Quantum mechanical numerical model of the interaction of dark atom with the nucleus of substance, taking into account non-point-like structure of interacting particles.

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“What comes beyond the Standard models?”

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Scenarios of hypothetical, stable, electrically charged particles.

*We consider composite dark matter scenario in which hypothetical, stable, relict, lepton-like, massive **X particles** with charge **-2n** (where n is any natural number) escape experimental discovery because they are coupled by Coulomb interactions with n primordial helium nuclei into neutral atom-like states of XHe (**X - helium**), called «dark atoms».*

*In the case of **n=1**, the X particle is called* O^{--}, *and the «dark atom» is called **O-helium.***

$$O^{--} + He^4 \to (OHe) + \gamma$$

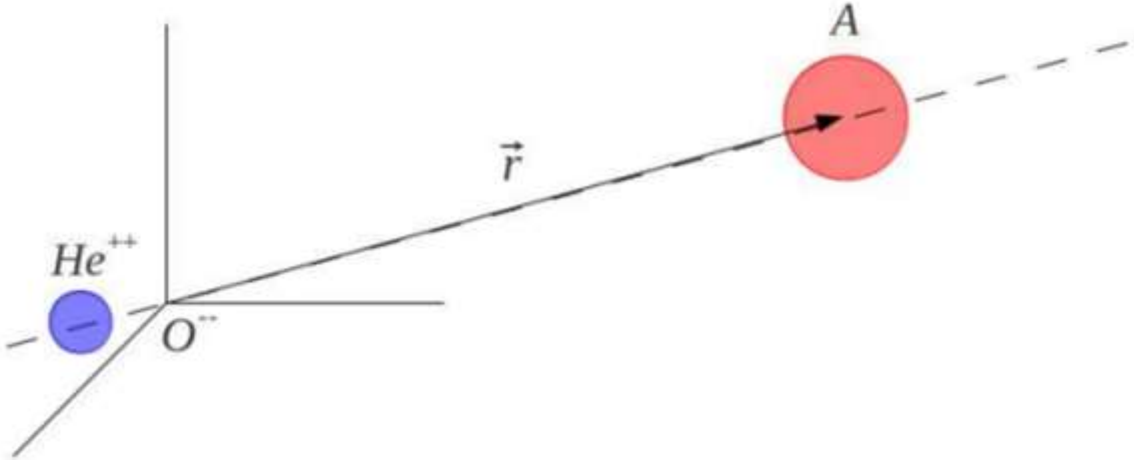


Illustration of the *OHe* dark atom and the outer nucleus of substance A.

The problem of the X – helium hypothesis.

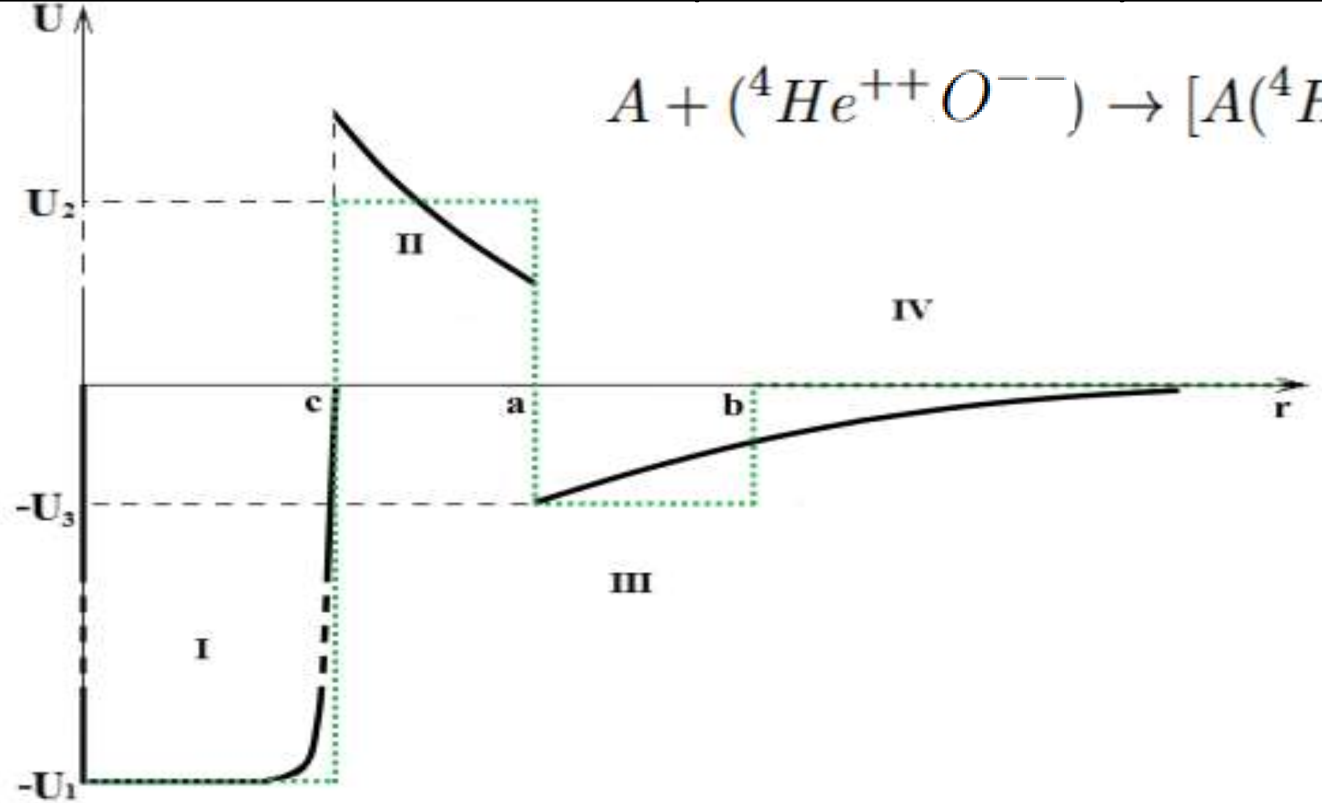
Dark atoms of XHe provide the modern density of non-relativistic matter and play the role of non-trivial form of strongly interacting dark matter.

The possibility of the existence of low-energy bound state of *XHe* with nuclei and the dominance of elastic processes in the dark atom scenario is based on the hypothesis of the presence of potential barrier and shallow potential well in the processes of interaction of *X*-helium with the nuclei of matter, which requires correct quantum mechanical justification.

Task: *development of numerical model of the interaction of X-helium with the nucleus of substance.*

Purpose of the work: reconstruction of the form of the total effective interaction potential of the *XHe*-nucleus system and calculation of the cross section of inelastic capture of *X*-helium by the nucleus of matter.

$$A + ({}^4He^{++}O^{--}) \rightarrow [A({}^4He^{++}O^{--})] + \gamma$$



Hypothetical effective potential of interaction of XHe with the nucleus of substance.

Experiments on the direct search for dark matter particles.

Detector	Nuclei	A	Z	Temperature	Detection
DAMA (/NaI + /LIBRA)	Na, I, Tl	23, 127, 205	11, 53, 81	300 K	13.7 σ
CoGeNT	Ge	70-74	32	70 K	2.8 σ
CDMS	Ge (Si)	70-74 (28-30)	32 (14)	Cryogenic	_
XENON100	Xe	124-134	54	Cryogenic	_
LUX	Xe	124-134	54	173 K	_
CRESST-III	Ca, W, O ($CaWO_4$)	40, 182-186, 16	20, 74, 8	Cryogenic	–
SuperCDMS	Ge, Si	70-74, 28-30	32, 14	Cryogenic	–
COSINE-100	Na, I	23, 127	11, 53	300 K	–
ANAIS-112	Na, I	23, 127	11, 53	300 K	–
LUX-ZEPLIN (LZ)	Xe	124-134	54	Cryogenic	–
XENONnT	Xe	124-134	54	Cryogenic	–
PandaX-4T	Xe	124-134	54	Cryogenic	–

$$\sigma v = \frac{f\pi\alpha}{m_p^2}\frac{3}{\sqrt{2}}\left(\frac{Z}{A}\right)^2\frac{T}{\sqrt{A m_p E}}$$

*) M. Y. Khlopov, A. G. Mayorov, and E. Y. Soldatov. Composite dark matter and puzzles of dark matter searches. International Journal of Modern Physics D, 19(08n10):1385–1395, 2010.

Stark effect.

$$\boxed{U_{St} = eZ_{\alpha}E\delta}$$

Quantum-mechanical calculation.

$$\boxed{\delta = \int_r \Psi^*_{OHe} \cdot r \cdot \Psi_{OHeNa} \cdot 4\pi r^2 dr}$$

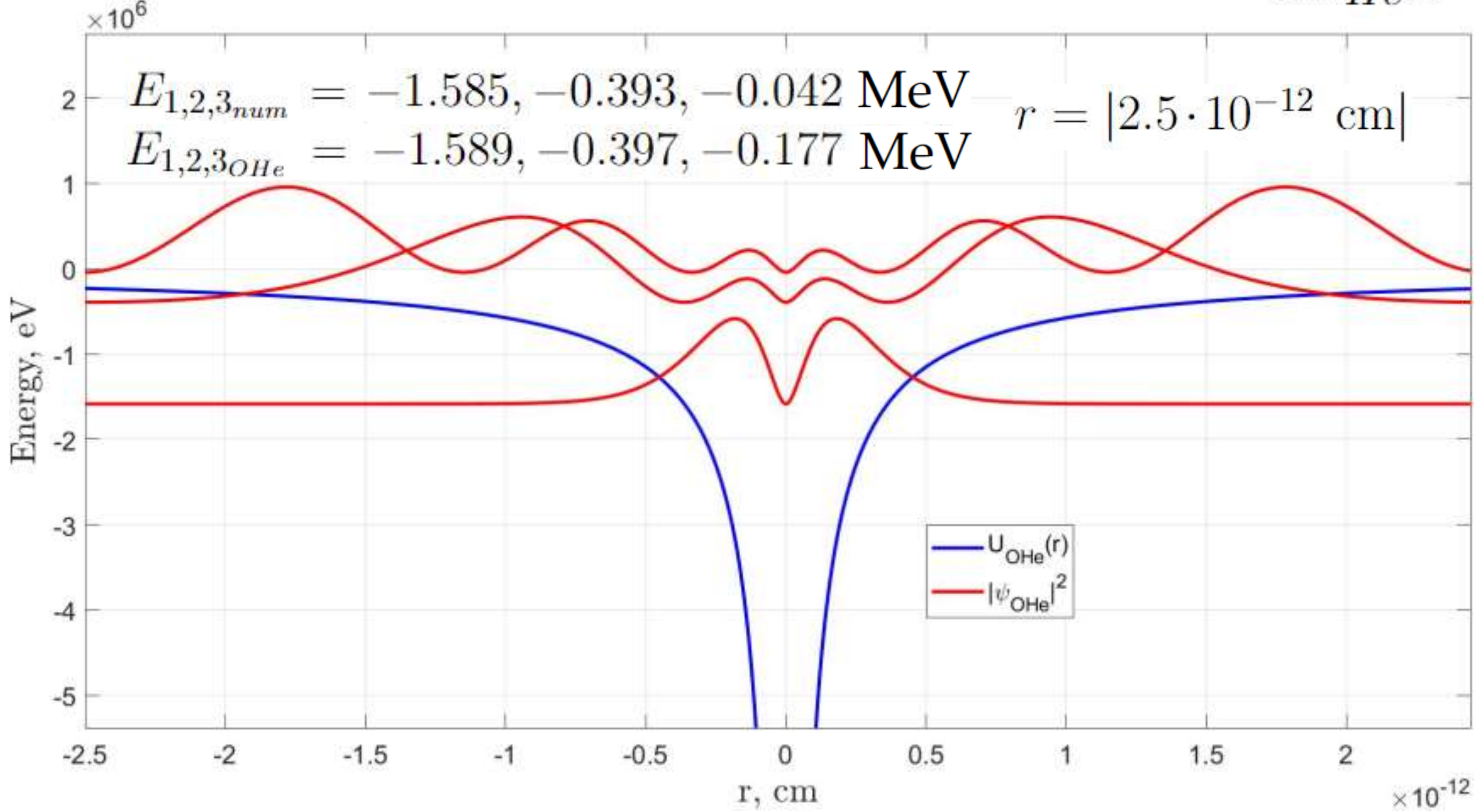
Isolated O-helium dark atom.

$$\boxed{\hat{H}_0\Psi(\vec{r}) = E_{OHe}\Psi(\vec{r})}$$

$$\boxed{\Delta_r\Psi(\vec{r}) + \frac{2m_{He}}{\hbar^2}\left(E_{OHe} + \frac{4e^2}{r}\right)\Psi(\vec{r}) = 0}$$

$$E_{n_{OHe}} = \frac{-8m_{He}\alpha^2}{n^2}\text{MeV}$$

$$-8m_{He}\alpha^2 \approx -1.6\,\text{MeV}$$



Eigenvalues of the Hamiltonian of the helium nucleus (the first 3 energy levels) in the potential of the OHe «dark atom» (blue solid line) and the graphs of the squared modulus of the wave function corresponding to these energy levels (red solid line).

Quantum - mechanical description of the three-body problem in the OHe-nucleus system.

$$\hat{H} = \hat{H}_0 + \hat{U}$$

$$\hat{U} = U_{kul}(|\vec{R}_{OA} - \vec{r}|) + U_{Nuc}(|\vec{R}_{OA} - \vec{r}|)$$

$$\hat{H}_0 = -\frac{\hbar^2}{2m_{He}}\Delta - \frac{4e^2}{r}$$

$$\hat{H}\Psi = E\Psi$$

$$\vec{R}_{HeA} = \vec{R}_{OA} - \vec{r}$$

$$\Delta\Psi(\vec{r}) + \frac{2m_{He}}{\hbar^2}\left(E + \frac{4e^2}{r} - U_{kul}(|\vec{R}_{OA} - \vec{r}|) - U_N(|\vec{R}_{OA} - \vec{r}|)\right)\Psi(\vec{r}) = 0$$

$$U_{Nuc}(|\vec{R}_{OA} - \vec{r}|) = -\frac{U_0}{1 + \exp\left(\frac{|\vec{R}_{OA} - \vec{r}| - R_{N_{nuc}} - R_{N_{He}}}{p}\right)}$$

$$U_{kul}(|\vec{R}_{OA} - \vec{r}|) = \begin{cases} \frac{2e^2 Z_{nuc}}{|\vec{R}_{OA} - \vec{r}|} & \text{for } |\vec{R}_{OA} - \vec{r}| > R_{p_{nuc}}, \\ \frac{2e^2 Z_{nuc}}{2R_{p_{nuc}}}\left(3 - \frac{|\vec{R}_{OA} - \vec{r}|^2}{R_{p_{nuc}}^2}\right) & \text{for } |\vec{R}_{OA} - \vec{r}| <= R_{p_{nuc}} \end{cases}$$

Adding centrifugal potential to quantum mechanical numerical model of the OHe - nucleus system.

$$\boxed{U_{rot_{(OHe-Na)}}(R) = \frac{\hbar^2 c^2 J_{(OHe-Na)}(J_{(OHe-Na)}+1)}{2\mu c^2 R^2},}$$

$$\vec{J}_{(OHe-Na)}(\rho) = \vec{l}_{(OHe-Na)}(\rho) + \vec{I}_{Na} + \vec{I}_{OHe},$$

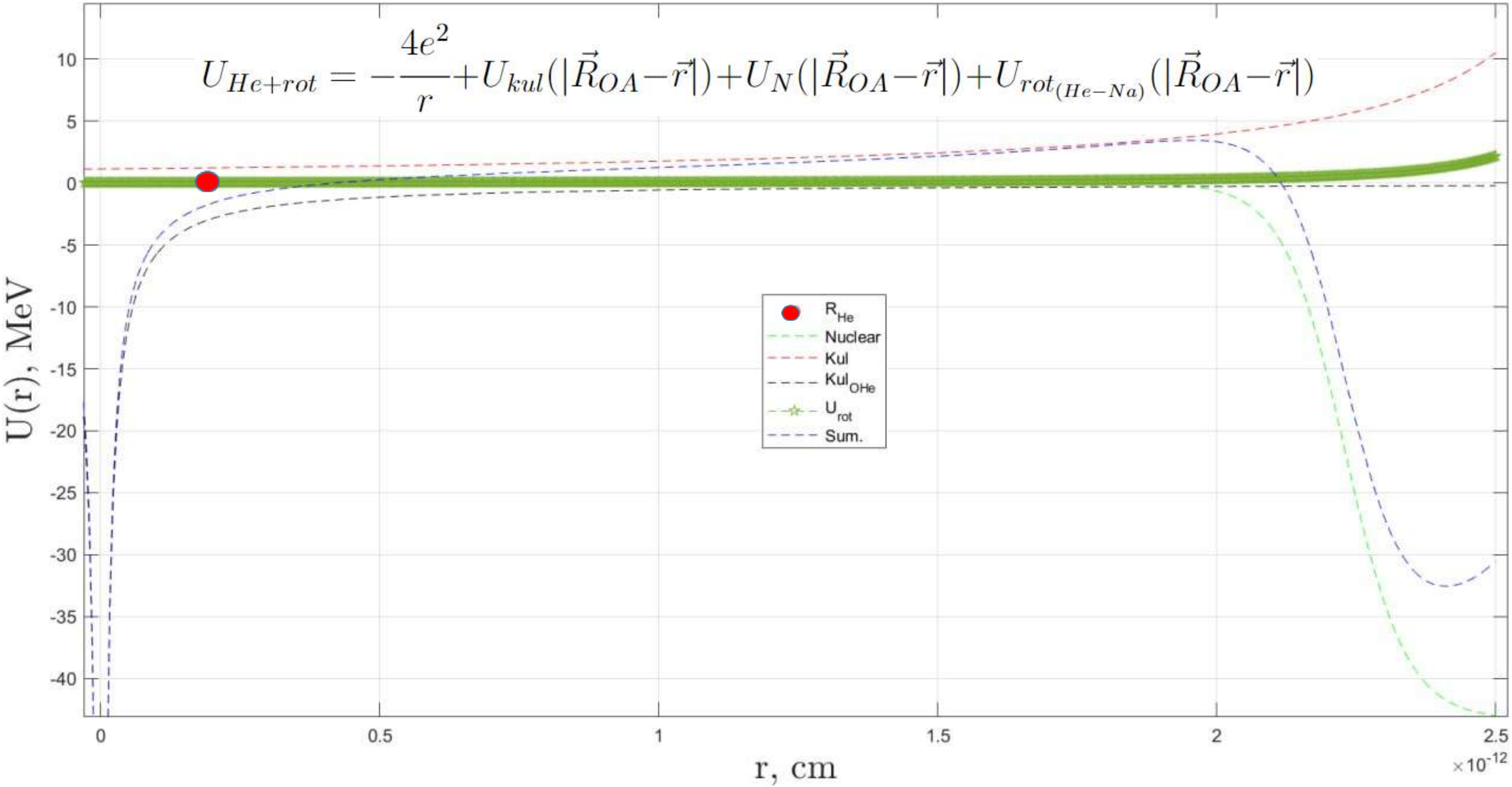
$$\boxed{\vec{J}_{(OHe-Na)} = \frac{\vec{3}}{2} + \vec{I}_{O^{--}}.}$$

$$\vec{I}_{OHe} = \vec{I}_{He} + \vec{I}_{O^{--}}.$$

$$\boxed{U_{He+rot} = -\frac{4e^2}{r} + U_{kul}(|\vec{R}_{OA}-\vec{r}|) + U_N(|\vec{R}_{OA}-\vec{r}|) + U_{rot_{(He-Na)}}(|\vec{R}_{OA}-\vec{r}|),}$$

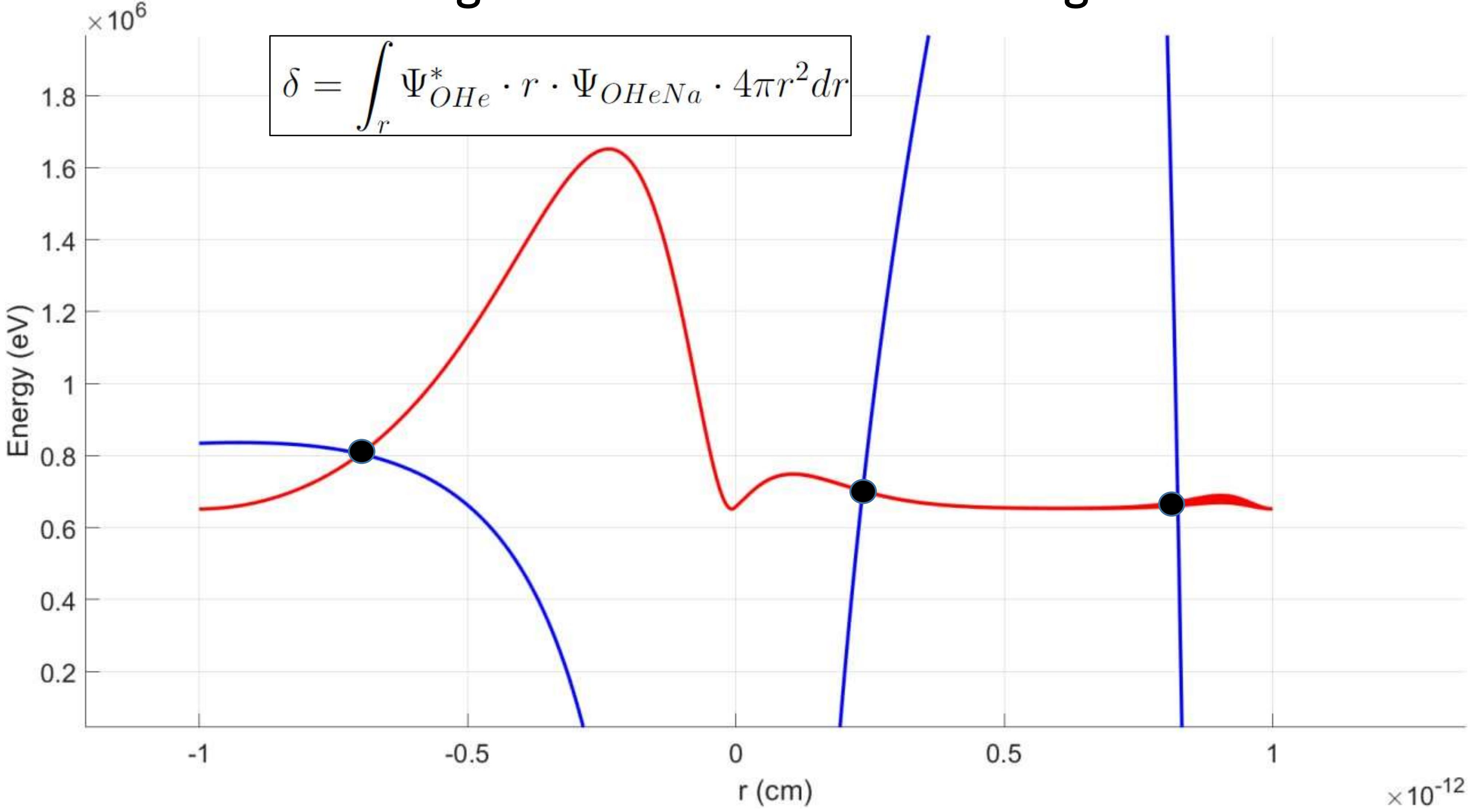
$$\boxed{U_{rot_{(He-Na)}}(|\vec{R}_{OA}-\vec{r}|) = \frac{\hbar^2 c^2 J_{(He-Na)}(J_{(He-Na)}+1)}{2m_{He}c^2|\vec{R}_{OA}-\vec{r}|^2},}$$

$$\boxed{\vec{J}_{(He-Na)} = \overrightarrow{3/2}.}$$

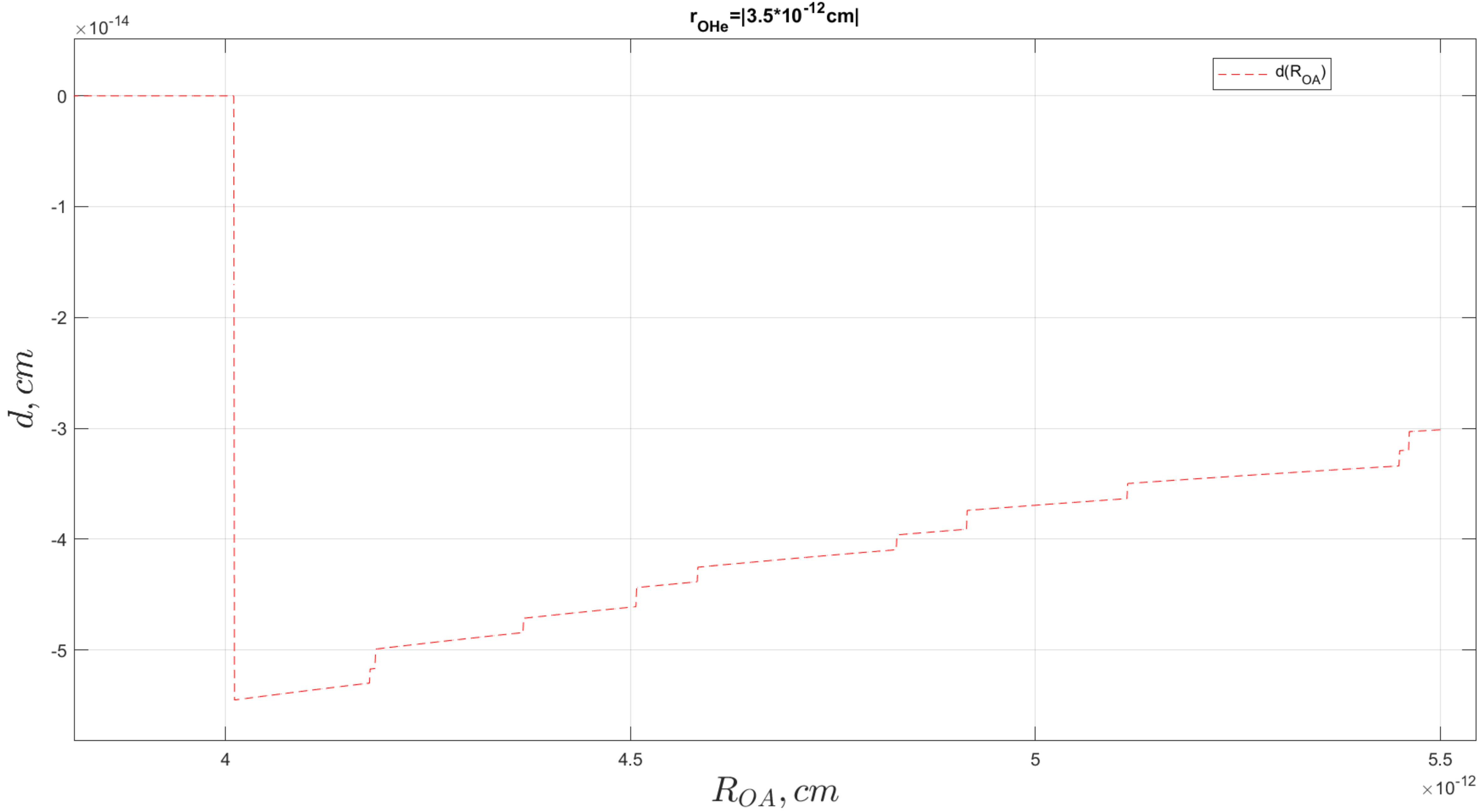


Potentials of Coulomb (red dotted line), nuclear (green dotted line) and centrifugal (green solid line) interaction between helium and the nucleus of substance of Na, potential of Coulomb interaction between helium and O^{--} particle (black dotted line) and the total interaction potential of the helium nucleus (blue dotted line) in the OHe - Na system at fixed R_{OA}. The red circle marks the value of the radius of the He nucleus.

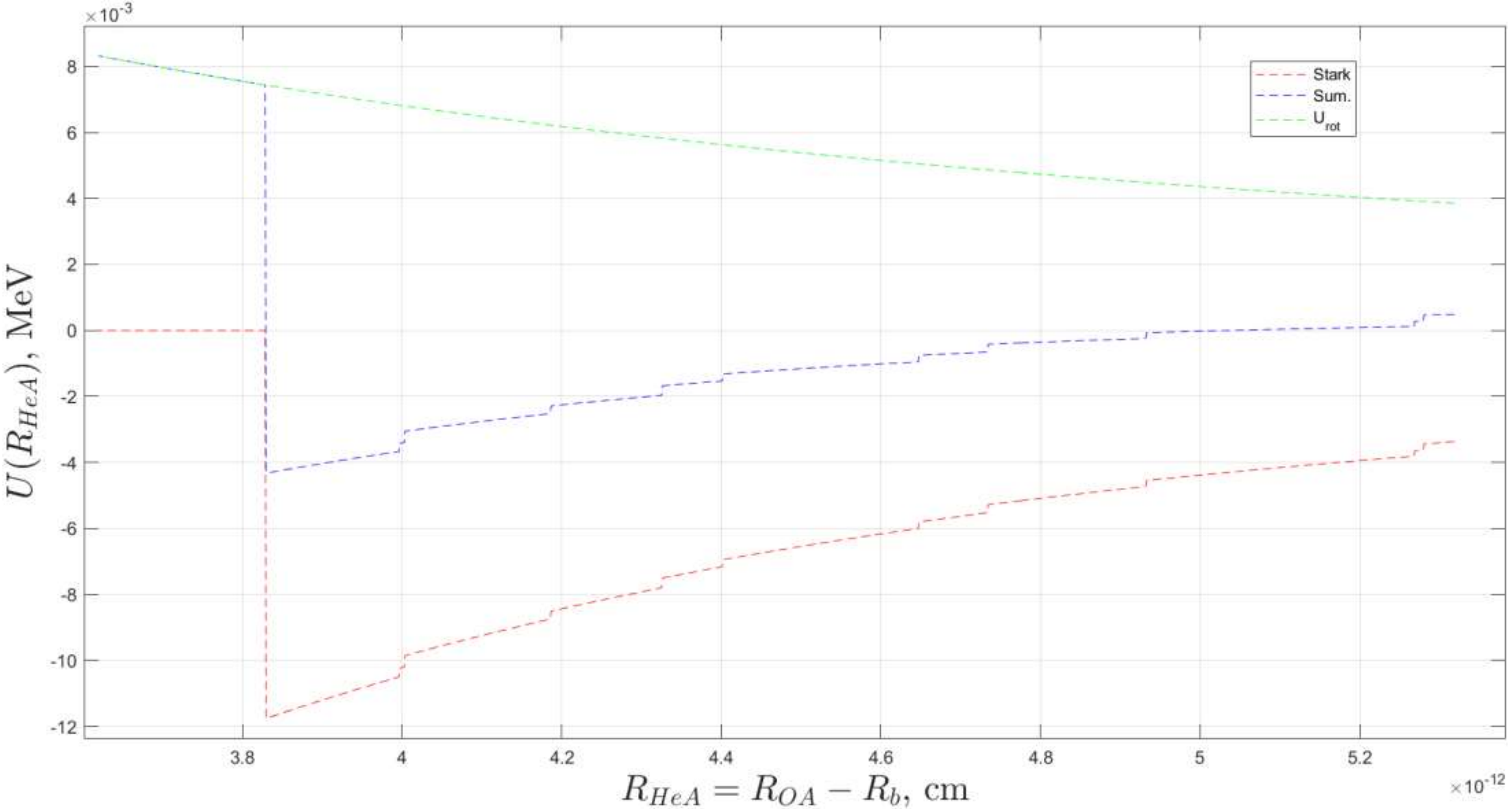
Integration limits for δ calculating.



The total potential of helium in the *O*He - Na system for fixed position of sodium R_{OA} (blue solid line), graph of the squared modulus of the wave function of the ground state of helium in polarized «dark atom» for fixed R_{OA} (red solid line), the intersection points of the graph of the total potential of helium and the graph of the squared modulus of the wave function of the ground state of helium (black circles).



Graph of the dependence of the dipole moment of polarized OHe atom (red dotted line) on the radius vector of the outer sodium nucleus R_{OA}.

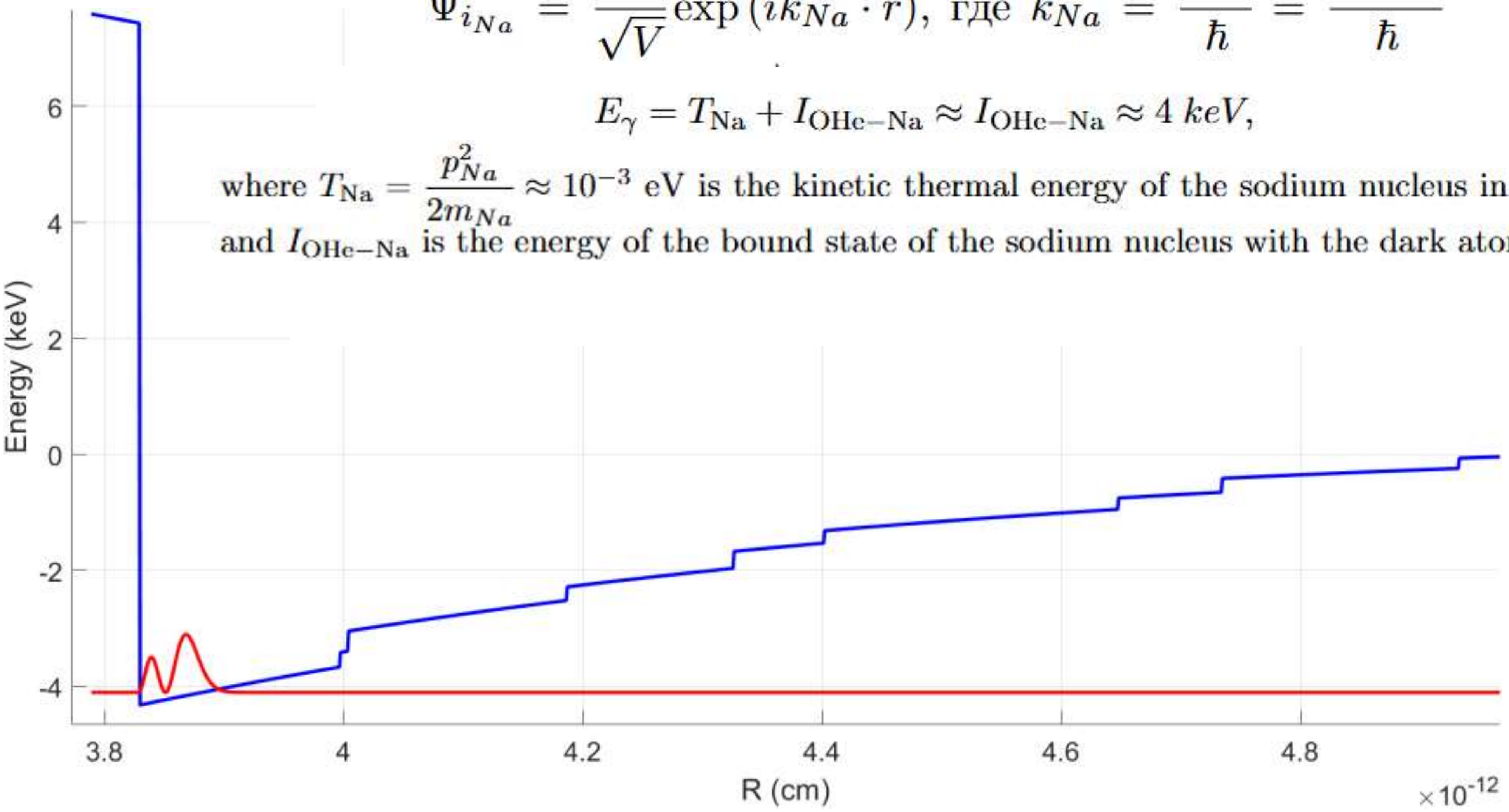


Interaction potentials in the OHe-Na system as a function of the distance between He, located in the Bohr orbit of the OHe atom, and the Na nucleus: the Stark potential (red dotted line), the centrifugal potential (green dotted line), and the total effective interaction potential (blue dotted line). This case corresponds to the total angular momentum for the interaction between OHe and the sodium nucleus, equal to $\overrightarrow{J_{(OHe-Na)}}$=3.

$$\Psi_{i_{Na}} = \frac{1}{\sqrt{V}} \exp\left(i\vec{k}_{Na} \cdot \vec{r}\right), \text{ где } \vec{k}_{Na} = \frac{\vec{p}_{Na}}{\hbar} = \frac{m_{Na}\vec{v}}{\hbar}$$

$$E_{\gamma} = T_{\mathrm{Na}} + I_{\mathrm{OHe-Na}} \approx I_{\mathrm{OHe-Na}} \approx 4\ keV,$$

where $T_{\mathrm{Na}} = \frac{p_{Na}^2}{2m_{Na}} \approx 10^{-3}$ eV is the kinetic thermal energy of the sodium nucleus in the free state and $I_{\mathrm{OHe-Na}}$ is the energy of the bound state of the sodium nucleus with the dark atom of OHe.



Graphs of the dependence of the total effective interaction potential of the OHe dark atom with the sodium nucleus (blue solid line) and the square of the modulus of the sodium wave function (red solid line) corresponding to the energy level of the ground state of sodium in this total effective interaction potential of the OHe-Na system, equal to 4.1 keV, on the radius vector of the sodium nucleus.

Cross-section of XHe inelastic capture by the nucleus of substance.

According to Fermi's Golden Rule, the probability of transition per unit of time from the initial state $|i\rangle$ to the set of final states $|f\rangle$ is, to first approximation, determined by the expression:

$$\Gamma_{i\to f} = \frac{2\pi}{\hbar}\left|\langle f|H'|i\rangle\right|^2 g(E_f), \qquad (1)$$

where $\langle f|H'|i\rangle$ is the matrix element of the disturbance H' between the final and initial states, and $g(E_f)$ - is the density of states at the energy of the final states E_f.

$$g(E_\gamma) = 2V\frac{d}{dE_\gamma}\left(\int\frac{d^3q}{(2\pi)^3}\right) = \frac{E_\gamma^2}{4\pi^3c^3\hbar^3}V\,d\Omega, \qquad (2)$$

where $\vec{q}$ is the wave vector of the photon.

The cross section of the inelastic capture of the sodium nucleus into the bound state of OHe-Na is expressed by the formula:

$$\sigma_{\mathrm{OHe-Na}} = \frac{\Gamma_{i\to f}}{j}, \qquad (3)$$

where $j = v_{\mathrm{OHe}}\cdot n_{\mathrm{Na}}$, - is the flux of sodium nuclei. Assuming that the volume of V is such that it contains one particle of the sodium nucleus $n_{\mathrm{Na}} = 1/V$, the flux of sodium nuclei can be expressed as: $j = v_{\mathrm{OHe}}/V = v/V$

The transition of the sodium nucleus to a bound state occurs when interacting with a photon, and in the dipole approximation, the H' perturbation between the final and initial states is given by:

$$H' = \frac{Z_{Na} e}{m_{Na}} A_0 (\vec{\epsilon} \cdot \vec{p}_{Na}), \tag{4}$$

where Z_{Na} is the charge number of the sodium nucleus, $\vec{\epsilon}$ is the polarization vector of the electromagnetic wave, and $A_0 = \sqrt{\frac{h^2}{2\varepsilon_0 E_\gamma V}}$ is the amplitude of the vector potential of the electromagnetic wave, where ε_0 is the dielectric constant of the vacuum.

$$\langle f|H'|i\rangle = \frac{Z_{Na} e}{m_{Na}} A_0 \left(-ih\vec{\epsilon} \int \Psi^*_{1_{Na}} \nabla \Psi_{i_{Na}} d^3\vec{r} \right), \tag{5}$$

where the gradient of the wave function of the initial state of the sodium nucleus $\nabla \Psi_{i_{Na}} = \frac{i\vec{k}_{Na}}{\sqrt{V}} \exp{(i\vec{k}_{Na} \cdot \vec{r})} = i\vec{k}_{Na} \Psi_{i_{Na}}$, therefore we can express the matrix element of the perturbation as follows:

$$\langle f|H'|i\rangle = \frac{Z_{Na} e}{m_{Na}} A_0 \frac{h}{\sqrt{V}} (\vec{\epsilon} \cdot \vec{k}_{Na}) \widetilde{\Psi^*}_{1_{Na}} (\vec{k}_{Na}), \tag{6}$$

where $\widetilde{\Psi^*}_{1_{Na}}(\vec{k}_{Na}) = \int \Psi^*_{1_{Na}} \exp{(i\vec{k}_{Na} \cdot \vec{r})} d^3\vec{r}$, - is the Fourier transform of the sodium wave function in the final OHe-Na bound state.

The scalar multiplication of the polarization vector of the electromagnetic wave and the wave vector of the sodium nucleus:

$$(\vec{\epsilon} \cdot \vec{k}_{Na}) = |\vec{k}_{Na}| \sin(\theta) = \frac{m_{Na} v}{h} \sin(\theta), \quad (7)$$

$$\sigma_{\mathrm{OHe-Na}} = \frac{\alpha}{\hbar c} Z_{Na}^2 \frac{v}{c} \frac{E_\gamma}{\pi} |\widetilde{\Psi^*}_{1_{Na}}(\vec{k}_{Na})|^2 \sin^2(\theta) d\Omega. \quad (8)$$

Integral over a solid angle:

$$\sin^2(\theta) d\Omega = \frac{8\pi}{3}. \quad (9)$$

Finally, the final expression for the cross section of inelastic capture of the sodium nucleus into the OHe-Na bound state is:

$$\sigma_{\mathrm{OHe-Na}} = \frac{8}{3} \frac{\alpha}{\hbar c} Z_{Na}^2 \frac{v}{c} E_\gamma |\widetilde{\Psi^*}_{1_{Na}}(\vec{k}_{Na})|^2, \quad (10)$$

$$\sigma_{\mathrm{OHe-Na}} \approx 2.3 \cdot 10^{-34}\ cm^2 = 2.3 \cdot 10^{-10}\ barn. \quad (11)$$

Comparison with the DAMA experiment.

$$f = 1.4 \cdot 10^{-3}$$

$$\boxed{\sigma v = \frac{f\pi\alpha}{m_p^2}\frac{3}{\sqrt{2}}\left(\frac{Z}{A}\right)^2\frac{T}{\sqrt{Am_pE}},}$$

$$\begin{array}{lcl} \langle\sigma v\rangle_{Na} & \approx & 5.8\cdot 10^{-31}\,\mathrm{cm}^3/\mathrm{s}, \\ \langle\sigma v\rangle_{I} & \approx & 1.9\cdot 10^{-31}\,\mathrm{cm}^3/\mathrm{s}. \end{array}$$

$$(\sigma_{\mathrm{OHe-Na}}\cdot v) = 8\frac{\alpha c}{\hbar c}Z_{Na}^2\frac{kT}{M_o c^2}E_\gamma|\widetilde{\Psi^*}_{1_{Na}}(\vec{k}_{Na})|^2.$$

$$(\sigma_{\mathrm{OHe-Na}}\cdot v) \approx 1.4\cdot 10^{-30}\,cm^3/sec.$$

$$R_{Quant-mech} \approx 0.51 + 3.49\times 10^{-2}\cos(\omega(t-t_0))\ \mathrm{counts/day\ kg}$$

$$R = 1\cdot\frac{\rho_O}{M_O}(\langle\sigma v\rangle_{Na} + \langle\sigma v\rangle_I)\cdot N_T, \quad \textbf{(12)}$$

$$\rho_O = \frac{M_O n_0}{320\cdot S_3\cdot 30^{1/2}}V_h + \frac{M_O n_0}{640\cdot S_3\cdot 30^{1/2}}V_E\cos(\omega(t-t_0)),$$

where $S_3 = M_O/1\,TeV$, and the mass of XHe is assumed $M_O = 2\,TeV$, the number of target N_T is the same for Na and Iodine nuclei: $N_T = 4.015\times 10^{24}$ nuclei per kg of NaI(Tl), the speed of Solar System is taken as $V_h = 233\cdot 10^5$ cm/s and the speed of Earth is $V_E = 30\cdot 10^5$ cm/s, local dark matter concentration in case of considered particles $n_0 = 1.5\cdot 10^{-4}\,\mathrm{cm}^{-3}$. Due to the large mass of the DAMA/LIBRA detectors (approximately 9.7 kg each), we can assume that the low-energy γ-rays are completely absorbed inside the detector.

$$\Delta R = (6.95\pm 0.45)\times 10^{-2}\ \mathrm{counts/(day\ kg)}$$

$$R_0 < 0.5\ \mathrm{counts/(day\ kg)}$$

$$\begin{aligned} R &\approx 2.951\cdot 10^{-6} + 2.011\cdot 10^{-7}\cos(\omega(t-t_0))\ \mathrm{counts/s\ kg} \\ &= 0.255 + 0.017\cos(\omega(t-t_0))\ \mathrm{counts/day\ kg} \end{aligned}$$

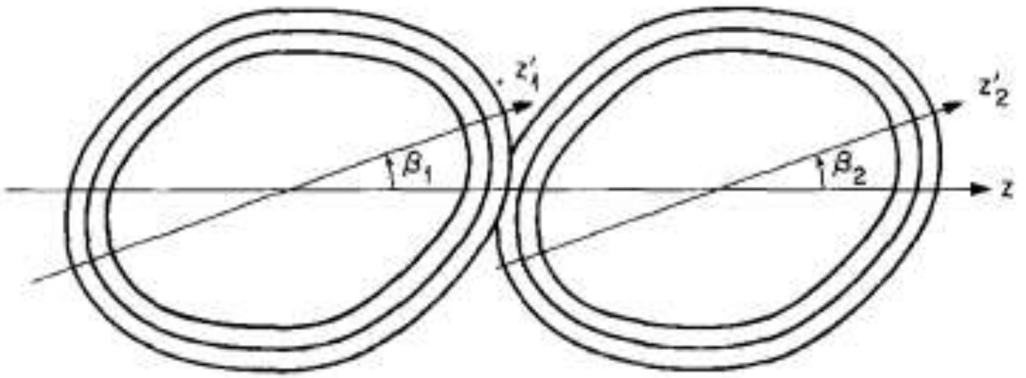
$$U = \sum_{l_1, l_2} U(l_1, l_2)$$

$$A^{(j)}_{lm}(k) = \int d^3 r \rho_j(\mathbf{r}) j_l(kr) Y_{lm}(\hat{r})$$

$$U(0,0) = \frac{2}{\pi} \int_0^\infty dk k^2 j_0(kr) \tilde{v}(k) A'^{(1)}_{00}(k) A'^{(2)}_{00}(k)$$

$$U(0,2) = \frac{2}{\pi} \sqrt{5} \int_0^\infty dk k^2 j_2(kr) \tilde{v}(k) [A'^{(1)}_{00}(k) A'^{(2)}_{20}(k)$$
$$\cdot P_2(\cos\beta_2) + A'^{(1)}_{20}(k) A'^{(2)}_{00}(k) P_2(\cos\beta_1)]$$

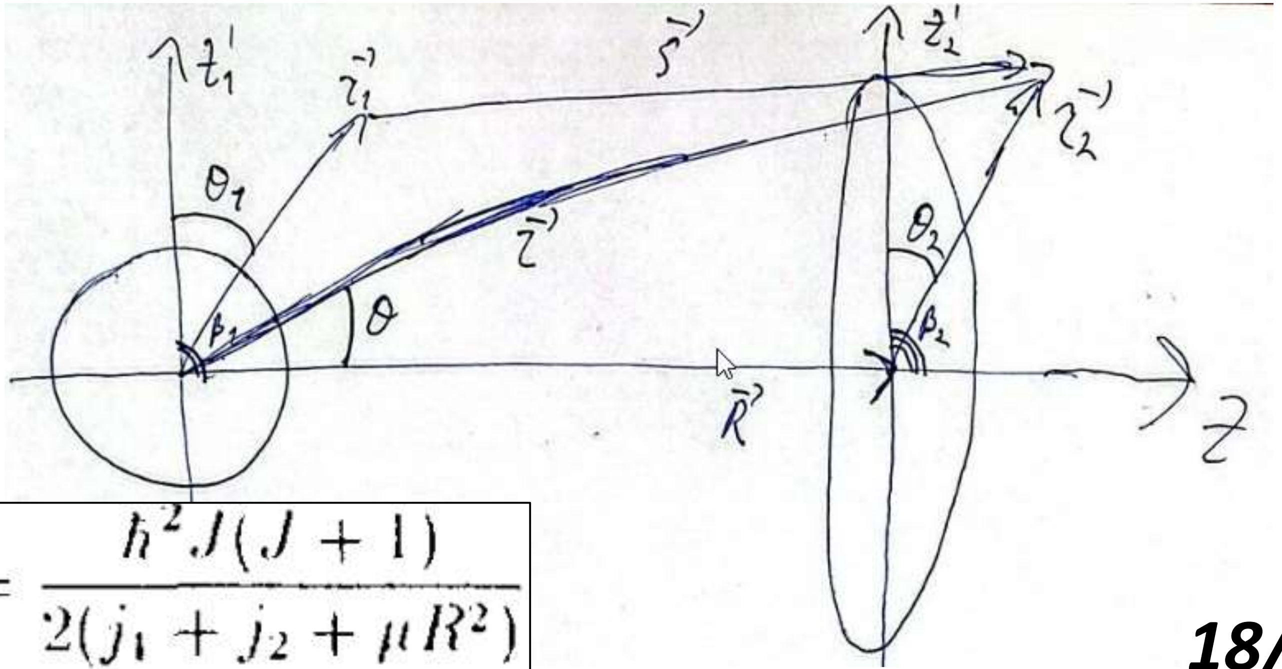
$$U(0,4) = \frac{6}{\pi} \int_0^\infty dk k^2 j_4(kr) \tilde{v}(k) [A'^{(1)}_{00}(k) A'^{(2)}_{40}(k)$$
$$\cdot P_4(\cos\beta_2) + A'^{(1)}_{40}(k) A'^{(2)}_{00}(k) P_4(\cos\beta_1)]$$



$$\rho_i(r,\theta) = \frac{\rho_0^i}{1 + \exp\left(\frac{r - R_i(\theta)}{a_i}\right)}$$

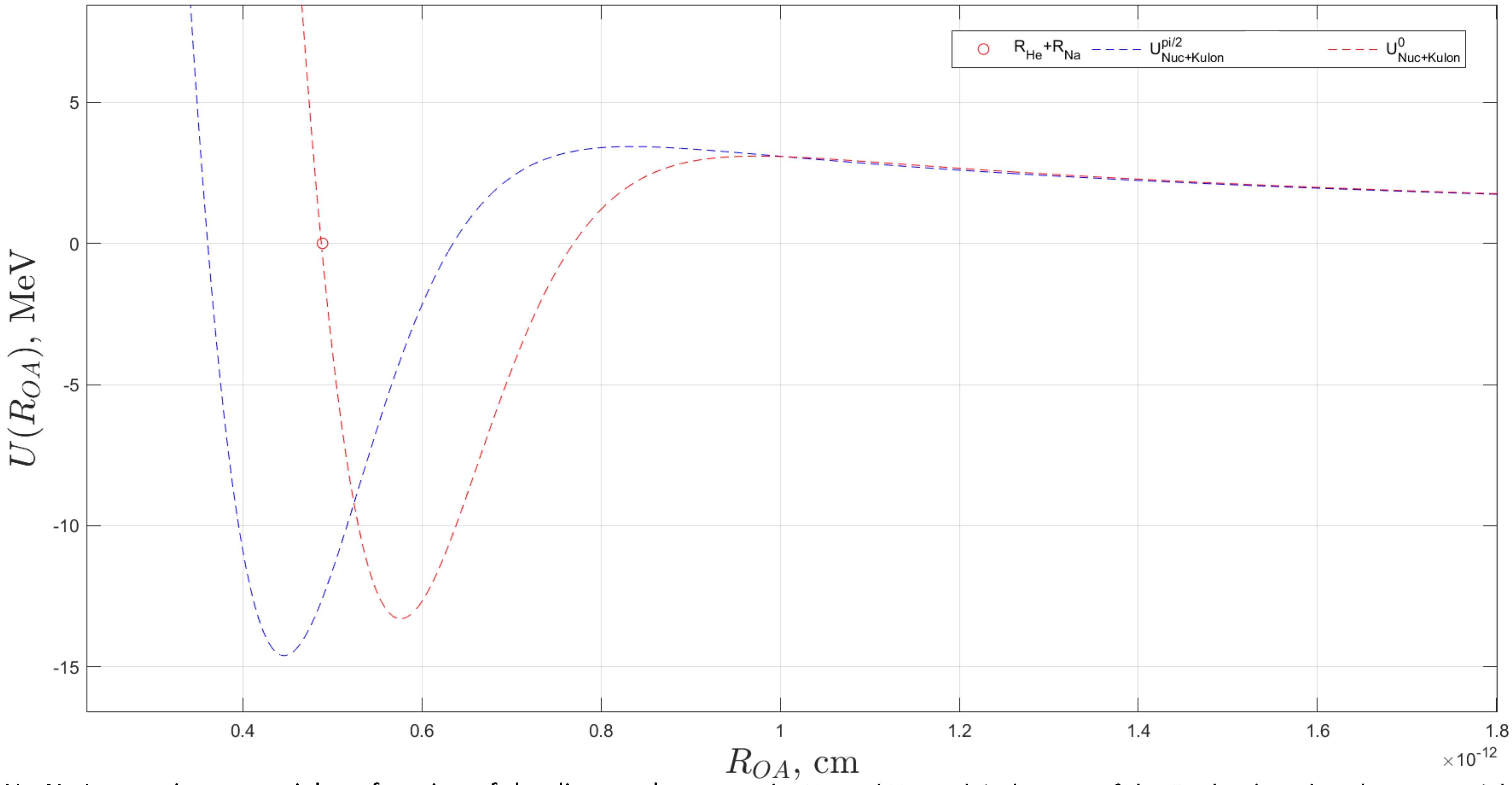
$$R_i(\theta) = R_0^i \left[1 + \delta_2^i Y_{2,0}(\theta) + \delta_4^i Y_{4,0}(\theta)\right]$$

$$\begin{aligned} U_N(R) &= C_0 \Bigg\{ \frac{F_{in} - F_{ex}}{\rho_{00}} \left(\int \rho_1^2(\mathbf{r}) \rho_2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \right. \\ &+ \left. \int \rho_1(\mathbf{r}) \rho_2^2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \right) + F_{ex} \int \rho_1(\mathbf{r}) \rho_2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \Bigg\}, \\ F_{in,ex} &= f_{in,ex} + f'_{in,ex} (N_1 - Z_1)/A_1 \cdot (N_2 - Z_2)/A_2, \end{aligned}$$



$$\mathcal{J}_{\mathrm{RB}} = \frac{2}{5} M R^2 \left(1 + \frac{1}{2}\sqrt{\frac{5}{4\pi}}\beta + \frac{25}{32\pi}\beta^2\right)$$

$$U_{rot}(R) = \frac{\hbar^2 J(J+1)}{2(j_1 + j_2 + \mu R^2)}$$



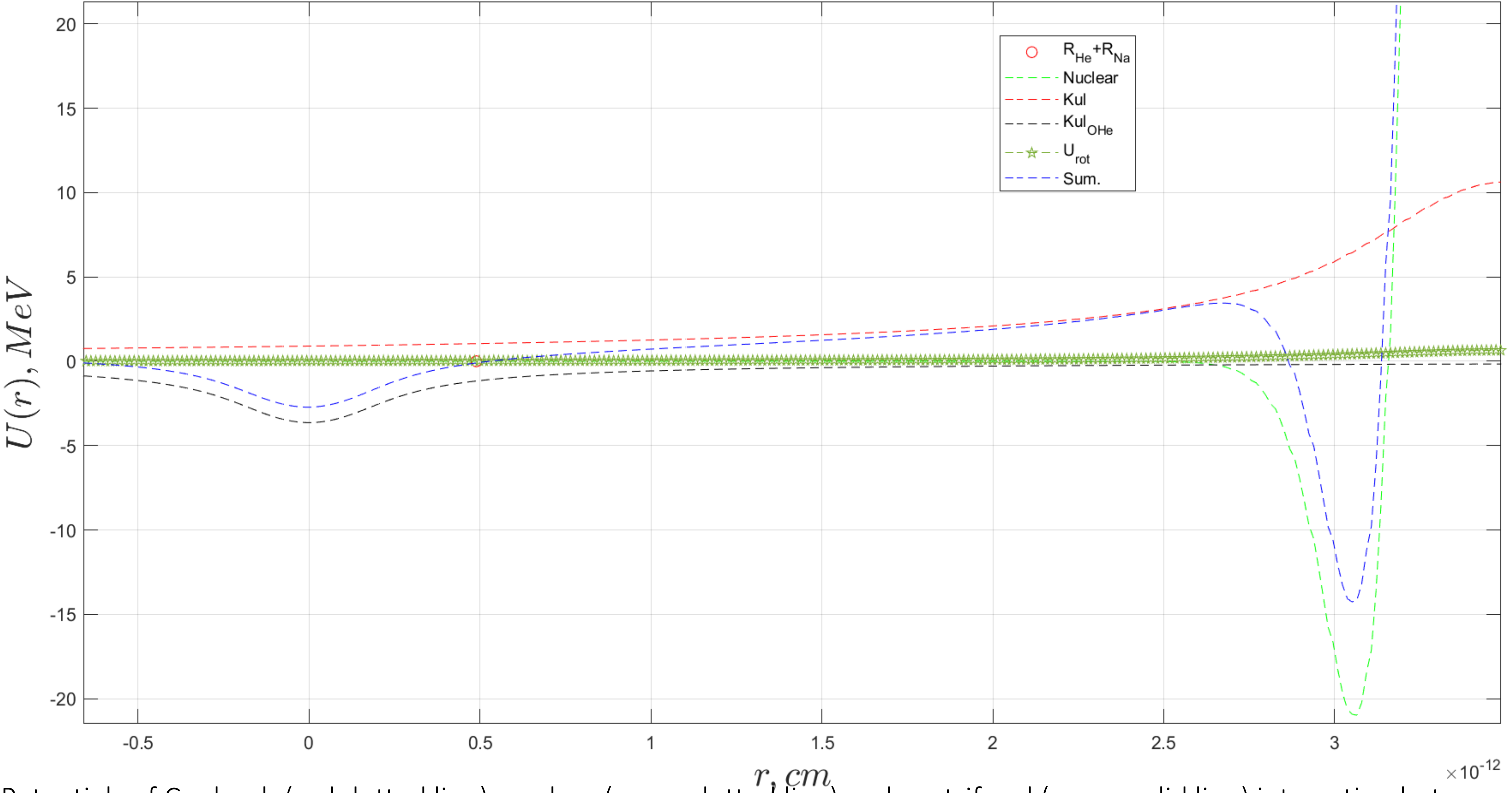
He-Na Interaction potentials as function of the distance between the He and Na nuclei: the sum of the Coulomb and nuclear potentials of the interaction of helium with sodium for the case of lateral collision (blue dotted line), the sum of the Coulomb and nuclear potentials of the interaction of helium with sodium for the case of axially symmetric configuration (red dotted line).

Conclusion.

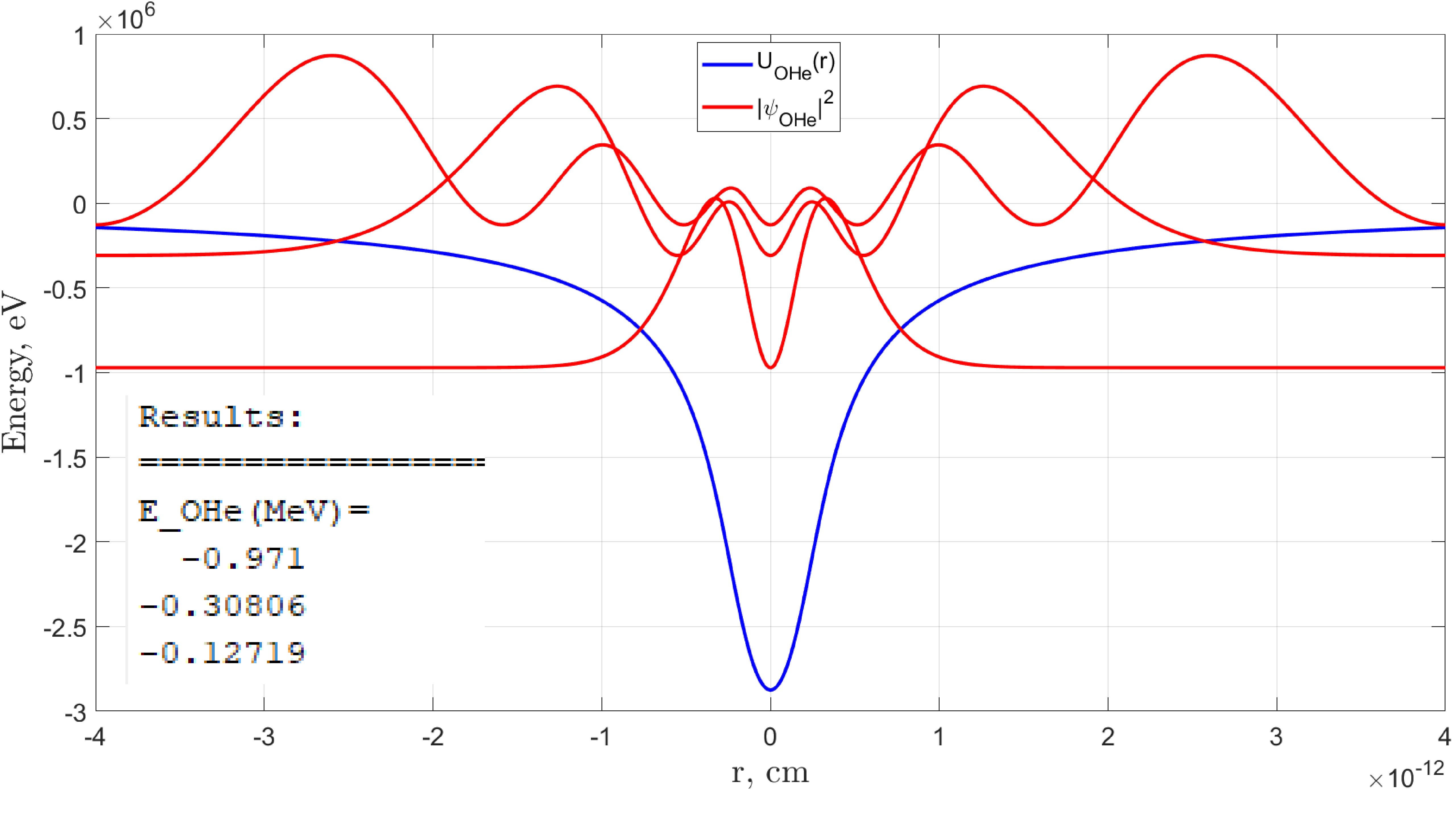
- Quantum mechanical numerical model of the interaction of the OHe dark atom with the nucleus of Na is constructed.
- The total effective interaction potential of sodium with dark atom in the OHe –Na system has been restored, the shape of which is consistent with the theoretically expected one.
- Expression is obtained for calculating the cross section of inelastic capture of the nucleus of substance into the OHe-nucleus bound state, the calculation of the rate of radiative capture for the sodium nucleus is consistent with the experimental restriction on the capture rate at mass of dark atom greater than 2 TeV.
- ***It is planned*** *to improve the quantum mechanical numerical model of the interaction of the X-helium dark atom with the nucleus of matter, taking into account the non-point-like structure of the interacting particles (the nucleus of matter and the dark atom), the inhomogeneities of the distributions of electric charge and nucleons in the nuclei and the deformation of the Na nucleus and its spatial configuration described by the angle between its axis of axial symmetry and the straight line connecting the centers of interacting particles. It is also planned the solution of the one-dimensional Schrodinger equation for the sodium nucleus in the total effective interaction potential of the XHe-nucleus system reconstructed using this quantum mechanical numerical model. After that, it is planned to calculate of the cross section of inelastic capture of XHe dark atom by sodium nucleus into a low-energy bound state of the XHe-nucleus system and comparison with the results of experiments on direct search for particles of dark matter.*

Thank you for your attention!

Additional slides



Potentials of Coulomb (red dotted line), nuclear (green dotted line) and centrifugal (green solid line) interaction between helium and the nucleus of substance of Na, potential of Coulomb interaction between helium and O^{--} particle (black dotted line) and the total interaction potential of the helium nucleus (blue dotted line) in the OHe - Na system at fixed R_{OA}. The red circle indicates the value of the sum of the radii of the He and Na nuclei.



×10^6
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
Energy, eV
U_OHe(r)
|ψ_OHe|^2
Results:
===================
E_OHe(MeV)=
-0.971
-0.30806
-0.12719
-4
-3
-2
-1
0
1
2
3
4
r, cm
×10^-12

Structure of the bound state of X-helium.

The structure of the bound state of X-helium depends on the value of the parameter:

$$\mathrm{a} = Z_{\boldsymbol{\alpha}} Z_X \boldsymbol{\alpha} \mathrm{A} m_p R_{nHe}$$

- At 0 < a < 1 bound state looks like Bohr atom with doubly negatively charged O^{--} particle in the core and *He* nucleus moving along Bohr orbit.

$$I_0 = \frac{Z_{O^{--}}^2 Z_{He}^2 \alpha^2 m_{He}}{2} \approx 1.6 \text{ MeV} \qquad R_b = \frac{\hbar c}{Z_{O^{--}} Z_{He} m_{He} \alpha} \approx 2 \cdot 10^{-13} \text{ cm}$$

- At 1 < a < ꚙ bound state looks like Thomson atoms, in which the nuclear body nHe vibrates around heavy negatively charged particle X.

Root mean square radii of distribution of neutrons and protons.

$$R_{N_{nuc,He}} = \sqrt{\frac{3}{5}R^2_{0N_{nuc,He}} + \frac{7\pi^2}{5}a^2_{N_{nuc,He}}}\sqrt{1+\frac{5b^2_{nuc,He}}{4\pi}}\ \text{fm},$$

$$R_{0N_{nuc,He}} = 0.953N^{1/3}_{nuc,He} + 0.015Z_{nuc,He} + 0.774\ \text{fm},$$

$$a_{N_{nuc,He}} = 0.446 + 0.072\frac{N_{nuc,He}}{Z_{nuc,He}}\ \text{fm}.$$

$$R_{p_{nuc}} = \sqrt{\frac{3}{5}R^2_{0p_{nuc}} + \frac{7\pi^2}{5}a^2_{p_{nuc}}}\sqrt{1+\frac{5b^2_{nuc}}{4\pi}}\ \text{fm},$$

$$R_{0p_{nuc}} = 1.322Z^{1/3}_{nuc} + 0.007N_{nuc} + 0.022\ \text{fm},$$

$$a_{p_{nuc}} = 0.449 + 0.071\frac{Z_{nuc}}{N_{nuc}}\ \text{fm}.$$

Electric potential of unpolarized X-helium.

$$\psi = \frac{e^{-r/r_0}}{\sqrt{\pi} r_0^{3/2}} \qquad en_p = \begin{cases} \dfrac{eZ_\alpha}{\frac{4}{3}\pi R_{nHe}^3} & \text{for } r < R_{nHe}, \\ 0 & \text{for } r > R_{nHe}. \end{cases}$$

$$\frac{1}{r}(\phi r)'' = -4\pi e\left(n_p + \frac{Z_X e^{-2r/r_0}}{\pi r_0^3}\right) \qquad U^e_{XHe} = eZ_A\phi$$

$$\phi = \begin{cases} -eZ_X e^{-2r/r_0}\left(\dfrac{1}{r_0} + \dfrac{1}{r}\right) & \text{for } r > R_{nHe}, \\ -eZ_X e^{-2r/r_0}\left(\dfrac{1}{r_0} + \dfrac{1}{r}\right) + \dfrac{eZ_X}{r} + \dfrac{eZ_\alpha}{R_{nHe}}\left(\dfrac{3}{2} - \dfrac{r^2}{2R_{nHe}^2}\right) & \text{for } r < R_{nHe}. \end{cases}$$

radius vector interval. That is, the radius vector of helium is given as $\vec{r} = [-a; a]$ and the radius vector of the outer nucleus of matter as $\vec{R}_{O\Lambda} = [c; b]$, where $a, c, b > 0$ and $a \leqslant b < c$. Thus, the fixed position of the nucleus of the substance, $\vec{R}_{O\Lambda}$, consistently takes values from the interval $[c; b]$, starting from point c to point b. And for each point $p^* \in [c; b]$, the distance between the helium nucleus and the nucleus of matter, $\vec{R}_{He\Lambda} = \vec{R}_{O\Lambda} - \vec{r}$, varies in the interval $\vec{R}_{He\Lambda} = [p^* + a; p^* - a]$. As the external nucleus approaches the dark matter atom, polarization of OHe intensifies, responding to the proximity of the nucleus.