Pre-geometric Gravity

Emergent Gravity from Topological Quantum Field Theory: Stochastic Gradient Flow Perspective away from the Quantum Gravity Problem

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The problem of Quantum Gravity

An alternative approach off the beaten path: gauge theory and emergent metric structure

References

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- Mielke (2011) 10.1103/PhysRevD.83.044004
- Westman & Zlosnik (2013) 10.1016/j.aop.2013.03.012

Outline

- physical motivations
- constructing the theory
- Hamiltonian analysis
- Topological BF theory and Gradient Flow
- conclusions and perspectives

Quantum Mechanics

General Relativity

$$\phi, \qquad \psi^a, \qquad A_\mu^a$$

$$[\hat{a}, \hat{a}^\dagger] = \mathbb{I}$$

$$S = \mathbb{I} + iT$$

$$\frac{d}{dt}\langle \hat{O} \rangle = \frac{1}{i\hbar}\langle \hat{O}, \hat{H} \rangle + \left\langle \frac{\partial}{\partial t}\hat{O} \right\rangle$$

$$g_{\mu\nu}, \qquad e_\mu^a, \qquad \omega_\mu^{ab}$$

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$$

$$\nabla_\lambda g_{\mu\nu} = 0$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Quantum Mechanics

General Relativity

gauge theory

metric structure

spontaneous symmetry breaking

local Lorentz transformation $\Lambda^a_b(x)$

Spin Connections

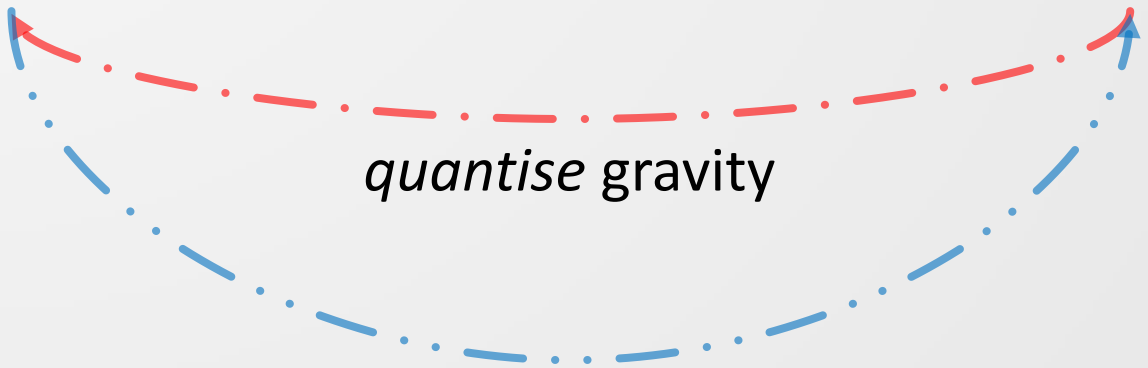
$$\omega^{ab}_\mu \to \Lambda^a_c \omega^{cd}_\mu (\Lambda^{-1})^b_d + \eta^{cd} \Lambda^a_c \partial_\mu (\Lambda^{-1})^b_d$$

gauge theory

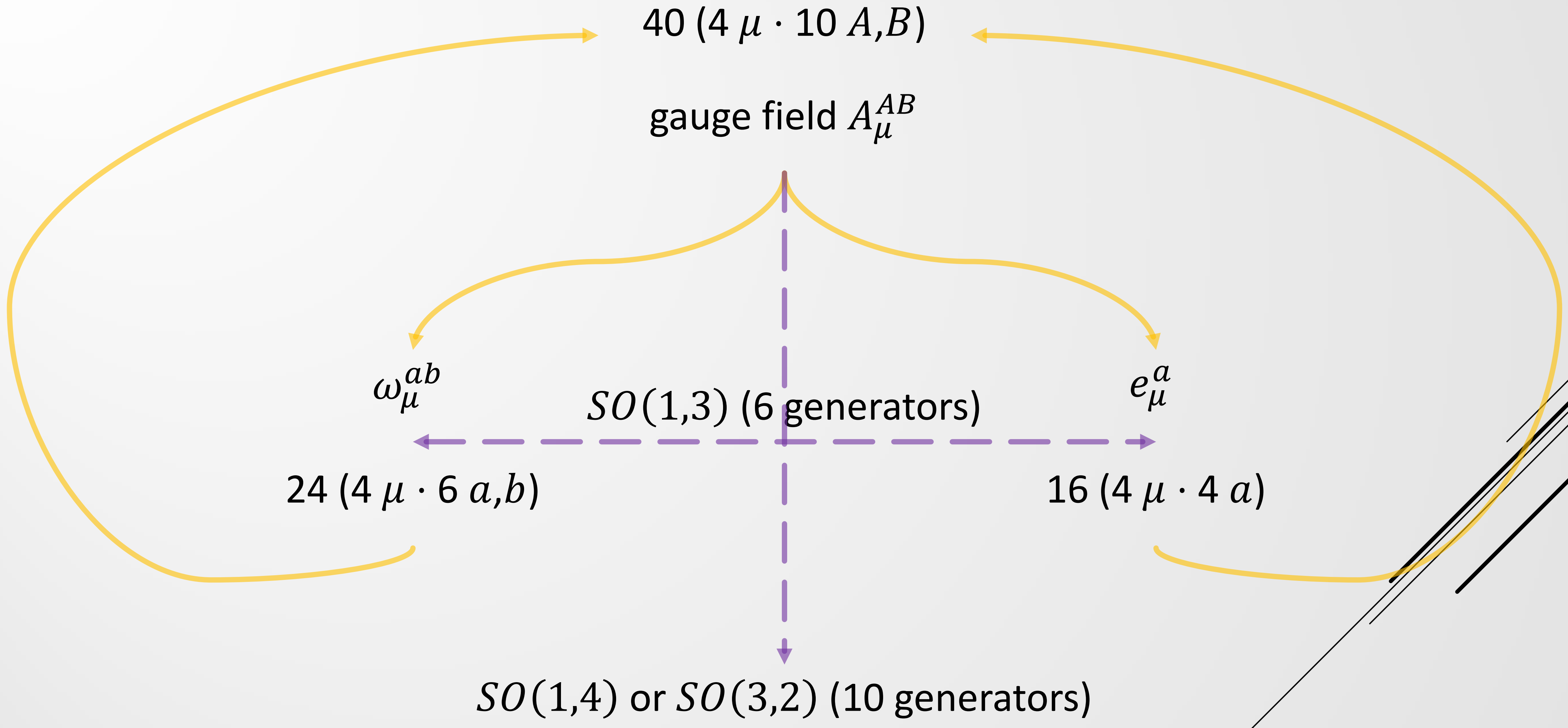
tetrads

$$e^a_\mu \to \Lambda^a_b e^b_\mu$$

metric structure



Unification Logic



SSB

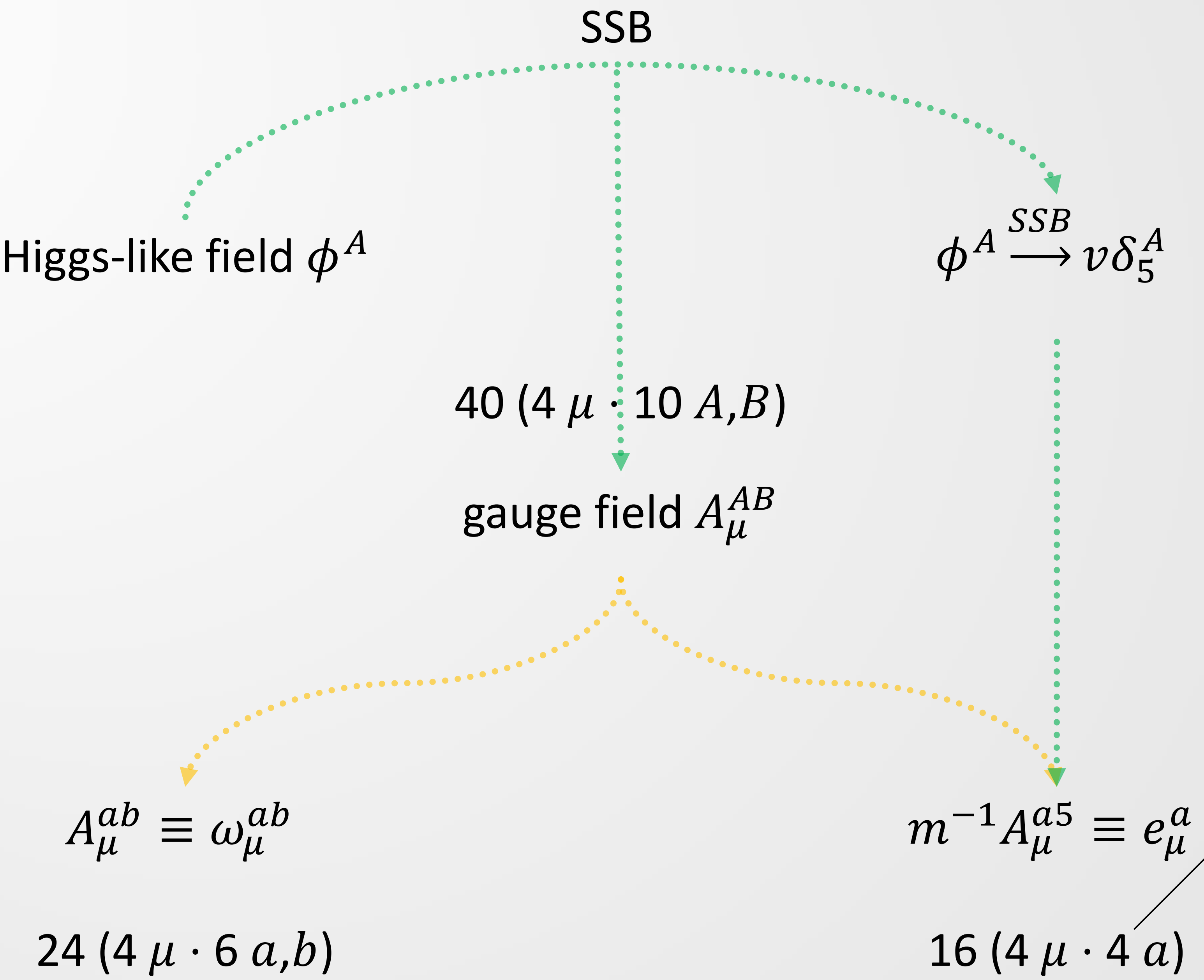
Higgs-like field ϕ^A

$\phi^A \xrightarrow{SSB} v\delta_5^A$

$SO(1,4)$ or $SO(3,2)$ (10 generators)

$SO(1,3)$ (6 generators)

SSB

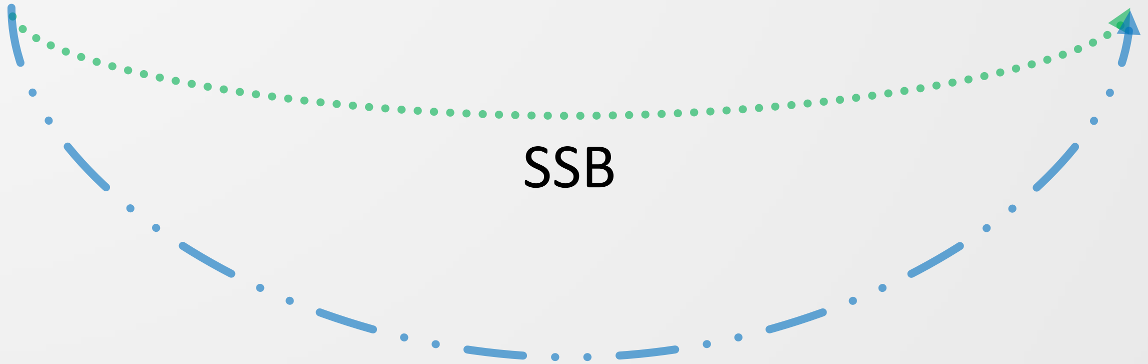

SSB
Higgs-like field ϕ^A
$\phi^A \xrightarrow{SSB} v\delta_5^A$
40 ($4\ \mu \cdot 10\ A,B$)
gauge field A_μ^{AB}
$A_\mu^{ab} \equiv \omega_\mu^{ab}$
$m^{-1}A_\mu^{a5} \equiv e_\mu^a$
24 ($4\ \mu \cdot 6\ a,b$)
16 ($4\ \mu \cdot 4\ a$)

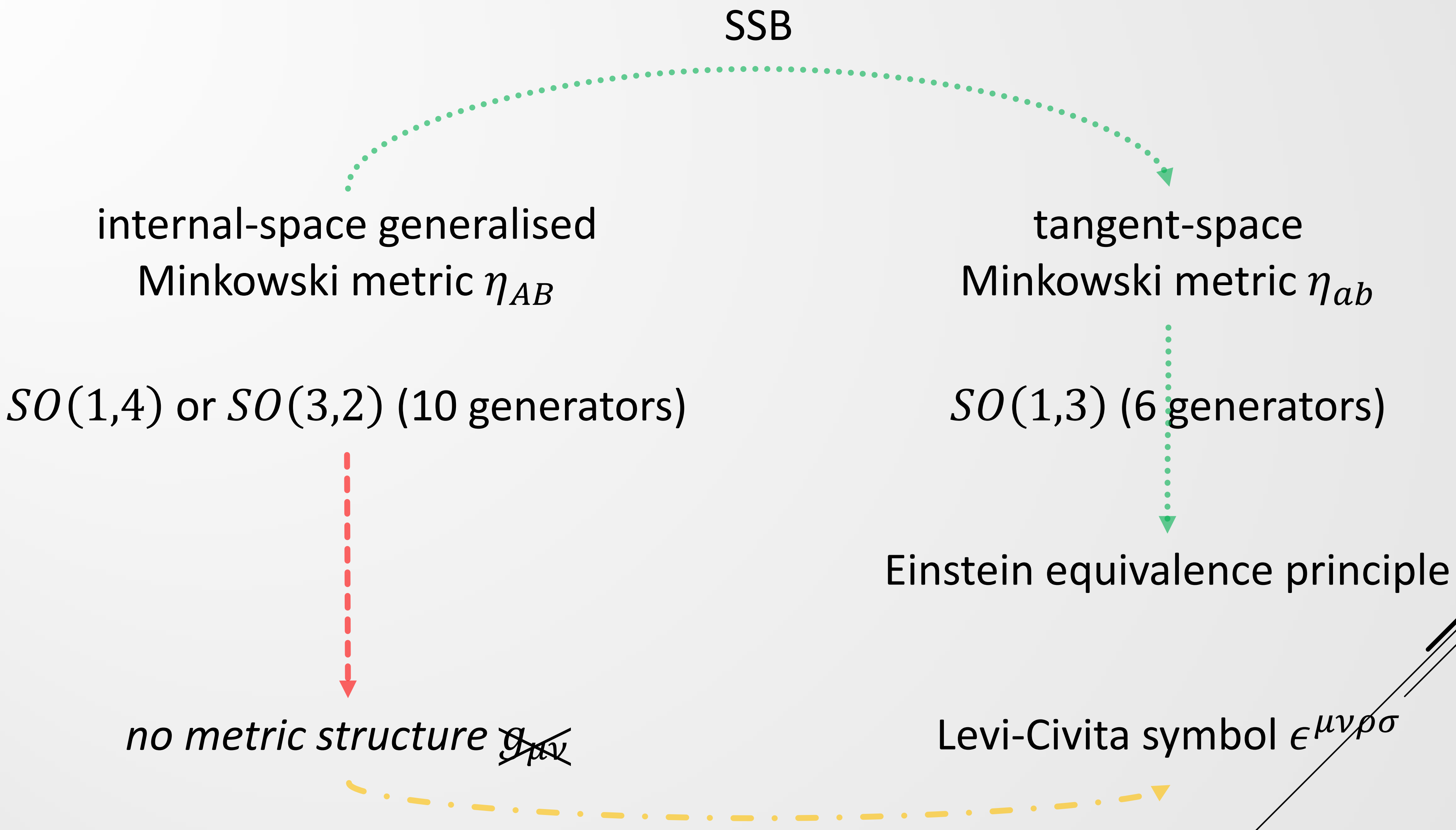
$A_\mu^{AB}, \quad \phi^A$

gauge theory

$g_{\mu\nu}, \quad e_\mu^a, \quad \omega_\mu^{ab}$

metric structure




SSB
internal-space generalised Minkowski metric η_{AB}
tangent-space Minkowski metric η_{ab}
$SO(1,4)$ or $SO(3,2)$ (10 generators)
$SO(1,3)$ (6 generators)
Einstein equivalence principle
no metric structure $g_{\mu\nu}$
Levi-Civita symbol $\epsilon^{\mu\nu\rho\sigma}$

Assumptions

- 4D spacetime
- principle of general covariance
- $\mathcal{L}_{\mathrm{EH}}$ and EEP after SSB
- only A_μ^{AB} and ϕ^A before SSB
- polynomiality in A_μ^{AB} and ϕ^A and their derivatives

Classical gravity

$$R_{\mu\nu}^{ab} = 2\left(\partial_{[\mu}\omega_{\nu]}^{ab} + \omega_{c[\mu}^{a}\omega_{\nu]}^{cb}\right)$$

$$\mathcal{L}_{\mathrm{EH}} = \frac{M_P^2}{2} e e_a^\mu e_b^\nu R_{\mu\nu}^{ab}$$

$$g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$$

$$e \equiv \det e_\mu^a$$

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \left(e_a^\mu e_b^\nu e_c^\rho e_d^\sigma - 4 e_a^\mu e_d^\nu e_c^\rho e_b^\sigma + e_c^\mu e_d^\nu e_a^\rho e_b^\sigma\right) R_{\mu\nu}^{ab} R_{\rho\sigma}^{cd}$$

Constructing the theory

rigidity, uniqueness, no so much space for arbitrary lagrangians (Addazi et al 2025)

Generlized Lovelock's theorem for pre-Geometric theories

$\epsilon^{\mu\nu\rho\sigma}$

$\epsilon^{\mu\nu\rho\sigma} F^{AB}_{\mu\nu}$

$\epsilon_{ABCDE}\epsilon^{\mu\nu\rho\sigma} F^{AB}_{\mu\nu}$

$\epsilon_{ABCDE}\epsilon^{\mu\nu\rho\sigma} F^{AB}_{\mu\nu} \nabla_\rho \phi^C \nabla_\sigma \phi^D$

$\epsilon_{ABCDE}\epsilon^{\mu\nu\rho\sigma} F^{AB}_{\mu\nu} F^{CD}_{\rho\sigma}$

$\epsilon_{ABCDE}\epsilon^{\mu\nu\rho\sigma} F^{AB}_{\mu\nu} \nabla_\rho \phi^C \nabla_\sigma \phi^D \phi^E$

$\epsilon_{ABCDE}\epsilon^{\mu\nu\rho\sigma} F^{AB}_{\mu\nu} F^{CD}_{\rho\sigma} \phi^E$

$$\nabla_\mu \phi^A = \partial_\mu \phi^A + A^A_{B\mu}\phi^B, \qquad A^A_{B\mu} \equiv \eta_{BC} A^{AC}_\mu$$

Two possible Lagrangians

$$\mathcal{L}_{\mathrm{W}} = k_{\mathrm{W}}\epsilon_{ABCDE}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{AB}\nabla_\rho\phi^C\nabla_\sigma\phi^D\phi^E$$

$$\mathcal{L}_{\mathrm{W}} \xrightarrow{SSB} \mathcal{L}_{\mathrm{EH}} - M_{\mathrm{P}}^2\Lambda e$$

$$M_{\mathrm{P}}^2 \equiv -8k_{\mathrm{W}}v^3m^2$$

$$\Lambda \equiv \pm 6m^2 = \mp\frac{3M_{\mathrm{P}}^2}{4k_{\mathrm{W}}v^3}$$

$$k_{\mathrm{W}} \sim -1\,[\phi]^{-3} \Rightarrow v \sim 10^{40}\,[\phi]$$

$$\mathcal{L}_{\mathrm{MM}} = k_{\mathrm{MM}}\epsilon_{ABCDE}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{AB}F_{\rho\sigma}^{CD}\phi^E$$

$$\mathcal{L}_{\mathrm{MM}} \xrightarrow{SSB} \mathcal{L}_{\mathrm{EH}} - M_{\mathrm{P}}^2\Lambda e + \lambda e\mathcal{G}$$

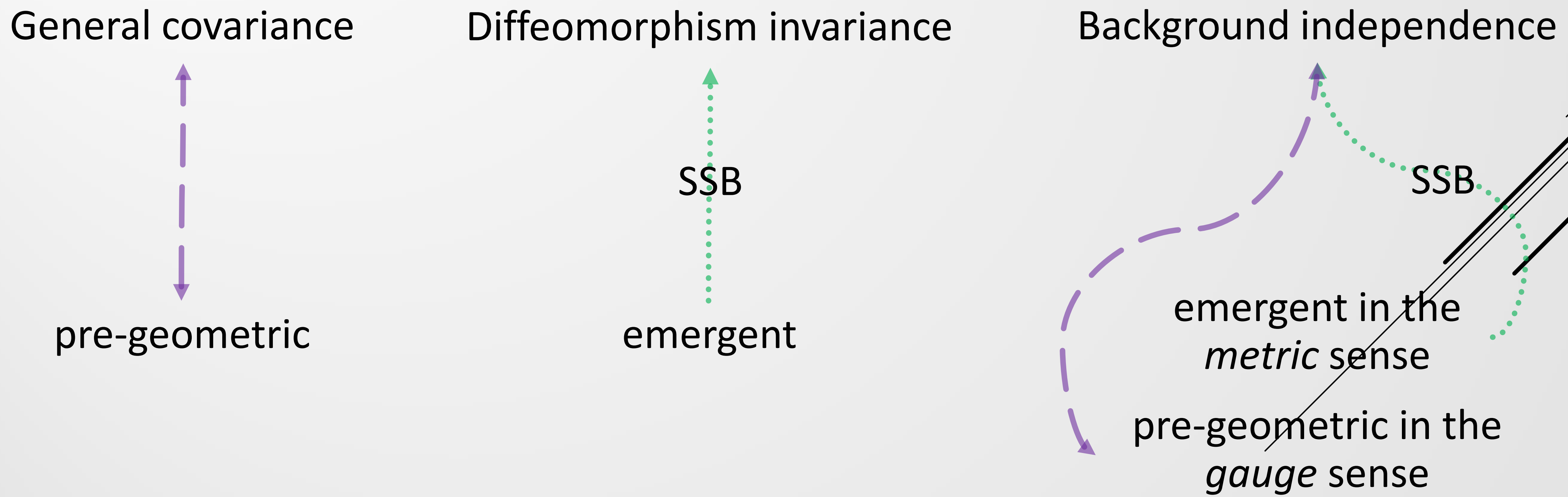
$$M_{\mathrm{P}}^2 \equiv \pm 32k_{\mathrm{MM}}vm^2$$

$$\Lambda \equiv \pm 3m^2 = \frac{3M_{\mathrm{P}}^2}{32k_{\mathrm{MM}}v}$$

$$\lambda \equiv -4k_{\mathrm{MM}}v$$

$$k_{\mathrm{MM}} \sim \pm 1\,[\phi]^{-1} \Rightarrow v \sim 10^{119}\,[\phi]$$

Emergence of (non)equivalent principles



A dictionary for emergent gravity

effective geometric quantities

$$\pm v^{-1} m^{-1} \nabla_\mu \phi^A \xrightarrow{SSB} e_\mu^a$$

$$4 v m J^{-1} w_A^\mu \xrightarrow{SSB} e_a^\mu$$

$$-24^{-1} v^{-5} m^{-4} J \xrightarrow{SSB} e = \sqrt{-g}$$

$$v^{-2} m^{-2} P_{\mu\nu} \xrightarrow{SSB} \eta_{ab} e_\mu^a e_\nu^b \equiv g_{\mu\nu}$$

$$16 v^2 m^2 J^{-2} \eta^{AB} w_A^\mu w_B^\nu \xrightarrow{SSB} \eta^{ab} e_a^\mu e_b^\nu \equiv g^{\mu\nu}$$

auxiliary fields

$$P_{\mu\nu} \equiv \eta_{AB} \nabla_\mu \phi^A \nabla_\nu \phi^B$$

$$w_A^\mu \equiv \pm \epsilon_{ABCDE} \epsilon^{\mu\nu\rho\sigma} \nabla_\nu \phi^B \nabla_\rho \phi^C \nabla_\sigma \phi^D \phi^E$$

$$J \equiv \epsilon_{ABCDE} \epsilon^{\mu\nu\rho\sigma} \nabla_\mu \phi^A \nabla_\nu \phi^B \nabla_\rho \phi^C \nabla_\sigma \phi^D \phi^E$$

Spontaneous symmetry breaking

symmetry-breaking potential

$$\mathcal{L}_{\mathrm{SSB}} = -k_{\mathrm{SSB}} v^{-4} |J| (\eta_{AB} \phi^A \phi^B \mp v^2)^2$$

$$\phi^A = v \delta_5^A$$

unitary gauge

$$\phi^A \xrightarrow{SSB} (v + \rho) \delta_5^A$$

Quantum fluctuations of the background geometry

total effective geometric quantities

$$\pm v^{-1}m^{-1}\nabla_\mu\phi^A \xrightarrow{SSB} \mathcal{E}_\mu^a$$

$$4vmJ^{-1}w_A^\mu \xrightarrow{SSB} \mathcal{E}_a^\mu$$

$$-24^{-1}v^{-5}m^{-4}J \xrightarrow{SSB} \mathcal{E} = \sqrt{-\mathcal{G}}$$

$$v^{-2}m^{-2}P_{\mu\nu} \xrightarrow{SSB} \eta_{ab}\mathcal{E}_\mu^a\mathcal{E}_\nu^b \equiv \mathcal{G}_{\mu\nu}$$

$$16v^2m^2J^{-2}\eta^{AB}w_A^\mu w_B^\nu \xrightarrow{SSB} \eta^{ab}\mathcal{E}_a^\mu\mathcal{E}_b^\nu \equiv \mathcal{G}^{\mu\nu}$$

'classical' and 'quantum' parts

$$\mathcal{E}_\mu^a \equiv e_\mu^a + \hat{e}_\mu^a, \qquad \mathcal{E}_a^\mu \equiv e_a^\mu + \hat{e}_a^\mu, \qquad \mathcal{E} \equiv e + \hat{e},$$

$$\mathcal{G}_{\mu\nu} \equiv g_{\mu\nu} + \hat{g}_{\mu\nu}, \qquad \mathcal{G}^{\mu\nu} \equiv g^{\mu\nu} + \hat{g}^{\mu\nu}$$

quantum fluctuations of the metric

$$\hat{g}_{\mu\nu} = \mathcal{G}_{\mu\nu} - g_{\mu\nu} \xleftarrow{SSB} v^{-2}m^{-2}P_{\mu\nu} - m^{-2}\eta_{ab}A_\mu^{a5}A_\nu^{b5}$$

Matter couplings and the equivalence principle

correspondence principle

$$\mathcal{L}_\Phi = \frac{1}{3} v^{-3} m^{-2} J^{-1} w_A^\mu w_B^\nu \eta^{AB} \partial_\mu \Phi \partial_\nu \Phi \xrightarrow{SSB} -\frac{1}{2}\sqrt{-g} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi$$

$$\mathcal{L}_\Psi = -\frac{i}{6} v^{-4} m^{-3} \bar{\Psi} w_A^\mu \gamma^A \nabla_\mu \Psi \xrightarrow{SSB} i\sqrt{-g} \bar{\Psi} \gamma^\mu \nabla_\mu \Psi$$

$$\mathcal{L}_G = \frac{8}{3} v^{-1} J^{-3} w_A^\rho w_B^\mu w_C^\sigma w_D^\nu \eta^{AB} \eta^{CD} \mathrm{tr}\left(G_{\mu\nu} G_{\rho\sigma}\right) \xrightarrow{SSB} -\frac{1}{4}\sqrt{-g} g^{\mu\rho} g^{\nu\sigma} \mathrm{tr}\left(G_{\mu\nu} G_{\rho\sigma}\right)$$

Gravitational Higgs mechanism

kinematics of the Higgs-like field

$$\mathcal{L}_\phi = \frac{1}{3} v^{-3} J^{-1} w_A^\mu w_B^\nu \eta^{AB} \eta_{CD} \nabla_\mu \phi^C \nabla_\nu \phi^D \xrightarrow{SSB} -\frac{1}{2} m^2 \sqrt{-g} g^{\mu\nu} \eta_{AB} \nabla_\mu \phi^A \nabla_\nu \phi^B + \cdots$$

$A_\mu^{ab} \xrightarrow{SSB} \omega_\mu^{ab}$ ⋯⋯▸ massless

$m^{-1} A_\mu^{a5} \xrightarrow{SSB} e_\mu^a$ ⋯⋯▸ massive, but actually…

$\phi^A \xrightarrow{SSB} (v + \rho)\delta_5^A$ ⋯⋯▸ massive

Gravitational Higgs mechanism

Λ sector

$$\mathcal{L}_{\mathrm{W}}: \cdots\cdots\rightarrow \mathcal{L}_\Lambda \equiv -M_{\mathrm{P}}^2\left(\pm 6m^2 - \frac{m^2}{4k_{\mathrm{W}}v}\right)e$$

$$\mathcal{L}_{\mathrm{MM}}: \cdots\cdots\rightarrow \mathcal{L}_\Lambda \equiv -M_{\mathrm{P}}^2\left(\pm 3m^2 \pm \frac{vm^2}{16k_{\mathrm{MM}}}\right)e$$

$$\mathcal{L}_\phi \xrightarrow{SSB} -\frac{1}{2}m^2\sqrt{-g}g^{\mu\nu}\eta_{AB}\nabla_\mu\phi^A\nabla_\nu\phi^B + \cdots = -\frac{1}{2}m^2\sqrt{-g}g^{\mu\nu}\eta_{ab}(vme_\mu^a)(vme_\nu^b) + \cdots$$

$$= -\frac{1}{2}m^2\sqrt{-g}g^{\mu\nu}\eta_{ab}(v^2m^2)e_\mu^a e_\nu^b + \cdots = -\frac{1}{2}v^2m^4\sqrt{-g}g^{\mu\nu}g_{\mu\nu} + \cdots = -2v^2m^4\sqrt{-g} + \cdots$$

Gravitational Higgs mechanism

$SO(1,4)$ $\quad$ ~~$SO(3,2)$~~

ρ sector

$$\mathcal{L}_\rho \equiv \mp\frac{1}{2}m^2\sqrt{-g}g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho + \sqrt{-g}\left[12k_{\mathrm{W}}v^2m^2\left(-e_a^\mu e_b^\nu R_{\mu\nu}^{ab} \pm 12m^2\right) - 4vm^4\right]\rho$$
$$+\sqrt{-g}\left[12k_{\mathrm{W}}vm^2\left(-e_a^\mu e_b^\nu R_{\mu\nu}^{ab} \pm 12m^2\right) - 96k_{\mathrm{SSB}}v^3m^4 - 2m^4\right]\rho^2 + \mathcal{O}(\rho^3)$$

$$m_\rho^2 = -24\frac{k_{\mathrm{SSB}}}{k_{\mathrm{W}}}M_{\mathrm{P}}^2 + \frac{36}{v^2}M_{\mathrm{P}}^2 + \frac{2}{3}\Lambda \approx \left(1.43\cdot 10^{38}k_{\mathrm{SSB}} + 2.07\cdot 10^{-42} + 2.83\cdot 10^{-84}\right)\,\mathrm{GeV}^2$$

$m_\rho \sim M_{\mathrm{P}}$ for $k_{\mathrm{SSB}} \sim 10^{-2}$ or $k_{\mathrm{SSB}} \ll 10^{116}$ and $v \sim 10$

Cosmological implications

- eternal and singularity-free pre-geometric universe before SSB
- problem of causality
- phase transition at $T_c \sim m_\rho \lesssim M_\mathrm{P}$ if $k_\mathrm{SSB} \lesssim 10^{-2}$
- supermassive boson ρ

Non-perturbative analysis of the # d.o..f

Hamiltonian formulation

conjugate momenta

$$\Pi_A = \frac{\delta(\mathcal{L}_{\mathrm{W}} + \mathcal{L}_{\mathrm{SSB}})}{\delta\dot{\phi}^A} = 2\epsilon_{ABCDE}\epsilon^{0ijk}\nabla_k\phi^D\phi^E\left[k_{\mathrm{W}}F_{ij}^{BC} - \frac{2k_{\mathrm{SSB}}}{v^4|J|}J\nabla_i\phi^B\nabla_j\phi^C(\eta_{FG}\phi^F\phi^G \mp v^2)^2\right],$$

$$\Pi_{AB}^i = \frac{\delta(\mathcal{L}_{\mathrm{W}} + \cancel{\mathcal{L}_{\mathrm{SSB}}})}{\delta\dot{A}_i^{AB}} = 2k_{\mathrm{W}}\epsilon_{ABCDE}\epsilon^{0ijk}\nabla_j\phi^C\nabla_k\phi^D\phi^E, \qquad \Pi_{AB}^0 = \frac{\delta(\mathcal{L}_{\mathrm{W}} + \cancel{\mathcal{L}_{\mathrm{SSB}}})}{\delta\dot{A}_0^{AB}} = 0$$

Lagrangian density

$$\mathcal{L}_{\mathrm{W}} + \mathcal{L}_{\mathrm{SSB}} = \Pi_{AB}^i F_{0i}^{AB} + \Pi_A\nabla_0\phi^A$$

Hamiltonian density

$$\mathcal{H}_{\mathrm{W}} = \Pi_{AB}^i\dot{A}_i^{AB} + \Pi_A\dot{\phi}^A - \mathcal{L}_{\mathrm{W}} - \mathcal{L}_{\mathrm{SSB}} = \Pi_{AB}^i\left(\partial_i A_0^{AB} - 2A_{C[0}^A A_{i]}^{CB}\right) - \Pi_A A_{B0}^A\phi^B$$

Hamiltonian

$$H_{\mathrm{W}} = -\int A_0^{AB}\left(\partial_i \Pi_{AB}^i + 2\Pi_{BC}^i A_{Ai}^C + \eta_{BC}\Pi_A \phi^C\right) d^3x + \int \partial_i\left(\Pi_{AB}^i A_0^{AB}\right) d^3x$$

$$H_{\mathrm{W}} \xrightarrow{SSB} \frac{M_{\mathrm{P}}^2}{4}\epsilon_{abcd}\epsilon^{0ijk}\left\{\int\left[\omega_0^{ab}\partial_i\left(e_j^c e_k^d\right) + 2\omega_{e0}^a \omega_i^{eb} e_j^c e_k^d\right] d^3x\right.$$

$$\left. + \int\left[\mp 4m^2 e_0^a e_i^b e_j^c e_k^d + e_0^a e_i^b R_{jk}^{cd} - \partial_i\left(\omega_0^{ab} e_j^c e_k^d\right)\right] d^3x\right\}$$

$$\Pi_A \xrightarrow{SSB} \Pi_a = \pm 2k_{\mathrm{W}} v^2 m \epsilon_{abcd}\epsilon^{0ijk} e_k^d\left(R_{ij}^{bc} \mp 2m^2 e_i^b e_j^c\right),$$

$$\Pi_{AB}^i \xrightarrow{SSB} \Pi_{ab}^i = 2k_{\mathrm{W}} v^3 m^2 \epsilon_{abcd}\epsilon^{0ijk} e_j^c e_k^d, \qquad \Pi_{AB}^0 \xrightarrow{SSB} 0$$

$$\Pi^i_{AB}\dot{A}^{AB}_i + \Pi_A\dot{\phi}^A = \Pi^i_{ab}\dot{A}^{ab}_i + 2m\Pi^i_{a5}\dot{A}^{a5}_i + \Pi_A\dot{\phi}^A$$

$$\xrightarrow{SSB} \Pi^i_{ab}\dot{\omega}^{ab}_i \cancel{+2m\Pi^i_{a5}\dot{e}^a_i} \cancel{+\dot{v}\Pi_A\delta^A_5}$$

$$\mathcal{L}_{\mathrm{W}} \xrightarrow{SSB} \mathcal{L}_{\mathrm{EH}} + \mathcal{L}_\Lambda$$

$$\mathcal{L}_{\mathrm{SSB}} \xrightarrow{SSB} 0$$

$$\mathcal{H}_{\mathrm{W}} \xrightarrow{SSB} \Pi^i_{ab}\dot{\omega}^{ab}_i - \mathcal{L}_{\mathrm{EH}} - \mathcal{L}_\Lambda$$

ADM formalism

$$\mathcal{H}_{\mathrm{ADM}} = \pi^i_{ab}\dot{\omega}^{ab}_i - \mathcal{L}_{\mathrm{EH}} - \mathcal{L}_\Lambda$$

$$\Pi^i_{ab} = \pi^i_{ab} \Leftrightarrow \textit{time gauge}$$

recovery of the canonical formulation of General Relativity after SSB

recovery of the canonical formulation of General Relativity after SSB

$$\omega_0^{ab}\mathcal{J}_{ab}$$

$$H_{\mathrm{W}} \xrightarrow{SSB} \frac{M_{\mathrm{P}}^2}{4}\epsilon_{abcd}\epsilon^{0ijk}\left\{\int\left[\omega_0^{ab}\partial_i\left(e_j^c e_k^d\right) + 2\omega_{e0}^{a}\omega_i^{eb}e_j^c e_k^d\right]d^3x\right.$$

$$\left.+\int\left[\mp 4m^2 e_0^a e_i^b e_j^c e_k^d + e_0^a e_i^b R_{jk}^{cd} - \partial_i\left(\omega_0^{ab}e_j^c e_k^d\right)\right]d^3x\right\}$$

$$\mathcal{H}_\Lambda = M_{\mathrm{P}}^2\Lambda e \qquad N\mathcal{H}_\perp + N^i\mathcal{H}_i \qquad \partial_i\left(\pi^i_{ab}\omega_0^{ab}\right)$$

$$H_{\mathrm{ADM}} = \int\left[N\mathcal{H}_\perp + N^i\mathcal{H}_i + \omega_0^{ab}\mathcal{J}_{ab} + \mathcal{H}_\Lambda + \partial_i\left(\pi^i_{ab}\omega_0^{ab}\right)\right]d^3x$$

Loop Quantum Gravity and BF theories

$$\Pi^i_{A0} \xrightarrow{SSB} \Pi^i_{a0} = \frac{M_P^2}{4}\epsilon_{0abc}\epsilon^{0ijk}e_j^b e_k^c = \frac{M_P^2}{2}\tilde{E}^i_a = \frac{M_P^2}{2}\sqrt{h}E^i_a$$ Ashtekar's "electric field"

$$S_{\mathrm{EH}} = \frac{M_P^2}{2}\int \mathrm{tr}(B \wedge F) = \frac{M_P^2}{4}\int \epsilon_{abcd}e^c \wedge e^d \wedge R^{ab}$$

$$\Leftrightarrow B_{ab}^{(\mathrm{EH})} = \frac{M_P^2}{4}\epsilon_{abcd}e^c \wedge e^d$$ *simplicity constraint*

$$H_{\mathrm{W}} \xrightarrow{SSB} H_{\mathrm{ADM}} \Leftrightarrow$$ *time gauge*

$$S_{\mathrm{W}} = \int \mathrm{tr}(B \wedge F) = -2k_{\mathrm{W}}\int \epsilon_{ABCDE}\nabla\phi^C \wedge \nabla\phi^D \phi^E \wedge F^{AB}$$

$$\Leftrightarrow B_{AB}^{(\mathrm{W})} = -2k_{\mathrm{W}}\epsilon_{ABCDE}\nabla\phi^C \wedge \nabla\phi^D\phi^E \xrightarrow{SSB} B_{ab}^{(\mathrm{W})} = B_{ab}^{(\mathrm{EH})}$$

Algebra of constraints

extended Hamiltonian density

$$\mathcal{H}_{\mathrm{W}}^{ext} = -A_0^{AB}\left(\partial_i \Pi_{AB}^i + 2\Pi_{BC}^i A_{Ai}^C + \eta_{BC}\Pi_A \phi^C\right) + \lambda^A Z_A + \lambda_i^{AB} Z_{AB}^i + \lambda_0^{AB} Z_{AB}^0$$
$$= \lambda^A Z_A + \lambda_i^{AB} Z_{AB}^i + \lambda_0^{AB} Z_{AB}^0 + \tilde{\lambda}_0^{AB} \dot{Z}_{AB}^0$$

A_0^{AB} is a *gauge* degree of freedom (λ_0^{AB} is undetermined)

constraint	first-class	second-class
primary	Z_{AB}^0	Z_A, Z_{AB}^i
secondary		$\dot{Z}_{AB}^0$

Degrees of freedom

- 90 dynamical variables: 10 A_0^{AB}, 30 A_i^{AB}, 5 ϕ^A, 10 Π^0_{AB}, 30 Π^i_{AB} and 5 Π_A
- 20 gauge choices: -10 A_0^{AB} and -10 Π^0_{AB}
- 10 independent first-class constraints: 10 Z^0_{AB}
- 44 independent second-class constraints: 30 Z^i_{AB}, 5 Z_A, 10 $\dot{Z}^0_{AB}$ and -1 H^{ext}_W

$$2\#\text{degrees of freedom}$$
$$= \#\text{variables} - 2\#\text{first class constraints} - \#\text{second class constraints}$$
$$= 90 - 20 - 20 - 44 = 6$$

3 degrees of freedom: $h_{\mu\nu}$ and ρ (like in scalar-tensor *metric* theories)

$$S_{\text{P-W}} = \int (B_{AB} \wedge F^{AB} + \epsilon_{ABCDE} B^{AB} \wedge B^{CD} \phi^E)$$

$$B^{(AB} \wedge B^{CD)} = 0,$$

$$\frac{\partial A^{AB}}{\partial s} = -i\mathcal{D}B^{AB} + \xi_g A^{AB} \,,$$
$$\frac{\partial B^{AB}}{\partial s} = -iF^{AB} - i\epsilon^{AB}{}_{CDE} B^{CD} \phi^E + \xi_f B^{AB} \,,$$
$$\frac{\partial \phi^A}{\partial s} = -i\epsilon^{A}{}_{BCDE} B^{BC} \wedge B^{DE} + \xi_s \phi^A \,,$$

$$\frac{\partial \phi^A}{\partial s} - \xi_s \phi^A \approx 0 \,.$$

$$\phi^A(s) \approx e^{-\frac{\sigma^2_{\xi_s}}{2}(s-s_0)} \phi^A_0(s),$$

$$\lim_{s \to s_0} \phi^A_0(s) \sim \frac{v}{2} \delta^A_5 [1 + \theta(s - s_0)] \,.$$

Conclusions

- gravity can emerge from SSB in a pre-geometric universe
- $SO(1,4) \xrightarrow{SSB} SO(1,3)$: emerging principles and energy scales of GR from $\mathcal{L}_{\mathrm{W}}$
- phase transition at $T_c \lesssim M_{\mathrm{P}}$ and supermassive boson ρ
- gravity as a BF theory with 2+1 degrees of freedom

Perspectives

- resolution of classical spacetime singularities, Quantum Cosmology and Inflation
- causality, propagators and matter couplings
- quantisation of gravity and UV completion of GR
- background independent quantisation, renormalisability, path integral

THANKS FOR YOUR ATTENTION!