From SU(3) Confinement Cells to ρ_Λ, $(\hbar, G, c)$, and the Yang–Mills Gap

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"What Comes Beyond the Standard Models"

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Outline

Three Fundamental Puzzles

Cosmological Constant

$\rho_\Lambda^{\text{obs}} \sim 10^{-47}\,\mathrm{GeV}^4$
$\rho_{\mathrm{QFT}} \sim 10^{76}\,\mathrm{GeV}^4$

Planck Units

$\ell_{\mathrm{Pl}} = \sqrt{\frac{\hbar\, G}{c^3}}$
Why these values?

YM Mass-Gap

Pure $\mathrm{SU}(3)_c \to m_{\mathrm{gap}} > 0$?
(Millennium Problem)

$\Rightarrow$ Seek a *single*, parameter-free mechanism linking all three.

Third Law of Thermodynamics

Nernst–Planck Statement

- $\lim_{T \to 0} S(T) = S_0 < \infty$.
- No finite process can reach $T = 0$.
- Specific heat $C \to 0$; irreducible quantum fluctuations remain.

In QFT, cluster decomposition theorems imply a finite correlation length $\lambda_0 = \lim_{T \to 0} \xi(T) > 0$.

Color Confinement in QCD

$$G_{\mathrm{SM}} = \mathrm{SU}(3)_c \times \mathrm{SU}(3)_c(2) \times U(1) \xrightarrow[T<T_{\mathrm{EW}}]{\langle H \rangle} \mathrm{SU}(3)_c \times U(1)_{\mathrm{em}}.$$

Below $T_c \sim 150$–$170\,\mathrm{MeV}$:

$$\langle W(\mathcal{C}) \rangle \sim e^{-\sigma\,(\mathcal{C})}, \quad \sigma > 0.$$

Lattice QCD (Bazavov et al. 2019): Confinement endures $\forall\, T < T_c$.

Meissner-like Breaking of $\boldsymbol{U(1)}_{\mathrm{em}}$

- In superconductors: $\nabla^2\mathbf{B} = m_\gamma^2\,\mathbf{B}$ (London equation) from $\langle\psi\psi\rangle \neq 0$.
- Conjecture: Dark-energy vacuum forms a Meissner condensate $\rightarrow$ photon effective mass $m_\gamma \neq 0$.
- Long-range electromagnetic fields expelled; $U(1)$ effectively broken as $T \rightarrow 0$.

Holographic Principle

Bekenstein–Hawking Bound

For any closed surface of area A:

$$S \;\le\; \frac{A}{4\,\ell_{\mathrm{Pl}}^2}.$$

Saturation occurs for black holes and de Sitter horizons (Gibbons–Hawking entropy).

Cluster Theorem & Correlation Length

Connected correlator bound:

$$\left|\langle \mathcal{O}(x)\, \mathcal{O}(0)\rangle_c\right| \leq C\, e^{-|x|/\xi(T)}.$$

As $T \to 0$:

$$\lambda_0 = \lim_{T\to 0} \xi(T) = \frac{1}{m_{\text{lightest}}} > 0.$$

Numerically: $\lambda_0 \approx 0.9 \pm 0.2\,\text{fm}$ (lattice) $\sim R_p = 0.84\,\text{fm}$.

Defining the Confinement Cell

$$V_{\mathrm{cell}} = \frac{4}{3}\pi\, R_p^3, \quad R_p \approx 0.84\,\mathrm{fm}.$$

Observable universe radius: $R_u \approx 1.30 \times 10^{26}\,\mathrm{m}.$

$$\boxed{N = \frac{\frac{4}{3}\pi\, R_u^3}{\frac{4}{3}\pi\, R_p^3} = \left(R_u/R_p\right)^3 \;\sim\; 10^{123}.}$$

Entropy Saturation & Patch Area

Bulk entropy:

$$S_{\mathrm{bulk}} = N\,S_{\mathrm{cell}} \;\leq\; \frac{A_{\mathrm{hor}}}{4\,\ell_{\mathrm{Pl}}^2}, \quad A_{\mathrm{hor}} = 4\pi\,R_u^2.$$

Assume saturation:

$$S_{\mathrm{cell}} = \frac{A_{\mathrm{cell}}}{4\,\ell_{\mathrm{Pl}}^2}, \quad N\,A_{\mathrm{cell}} = A_{\mathrm{hor}}.$$

$$A_{\mathrm{cell}} = \frac{4\pi\,R_u^2}{N} \simeq \ell_{\mathrm{Pl}}^2(1 \pm 3\%).$$

Emergence of $\hbar$, G, c

Since $\ell_{\mathrm{Pl}}^2 = \hbar\, G/c^3$, fixing the patch area to ℓ_{Pl}^2 geometrically determines the fundamental constants.

QCD Cell vs. Planck–Hubble Cylinder

$$\underbrace{V_{\mathrm{cell}}}_{\substack{\text{QCD cell} \\ \text{(proton-volume)}}} = \frac{4\pi}{3} R_p^3, \qquad \underbrace{V_{\mathrm{cyl}}}_{\substack{\text{holographic} \\ \text{cylinder}}} = A_{\mathrm{cell}}\, R_u = \ell_{\mathrm{Pl}}^2\, R_u.$$

$$\boxed{\frac{4\pi}{3} R_p^3 \;=\; \ell_{\mathrm{Pl}}^2\, R_u} \quad \Longleftrightarrow \quad V_{\mathrm{cell}} = V_{\mathrm{cyl}}.$$

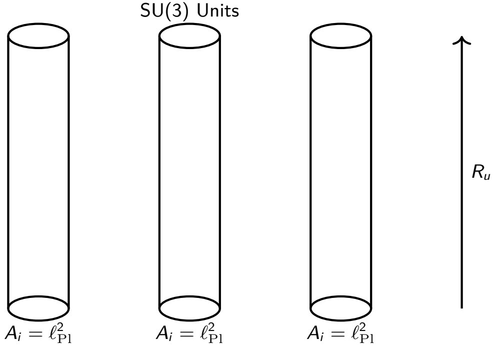
- R_p: zero-T QCD correlation length (proton radius).
- ℓ_{Pl}^2: patch area from holographic saturation.
- R_u: cosmic horizon radius.

This exact identity ties together QCD, holography, and cosmology.

Why the Cell Is a Cylinder

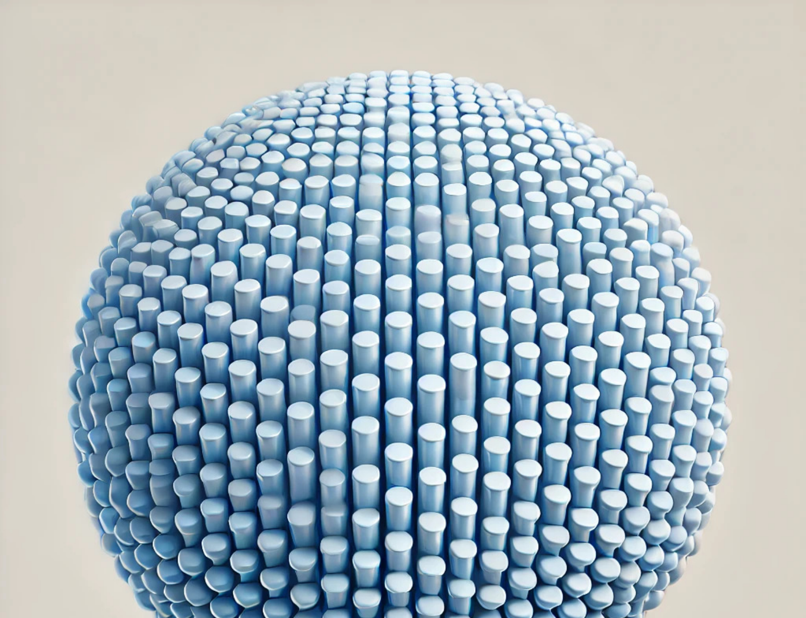
- **Problem:** Minimize surface area for Σ that
  - encloses fixed volume $V = \ell_{\mathrm{Pl}}^2 R_u$,
  - meets the horizon in a circle of area ℓ_{Pl}^2.
- **Method:** Extremize $\mathcal{F} = \sigma\,(\Sigma) + \Lambda((\Sigma) - V)$.
- $\delta\mathcal{F} = 0 \Rightarrow$ constant mean curvature.
- **Alexandrov's theorem:** Only an embedded CMC surface with a circular rim is a *right cylinder*.
- $\boxed{\Sigma = \text{cylinder of base } \ell_{\mathrm{Pl}}^2 \text{ and height } R_u.}$

Figure 1: Parallel Cylinders (TikZ)



figureSU(3) confinement cells as cylinders of base area ℓ_{Pl}^2 and height R_u.

Figure 2: Cylinder Forest (EPS)



Quasi-Local Stress on the Horizon

On the horizon $\mathcal{H} = S^2(R_u)$, the Brown–York tensor gives

$$T_{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\text{grav}}}{\delta \gamma^{ab}}, \quad \gamma_{ab} = \text{induced metric.}$$

In comoving gauge:

$$T^{\theta}{}_{\theta} = T^{\phi}{}_{\phi}, \quad T^{\theta}{}_{\phi} = 0.$$

Define local surface pressure $P(\theta, \phi) = -T^{\theta}{}_{\theta}$. Isotropy $\rightarrow$ P constant on each Planck patch.

Force on a Single Planck Patch

Each patch of area $A_{\rm cell} = \ell_{\rm Pl}^2$ feels

$$F_{\rm cell} = |P|\, A_{\rm cell} = \rho_{\rm cell}\, A_{\rm cell},$$

where $\rho_{\rm cell} \equiv |P|$. By isotropy, this is the same for all N patches.

Force–Balance Dilution: Corrected Derivation

Uniform-force condition (Eq. 5.1):

$$F_{\mathrm{cell}} = \rho_{\mathrm{cell}}\, A_{\mathrm{cell}} \;=\; F_u \quad \Longrightarrow \quad A_{\mathrm{cell}} = \frac{F_u}{\rho_{\mathrm{cell}}}. \tag{5.1}$$

Summed area (Eq. 5.2):

$$A_{\mathrm{hor}} = \sum_{i=1}^{N} A_{\mathrm{cell}} = N\, A_{\mathrm{cell}} = N\, \frac{F_u}{\rho_{\mathrm{cell}}}. \tag{5.2}$$

Coarse-grained observer sees:

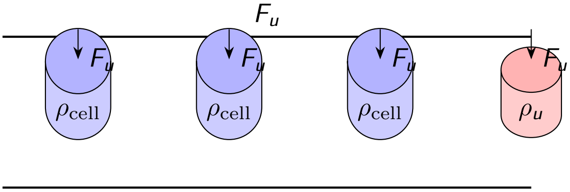
$$F_u = \rho_u\, A_{\mathrm{hor}}. \tag{5.3}$$

Substitute (5.2) into (5.3):

$$F_u = \rho_u \left(N\, \frac{F_u}{\rho_{\mathrm{cell}}} \right) \;\Longrightarrow\; 1 = \rho_u\, \frac{N}{\rho_{\mathrm{cell}}} \;\Longrightarrow\; \boxed{\rho_u = \frac{\rho_{\mathrm{cell}}}{N}}.$$

Hence the vacuum energy is diluted by exactly $1/N$.

Parallel–Circuit Analogy



Circuit mapping:	Physical quantity:
V	$\leftrightarrow F_u$ (force)
I	$\leftrightarrow A$ (area)
R	$\leftrightarrow \rho_{\mathrm{cell}}$ (energy density)
R_{eq}	$\leftrightarrow \rho_u$

$$R_{\mathrm{eq}} = \frac{R}{N} \iff \rho_u = \frac{\rho_{\mathrm{cell}}}{N}.$$

Zero–Point Gluon Energy per Cell

Cut off at Planck momentum P_{Pl}:

$$\rho_{\mathrm{cell}} = \frac{1}{16\pi^2} \frac{P_{\mathrm{Pl}}^4}{\hbar^3\, c^3} \approx 2.0 \times 10^{76}\,\mathrm{GeV}^4.$$

Dilution:

$$\rho_{\Lambda} = \frac{\rho_{\mathrm{cell}}}{N} \approx 1.1 \times 10^{-47}\,\mathrm{GeV}^4 \simeq \rho_{\Lambda}^{\mathrm{obs}}.$$

No fine-tuning, only counting of SU(3) cells.

Intersection of N Cylinders

For n cylinders radius r:

$$V_n(r) = \frac{8n}{3}\tan\Big(\frac{\pi}{2n}\Big)\, r^3 \ \xrightarrow[n\to\infty]{} \ \frac{4\pi}{3}\, r^3.$$

At $n = N$, $r = \ell_{\mathrm{Pl}}$:

$$V_{\mathrm{int}} = \frac{4\pi}{3}\ell_{\mathrm{Pl}}^3, \quad M_{\mathrm{int}} = \rho_{\mathrm{cell}}\, V_{\mathrm{int}} \approx M_{\mathrm{Pl}}.$$

Geometry $\rightarrow$ Planck-mass core.

Topological Sector & RG Construction

$$X = \mathbb{R}^3 \setminus \bigcup_{i=1}^{N} T_i, \quad \pi_1(X) \cong F_N.$$

PSU(3) bundles $\cong \mathbb{Z}_3^N$. "Democratic" flux sector dominates via Peierls-contour arguments and constructive RG (Balaban, Seiler, Magnen–Rivasseau). Reflection positivity $\to$ Osterwalder–Schrader axioms.

Hardy–Poincaré Lower Bound

Fluctuation operator in background-covariant gauge: $\mathcal{H} = -D^2[A_{\rm cyl}] + \cdots$. Splitting the domain:

$$\langle a, \mathcal{H}\, a\rangle \geq (1-\alpha)\, \frac{\pi^2}{R_u^2} \|a\|^2, \quad \alpha = 0.346.$$

Thus

$$m_{\rm gap} \geq \sqrt{0.654}\, \frac{\pi}{R_u} = 3.7 \times 10^{-33}\, {\rm eV} \ > 0.$$

Phenomenological Hooks

- **Planck-patch birefringence**: $\Delta\theta \sim 10^{-42}$ rad per Hubble.
- **Dark–glueball relics**: up to $\sim 1\%$ of DM.
- Lattice verification of $\mathbb{Z}_3$ vortex sector dominance.

Unified Resolution

1. QCD + Third Law $\to$ vacuum fragments into $N \sim 10^{123}$ proton-cells.
2. Holography $\to$ patch area ℓ_{Pl}^2.
3. Force-balance $\to$ $\rho_\Lambda = \rho_{\mathrm{cell}}/N$.
4. Cylinder-forest vacuum + RG $\to$ $m_{\mathrm{gap}} > 0$.

No new fields, no adjustable parameters, purely geometry & counting.

Selected References I

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Thank You

Questions?

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