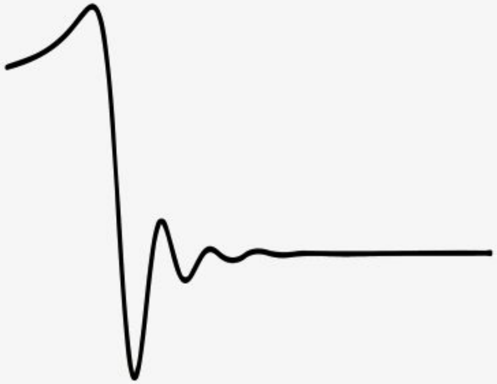
The Merger History of Primordial Black Holes with GWTC and Their Gravitational-Wave Signatures



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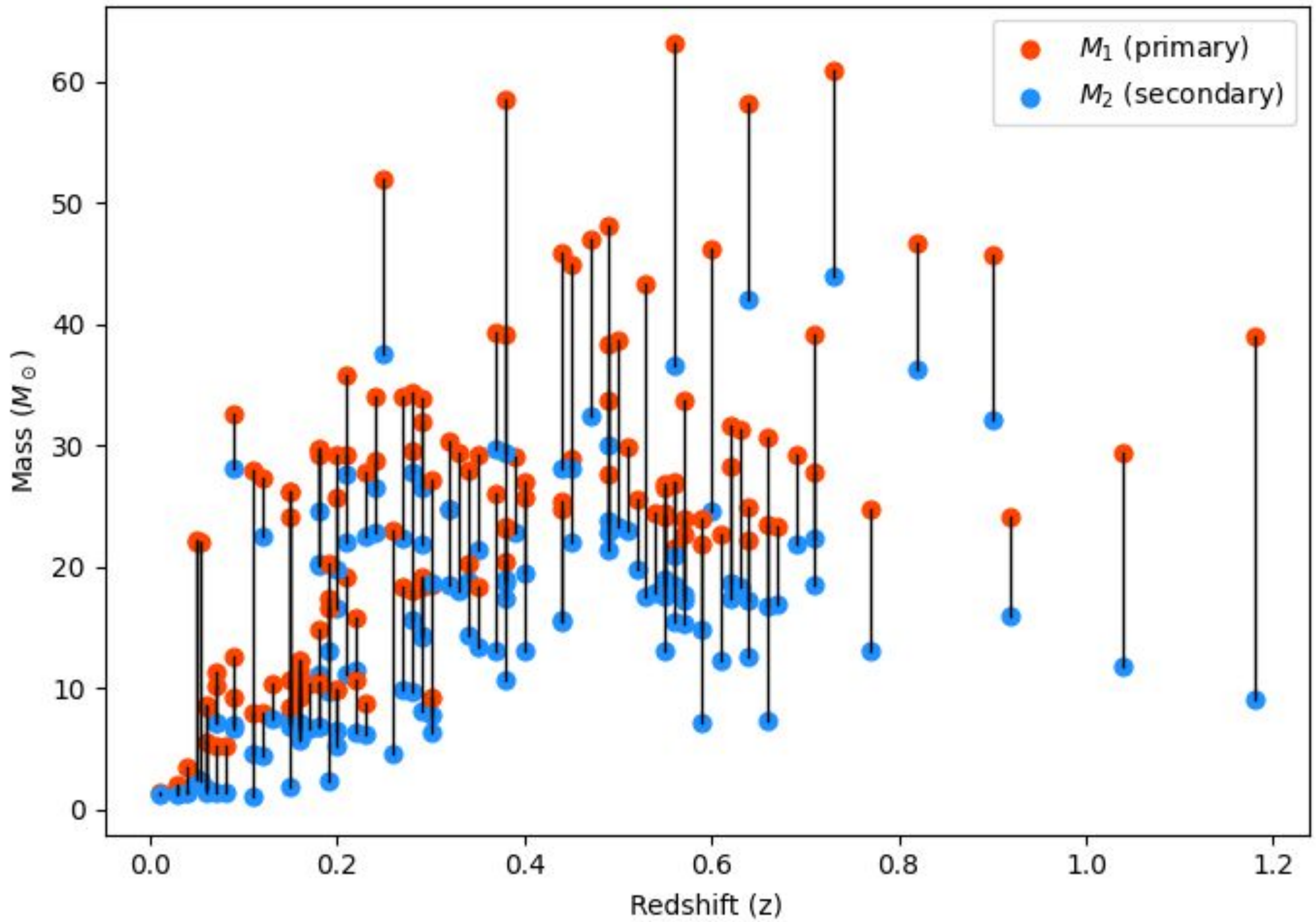
Motivation

- **Unresolved Origin of BBH Mergers:**
  Despite numerous BBH merger detections, the progenitor origin—astrophysical or primordial—remains uncertain.

- **Primordial Black Holes as Dark Matter Candidates:**
  PBHs, hypothesized to form in the early universe, offer a unique probe of fundamental physics and may constitute a non-negligible fraction of dark matter.

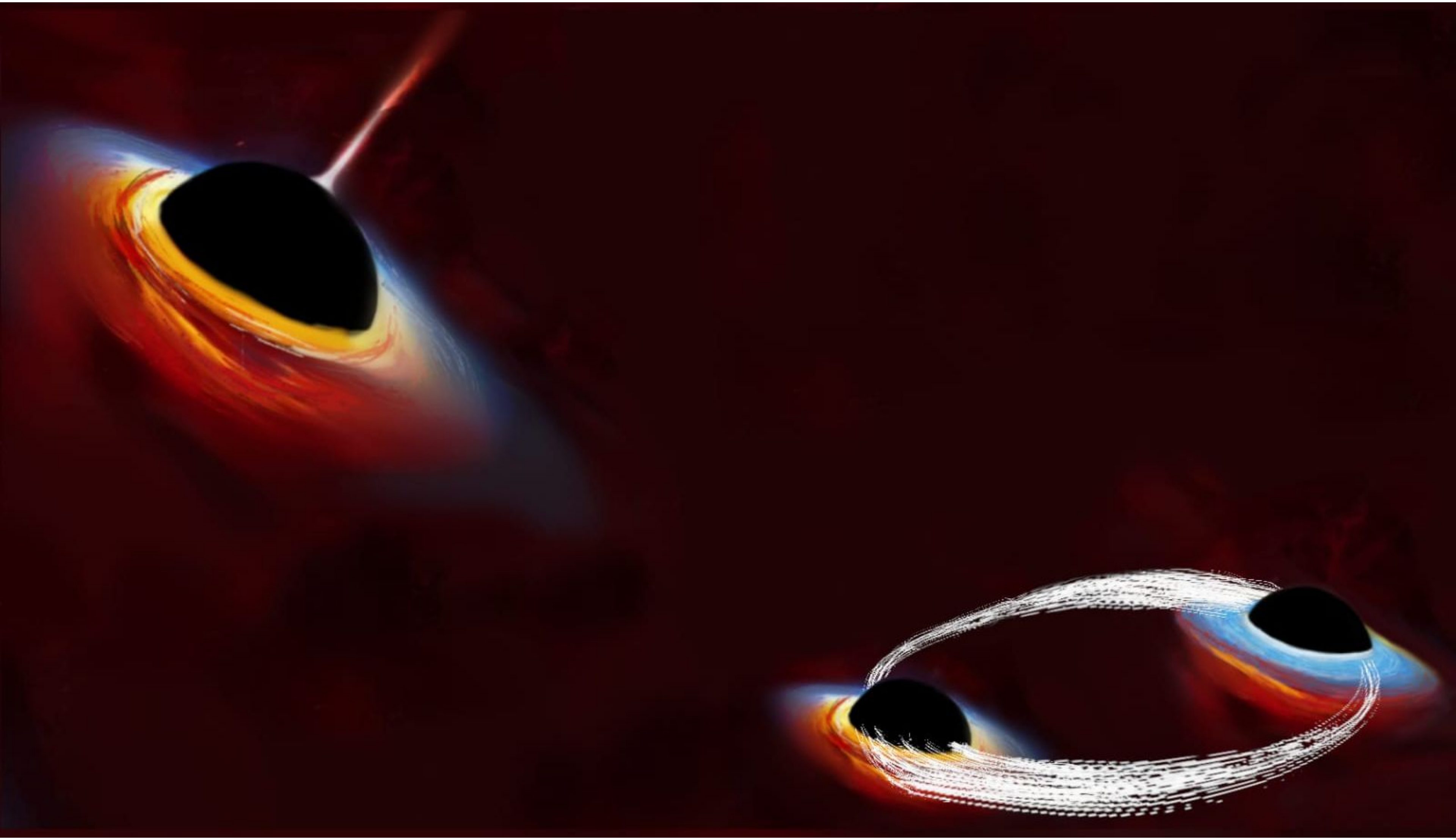
- **Influence of Three-Body Dynamics:**
  Gravitational interaction with a third PBH is crucial for binary formation, as it sets merger timescales and shapes the merger rate distribution.

- **Impact on Fundamental Physics and Cosmology:**
  Constraining PBH properties through GW observations can illuminate early universe conditions, dark matter composition, and the origin of black holes.



The variation of detected binary black hole mergers as a function of redshift by LIGO, Virgo and Kagra collaboration.

Road Map of the work



Schematic representation of the fundamental idea underlying this study.

Model of the Merger Rate Density Distribution

The merger rate density for PBH binaries is

$$\mathcal{R}(t, m_i, m_j) = \int R_l(t, m_i, m_j, m_l)\, \mathrm{d}m_l,$$

where the integrand R_l is

$$R_l(t, m_i, m_j, m_l) = 1.32 \times 10^6 \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} \left(\frac{f_{\mathrm{PBH}}}{m_{\mathrm{PBH}}}\right)^{\frac{53}{37}} m_l^{-\frac{21}{37}} (m_i m_j)^{\frac{3}{37}} (m_i + m_j)^{\frac{36}{37}} F(m_i) F(m_j) F(m_l)$$

The total merger rate density is

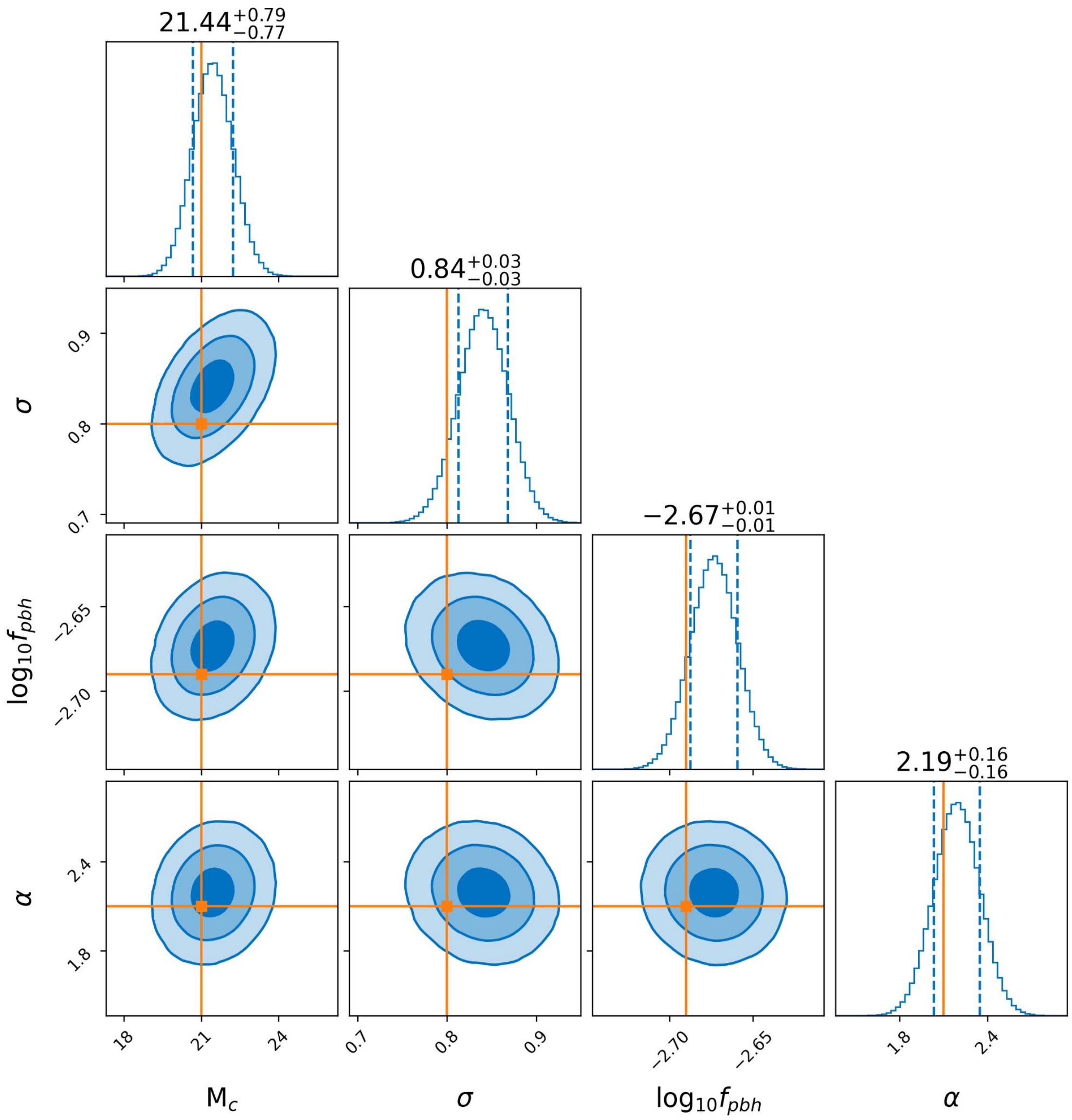
$$\mathcal{R}(t) = \int \mathcal{R}(t, m_i, m_j)\, \mathrm{d}m_i\, \mathrm{d}m_j.$$

where, $F(m) = \frac{P(m)}{m_{\mathrm{PBH}}} m$

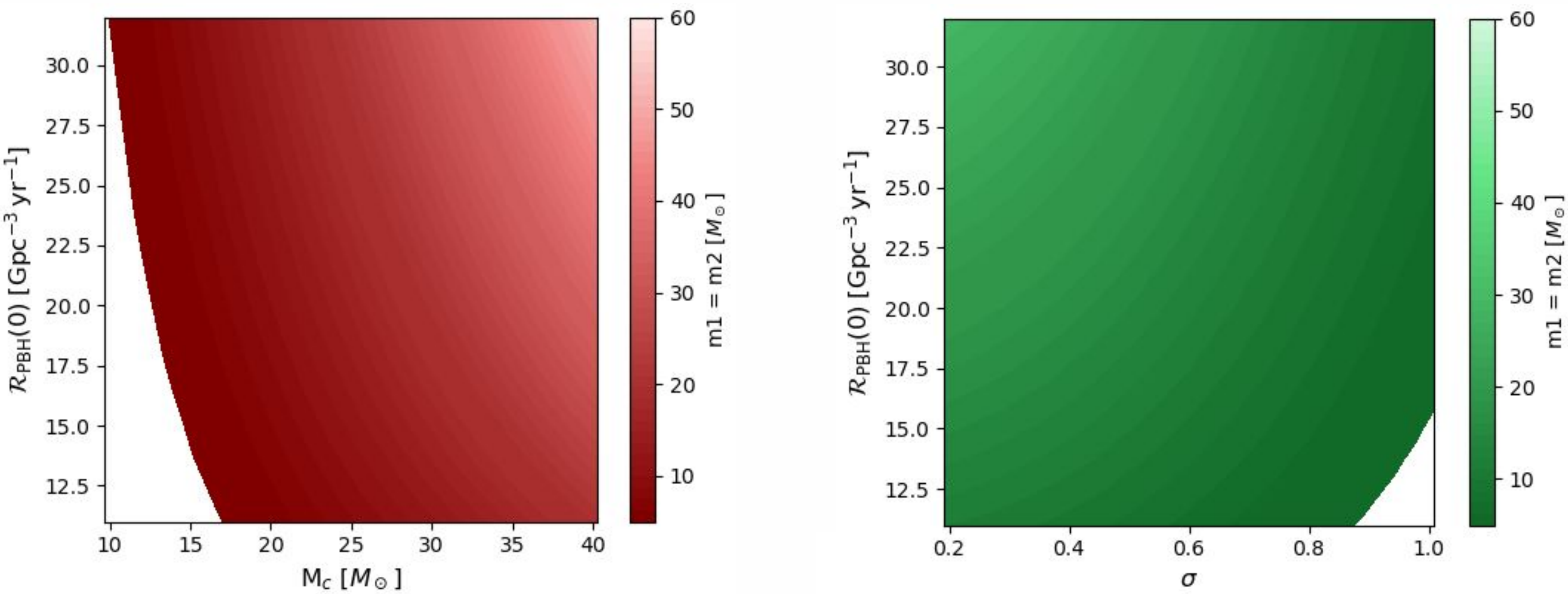
Bayesian Inference of Parameters

For a set of population parameters $\Lambda = \{M_c, \sigma, f_{\rm PBH}, \alpha\}$, we evaluate the likelihood of N observed BBH merger events as :

$$\mathcal{L}(d \mid \Lambda) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\mathcal{R}_{\rm PBH}(m_1^{(i)}, m_2^{(i)}, z^{(i)} \mid \Lambda)}{\mathcal{N}(\Lambda)}$$



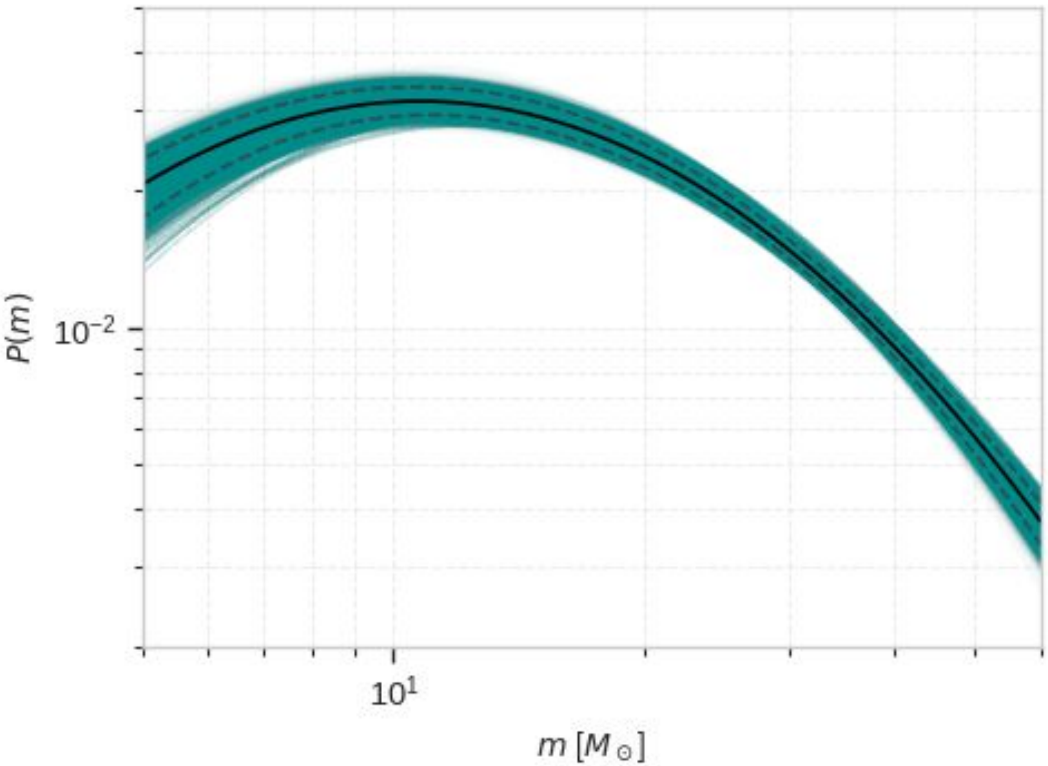
Dependence of Individual Parameters



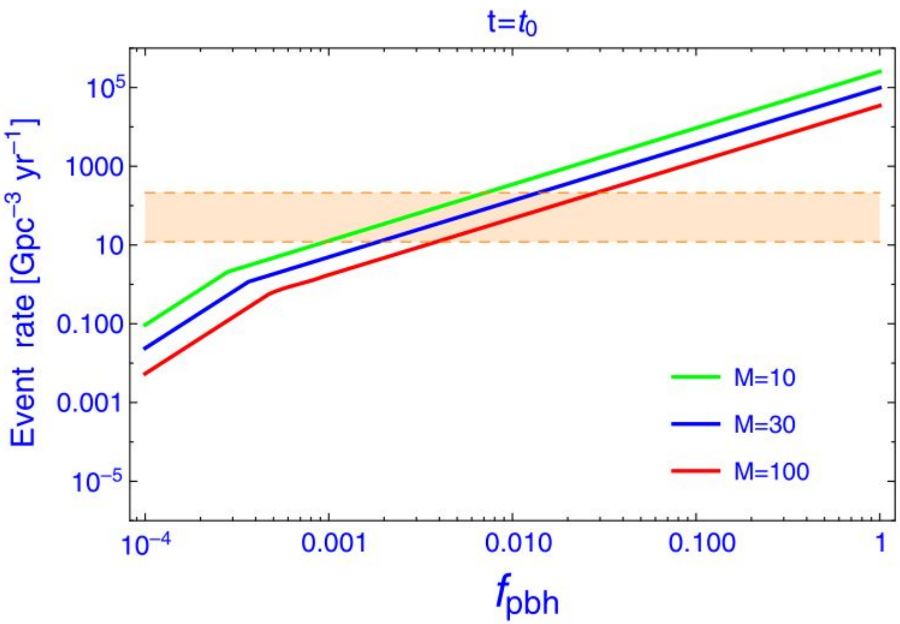
The panels shows that the merger rate $\mathcal{R}_{PBH}(z)$ increases with increasing M_c and decreases with increasing σ.

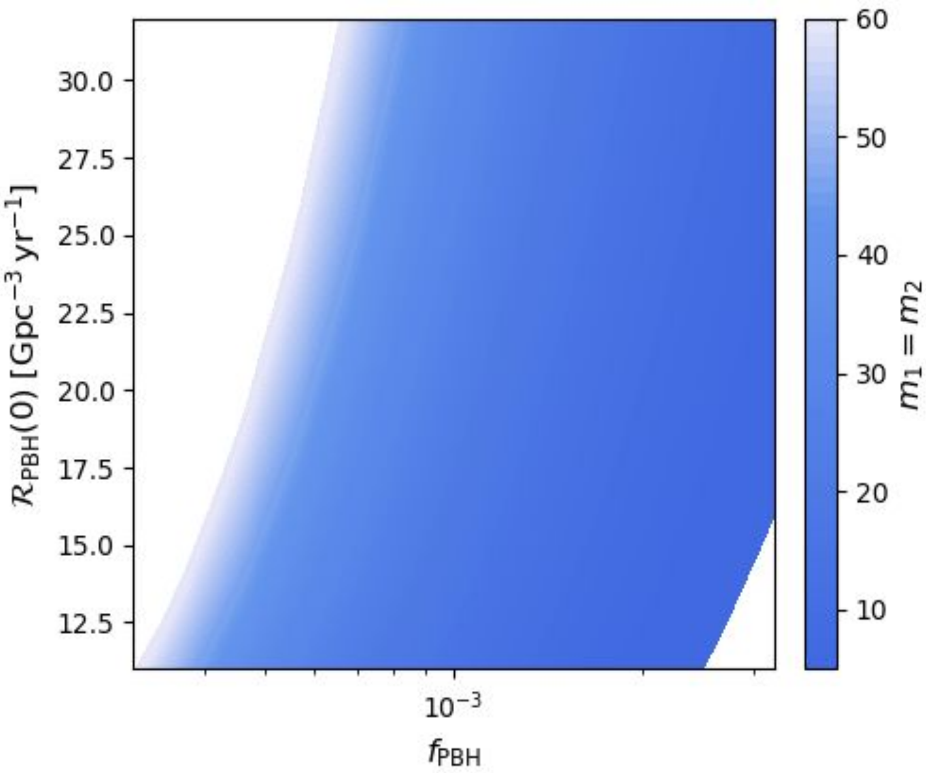
Mass function

$$P(m) = \frac{1}{\sqrt{2\pi}\,\sigma\, m} \exp\left[-\frac{(\ln(m/M_c))^2}{2\sigma^2}\right]$$



Posterior predictive distributions for PBHs assuming a log-normal mass function, inferred from the GW Transient catalog via Bayesian analysis. The black solid line denotes the median, and dashed lines indicate the 90\% credible interval.

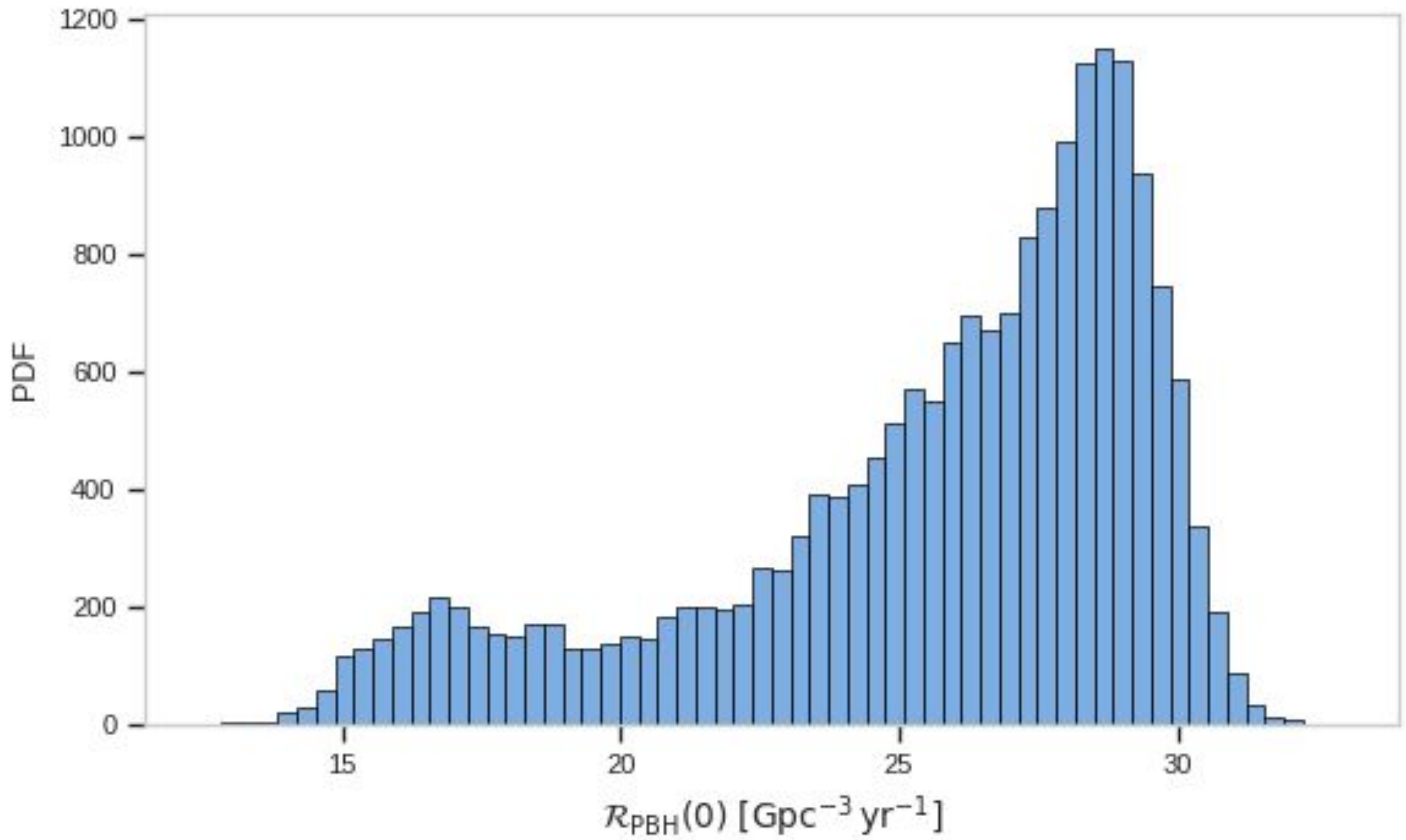




Left panel shows the variation of f_{PBH} for single event while the right panel shows the variation for multiple merger event.

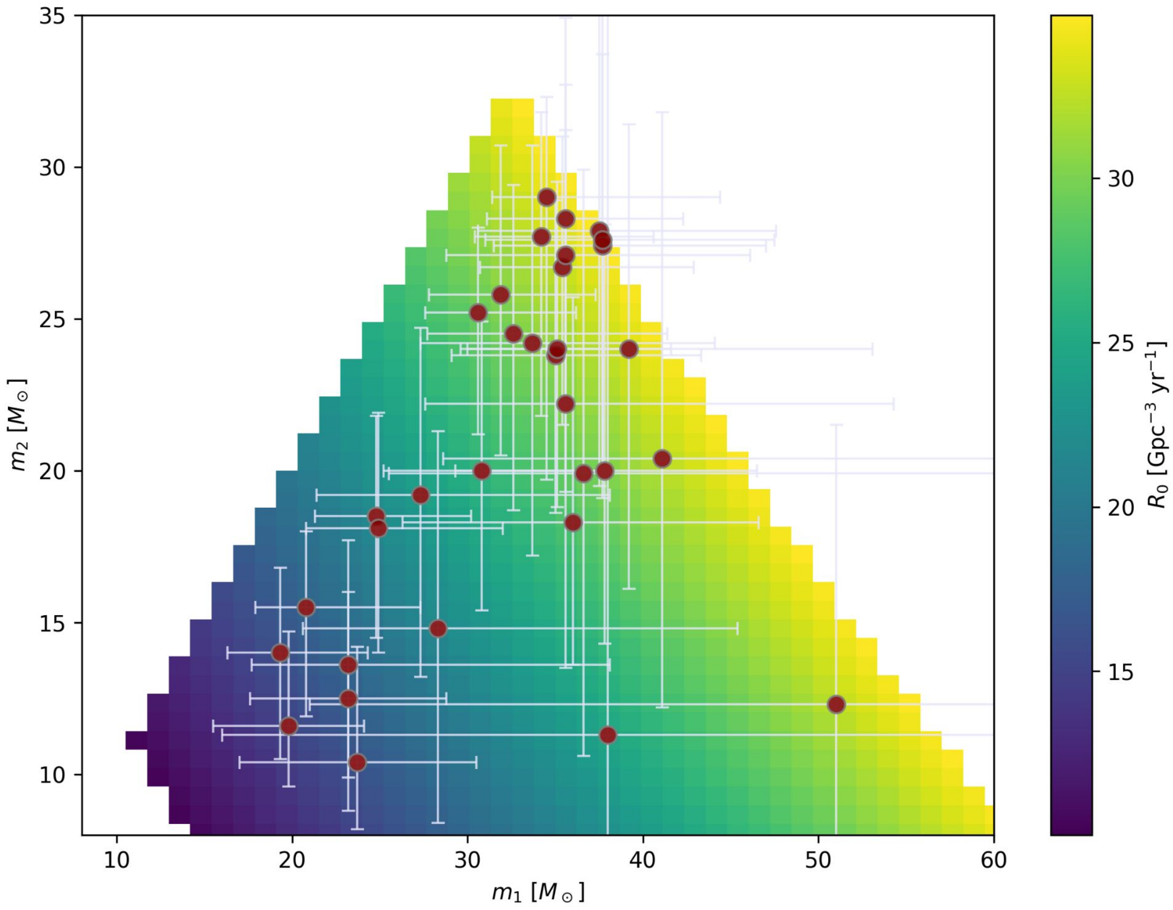
*Left panel fig taken from Liu et al. (Eur. Phys. J. C (2019) 79:717)

Variation of local Event Rate



Histogram plot of predicted local PBH binary merger rate

Mass variation



Two-dimensional merger rate distributions.

Stochastic Background

The normalized GW energy density can be represented by the dimensionless quantity $\Omega_{\rm GW}(f)$ can be written as,

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f},$$

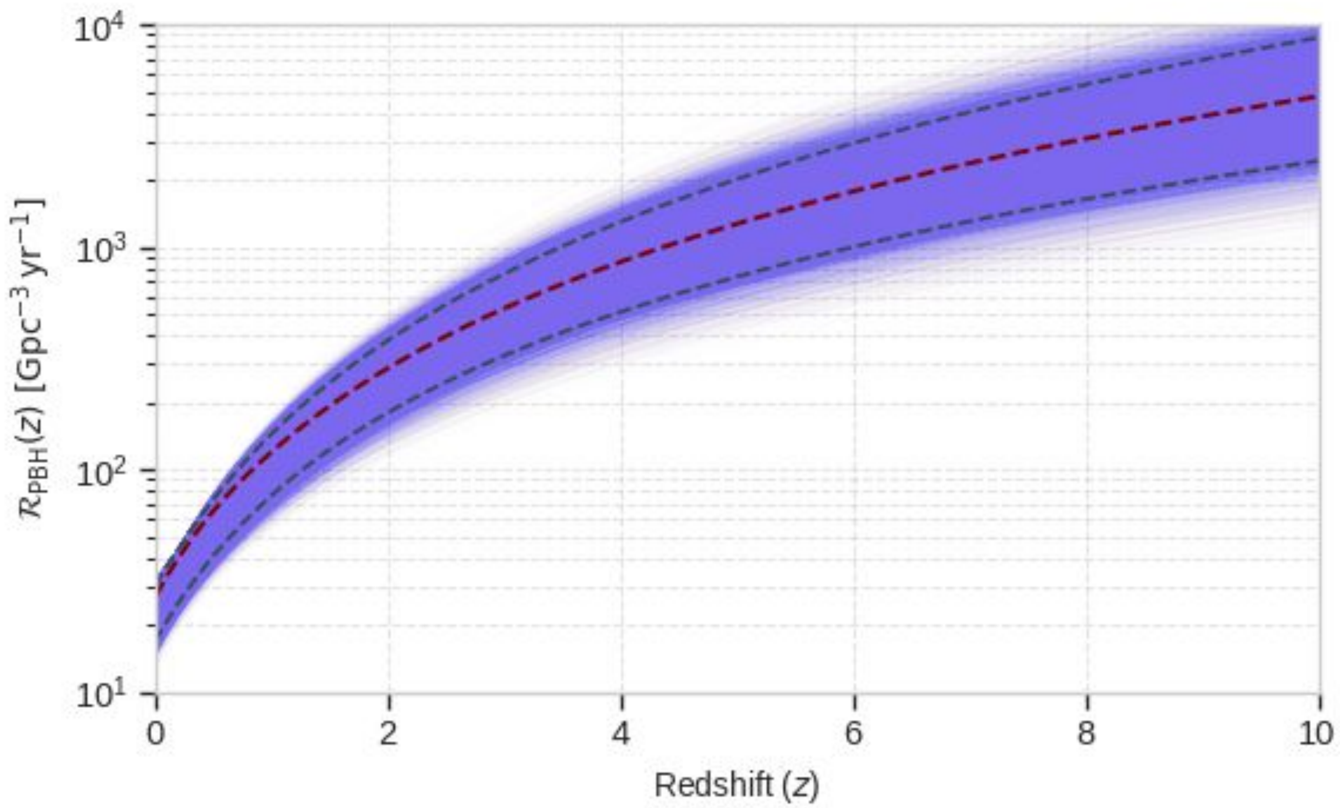
For a stochastic background originating from a population of BBH mergers, the spectral shape of $\Omega_{\rm GW}(f)$ can be computed by integrating the redshift-dependent merger rate and the GW energy spectrum emitted by each event as:

$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c H_0}\int_0^{z_{\rm max}} \frac{\mathcal{R}_{PBH}(z)}{(1+z)E(\Omega,z)}\frac{dE_{\rm GW}}{df_s}dz$$

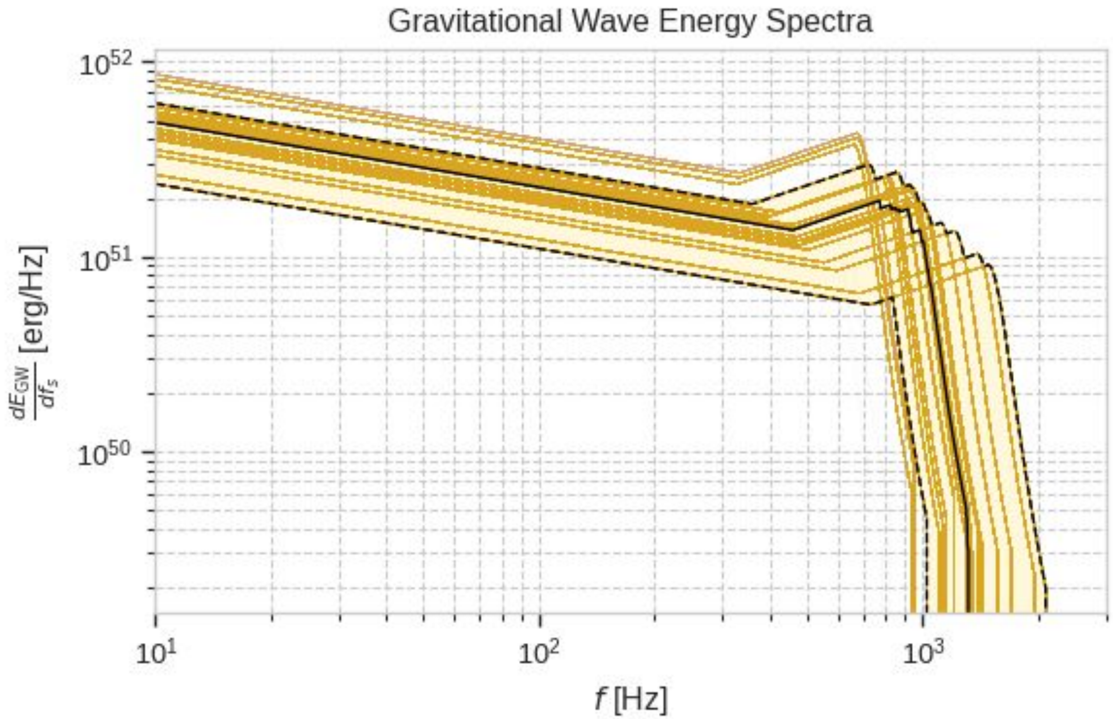
Where, $E(\Omega, z) = \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}.$

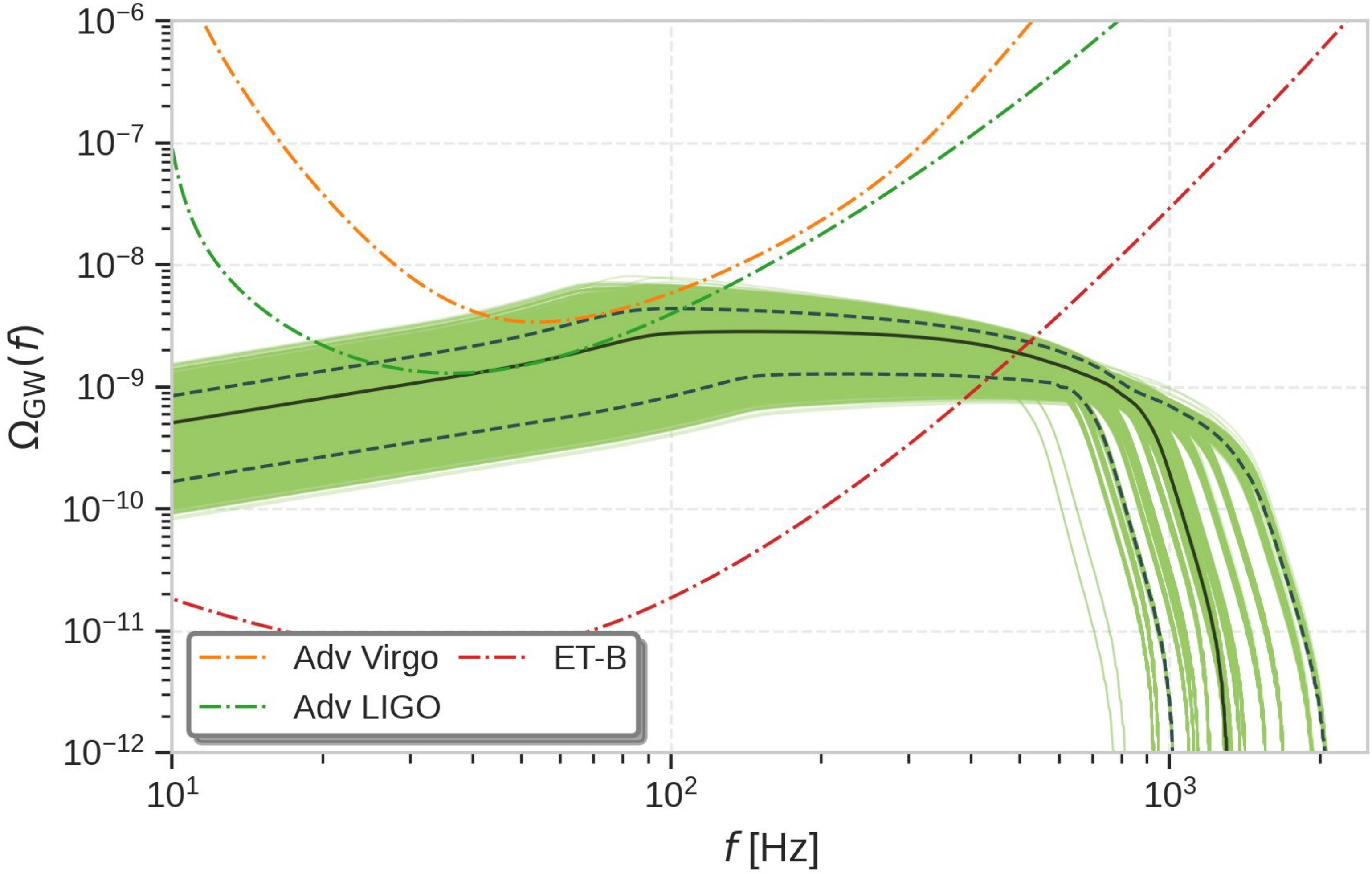
The redshift evolution of the PBH merger rate is

$$\mathcal{R}_{\rm PBH}(z) = \mathcal{R}_{\rm PBH}(0, m_1, m_2)(1+z)^{\alpha},$$



$$\frac{dE_{\rm GW}}{df_s} = A \begin{cases} f_s^{-1/3}, & f_s < f_{\rm merge} \\ f_{\rm merge}^{-1} f_s^{2/3}, & f_{\rm merge} \leq f_s < f_{\rm ring} \\ \dfrac{f_s^2 f_\omega^4}{f_{\rm merge} f_{\rm ring}^{4/3} [4(f_s - f_{\rm ring})^2 + f_\omega^2]^2}, & f_{\rm ring} \leq f_s < f_{\rm cut} \end{cases}$$





10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
$\Omega_{GW}(f)$
10^1
10^2
10^3
f [Hz]
Adv Virgo
ET-B
Adv LIGO

Detectable Analysis

$$\mathrm{SNR}^2 = 2T \int_0^\infty df \, \frac{\gamma^2(f)\Omega_{\mathrm{GW}}^2(f)}{f^6 P_1(f) P_2(f)} \left(\frac{3H_0^2}{10\pi^2}\right)^2$$

- **Advanced LIGO** and **Advanced Virgo** exhibit substantially lower sensitivities, with mean SNRs of 0.0417 and 0.0206, respectively. No events in the sample cross the typical detectability threshold for either detector.

- **Einstein Telescope (ET)** demonstrates significantly enhanced sensitivity to the simulated PBH signals, with a mean and median SNR of 3.24. Approximately 81% of events are detectable by ET.

Summary

- The local PBH binary merger rate inferred from the data is consistent with the estimates reported by the LVK collaboration, providing further support for the PBH merger scenario.

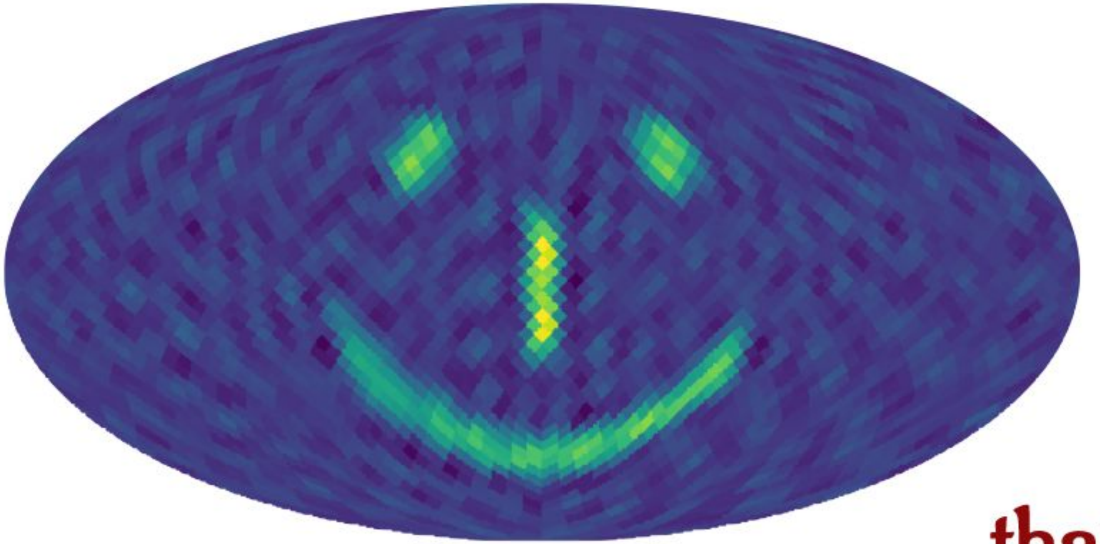
- The merger rate is found to decrease with increasing central mass while mass function width and PBH abundance increases strongly, with higher merger rates generally associated with lower PBH masses.

- The predicted SGWB from unresolved PBH mergers has a mean total energy density of $\Omega_{GW} = 1.67 \times 10^{-6}$, peaking near 158 Hz.

- Detectability forecasts indicate that current detectors (Advanced LIGO and Virgo) are unlikely to observe this SGWB signal, whereas the future Einstein Telescope (ET-B) will have sufficient sensitivity to probe or constrain the PBH contribution to the SGWB.

- These results highlight the potential of next-generation gravitational wave observatories to detect or further constrain the PBH dark matter hypothesis and demonstrate the importance of SGWB measurements for probing the early universe.

and we expect many more stronger signals. . .



thank you..