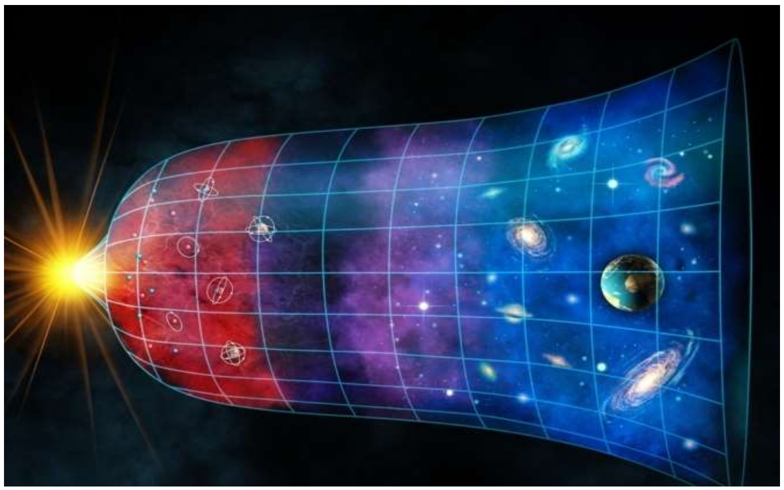
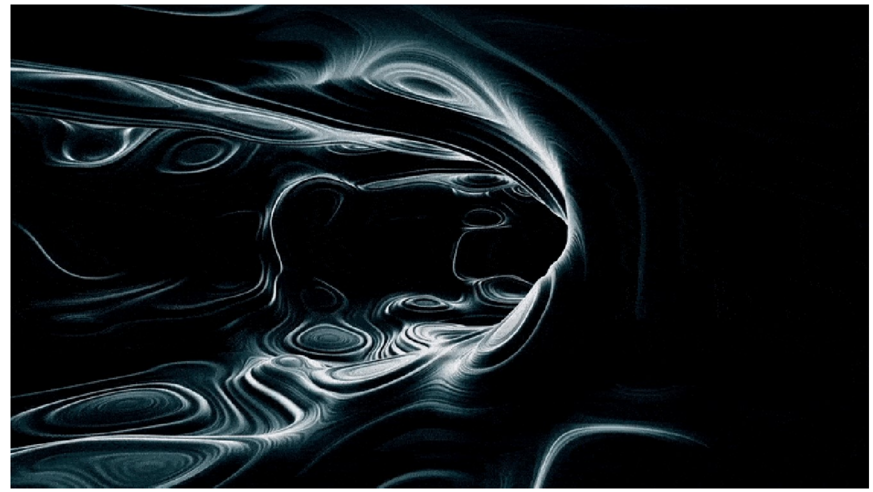
28th International Workshop

“What Comes Beyond the Standard Models?”

(July 2025)





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Resolving the Cosmological Constant problem via quantum space-time uncertainty

Relating Cosmological Constant Problem to Quantum space-time Uncertainty

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Unraveling the mystery of the cosmological constant: Does spacetime uncertainty hold the key?[a]

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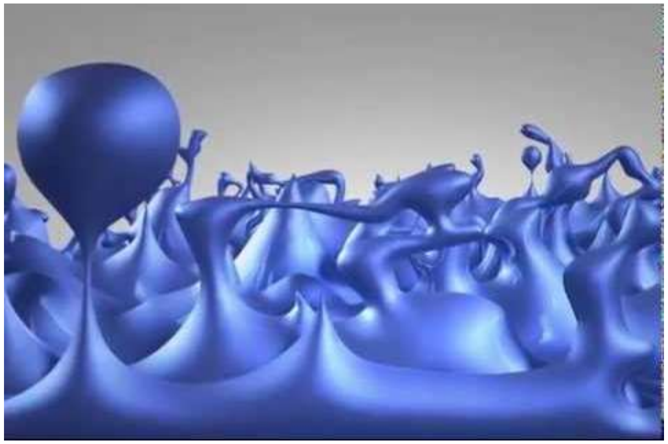


Essay received honorable mention in the 2023 Gravity Research Foundation Competition.

Outline

- Brief review of the Cosmological Constant problem
- Planck scale (L_{Pl}) implies quantum uncertainty in space-time metric
- Heuristic argument: Einstein power emergent from Planck scale
- Replace L_{Pl} cut-off with phenomenological length scale (L_{Z}) obtained from vacuum energy causing **observed** expansion
- L_{Z} is geometric mean of L_{Pl} and the observable universe, L_{U}
- L_{Z} is consistent with uncertainty in macroscopic quantum systems
- **Effective** QFT vacuum energy at Planck scale now matches cosmological vacuum energy
- Time-dependence of space-time uncertainty in an FLRW universe and evolution of the Cosmological "Constant"

QFT vacuum energy density

Vacuum energy density from Quantum Field Theory (QFT)

$$\rho_{\text{QFT}} = \frac{1}{(2\pi\hbar)^3}\int_0^{P_Z}\left(\tfrac{1}{2}\hbar\omega\right)d^3p = \frac{1}{16\pi^3\hbar^3}\int_0^{P_Z}\sqrt{p^2c^2+m^2c^4}\;d^3p$$

where P_Z is the cut-off momentum. (Weinberg, 1989)

- **Lorentz flat space-time geometry is assumed**.

$$\eta_{\mu\nu}p^{\mu}p^{\nu} = -m^2c^2 \quad \rightarrow \quad E^2 = m^2c^4 + p^2c^2$$

- This vacuum energy is used to predict cosmological expansion of the universe using Einstein field equations of GR, therefore the **assumption of flat space-time is inherently flawed**.

Comparing QFT and GR vacuum energy

Spherical momentum space ($mc << p$):

$$\rho_{\mathrm{QFT}} \approx \frac{c}{16\pi^3\hbar^3}\int_0^{P_Z} p\left(4\pi\, p^2 d^3 p\right) = \frac{c}{16\pi^2\hbar^3} P_Z^4 = \frac{\hbar c}{16\pi^2 L_Z^4}$$

where De Broglie relation, $P_Z = \hbar / L_Z$, gives L_Z as cut-off length.

- **Planck scale cut-off**: $L_Z = L_{\mathrm{Pl}} = \sqrt{G\hbar / c^3}$.

$$\boxed{\rho_{\mathrm{QFT}} \approx \frac{c^7}{16\pi^2 G^2 \hbar} \sim 10^{111}\,\tfrac{\mathrm{J}}{\mathrm{m}^3}}$$ or $10^{71}\,\mathrm{GeV}^4$ in natural units.

- **Compare to GR**: $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \rightarrow \quad G_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} + T_{\mu\nu}^{\Lambda}\right)$

  where $T_{\mu\nu}^{\Lambda} \equiv -\frac{c^4 \Lambda}{8\pi G} g_{\mu\nu} \quad \rightarrow \quad T_{00}^{\Lambda} \approx \frac{c^4\Lambda}{8\pi G}$

$$\boxed{\rho_{\Lambda} \equiv \frac{c^4 \Lambda}{8\pi G} \sim 10^{-10}\,\tfrac{\mathrm{J}}{\mathrm{m}^3}}$$ or $10^{-47}\,\mathrm{GeV}^4$ in natural units.

Cosmological Constant problem and Planck scale

Ratio of energy densities: $\frac{\rho_{\mathrm{QFT}}}{\rho_{\Lambda}} \approx \frac{c^3}{Gh\Lambda} \sim \frac{10^{111}\ \mathrm{J}/\mathrm{m}^3}{10^{-10}\ \mathrm{J}/\mathrm{m}^3} = 10^{121}$

Reasons to question this result:

- Lorentz flat space-time geometry was assumed in **expanding** universe.
- G may vary with cosmological time: $G \sim 1/t$ (Dirac, 1937)
- c (and α) may vary with cosmological time (Bekenstein, 1982.)
- Planck length scale is not necessarily fundamental.

Historically, Planck quantities are not based on physical relationships but dimensional analysis as originally done by Planck.*

$$[G] = \mathrm{M}^{-1}\,\mathrm{L}^{3}\,\mathrm{T}^{-2}, \quad [h] = \mathrm{M}^{1}\,\mathrm{L}^{2}\,\mathrm{T}^{-1}, \quad [c] = \mathrm{M}^{0}\,\mathrm{L}^{1}\,\mathrm{T}^{-1}$$

*Planck (1899), Meschini (2007), Tank (2011)

Planck quantities by Planck himself

Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 annimmt, so erhält man als Einheit der Länge die Grösse:

$$\sqrt{\frac{bf}{c}} = 4.13 \cdot 10^{-33} \text{ cm},$$

als Einheit der Masse:

$$\sqrt{\frac{bc}{f}} = 5.56 \cdot 10^{-5} \text{ gr},$$

als Einheit der Zeit:

$$\sqrt{\frac{bf}{c^5}} = 1.38 \cdot 10^{-43} \text{ sec},$$

als Einheit der Temperatur:

$$a \sqrt{\frac{c^5}{bf}} = 3{,}50 \cdot 10^{32} \text{ Cels.}$$

M. Planck, "Über irreversible Strahlungsvorgänge," Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin (1899).

<u>Deriving Planck quantities from *physical* *r*elationships</u>

Compton wavelength: $\lambda_C = \dfrac{h}{mc}$
(length scale of a **quantum** particle)

Schwarzschild radius: $R_S = \dfrac{2Gm}{c^2}$
(**classical** length scale of a black hole)

Equating λ_c (**quantum scale**) and R_S (**classical scale**) leads to Planck mass and Planck length:

$$m_{PL} = \sqrt{\frac{hc}{G}} \sim 10^{-8}\ \text{kg} \qquad L_{PL} = \sqrt{\frac{Gh}{c^3}} \sim 10^{-35}\ \text{m}$$

This is a **mesoscale** (classical) mass scale but a **miniscule** (quantum) length scale which implies an important connection between classical and quantum scales.

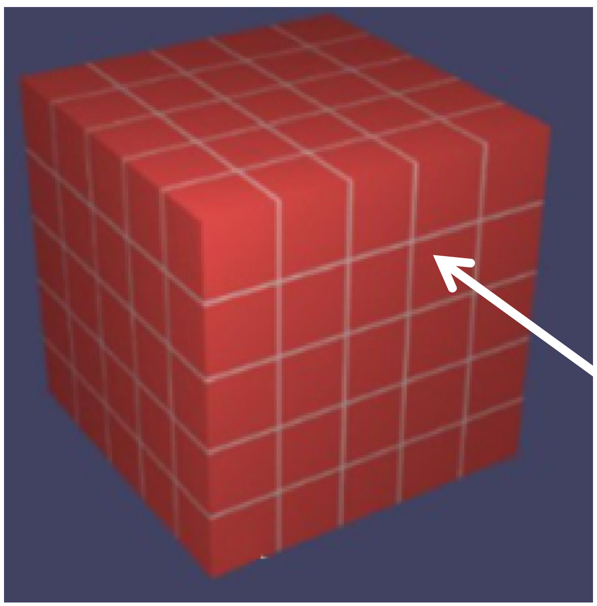
This implies that ignoring the quantum uncertainty of space-time could be the cause for the Cosmological Constant Problem.

Planck Mass (classical) compared to Planck Length (quantum)

Planck volume:

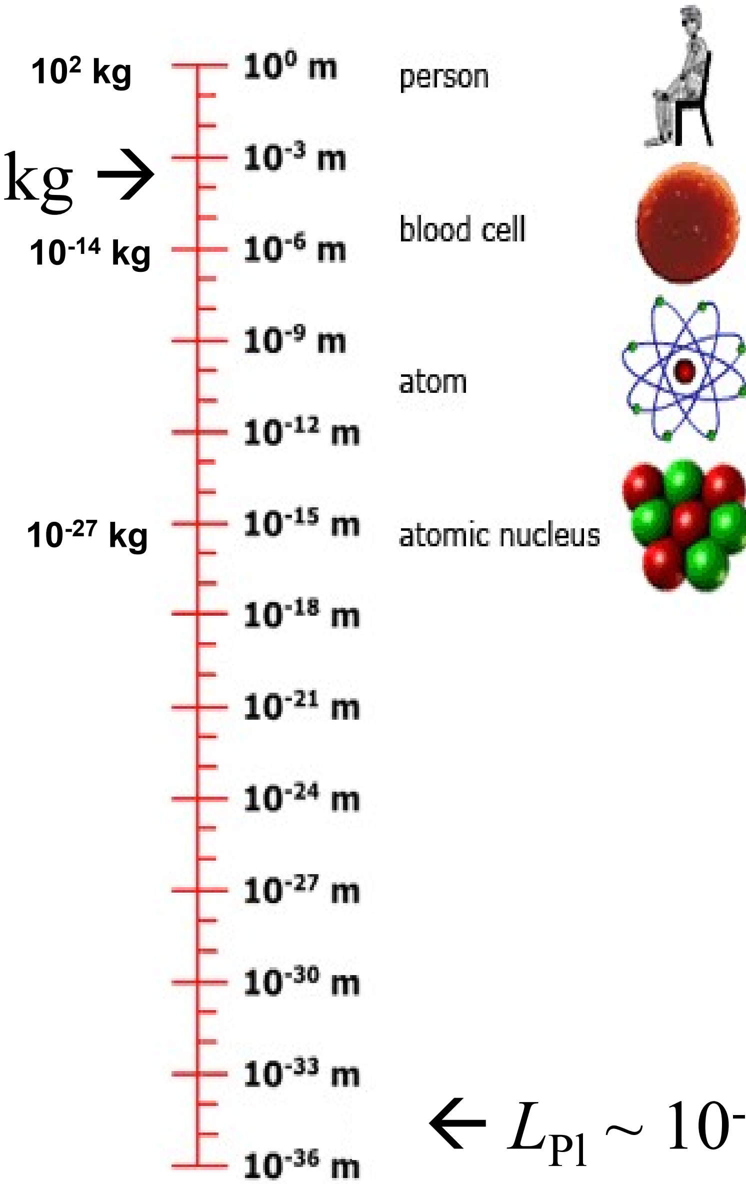
$$V_{\mathrm{Pl}} = L_{\mathrm{Pl}}^3 \sim 10^{-105}\ \mathrm{m}^3$$

Planck scale predicts a **classical** mass scale enclosed in a **quantum** volume scale (Planck unit cell).



Planck volume
(unit cell)

$m_{\mathrm{Pl}} \sim 10^{-8}$ kg →



← $L_{\mathrm{Pl}} \sim 10^{-35}$ m

Classical Einstein power emergent from Planck quantities

Planck energy: $E_{\mathrm{p}} = m_{\mathrm{P}} c^2 = \sqrt{hc^5 / G} \sim 10^9\,\mathrm{J} \sim 10^{28}\,\mathrm{eV}$

Planck time: $t_{\mathrm{PL}} = \dfrac{L_{\mathrm{PL}}}{c} = \sqrt{\dfrac{Gh}{c^5}} \sim 10^{-43}\,\mathrm{s}$

Planck power: $P_{\mathrm{P}} = \dfrac{E_{\mathrm{P}}}{t_{\mathrm{P}}} = \dfrac{\sqrt{hc^5 / G}}{\sqrt{Gh / c^5}} = \dfrac{c^5}{G} \sim 10^{52}\,\mathrm{W}$

Notice Planck's constant (**quantum**) cancels and the Result is only in terms of G and c which are constants in **classical** General Relativity. Therefore, the power can be named the **Einstein** power: $P_{\mathrm{E}} = c^5 / G$

Coefficient of classical GR emergent from Planck quantities

Planck/Einstein power is related to force by $P_{pl} = F_{pl}c$. Then

$$F_E = \frac{P_E}{c} = \frac{c^4}{G} \sim 10^{44}\,N$$

Amazingly, this turns to be the constant that appears in Einstein's General Relativity (GR).

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu} \quad \rightarrow \quad G^{\mu\nu} = \frac{8\pi}{F_E} T^{\mu\nu}$$

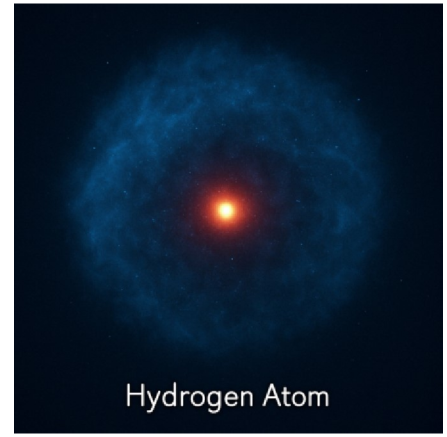
- The coefficient of GR (a classical theory) emerges from the Planck scale (which is quantum mechanical). It is "hidden" in the theory of GR.

- This may imply that ignoring the **explicit** role of **quantum** uncertainty of space-time (involving *h*) in the **classical** theory of GR is the cause of the Cosmological Constant Problem.

Newtonian space-time uncertainty

Proposal: Relate fixed cut-off to spacetime uncertainty.

- Mass distribution shapes curved space-time (GR).
- Distribution of quantum mass has fundamental uncertainty. (Example: hydrogen atom cloud.)
- It follows that curved space-time must have a fundamental uncertainty (space-time "fuzziness").*



$$\Delta g_N \equiv \frac{U_{\text{grav}}}{U_{\text{rest}}} = \frac{m\phi_N}{mc^2} = \cdots = \frac{L_{\text{Pl}}^2}{L^2}$$

where we use $\phi_N = Gm/L$, $E = mc^2$, and $E = hc/L$.

- $\Delta g_N \approx 0$ for $L >> L_{pl}$: insignificant space-time uncertainty
- $\Delta g_N = 1$ for $L = L_{Pl}$: max uncertainty (quantum space-time foam at Planck scale)

*Adler (2010), Regge (1958), Ng and van Dam (1995, 2000), Christiansen, et al (2011), Mead (1964, 1966), Vilkovisky (1992), DeWitt (1964), Garay (1999)

Phenomenological cut-off length scale

Equate $\rho_{\mathrm{QFT}} \approx \dfrac{\hbar c}{16\pi^2 L_Z^4}$ and $\rho_\Lambda \approx \dfrac{c^4 \Lambda}{8\pi G}$ then solve for L_Z.

$$L_Z \approx \left(\frac{G\hbar}{2\pi c^3 \Lambda} \right)^{1/4} \approx 2\times 10^{-5}\,\mathrm{m}$$

This is a **phenomenological** length scale obtained using the **observed** vacuum energy determined by the expansion of the universe (via the measured value of Λ).

Interpretation: $L_Z = \sqrt{L_{\mathrm{Pl}} L_{\mathrm{U}}}$, where L_{U} is the radius of the observable universe. This is the geometric mean of the smallest and largest length scales of the universe.*

* Zel’dovich and Krasinski (1968), Freidel, et. al.(2023), Tello, et. al. (2023)

Uncertainty principle on macroscopic length scales

📍 $\Delta x \, \Delta p \geq \frac{\hbar}{2}$

System	Scale	Quantum Feature	Role of Uncertainty Principle
BECs	~10–100 µm	Macroscopic wavefunction coherence	Wavefunction delocalization and thermal de Broglie overlap
Optomechanics	~50–500 µm	Quantum control of mechanical motion	Limits sensitivity in displacement sensing; governs backaction
Quantum Drum	~30 µm	Quantized vibrational states	Demonstrates macroscopic quantum superposition in mechanical modes

Uncertainty principle on macroscopic length scales

⏳ $\Delta E \, \Delta t \geq \frac{\hbar}{2}$

System	Scale	Quantum Feature	Role of Uncertainty Principle
Cavity QED	~100 µm cavity λ	Quantized light–matter interactions	Zero-point energy and mode fluctuations from confined fields
THz Photons	~100 µm λ	Single-photon THz quantum optics	Sets spectral linewidth and coherence time
Flux Qubits	~100 µm loop	Macroscopic current tunneling	Governs tunneling rates and coherence lifetimes

🔁 $\Delta \Phi \, \Delta Q \geq \frac{\hbar}{2}$

System	Scale	Quantum Feature	Role of Uncertainty Principle
SQUIDs	~10–200 µm	Superposition of flux/current states	Enables tunneling between flux states; quantization of magnetic flux

Effective Planck scale QFT vacuum energy density due to suppression by quantum space-time uncertainty

Use $\rho_{\mathrm{QFT}} \approx \dfrac{\hbar c}{16\pi^2 L_Z^4}$ and $L_Z^2 = \dfrac{L_{\mathrm{Pl}}^2}{\Delta g_N}$ to obtain **effective** QFT energy density:

$$\boxed{\rho_{\mathrm{QFT}} \approx \frac{\hbar c}{16\pi^2 L_{\mathrm{Pl}}^4}(\Delta g_{\mathrm{N}})^2} \quad \text{where} \quad (\Delta g_{\mathrm{N}})^2 = \left(\frac{L_{\mathrm{Pl}}^2}{L_Z^2}\right)^2 \sim 10^{-121}$$

- QFT vacuum energy at Planck scale is drastically suppressed by quantum space-time uncertainty so that it **appears** as if there is a cut-off at L_z.

- The L_z cut-off does NOT mean QFT fails below that length scale. Rather, higher energies (below L_Z) are "smeared out" by this space-time uncertainty.

- **Effective** QFT vacuum energy at Planck scale is corrected by $(\Delta g_N)^2 \sim 10^{-121}$ and now matches **observed** cosmological vacuum energy so $\rho_{\mathrm{QFT}} = \rho_\Lambda$.

Therefore, the Cosmological Constant problem is resolved by acknowledging the role of quantum space-time uncertainty.

Space-time metric for expanding universe

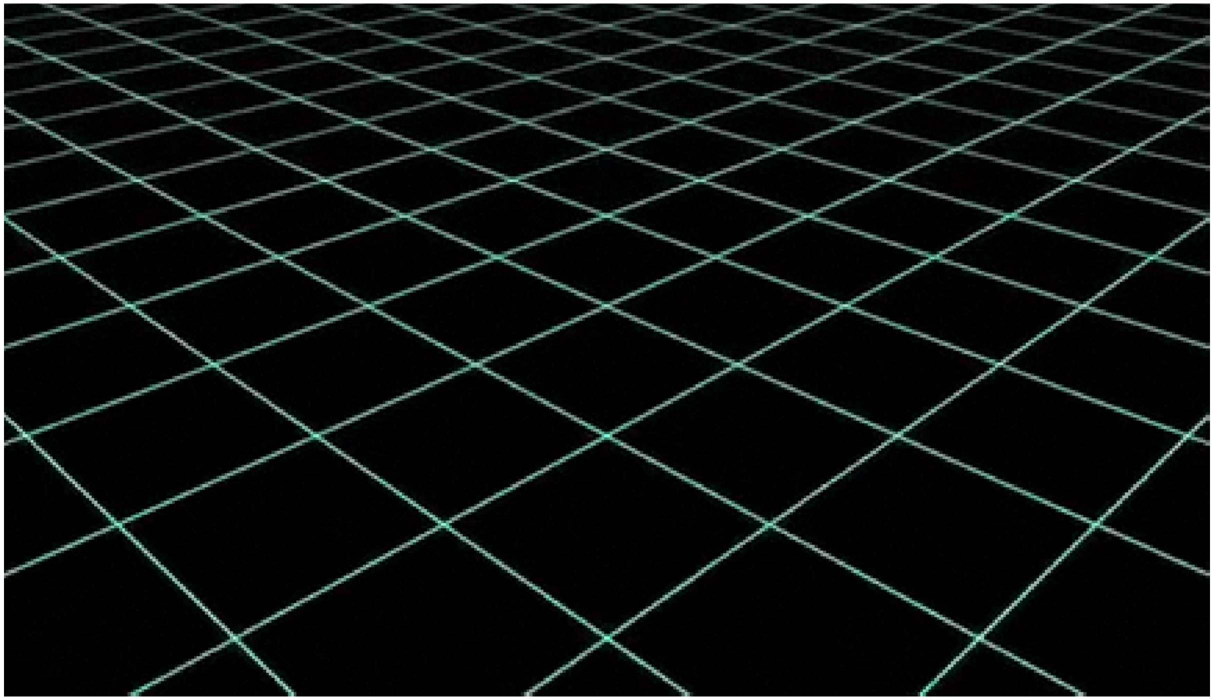
Recall: Flat (Minkowski) space-time metric:

$$ds^2 = \eta_{00} c^2 dt^2 + \eta_{ij} dx^i dx^j$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now consider **stretching** space-time:

$$ds^2 = g_{00}(\vec{x}, t) c^2 dt^2 + g_{ij}(\vec{x}, t) dx^i dx^j$$



Vacuum energy density in FLRW universe

Generalize invariant to **curved** space-time: $g_{\mu\nu} p^{\mu} p^{\nu} = -m^2 c^2$

Solve for energy: $E = \dfrac{-cg_{0i} p^i + c\sqrt{\left(g_{0i} p^i\right)^2 - g_{00}\left(g_{ij} p^i p^j + m^2 c^2\right)}}{g_{00}}$

Use FLRW metric with zero spatial curvature ($k = 0$) which is

$$g_{00} = -1, \quad g_{0i} = 0, \quad g_{ij} = a^2(t)\delta_{ij}$$

Then energy becomes $E = \sqrt{m^2 c^4 + a^2(t) p^2 c^2}$

Vacuum energy in spherical momentum space ($mc << p$):

$$\rho_{\text{QFT}} = \frac{c}{8\pi^3 \hbar^3} \int_0^{P_Z} a(t) p \; d^3 p$$

Vacuum energy density in FLRW universe

For universe dominated by dark energy: $a(t) = a_0 e^{\pm 2\lambda t}$, where

- $\lambda = H(t_0)\sqrt{\Omega_\Lambda(t_0)}$ and $H = a^{-1}\dfrac{da}{dt}$ is Hubble parameter.
- t_0 is the current epoch of the universe.
- $\Omega_\Lambda(t_0) = \dfrac{\rho(t_0)}{\rho_c}$, where ρ_c is the critical mass density
- Since $\rho_c\,(t_0) \approx \rho_c$ in the current epoch, then $\lambda \approx H(t_0) \equiv H_0$

$$\rho_{\mathrm{QFT}} = \frac{c a_0 e^{\pm 2\lambda t}}{4\pi^3 \hbar^3} \int_0^{P_Z} p^3\, dp = \frac{c a_0 e^{\pm 2\lambda t}}{16\pi^2 \hbar^3} P_Z^4$$

Time-scale obtained from energy density

Normalize to $a_0 = 1$, use time scale $t = t_Z$, and $P_Z = \hbar / L_Z$ where $L_Z \approx 2\times10^{-5}$ m.

$$t_Z = \frac{1}{2H_0}\ln\left(\frac{16\pi^2 L_Z^4 \rho_\Lambda}{\hbar c}\right) \sim 10^9\,\mathrm{y}r$$

- **Therefore, the time scale is the age of the universe.**
- This is more fitting that than the Planck time, $t_{\mathrm{Pl}} \sim 10^{-44}$ s, associated with the usual choice of using a Planck cut-off momentum, $P_{Pl} = h / L_{Pl}$, where $L_{\mathrm{Pl}} = ct_{\mathrm{Pl}}$.

Proper length of a wordline in a FLRW Universe

The proper length of a wordline is given by

$$L_{\text{proper}} = \int_0^{L_C} \sqrt{-g_{\mu\nu} dx^\mu dx^\nu} = \int_0^{L_C} \sqrt{c^2 dt^2 - a^2(t) dx^2} = c\int_0^{t_C} \gamma_{\text{FLRW}}^{-1} dt$$

where $\gamma_{\text{FLRW}} \equiv \left[1 - a^2(t) v^2 / c^2\right]^{-1/2}$ is a Lorentz factor in FLRW space-time.

The bounds of the integral go from the start of the universe ($t = 0$) to a coordinate time t_C associated with a coordinate length $L_C = c\, t_C$.

Using $\lambda \approx H_0$ and $a_0 = 1$ in $a(t) = a_0 e^{2H_0 t}$, then integrating leads to

$$L_{\text{proper}} = \frac{c}{4H_0}\left[2\left(\sqrt{1-\beta e^{4H_0 t_C}} - \sqrt{1-\beta^2}\right) - \ln\left(\frac{1+\sqrt{1-\beta^2 e^{4H_0 t_C}}}{1-\sqrt{1-\beta^2 e^{4H_0 t_C}}}\,\frac{1-\sqrt{1-\beta^2}}{1+\sqrt{1-\beta^2}}\right)\right]$$

where $\beta \equiv v/c$. For slow observers ($v \ll c$), the expression reduces to

$$L_{\text{proper}} \approx ct_C - \frac{c}{8H_0}\left(\frac{v}{c}\right)^2 \left(e^{4H_0 t_C} - 1\right)$$

Space-time uncertainty in a FLRW Universe

Similar to $\Delta g_{\mathrm{N}} = L_{\mathrm{Pl}}^2 / L_{\mathrm{Z}}^2$, we define the FLRW space-time uncertainty as

$$\boxed{\Delta g_{\mathrm{FLRW}} \equiv L_{\mathrm{Pl}}^2 / L_{\mathrm{proper}}^2}$$

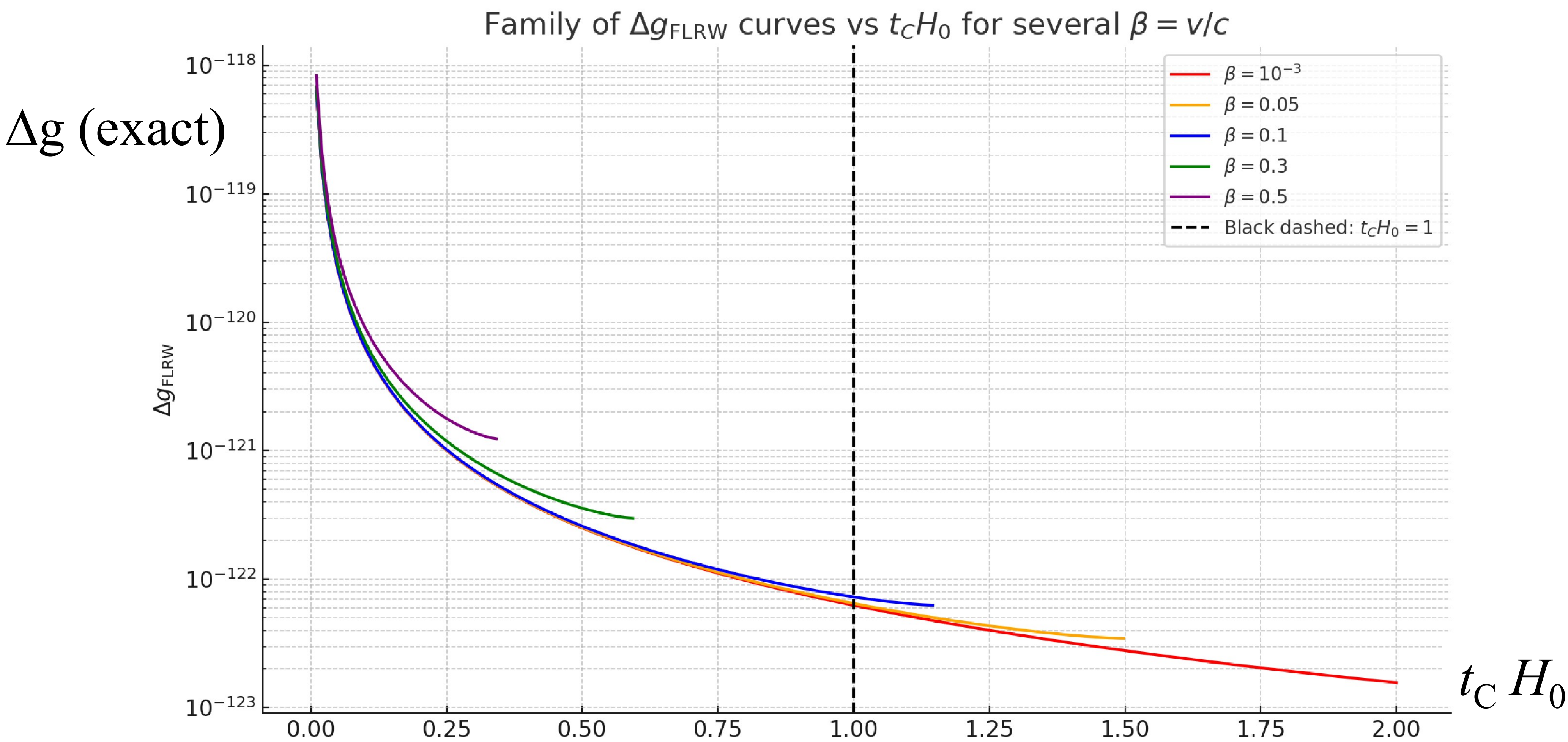
Since L_{proper} is Lorentz invariant then so is Δg_{FLRW}. Using L_{proper} from the previous slide gives a coordinate-dependent expression for a moving observer:

$$\boxed{\Delta g_{\mathrm{FLRW}} = \frac{16 H_0^2 L_{\mathrm{Pl}}^2}{c^2}\left[2\left(\sqrt{1-\beta\, e^{4H_0 t_{\mathrm{C}}}} - \sqrt{1-\beta^2}\right) - \ln\left(\frac{1+\sqrt{1-\beta^2 e^{4H_0 t_{\mathrm{C}}}}}{1-\sqrt{1-\beta^2 e^{4H_0 t_{\mathrm{C}}}}} \cdot \frac{1-\sqrt{1-\beta^2}}{1+\sqrt{1-\beta^2}}\right)\right]^{-2}}$$

This has a complicated time-dependence which is plotted on the next slide. For slow observers ($v << c$), the expression simplifies to

$$\Delta g_{\mathrm{FLRW}} \approx \frac{L_{\mathrm{Pl}}^2}{c t_{\mathrm{C}}^2}\left[1 + \frac{1}{4 H_0 t_{\mathrm{C}}}\left(\frac{v}{c}\right)^2 \left(e^{4H_0 t_{\mathrm{C}}} - 1\right)\right]$$

Evolution of FLRW space-time uncertainty



In all cases, Δg starts at large values for small t_C (early universe) and decreases with cosmological time. Curves with $\beta < 0.1$ are cut off before $t = t_C H_0$ (the age of the observable universe) because L_{proper} becomes imaginary.

Features of this model

$$\Delta g_{\mathrm{FLRW}} \approx \frac{L_{\mathrm{Pl}}^2}{c t_{\mathrm{C}}^2}\left[1+\frac{1}{4H_0 t_{\mathrm{C}}}\left(\frac{v}{c}\right)^2\left(e^{4H_0 t_{\mathrm{C}}}-1\right)\right] \quad \text{for v << c}$$

- **For an expanding FLRW universe, Δg_{FLRW} decreases with t_{C}.** Thus quantum space-time fluctuations diminish with cosmological time and the universe becomes increasingly more classical.

- **Observers with larger β will observe a larger Δg_{FLRW}.** Such observers essentially sample larger segments of space-time and hence the effect of space-time uncertainty becomes more pronounced.

- **There is no discernible distinction between curves having $\beta < 10^{-3}$.** This means the model is practically independent of velocity for all observers with $v \lesssim 10^5$ m/s. Hence, we may simply set $v = 0$ and use

$$\boxed{\Delta g_{\mathrm{FLRW}} \approx \frac{L_{\mathrm{Pl}}^2}{c^2 t_{\mathrm{C}}^2}}$$

Revisiting the Cosmological Constant (CC) problem

Recall that we previously had $\Delta g_{\mathrm{N}} = L_{\mathrm{Pl}}^2 / L_{\mathrm{Z}}^2$ and $L_{\mathrm{Z}} = \sqrt{L_{\mathrm{Pl}} L_{\mathrm{U}}}$ which gives $\Delta g_{\mathrm{N}} = L_{\mathrm{Pl}} / L_{\mathrm{U}}$. For $t_{\mathrm{C}} \sim 1/H_0$, then $L_{\mathrm{C}} \sim c/H_0 = L_{\mathrm{U}}$ and $\Delta g_{\mathrm{FLRW}} \approx (\Delta g_{\mathrm{N}})^2$.

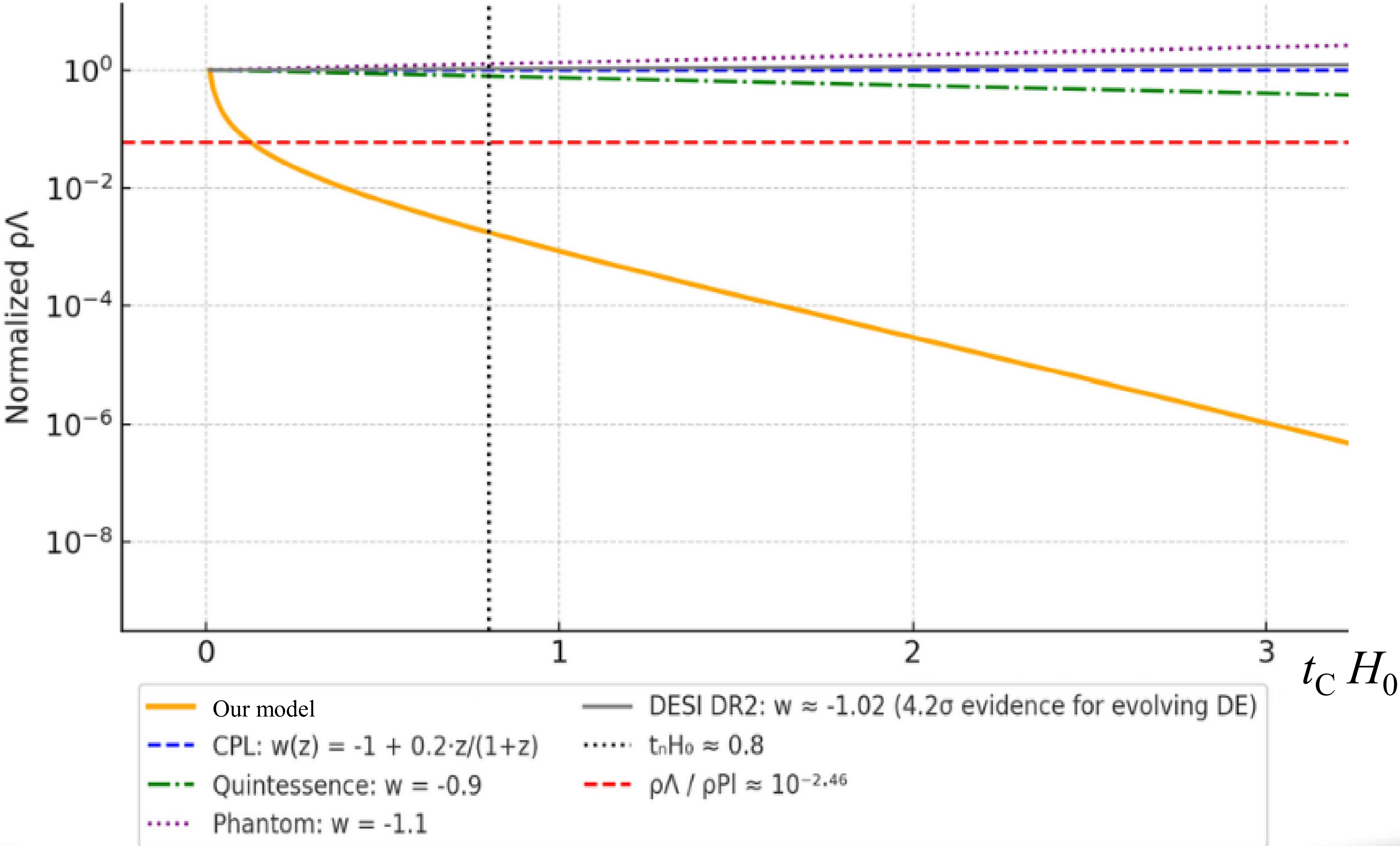
Thus, the QFT vacuum energy density becomes

$$\boxed{\rho_{\mathrm{QFT}} \approx \frac{\hbar c}{16\pi^2 L_{\mathrm{Pl}}^4} (\Delta g_{\mathrm{N}})^2 \approx \frac{\hbar c}{16\pi^2 L_{\mathrm{Pl}}^4} \Delta g_{\mathrm{FLRW}}} \quad \text{where} \quad \Delta g_{\mathrm{FLRW}} \approx \frac{H_0^2 L_{\mathrm{Pl}}^2}{c^2} \sim 10^{-122}$$

The value of Δg_{FLRW} matches the CC problem within an order of magnitude: $\frac{\rho_{\mathrm{QFT}}}{\rho_\Lambda} \sim 10^{121}$. In fact, using $L_{\mathrm{PL}}^2 = \frac{G\hbar}{c^3}$, $\rho_\Lambda = \frac{c^4 \Lambda}{8\pi G}$, and $\Lambda = \frac{3H_0^2}{c^2}$ leads to $\rho_{\mathrm{QFT}} \approx \frac{1}{6\pi} \rho_\Lambda \sim 10^{-1} \rho_\Lambda$.

Lastly, since $\rho_{\mathrm{QFT}} \propto \Delta g_{\mathrm{FLRW}}$ evolves with cosmological time, then so does $\rho_\Lambda \propto \Lambda$. Hence, this model predicts Λ is not a constant but more like the Hubble *parameter* (H) which is only a “constant” (H_0) in our current epoch.

Comparison to other models



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