The Gravitational-Wave Aspect of PBH-Catalyzed Phase Transitions

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Outline

- Cosmological Phase Transitions
- PBH’s Catalytic Effect
- Gravitational Waves
- Insights from PTA SGWB

Cosmological Phase Transition

Motivations for First-order Phase Transitions

Theoretical points:

Fundamental theory, dark sector theory, baryon asymmetry, ……

To probe BSM via GWs:

Since BSM is necessary for FOPT

To probe the early universe via GWs:

Phase Transition is a result of competition between cosmic expansion and bubble nucleation rate

Thermal Phase Transitions

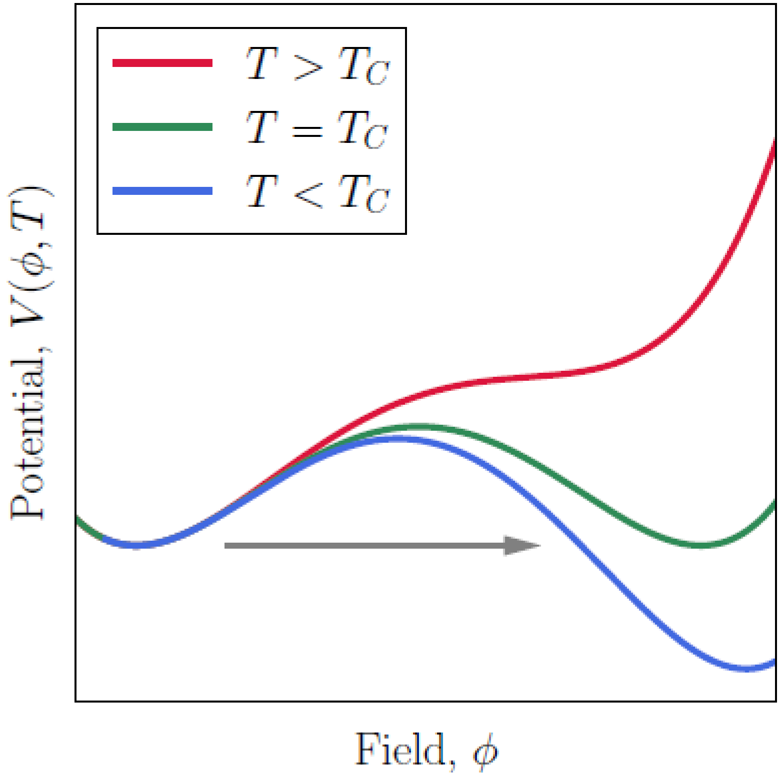
- **potential changed by temperature**
- Need **quantum tunneling** (pure universe)

Tunneling Rate per unit time and volume

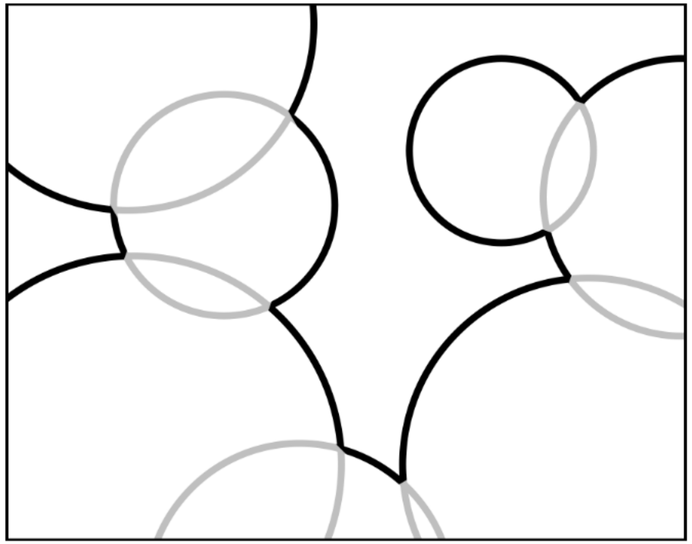
$$\Gamma(T) \approx T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-\frac{S_3}{T}\right)$$

$S_3(T)$ is instanton solution correspond to bubble profile

Could be solved numerically, e.g. **CosmoTransitions,**
or analytically with thin-wall approximation



Bubbles in First-order PT (FOPT)



Critical bubble radius

$$R_c \sim 1/\Delta\langle\phi\rangle$$

Bubble wall velocity

$$v_{wall} \sim c$$ For strong PT

Bubble wall profile

Movement of high energy bubble wall

Bubble Collision Gravitational Waves

Parameters

Nucleation temperature T_n: One bubble in unit Hubble volume

$$\int_{t_c}^{t_n} dt' \frac{\Gamma(t')}{H^3(t)} = 1 \sim \Gamma(T_n) = H^4(T_n)$$

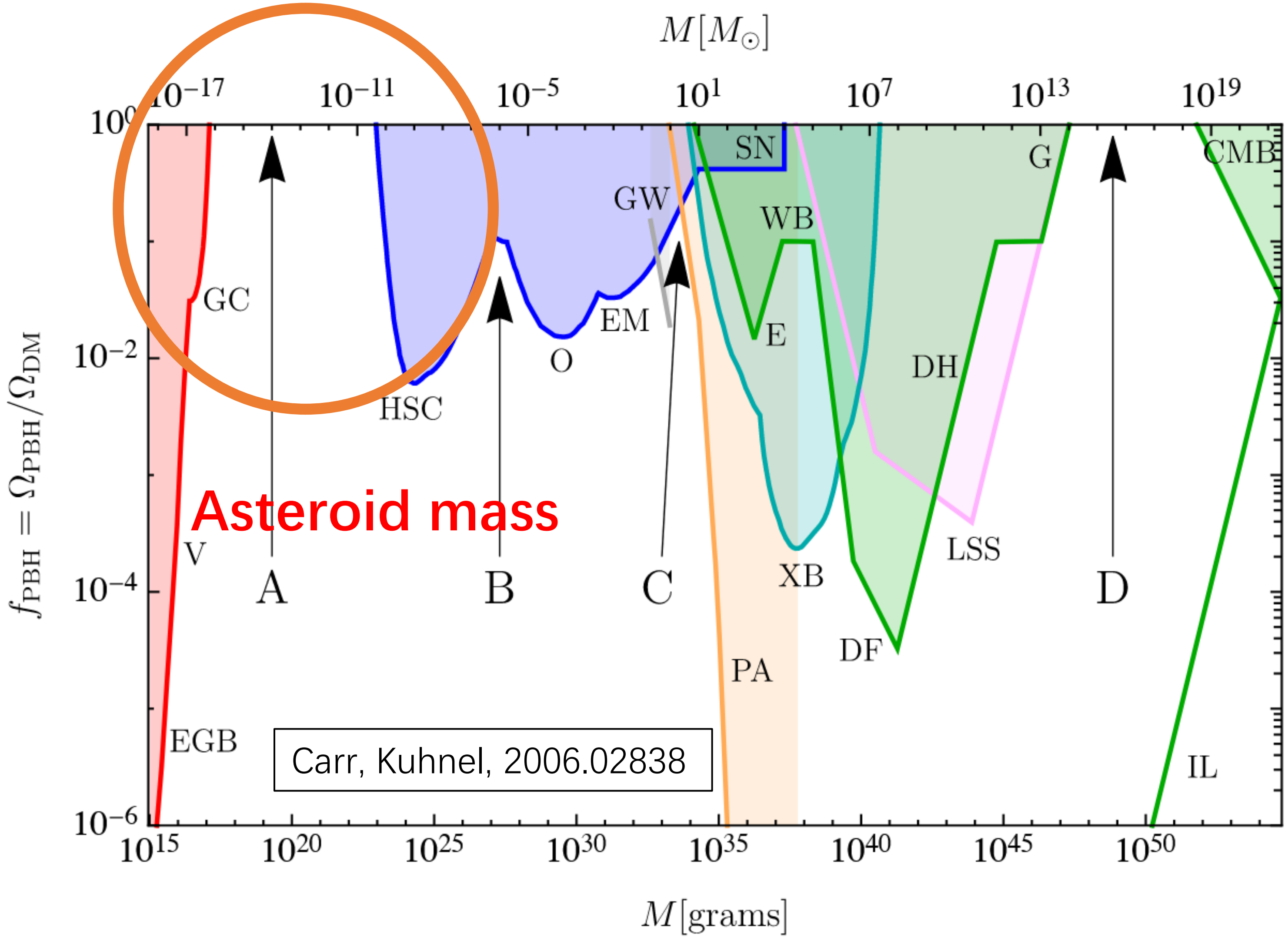
Vacuum energy density ρ_V **Radiation energy density $\rho_{rad} \sim T^4$**

Inverse duration: β: $$\beta = \frac{d\, ln(\Gamma)}{dt}\Big|_{T_n}$$

Near T_n $$\Gamma = H^4(T_n) e^{\beta(t-t_n)}$$

PBH’s catalytic effect

Primordial Black Holes

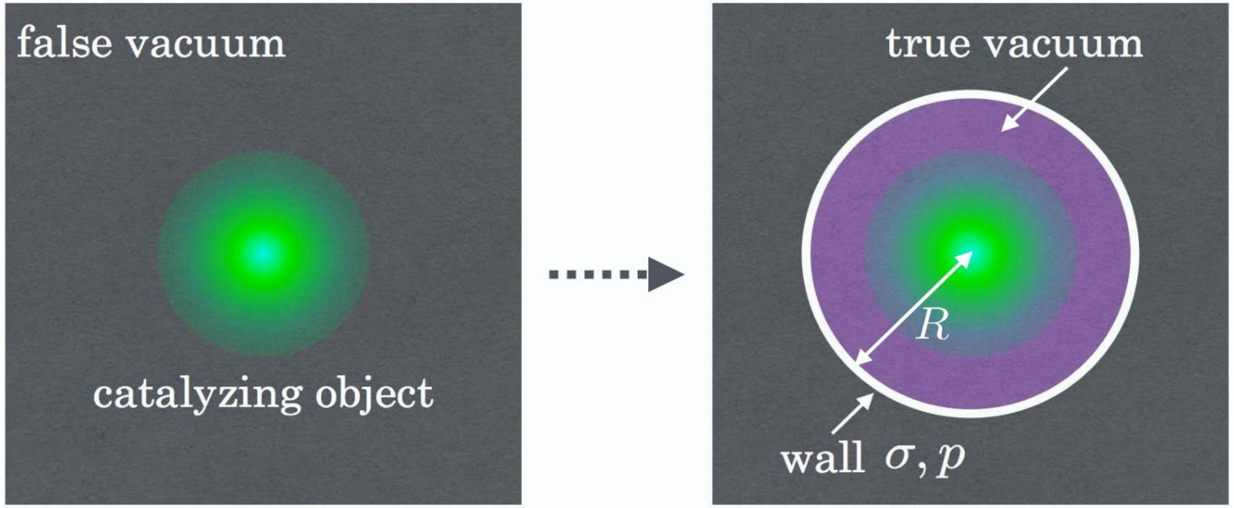


Dark matter candidate

PBHs are formed in the early universe

May formed before a PT

Catalytic Effect: Impurity in the universe



Oshita, Yamada, Yamaguchi, 1808.01382

Scalar Field: from **False Vacuum** to **True Vacuum**

Metric Field: **Schwarzschild-de Sitter spacetime**

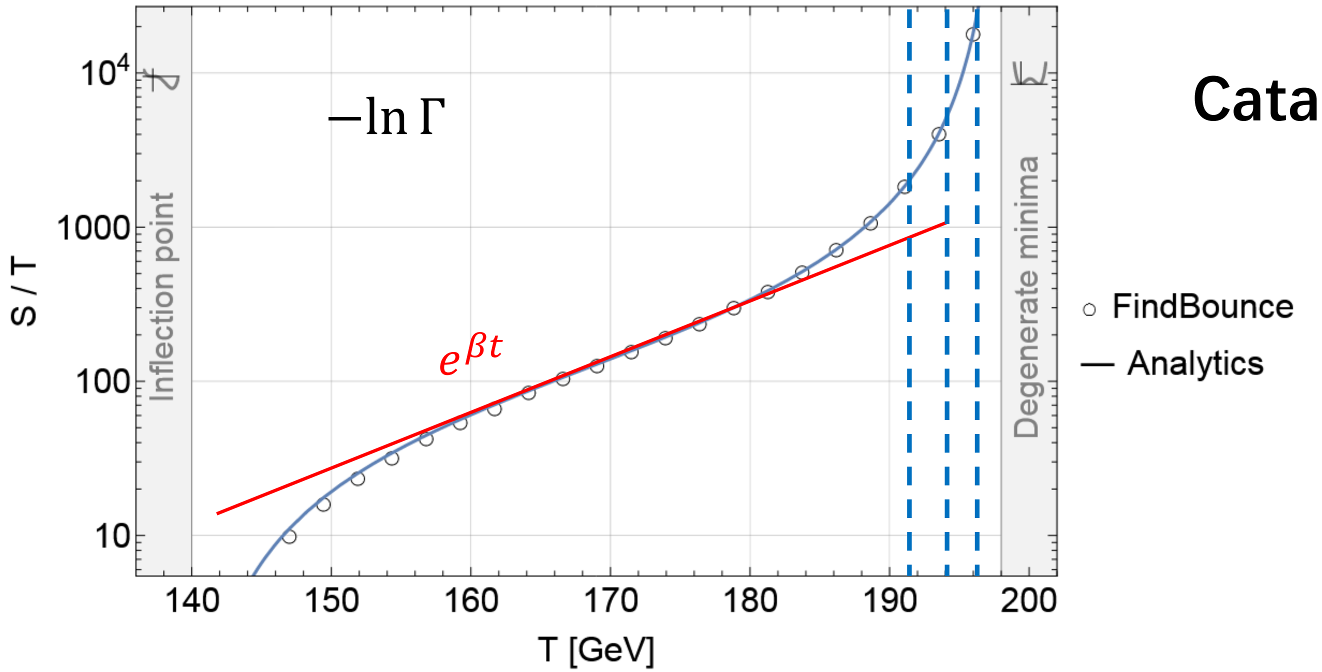
$$ds^2 = -f_{\mathrm{SdS}}(r)dt^2 + \frac{dr^2}{f_{\mathrm{SdS}}(r)} + r^2 d\Omega,$$

from $\boldsymbol{\Lambda}_+$ to $\boldsymbol{\Lambda}_-$ $\quad$ $\boldsymbol{\rho_V = \Lambda_+ - \Lambda_-}$

$$f_{\mathrm{SdS}}(r) = 1 - \frac{M_{\mathrm{BH}}}{4\pi r} - \frac{\Lambda r^2}{3},$$

Catalytic Effect

Catalytic strength depend on PBH mass M_{PBH}, vacuum energy density ρ_V, and bubble wall tension σ_w.



Catalytic effect is usually very strong

$$S_{pbh}/S < 1/10$$

background $S/T < \sim 140$

PBH $S/T < \sim 1000$

Approximation: **Each PBH induce one bubble at critical temperature T_c**

Catalytic Effect

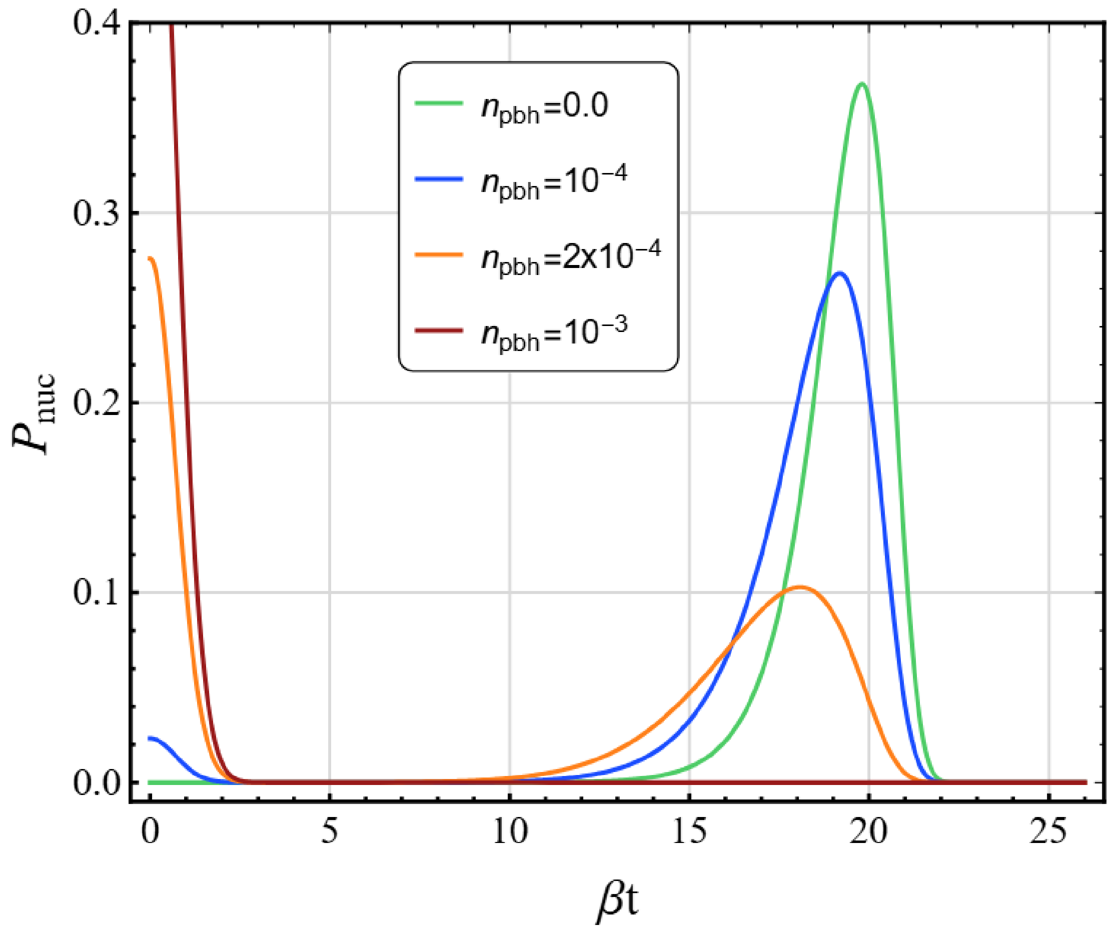
Nucleation Rate

$$\Gamma = \Gamma_b + \Gamma_{PBH}$$

$$\Gamma_b = H^4(T_n)e^{\beta(t-t_n)}$$

$$\Gamma_{PBH} = n_{pbh}H^3\delta(t-t_c)$$

Averaged PBH number per unit Hubble patch



Gravitational Waves

GW from FOPTs

The approximate form on dimension ground:

$$\mathrm{E_{GW}} \sim G v_w^3 \kappa^2 \rho_V^2 \beta^{-5}$$

$$\mathrm{E_V} \sim \rho_V v_w^3 \beta^{-3}$$

$$\frac{\rho_{GW}}{\rho_{tot}} \sim \frac{\rho_V}{\rho_{tot}} \frac{E_{GW}}{E_V} \sim \kappa^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H}\right)^{-2}$$

$$\Omega_{GW} = \frac{1}{\rho_{tot}} \frac{d\rho_{GW}}{d\ln f} \sim \boxed{\kappa^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H}\right)^{-2}} g(f\beta^{-1})$$

$$\alpha = \rho_V / \rho_{rad}$$

$$\rho_{tot} = \rho_V + \rho_{rad}$$

$$H = \sqrt{\rho_{tot}} / M_{pl}$$

Strong PT:

$$\alpha \gg 1$$

$$\kappa \sim 1$$

GW from FOPTs

PBHs affect the PT GW only through affect the bubble nucleation rate

v_w, ρ_V, κ will not be affected PBHs

Only the timescale β^{-1} will be affected

In strong PT, $v_w \sim c = 1$ and $\beta^{-1} \sim R_{sep}$ Mean bubble separation

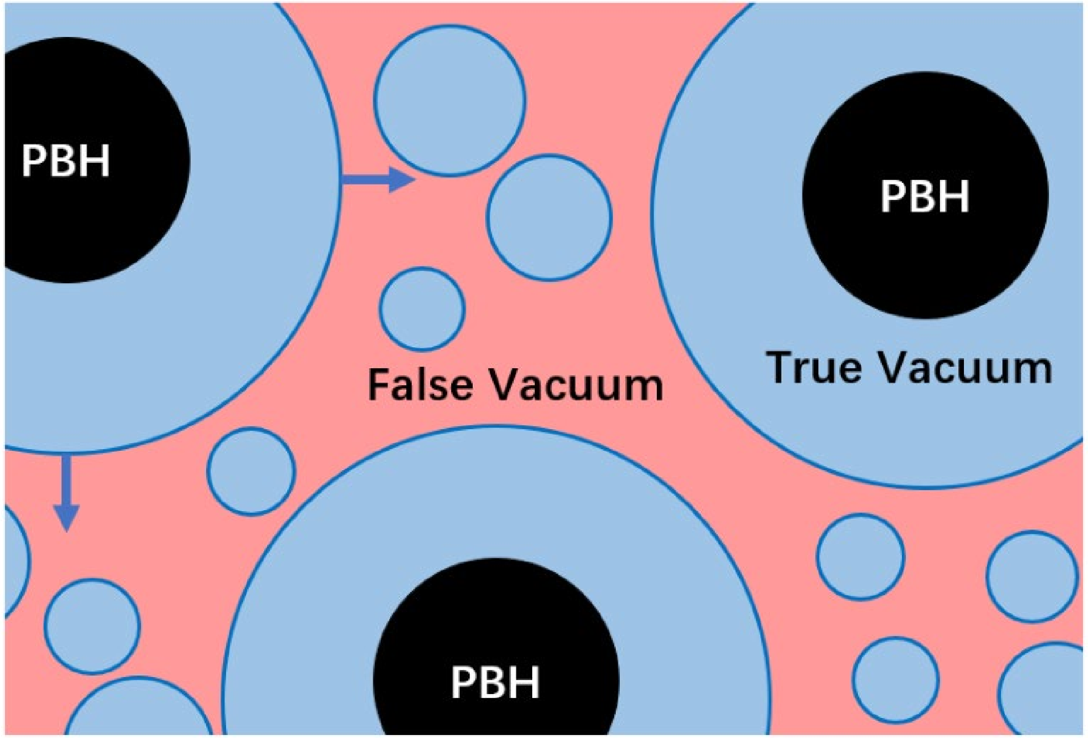
GW from FOPTs

$$R_{sep} = (n_{bubble})^{-1/3} = \left(\int_{t_c}^{t_p} dt\, \Gamma(t) F(t)\right)^{-1/3}$$

$F(t)$ is false vacuum fraction

t_p is percolation time: $F(t_p) \approx 0.7$

$$\frac{\beta}{\beta\ (n_{pbh} = 0)} = \left(\frac{R_{sep}}{R_{sep}(n_{pbh} = 0)}\right)^{-1}$$



GW from FOPTs

$$\frac{\Omega_p}{\Omega_p(n_{pbh}=0)} = \left(\frac{\beta}{\beta(n_{pbh}=0)}\right)^{-2} \qquad \frac{f_p}{f_p\,(n_{pbh}=0)} = \frac{\beta}{\beta(n_{pbh}=0)}$$

In high PBH number density limit:

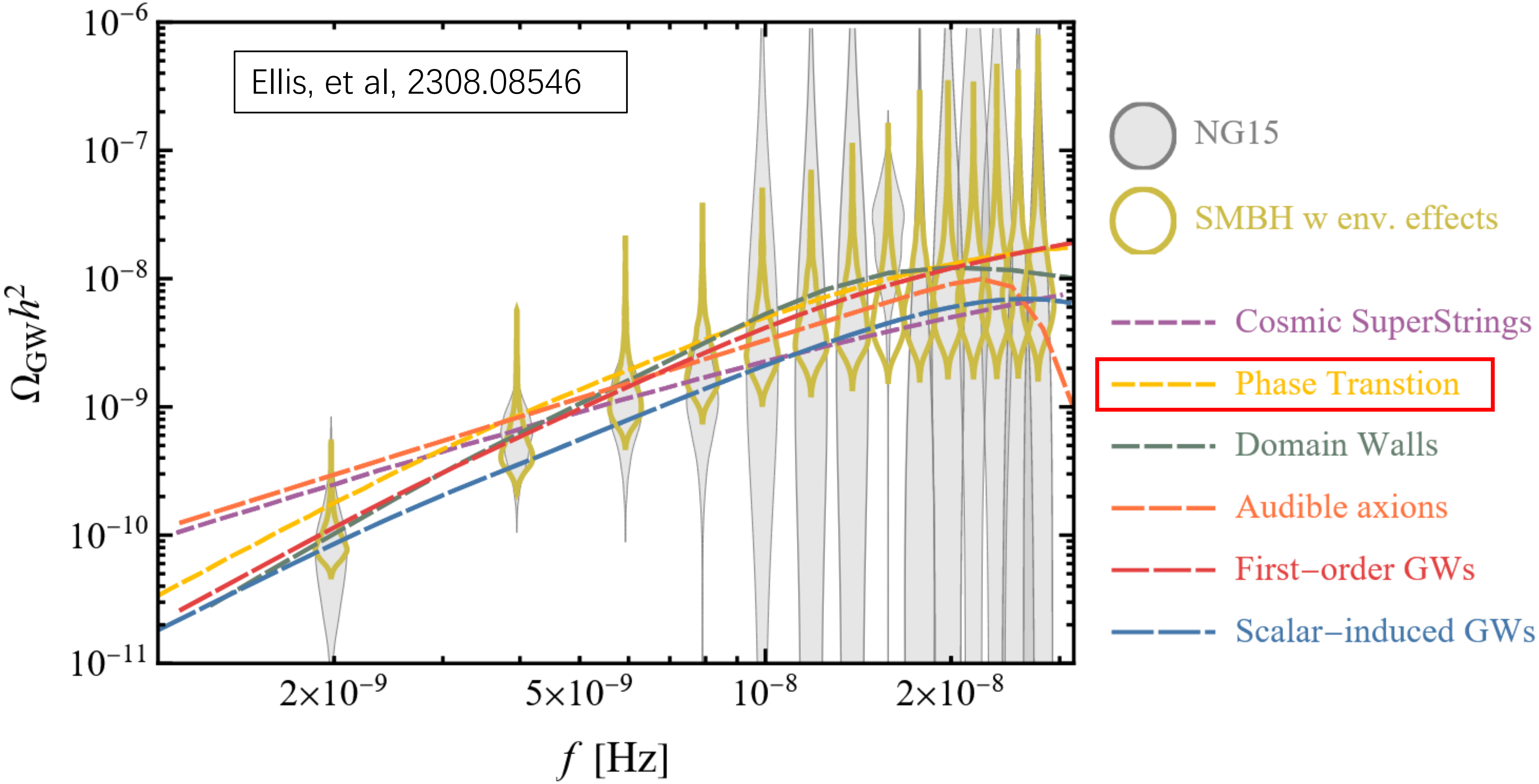
$$\frac{\beta}{\beta(n_{pbh}=0)} \approx 4\, n_{pbh}^{\frac{1}{3}} H/\beta(n_{pbh}=0)$$

Leading to suppressed GW signals

Insight from PTA SGWB

SGWB Observed by PTA

[The NANOGrav 15-Year Data Set]



Strong Phase Transition

$\alpha \gg 1$

$T \sim 0.1$ GeV

$\beta / H \sim O(10)$

Number Density of PBHs

$$n_{pbh} \approx 1.3 \times 10^{-8} \left(\frac{M_{sun}}{M_{PBH}}\right)\left(\frac{f_{PBH}}{1.0}\right)\left(\frac{0.1GeV}{T}\right)^{3}\left(\frac{g_*}{100}\right)^{-1/2}$$

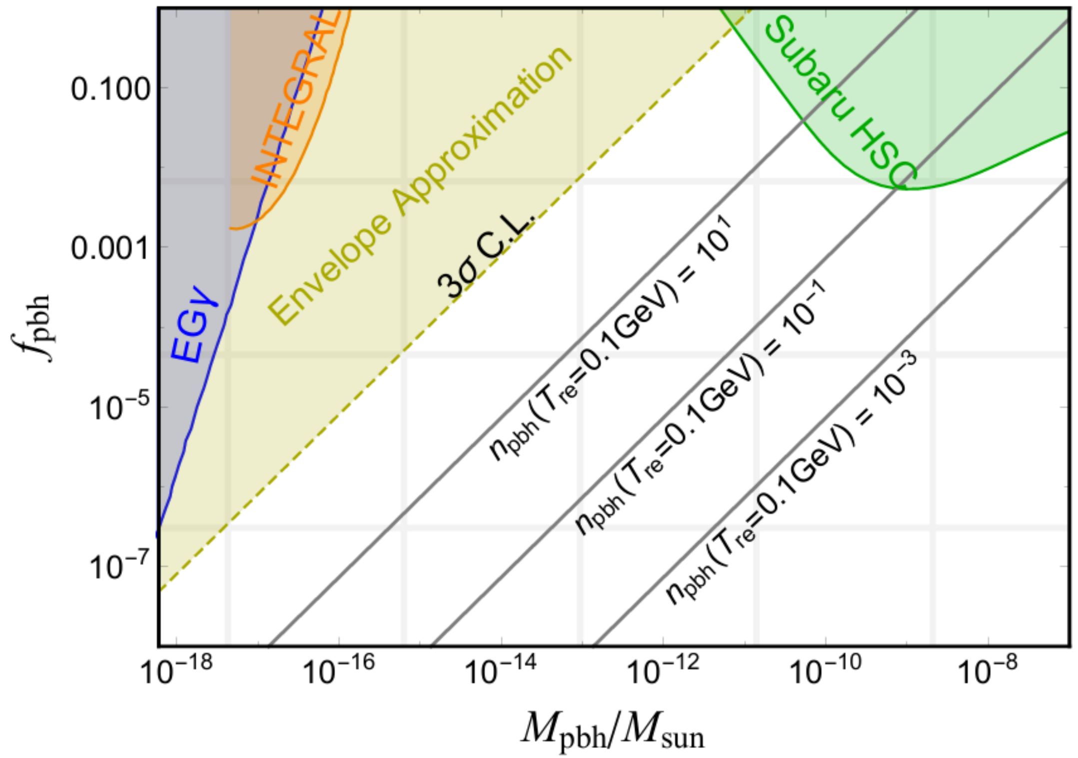
$0.1GeV$ Asteroid-mass PBH as whole dark matter

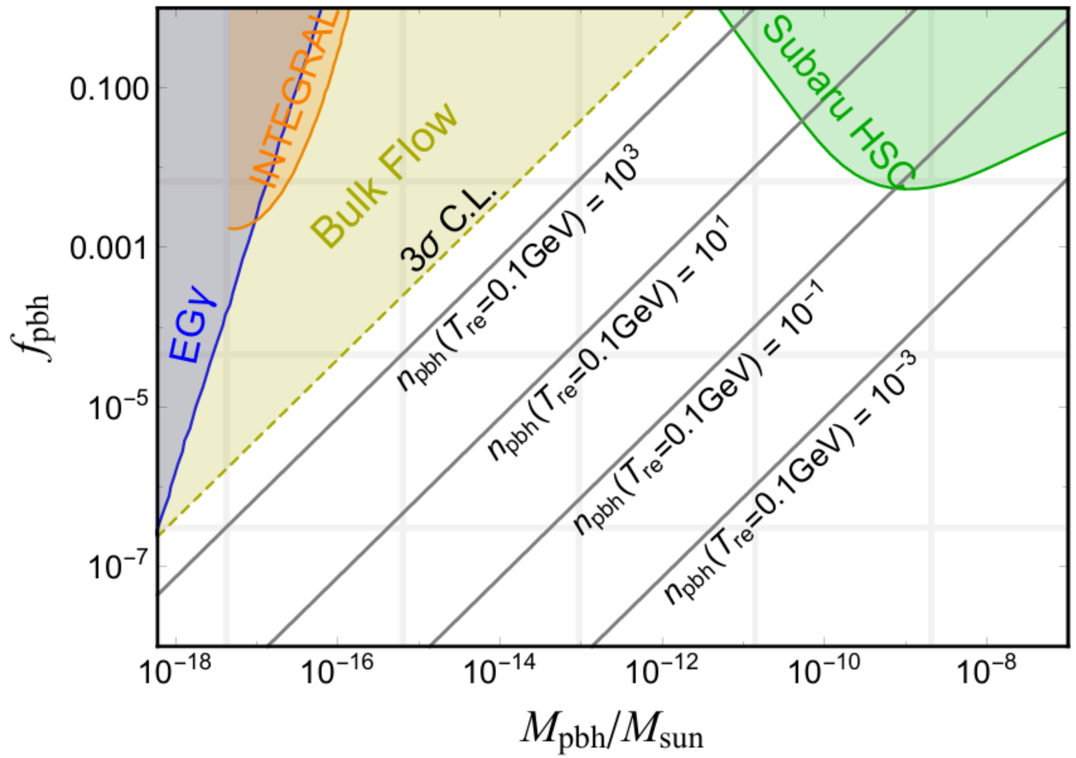
M_{PBH}	n_{pbh}
$10^{-14}M_{sun}$	10^{6}
$10^{-16}M_{sun}$	10^{8}

High number density

Suppressed GW signals

Insight from PTA data





Asteroid-mass PBH as whole dark matter

Incompatible

NanoHertz SGWB comes from PT GWs

Summary

- PBH can affect PT GWs through catalytic effect
- Asteroid-mass PBH as whole dark matter conflict with PT interpretation of PTA data

Thank you!

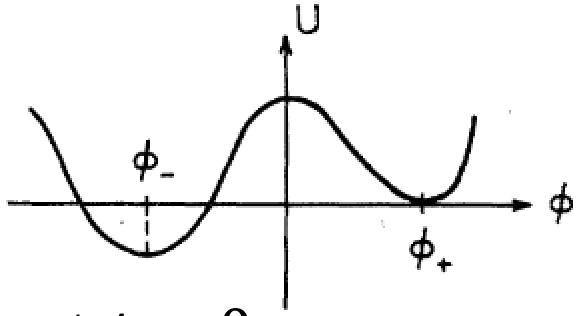
Instanton

Field equation at Euclidean space:

$$\left(\frac{\partial^2}{\partial\tau^2}+\nabla^2\right)\phi=+U'(\phi)$$

Bounce solution:

$$\phi(\tau=\pm\infty)=\phi_+$$



Boundary choose to be at $t=0$

$$\frac{d\phi}{d\tau}(t=0)=0$$

$O(4)$ symmetry: $\rho=\sqrt{x^2+\tau^2}$

$$\frac{d^2\phi}{d\rho^2}+\frac{3}{\rho}\frac{d\phi}{d\rho}=U'(\phi)$$

$$\phi(\rho=\infty)=\phi_+,\ \frac{d\phi}{d\rho}(\rho=0)=0$$

$$B=S_E=2\pi^2\int_0^\infty\rho^3 d\rho\left[\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2+U\right]$$

$$\Gamma/\mathrm{V}=A\exp\left(-\frac{B}{\hbar}\right)(1+O(\hbar))$$

Thermal correction to potential: EWPT as an example

Masses depend on Higgs, e.g. $m_h^2 \sim \lambda(3h^2 - v_{ew}^2), m_f \sim g_Y h$.

$V = V_0(h) + f_{plasma}(h,T)$ Free energy of thermal particle in equilibrium

Coleman-Weinberg potential

$f_{plasma} = T^4 \left[\sum_B J_B\left(\frac{M_B}{T}\right) + \sum_F J_F\left(\frac{M_F}{T}\right)\right]$, B, F denote for bosons and fermions

$$J_B\left(\frac{m}{T}\right) = -\frac{\pi^2}{90} + \frac{1}{24}\left(\frac{m}{T}\right)^2 - \frac{1}{2(4\pi)^2}\left(\frac{m}{T}\right)^4\left(ln\left(\frac{1}{\pi}\frac{m}{T}\,e^{\gamma_E}\right) - \frac{3}{4}\right) + O\left(\left(\frac{m}{T}\right)^6\right),$$

$m = m(h)$

$$J_F\left(\frac{m}{T}\right) = -\frac{7}{8}\frac{\pi^2}{90} + \frac{1}{48}\left(\frac{m}{T}\right)^2 - \frac{1}{2(4\pi)^2}\left(\frac{m}{T}\right)^4\left(ln\left(\frac{1}{\pi}\frac{m}{T}\,e^{\gamma_E}\right) - \frac{3}{4}\right) + O\left(\left(\frac{m}{T}\right)^6\right),$$

Finite-temperature field theory

$$V = \frac{\boldsymbol{D}}{2}(T^2 - \boldsymbol{a})h^2 - \frac{\boldsymbol{A}}{3}(T + \boldsymbol{b})h^3 + \frac{\lambda}{4}h^4 - g_{eff}\frac{\pi^2}{90}T^4$$

precise formula also including **thermal loop** contribute have impact on $\boldsymbol{D}, \boldsymbol{a}, \boldsymbol{A}, \boldsymbol{b}$

GW Spectrum Shape

GWs from FOPT

Sources:

- Sound Waves
- Bubble collision
- magnetohydrodynamic (MHD) turbulence

Typical frequency: $f_\star \sim \beta \sim H_\star \dfrac{\beta}{H_\star}$

$$f_0 = \frac{a(t_\star)}{a(t)} f_\star \sim 10^{-5} \left(\frac{g_\star(T_\star)}{106}\right)^{\frac{1}{6}} \left(\frac{T_\star}{100\text{GeV}}\right) \frac{\beta}{H_\star}\ \text{Hz}$$