Spontaneous baryogenesis with large misalignment

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Goal

The goal of this work is to consider arbitrary initial phase of Nambu-Goldstone boson and calculate corresponding baryon asymmetry.

Model is described via Lagrangian:

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi) + i\overline{Q}\gamma^\mu \partial_\mu Q + i\overline{L}\gamma^\mu \partial_\mu L - m_Q \overline{Q}Q - m_L \overline{L}L + g(\Phi \overline{Q}L + \Phi^* \overline{L}Q), \quad (1)$$

where

$$V(\Phi) = \lambda[\Phi^*\Phi - f^2/2]^2. \quad (2)$$

Symmetry breaking

After symmetry breaking which happens at energy scale f field aquire its vacuum expectation value

$$\langle\Phi\rangle = \frac{f}{\sqrt{2}} e^{i\phi/f}, \tag{3}$$

after symmetry breaking the Lagrangian is as follows ($\theta = \phi/f$):

$$\mathcal{L} = \frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + i\overline{Q}\gamma^\mu\partial_\mu Q + i\overline{L}\gamma^\mu\partial_\mu L - m_Q\overline{Q}Q - m_L\overline{L}L + \\ + \frac{gf}{\sqrt{2}}(\overline{Q}Le^{i\theta} + \overline{L}Qe^{-i\theta}) - V(\theta), \tag{4}$$

it has following symmetries:

$$Q \to e^{i\alpha}Q,\ L \to L,\ \theta \to \theta + \alpha. \tag{5}$$

Baryon current

Let us transform Lagrangian by setting $\alpha = -\theta$, then we obtain:

$$\mathcal{L} = \frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + i\overline{Q}\gamma^\mu\partial_\mu Q + i\overline{L}\gamma^\mu\partial_\mu L - m_Q\overline{Q}Q - m_L\overline{L}L + \\ + \frac{gf}{\sqrt{2}}(\overline{Q}L + \overline{L}Q) + \partial_\mu\theta\overline{Q}\gamma^\mu Q - \Lambda^4(1-\cos\theta). \quad (6)$$

Why arbitrary initial phase?

$$f(\theta_i, t = 60H_\star^{-1}) = \frac{1}{\sqrt{2\pi}\sigma'} \exp\left(-\frac{(\theta_i - \theta_u)^2}{2\sigma'^2}\right) , \tag{7}$$

where $\sigma' = \frac{H_\star}{2\pi f}\sqrt{60}$. Here, we have assumed that the duration of inflation is $60H_\star^{-1}$. The probability for $|\theta_i - \theta_u|$ to be more than π by the end of inflation is given by:

$$P(|\theta_i - \theta_u| > \pi) = 1 - \mathrm{erf}(\pi) \approx 10^{-5} , \tag{8}$$

if we assume that $f \approx H_\star$. By the end of inflation we are given with e^{180} initially casually independent regions, thus, the number of regions, where $|\theta_i - \theta_u| > \pi$ could be estimated in the following way:

$$n_{\mathrm{regions}} = e^{180} P(|\theta_i - \theta_u| > \pi) \gg 1 , \tag{9}$$

Equation of motion in flat space

Semiclassical equation of motion in Minkowski metric is as follows:

$$\ddot{\theta} + \frac{\Lambda^4}{f^2} \sin\theta = \frac{g^2}{\pi^2} \lim_{\omega\to\infty} \int_{-\infty}^{0} dt' \left[\frac{\sin^2 \omega t'}{t'} \right] \times \\ \times \left[\ddot{\theta}(t+t') \cos\Delta\theta - \dot{\theta}^2(t+t') \sin\Delta\theta \right] \equiv \mathcal{I}. \quad (10)$$

In order to solve it we truncate the limit

$$\frac{\omega f}{\Lambda^2} \gg 1 \text{ and } \omega < \infty. \quad (11)$$

Let us integrate $\mathcal{I}(t)$ by parts:

$$\mathcal{I}(t) = \frac{g^2}{\pi^2}\frac{\sin^2 \omega t'}{t'}\dot{\theta}(t+t')\cdot\cos\left[\Delta\theta\right]\Big|_{-\infty}^{0} - \\ - \frac{g^2}{\pi^2}\int_{-\infty}^{0}\dot{\theta}(t+t')\cdot\cos\left[\theta(t+t')-\theta(t)\right]\cdot\left(\frac{\omega\sin\left(2\omega t'\right)}{t'} - \frac{\sin^2\left(\omega t'\right)}{t'^2}\right). \quad (12)$$

Recall known representations of Dirac delta function:

$$\frac{\omega\sin\left(2\omega t'\right)}{t'} - \frac{\sin^2\left(\omega t'\right)}{t'^2} \approx \pi\omega\delta(t'). \quad (13)$$

We obtain

$$\ddot{\theta} + \frac{g^2\omega}{\pi}\dot{\theta} + \frac{\Lambda^4}{f^2}\sin\theta = 0. \quad (14)$$

Let us consider dimensionless derivative with respect to $\Lambda^2 t/f$ (represented by prime):

$$\theta'' + \frac{g^2\omega f}{\Lambda^2\pi}\theta' + \sin\theta = 0. \tag{15}$$

Let us introduce the notation $\Gamma = \dfrac{g^2\omega f}{\Lambda^2\pi}$ and treat it as a free parameter.
Given with the solution of eq. (15), one can calculate number density of baryons and antibaryons:

$$n_{B,\overline{B}} = \frac{g^2 f^2}{2\pi^2}\int_0^m \omega^2 d\omega \left|\int_{-\infty}^{+\infty} e^{2i\omega t \pm i\theta(t)} dt\right|^2. \tag{16}$$

Numerical solution in Minkowski metric

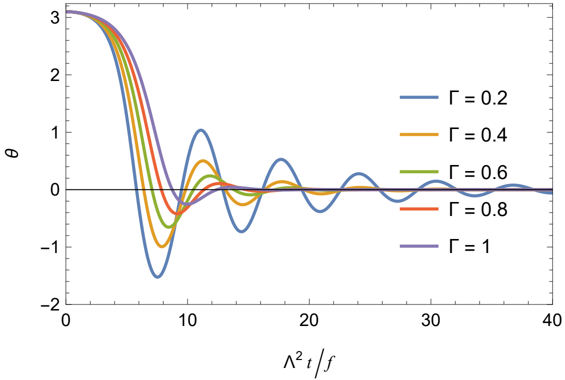

Figure 1: Numerical solution of equation (15) with initial conditions $\theta_{in} = 3.1$ and $\dot{\theta}_{in} = 0$. We see nearly aperiodic behavior for $\Gamma = 1$, thus considered values of Γ are suffucient to study its impact on baryon asymmetry.

Baryon asymmetry in flat space

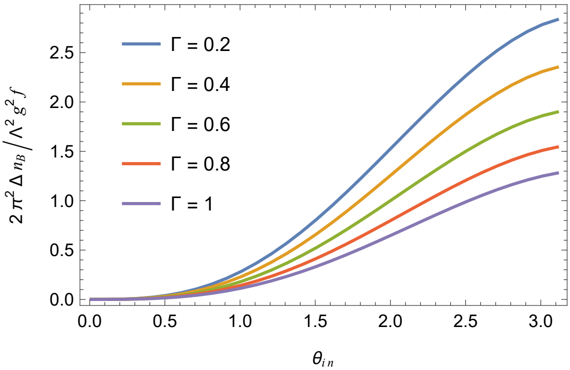

Figure 2: Baryon asymmetry $\Delta n_B = n_B - n_{\overline{B}}$ as a function of initial phase for different values of Γ.

Validation of approach

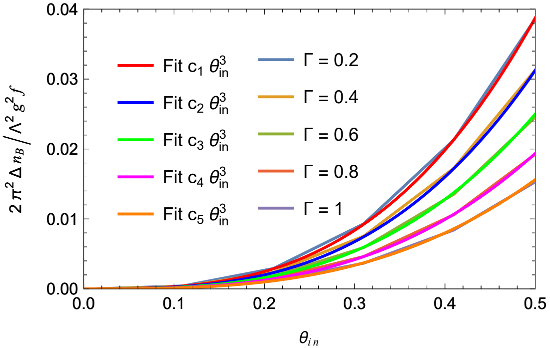

Figure 3: Baryon asymmetry in Minkowski metric for small initial phase for different values of decay rate Γ with cubic fit functions. Made for validation of our approach. Values of c_i are as follows: $c_1 \approx 0.31$, $c_2 \approx 0.25$, $c_3 \approx 0.2$, $c_4 \approx 0.155$, $c_5 \approx 0.125$.

Dynamics in conformal FLRW

In conformal FLRW ($g_{\mu\nu} = a^2\eta_{\mu\nu}$) the Lagrangian might be rewritten as follows (fermions are redefined $\psi \to \psi/a^{3/2}$):

$$\mathcal{L} = \frac{1}{2}f^2a^2\partial_\mu\theta\partial^\mu\theta + i\overline{Q}\partial_\mu\gamma^\mu Q + i\overline{L}\partial_\mu\gamma^\mu L + \partial_\mu\theta\overline{Q}\gamma^\mu Q + \\ + gfa(\overline{Q}L - \overline{L}Q) - a^4U(\theta) - am_Q\overline{Q}Q - am_L\overline{L}L. \quad (17)$$

Semiclassical equation is as follows:

$$\partial_\mu(a^2\partial^\mu\theta) + a^4\frac{\Lambda^4}{f^2}\sin\theta = -\frac{4g^2}{\pi^2}a(\tau)\int_0^\infty \omega^2 d\omega \int_{-\infty}^0 a(\tau+\tau')\times \\ \times \sin(2\omega\tau')\sin[\theta(\tau+\tau') - \theta(\tau)]d\tau'. \quad (18)$$

Integrating by parts and utilizing similar procedure as in Minkowski space we obtain

$$\ddot{\theta} + \left(2\frac{\dot{a}}{a} + \frac{g^2\omega}{\pi} a^2 \right) \dot{\theta} + a^4 \frac{\Lambda^4}{f^2} \sin\theta = 0. \tag{19}$$

Consider dimensionless variable ($\tau \to \Lambda^2 \tau / f = \eta$): (recall that for RD stage $a \propto \tau$.)

$$\theta'' + \left(\frac{2}{\eta} + \Gamma\eta^2 \right) \theta' + \eta^4 \sin\theta = 0, \tag{20}$$

where $\Gamma = g^2 f\omega / \pi\Lambda^2$.

Solution of EOM in conformal FLRW (1)

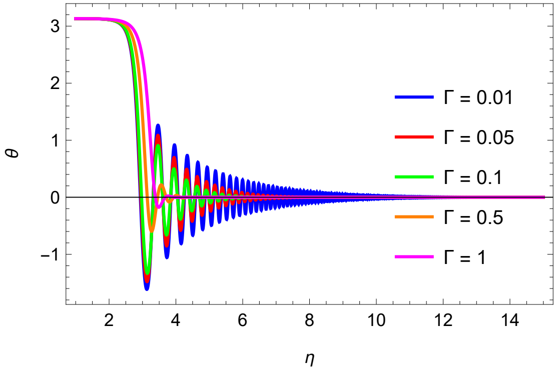

Figure 4: Numerical solution of eq. (20) with initial phase close to π for different values of Γ.

Solution of EOM in conformal FLRW (2)

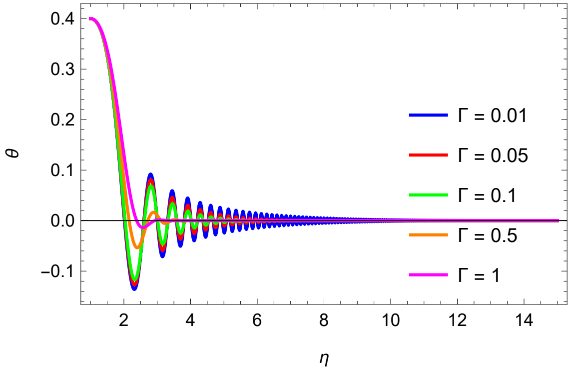

Figure 5: Numerical solution of eq. (20) with small initial phase for different values of Γ.

Baryon number density in conformal FLRW

Baryon number density is given by:

$$n(Q,\overline{L}) = \frac{1}{V}\sum_{s_Q,s_{\overline{L}}}\int \widetilde{dp}\widetilde{dq}\left|\langle Q(p,s_Q),\overline{L}(q,s_{\overline{L}})|i\frac{gf}{\sqrt{2}}\int d^4x\, a(\tau)\overline{Q}(x)L(x)e^{i\theta(\tau)}|0\rangle\right|^2, \tag{21}$$

which implies

$$n_{b,\overline{b}}(\tau) = \frac{g^2f^2}{2\pi^2a^3(\tau)}\int_0^{\Lambda^2/f}\omega^2 d\omega\left|\int_{\tau_{in}}^{\tau} d\tau'\, a(\tau')e^{2i\omega\tau'\pm i\theta(\tau')}\right|^2. \tag{22}$$

We will illustrate

$$a^3(\infty)n_{b,\overline{b}}(\infty) = \frac{g^2f^2}{2\pi^2}\int_0^{\Lambda^2/f}\omega^2 d\omega\left|\int_{\tau_{in}}^{\infty} d\tau'\, a(\tau')e^{2i\omega\tau'\pm i\theta(\tau')}\right|^2. \tag{23}$$

Baryon asymmetry in conformal FLRW

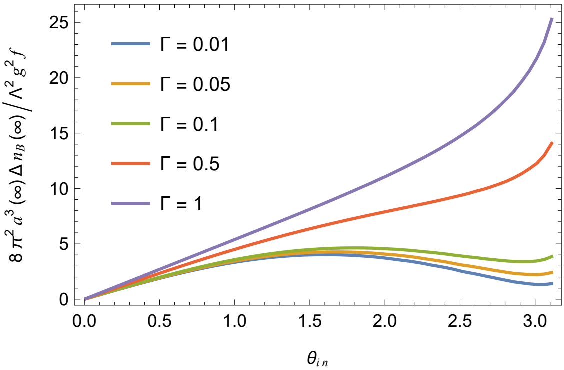

Figure 6: Baryon asymmetry $a^3 \Delta n_B = a^3(n_B - n_{\overline{B}})$ as a function of initial phase for different values of Γ.

Results

Minkowski metric:

- Particle production slow down as initial phase grows.
- Decay rate Γ does not affect asymmetry significantly.

FLRW metric:

- For small initial phase asymmetry is proportional to θ_{in}.
- Decay rate Γ could affect asymmetry significantly.

Thank you for your attention!