Effective Frameworks for Dark Energy Evolution

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Outline of the talk

(A) Introduction: Our Universe

** Observations, Cosmological models, and Theory*

***The Cosmic budget*

****Modern Cosmology*

(B) Is Dark Energy Changing

(C) DESI DR2 results

(D) Dynamical Dark Energy Models

(E) Diagnostics of Dark Energy models

(F) Summary and Key takeaways

(G) References

(A) Intro...: Our Universe

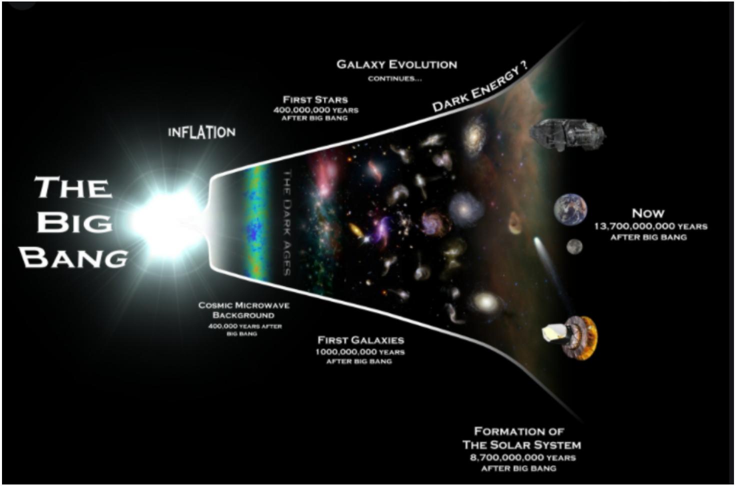
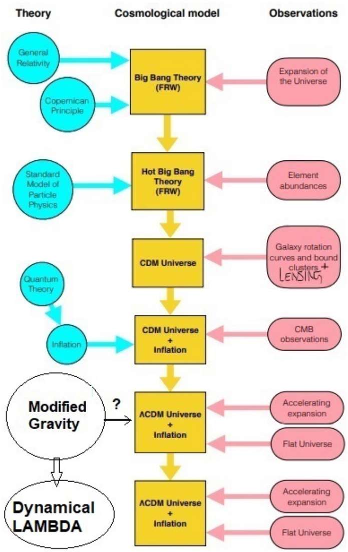


Figure: Credit: NASA/ESA

Intro...: Observations, Models, and Theory



Type Ia Supernovae: Standard candles ---> luminosity distance

BAO: Standard rulers -----> angular and Hubble distances

Cosmic Chronometers (CC): H(z) via galaxy ages

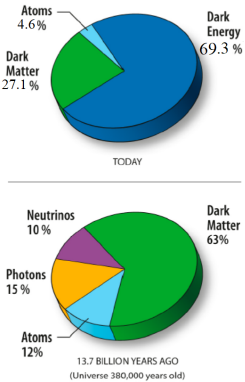
Strong Lensing Time Delays: Angular diameter distance

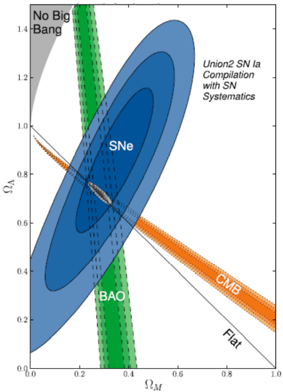
DESI DR2, DES-SN5YR, Planck, Pantheon Plus - precision cosmology

RECENT DESI DR2

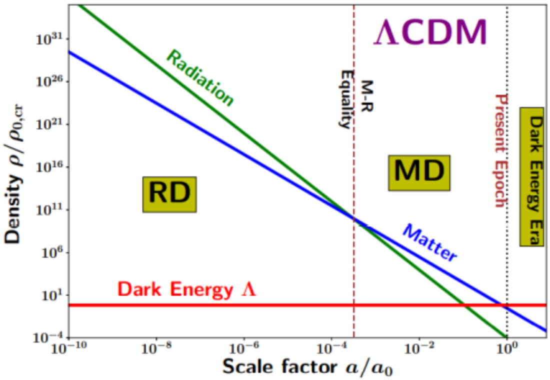
Intro...: The cosmic budget of energy densities

The cosmic recipe from Planck **A&A 641, A6 (2020)**





Cosmic budget of energy densities in ΛCDM



Intro...: Modern Cosmology in a nutshell

- (*Foundation: General Relativity + Cosmological Principle*)

- $$\mathcal{A} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda) + \mathcal{A}_m \tag{1}$$

- **Cosmological Principle:** The universe is **spatially homogeneous** and it is **spatially isotropic** at large scale (few Mpc), with metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{2}$$

where $K = +1, 0, -1$ and $a(t)$ is the scale factor of expansion.

Cont...: Friedmann Equations

First Friedmann eq.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{3}$$

where:

- H is the Hubble parameter, $a(t)$ is the scale factor,
- ρ is the total energy density (matter + radiation + dark energy),
- k is the curvature index (0 for flat, +1 for closed, −1 for open),

The **second Friedmann equation** describes the acceleration of cosmic expansion:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{4}$$

where p is the pressure of the cosmic fluid.

$$\text{if} \;\; \rho + 3P < 0 \Rightarrow \ddot{a} > 0 \;\; \text{then} \;\; w = \frac{P}{\rho} < -1/3.$$

For cosmic accelerated expansion $w < -1/3$. For ΛCDM, $w = -1$ (constant).

(B) Is Dark Energy changing? Cosmic tug of war

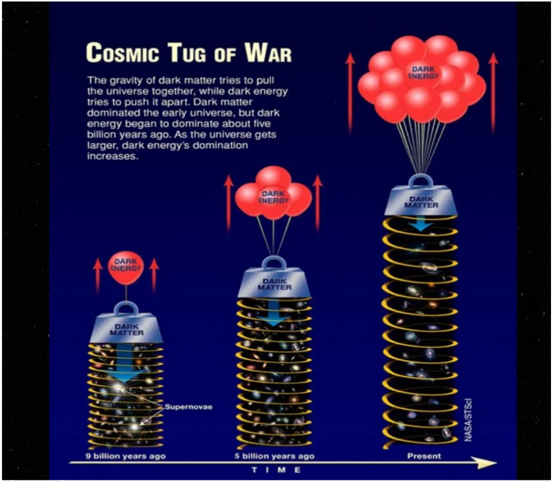


Figure: As the Universe grows, the total density of matter goes down. Credit: NASA

(B) Is Dark Energy changing?

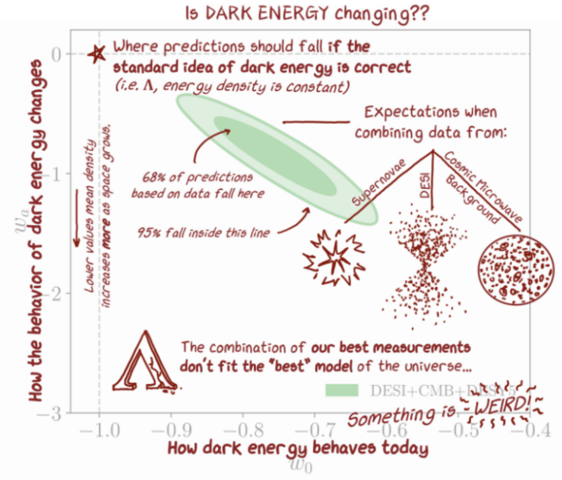


Figure: The Universe grows, so the total density of matter goes down. But dark energy is different. Credit: Claire Lamman

(C) DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints (arXiv:2503.14738v2 [astro-ph.CO] 26 Mar 2025)

- DESI collaboration report baryon acoustic oscillation (BAO) measurements from more than 14 million galaxies and quasars drawn from the Dark Energy Spectroscopic Instrument (DESI) Data Release 2 (DR2), based on three years of operation ,
- The results are well described by a flat ΛCDM model, but the parameters preferred by BAO are in mild, 2.3σ tension with those determined from the cosmic microwave background (CMB),
- A preference for dynamical dark energy scenarios, as opposed to the standard ΛCDM framework, is strongly supported.

Cont...Dark Energy Equation of state and DESI

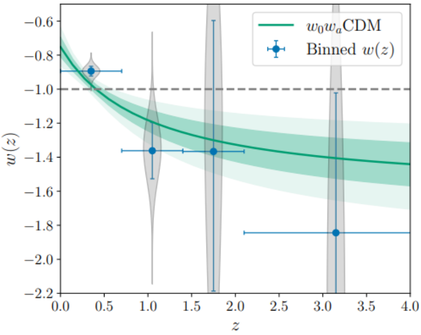


Figure: Results favor Dynamical nature of dark energy, i.e., dark energy is changing.

(D) Dynamical dark energy

Mostly dynamical dark energy models are categorized in three types:

- (1) Time varying Cosmological Constant model,
- (2) Dark energy models with a parameterized EoS,
  (i) **the constant w (wCDM) model**,
  (ii) **the Chevallier–Polarski–Linder (CPL) model**, and
  (iii) **the Jassal–Bagla–Padmanabhan (JBP) model,**
- (3) Effective dark energy models (MG theories).

(1). Time varying Cosmological Constant model

- The Einstein equation with a cosmological constant is written as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{5}$$

- To have a time variable cosmological constant $\Lambda = \Lambda(t)$, one can move the cosmological constant to the right hand side of Eq. (5) and take $\tilde{T}_{\mu\nu} = T_{\mu\nu} - \frac{\Lambda(t)}{8\pi G} g_{\mu\nu}$ as the total energy-momentum tensor.
- The Friedmann equation as usual can be written as, in a spacial flat FLRW universe,

$$H^2 = \frac{1}{3M_P^2} \left(\rho_m + \rho_\Lambda\right). \tag{6}$$

- The effective equation of state of dark energy

$$\begin{aligned} w_\Lambda^{eff} &= \frac{p_\Lambda^{eff}}{\rho_\Lambda} \\ &= = -1 - \frac{1}{3} \frac{d \ln \rho_\Lambda}{d \ln a}. \end{aligned} \tag{7}$$

(2) Dark energy models with a parameterized EoS

- In this category, we explore three dark energy models with parameterized EoS:
  (i) **the constant w (wCDM) model**,
  (ii) **the Chevallier–Polarski–Linder (CPL) model**, and
  (iii) **the Jassal–Bagla–Padmanabhan (JBP) model.**

- (i) **Constant w parametrization**: the dark energy EoS is described by a constant w (can be any real number). If $w < -1$, the energy density of dark energy increase as the universe expands (Big Rip). If $-1 < w < -1/3$, the universe sustains a gradually weakening acceleration. For cases where $w \geq -1/3$, the universe reverts to the decelerated expansion.

- The wCDM model introduces an additional degree of freedom compared to ΛCDM, which allows for more flexible fitting to the observational data. For this model, the reduced Hubble parameter is given by

$$E(z) = \left[\Omega_{\mathrm{m}}(1+z)^3 + (1-\Omega_{\mathrm{m}})(1+z)^{3(1+w)}\right]^{1/2}. \tag{8}$$

Cont...

- (ii) **The Chevallier–Polarski–Linder parametrization**: It parameterizes the dark energy EoS as a linear function of the scale factor a, given by

$$w(z) = w_0 + w_a(1-a),$$

where w_0 and w_a are free parameters.

- For this model, the reduced Hubble parameter is given by

$$E(z) = \Big[\Omega_\mathrm{m}(1+z)^3 \\ +(1-\Omega_\mathrm{m})(1+z)^{3(1+w_0+w_a)} \exp\left(-\frac{3w_a z}{1+z}\right)\Big]^{1/2}. \tag{9}$$

Cont...: **(iii) The Jassal-Bagla-Padmanabhan parametrization**

- In this model, the reduced Hubble parameter is calculated by

$$E(z) = \Big[\Omega_{\rm m}(1+z)^3 + (1-\Omega_{\rm m})(1+z)^{3(1+w_0)} \exp\left(\frac{3w_a z^2}{2(1+z)^2}\right)\Big]^{1/2}. \tag{10}$$

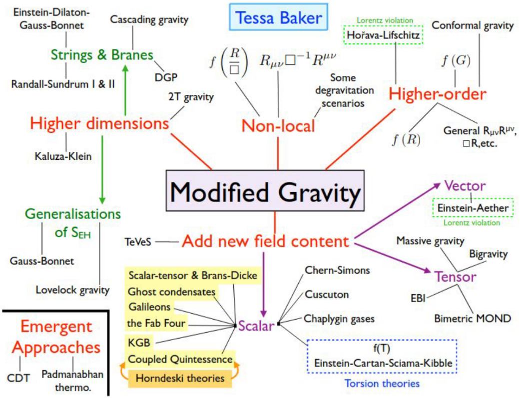
- It parameterizes the dark energy EoS as

$$w(z) = w_0 + w_a z/(1+z)^2$$

, where w_0 and w_a are also free parameters.

3. Effective dark energy models (Modified Gravity)

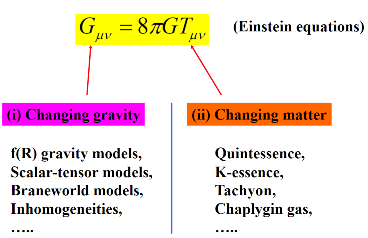
- In physics, the observed acceleration of the universe is commonly attributed to one of two possibilities: the presence of dark energy or the deviations from Einstein's general relativity on cosmological scales. The latter, referred to as MG theories, can create effective dark energy scenarios mimicking the behavior of actual dark energy. As a crucial avenue for explaining the acceleration, it is imperative to extend investigation into MG models.



Tree diagram of modified theories of gravity

Credit : 1512.05356

Effective dark energy models (cont...)



A general functional representation of Ricci scalar, called $f(R)$ gravity.

- In this model, the reduced Hubble parameter is calculated by

$$3H^2 = 8\pi G\rho_m + \left[\frac{RF - f(R)}{2} - 3H\dot{F} - 3H^2(F-1)\right], \tag{11}$$

- It parameterizes the dark energy EoS as

$$w_{eff} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}. \tag{12}$$

Cont...*Dynamical dark energy: quintessence field*

- For spatially homogeneous quintessence we have
  - energy density $\rho_\phi = T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$.
  - pressure $p_\phi = T^1_1 = \frac{1}{2}\dot{\phi}^2 - V(\phi)$.
  - Friedmann eqations in $K = 0$ universe: $H^2 = \frac{8\pi G}{3}\rho_\phi$ and $\frac{\ddot{a}}{a} = \frac{8\pi G}{3}[\dot{\phi}^2 - V(\phi)]$.
  - universe accelerate when $V(\phi)$ dominate over kinetic term $\dot{\phi}^2$ i.e. $\dot{\phi}^2 < V(\phi)$.
- Continuity gives to scaling solution $\rho_\phi = \rho^0_\phi \exp\left[-3\int\left(1 + w_\phi \frac{da}{a}\right)\right]$.
  - under slow-roll limit i.e. $\dot{\phi}^2 \ll V(\phi)$ corresponds to $w_\phi = -1$ gives $\rho_\phi =$ constant.
  - for stiff matter i.e. $\dot{\phi}^2 \gg V(\phi)$ corresponds to $w_\phi = +1$, in which energy density evolves $\rho_\phi \propto a^{-6}$.
- For power law expansion $a(t) \propto t^n$, (accelerated expansion possible for $n > 1$) we have

$$V = \frac{3H^2}{8\pi G}\left[1 + \frac{\dot{H}}{3H^2}\right]; \quad \phi(t) = \int\left(-\frac{\dot{H}}{4\pi G}\right)^2 dt.$$

- Power law expansion corresponds to exponential potential of quintessence in which field evolves as $\phi \propto \ln t$.

Cont...*Dynamical dark energy: Tachyon field*

- Rolling tachyon condensates, in a class of string theories, may have interesting cosmological consequences.
- Decay of D-branes produces a pressureless gas with finite energy density that resembles classical dust.
- Tachyon as a scalar field from which it is possible to obtain viable models of dark energy.
- Action of tachyon is given as $\mathcal{A} = -\int V(\phi)\sqrt{-det(-g_{ab} + \partial_a\phi\partial_b\phi)}\sqrt{-g}d^4x,$
- Variation of action with respect to ϕ gives following equation of motion

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{1}{V(\phi)}\frac{dV(\phi)}{d\phi} = 0$$

- The energy-momentum tensor gives the energy density and pressure for spatially homogeneous field

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}; \quad p_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2}.$$

Cont... *Tachyon field*

- For tachyon scalar field the accelerated expansion occurs when $\dot{\phi}^2 < 2/3$.
- Equation of state of tachyon varies between 0 and -1 in which $\rho_\phi \propto a^{-m}$ with $0 \leq m \leq 3$.
- Like quintessence it can be be written $V(\phi)$ and ϕ in terms of H and $\dot{H}$ as

$$V = \frac{3H^2}{8\pi G}\left[1 + \frac{2\dot{H}}{3H^2}\right]^{1/2}; \quad \phi(t) = \int \left(-\frac{2\dot{H}}{3H^2}\right)^{1/2} dt.$$

- The the tachyon potential giving the power-law expansion $a(t) \propto t^n$ is

$$V(\phi) = \frac{2H}{4\pi G}\left(1 - \frac{2}{3n}\right)^{1/2} \phi^{-2}.$$

- In this case the evolution of the tachyon is given by $\phi(t) = \sqrt{\frac{2}{3n}}t$.
- Tachyon potentials which are not steep compared to $V(\phi) \propto \phi^{-2}$ lead to an accelerated expansion.

Cont..*Dynamical dark energy: Phantom field*

- Historically, phantom fields were first introduced in Hoyle's version of the steady state theory.
- Recent observational data[1] indicates that the equation of state parameter w lies in a narrow strip around $w = -1$ and is quite consistent with being below this value.
- The simplest explanation for the phantom dark energy is provided by a scalar field with a negative kinetic energy.
- Energy density and pressure for spatially homogeneous phantom field are

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi); \qquad p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi).$$

- For $\dot{\phi}^2/2 < V(\phi)$ the equation of state parameter of phantom is $w_\phi < -1$.
- the fundamental origin of the phantom field still poses an interesting challenge for theoreticians.

[1] P. S. Corasaniti *et. al*; *Phys. Rev. D*, **70**, (2004)083006; U. Alam *et. al*; *Mon. Not. R. Astron. Soc.* **354**, (2004)275.

Diagnostics of dark energy

- The available cosmological data do not fix a microscopic theory of dark energy.
- They can tell us about the macroscopic density parameter and in a way about the equation of state.
- But the overall uncertainty reflects in a range of phenomenological models.
- This range can be reduced by choosing only those models which do not violate any existing fundamental theories.
- The next step should be to test the model against the massive cosmological data. This is step one.
- Some authors[2] have built the (so-called $O_m(x) = \frac{E^2(x)-1}{x^3-1}$) diagnostics to distinguishing cosmological constant from the dynamical dark energy of $w \neq -1$. Here $x = 1 + z = a_0/a$, and $E^2(x) = H^2(x)/H_0^2$
  - If $O_m(x) = \Omega_m^0$ the dark energy is just the cosmological constant.
  - If $O_m(x) > \Omega_m^0$, it must be quintessence.
  - If $O_m(x) < \Omega_m^0$ it turns out to be phantom energy.

[2] V. Sahni, A. Shafieloo and A. A. Starobinsky; *Phys. Rev. D*, **78**, (2008)103502.

Summary and takeaway

- As an update on the initial findings of DESI, the new results provide the first hint of potential deviations from a cosmological constant EoS ($w = -1$), which, if confirmed with significance $> 5\sigma$, will falsify the ΛCDM model.

- Such Dynamical models can explain the Hubble tension.

- Dynamical models are promising frameworks to:

  *Address the Hubble tension.

  *Provide deeper connections to dark matter.

  *Open avenues for modified gravity and scalar field theories.

References

- Y. Wang, **Dark Energy** (Wiley-vch) 2010, ISBN: 978-3-527-40941-9.
- Peng-Ju Wu, Comparison of dark energy models using late-universe observations, arXiv:2504.09054v1 [astro-ph.CO] 12 Apr 2025.
- Vipin K. Sharma et al., Probing Generalized Emergent Dark Energy with DESI DR2, arXiv:2507.00835v2 [astro-ph.CO].
- DESI collaboration, DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints, arXiv:2503.14738v2 [astro-ph.CO] 26 Mar 2025.

Thank You

SNIa

Type Ia Supernovae (SNe Ia) are reliable **standard candles**, due to their nearly uniform peak luminosity.

They provide a measure of the **luminosity distance** $D_L(z)$ via the distance modulus:

$$\mu(z) = m_B - M_B = 5\log_{10}\left(\frac{D_L(z)}{\mathrm{Mpc}}\right) + 25$$

where m_B is the observed magnitude, M_B is the absolute magnitude (nuisance parameter), and $\mu(z)$ is the distance modulus.

In a flat Universe, $D_L(z)$ is given by:

$$D_L(z) = (1+z)\int_0^z \frac{c}{H(z')}dz'$$

BAO

What is BAO? Baryon Acoustic Oscillations (BAO) are regular, periodic fluctuations in the density of visible baryonic matter in the Universe, caused by sound waves (acoustic waves) propagating in the early Universe's hot plasma. These oscillations leave a characteristic scale known as the **sound horizon**, which serves as a "standard ruler" for cosmological distance measurements.

Sound Horizon at Drag Epoch:

$$r_d = \int_{z_d \approx 1060}^{\infty} \frac{c_s(z)}{E(z)}\, dz$$

with

$$c_s \approx c \left(3 + \frac{9\rho_b}{4\rho_\gamma}\right)^{-0.5}$$

- r_d: comoving sound horizon at drag epoch z_d
- $c_s(z)$: sound speed in the photon–baryon fluid
- ρ_b and ρ_γ : baryon and energy density
- Planck (2018): $r_d = 147.09 \pm 0.26\,\mathrm{Mpc}$

Standard rulers

Having defined the sound horizon, we can now explore how it serves as a **standard ruler** for measuring cosmic distances. The characteristic scale is set by the comoving sound horizon r_d at the drag epoch. Observables derived from BAO include:

1. **Transverse (Radial) Distance:**

$$D_H(z) = \frac{c}{H(z)} = \text{line-of-sight distance scale}$$

2. **Angular Diameter (Transverse) Distance:**

$$D_M(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')} = \text{transverse separation}$$

3. **Volume-Averaged Distance:**

$$D_V(z) = \left[z\, D_H(z)\, D_M^2(z) \right]^{1/3}$$

These distances are often constrained via the ratios:

$$\frac{D_H(z)}{r_d}, \quad \frac{D_M(z)}{r_d}, \quad \frac{D_V(z)}{r_d}$$

H_0 tension

1. **Simple Version (most common)**
   - Hubble tension = Planck H_0 vs SH0ES H_0
   - SH0ES gives $H_0 \sim 73\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, Planck (CMB) gives $H_0 \sim 67\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, They don't match "that's the tension"
2. **Sound Horizon Mismatch (Reframed Tension)**
   - Instead of H_0, the tension is in the sound horizon r_s.
   - SH0ES $H_0 \sim 73$ + BAO data $\Rightarrow r_s \sim 137$ Mpc
   - Planck (CMB) + ΛCDM $\Rightarrow r_s \sim 147$ Mpc
   - Mismatch: $\sim$10 Mpc in r_s "that's is tension"