How do the “basis vectors”, describing the internal spaces of fermion and boson fields with the odd (for fermion) and even (for boson) products of γ^a’s, explain all the observed second quantised fermion and boson fields and the interactions among fields, with the gravity included.

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Bled 4-17 of July, 2025 Bled Slovenia

July 5, 2025

- The odd and even "basis vectors", the superposition of odd and even products of γ^a's,
- describing the internal spaces of fermion and boson fields, respectively,
- offer in even-dimensional spaces like in $d = (13 + 1)$, the description of internal spaces of quarks and leptons and, antiquarks and antileptons appearing in families,
- as well as of all the corresponding gauge fields: photons, weak bosons, gluons, Higgs's scalars and gravitons,
- which not only explain all the assumptions of the *standard model*, and make several predictions,
- but also explains the existence of the graviton gauge fields,
- explaining the second quantization postulates for fermion and boson fields.

- **The theory offers, representing the Feynman diagrams, an elegant and promising illustration of the interaction between fermion and boson second-quantised fields.**
- **Leaving several open questions, like:**
  Can the theory of fermion, and boson second quantised fields, with the internal spaces described by "basis vectors", extended to strings, or to odd dimensional spaces, achieve renormalizability?

Although it looks like that we know almost everything about our Universe, due to the fact that the theories and models are to high extend supported by the experiments and the cosmological observations,
it is also true that we do not know why and how the universe has started,
what caused the exponential grow of the size of the universe,
what is happening in the black holes,
we namely do not know how to treat the second quantized gravity.

More than **50 years ago** the **electroweak (and colour) standard model** offered an **elegant new step** in **understanding the origin of fermions and bosons** by **postulating**, **supported by theories, models, experiments and cosmological observations**:

- *** The existence** of **massless family members** with the **charges** in the **fundamental representation of the groups** -
  o the **coloured triplet quarks and colourless leptons**,
  o the **left handed members as the weak charged doublets**,
  o the **right handed weak chargeless members**,
  o the **left handed quarks distinguishing in the hyper charge from the left handed leptons**,
  o each right handed member having a different hyper charge.
  o * NO right handed neutrinos and NO left handed antineutrinos.
- **The existence** of **massless families .**

- **The existence** of **massless vector gauge fields** to the observed **charges** of the **fermion family members**,
  o Three massless vector fields, **the gauge fields of the three charges**.
  They all are vectors in $d = (3 + 1)$, **in the adjoint representations with respect to the weak SU(2), colour SU(3) and hyper U(1) charges**.
- The **existence of a massive scalar field - the higgs**,
  o carrying the weak charge $\pm\frac{1}{2}$ and the hyper charge $\mp\frac{1}{2}$, breaking the weak and the hyper charge and correspondingly breaking the **mass protection**.
- The **existence** of the **Yukawa couplings**, taking care of
  o the properties of **fermions** and
  o the masses of the **heavy bosons**.

- There is the gravitational field in d=(3+1), determined by the vielbeins and spin connections, with the space index in (d=3+1).
- There are the Dirac prescriptions for the second quantized fermion and boson fields.
- There are several trials to explain the appearance of families of quarks and leptons.
- There are several trials to explain the appearance of the inflation of the universe.
- There are several trials to make a next step beyond both standard models, electroweak and cosmological.
- There are several trials to make the theories renormalizable, without anomalies.

- **The *standard model* assumptions have been confirmed without offering surprises.**
- **The last unobserved field as a field, the Higgs's scalar, detected in June 2012, was confirmed in March 2013.**
- **The waves of the gravitational field were detected in February 2016 and again 2017.**

The assumptions of the *standard model* remain unexplained.

- **There are several cosmological observations which do not look to be explainable within the** *standard model*,
- the **second quantization of fermion and boson fields are postulated**,
- the **second quantization of the gravitational field is not yet even postulated**,
- the **used groups used in the standard model are postulated**,
- ...

- **The** *standard model* **assumptions have in the literature several explanations, mostly with new assumptions made.**
- The **string theorists promise the theory offering the understanding the nature.**
- **We might agree that the elementary constituents are two kinds of fields: Anti-commuting fermion and commuting boson fields, both assumed to be second quantized fields.**

- **Let us present a new trial, describing the internal spaces of fermion and boson fields in an unique way, with the odd "basis vectors" (superposition of odd products of γ^a's for fermions) and**

  even "basis vectors" (superposition of even products of γ^a's for bosons),

  explaining properties of all fields, with gravitons included.

The idea started 1992.

First, it was applied on fermion fields, recognizing that knowing the "basis vectors" describing the internal space of fermions, which appear in irreducible representations representing families.

(It is easy to find the Pauli matrices in our way of describing **fermions**, in any even dimension, although Pauli matrices are in our way of describing **fermions**, not needed.)

The idea that
fermions and bosons
can be described in an equivalent way,
although fermions appear in families and have their
Hermitian conjugated partners in a separate group,
while bosons appear in two separate groups,

become clear only around three years ago.

- **In the unique description of boson and fermion second quantized fields, with odd “basis vectors” for fermions** and **with even “basis vectors” for bosons,**

  there are the same number of fermion and boson “basis vectors” and their Hermitian conjugated partners, manifesting a kind of supersymmetry.

- **The internal space, involved in creating our universe, has** $d \geq (13+1)$ and **the ordinary space is active in** $d = (3+1)$ **and no symmetry is broken, both “basis vectors” – and their Hrmitian conjugated partners — have the same number of states.**

- **The Spin-Charge-Family theory, using this description,**

  offers the explanation for all the assumptions of the *standard model*,

  including the second quantisation postulates of Dirac,

  the appearance of the scalar fields, explaining several cosmological observations,
  like the **dark matter**,
  the **matter-antimatter asymmetry,**

  making several predictions, like the existence of the **fourth family to the observed three,**
  offering explanation for the appearance of the graviton.

How does this proposed theory — called the spin-charge-family — proceeds?

- **Making a choice that all "basis vectors" are eigenvectors of all chosen Cartan subalgebra members,**

$$\mathbf{S}^{03}, \mathbf{S}^{12}, \mathbf{S}^{56}, \cdots, \mathbf{S}^{d-1\,d},$$
$$\tilde{\mathbf{S}}^{03}, \tilde{\mathbf{S}}^{12}, \tilde{\mathbf{S}}^{56}, \cdots, \tilde{\mathbf{S}}^{d-1\,d},$$
$$\mathbf{S}^{ab} = \mathbf{S}^{ab} + \tilde{\mathbf{S}}^{ab}.$$

and **arrange the "basis vectors" to be products** of **nilpotents and projectors**

$$\overset{ab}{(\mathbf{k})} := \frac{1}{2}(\gamma^{a} + \frac{\eta^{aa}}{ik}\gamma^{b}),$$
$$\overset{ab}{[\mathbf{k}]} := \frac{1}{2}(1 + \frac{i}{k}\gamma^{a}\gamma^{b}),$$

then **"basis vectors" of fermions have an odd number of nilpotents** , and **"basis vectors" of bosons have an even number of nilpotents** .

Let me first represent in terms of nilpotents and projectors one octet of colourless leptons $((\tau^3 = 0, \tau^8 = 0))$:

i		$\|{}^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(7,1)} = 1,\ \Gamma^{(6)} = -1,$ of leptons							
1	ν_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \| \stackrel{56}{(+)}\stackrel{78}{(+)} \|\| \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
2	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \| \stackrel{56}{(+)}\stackrel{78}{(+)} \|\| \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
3	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \| \stackrel{56}{[-]}\stackrel{78}{[-]} \|\| \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
4	e_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \| \stackrel{56}{[-]}\stackrel{78}{[-]} \|\| \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
5	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \| \stackrel{56}{[-]}\stackrel{78}{(+)} \|\| \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
6	e_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \| \stackrel{56}{[-]}\stackrel{78}{(+)} \|\| \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
7	ν_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \| \stackrel{56}{(+)}\stackrel{78}{[-]} \|\| \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
8	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \| \stackrel{56}{(+)}\stackrel{78}{[-]} \|\| \stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

All leptons have an odd number of nilpotents the rest are projectors.
Each nilpotent and each projector is the eigenvector of one of the Cartan subalgebra members, so that each nilpotent is the eigenvector of all Cartan subalgebra members.

Before explaining the details of the theory, let me show how is presented one of weak bosons,

$$ {}^{I}\hat{\mathcal{A}}^{\dagger}_{\mathbf{weak}\,\nu_{\mathbf{R}}\to \mathbf{e_R}}(\equiv \stackrel{03}{[+\mathbf{i}]}\stackrel{12}{[+]}\stackrel{56}{(-)}\stackrel{78}{(-)}\stackrel{9\,10}{[+]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}) $$

The weak boson has an even number of nilpotents.

Applying on ν_L transforms it to e_L, offering it an integer weak charge.

- These description is elegant and simple to use.
- Analysing the "basis vectors" with respect to the symmetry of the Standard Model groups: $SO(3,1)\times SU(2)\times SU(2)\times SU(3)\times U(1)$,
  they manifest in $d=(3+1)$ all the quantum numbers of the second quantized boson fields observed in $d=(3+1)$, and all the quantum numbers of the second quantized fermion fields observed in $d=(3+1)$, with the second quantized graviton fields included.
- Choosing the simplest action for fermions and bosons, we can describe properties of the so far observed boson and fermion fields.
- Knowing the fermion and boson states, it is easy to calculate Pauli matrices in any dimension, although it is not needed, it is easier to work with states.
- There are open problems waiting to be solved.

- **Let us make a brief introduction into the spin-charge-family theory.**

- There are **two kinds of the Clifford algebra objects in any d . I recognized that in Grassmann space.**

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  θ^a**'s and** p^{θ}_a**'s**, $p^{\theta}_a = \frac{\partial}{\partial \theta_a}$

  with the property

  $(\theta^a)^\dagger = \eta^{aa} \frac{\partial}{\partial \theta_a}$.

$$\{\theta^a, \theta^b\}_+ = 0\,, \qquad \{\frac{\partial}{\partial \theta_a}, \frac{\partial}{\partial \theta_b}\}_+ = 0\,,$$

$$\{\theta_a, \frac{\partial}{\partial \theta_b}\}_+ = \delta_{ab}\,,\ (a,b) = (0,1,2,3,5,\cdots,d)\,.$$

Making a choice

$$(\theta^a)^\dagger = \eta^{aa} \frac{\partial}{\partial \theta_a}\,, \quad \text{leads to} \quad (\frac{\partial}{\partial \theta_a})^\dagger = \eta^{aa} \theta^a\,,$$

with $\eta^{ab} = diag\{1,-1,-1,\cdots,-1\}$.

i. The **Dirac** γ^a (recognized 90 years ago in $d = (3+1)$).

ii. The **second one:** $\tilde{\gamma}^a$,

$$\gamma^a = (\theta^a - i\, p^{\theta a})\,, \quad \tilde{\gamma}^a = i\,(\theta^a + i\, p^{\theta a})\,,$$

- The two kinds of the **Clifford algebra objects** anticommute as follows

$$\{\gamma^{\mathbf{a}}, \gamma^{\mathbf{b}}\}_{+} = 2\eta^{\mathbf{ab}} = \{\tilde{\gamma}^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_{+},$$
$$\{\gamma^{\mathbf{a}}, \tilde{\gamma}^{\mathbf{b}}\}_{+} = 0,$$

- the **postulate**

$$(\tilde{\gamma}^{\mathbf{a}}\mathbf{B} = \mathbf{i}(-)^{\mathbf{n_B}}\mathbf{B}\gamma^{\mathbf{a}})\,|\psi_0>,$$
$$(\mathbf{B} = a_0 + a_a\gamma^a + a_{ab}\gamma^a\gamma^b + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\ldots\gamma^{a_d})|\psi_o>$$

with $(-)^{n_B} = +1, -1$, if B has a Clifford even or odd character, respectively, $|\psi_o>$ is a vacuum state on which the operators γ^a **apply**,
reduces the Clifford space for fermions for the factor of two, from 2×2^d **to** 2^d,
while the operators $\tilde{\gamma}^a\tilde{\gamma}^b = -2i\tilde{S}^{ab}$ **define the family quantum numbers**.

- It is convenient to write all the **"basis vectors", describing the internal space of either fermion fields or boson fields as products of nilpotents and projectors, which are eigenvectors of the chosen Cartan subalgebra**

$$S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d},$$
$$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d},$$
$$\mathbf{S}^{ab} = S^{ab} + \tilde{S}^{ab}.$$

nilpotents

$$S^{ab}\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b) = \frac{k}{2}\,\frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b), \quad \stackrel{\mathbf{ab}}{(\mathbf{k})} := \frac{\mathbf{1}}{\mathbf{2}}(\gamma^{\mathbf{a}} + \frac{\eta^{\mathbf{aa}}}{\mathbf{ik}}\gamma^{\mathbf{b}}),$$

projectors

$$S^{ab}\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b) = \frac{k}{2}\,\frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b), \quad \stackrel{\mathbf{ab}}{[\mathbf{k}]} := \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{1} + \frac{\mathbf{i}}{\mathbf{k}}\gamma^{\mathbf{a}}\gamma^{\mathbf{b}}),$$

$$(\stackrel{\mathbf{ab}}{(\mathbf{k})})^2 = \mathbf{0}, \qquad (\stackrel{\mathbf{ab}}{[\mathbf{k}]})^2 = \stackrel{\mathbf{ab}}{[\mathbf{k}]},$$

$$\stackrel{\mathbf{ab}}{(\mathbf{k})}^{\dagger} = \eta^{\mathbf{aa}}\stackrel{\mathbf{ab}}{(-\mathbf{k})}, \quad \stackrel{\mathbf{ab}}{[\mathbf{k}]}^{\dagger} = \stackrel{\mathbf{ab}}{[\mathbf{k}]}.$$

$$
\begin{aligned}
S^{ab} \stackrel{ab}{(k)} &= \frac{k}{2} \stackrel{ab}{(k)}, \quad S^{ab} \stackrel{ab}{[k]} = \frac{k}{2} \stackrel{ab}{[k]}, \\
\tilde{S}^{ab} \stackrel{ab}{(k)} &= \frac{k}{2} \stackrel{ab}{(k)}, \quad \tilde{S}^{ab} \stackrel{ab}{[k]} = -\frac{k}{2} \stackrel{ab}{[k]}.
\end{aligned}
$$

- γ^a **transforms** $\stackrel{ab}{(k)}$ **into** $\stackrel{ab}{[-k]}$**, never to** $\stackrel{ab}{[k]}$**.**
- $\tilde{\gamma}^a$ **transforms** $\stackrel{ab}{(k)}$ **into** $\stackrel{ab}{[k]}$, **never to** $\stackrel{ab}{[-k]}$**.**

$$
\begin{aligned}
\gamma^a \stackrel{ab}{(k)} &= \eta^{aa} \stackrel{ab}{[-k]}, \gamma^b \stackrel{ab}{(k)} = -ik \stackrel{ab}{[-k]}, \gamma^a \stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, \gamma^b \stackrel{ab}{[k]} = -ik\eta^{aa} \stackrel{ab}{(-k)}, \\
\tilde{\gamma}^a \stackrel{ab}{(k)} &= -i\eta^{aa} \stackrel{ab}{[k]}, \tilde{\gamma}^b \stackrel{ab}{(k)} = -k \stackrel{ab}{[k]}, \tilde{\gamma}^a \stackrel{ab}{[k]} = i \stackrel{ab}{(k)}, \tilde{\gamma}^b \stackrel{ab}{[k]} = -k\eta^{aa} \stackrel{ab}{(k)}, \\
\stackrel{ab}{(k)} \stackrel{ab}{(-k)} &= \eta^{aa} \stackrel{ab}{[k]}, \stackrel{ab}{[k]} \stackrel{ab}{(k)} = \stackrel{ab}{(k)}, \stackrel{ab}{(k)} \stackrel{ab}{[-k]} = \stackrel{ab}{(k)}, ** \\
\stackrel{ab}{(k)} \stackrel{ab}{[k]} &= 0, \stackrel{ab}{[k]} \stackrel{ab}{(-k)} = 0, \stackrel{ab}{[k]} \stackrel{ab}{[-k]} = 0, **
\end{aligned}
$$

- There are the Clifford odd "basis vectors", that is the "basis vectors" with an odd number of nilpotents, at least one, the rest are projectors, such "basis vectors" anti commute among themselves.
- There are the Clifford even "basis vectors", that is the "basis vectors" with an even number of nilpotents, the rest are projectors, such "basis vectors" commute among themselves.
- There are the $2^{\frac{d}{2}-1}$ Clifford odd "basis vectors" appearing in $2^{\frac{d}{2}-1}$ families and the same number of their Hermitian conjugated partners, $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$.
- There are $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ Clifford even "basis vectors" appearing in two orthogonal groups.

- **Let us see how does one family of the Clifford odd "basis vector" in $d = (13+1)$ look like, if spins in $d = (13+1)$ are analysed with respect to the Standard Model groups: $SO(3,1)\times$ $SU(2)\times SU(2)\times$ $SU(3)\times U(1)$.**

- **One irreducible representation** , this means one **family, contains $2^{\frac{(13+1)}{2}-1} = 64$ members which include all the family members, quarks and leptons with the right handed neutrinos included, as well as all the antimembers, antiquarks and antileptons, reachable by either S^{ab} (or by $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ on a family member)**.

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S^{ab} generate **all the members of one family**. The **eightplet** (represent. of $SO(7,1)$) **of quarks of a particular colour charge. All are Clifford odd "basis vectors"** , with $SU(3) \times U(1)$ **part** **($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and $\tau^{41} = 1/6$)**

i		$\|^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Octet, $\Gamma^{(7,1)} = 1$, $\Gamma^{(6)} = -1$, of quarks							
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \| \stackrel{56}{(+)}\stackrel{78}{(+)} \|\| \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \| \stackrel{56}{(+)}\stackrel{78}{(+)} \|\| \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \| \stackrel{56}{[-]}\stackrel{78}{[-]} \|\| \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \| \stackrel{56}{[-]}\stackrel{78}{[-]} \|\| \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \| \stackrel{56}{[-]}\stackrel{78}{(+)} \|\| \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \| \stackrel{56}{[-]}\stackrel{78}{(+)} \|\| \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \| \stackrel{56}{(+)}\stackrel{78}{[-]} \|\| \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \| \stackrel{56}{(+)}\stackrel{78}{[-]} \|\| \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\mathbf{u_R}$ of the 1^{st} row into $\mathbf{u_L}$ of the 7^{th} row, and $\mathbf{d_R}$ of the 4^{rd} row into $\mathbf{d_L}$ of the 6^{th} row, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} **generate** all the **members of one family of quarks, leptons, antiquarks, antileptons. Here is the eightplet (represent. of** $SO(7,1)$**) of the colour chargeless leptons.**

The $SO(7,1)$ part is identical with the one of quarks, while the $SU(3)\times U(1)$ part is: $\tau^{33}=0$, $\tau^{38}=0$, $\tau^{41}=-\frac{1}{2}$.

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	Q
		Octet, $\Gamma^{(7,1)}=1,\ \Gamma^{(6)}=-1,$ of leptons							
1	$\nu_{\mathbf{R}}$	$\stackrel{03}{(+\mathbf{i})}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	**1**	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
2	ν_R	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	**1**	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0
3	e_R	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	**1**	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
4	$\mathbf{e_R}$	$\stackrel{03}{[-\mathbf{i}]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	**1**	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	-1	-1
5	e_L	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	**-1**	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
6	$\mathbf{e_L}$	$\stackrel{03}{(+\mathbf{i})}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	**-1**	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
7	$\nu_{\mathbf{L}}$	$\stackrel{03}{[-\mathbf{i}]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	**-1**	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
8	ν_L	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}$	**-1**	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0

$\gamma^{\mathbf{0}}\gamma^{\mathbf{7}}$ and $\gamma^{\mathbf{0}}\gamma^{\mathbf{8}}$ transform $\nu_{\mathbf{R}}$ of the 1^{st} line into $\nu_{\mathbf{L}}$ of the 7^{th} line, and $\mathbf{e_R}$ of the 4^{rd} line into $\mathbf{e_L}$ of the 6^{th} line, doing what the Higgs scalars and γ^0 do in the *standard model*.

S^{ab} **generate also all the anti-eightplet** (repres. of $SO(7,1)$) of **anti-quarks of the anti-colour charge belonging to the same family of the Clifford odd basis vectors** . **(**$\tau^{33} = -1/2$, $\tau^{38} = -1/(2\sqrt{3})$, $\tau^{41} = -1/6$**).**

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	Y	τ^4
		Antioctet, $\Gamma^{(7,1)} = -1$, $\Gamma^{(6)} = 1$, of antiquarks							
33	$\bar{\mathbf{d}}^{\bar{\mathbf{c}}1}_{\mathbf{L}}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
34	$\bar{d}^{\bar{c}1}_{L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
35	$\bar{u}^{\bar{c}1}_{L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \mid \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
36	$\bar{\mathbf{u}}^{\bar{\mathbf{c}}1}_{\mathbf{L}}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \mid \stackrel{56}{[-]}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	- 1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$
37	$\bar{\mathbf{d}}^{\bar{\mathbf{c}}1}_{\mathbf{R}}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
38	$\bar{d}^{\bar{c}1}_{R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \mid \stackrel{56}{(+)}\stackrel{78}{[-]} \Vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
39	$\bar{u}^{\bar{c}1}_{R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$
40	$\bar{\mathbf{u}}^{\bar{\mathbf{c}}1}_{\mathbf{R}}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \mid \stackrel{56}{[-]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{[-]}\;\stackrel{11\,12}{[+]}\;\stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$-\frac{1}{6}$

$\gamma^0\gamma^7$ and $\gamma^0\gamma^8$ transform $\bar{\mathbf{d}}_{\mathbf{L}}$ of the 1^{st} line into $\bar{\mathbf{d}}_{\mathbf{R}}$ of the 5^{th} line, and $\bar{\mathbf{u}}_{\mathbf{L}}$ of the 4^{rd} line into $\bar{\mathbf{u}}_{\mathbf{R}}$ of the 8^{th} line.

- The **Clifford odd "basis vector"** describing the internal space of **quark** $u^{c1\dagger}_{\uparrow R}, \Leftrightarrow b^{1\dagger}_{1} :=$
$\stackrel{03}{(+i)}\stackrel{12}{[+]}\,|\,\stackrel{56}{[+]}\stackrel{78}{(+)}\,||\,\stackrel{9\,10}{(+)}\,\stackrel{11\,12}{[-]}\,\stackrel{13\,14}{[-]}\,,$
has the Hermitian conjugated partner equal to
$u^{c1}_{\uparrow R} \Leftrightarrow (\mathbf{b}^{1\dagger}_{1})^{\dagger} = \stackrel{13\,14}{[-]}\,\stackrel{11\,12}{[-]}\,\stackrel{9\,10}{(-)}\,||\,\stackrel{78}{(-)}\stackrel{56}{[+]}\,|\,\stackrel{12}{[+]}\stackrel{03}{(-\mathbf{i})}$, both with an odd number of nilpotents,
both are the Clifford odd objects — forming two separate groups.

Anti-commutation relations for Clifford odd "basis vectors", representing the internal space of fermion fields of quarks and leptons ($i = (u^{c,f,\uparrow,\downarrow}_{R,L}, d^{c,f,\uparrow,\downarrow}_{R,L}, \nu^{f,\uparrow,\downarrow}_{R,L}, e^{f,\uparrow,\downarrow}_{R,L})$), and anti-quarks and anti-leptons, with the family quantum number f.

- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = \delta_{\mathbf{f}\,\mathbf{f}'}\,\delta^{\mathbf{mk}}\,|\psi_\mathbf{o}>$,
- $\{\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}}_{\mathbf{f}'}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\{\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}, \mathbf{b}^{\mathbf{k}\dagger}_{\mathbf{f}'}\}_{*_\mathbf{A}+}|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\mathbf{b}^{\mathbf{m}}_{\mathbf{f}}\,|\psi_\mathbf{o}> = 0\cdot|\psi_\mathbf{o}>$,
- $\mathbf{b}^{\mathbf{m}\dagger}_{\mathbf{f}}\,|\psi_\mathbf{o}> = |\psi^{\mathbf{m}}_{\mathbf{f}}>$,

$$|\psi_\mathbf{o}> = \stackrel{03}{[-\mathbf{i}]}\stackrel{12}{[-]}\stackrel{56}{[-]}\cdots\stackrel{13\,14}{[-]}\;|\mathbf{1}>$$

define the vacuum state for quarks and leptons and antiquarks and antileptons of the family $\mathbf{f}$.

- The **Clifford even "basis vectors"**, having an even number of nilpotents, describe the internal space of the corresponding **boson** field. The **gluon** field, for example, ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}$, which transforms the u_R^{c1} into u_R^{c2} looks like: ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}$ $(\equiv \stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{(-)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]})$.
  If it algebraically multiplies on u_R^{c1} $(\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]})$ it follows

$$
{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c1}\to u_R^{c2}}(\equiv \stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{(-)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}) *_A
$$

$$
u_R^{c1\dagger}, (\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) \rightarrow
$$

$$
u_R^{c2\dagger}, (\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{(+)}\stackrel{9\,10}{[-]}\stackrel{11\,12}{(+)}\stackrel{13\,14}{[-]}),
$$

It follows

$$
{}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger}_{\mathbf{gl\; u_R^{c1}\to u_R^{c2}}} = \mathbf{u_R^{c2\dagger}} *_{\mathbf{A}} (\mathbf{u_R^{c1\dagger}})^{\dagger},
$$

$$
{}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\; u_R^{c2}\to u_R^{c1}} (\equiv \stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[+]}\stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{[-]}\,) *_A \; u_R^{c2\dagger} \to u_R^{c1\dagger} \,,
$$

$$
{}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger}_{\mathbf{gl}\; \mathbf{u_R^{c2}}\to \mathbf{u_R^{c1}}} = \mathbf{u_R^{c1\dagger}} \; *_{\mathbf{A}} \; (\mathbf{u_R^{c2\dagger}})^{\dagger} \,.
$$

Knowing the "basis vectors" of fermions we know also the "basis vectors" of their gauge fields bosons.

- These gluons ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gl\, u_R^{ci} \to u_R^{cj}} = u_R^{cj\dagger} *_A (u_R^{ci\dagger})^{\dagger}$ transform quarks of a particular colour charge to quarks the rest colour charges.

  Let us notice that they all are expressed as the algebraic product of a family member and one of the Hermitian conjugated partner.

- We can in an equivalent way express the weak boson ${}^{I}\hat{\mathcal{A}}^{\dagger}_{weak\, u_R^{ci} \to d_R^{ci}} = d_R^{ci\dagger} *_A (u_R^{ci\dagger})^{\dagger}$ transforming quarks of a particular weak charge to quarks of another weak charge (keeping the colour charges unchanged).

- **Let us look for a**

  photon ${}^{I}\hat{\mathcal{A}}^{\dagger}_{ph\,\nu_L \to \nu_L} = \nu^{\dagger}_{L} *_A (\nu^{\dagger}_{L})^{\dagger}$,

  able (carrying no charges) to transfer the momentum in ORDINARY space to $\nu^{\dagger}_{L}$**,** but not in internal space, since **we find all the quantum numbers — that is all the eigenvalues of the Cartan subalgebra members — equal zero:**

$$\mathbf{S}^{ab} = S^{ab} + \tilde{S}^{ab}\,.$$

while

$$\mathbf{S}^{ab} \overset{ab}{(\mathbf{k})} = \frac{k}{2}\overset{ab}{(\mathbf{k})}, \quad \mathbf{S}^{ab} \overset{ab}{[\mathbf{k}]} = \frac{k}{2}\overset{ab}{[\mathbf{k}]}, \quad \tilde{\mathbf{S}}^{ab}\overset{ab}{(\mathbf{k})} = \frac{k}{2}\overset{ab}{(\mathbf{k})}, \quad \tilde{\mathbf{S}}^{ab}\overset{ab}{[\mathbf{k}]} = -\frac{k}{2}\overset{ab}{[\mathbf{k}]}\,.$$

$$\text{the } \mathbf{photon}\,{}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger}_{\mathbf{ph}\,\nu_{\mathbf{L}} \to \nu_{\mathbf{L}}} (\equiv \overset{03}{[-\mathbf{i}]}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{[-]}\overset{9\,10}{[+]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}),$$

$$\text{the } \mathbf{photon}\,{}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger}_{\mathbf{ph}\,\mathbf{e}^{-}_{\mathbf{L}} \to \mathbf{e}^{-}_{\mathbf{L}}} (\equiv \overset{03}{[-\mathbf{i}]}\overset{12}{[+]}\overset{56}{[-]}\overset{78}{[+]}\overset{9\,10}{[+]}\;\overset{11\,12}{[+]}\;\overset{13\,14}{[+]}).$$

can not transfer any internal spin to fermions.

There is the second kind of the even "basis vectors", having as well an even number of nilpotents, and consequently commute, describing the internal space of boson fields; they are orthogonal to all ${}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$.

- We call them ${}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$.
  ${}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$ transform a family member of a particular family of fermions to the same family member of all the rest families of quarks and leptons and antiquarks and antileptons.

  Let $e^{-\dagger}_{L\uparrow f=1}$ be $(\equiv\overset{03}{[-i]}\overset{12}{[+]}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})$, and $e^{-\dagger}_{L\uparrow f=2}$ be $(\equiv\overset{03}{(-)}\overset{12}{(+)}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)})$.

  It follows that ${}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$ apply fermions from the right hand side

  $$e^{-\dagger}_{L\uparrow f=2}\,(\equiv\overset{03}{(-)}\overset{12}{(+)}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}) \;*_A\; {}^{\mathbf{II}}\hat{\mathcal{A}}^{\dagger m}_{\mathbf{f}}$$
  $$(\equiv\overset{\mathbf{03}}{(+)}\overset{\mathbf{12}}{(-)}\overset{\mathbf{56}}{[+]}\,\overset{\mathbf{78}}{[-]}\overset{\mathbf{9\,10}}{[-]}\,\overset{\mathbf{11\,12}}{[-]}\;\overset{\mathbf{13\,14}}{[-]}\,) \rightarrow e^{-\dagger}_{L\uparrow f=1}$$
  $$(\equiv\overset{03}{[-i]}\overset{12}{[+]}\overset{56}{(-)}\overset{78}{(+)}\overset{9\,10}{(+)}\overset{11\,12}{(+)}\overset{13\,14}{(+)}).$$

  $$\gamma^{\mathbf{a}}\overset{\mathbf{ab}}{(\mathbf{k})} = \eta^{aa}\overset{\mathbf{ab}}{[-\mathbf{k}]}\,,\; \gamma^{\mathbf{a}}\overset{\mathbf{ab}}{[\mathbf{k}]} = \overset{\mathbf{ab}}{(-\mathbf{k})}\,,\; \tilde{\gamma}^{\mathbf{a}}\overset{\mathbf{ab}}{(\mathbf{k})} = -i\eta^{aa}\overset{\mathbf{ab}}{[\mathbf{k}]}\,,\; \tilde{\gamma}^{\mathbf{a}}\overset{\mathbf{ab}}{[\mathbf{k}]} = i\overset{\mathbf{ab}}{(\mathbf{k})}.$$
  $$\overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{(-\mathbf{k})} = \eta^{aa}\overset{\mathbf{ab}}{[\mathbf{k}]}\,,\; \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{(\mathbf{k})} = \overset{\mathbf{ab}}{(\mathbf{k})}\,,\; \overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{[-\mathbf{k}]} = \overset{\mathbf{ab}}{(\mathbf{k})}\,,\; \overset{\mathbf{ab}}{(\mathbf{k})}\overset{\mathbf{ab}}{[\mathbf{k}]} =$$
  $$\mathbf{0}\,,\; \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{(-\mathbf{k})} = \mathbf{0}\,,\; \overset{\mathbf{ab}}{[\mathbf{k}]}\overset{\mathbf{ab}}{[-\mathbf{k}]} = \mathbf{0}\,.$$

Also ${}^{II}\hat{\mathcal{A}}^{\dagger m}_{f}$ can be expressed as algebraic products of the Hermitian conjugate fermion fields and **one of the odd "basis vector'.**

- **photon** ${}^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{-\dagger}e^{-}} = (e_L^{-\dagger})^{\dagger} *_A e_L^{-\dagger} =$
  photon ${}^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{+\dagger}e^{+}} = (e_R^{+\dagger})^{\dagger} *_A e_R^{+\dagger}$

- **The members of each of these two groups have the property**

$$ {}^{i}\hat{\mathcal{A}}^{m\dagger}_{f} *_A {}^{i}\hat{\mathcal{A}}^{m'\dagger}_{f`} \rightarrow \begin{cases} {}^{i}\hat{\mathcal{A}}^{m\dagger}_{f`}\,, i=(I,II) \\ \qquad \text{or zero}\,. \end{cases} $$

$$ {}^{I}\hat{\mathcal{A}}^{m\dagger}_{f} *_A \hat{b}^{m'\dagger}_{f`} \rightarrow \begin{cases} \hat{b}^{m\dagger}_{f`}\,, \\ \text{or zero}\,, \end{cases} $$

$$ \hat{b}^{m\dagger}_{f} *_A {}^{II}\hat{\mathcal{A}}^{m'\dagger}_{f`} \rightarrow \begin{cases} \hat{b}^{m\dagger}_{f``}\,, \\ \text{or zero}\,, \end{cases} $$

- All bosons "basis vectors", ${}^{I}\hat{\mathcal{A}}^{m\dagger}_{f}$ and ${}^{II}\hat{\mathcal{A}}^{m\dagger}_{f}$ (describing internal spaces of boson fields) are expressible as algebraic products of "basis vectors" and their Hermitian conjugated partners as $\hat{b}^{m'\dagger}_{f`} *_A (\hat{b}^{m''\dagger}_{f``})^{\dagger}$ or as $(\hat{b}^{m'\dagger}_{f`})^{\dagger} *_A \hat{b}^{m''\dagger}_{f``}$.
- Knowing "basis vectors" of fermions appearing in families we know all the boson fields as well.
- This is true for "basis vectors" of scalars AND vectors.
- The vector gauge fields and the scalar gauge fields carry in a tensor products with the basis in ordinary space the space index α, which is for the vector gauge fields equal to $\alpha = \mu = (0, 1, 2, 3)$ and for scalars $\alpha = \sigma = (5, 6, 7, 8, ..., d)$.

In this talk, the internal spaces of all the boson gauge fields, vectors and scalars are discussed in order to try to understand the second quantized fermion and boson fields in this new way.
The even "basis vectors" are identical for vector gauge fields and the scalar gauge fields.

- The even "basis vectors" form in a tensor product with the basis in ordinary space, the creation and annihilation operators, which manifest the commuting properties of the second-quantised boson fields, explaining the second quantisation postulates for boson fields.
  The even "basis vectors" have all the properties of boson gauge fields: Integer spins for the Cartan subalgebra members of the Lorentz algebra in the internal space of bosons.
- They appear in two orthogonal groups, each with $2^{\frac{d}{2}-1} \times 2^{\frac{d}{2}-1}$ members for even d. Each group has their Hermitian conjugate partners within the same group.

- **The assumption that the internal spaces of fermion and boson fields are describable by the odd and even "basis vectors", respectively, leads to the conclusion that the internal space of gravitons must also be described by even "basis vectors",**

  We again arrange them to be the superposition of even products of γ^a**'s, arranged to be eigenvectors of all the Cartan subalgebra members and are arranged correspondingly to be products of an even number of nilpotents and the rest of projectors.**

- Let us, therefore, try to describe "gravitons" in an equivalent way as we do for the so far observed gauge fields that is, in terms of ${}^{i}\hat{\mathcal{A}}^{m\dagger}_{f}$, $i = I, II$, starting with $i = I$.

  We expect correspondingly that "gravitons" have no weak and no colour "charges", as also photons do not have any charge from the point of view of $d = (3+1)$. However, "gravitons" can have the spin and handedness (non-zero $\mathcal{S}^{03}$ and $\mathcal{S}^{12}$) in $d = (3+1)$.

- As all the boson gauge fields, manifesting in $d = (3+1)$ as the vector gauge fields of the corresponding quarks and leptons and antiquarks and antileptons, also the creation operators for "gravitons" must carry the space index α, describing the α component of the creation operators for "gravitons" in the ordinary space. Since we pay attention to the vector gauge fields in $d = (3+1)$ α must be $\mu = (0, 1, 2, 3)$.

We find that the **"basis vector"** of the **"graviton"** ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\, u_R^{c1\dagger} \to u_R^{c1\dagger}}$ applies on $u_R^{c1\dagger}$ as follows

$$
{}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger}_{\mathbf{gr}\, \mathbf{u}^{\mathbf{c1}\dagger}_{\mathbf{R},\uparrow} \to \mathbf{u}^{\mathbf{c1}\dagger}_{\mathbf{R},\downarrow}} (\equiv \overset{03}{(-\mathbf{i})} \overset{12}{(-)} \overset{56}{[+]} \overset{78}{[+]} \overset{9\,10}{[+]} \overset{11\,12}{[-]} \overset{13\,14}{[-]}) *_{\mathbf{A}}
$$

$$
\mathbf{u}^{\mathbf{c1}\dagger}_{\mathbf{R},\uparrow} (\equiv \overset{03}{(+\mathbf{i})} \overset{12}{[+]} \overset{56}{[+]} \overset{78}{(+)} \overset{9\,10}{(+)} \overset{11\,12}{[-]} \overset{13\,14}{[-]}) \to
$$

$$
\mathbf{u}^{\mathbf{c1}\dagger}_{\mathbf{R},\downarrow}, (\equiv \overset{03}{[-\mathbf{i}]} \overset{12}{(-)} \overset{56}{[+]} \overset{78}{(+)} \overset{9\,10}{(+)} \overset{11\,12}{[-]} \overset{13\,14}{[-]}),
$$

$$
{}^{\mathbf{I}}\hat{\mathcal{A}}^{\dagger}_{\mathbf{gr}\, \mathbf{u}^{\mathbf{c1}\dagger}_{\mathbf{R},\uparrow} \to \mathbf{u}^{\mathbf{c1}\dagger}_{\mathbf{R},\downarrow}} = \mathbf{u}^{\mathbf{c1}\dagger}_{\mathbf{R},\downarrow} *_{\mathbf{A}} (\mathbf{u}^{\mathbf{c1}\dagger}_{\mathbf{R},\uparrow})^{\dagger},
$$

The creation operators for "gravitons" must carry the space index α, describing the α component of the creation operators for "gravitons" in the ordinary space. Since we pay attention to the vector gauge fields in $d = (3+1)$ α must be $\mu = (0, 1, 2, 3)$.

The creation operator for "graviton" manifesting in $d = (3+1)$ must correspondingly be of the kind

$$ {}^{I}\hat{\mathcal{A}}^{\dagger}_{\mathbf{gr}\,\mu} = \overset{03}{(\pm \mathbf{i})}\overset{12}{(\pm)}\overset{56}{[\pm]} \ldots \overset{11\,12}{[\pm]}\ \overset{13\,14}{[\pm]}\, {}^{I}\mathcal{C}_{\mathbf{gr}\,\mu}\,, $$

with the only nilpotents in the first two factors, with eigenvalues of S^{03} and S^{12} equal to $\pm i$ and ± 1, respectively, while all the rest of the factors are projectors with the corresponding eigenvalues of the Cartan subalgebra members equal zero, $\mathcal{S}^{ab} = 0$.

The "basis vectors" of "gravitons" can offer to fermions on which they apply the integer spin $\mathcal{S}^{12} = \pm 1$ and $\mathcal{S}^{03} = \pm i$.

The product of two "basis vectors" of "gravitons" can lead to the "basis vector" with only projectors, or to zero.

The product of two "basis vectors" of "gravitons" leads to the object being indistinguishable from the "basis vectors" of photons.

"Gravitons" ${}^{I}\hat{\mathcal{A}}^{\dagger}_{gr\,\mu}$, like photons, weak bosons and gluons offer fermions also the momentum in ordinary space.

Feynman diagrams for scattering fermions and bosons

We treat massless fermions and boson fields.

We do not discuss the breaks of symmetries, bringing masses to all fermions and boson fields, except to gravitons, photons and gluons; also the coupling constants are not discussed.

We pay attention primarily to internal spaces of fields.

We assume that the scattering objects have non-zero starting momentum (only) in ordinary $(3+1)$ space, the sum of which is conserved.

Let be pointed out that since the proposed theory differs from the usual second quantization procedure for fermions and bosons:

i. The anti-commuting "basis vectors" describing internal spaces of fermions appear in $2^{\frac{d}{2}-1}$ families,

ii. *each family*, having $2^{\frac{d}{2}-1}$ members, includes "basis vectors" of fermions and anti-fermions,

iii. The Hermitian conjugated "basis vectors" of fermions appear in a separate group,

iv. The commuting "basis vectors" describing internal spaces of boson fields appear in two orthogonal groups, having their Hermitian conjugated partners within each group,

v. One group causes transformations among family members – the same "basis vectors" cause transformations among members within all the families,

vi. The second group causes transformations of a particular family member among all families – the same "basis vectors" cause transformations of any family member,

vii. ''"basis vectors" describing internal spaces of boson fields can be described as algebraic products of "basis vectors" of fermions and the Hermitian conjugated partners

The Feynman diagrams in this theory differ from the Feynman diagrams as usually presented,

Offering a possibility for different understanding the second quantization of fermion and boson fields.

Let us study the electron - positron annihilation, when the internal space of fermions and bosons concerns $d = (13 + 1)$, while all the fields have non zero momenta only in ordinary $(3 + 1)$ space-time.

The "basis vector" of an electron with spin up carrying the charge $Q = -1$, will be named $e_L^{-\dagger}$, the "basis vector" of its positron (its anti-particle) carrying $Q = +1$ (both belong to the same family, will be named $e_R^{+\dagger}$.

Let the electron carry the momentum in ordinary space $\vec{p}_1$, while the positron carry the momentum in ordinary space $\vec{p}_2$.

We do not need to know all the even "basis vectors", just those with which the electron and positron interact.

We need to know photons to which the electron and positron with momentum $\vec{p}_1 + \vec{p}_2$ transfer momenta, remaining as two "basis vectors" of electron and positron (without any momentum in ordinary space). And we need to know the photon exchanged by the electron and positron,

A **photon** ${}^{I}\hat{\mathcal{A}}^{\dagger}_{phee^{\dagger}}$ **to which the electron** $e_L^{-\dagger}$ **transfers its momentum is generated by** $e_L^{-\dagger} \; *_A \; (e_L^{-\dagger})^{\dagger}$**, a photon** ${}^{I}\hat{\mathcal{A}}^{\dagger}_{phpp^{\dagger}}$ **to which the positron** $e_R^{+\dagger}$ **transfers its momentum is generated by** $e_R^{+\dagger} *_A \; (e_R^{+\dagger})^{\dagger}$**. The electron and positron will exchange the photon** ${}^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{\dagger}e} = (e_L^{-\dagger})^{\dagger} \; *_A \; e_L^{-\dagger} = (e_R^{+\dagger})^{\dagger} \; *_A$ $e_R^{+\dagger}$.

Let us show how the three photons' "basis vectors" look like:

$$
e_L^{-\dagger}(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}) *_A (e_L^{-\dagger})^{\dagger}(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(+)}\stackrel{78}{(-)}\stackrel{9\,10}{(-)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)})
$$

$$
= {}^{I}\hat{\mathcal{A}}^{\dagger}_{phee^{\dagger}}(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{[-]}\stackrel{78}{[+]}\stackrel{9\,10}{[+]}\stackrel{11\,12}{[+]}\stackrel{13\,14}{[+]}),
$$

$$
e_R^{+\dagger}(\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[-]}\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) *_A (e_R^{+\dagger})^{\dagger}(\equiv \stackrel{03}{(-i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[-]}\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]})
$$

$$
= {}^{I}\hat{\mathcal{A}}^{\dagger}_{phpp^{\dagger}}(\equiv \stackrel{03}{[+i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[-]}\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}),
$$

$$
(e_L^{-\dagger})^{\dagger}(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(+)}\stackrel{78}{(-)}\stackrel{9\,10}{(-)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}) *_A e_L^{-\dagger}(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{(-)}\stackrel{78}{(+)}\stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)})
$$

$$
= {}^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{\dagger}e}(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{[+]}(\stackrel{78}{[-]}\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}),
$$

$$
(e_R^{+\dagger})^{\dagger}(\equiv \stackrel{03}{(-i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[-]}\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}) *_A e_R^{+\dagger}(\equiv \stackrel{03}{(+i)}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[-]}\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]})
$$

$$
= {}^{II}\hat{\mathcal{A}}^{\dagger}_{php^{\dagger}p}(\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[-]}\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}).
$$

We read from the last two relations :

$$ {}^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{\dagger}e} = {}^{II}\hat{\mathcal{A}}^{\dagger}_{php^{\dagger}p} $$

are namely identical:

$$ {}^{II}\hat{\mathcal{A}}^{\dagger}_{phe^{\dagger}e} = (e_L^{-\dagger})^{\dagger} *_A e_L^{-\dagger} $$

$$ {}^{II}\hat{\mathcal{A}}^{\dagger}_{php^{\dagger}p} = (e_R^{+\dagger})^{\dagger} *_A e_R^{+\dagger} . $$

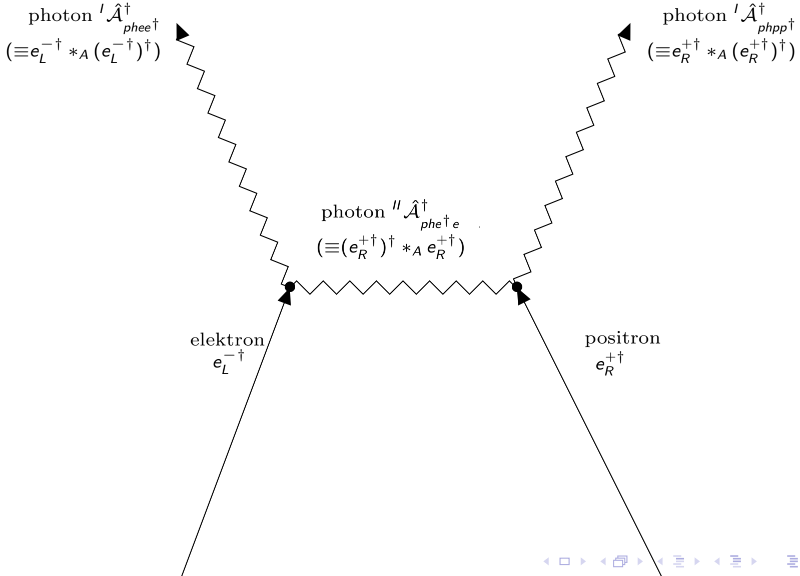
Let me add that also

$(u_L^{c1})^{\dagger} *_A u_L^{c1} = {}^{II}\hat{\mathcal{A}}^{\dagger}_{ph(u_L^{c1})^{\dagger}u_L^{c1}} (\equiv \stackrel{03}{[-i]}\stackrel{12}{[+]}\stackrel{56}{[+]}\stackrel{78}{[-]}\stackrel{9\,10}{[-]}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]})$, **as well as for the corresponding antiparticle.**

Quarks and leptons exchange the same

$$ {}^{II}\hat{\mathcal{A}}^{\dagger}_{ph(u_L^{c1})^{\dagger}u_L^{c1}} = {}^{II}\hat{\mathcal{A}}^{\dagger}_{php^{\dagger}p} . $$

Let us see how does the annihilation of electron and positron look like.



The **"basis vectors"** of two **photons** taking away the momenta $\vec{p}_1$ and $\vec{p}_2$, named ${}^{I}\hat{\mathcal{A}}^{\dagger}_{phee^{\dagger}}$ and ${}^{I}\hat{\mathcal{A}}^{\dagger}_{phpp^{\dagger}}$, respectively, are represented by $e_L^{-\dagger} \; *_A \; (e_L^{-\dagger})^{\dagger}$ and $e_R^{+\dagger} \; *_A \; (e_R^{+\dagger})^{\dagger}$, respectively.

The **"basis vector" of a photon,** ${}^{II}\hat{\mathcal{A}}^{\dagger}_{\mathbf{phe^{\dagger}e}} = {}^{II}\hat{\mathcal{A}}^{\dagger}_{php^{\dagger}p}$, exchanged by $e_L^{-\dagger}$ and $e_R^{+\dagger}$, is equal to $(e_L^{-\dagger})^{\dagger} \; *_A \; e_L^{-\dagger} = (e_R^{+\dagger})^{\dagger} \; *_A \; e_R^{+\dagger}$ (due to the fact that $e_L^{-\dagger}$ and $e_R^{+\dagger}$ belong to the same family).

This exchange results in transferring the momenta $\vec{p}_1$ and $\vec{p}_2$ from $e_L^{-\dagger}$ and $e_R^{+\dagger}$, to the two **photons** ${}^{I}\hat{\mathcal{A}}^{\dagger}_{phee^{\dagger}}$ and the **"basis vectors"** $e_L^{-\dagger}$ and $e_R^{+\dagger}$ without momenta in ordinary space.

Analysing the properties of fermion and boson fields from the point of view how they manifest in $d = (3+1)$, the proposed theory discussed in this talk promises to be the right step to better understanding the laws of nature in our universe.

We start with the assumption that the internal spaces wait for the "push" in ordinary space (in the case of our universe, the "push" was made in $d = (3+1)$ making active internal spaces of $d \geq (13+1)$).
To make the discussions of the internal spaces of fermion and boson fields (manifesting in $d = (3+1)$ quarks and leptons and antiquarks and antileptons appearing in families, while boson fields appear in two orthogonal groups manifesting vector and scalar gauge fields with graviton included transparent, we assume that the ordinary $d = (3+1)$ space-time is flat, so that the interpretation of the Feynman diagrams make sense.

Let us briefly review what we have learned in this talk, assuming the space in $d=(3+1)$ is flat, the internal spaces of fermion and boson fields manifesting $(13+1)$ dimensions are described by the odd "basis vectors" for fermions and antifermions (both appear within the same family, correspondingly, there is no Dirac sea in this theory), and by the even "basis vectors" for bosons. The fermion "basis vectors" appear in $d=2(2n+1)$ and $d=4n$ dimensional spaces in $2^{\frac{d}{2}-1}$ families, each family having $2^{\frac{d}{2}-1}$ members, their Hermitian conjugated partners having as well $2^{\frac{d}{2}-1}\times 2^{\frac{d}{2}-1}$ members. The boson fields appear in two orthogonal groups, each with $2^{\frac{d}{2}-1}\times 2^{\frac{d}{2}-1}$ members, having their Hermitian conjugated partners within the same group.

Can this proposal help to better understand our universe?