized fermion fields.
- There are in even dimensional spaces two times $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ Clifford even "basis vectors", with their Hermitian conjugated partners within the same group.

  In a tensor product with the basis in ordinary space the Clifford even "basis vectors" form commuting creations and annihilation operators, fulfilling the postulates of the second quantized boson fields.

We want to see whether our description of the **second quantized boson** and **fermion** fields can in the low energy limit be related to the **string theories of type II** for $d = (9+1)$.

- Let us start with the generation of the **Clifford even "basis vectors" for** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ from the algebraic products of the **Clifford odd "basis vectors"** and their **Hermitian conjugated partners**, $\hat{b}^{m\dagger}_{f} *_{A} (\hat{b}^{m'\dagger}_{f})^{\dagger}$, for a particular case when $d = (5+1)$

  to learn what we can expect after comparing these relations with the similar relations in the **string theory**.
- Let us treat ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ as the algebraic product, $(\hat{b}^{m\dagger}_{f})^{\dagger} *_{A} \hat{b}^{m\dagger}_{f}$, again for the case when $d = (5+1)$.

  The generalization to any even d is not difficult.

- Looking at the properties of **both kinds** of the **Clifford even "basis vectors"**, ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, manifesting momentum in only transverse dimensions (with $\mathcal{S}^{12}$ not equal 0), we found that to **both groups** of the **Clifford even "basis vectors"** **all family members** m **and all families** f contribute:

  a. To ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$, manifesting non zero transverse momentum, $S^{12} = \pm 1$, only half of possibilities contribute $(\frac{1}{2} \times 2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1})$ of $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`})^{\dagger}$, the other half possibilities contribute to $\mathcal{S}^{12} = 0$. (Each **family** f' **of** $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`})^{\dagger}$ generates the same eight **Clifford even** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$) as are the ones presented in Table below for $f` = 1$.)

  b. Again, to ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f} = (\hat{b}^{m'\dagger}_{f`})^{\dagger} *_A \hat{b}^{m'\dagger}_{f`'}$, only half of possibilities contribute, the other half contribute to $\mathcal{S}^{12} = 0$.
  (Each **family member** m' generates in $(\hat{b}^{m'\dagger}_{f`})^{\dagger} *_A \hat{b}^{m'\dagger}_{f`'}$ the same eight **Clifford even** ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ as are the ones presented in the second Table below for $m' = 1$.)

The below Table is presenting the Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ (they are products of one projector and two nilpotents), as the algebraic products of the Clifford odd "basis vectors" and their Hermitian conjugated partners (which are products of one nilpotent and two projectors or of three nilpotents.)

They represent $\frac{1}{2}$ of $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ members, the ones with $S^{12} = \pm 1$.

We shall see four Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ with spin $= \pm 1$ and $S^{03} = \pm i$, while $S^{56} = 0$. They represent gravitons. The "true " gravitons have in internal spaces $S^{12} = \pm 1$ $S^{03} = \pm i$ all other $S^{ab} = 0$, all others described by projectors.

$\mathcal{S}^{12}$	symbol	${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} = \hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f`})^\dagger$
1	$\star\star$	${}^{I}\hat{\mathcal{A}}^{1\dagger}_{1} = \hat{b}^{1\dagger}_{1} *_A (\hat{b}^{4\dagger}_{1})^\dagger$ $\stackrel{03}{[+i]}\stackrel{12}{(+)}\stackrel{56}{(+)} = \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]} *_A \stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{(+)}$
1	$\ddagger$ **graviton**	${}^{I}\hat{\mathcal{A}}^{3\dagger}_{1} = \hat{b}^{3\dagger}_{1} *_A (\hat{b}^{4\dagger}_{1})^\dagger$ $\stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{[-]} = \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)} *_A \stackrel{03}{(-i)}\stackrel{12}{(+)}\stackrel{56}{(+)}$
1	$\odot\odot$ **graviton**	${}^{I}\hat{\mathcal{A}}^{1\dagger}_{4} = \hat{b}^{1\dagger}_{1} *_A (\hat{b}^{2\dagger}_{1})^\dagger$ $\stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{[+]} = \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]} *_A \stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{[+]}$
1	$\otimes$	${}^{I}\hat{\mathcal{A}}^{3\dagger}_{4} = \hat{b}^{3\dagger}_{1} *_A (\hat{b}^{2\dagger}_{1})^\dagger$ $\stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{(-)} = \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)} *_A \stackrel{03}{[-i]}\stackrel{12}{(+)}\stackrel{56}{[+]}$
-1	$\otimes$	${}^{I}\hat{\mathcal{A}}^{2\dagger}_{2} = \hat{b}^{2\dagger}_{1} *_A (\hat{b}^{3\dagger}_{1})^\dagger$ $\stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{(+)} = \stackrel{03}{[-i]}\stackrel{12}{(-)}\stackrel{56}{[+]} *_A \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(+)}$
-1	$\ddagger$ **graviton**	${}^{I}\hat{\mathcal{A}}^{4\dagger}_{2} = \hat{b}^{4\dagger}_{1} *_A (\hat{b}^{3\dagger}_{1})^\dagger$ $\stackrel{03}{(+i)}\stackrel{12}{(-)}\stackrel{56}{[-]} = \stackrel{03}{(+i)}\stackrel{12}{(-)}\stackrel{56}{(-)} *_A \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(+)}$
-1	$\odot\odot$ **graviton**	${}^{I}\hat{\mathcal{A}}^{2\dagger}_{3} = \hat{b}^{2\dagger}_{1} *_A (\hat{b}^{1\dagger}_{1})^\dagger$ 03 12 56 [illegible] = 03 12 56 [illegible] 03 12 56 [illegible]

Table presenting the Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ (they are products of one projector and two nilpotents), as the algebraic products of the Clifford odd "basis vectors" and their Hermitian conjugated partners (which are products of one nilpotent and two projectors or of three nilpotents.) They represent $\frac{1}{2}$ of $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ members, the ones with $S^{12} = 0$.

We shall see four Clifford even "basis vectors" ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ which are products of only projectors.
They carry all the internal quantum numbers equal to zero.
They can represent photons.
The "true " photons have in internal spaces all $S^{ab} = 0$, all described by projectors.

$\mathcal{S}^{12}$	*symbol*	${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ $=$	$\hat{b}^{m'\dagger}_{f^{`}} *_A (\hat{b}^{m''\dagger}_{f^{`}})^\dagger$
0	$\triangle$	${}^{I}\hat{\mathcal{A}}^{2\dagger}_{1} =$ $\stackrel{03}{(-i)}\,\stackrel{12}{[-]}\stackrel{56}{(+)}$	$\hat{b}^{2\dagger}_{1} *_A (\hat{b}^{4\dagger}_{1})^\dagger$ $\stackrel{03}{[-i]}\,\stackrel{12}{(-)}\stackrel{56}{[+]} *_A \stackrel{03}{(-i)}\,\stackrel{12}{(+)}\stackrel{56}{(+)}$
0	$\bigcirc$ **photon**	${}^{I}\hat{\mathcal{A}}^{4\dagger}_{1} =$ $\stackrel{03}{[+i]}\,\stackrel{12}{[-]}\stackrel{56}{[-]}$	$\hat{b}^{4\dagger}_{1} *_A (\hat{b}^{4\dagger}_{1})^\dagger$ $\stackrel{03}{(+i)}\,\stackrel{12}{(-)}\stackrel{56}{(-)} *_A \stackrel{03}{(-i)}\,\stackrel{12}{(+)}\stackrel{56}{(+)}$
0	$\bullet$	${}^{I}\hat{\mathcal{A}}^{1\dagger}_{2} =$ $\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\stackrel{56}{(+)}$	$\hat{b}^{1\dagger}_{1} *_A (\hat{b}^{3\dagger}_{1})^\dagger$ $\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\stackrel{56}{[+]} *_A \stackrel{03}{[-i]}\,\stackrel{12}{[+]}\stackrel{56}{(+)}$
0	$\bigcirc$ **photon**	${}^{I}\hat{\mathcal{A}}^{3\dagger}_{2} =$ $\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\stackrel{56}{[-]}$	$\hat{b}^{3\dagger}_{1} *_A (\hat{b}^{3\dagger}_{1})^\dagger$ $\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\stackrel{56}{(-)} *_A \stackrel{03}{[-i]}\,\stackrel{12}{[+]}\stackrel{56}{(+)}$
0	$\bigcirc$ **photon**	${}^{I}\hat{\mathcal{A}}^{1\dagger}_{3} =$ $\stackrel{03}{[+i]}\,\stackrel{12}{[+]}\stackrel{56}{[+]}$	$\hat{b}^{1\dagger}_{1} *_A (\hat{b}^{1\dagger}_{1})^\dagger$ $\stackrel{03}{(+i)}\,\stackrel{12}{[+]}\stackrel{56}{[+]} *_A \stackrel{03}{(-i)}\,\stackrel{12}{[+]}\stackrel{56}{[+]}$
0	$\bullet$	${}^{I}\hat{\mathcal{A}}^{3\dagger}_{3} =$ $\stackrel{03}{(-i)}\,\stackrel{12}{[+]}\stackrel{56}{(-)}$	$\hat{b}^{3\dagger}_{1} *_A (\hat{b}^{1\dagger}_{1})^\dagger$ $\stackrel{03}{[-i]}\,\stackrel{12}{[+]}\stackrel{56}{(-)} *_A \stackrel{03}{(-i)}\,\stackrel{12}{[+]}\stackrel{56}{[+]}$
0	$\bigcirc$ **photon**	${}^{I}\hat{\mathcal{A}}^{2\dagger}_{4} =$ 03 12 56	$\hat{b}^{2\dagger}_{1} *_A (\hat{b}^{2\dagger}_{1})^\dagger$ 03 12 56 03 12 56

Also ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f} = (\hat{b}^{m\dagger}_{f`})^{\dagger} *_{A} \hat{b}^{m\dagger}_{f``}$, can in $d = (3+1)$ represent "gravitons" and "photons".

While ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ transform the family members among themselves, transform ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ a particular family member into the same family member belonging to a different family.

Let us now try to relate the description of the internal spaces of bosons described by the Clifford even "basis vectors", as explained so far and the way how *string theories* **consider the IIA or IIB superstring model for closed RNS strings** Ref. Kevin., where the spectrum is described.

We find in $d = (9+1)$, according to what it is discussed so far the case that we are interested only on those internal degrees of freedom of the Clifford even basis vectors of each of the two kinds, ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$, which manifest momentum in only transverse dimensions, that means
$\frac{1}{2} \times 2^{\frac{d=10}{2}-1} \times 2^{\frac{d=10}{2}-1} = 8 \times 16 = 128$ of ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ **and** 128 **of** ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$**, together** 256 **of both kinds of the Clifford even "basis vectors", representing the boson fields.**
These are also possibilities suggested in Ref. Kevin for closed strings in $d = (9+1)$; for the left-right movers or right-left movers forming the closed boson strings of AII and BII kind, manifesting the momentum in only transverse dimensions they found 256 **possibilities.**

Let me conclude this presentation recognizing:

- We, Holger and me, we started this contribution to show whether, and if yes, to which extent can the description of internal spaces of **fermions and bosons with the Clifford anticommuting and the Clifford commuting "basis vectors"**, **respectively, agree with the low energy limit of the** *string theories* **considered in the IIA or IIB superstring model for closed RNS strings**, Ref. Kevin., where the spectrum is described.
- **Having internal space of boson second quantized fields described by** ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ (manifesting non zero transverse momentum, $S^{12}=\pm 1$) **expressed as** $\hat{b}^{m'\dagger}_{f'} *_A (\hat{b}^{m''\dagger}_{f'})^{\dagger}$ **or by** ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ (manifesting non zero transverse momentum, $S^{12}=\pm 1$) **expressed as** $(\hat{b}^{m'\dagger})^{\dagger} *_A \hat{b}^{m''\dagger}_{f'}$, **(resembling left and right movers in** *string theories* **of the IIA or IIB superstring model for closed RNS strings),**

it turns out that while such left and right movers with transverse momentum, $S^{12} = \pm 1$, can describe gravitons they can not describe photons, weak bosons, gluons which have **two nilpotents only in the weak region** (S^{56}, S^{78}), **colour region** $(S^{9\,10}, S^{11\,12}, S^{13\,14})$, **or only projectors in the case of photons.**

It is that the relation between strings and the description of the internal spaces of the boson second quantized fields by ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ **and** ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ **need more understanding. In particular since we need to relate creation and annihilation operators and not only the internal spaces of boson fields.**