Algebraic Treatment of Infinities in Integrals

Resolving Infinities in $\int x^{-1}dx$ and $\int x^{dx}$ through Inherent Boundary Entanglement and Landauer's Principle Reveals Adiabatic Character of Definite Integration

Elia Dmitrieff
SI JAM Labs, Kazakhstan

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Introduction: Algebraic Infinities

- In standard analysis, ∞ is a limit statement, not a number.
- Alternate frameworks treat infinities as formal algebraic entities.
- ζ**-Function Regularization:** Assigns finite values to divergent series via analytic continuation (e.g., $\sum n = -1/12$) [1].
- **Wheel Theory:** Direct algebraic division by zero is valid by extending fields into structures where $0/0$ is a tracked baseline [2].
- **Our Framework:** Treats infinities as negative powers of zero (0^{-1}) that can be formally manipulated alongside standard functions.

Problem 1: $\int x^{-1}dx$ and Infinity Absorption

We evaluate the classic divergence at $n = -1$ through a formal limit process:

$$\int x^{-1}dx = \lim_{n \to -1} \int x^n dx = \lim_{n \to -1} \left(\frac{x^{n+1}}{n+1} + C \right)$$

By adding and subtracting 1 in the numerator, we isolate the natural logarithm:

$$\lim_{n \to -1} \frac{(x^{n+1} - 1) + 1}{n+1} + C = \ln|x| + \lim_{n \to -1} \frac{1}{n+1} + C$$

In an algebraic setting, $\lim_{n \to -1} \frac{1}{n+1}$ behaves as a constant first-order infinity (∞).

Problem 1: Solution

The Absorbing Constant

Because the infinity (∞) is independent of x, it can be completely absorbed by the arbitrary integration constant C:

$$C^* = C + \infty \quad \text{where } C^* \in \mathbb{R} \cup \{\infty\}$$

Differentiating this sum preserves the true baseline derivative since the derivative of a constant infinity is 0:

$$\frac{d}{dx}(\ln|x| + C^*) = \frac{1}{x} + 0 = \frac{1}{x}$$

Thus, the function survives unswallowed.

Problem 2: Non-Standard $\int x^{dx}$

Consider an integral where the differential sits inside the exponent: $\int x^{dx}$. Using the identity $x^{dx} = e^{\ln(x)dx}$, we expand via Taylor series:

$$e^{\ln(x)dx} = 1 + \ln(x)dx + \frac{(\ln(x)dx)^2}{2!} + \dots$$

Under standard additive integration frameworks without adding an external dx weighting factor:

$$\int x^{dx} = \int 1 + \int \ln(x)dx$$

Problem 2: Indefinite vs. Definite Forms

- **Indefinite Integral:** The $\int 1$ term behaves as a discrete infinite summation of constants $(\infty = 0^{-1})$. By our absorption rule:

$$\int x^{dx} = \infty + x\ln(x) - x + C = x\ln(x) - x + C^*$$

- **The Boundary Dilemma:** To evaluate the definite integral $\int_a^b x^{dx}$, we must transition from an indefinite form to a definite boundary evaluation.
- Traditional substitution can create an ambiguous $(+\infty - \infty)$ form if evaluations are treated independently.

The Informational Cost of Calculus

- **Landauer's Principle:** In physics and information theory, making a discrete choice or erasing an undefined state requires a local reduction of entropy, dissipating physical heat ($\Delta S > 0$).
- **The Standard Assumption:** Textbooks often imply we must arbitrarily select a concrete value for the constant (e.g., $C = 0$) to evaluate a definite integral.
- **The Quantum/Relativistic Conflict:** Forcing a local informational choice would emit entropy, disrupting highly sensitive quantum or relativistic frameworks.

Definite Integration as an Adiabatic Process

Calculus Without Choice

Because the final definite volume is invariant under any choice of C, the FTC allows us to **entirely avoid** selecting a concrete value.

- Instead of localizing C, the calculus machinery subtracts two unknown, non-deterministic, but **entangled** values.
- Like space-like separated quantum particles sharing a single source event, the boundaries maintain state identity across the interval.
- By operating strictly on entangled uncertainties, integration remains a perfectly **adiabatic process** (zero entropy flow).

Bypassing the Impossible Choice in $\int x^{dx}$

When an anti-derivative contains an infinite, non-numerical term ($\infty = 0^{-1}$), making a localized numerical choice is mathematically impossible.

$$\text{Frozen Anti-derivative: } F(x) = (x\ln(x) - x) + \infty_{\text{entangled}} + C_{\text{entangled}}$$

- The intrinsic entanglement of the FTC shields the operation:

$$\int_a^b x^{dx} = [F(b) + \infty_{\text{entangled}} + C_{\text{entangled}}] - [F(a) + \infty_{\text{entangled}} + C_{\text{entangled}}]$$

The Adiabatic Cancellation

The non-numerical infinities and integration constants cancel perfectly out of structural necessity, yielding a real-valued geometric residue:

$$\int_a^b x^{dx} = (b\ln(b) - b) - (a\ln(a) - a)$$

Conclusion and Open Horizons

The investigation into $\int x^{-1}dx$ and $\int x^{dx}$ through an algebraic treatment of infinities reveals that standard calculus notation contains deep, structural safety nets.
By moving away from purely dynamic limit-based frameworks and treating singular poles as structured algebraic expressions (such as 0^{-1}), we expose a more robust relationship between indefinite variables and definite boundaries.

Conclusion and Open Horizons

Our results demonstrate two key mechanisms of regularization within additive calculus:

1. **Constant Infinity Absorption:** In indefinite integration, uniform, non-spatial infinities can be successfully bundled into the integration constant C. This allows equations like $\int x^{-1}dx$ to preserve their core transcendental geometry $(\ln|x|)$ rather than being immediately swallowed by a divergent state.
2. **Intrinsic Boundary Entanglement:** The evaluation of definite integrals does not suffer from indeterminate chaos $(\infty - \infty)$ because the Fundamental Theorem of Calculus fundamentally enforces state correlation. The anti-derivatives at the upper and lower bounds are structurally entangled, ensuring that any unchosen or non-deterministic properties cancel themselves out across space with absolute precision.

Conclusion and Open Horizons

This structural entanglement bridges a conceptual gap between mathematical architecture and quantum realities, proving that the identity of a function is preserved even when sampled across space-like separated intervals. Future inquiries may look toward formalizing this entanglement via operator theory, or establishing explicit mappings to modern algebraic systems like Wheel Theory to see if similar boundary correlations can resolve division-by-zero singularities in multi-dimensional vector fields.

References

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